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On r-Edge Regular Neutrosophic Graphs

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Abstract: We approach learning characteristic on a neutrosophic graph such as r-edge regular neutrosophic graph, strongly edge regular neutrosophic graph and absolute degree of vertex since a neutrosophic set $NS = \{\langle x, NS_{\mathfrak{T}}(x), NS_{\mathrm{I}}(x), NS_{\mathfrak{F}}(x)\rangle; x \in X\}$ of a universe set A. We discuss different aspects of these graphics in this article. We've also included several examples to help you understand these concepts.

Keywords: Neutrosophic set, r-edge regular neutrosophic graph, strongly edge regular neutrosophic graph, absolute degree of vertex.

1. Introduction

By changing the definition of the fuzzy set, Smarandache [2] presented the neutrosophic set. Any vague real-life problem can be solved using the neutrosophic set, which can function with uncertain, indeterminate, unclear, and inconsistent details. It's essentially a hybrid of the crisp set, Type 1 fuzzy set, and the IFS. The truth, indeterminate, and false membership degrees of any object are used to define it. These three membership degrees are independent of one another and always fall within the range of [0, 1+], i.e. a nonstandard unit interval. Numerous scholars have long become more interested in neutrosophic graph theory, such as Ye [3] and Yang et al. [5]. Borzooei [1], Azadi et al. [9], Arkam [6] and Poulik, S., Ghorai, G [9-14] . The vertex degree is a useful way to define a vertex's total number of relationships in a graph, and it can also be utilised evaluate the graph. In a fuzzy graph, Gani and Lathi raised the concepts of irregularity, total irregularity, and total degree. Maheswari and Sekar suggested the d_2-vertex term and defined several assets of the d_2-vertex degree of a fuzzy graph. Darabian et al. introduced the d_m-regular vague graph, the td_m-regular vague graph, the m-highly irregular vague graph, and the m-highly complete irregular vague graph, as well as some of their attributes. In this article, we look at neutrosophic graphs using certain r-edge regularity and absolute degree of vertex properties. The purpose of this work is to generalise an idea from neutrosophic graph.

2. Preliminaries

2.1. Definition [7]

A graph $G = (V, E)$ is really an ordered pair made up of a non-empty vertex set V, another edge set E, and a link that connects each edge across two end points.

2.2. Definition [7]
Consider the graph $G = (V, E)$. Since $\mathfrak{A} \subseteq$ V and $\mathfrak{B} \subseteq$ E then $\vartheta_G = (\mathfrak{A}, \mathfrak{B})$ could very well be a sub graph of G.

2.3. Definition [11]
A function $\mu : \mathfrak{A} \to [0, 1]$. Defines a fuzzy set on a set $\mathfrak{A}$.

2.4. Definition
A fuzzy graph $G = (\sigma, \mu)$ is called complete fuzzy graph if $\mu(a, b) = \min\{\sigma(a), \sigma(b)\}, \forall a, b \in \sigma$.

2.5. Definition
A fuzzy graph $G = (\sigma, \mu)$ is called strong fuzzy graph if $\mu(a, b) = \min\{\sigma(a), \sigma(b)\}$ $\forall a, b \in \mu$.

2.6. Definition
The complement of a fuzzy graph $G = (\sigma, \mu)$ is a fuzzy graph and it is represented as $G^c = (\sigma^c, \mu^c)$, where $\sigma^c = \sigma$ and $\mu^c(a, b) = min\{\sigma(a), \sigma(b)\} - \mu(a, b)$.

2.7. Definition [7]
A fuzzy graph $\mathfrak{A}_G = (V, \lambda, \mu)$ is a non-empty set V together with pair of functions $\lambda: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that for all $a, b \in V, \mu(a, b) \le \min\{\lambda(a), \lambda(b)\}$ where $\lambda(a)$ and $\mu(ab)$ represent the membership value of the vertex a and the edge a, b in $\mathfrak{A}_G$, respectively. The underlying crisp graph of the fuzzy graph $\mathfrak{A}_G = (V, \lambda, \mu)$ is denoted by $\mathfrak{A}_{G^*} = (V, \lambda^*, \mu^*)$ where $\lambda^* = \{a \in V; \lambda(a) > 0\}$ and $\mu^* = \{ab \in V \times V; \mu(ab) > 0\}$. Thus for underlying fuzzy graph $\lambda^* = V$.

2.8. Definition [14]
An intuitionistic fuzzy graph is a pair Let $G = (V, E)$ of a graph $G^* = (V, E)$ where $\mathfrak{A} = (\mathfrak{A}_\mu, \mathfrak{A}_\lambda)$an intuitionistic fuzzy set on V is and $\mathfrak{B} = (\mathfrak{B}_\mu, \mathfrak{B}_\lambda)$ is an intuitionistic fuzzy relation on E such that $\mathfrak{B}_\mu(ab) \le \min\{\mathfrak{A}_\mu(a), \mathfrak{A}_\mu(b)\}, \mathfrak{B}_\lambda(ab) \ge \max\{\mathfrak{A}_\lambda(a), \mathfrak{A}_\lambda(b)\}$ for all a, b *in* V. The underlying crip graph of $G = (\mathfrak{A}, \mathfrak{B})$ is the crisp graph $G^* = (V, E)$,where $V = \{a; \mathfrak{A}_\lambda(a) > 0 \text{ or } \mathfrak{A}_\lambda(a) = 0\}$ and $E = \{ab; \mathfrak{B}_\mu(ab) > 0 \text{ or } \mathfrak{B}_\mu(ab) = 0\}$

2.9. Definition [3]
A neutrosophic graph is of the form $G = (V, E)$ where

1. V such that $\mathfrak{T}_1: \mathfrak{A} \to [0,1]$, $I_1: \mathfrak{A} \to [0,1]$ and $\mathfrak{F}_1: \mathfrak{A} \to [0,1]$ denote the degree of membership, degree of indeterminacy and non-membership of the element $v_i \in V$, respectively, and $0 \le \mathfrak{T}_i(v_i) + I_i(v_i) + \mathfrak{F}_i(v_i) \le 3$, for every $v_i \in V, (i = 1,2, \dots \dots, n)$.

2. $E \subseteq V \times V$, Where $\mathfrak{T}_2: \mathfrak{A} \to [0,1]$, $I_2: \mathfrak{A} \to [0,1]$ and $\mathfrak{F}_2: \mathfrak{A} \to [0,1]$ such that $\mathfrak{T}_2(v_i, v_j) \le \min\{\mathfrak{T}_1(v_i), \mathfrak{T}_1(v_j)\}, I_2(v_i, v_j) \ge \max\{I_1(v_i), I_1(v_j)\}$, and $\mathfrak{F}_2(v_i, v_j) \ge \max\{\mathfrak{F}_1(v_i), \mathfrak{F}_1(v_j)\}$ and $0 \le \mathfrak{T}_i(v_i, v_j) + I_i(v_i, v_j) + \delta_i(v_i, v_j) \le 3$ for every $v_i, v_j \in E, (i, j = 1,2, \dots \dots, n)$.

2.10. Example
Consider a neutrosophic graph G, such that $\mathfrak{A} = \{a, b, c, d, e\}$ and $\mathfrak{B} = \{ab, ac, cb, ce, ed, bd, cd, eb\}$ as in Figure 1.

