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Integrity and Domination Integrity in Neutrosophic Soft Graphs

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Abstract: Vagueness and uncertainty are two distinct models are represented by Fuzzy sets and Soft sets. The combination of Soft sets and simple graphs produces soft graphs which is also an interesting concept to deal with uncertainty problems. Any communication network can be modeled as a graph whose nodes are the processors (stations) and a communication link as an edge between corresponding nodes. The stability of a communication network is a very important factor for the network designers to reconstruct the it after the failure of certain stations or communication links. Two essential quantities in an analysis of the vulnerability of a communication network are (1) the number of nodes that are not functioning and (2) the size of a maximum order of a remaining sub network within which mutual communications can still occur. C. A. Barefoot, et. al. [13] introduced the concept of integrity. The extension of such a vulnerability parameter is studied in fuzzy graphs. Since neutrosophic soft graphs are the most generalized network structure where we can define and study the importance of the vulnerability parameters is made in this manuscript. Also, we introduce the domination integrity of neutrosophic soft graphs and explain with suitable examples. Few bounds are obtained.

Keywords: Soft graph, Neutrosophic soft graphs, Integrity, Domination integrity.

1. Introduction

The problems deal with vagueness and uncertainty can be modelled by using two different soft tools namely f**uzzy set** defined by Zadeh [48] in 1965 and **soft set** defined by Molodtsov [31] in 1999. The **intuitionistic fuzzy set** is the generalization of fuzzy set was introduced by Atanassov [2-4]. It depends on a membership function and a non membership function. Any real time problems which consist of involving imprecise, indeterminacy and inconsistent data can be represented as the **neutrosophic se**t, introduced by Smarandache [38]. This is the generalization of classical sets and fuzzy sets. The degree of acceptance deals in fuzzy sets, membership (truth) function and a non-membership (falsity) function deals in intuitionistic fuzzy set, neutrosophic set deals truth-membership, indeterminacy-membership, and falsity-membership. The **rough soft sets, soft rough sets, and soft-rough fuzzy sets** are obtained from soft sets with rough sets and fuzzy sets. Feng et al. [18 -20] and Ali [7] introduced these soft tools in the consecutive years 2010 and 2011. In 2014, Rajesh Thumbakara et. al.[33] introduced **soft graphs.** They defined soft graph homomorphism, soft tree and soft complete graph and discussed their properties also. Ali et al. [7] discussed the fuzzy sets and fuzzy soft sets induced by soft sets.

In 1736, **graph theory** was defined by Euler. **Fuzzy graph** was introduced by Azriel Rosenfied in 1975[29 & 35]. Muhammad Akram et.al. [6] defined **fuzzy soft graphs i**n 2015**.** Also, they have investigated the properties **of strong, complete** and **regular fuzzy soft graphs.** Guven et. al. [25] introduced an idea about neutrosophic soft graphs and its application. Shannon and Atanassov [37] defined the **intuitionistic fuzzy graph** (IFG). A.M.Shyla [46] introduced the concept of **Intuitionistic Fuzzy Soft graph** in 2016. Ghorai. G. et. al.[21] modelled **the neutrosophic graphs** in 2017. Akram [6] established the certain notions including **neutrosophic soft graphs**, strong neutrosophic soft graphs, and complete neutrosophic soft graphs.

Graphs are the most important and essential tool in the modern communication world which has communication nodes and links. The stability of such communication networks can be measured by vulnerability parameters like connectivity, toughness [11], tenacity [16], rupture degree, scattering number, integrity [13-15], domination integrity [39-42], etc. Two essential quantities in an analysis of the vulnerability of a communication network are (1) the number of nodes that are not functioning and (2) the size of a maximum order of a remaining sub network within which mutual communications can still occur. C. A. Barefoot, et. al. [13-14] introduced the concept of integrity. It is a useful measure of vulnerability and it is defined as follows. $I(G) = min\{|S| + m(G - S) : S \subset V(G)\}$, where $m(G - S)$ denotes the order of the largest component in G – S.

Integrity measures not only the difficulty to break down the network but also the damage caused. A small group of people have effective communication links with other members of the organization and they take important decisions in an administrative set up. Domination in graphs provides a model for such a concept. A minimum dominating set of nodes provides a link with the rest of the nodes in a network, If the removal of such a set, results huge impact in the network. That is, the decision-making process is paralyzed but also the communication between the remaining members is minimized. The damage will be more when the dominating sets of nodes are under attack.

This motivated to study the concept of domination integrity when the sets of nodes disturbed are dominating sets. Sundareswaran et. al. introduced the concept of Domination Integrity of a graph and studied in [39] as another measure of vulnerability of a graph which is defined as follows $DI(G) = min\{|S| + m(G - S)\}$, where S is a dominating set of G and $m(G - S)$ denotes the order of the largest component in $G - S$ and is denoted by $DI(G)$. M. Saravanan et. al. extended the idea of vulnerability parameters in fuzzy graphs [42 - 44]. They explained a real time application for the domination integrity [45]. There are different versions of domination integrity were introduced in the literature such as Domination Weak Integrity in graphs [47], Geodomination integrity [12], Connected domination integrity in graphs [27] and Total Edge Domination Integrity in graphs [8].

