NSS

Neutrosophic Sets and Systems, Vol. 53, 2023

University of New Mexico

Fundamentals of Picture Fuzzy Hypersoft Set with Application

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Abstract. Theory of picture fuzzy soft set and generalized picture fuzzy soft sets(GPFSS) extended to picture fuzzy hypersoft sets (PFHSS) and generalized picture fuzzy hypersoft set (GPFHSS) respectively handle the uncertainties and multi-attribute values in the material during evaluation. The main focus of this research work is to initiate and learn new operations, along with properties and examples of PFHSS and GPFHSS. Several basic operations PFHSS are defined and also prove De Morgans laws for PFHSS. Furthermore, we construct an algorithm using GPFHSS and a new expectation score function for the positive value of the score function that is useful for ranking different MADM problems. We conclude from this study the proposed outlook used to manipulate the uncertainties and multi-attribute values decision-making problems.

Keywords: SS, HS, PFS, PFSS, GPFSS, PFHSS, GPFHSS.

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1. **Introduction**

Many researchers are intrigued by the Molodtsov [1] softset ($\mathbf{S}_s$) for specific applications in data analysis, cryptography, and distributed storage. Later, the work was expanded upon and some of its foundational ideas and set-theoretic operations were examined by Maji et al. [4] and Zou et al [5]. Picture fuzzy set was proposed by Couge [6]. Positive, neutral, and negative degree are the elements of PFS. Later Yang et al. [7] combine picture fuzzy set and soft set and introduce the new concept of picture fuzzy soft set. In 2018, the theory of the HS set was introduced by Smarandache [16]. It is an extension of a soft set. The basic operations of a HS such as HS containments, Zero HS, aggregation operators along with HS set relation, sub relation, complement relation, function, matrices, and operations on HS matrices discussed by Saeed et al. [17]. In 2020 Raman et al. [23] offers the concept of a hybrid HS set structure of FS, IFS and Neutrosophy sets. The concept of convexity and concavity on a HS set proposed by Rahman et al. [24] in 2020.

1.1. *Literature Review*

In 1965 [2], Zadeh's fuzzy set theory brought about a significant generalisation in mathematics. The membership function aids in the invention of the FS structure. By including a non-membership function, Atanassov [3] extended a fuzzy set structure to an intuitionistic fuzzy set in 1986. IFS lessens the challenges associated with dealing with fuzzy, uncertain, and incomplete information. A soft set theory for solving problems involving uncertainty and decision-making was developed by Molodtsov [1]. By fusing the ideas of soft sets and fuzzy sets, Maji et al. [4] produced fuzzy soft Sets. PFS was developed by Coung et al. [6] to deal with inconsistencies in real-world data. Favorable , neutraland negative degreemake up PFS.Voting is a PFS example, as is the process of conducting elections. To deal with ambiguity, Smarandache [16] developed a novel method. He made the soft to HS more general by breaking the function down into several decision functions. In 2019 Jaber et al. presented an algorithm with the help of extended intersection of GPFS and PFDWA for solving MADM problems.

NO	Structure	Authors	Year	Properties
01	Fuzzy Set	Zadeh [2]	1965	Each individual in universal set is assign to membership value between [0,1].
02	Intuitionistic Fuzzy Set	Atanassov [3]	1983	It describes the membership degree and non-membership degree of an element to a set.
03	Soft set	Molodtsov [1]	1999	It deals with uncertainy in parametric manner.
04	Fuzzy Soft set	Maji et al [4]	2001	Fuzzy values are assign to each power set of universal set.
05	Picture Fuzzy Set	Cuong [?]	2013	Handling issue of inconsistent information.
06	Picture Fuzzy Soft Set	Yang [28]	2015	combination of PFS and SS.
07	Hypersoft sets	Smarandache [16]	2018	Extension of SS.
08	Fuzzy Hypersoft Set	Yolcu et al	2021	Each element in power set have a fuzzy membership degree.
09	Intuitionistic Fuzzy Hypersoft Set	Yolcu et al [36]	2021	Combination of intuitionistic Fuzzy set and Hypersoft Set.
10	Generalized Picture Fuzzy Soft set	Jaber et al. [25]	2019	Hybrid modal of PFSS and PFS in which extra information is given in form of PFS in output for accuracy of results in decision making.