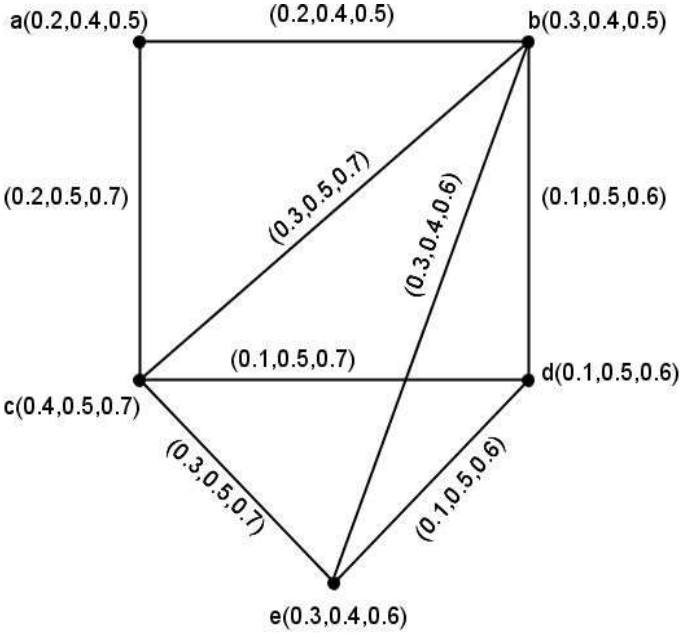


Figure 1: An example of neutrosophic graph

3. Neutrosophic graph in r-edge regular

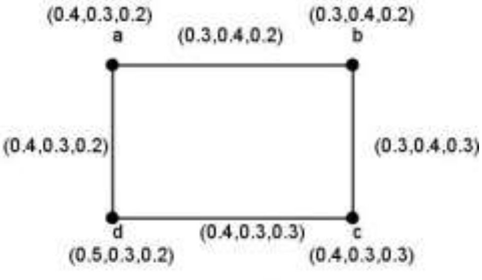
3.1. Definition

A graph $G^* = (\mathfrak{A}, \mathfrak{B})$ with a neutrosophic graph $G = (\mathfrak{A}, \mathfrak{B})$ is said to be strong if, for all $\mathfrak{a}\mathfrak{b} \in \mathfrak{A}$ and

1. $\mathfrak{B}_{\mathfrak{T}}(\mathfrak{a}\mathfrak{b}) = \min\{\mathfrak{A}_{\mathfrak{T}}(\mathfrak{a}), \mathfrak{A}_{\mathfrak{T}}(\mathfrak{b})\}$
2. $\mathfrak{B}_{\mathrm{I}}(\mathfrak{a}\mathfrak{b}) = \max\{\mathfrak{A}_{\mathrm{I}}(\mathfrak{a}), \mathfrak{A}_{\mathrm{I}}(\mathfrak{b})\}$
3. $\mathfrak{B}_{\mathfrak{F}}(\mathfrak{a}\mathfrak{b}) = \max\{\mathfrak{A}_{\mathfrak{F}}(\mathfrak{a}), \mathfrak{A}_{\mathfrak{F}}(\mathfrak{b})\}$

3.2. Example

Consider a neutrosophic graph G, such that $\mathfrak{A} = \{a, b, c, d\}$ and $\mathfrak{B} = \{ab, bc, cd, da\}$ as in figure 2.



Strong neutrosophic graph

Figure 2

3.3. Definition

A graph $G^* = (\mathfrak{A}, \mathfrak{B})$ with a neutrosophic graph $G = (\mathfrak{A}, \mathfrak{B})$ is said to be complete if, for all $\mathfrak{a}\mathfrak{b} \in \mathfrak{B}$ and

1. $\mathfrak{B}_{\mathfrak{T}}(\mathfrak{a}\mathfrak{b}) = \min\{\mathfrak{A}_{\mathfrak{T}}(\mathfrak{a}), \mathfrak{A}_{\mathfrak{T}}(\mathfrak{b})\}$
2. $\mathfrak{B}_{\mathrm{I}}(\mathfrak{a}\mathfrak{b}) = \max\{\mathfrak{A}_{\mathrm{I}}(\mathfrak{a}), \mathfrak{A}_{\mathrm{I}}(\mathfrak{b})\}$
3. $\mathfrak{B}_{\mathfrak{F}}(\mathfrak{a}\mathfrak{b}) = \max\{\mathfrak{A}_{\mathfrak{F}}(\mathfrak{a}), \mathfrak{A}_{\mathfrak{F}}(\mathfrak{b})\}$.

3.4. Example

Consider a neutrosophic graph G, such that $\mathfrak{A} = \{\text{a, b, c, d}\}$ and $\mathfrak{B} = \{\text{ab, bc, cd, da, ac, db}\}$ as in figure 3.

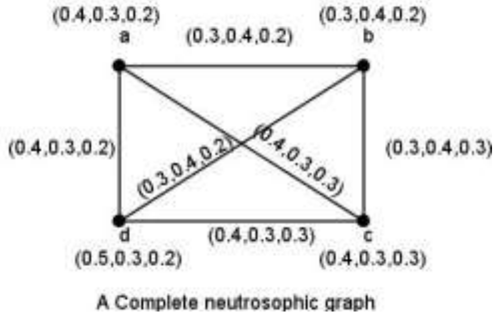


Figure 3

3.5. Definition

A complement of a neutrosophic graph $G = (\mathfrak{A}, \mathfrak{B})$ is a neutrosophic graph $\bar{G} = (\bar{\mathfrak{A}}, \bar{B})$, where $\bar{\mathfrak{A}} = (\bar{\mathfrak{A}}_{\mathfrak{T}}(\text{a}),\ \bar{\mathfrak{A}}_{I}(\text{a}), \bar{\mathfrak{A}}_{\mathfrak{F}}(\text{a}))$ and $\bar{\mathfrak{B}} = (\bar{\mathfrak{B}}_{\mathfrak{T}}(\text{a}),\ \bar{\mathfrak{B}}_{I}(\text{a}), \bar{\mathfrak{B}}_{\mathfrak{F}}(\text{a}))$

Here,

1. $\bar{\mathfrak{B}}_{\mathfrak{T}}(\text{ab}) = \min\{\mathfrak{A}_{\mathfrak{T}}(\text{a}), \mathfrak{A}_{\mathfrak{T}}(\text{b})\} - \mathfrak{B}_{\mathfrak{T}}(\text{ab})$
2. $\bar{\mathfrak{B}}_{\mathrm{I}}(\text{ab}) = \max\{\mathfrak{A}_{\mathrm{I}}(\text{a}), \mathfrak{A}_{I}(\text{b})\} - \mathfrak{B}_{\mathrm{I}}(\text{ab})$
3. $\bar{\mathfrak{B}}_{\mathfrak{F}}(\text{ab}) = \max\{\mathfrak{A}_{\mathfrak{F}}(\text{a}), \mathfrak{A}_{\mathfrak{F}}(\text{b})\} - \mathfrak{B}_{\mathfrak{F}}(\text{ab})$ for all a, b $\epsilon\ \mathfrak{B}$.

3.6. Example

Consider a neutrosophic graph G, such that $\mathfrak{A} = \{\text{a, b, c, d}\}$ and $\mathfrak{B} = \{\text{ac, ad, db, bc}\}$ as in figure 4.