This motivated us to introduce the concept of integrity and domination integrity in neutrosophic fuzzy soft graphs. Also, we prove certain properties of these new parameter concepts are described with suitable examples.

In the second section, we provide all the basic definitions and results related to our article. The definitions of the Integrity and Domination integrity in Fuzzy graphs were stated in the third section and in the fourth section, we introduce the concept of Integrity and Domination integrity in Neutrosophic graphs. At the end of the article, we give the conclusion of our work and discuss the future work.

2. Preliminaries

In this section, we provide all the basic definitions and results in the literature.

Definition 2.1 [21]

A **neutrosophic graph** is of the form $G^* = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$ & $\mu = (T_2, I_2, F_2)$

(i) $V = \{v_1, v_2, v_3, \dots v_n\}$ such that $T_1 : V \longrightarrow [0,1]$, $I_1 : V \longrightarrow [0,1]$ and $F_1 : V \rightarrow [0,1]$ denote the degree of truth-membership function , indeterminacy –membership function and falsity-membership function of the vertex $v_1 \in V$ respectively and $0 \leq T_i(v) + I_i(v) + F_1(v) \leq 3, \forall\, v_i \in V$ ($i = 1,2\,,3, \dots n$).

(ii) $T_3 : V \times V \longrightarrow [0,1]$, $I_2 : V \times V \longrightarrow [0,1]$ and $F_2 : V \times V \rightarrow [0,1]$ where $T_2(v_i, v_j), I_2(v_i, v_j)$ and $F_2(v_i, v_j)$ denote the degree of truth-membership function , indeterminacy –membership function and falsity-membership function of the edge (v_i, v_j) respectively such that for every edge (v_i, v_j),

$$T_2(v_i, v_j) \leq \min\{T_1(v_i), T_1(v_j)\},$$
$$I_2(v_i, v_j) \leq \min\{I_1(v_i), I_1(v_j)\},$$
$$F_2(v_i, v_j) \leq \max\{F_1(v_i), F_1(v_j)\},$$

and $T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$

Definition 2.2 [33]

Let $G = (V, E)$ be a simple graph, A any nonempty set. Let R an arbitrary relation between elements of A and elements of V. That is $\subseteq A \times V$. A set valued function $F : A \dashrightarrow P(V)$ can be defined as $F(x) = y \in V \mid xRy\}$. The pair (F, A) is a soft set over V. Let (F, A) be a soft set over V. Then (F, A) is said to be a **soft graph** of G if the subgraph induced by F(x) in $G, F(x)$ is a connected subgraph of G for all $x \in A$. The set of all soft graph of G is denoted by $SG(G)$.

Definition 2.3 [6]

A **neutrosophic soft graph** $G = (G^*, F, K, A)$ is an ordered four tuple if it satisfies the following conditions:

i. $G^* = (V, E)$ is a simple graph,
ii. A is a nonempty set of parameters,
iii. (F, A) is a neutrosophic soft set over V,
iv. (K, A) is a neutrosophic soft set over E,
v. $(F(e), K(e))$ is a neutrosophic graph of $G^*$for all $e \in A$. That is

$$T_{K(e)}(xy) \leq \min\{T_{F(e)}(x), T_{F(e)}(y)\} ;$$
$$I_{K(e)}(xy) \leq \min\{I_{F(e)}(x), I_{F(e)}(y)\} ;$$
$$F_{K(e)}(xy) \leq \max\{F_{F(e)}(x), F_{F(e)}(y)\} ;$$

such that $0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 3, \forall e \in A, x, y \in V$.

S. Satham Hussain et. al. defined in [36] degree and total degree of a vertex v in a neutrosophic soft graph G, order and size of a neutrosophic soft graph G. Also, they introduced vertex, edge and cardinality of a neutrosophic graph G.

Definition 2.4 [36]

Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. Then the **degree of a vertex** $u \in G$ is a sum of degree truth membership, sum of indeterminacy membership and sum of falsity membership of all those edges which are incident on vertex u denoted by $d(u) = (d_{TJ(e)}(u), d_{IJ(e)}(u), d_{FJ(e)}(u))$ where

$d_{TJ(e)}(u) = \Sigma_{e\in A}(\Sigma_{u\neq v\in V} T_{K(e)}(u,v))$ called the degree of truth membership vertex

$d_{IJ(e)}(u) = \Sigma_{e\in A}(\Sigma_{u\neq v\in V} I_{K(e)}(u,v))$ called the degree of indeterminacy membership vertex

$d_{FJ(e)}(u) = \Sigma_{e\in A}(\Sigma_{u\neq v\in V} F_{K(e)}(u,v))$ called the degree of falsity membership vertex for all $e \in A, u, v \in V$.