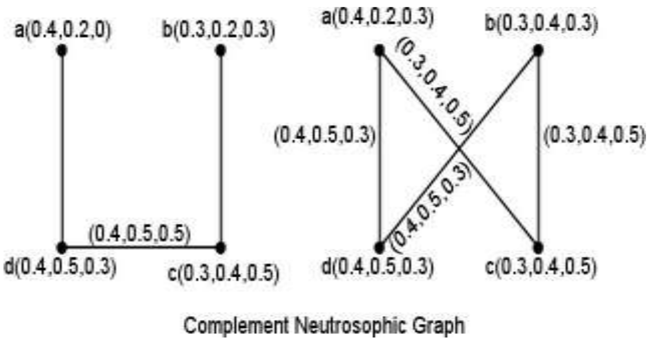


Figure 4

3.7. Definition

The absolute degree of any vertex an is determined by if $G = (\mathfrak{A}, \mathfrak{B})$ is a neutrosophic graph.

$\mathfrak{D}_{\text{a}} = (\mathfrak{T}_{\mathrm{D}}(\text{a}),\ \mathrm{I}_{\mathrm{D}}(\text{a}),\ F_{\mathrm{D}}(\text{a}))$, where

1. $\mathfrak{T}_{\mathrm{D}}(\text{a}) = \sum \mathfrak{B}_{\mathfrak{T}}(\text{ab});\ \text{a} \neq \text{b}, \text{ab} \in E$
2. $\mathrm{I}_{\mathrm{D}}(\text{a}) = \sum \mathfrak{B}_{\mathrm{I}}(\text{ab});\ \text{a} \neq \text{b}, \text{ab} \in E$
3. $\mathfrak{F}_{\mathrm{D}}(\text{a}) = \sum \mathfrak{B}_{\mathfrak{F}}(\text{ab});\ \text{a} \neq \text{b}, \text{ab} \in E$

And hence $\mathfrak{D}_{\text{a}} = \left| \sum_{\substack{\text{a}\neq\text{b} \\ \text{a}\in V}} \mathfrak{B}_{\mathfrak{T}}(\text{ab}) - \sum_{\substack{\text{a}\neq\text{b} \\ \text{a}\in V}} \mathfrak{B}_{\mathrm{I}}(\text{ab}) - \sum_{\substack{\text{a}\neq\text{b} \\ \text{a}\in V}} \mathfrak{B}_{\mathfrak{F}}(\text{ab}) \right|$

3.8. Example

Let $G^* = (\mathfrak{A}, \mathfrak{B})$, where $\mathfrak{A} = \{\mathrm{a, b, c, d, e}\}$ and $\mathfrak{B} = \{\mathrm{ab, ae, bd, ad, de}\}$, then

$$D_{\mathrm{a}} = |(0.3 + 0.2) - (0.5 + 0.5) - (0.6 + 0.6)| = 1.8$$
$$D_{\mathrm{b}} = |(0.4 + 0.3) - (0.6 + 0.5) - (0.7 + 0.6)| = 1.7$$
$$D_{\mathrm{ç}} = |(0.3 + 0.4 + 0.2) - (0.6 + 0.6 + 0.6) - (0.7 + 0.7 + 0.7)| = 3$$
$$D_d = |(0.2 + 0.2 + 0.2) - (0.5 + 0.5 + 0.6) - (0.6 + 0.6 + 0.7)| = 2.9$$
$$D_e = |(0.3 + 0.2) - (0.6 + 0.5) - (0.7 + 0.6)| = 1.9$$

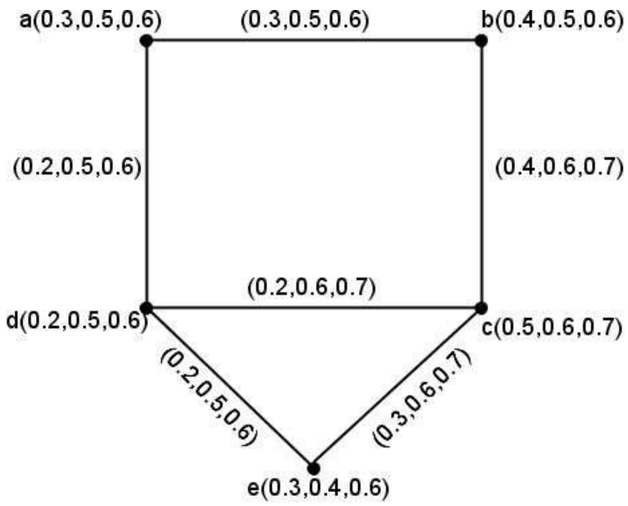


Figure 2: Graph G which id defined in Example 3.8

Figure 5

3.9 Definition

If $G^* = (V, E)$ is a crisp graph and i = a, b is an edge in G^*, then $\mathcal{D}_{\mathrm{e}} = \mathcal{D}_{\mathrm{a}} + \mathcal{D}_{\mathrm{b}} - 2$ is the degree of the e ∈ E.

3.10. Definition

Let $G = (\mathfrak{A}, \mathfrak{B})$ be a neutrosophic graph. $\mathcal{D}_N(\mathrm{a}) = (N_{\mathfrak{A}}(\mathrm{a}) + N_{\mathfrak{B}}(\mathrm{a}))$ is the degree neighbourhood of a vertex. Where $N_{\mathfrak{A}}(\mathrm{a}) = \sum_{\mathrm{b} \in N(\mathrm{a})} \mathfrak{A}_{\mathfrak{T}}(\mathrm{b}), N_{\mathfrak{A}}(\mathrm{a}) = \sum_{\mathrm{b} \in N(\mathrm{a})} \mathfrak{A}_{\mathrm{I}}(\mathrm{b})$. $\mathrm{a}nd\ N_B(\mathrm{a}) = \sum_{\mathrm{b} \in N(\mathrm{a})} \mathfrak{B}_{\mathfrak{F}}(\mathrm{b})$.

3.11. Definition

In a neutrosophic graph G = ($\mathfrak{A}$, $\mathfrak{B}$) an edge's total open neighbourhood degree $\mathrm{ab} \in$ E is known as $\mathfrak{T}_{\mathrm{D}}(\mathrm{ab}) = \left(T_{\mathrm{D}}\mathfrak{A}_{\mathfrak{T}}(\mathrm{ab}), T_{\mathrm{D}}\mathfrak{A}_{\mathrm{I}}(\mathrm{ab}), T_{\mathrm{D}}\mathfrak{A}_{\mathfrak{F}}(\mathrm{ab})\right)$

Where,

$$T_{\mathrm{D}}\mathfrak{A}_{\mathfrak{T}}(\mathrm{ab}) = T_{\mathrm{D}}(\mathrm{a}) + T_D(\mathrm{b}) - \mathfrak{B}_{\mathfrak{T}}(\mathrm{ab})$$
$$T_{\mathrm{D}}\mathfrak{A}_{\mathrm{I}}(\mathrm{ab}) = T_{\mathrm{D}}(\mathrm{a}) + T_{\mathrm{D}}(\mathrm{b}) - \mathfrak{B}_{\mathrm{I}}(\mathrm{ab})$$
$$T_{\mathrm{D}}\mathfrak{A}_{\mathfrak{F}}(\mathrm{ab}) = T_{\mathrm{D}}(\mathrm{a}) + T_{\mathrm{D}}(\mathrm{b}) - \mathfrak{B}_{\mathfrak{F}}(\mathrm{ab})$$

An edge's minimum total open neighbourhood degree is equal to $\Delta_{TE} = \min\{T_{\mathrm{D}}(\mathrm{ab}); \mathrm{ab} \in E\}$
An edge's minimum total open neighbourhood degree is equal to $\Delta_{IE} = \min\{I_{\mathrm{D}}(\mathrm{ab}); \mathrm{ab} \in E\}$ and
An edge's maximum open neighbourhood degree is known as $F_{TE} = \max\{F_{\mathrm{D}}(\mathrm{ab}); \mathrm{ab} \in E\}$.