Definition 2.5 [36]

Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. Then the **total degree of a vertex** $u \in G$ is a sum of degree truth membership, sum of indeterminacy membership and sum of falsity membership of all those edges which are incident on vertex u denoted by $td(u) = (td_{TJ(e)}(u), td_{IJ(e)}(u), td_{FJ(e)}(u))$ where

$td_{TJ(e)}(u) = \Sigma_{e\in A}(\Sigma_{u\neq v\in V} T_{K(e)}(u,v) + T_{J(e)}(u,v)$) called the degree of truth membership vertex

$td_{IJ(e)}(u) = \Sigma_{e\in A}(\Sigma_{u\neq v\in V} I_{K(e)}(u,v) + I_{J(e)}(u,v)))$ called the degree of indeterminacy membership vertex

$td_{FJ(e)}(u) = \Sigma_{e\in A}(\Sigma_{u\neq v\in V} F_{K(e)}(u,v) + F_{J(e)}(u,v))$ called the degree of falsity membership vertex for all $e \in A, u, v \in V$.

Definition 2.6 [36]

The **order** of a neutrosophic soft graph G is

$$Ord(G) = \Sigma_{e_i\in A}(\Sigma_{x\in V} T_{J(e_i)}(e_i)(x), \Sigma_{x\in V} I_{J(e_i)}(ei)(x), \Sigma_{x\in V} I_{J(e_i)}(ei)(x).$$

Definition 2.7 [36]

The **size** of a neutrosophic soft graph G is

$$S(G) = \Sigma_{e_i\in A}(\Sigma_{xy\in V} T_{K_{e_i}}(e_i)(xy), xy \in V, \Sigma_{xy\in V} I_{K_{e_i}}(e_i)(xy), \Sigma_{xy\in V} F_{e_i}(e_i)(xy)$$

Definition 2.8 [36]

Let $G = (G^*, J, K, A)$ be an neutrosophic soft graph. Then **cardinality** of G is defined to be

$$|G| = \Sigma_{e\in A} |\Sigma_{v_i\in V} \frac{1 + T_{J(e)}(x) + I_{J(e)}(x) - F_{J(e)}(x)}{2}| + | \Sigma_{v_i, v_j\in V} \frac{1 + T_{J(e)}(xy) + I_{J(e)}(xy) - F_{J(e)}(xy)}{2}|$$

Definition 2.9 [36]

Let $G = (G, J, K, A)$ be an neutrosophic soft graph, then **vertex cardinality of *G*** is defined to be

$$|V| = \Sigma_{e\in A}|\Sigma_{v_i\in V} \frac{1 + T_{J(e)}(x) + I_{J(e)}(x) - F_{J(e)}(x)}{2}|$$

Definition 2.10 [36]

Let $G = (G*, J, K, A)$ be an neutrosophic soft graph, then **edge cardinality of G** is defined to be

$$|E| = \Sigma_{e\in A}|\Sigma_{xy\in E} \frac{1 + T_{K(e)}(xy) + I_{K(e)}(xy) - F_{K(e)}(xy)}{2}|$$

Definition 2.11 [36]

An arc (u, v) is said to be **strong arc**, if $T_{K(e)}(u,v) \geq T^{\infty}_{K(e)}(u,v)$ and $I_{K(e)}(u,v) \geq I^{\infty}_{K(e)}(u,v)$ and $F_{K(e)}(u,v) \geq F^{\infty}_{K(e)}(u,v)$.

Clearly, if u,v are connected by means of path of length k then $T^{k}_{K(e)}(v_i, v_j)$ is defined as

$$sup\{T_{K(e)}(u, v_1) \wedge T_{K(e)}(v_1, v_2) \wedge T_{K(e)}(v_1, v_3) \wedge \ldots \wedge T_{K(e)}(v_{k-1}, v_k)/u, v, v_1, \ldots v_{k-1}, v \in V\},$$

$I^{k}_{k(e)}(v_i, v_j)$ is defined as

$$inf\{I_{K(e)}(u,v_1) \vee I_{K(e)}(v_1,v_2) \vee I_{K(e)}(v_1,v_3) \vee \ldots \vee I_{K(e)}(v_{k-1},v_k)/u,v,v_1,\ldots v_{k-1},v \in V\}$$ and

$F^{k}_{K(e)}(v_i, v_j)$ is defined as

$$inf\{F_{K(e)}(u,v_1) \vee F_{K(e)}(v_1,v_2) \vee F_{K(e)}(v_1,v_3) \vee \ldots \vee F_{K(e)}(v_{k-1},v_k)/u,v,v_1,\ldots v_{k-1},v \in V\}, e \in A.$$

Definition 2.12 [36]

Let $G = (G*, J, K, A)$be a neutrosophic soft graph on V. Let $u, v \in V$, we say that u dominates v in G if there exists a strong arc between them.

Definition 2.13 [36]

Given $S \subset V$ is called a **dominating set** in G if for every vertex $v \in V - S$ there exists a vertex $u \in S$ such that u dominates v. for all $e \in A, u, v \in V$.

Definition 2.14 [36]

A **dominating set** S of a neutrosophic soft graph $G = (G*, J, K, A)$ is said to be **minimal dominating set** if no proper subset of S is a dominating set, for all $e \in A, u, v \in V$.

Definition 2.15 [R.Dhavaseelan et. al.17]

A neutrosophic graph $\mathrm{G} = (\mathrm{G}*, \mathrm{J}, \mathrm{K}, \mathrm{A})$ is called **Strong Neutrosophic graph** if

$$T_{K(e)}(xy) = \min\{T_{F(e)}(x), T_{F(e)}(y)\} ;$$
$$I_{K(e)}(xy) = \min\{I_{F(e)}(x), I_{F(e)}(y)\} ;$$
$$F_{K(e)}(xy) = \max\{F_{F(e)}(x), F_{F(e)}(y)\} \ \forall e \in A, x, y \in V$$

Definition 2.16 [36]

A neutrosophic soft graph G is a **strong neutrosophic soft graph** if H(e) is a strong neutrosophic graph for all e $\in$ A.

Definition 2.17 [36]

Let $G = (G*, J, K, A)$ be a **strong neutrosophic soft graph** and $v \in V$. Then the strong degree and the **strong neighborhood degree** of v are defined, respectively

$$d_S(v) = \Sigma_{e\in A}(\Sigma_{u\in N_s(v)} T_{K(e)}(uv), \Sigma_{u\in N_s(v)} I_{K(e)}(uv), \Sigma_{u\in N_s(v)} F_{K(e)}(uv))$$
$$d_s N(v) = \Sigma_{e\in A}(\Sigma_{u\in N_s(v)} T_{J(e)}(uv), \Sigma_{u\in N_s(v)} I_{J(e)}(uv), \Sigma_{u\in N_s(v)} F_{J(e)}(uv))$$

The strong degree cardinality of v are defined by

$$|d_s(v)| = \sum_{e\in A}\Big(\sum_{u\in N_S(v)} \frac{1 + T_{K(e)}(u,v) + I_{K(e)}(u,v) - F_{K(e)}(u,v)}{2}\Big)$$

The minimum and maximum strong degree of G are defined, respectively as

$\delta_s(G) = \wedge |d_s(v)|, \forall v \in V$ and $\Delta_s(v) = \vee |d_s(v)|, \forall v \in V, e \in A$

Definition 2.18 [36]

The **strong degree** cardinality and the strong neighborhood degree cardinality of v are defined by

$$|d_s(v)| = \Sigma_{e\in A}(\Sigma_{u\in N_s(v)} \frac{1+T_{K(e)}(u,v)+I_{K(e)}(u,v)-F_{K(e)}(u,v)}{2})$$

$$|d_s N(v)| = \Sigma_{e\in A}(\Sigma_{u\in N_s(v)} \frac{1+T_{J(e)}(u,v)+I_{J(e)}(u,v)-F_{J(e)}(u,v)}{2})$$

Definition 2.19 [36]

Two vertices in a neutrosophic soft graph $G = (G *, J, K, A)$are said to be an **independent** if there is no strong arc between them.

Definition 2.20 [36]

Given $S \subset V$ is said to be **independent set** of G if $T_{K(e)}(u,v) < T^{\infty}_{K(e)}(u,v)$ and $I_{K(e)}(u,v) < I^{\infty}_{K(e)}(u,v)$ and $F_{K(e)}(u,v) < F^{\infty}_{K(e)}(u,v)$ $\forall e \in A, u, v \in S$.

Definition 2.21 [36]

An independent set S of G in a neutrosophic soft graph is said to be **maximal independent**, if for every vertex $v \in V - S$, the set $S \cup \{v\}$ is not independent.

Definition 2.22 [36]

The minimum cardinality among all maximal independent set is called lower independence number of G, and it is denoted by $\Sigma_{e\in A}(iNS(G))$. The maximum cardinality among all maximal independent set is called lower independence number of G, and it is denoted by $\Sigma_{e\in A}(INS(G))$.

Muhammad Akram and Sundas Shahzadi gave the following definitions [6]

Definition 2.23 [6]

A neutrosophic soft graph $G' = = (G *, J', K', A')$ is called a **neutrosophic soft subgraph** of G $= (G *, J, K, A)$ if i. $A' \subseteq A$

$ii. K'_e \subseteq K_e$, that is $T_{K'_e}(x) \leq T_{K_e}(x), , I_{K'_e}(x) \leq I_{K_e}(x), F_{K'_e}(x) \geq F_{K_e}(x)$

$iii. J'_e \subseteq J_e$, that is $T_{J'_e}(x) \leq T_{J_e}(x), , I_{J'_e}(x) \leq I_{J_e}(x), F_{J'_e}(x) \geq F_{J_e}(x)$ for all $e \in A$.