3.12. Definition

Let G = ($\mathfrak{A}$, $\mathfrak{B}$) be a neutrosophic graph. The degree neighbourhood of a vertex a is defined as $D_N = \left(N_T(a), N_I(a), N_F(a)\right)$, Where $N_T(a) = \sum_{b \in N_T(a)} T\,(b),\ N_I(a) = \sum_{b \in N_I(a)} I\,(b)\ and\ N_F(a) = \sum_{b \in N_F(a)} F\,(b)$.

3.13. Definition

Assume $G = (\mathfrak{A}, \mathfrak{B})$ is a neutrosophic graph on G^*.

1. G also is an r-edge regular neutrosophic graph if all of its edges get the same neighbourhood degree r.
   In a neutrosophic graph $G = (\mathfrak{A}, \mathfrak{B})$, the open neighbourhood degree of an edge $a\mathfrak{h} \in E$ is classified as $D_{a\mathfrak{h}} = (D_{\mathfrak{A}\mathfrak{T}}(a\mathfrak{h}), D_{\mathfrak{A}I}(a\mathfrak{h}), D_{\mathfrak{A}\mathfrak{F}}(a\mathfrak{h})$ such that;
   $D_{\mathfrak{A}\mathfrak{T}}(a\mathfrak{h}) = \mathfrak{T}_D(a) + \mathfrak{T}_D(\mathfrak{h}) - 2\mathfrak{B}_{\mathfrak{T}}(a\mathfrak{h}), D_{\mathfrak{A}I}(a\mathfrak{h}) = I_D(a) + I_D(\mathfrak{h}) - 2\mathfrak{B}_I(a\mathfrak{h})$,
   and $D_{\mathfrak{A}\mathfrak{F}}(a\mathfrak{h}) = \mathfrak{F}_D(a) + \mathfrak{F}_D(\mathfrak{h}) - 2\mathfrak{B}_{\mathfrak{F}}(a\mathfrak{h})$.

3.14. Example

Consider a neutrosophic graph $G = (\mathfrak{A}, \mathfrak{B})$ such that

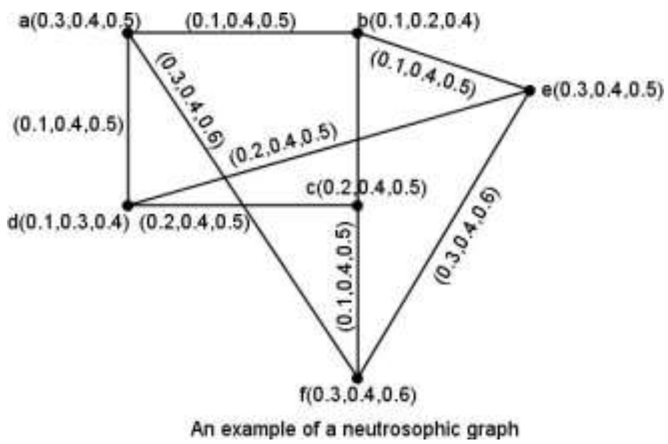


Figure 6

$D_a = (0.5, 1.2, 1.6)$ $D_{\mathfrak{h}} = (0.3, 1.2, 1.5)$ $D_{\varsigma} = (0.5, 1.2, 1.6)$
$D_d = (0.5, 1.2, 1.5)$ $D_e = (0.6, 1.2, 1.6)$ $D_f = (0.8, 1.2, 1.8)$

$$D_{\mathfrak{A}T}(a\mathfrak{h}) = \mathfrak{T}_D(a) + \mathfrak{T}_D(\mathfrak{h}) - 2\mathfrak{B}_{\mathfrak{T}}(a\mathfrak{h}) = 0.5 + 0.3 - 2(0.1) = 0.6$$
$$D_{\mathfrak{A}I}(a\mathfrak{h}) = I_D(a) + I_D(\mathfrak{h}) - 2\mathfrak{B}_I(a\mathfrak{h}) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$D_{\mathfrak{A}\mathfrak{F}}(a\mathfrak{h}) = \mathfrak{F}_D(a) + \mathfrak{F}_D(\mathfrak{h}) - 2\mathfrak{B}_{\mathfrak{F}}(a\mathfrak{h}) = 1.6 + 1.5 - 2(0.5) = 2.1$$
$$D_{a\mathfrak{h}} = (D_{\mathfrak{A}\mathfrak{T}}(a\mathfrak{h}), D_{AI}(a\mathfrak{h})\ D_{AF}(a\mathfrak{h})) = (0.6, 1.6, 2.1)$$

$$D_{\mathfrak{A}\mathfrak{T}}(\mathfrak{h}\varsigma) = \mathfrak{T}_D(\mathfrak{h}) + \mathfrak{T}_D(\varsigma) - 2\mathfrak{B}_{\mathfrak{T}}(\mathfrak{h}\varsigma) = 0.3 + 0.5 - 2(0.1) = 0.6$$
$$D_{\mathfrak{A}I}(\mathfrak{h}\varsigma) = I_D(\mathfrak{h}) + I_D(\varsigma) - 2\mathfrak{B}_I(\mathfrak{h}\varsigma) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$D_{\mathfrak{A}\mathfrak{F}}(\mathfrak{h}\varsigma) = \mathfrak{F}_D(\mathfrak{h}) + \mathfrak{F}_D(\varsigma) - 2\mathfrak{B}_{\mathfrak{F}}(\mathfrak{h}\varsigma) = 1.5 + 1.6 - 2(0.5) = 2.1$$
$$D_{\mathfrak{h}\varsigma} = (D_{\mathfrak{A}\mathfrak{T}}(\mathfrak{h}\varsigma), D_{\mathfrak{A}I}(\mathfrak{h}\varsigma)\ D_{\mathfrak{A}\mathfrak{F}}(\mathfrak{h}\varsigma)) = (0.6, 1.6, 2.1)$$

$$D_{\mathfrak{A}\mathfrak{T}}(\varsigma d) = \mathfrak{T}_D(\varsigma) + \mathfrak{T}_D(d) - 2\mathfrak{B}_{\mathfrak{T}}(\varsigma d) = 0.5 + 0.5 - 2(0.2) = 0.6$$
$$D_{\mathfrak{A}I}(\varsigma d) = I_D(\varsigma) + I_D(d) - 2\mathfrak{B}_I(\varsigma d) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$D_{\mathfrak{A}\mathfrak{F}}(\varsigma d) = \mathfrak{F}_D(\varsigma) + \mathfrak{F}_D(d) - 2\mathfrak{B}_{\mathfrak{F}}(\varsigma d) = 1.6 + 1.5 - 2(0.5) = 2.1$$
$$D_{\varsigma d} = (D_{\mathfrak{A}T}(\varsigma d), D_{\mathfrak{A}I}(\varsigma d)\ D_{\mathfrak{A}\mathfrak{F}}(\varsigma d)) = (0.6, 1.6, 2.1)$$