Definition 2.24 [6]

The neutrosophic soft graph $G_1 = (G *, J_1, K_1, B)$ is called **spanning neutrosophic soft subgraph** of $G = (G *, J, K, A)$ if

(i) $B \subseteq A$,

(ii) $T_{F_1(e)}(v) = T_{J(e)}(v), I_{J_1(e)}(v) = I_{J(e)}(v), F_{J_1(e)}(v) = F_{J(e)}(v)$ for all $e \in A, v \in V$

Definition 2.25 [6]

The **complement of a neutrosophic soft graph** $G = (J, K, A)$ denoted by $G^c = (J^c, K^c, A^c)$ is defined as follows:

(i) $A^c = A$,

$(ii) J^c(e) = J(e)$,

$(iii) T_{K^c}(e)(u,v) = T_{J(e)}(u) \wedge T_{J(e)}(v) - T_{K(e)}(u,v)$,

$(iv) I_{K^c}(e)(u,v) = I_{J(e)}(u) \wedge I_{J(e)}(v) - I_{K(e)}(u,v)$,

$(v) F_{K^c}(e)(u,v) = F_{J(e)}(u) \vee F_{J(e)}(v) - F_{K(e)}(u,v)$, for all $u, v \in V, e \in A$.

Definition 2.26 [6]
A neutrosophic soft graph G is **self-complementary** if $G \approx G^c$.

Definition 2.27 [6]
A neutrosophic soft graph G is a complete neutrosophic soft graph if $H(e)$ is a **complete neutrosophic graph** of G for all $e \in A$,

$$T_{K(e)}(uv) = min\{T_{F(e)}(u), T_{F(e)}(v)\}$$
$$I_{K(e)}(uv) = \min\{I_{F(e)}(u), I_{F(e)}(v)\} \; and$$
$$F_{K(e)}(uv) = max\{F_{F(e)}(u), F_{F(e)}(v)\}$$
$$\forall u, v \in V, e \in A.$$

3. Integrity and Domination integrity in Fuzzy graphs

Saravanan et. al.[33 - 36] introduced the idea of the vulnerability parameter namely integrity and domination integrity in fuzzy graphs.

Definition 3. 1 [41]
Let $G = (\sigma, \mu)$ be a fuzzy graph. The **integrity** of G, denoted by $\check{I}(G)$, is defined as $\check{I}(G) = \min\{|S| + m(G-S)\}$ where $|S| = \sum_{u\in S}\sigma(u)$ denotes the cardinality of S, and $m(G-S) = \sum_{u\in V(G-S)}\sigma(v)$ is order of the biggest component of $G - S$ [41 - 43].

Definition 3.2 [35]
Let $G = (\sigma, \mu)$ be a fuzzy graph. The **domination integrity** of G, denoted by $\widetilde{DI}(G)$, is defined as $\widetilde{DI}(G), = \min\{|S| + m(G-S)\}$, S is the dominating set of G and $|S| = \sum_{u\in S}\sigma(u)$ denotes the cardinality of S, and $m(G-S) = \sum_{u\in V(G-S)}\sigma(v)$ is order of the biggest component of $G - S$ [33 - 36].

4. Integrity and Domination integrity in Neutrosophic soft graphs

In the crisp graph, membership values of vertex and edge are the same. In fuzzy, intuitionistic fuzzy graphs and neutrosophic graph, the membership values of vertices and edges have their own importance depending on the situation like uncertainty, indeterminacy, and falsity. This motivates to define these vulnerability parameters in neutrosophic fuzzy graphs. Also, it gives more accurate values in the real time problems especially in decision making process.

Definition 4.1
Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. The **integrity** of G, denoted by $\ddot{I}(G)$ is defined as $\ddot{I}(G) = \min\{|S| + m(G-S)\}$ where $|S| = \Sigma_{e\in A}\left|\Sigma_{v_i\in S}\frac{1+T_{J(e)}(x)+I_{J(e)}(x)-F_{J(e)}(x)}{2}\right|$ denotes the cardinality of S, and $m(G-S) = \Sigma_{e\in A}|\Sigma_{v_i\in V(G-S)}\frac{1+T_{J(e)}(x)+I_{J(e)}(x)-F_{J(e)}(x)}{2}|$ is order of the biggest component of $G - S$.

Definition 4.2
Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. The **domination integrity** of G, denoted by $\ddot{DI}(G)$, is defined as $\ddot{DI}(G) = \min\{|S| + m(G-S)\}$ and S is a dominating set of G, where $|S| =$

$\Sigma_{e\in A}|\Sigma_{v_i\in S}\frac{1+T_{J(e)}(x)+I_{J(e)}(x)-F_{J(e)}(x)}{2}|$ denotes the cardinality of S, and $m(G-S) = \Sigma_{e\in A}|\Sigma_{v_i\in V(G-S)}\frac{1+T_{J(e)}(x)+I_{J(e)}(x)-F_{J(e)}(x)}{2}|$ is order of the biggest component of $G - S$.

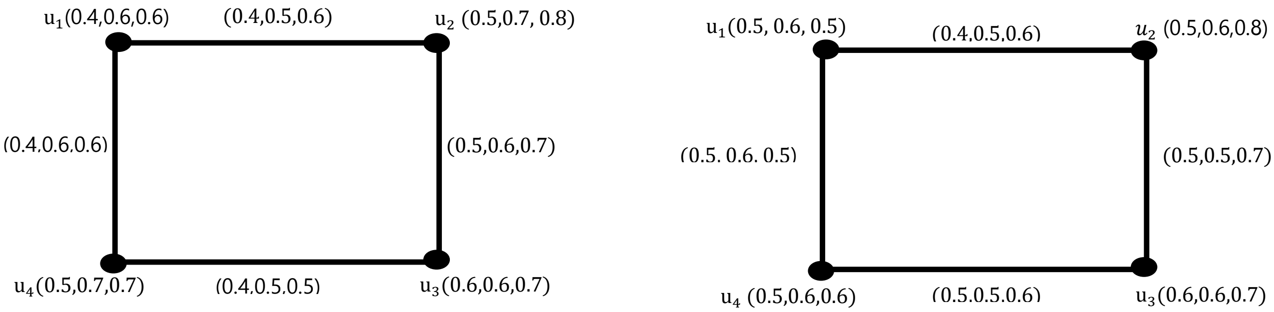
Definition 4.3

An Ï **-set** of $G = (G^*, J, K, A)$ is any (strict) subset S of V(G) for which Ï(G) = min{ |S| + m(G − S)}.

Definition 4.4

An D̈Ï(G)**-set** of $G = (G^*, J, K, A)$ is any (strict) subset S of V(G) for which D̈Ï(G) = min{ |S| + m(G − S)}.

Example : 4.5



$H(e_1)$ Figure 1 $H(e_2)$

S	$\|S\|$	$m(G-S)$	Ï(G)
$S_1 = \{u_1, u_3\}$	1.4	$.7$ for $\{u_2\}$ $.75$ for $\{u_4\}$	2.1 2.15
$S_2 = \{u_2, u_4\}$	1.45	$.7$ for $\{u_1\}$ $.75$ for $\{u_3\}$	2.1 2.15
$S_3 = \{u_1, u_2\}$	1.4	1.5 for $\{u_3, u_4\}$	2.9
$S_4 = \{u_1, u_4\}$	1.45	1.45 for $\{u_2, u_3\}$	2.9

Among all these subsets, S_1 is a Ï **-set** of G and Ï(G) = 2.1 corresponding to the parameter e_1

For e_2

S	$\|S\|$	$m(G-S)$	Ï(G)
$S_1 = \{u_1, u_3\}$	1.55	$.65$ for $\{u_2\}$ $.75$ for $\{u_4\}$	2.2 2.3
$S_2 = \{u_2, u_4\}$	1.4	$.8$ for $\{u_1\}$ $.75$ for $\{u_3\}$	2.2 2.15
$S_3 = \{u_1, u_2\}$	1.45	1.5 for $\{u_3, u_4\}$	2.95
$S_4 = \{u_1, u_4\}$	1.55	1.4 for $\{u_2, u_3\}$	2.95

Among all these subsets, S_1 and S_2 are the Ï **-sets** of G and Ï(G) = 2.2 corresponding to the parameter e_2

Example: 4.6

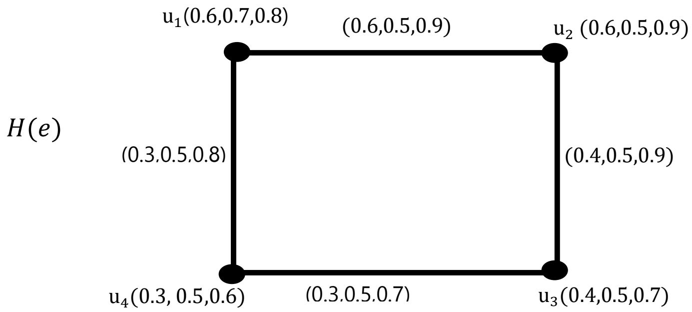


Figure 2

In Figure 2, corresponding to the parameter $H(e)$, $\{(u_1, u_2), (u_2, u_3), (u_3, u_4), (u_1, u_4)\}$ are the dominating sets.

S	$\|S\|$	$m(G - S)$	D̈Ï(G)
$S_1 = \{u_1, u_2\}$	1.35	1.2 for $\{u_3, u_4\}$	2.55
$S_2 = \{u_2, u_3\}$	1.2	1.35 for $\{u_1, u_4\}$	2.35
$S_3 = \{u_3, u_4\}$	1.2	1.5 for $\{u_1, u_2\}$	2.9
$S_4 = \{u_1, u_4\}$	1.35	1.45 for $\{u_2, u_3\}$	2.9
$S_5 = \{u_1, u_3\}$	1.35	0.6 for $\{u_2\}$ 0.6 for $\{u_4\}$	1.95
$S_5 = \{u_2, u_4\}$	1.2	0.6 for $\{u_3\}$ 0.75 for $\{u_1\}$	$Min\{1.8,1.95\}$ $= 1.8$

Among all these subsets, S_5 is a D̈Ï **-set** of G and D̈Ï(G) $= 1.8$ corresponding to the parameter e. In this neutrosophic graph G $\ddot{I}(G) = \ddot{D}\ddot{I}(G)$.