$$D_{\mathfrak{A}\mathfrak{T}}(da) = \mathfrak{T}_D(d) + \mathfrak{T}_D(a) - 2\mathfrak{B}_{\mathfrak{T}}(da) = 0.4 + 0.4 - 2(0.1) = 0.6$$
$$D_{\mathfrak{A}I}(da) = I_D(d) + I_D(a) - 2\mathfrak{B}_I(da) = 0.9 + 1.3 - 2(0.4) = 1.4$$
$$D_{\mathfrak{A}\mathfrak{F}}(da) = \mathfrak{F}_D(d) + \mathfrak{F}_D(a) - 2\mathfrak{B}_{\mathfrak{F}}(da) = 1.9 + 2.1 - 2(0.6) = 2.8$$
$$D_{da} = (D_{\mathfrak{A}\mathfrak{T}}(da), D_{\mathfrak{A}I}(da), D_{\mathfrak{A}\mathfrak{F}}(da)) = (0.6, 1.4, 2.8)$$

$$D_{\mathfrak{A}\mathfrak{T}}(af) = \mathfrak{T}_D(a) + \mathfrak{T}_D(f) - 2\mathfrak{B}_{\mathfrak{T}}(af) = 0.5 + 0.8 - 2(0.3) = 1.1$$

$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}f) = \mathrm{I}_{\mathrm{D}}(\mathrm{a}) + \mathrm{I}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathrm{I}}(\mathrm{a}f) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{a}f) = \mathfrak{F}_{\mathrm{D}}(\mathrm{a}) + \mathfrak{F}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathfrak{F}}(\mathrm{a}f) = 1.6 + 1.8 - 2(0.6) = 2.2$$

$$\mathfrak{D}_{\mathrm{a}f} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}f), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}f)\ \mathfrak{D}_{\mathfrak{A}F}(\mathrm{a}f)) = (1.1, 1.6, 2.2)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(ef) = \mathfrak{T}_{\mathrm{D}}(e) + \mathfrak{T}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathfrak{T}}(ef) = 0.6 + 0.8 - 2(0.3) = 0.8$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(ef) = \mathrm{I}_{\mathrm{D}}(e) + \mathrm{I}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathrm{I}}(ef) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(ef) = \mathfrak{F}_{\mathrm{D}}(e) + \mathfrak{F}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathfrak{F}}(ef) = 1.6 + 1.8 - 2(0.6) = 2.2$$
$$\mathfrak{D}_{ef} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(ef), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(ef)\ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(ef)) = (0.8, 1.6, 2.2)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(çf) = \mathfrak{T}_{\mathrm{D}}(ç) + \mathfrak{T}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathfrak{T}}(çf) = 0.5 + 0.8 - 2(0.2) = 0.9$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(çf) = \mathrm{I}_{\mathrm{D}}(ç) + \mathrm{I}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathrm{I}}(çf) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(çf) = \mathfrak{F}_{\mathrm{D}}(ç) + \mathfrak{F}_{\mathrm{D}}(f) - 2\mathfrak{B}_{\mathfrak{F}}(çf) = 1.6 + 1.8 - 2(0.6) = 2.2$$
$$\mathfrak{D}_{çe} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(çf), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(çf)\ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(çf)) = (0.9, 1.6, 2.2)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(be) = \mathfrak{T}_{\mathrm{D}}(b) + \mathfrak{T}_{\mathrm{D}}(e) - 2\mathfrak{B}_{\mathfrak{T}}(be) = 0.3 + 0.6 - 2(0.1) = 0.7$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(be) = \mathrm{I}_{\mathrm{D}}(b) + \mathrm{I}_{\mathrm{D}}(e) - 2\mathfrak{B}_{\mathrm{I}}(be) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(be) = \mathfrak{F}_{\mathrm{D}}(b) + \mathfrak{F}_{\mathrm{D}}(e) - 2\mathfrak{B}_{\mathfrak{F}}(be) = 1.5 + 1.6 - 2(0.5) = 2.1$$
$$\mathfrak{D}_{be} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(be), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(be)\ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(be)) = (0.7, 1.6, 2.1)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(de) = \mathfrak{T}_{\mathrm{D}}(d) + \mathfrak{T}_{\mathrm{D}}(e) - 2\mathfrak{B}_{\mathfrak{T}}(de) = 0.5 + 0.6 - 2(0.2) = 0.7$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(de) = \mathrm{I}_{\mathrm{D}}(d) + \mathrm{I}_{\mathrm{D}}(e) - 2\mathfrak{B}_{\mathrm{I}}(de) = 1.2 + 1.2 - 2(0.4) = 1.6$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(de) = \mathfrak{F}_{\mathrm{D}}(d) + \mathfrak{F}_{\mathrm{D}}(e) - 2\mathfrak{B}_{\mathfrak{F}}(de) = 1.5 + 1.6 - 2(0.5) = 2.1$$
$$\mathfrak{D}_{be} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(de), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(de)\ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(de)) = (0.7, 1.6, 2.1)$$