Example: 4.7

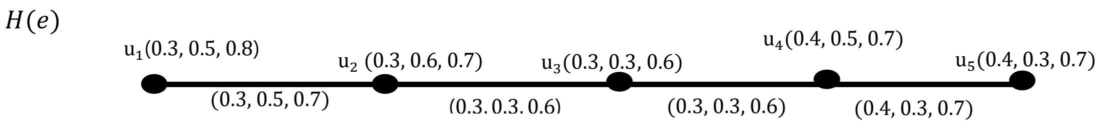


Figure 3

In Figure 2, corresponding to the parameter $H(e)$, $\{(u_2, u_4)\}$ are the dominating sets

S	$\|S\|$	$m(G - S)$	D̈Ï(G)
$S_1 = \{u_2, u_4\}$	1.2	$.5$ for $\{u_1\}$ $.5$ for $\{u_3\}$ $.7$ for $\{u_5\}$	1.7

S	$\|S\|$	$m(G - S)$	Ï(G)

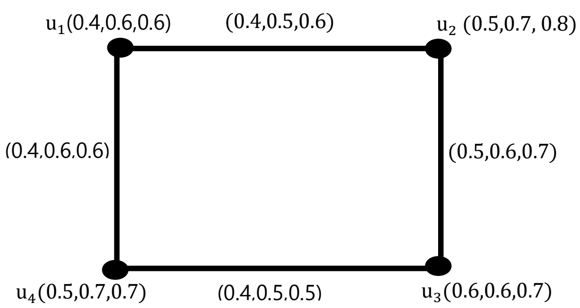
$S_1 = \{u_3\}$	.5	1.1 for $\{u_1, u_2\}$ 1.1 for $\{u_4, u_5\}$	1.6 1.6

In crisp graph , $I(G) \leq DI(G)$. But there is no relationship between these parameters in Fuzzy as well Neutrosophic soft graphs.

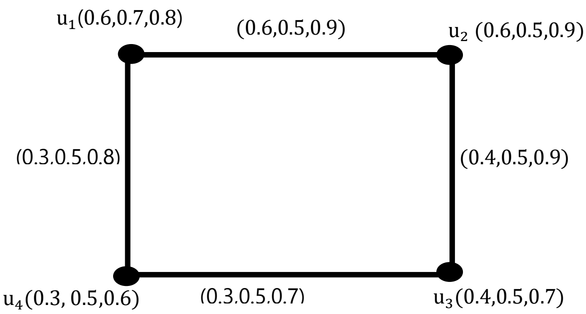
Definition 4.8

Let $G = (G^*, J, K, A)$ be a neutrosophic soft graph. A subset $S \subset V(G)$ is said to be a **vertex covering** of G if S contains at least one end of every strong arcs of G. A vertex covering S of G is called a minimal vertex covering if no subset of S is a vertex covering. The minimum cardinality of among all minimal vertex covering of G is called its vertex covering number and is denoted by $\Sigma_{e\in A}(cNS(G))$. .

Note: In Neutrosophic soft graphs, independent set may contain arcs which are not a strong arcs.



$H(e_1)$



$H(e_2)$

In $H(e_1)$, u_3u_4 is not a strong. So, independent set $S = \{u_3, u_4\}$ and vertex covering set $W = \{u_1, u_2\}$

In $H(e_2)$, all are strong arcs. Therefore, independent set $S = \{u_1, u_3\}$ and vertex covering set $W = \{u_3, u_4\}$

Theorem 4.9

Let G be neutrosophic soft graph. Then $\Sigma_{e\in A}(INS(G)) + \Sigma_{e\in A}(cNS(G)) = |V(G)|$.

Proof.

Let S be a maximum independent set of a neutrosophic soft graph G and W be a minimum vertex covering of G. Hence $\Sigma_{e\in A}(INS(G)) + \Sigma_{e\in A}(cNS(G)) = |V(G)|$.

Definition 4.10

A neutrosophic soft graph G is said to be **strong arc neutrosophic soft graph** if every arc in G is a strong arc.

Theorem 4.11

Let G be strong arc neutrosophic soft graph. Then $\ddot{I}(G) \leq \ddot{D}\ddot{I}(G) \leq |V(G)|$. Also $\ddot{I}(G) \leq \ddot{D}\ddot{I}(G) \leq |V(G)| - \Sigma_{e\in A}(cNS(G)) + 1$.

Proof.

In strong neutrosophic graph, every arc is a strong arc. Therefore, $\ddot{I}(G) \leq \ddot{DI}(G)$. Let S be vertex covering in G. Then, clearly the induced graph of $G - S$ is an independent set, say T. Hence the removal of S results totally independent vertices (isolates). Therefore, $m(G - S) = 1$. Hence $|V(G)| - \Sigma_{e\in A}\big(cNS(G)\big) + 1$.

Theorem 4.12

For any neutrosophic soft graph, $\Sigma_{e\in A}\left(d_{NS}(G)\right) \leq \ddot{DI}(G)$.

Proof.