$$\mathfrak{D}_{\mathfrak{A}T}(\mathrm{a}ꞕ) = \mathfrak{T}_{\mathrm{D}}(\mathrm{a}) + \mathfrak{T}_{\mathrm{D}}(ꞕ) - \mathfrak{B}_{\mathfrak{T}}(\mathrm{a}ꞕ) = 0.5 + 0.3 - (0.1) = 0.7$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}ꞕ) = \mathrm{I}_{\mathrm{D}}(\mathrm{a}) + \mathrm{I}_{\mathrm{D}}(ꞕ) - \mathfrak{B}_{\mathrm{I}}(\mathrm{a}ꞕ) = 1.2 + 1.2 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{a}ꞕ) = \mathfrak{F}_{\mathrm{D}}(\mathrm{a}) + \mathfrak{F}_{\mathrm{D}}(ꞕ) - \mathfrak{B}_{\mathfrak{F}}(\mathrm{a}ꞕ) = 1.6 + 1.5 - (0.5) = 2.6$$
$$\mathfrak{D}_{\mathrm{a}ꞕ} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}ꞕ), \mathfrak{D}_{AI}(\mathrm{a}ꞕ)\ \mathfrak{D}_{AF}(\mathrm{a}ꞕ)) = (0.7, 2.0, 2.6)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(ꞕç) = \mathfrak{T}_{\mathrm{D}}(ꞕ) + \mathfrak{T}_{D}(ç) - \mathfrak{B}_{\mathfrak{T}}(ꞕç) = 0.3 + 0.5 - (0.1) = 0.7$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(ꞕç) = \mathrm{I}_{\mathrm{D}}(ꞕ) + \mathrm{I}_{\mathrm{D}}(ç) - \mathfrak{B}_{\mathrm{I}}(ꞕç) = 1.2 + 1.2 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(ꞕç) = \mathfrak{F}_{\mathrm{D}}(ꞕ) + \mathfrak{F}_{\mathrm{D}}(ç) - \mathfrak{B}_{\mathfrak{F}}(ꞕç) = 1.5 + 1.6 - (0.5) = 1.6$$
$$\mathfrak{D}_{ꞕç} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(ꞕç), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(ꞕç)\ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(ꞕç)) = (0.7, 2.0, 1.6)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(çd) = \mathfrak{T}_{\mathrm{D}}(ç) + \mathfrak{T}_{\mathrm{D}}(d) - \mathfrak{B}_{\mathfrak{T}}(çd) = 0.5 + 0.5 - (0.2) = 0.8$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(çd) = \mathrm{I}_{\mathrm{D}}(ç) + \mathrm{I}_{\mathrm{D}}(d) - \mathfrak{B}_{\mathrm{I}}(çd) = 1.2 + 1.2 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(çd) = \mathfrak{F}_{\mathrm{D}}(ç) + \mathfrak{F}_{\mathrm{D}}(d) - \mathfrak{B}_{\mathfrak{F}}(çd) = 1.6 + 1.5 - (0.5) = 2.7$$
$$\mathfrak{D}_{çd} = (\mathfrak{D}_{\mathfrak{A}T}(çd), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(çd)\ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(çd)) = (0.8, 2.0, 2.7)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{da}) = \mathfrak{T}_{\mathrm{D}}(\mathrm{d}) + \mathfrak{T}_{\mathrm{D}}(\mathrm{a}) - \mathfrak{B}_{\mathfrak{T}}(\mathrm{da}) = 0.4 + 0.4 - (0.1) = 0.9$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{da}) = \mathrm{I}_{\mathrm{D}}(\mathrm{d}) + \mathrm{I}_{\mathrm{D}}(\mathrm{a}) - \mathfrak{B}_{\mathrm{I}}(\mathrm{da}) = 0.9 + 1.3 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{da}) = \mathfrak{F}_{\mathrm{D}}(\mathrm{d}) + \mathfrak{F}_{\mathrm{D}}(\mathrm{a}) - \mathfrak{B}_{\mathfrak{F}}(\mathrm{da}) = 1.9 + 2.1 - (0.6) = 2.6$$
$$\mathfrak{D}_{da} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{da}), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{da}), \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{da})) = (0.6, 1.4, 2.8)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}f) = \mathfrak{T}_{\mathrm{D}}(\mathrm{a}) + \mathfrak{T}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathfrak{T}}(\mathrm{a}f) = 0.5 + 0.8 - (0.3) = 1.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}f) = \mathrm{I}_{\mathrm{D}}(\mathrm{a}) + \mathrm{I}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathrm{I}}(\mathrm{a}f) = 1.2 + 1.2 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{a}f) = \mathfrak{F}_{\mathrm{D}}(\mathrm{a}) + \mathfrak{F}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathfrak{F}}(\mathrm{a}f) = 1.6 + 1.8 - (0.6) = 2.8$$
$$\mathfrak{D}_{\mathrm{a}f} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}f), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}f)\ \mathfrak{D}_{\mathfrak{A}F}(\mathrm{a}f)) = (1.0, 2.0, 2.8)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(ef) = \mathfrak{T}_{\mathrm{D}}(e) + \mathfrak{T}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathfrak{T}}(ef) = 0.6 + 0.8 - (0.3) = 1.1$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(ef) = \mathrm{I}_{\mathrm{D}}(e) + \mathrm{I}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathrm{I}}(ef) = 1.2 + 1.2 - (0.4) = 2.0$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(ef) = \mathfrak{F}_{\mathrm{D}}(e) + \mathfrak{F}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathfrak{F}}(ef) \ = 1.6 + 1.8 - (0.6) = 2.8$$
$$\mathfrak{D}_{ef} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(ef), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(ef)\ \ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(ef)) \ = (1.1, 2.0, 2.8)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(çf) = \mathfrak{T}_{\mathrm{D}}(ç) + \mathfrak{T}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathfrak{T}}(çf) = 0.5 + 0.8 - (0.2) = 1.1$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(çf) \ = \mathrm{I}_{\mathrm{D}}(ç) + \mathrm{I}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathrm{I}}(çf) \quad = 1.2 + 1.2 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(çf) = \mathfrak{F}_{\mathrm{D}}(ç) + \mathfrak{F}_{\mathrm{D}}(f) - \mathfrak{B}_{\mathfrak{F}}(çf) \ = 1.6 + 1.8 - (0.6) = 2.8$$
$$\mathfrak{D}_{çe} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(çf), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(çf)\ \ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(çf)) \ = (1.1,2.0, 2.8)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(be) = \mathfrak{T}_{\mathrm{D}}(b) + \mathfrak{T}_{\mathrm{D}}(e) - \mathfrak{B}_{\mathfrak{T}}(be) = 0.3 + 0.6 - (0.1) = 0.8$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(be) = \mathrm{I}_{\mathrm{D}}(b) + \mathrm{I}_{\mathrm{D}}(e) - \mathfrak{B}_{\mathrm{I}}(be) \quad = 1.2 + 1.2 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(be) = \mathfrak{F}_{\mathrm{D}}(b) + \mathfrak{F}_{\mathrm{D}}(e) - \mathfrak{B}_{\mathfrak{F}}(be) \ = 1.5 + 1.6 - (0.5) = 2.6$$
$$\mathfrak{D}_{be} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(b), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(e)\ \ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(be)) \ = (0.8, 2.0, 2.6)$$

$$\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(de) = \mathfrak{T}_{\mathrm{D}}(d) + \mathfrak{T}_{\mathrm{D}}(e) - \mathfrak{B}_{\mathfrak{T}}(de) = 0.3 + 0.6 - (0.1) = 0.9$$
$$\mathfrak{D}_{\mathfrak{A}\mathrm{I}}(de) = \mathrm{I}_{\mathrm{D}}(d) + \mathrm{I}_{\mathrm{D}}(e) - \mathfrak{B}_{\mathrm{I}}(de) \quad = 1.2 + 1.2 - (0.4) = 2.0$$
$$\mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(de) = \mathfrak{F}_{\mathrm{D}}(d) + \mathfrak{F}_{\mathrm{D}}(e) - \mathfrak{B}_{\mathfrak{F}}(de) \ = 1.5 + 1.6 - (0.5) = 2.6$$
$$\mathfrak{D}_{be} = (\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(de), \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(de)\ \ \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(de)) \ = (0.9, 2.0, 2.6)$$

3.15. Definition

A neutrosophic graph G is a totally (r_1, r_2, r_3) -edge regular neutrosophic graph if every edge has the same total degree (r_1, r_2, r_3).

3.16. Definition

A neutrosophic graph is $G = (\mathfrak{A}, \mathfrak{B})$. G is shown to be a regular neutrosophic graph of degree (r_1, r_2, r_3) if every vertex does have the same degree(r_1, r_2, r_3).

3.17. Definition

If any vertex in a neutrosophic graph $G = (\mathfrak{A}, \mathfrak{B})$has the same degree$(r_1, r_2, r_3)$, then G is called a (r_1, r_2, r_3) edge regular neutrosophic graph.

3.18. Theorem

$G = \sum_{\mathrm{a}_i\mathrm{a}_j \in E} D_{\mathrm{a}_i\mathrm{a}_j} = \sum_{\mathrm{a}_i \in V} \mathfrak{D}_{\mathrm{a}_i}$ if G is an edge regular neutrosophic graph on a cycle G^*.

Proof

Since G is an edge regular neutrosophic graph, thus

$$\sum_{i=1}^{n} \mathfrak{D}_{\mathrm{a}_i\,\mathrm{a}_{j+1}} = \left(\sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j)\,, \sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j), \sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j)\right)$$
$$\sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j) = \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_1\mathrm{a}_2) + \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_2\mathrm{a}_3) + \cdots + \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_n\mathrm{a}_1)$$

Since,

$$\begin{aligned} \mathrm{a}_{n+1} &= \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_1) + \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_2) - 2\mathfrak{B}(\mathrm{a}_1\mathrm{a}_2) + \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_2) + \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_3) - 2\mathfrak{B}(\mathrm{a}_2\mathrm{a}_3) + \cdots \\ &\quad + \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_n) + \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_1) - 2\mathfrak{B}(\mathrm{a}_n\mathrm{a}_1) \\ &= 2\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_1) + 2\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_2) + 2\mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_n) - 2(\mathfrak{B}(\mathrm{a}_1\mathrm{a}_2) + \mathfrak{B}(\mathrm{a}_2\mathrm{a}_3) + \mathfrak{B}(\mathrm{a}_n\mathrm{a}_1)) \\ &= 2\textstyle\sum_{\mathrm{a}_i \in V} \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_1) + 2\sum_{i=1}^{n}(\mathrm{a}_i\mathrm{a}_{i+1}) - 2\sum_{i=1}^{n} B_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_{i+1}) \\ &= \textstyle\sum_{\mathrm{a}_i \in V} \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_i) \end{aligned}$$

Then

$$\sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_{j+1}) \ = \ \sum_{\mathrm{a}_i \in V} \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_i)$$

Similarly

$$\sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}_i\mathrm{a}_{i+1}) \ = \ \sum_{\mathrm{a}_i \in V} \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}_i)$$
$$\sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_{i+1}) \ = \ \sum_{\mathrm{a}_i \in V} \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{a}_i) \qquad \sum_{i=1}^{n} \mathfrak{D}(\mathrm{a}_i\mathrm{a}_{i+1}) \quad = \left(\sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_{i+1})\,, \sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathrm{I}}(\mathrm{a}_i\mathrm{a}_{i+1}), \sum_{i=1}^{n} \mathfrak{D}_{\mathfrak{A}\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_{i+1})\right)$$

$\sum_{i=1}^{n} Ɗ(\mathrm{a}_i\mathrm{a}_{i+1}) = \sum_{\mathrm{a}_{i\in V}} Ɗ_{\mathrm{a}_i}$.

3.19. Lemma

If G is an edge regular neutrosophic graph on G^*, then $\sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ(\mathrm{a}_i\mathrm{a}_j) =$
$\left(\sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), \sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j), \sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j)\right)$
Since, $Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j) = Ɗ_{G^*}(\mathrm{a}_i) + Ɗ_{G^*}(\mathrm{a}_j) - 2, for\ all\ \mathrm{a}_i\mathrm{a}_j \in E$.

3.20. Proposition

If G is an r-regular G^* edge regular neutrosophic graph, then $\sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ(\mathrm{a}_i\mathrm{a}_{i+1}) = \Big((r-1)\sum_{\mathrm{a}_i} Ɗ_{\mathfrak{A}_{\mathfrak{T}}}(\mathrm{a}_i), (r-1)\sum_{\mathrm{a}_i} Ɗ_{\mathfrak{A}_I}(\mathrm{a}_i), (r-1)\sum_{\mathrm{a}_i} Ɗ_{\mathfrak{A}_{\mathfrak{F}}}(\mathrm{a}_i)\Big)$.

Proof

By lemma we obtain $\sum Ɗ(\mathrm{a}_i\mathrm{a}_j) =$
$\left(\sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), \sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j), \sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j)\right)$

$$= \begin{pmatrix} \sum(Ɗ_{G^*}(\mathrm{a}_i) + Ɗ_{G^*}(\mathrm{a}_j) - 2)\,\mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), \\ \sum(Ɗ_{G^*}(\mathrm{a}_i) + Ɗ_{G^*}(\mathrm{a}_j) - 2)\,\mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j), \\ \sum(Ɗ_{G^*}(\mathrm{a}_i) + Ɗ_{G^*}(\mathrm{a}_j) - 2)\,\mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j) \end{pmatrix}$$

We know G^* is regular then the degree of every vertex in G^* is r , this means that $Ɗ_{G^*}(\mathrm{a}_i) = r$ and hence,

$$\sum Ɗ(\mathrm{a}_i\mathrm{a}_{i+1}) = \Big((r+r-2)\sum \mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), (r+r-2)\sum \mathfrak{B}_I(\mathrm{a}_i\mathrm{a}_j), (r+r-2)\sum \mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j)\Big)$$
$$= \big(2(r-1)\sum \mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), 2(r-1)\sum \mathfrak{B}_I(\mathrm{a}_i\mathrm{a}_j), 2(r-1)\sum \mathfrak{B}_F(\mathrm{a}_i\mathrm{a}_j)\big)$$
$$\sum_{\mathrm{a}_i\mathrm{a}_j\in E}^{n} Ɗ(\mathrm{a}_i\mathrm{a}_{i+1}) = \Big((r-1)\sum_{\mathrm{a}_i} Ɗ_{\mathfrak{A}_{\mathfrak{T}}}(\mathrm{a}_i), (r-1)\sum_{\mathrm{a}_i} Ɗ_{\mathfrak{A}_I}(\mathrm{a}_i), (r-1)\sum_{\mathrm{a}_i} Ɗ_{\mathfrak{A}_F}(\mathrm{a}_i)\Big)$$

3.21. Corollary

$\sum T_Ɗ(\mathrm{a}_i\mathrm{a}_i) = \begin{pmatrix} \sum Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), \\ \sum Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j), \\ \sum Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j) \end{pmatrix}$, where G is a regular neutrosophic graph with edges on G^*.

Proof

$\sum T_Ɗ(\mathrm{a}_i\mathrm{a}_i) = \left(\sum T_Ɗ\,\mathfrak{A}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j) + \sum T_Ɗ\,\mathfrak{A}_I(\mathrm{a}_i\mathrm{a}_j) + \sum T_Ɗ\,\mathfrak{A}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j)\right)$
$= \left(\sum T_Ɗ\,\mathfrak{A}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j) + \mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), \sum T_Ɗ\,\mathfrak{A}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j) + \mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j), \sum T_Ɗ\,\mathfrak{A}_F(\mathrm{a}_i\mathrm{a}_j) + \mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j)\right) =$
$\left(\sum Ɗ_{\mathfrak{A}_{\mathfrak{T}}}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j), \sum Ɗ_{\mathfrak{A}_I}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_I(\mathrm{a}_i\mathrm{a}_j), \sum Ɗ_{\mathfrak{A}_F}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j)\right)$
By lemma, we get

$$\sum T_Ɗ(\mathrm{a}_i\mathrm{a}_i) = \begin{pmatrix} \sum Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathfrak{T}}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_T(\mathrm{a}_i\mathrm{a}_j) \\ \sum Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_{\mathrm{I}}(\mathrm{a}_i\mathrm{a}_j) \\ \sum Ɗ_{G^*}(\mathrm{a}_i\mathrm{a}_j)\,\mathfrak{B}_F(\mathrm{a}_i\mathrm{a}_j) + \sum \mathfrak{B}_{\mathfrak{F}}(\mathrm{a}_i\mathrm{a}_j) \end{pmatrix}$$

3.22. Definition

When a neutrosophic graph G is strongly regular, it means:

1. G is a regular neutrosophic graph (r_1, r_2, r_3)
2. $\mathrm{a}_i, \mathrm{a}_j$ of is the number of the member values of the general neighbourhood vertices of any pair of adjacent and non-adjacent vertices. G has the same weight and is denoted by the

symbols $\alpha = (\alpha_1, \alpha_2), \beta = (\beta_1, \beta_2)$. $SN_G = (r, \alpha, \beta)$ represents a strongly neutrosophic graph G.