The domination integrity number of a neutrosophic soft graph G depends upon the dominating set S and the corresponding maximum order of the component of $G - S$. This implies that $\Sigma_{e\in A}\left(d_{NS}(G)\right) < \ddot{DI}(G)$. The equality holds only when all the vertices of a neutrosophic soft graph. Hence $\Sigma_{e\in A}\left(d_{NS}(G)\right) \leq \ddot{DI}(G)$.

Theorem 4.13

For any strong arc neutrosophic soft graph, $\delta_s(G) + 1 \leq \ddot{I}(G) \leq \ddot{DI}(G)$.

Proof.

Let G be a strong neutrosophic soft graph. Let S be a subset of V(G) . Let u ∈ V(G) be a minimum strong degree vertex of G. Let $|d_s(v)| = \delta_s(G)$. Then, after the removal of the vertices in S from G, we get $m(G - S) \geq 1$ which gives the result $\delta_s(G) + 1 \leq \ddot{I}(G)$.

Theorem 4.14

Let $G' = = (G *, J', K', A')$ is called a neutrosophic soft subgraph of $G = (G *, J, K, A)$. Then $\ddot{I}(H) \leq \ddot{I}(G)$.

Proof.

Let $G' = = (G *, J', K', A')$ is called a neutrosophic soft subgraph of $G = (G *, J, K, A)$. Clearly, $|V(H)| \leq |V(G)|$ (by subgraph definition, at least one vertex, $v \in$ H which has less membership value comparing with membership value of G, otherwise $|V(G)| \leq |V(H)|$). Moreover, for any neutrosophic soft graph $H, \ddot{I}(H) \leq |H| < |G|$.

Suppose $\ddot{I}(G) > \ddot{I}(H)$ for an integrity set S of H. Then $m(H - S) < \ddot{I}(G) - |S|$. If S is also an integrity set of G, then $m(H - S) < m(G - S)$, which is impossible, since H is sub set of G. If S is not an integrity set of G then $\ddot{I}(G) - |S| < m(G - S)$, this is a contradiction. Hence any integrity set S of G is such that $\ddot{I}(H) \leq \ddot{I}(G)$.

Theorem 4.15

Let $G = (G *, J, K, A)$ be a complete neutrosophic soft graph. Then $\ddot{I}(G) = |V(G)| = \ddot{DI}(G)$.

Proof.

Clearly, in complete neutrosophic soft graph, all the vertices are adjacent with the remaining set of vertices. Therefore, after the removal of any subset S of vertices from G, $m(G - S) = |V(G)| - |S|$.

Theorem 4.16

If $G = (J, K, A)$ is a strong neutrosophic soft graph and its complement $G^c = (J^c, K^c, A^c)$, then $I(G \cup G^c) = |V(G)|$.

Proof.
Let G be a strong neutrosophic graph and G^c be the complement of G. By proposition 3.34[6], $G \cup G^c$ is a complete neutrosophic soft graph. Hence $I(G \cup G^c) = |V(G)|$.

Theorem 4.17
Let G_1 and G_2 be two connected neutrosophic soft graphs and $G = G_1 \cup G_2$ with $|G_1| \geq |G_2|$, then vertex integrity of G is given by
$\ddot{I}(G) = min\{|G_1|, \ddot{I}(G_1), |S| + |V(G_2)|, |S| + max\{m(G_1 - S), m(G_2 - S)\}\}$ where S is $\ddot{I}$- set of G.
Proof.
Let G_1 and G_2 be two connected neutrosophic soft graphs and $G = G_1 \cup G_2$ with $|G_1| \geq |G_2|$. Assume that $|G_1| > |G_2|$. In this case integrity set S of G is either vertices from G1 or G2 or both or empty. Since $|G_1| \geq |G_2|$, S cannot contain vertices from G_2 alone.
Based on each case which is mentioned above, we get the result.

Theorem 4.18
Let G_1 and G_2 be two connected neutrosophic soft graphs and $G = G_1 + G_2$ with $V_1 \cap V_2 \neq \emptyset$. Then $\ddot{I}(G) = min\{\ddot{I}(G_1) + |V(G_2)|, \ddot{I}(G_2) + |V(G_1)|\}$.
Proof.
Let G_1 and G_2 be two complete neutrosophic soft graphs. Clearly, G is a complete neutrosophic soft graph. Therefore, $\ddot{I}(G) = \ddot{I}(G_1) + \ddot{I}(G_2) = \ddot{I}(G_1) + |V(G_2)| = |V(G_1)| + \ddot{I}(G_2)$. If we take all the vertices of G_1 in the $\ddot{I}$-set of G, then induced graph G_2 is a single connected component, since every vertex from G_1 is linked with G_2 with an edge. In the similar manner, we consider G_2. Moreover, other subsets of $V(G)$, $m(G - S)$contains all the remaining vertices of G. Hence the theorem

5. Conclusion

In this present work, we introduced the concept of integrity and domination integrity in neutrosophic soft graphs and calculated the certain bounds of these new parameters. In our future work, we will study the applications of these new parameters in neutrosophic real time networks for decision making problems.

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