3.23. Theorem

If G is a complete neutrosophic graph with constant functions $(\mathfrak{A}_{\mathfrak{T}}, \mathfrak{A}_{\mathrm{I}}, \mathfrak{A}_{\mathfrak{F}})$ and $(\mathfrak{B}_{\mathfrak{T}}, \mathfrak{B}_{\mathrm{I}}, \mathfrak{B}_{\mathfrak{F}})$ then G is a highly normal neutrosophic graph.

Proof

Since that $(\mathfrak{A}_{\mathfrak{T}}, \mathfrak{A}_{\mathrm{I}}, \mathfrak{A}_{\mathfrak{F}})$ and $(\mathfrak{B}_{\mathfrak{T}}, \mathfrak{B}_{\mathrm{I}}, \mathfrak{B}_{\mathfrak{F}})$ are constant function, then
$\mathfrak{A}_{\mathfrak{T}}(a_i) = k,\ \ A_I(a_i) = s, \mathfrak{A}_F(a_i) = t$ and $\mathfrak{A}_{\mathfrak{T}}(a_i a_j) = d_1,\ \ \mathfrak{A}_{\mathrm{I}}(a_i a_j) = d_2, \mathfrak{A}_{\mathfrak{F}}(a_i a_j) = d_3$.
Such that k, s, t, d_1, d_2, d_3 are constant, and G is complete, then

$$\mathcal{D}_{a_i a_j} = \big((n-1)d_1, (n-1)d_2, (n-1)d_3\big),$$

Therefore G is

$$\sum \mathfrak{A}_{\mathfrak{T}}(a_i a_j), \sum \mathfrak{A}_{\mathrm{I}}(a_i a_j), \sum \mathfrak{A}_{\mathfrak{F}}(a_i a_j)) = \mathcal{D}_{\mathfrak{A}_{\mathfrak{T}}}(a_i a_j), \mathcal{D}_{\mathfrak{A}_{\mathrm{I}}}(a_i a_j), \mathcal{D}_{\mathfrak{A}_{\mathfrak{F}}}(a_i a_j)$$
$$= \big((n-1)d_1, (n-1)d_2, (n-1)d_3\big),$$

On the other hand, in a regular neutrosophic graph with n vertices, the sum of $\mathfrak{T}$, I,F of common neighbourhood vertices of any pair of adjacent vertices $\alpha = (n-2)k, (n-2)s, (n-2)t$ is equal, and the sum of $\mathfrak{T}$, I,F values common neighbourhood vertices of any pair of non-adjacent vertices $\beta = 0$ is equal.

3.24. Theorem

G^C is a (r_1, r_2, r_3) regular if G is a strongly regular neutrosophic graph that is strong.

Proof

We know G is strong, then

$$\mathfrak{B}^C{}_{\mathfrak{T}}(a_i a_j) = \begin{cases} 0 & a_i a_j \in \mathfrak{B} \\ \min\{\mathfrak{A}_{\mathfrak{T}}(a_i), \mathfrak{A}_{\mathfrak{T}}(a_j)\} & a_i a_j \notin \mathfrak{B} \end{cases}$$
$$\mathfrak{B}^C{}_I(a_i a_j) = \begin{cases} 0 & a_i a_j \in \mathfrak{B} \\ \min\{\mathfrak{A}_{\mathrm{I}}(a_i), \mathfrak{A}_{\mathrm{I}}(a_j)\} & a_i a_j \notin \mathfrak{B} \end{cases}$$
$$\mathfrak{B}^C{}_F(a_i a_j) = \begin{cases} 0 & a_i a_j \in \mathfrak{B} \\ \min\{\mathfrak{A}_{\mathfrak{F}}(a_i), \mathfrak{A}_{\mathfrak{F}}(a_j)\} & a_i a_j \notin \mathfrak{B} \end{cases}$$

Also,

$$\mathcal{D}^C{}_G(a_i) = \big(\mathfrak{T}\mathcal{D}^C{}_G(a_i), \mathrm{I}\mathcal{D}^C{}_G(a_i), \mathfrak{F}\mathcal{D}^C{}_G(a_i)\big),$$

Such that,

$$\mathfrak{T}\mathcal{D}^C{}_G(a_i) = \sum_{a_i \neq a_j} \mathfrak{B}^C{}_{\mathfrak{T}}(a_i a_j)$$
$$= \textstyle\sum_{a_i \neq a_j} min\left(\mathfrak{A}^C{}_{\mathfrak{T}}(a_i), \mathfrak{A}^C{}_{\mathfrak{T}}(a_j)\right) = r_1$$
$$\mathrm{I}\mathcal{D}^C{}_G(a_i) = \sum_{a_i \neq a_j} \mathfrak{B}^C{}_{\mathrm{I}}(a_i a_j)$$
$$= \textstyle\sum_{a_i \neq a_j} min\left(\mathfrak{A}^C{}_{\mathrm{I}}(a_i), \mathfrak{A}^C{}_{\mathrm{I}}(a_j)\right) = r_2$$
$$\mathfrak{F}\mathcal{D}^C{}_G(a_i) = \sum_{a_i \neq a_j} \mathfrak{B}^C{}_{\mathfrak{F}}(a_i a_j)$$
$$= \textstyle\sum_{a_i \neq a_j} min\left(\mathfrak{A}^C{}_{\mathfrak{F}}(a_i), \mathfrak{A}^C{}_{\mathfrak{F}}(a_j)\right) = r_3$$

For all $a_i \in V$. Thus $\mathcal{D}(a_i a_j) = (r_1,\ r_2,\ r_3)$. Hence G^C is (r_1, r_2, r_3) regular neutrosophic graph.

Application

The models of graph are used in wide application in much area of computer science, mathematical models of social sciences. These graph models need to incorporate more structure than simply the adjacency between vertices. In the discussion of set behaviour, it is observed that certain people can influence thinking of others. Each element of a set is represented by a node. There is a directed path from node a to node b when the member represented by node a influence the node represented by node b.

In any social set all the nodes can never be members of the group always. Any node can be removed from the set at any time if his/her activity is against the set. Each node of the set is represented by a vertex and every vertex has two values; the first value represents the power of the node in the set which means how much it possess to control the set, the second value represents the power of the node in the set when it became removed itself from the set.

Each path has also two values such that the first component represents the influence by the first node over the second node when the first node is element of the set. The second component represents the influence by the first node over the second node when the first node is non-member of the set. Any different neutrosophic graph needs large data for training to be able to help in decision making technology and science. The new style which is generalized in this research is based on the pattern of unique cases that can help us to make a better choice in the contrast to the established solutions of neutrosophic graph.

Conclusion

The main contribution of this manuscript is to introduce the idea of regularity in neutrosophic graph theory. In it paper, we have described the idea over the on regular neutrosophic graph. Some unique kinds on neutrosophic graphs certain as the regular, regular strong, r-edge regular neutrosophic graph, strongly edge regular, neutrosophic graph and absolute degree of vertex, have been introduced here. We bear additionally provided some sufficient standards for r-edge regular neutrosophic graph and strongly edge regular.

In the future, we pleasure focal point about the education on neutrosophic intersection graphs, neutrosophic interval graphs, neutrosophic hyper graphs, or therefore on. The notion over the neutrosophic graph execute stay ancient of countless areas regarding expert systems, image processing, computer networks, and communal systems.

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