1.2. *Motivation*

The concept of IFS invented by Atanassov [40] has membership and nonmembership degrees. IFS was not playing a role in handling inconsistency-like voting problems. Overcomes such types of difficulties Cuong [[6], [**?**]] defined the notion of PFS and basic operations, opened a new area of research in decision-making problems. An important hybrid modal that generalized SS to PFSS discussed in [7] obtained effective outcomes in DM. In [29] generalized picture distance measure is used to investigate hidden knowledge from a mass of data sets. In 2017 Peng et al. [13] proposed an algorithm using distance measure between PFS.
In 2018 HSS defined by Smarandache [16] which is a generalization of SS by transforming the mapping into a multi-attribute mapping. Saeed et al. [17] gave an idea of fundamentals of HS like union, intersection, containment, null, and compliment. The concept of HS points introduced by Abbas et al. [18]. Yolcu et al. extend the idea of HS to FHS and IFHS. They also discussed the role of FHS and IFHS in decision-making problems. With the help of IFHS, an algorithm developed by Zulqernain et al. for the solution of MADM problems [22]. Rehman et al. [23] proposed the idea of HS with the complex fuzzy set also introduced the theory of concave and convex HS [24]. The main motivation of using Hypersoft Set (HSS) is that when the attributes are more than one and further bisected, the circumstance of a soft set cannot handle such types of cases. So, there is a worth requirement to define a new approach to solve these. Decision-making methods help experts to choose a suitable alternative by analyzing the effectiveness of the alternatives. Having motivation from the work in [25], we extend the existing theory of PFSS to PFHSS to make it adequate for multi-attribute valued function. All the new proposed operations and properties are equipped with illustrated examples.
In section 2 our center of attention are some basic definitions which are useful in this paper. Section 3, the concept of a PFHSS with its properties is presented. In Section 4 and 5, we present definition of GPFHSS and operation of GPFHSS understand by an example. In Section 6 and 7, an algorithm is examined and ranking the companies. Section 9 wind up the paper.

2. **Preliminaries**

In this section, we define basic definitions of IFS, PFS, SS, HS, PFSS, Score function and PFDWA.

In 1986 Atanassov [40] include non membership function in fuzzy set obtained IFS. It overcomes defects of fuzzy sets.

Definition 2.1. [40]
An IFS X on universe of discourse $X = \{x^{\varsigma}_{\kappa 1}, x^{\varsigma}_{\kappa 2}, ..., x^{\varsigma}_{\kappa n}\}$ is defined as :

$$\tilde{L}^{\varsigma}_{\kappa} = \{\langle \ddot{\mu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}), \ddot{\nu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i})\rangle | x^{\varsigma}_{\kappa i} \in X\}$$

where $\ddot{\mu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}) : X \longrightarrow [0,1]$ denotes membership degree of $x^{\varsigma}_{\kappa i}$ in $\tilde{L}^{\varsigma}_{\kappa}$ $\ddot{\nu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}) : X \longrightarrow [0,1]$ is non membership degree of $x^{\varsigma}_{\kappa i}$ in $\tilde{L}^{\varsigma}_{\kappa}$ and $0 \leq \ddot{\mu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}) + \ddot{\nu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}) \leq 1$, $\forall x^{\varsigma}_{\kappa i} \in X$ $\pi_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}) = 1 - \ddot{\mu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}) - \ddot{\nu}_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i})$ represent hesitancy degree of $x^{\varsigma}_{\kappa i}$ in $\tilde{L}^{\varsigma}_{\kappa}$, $\forall x^{\varsigma}_{\kappa i} \in X$, $0 \leq \pi_{\tilde{L}^{\varsigma}_{\kappa}}(x^{\varsigma}_{\kappa i}) \leq 1$

In 2013 coung [42] introduced PFS to solve inconsistent information in real life. The procedure of voting is a good example to understand the concept of PFS.

Definition 2.2. [?]

A PFS on universe of discourse $X = \{x^{\varsigma}_{\kappa 1}, x^{\varsigma}_{\kappa 2}, ..., x^{\varsigma}_{\kappa n}\}$ is defined as:

$$N^{\varsigma}_{\kappa 1} = \{\langle \ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}), \ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}), \ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i})\rangle | x^{\varsigma}_{\kappa i} \in X\}$$

where $\ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) : X \longrightarrow [0,1]$, $\ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) : X \longrightarrow [0,1]$ and $\ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) : X \longrightarrow [0,1]$ represent degree of membership, neutral and non membership function $x^{\varsigma}_{\kappa i}$ in $N^{\varsigma}_{\kappa 1}$ respectively. Also $0 \leq \ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) + \ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) + \ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) \leq 1$, $\forall x^{\varsigma}_{\kappa i} \in X$.
$\rho_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) = 1 - \ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) - \ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i}) - \ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(x^{\varsigma}_{\kappa i})$ represent refusal membership degree of $x^{\varsigma}_{\kappa i}$ in $N^{\varsigma}_{\kappa 1}$, $\forall x^{\varsigma}_{\kappa i} \in X$ The set of all picture fuzzy subsets on universe of discourse $N^{\varsigma}_{\kappa 1}$ is denoted by PFSs(X).

Some basic operations of PFS is discussed as follows

Definition 2.3. [?]

The operations between two PFS $N^{\varsigma}_{\kappa 1} = \{\langle \ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(g), \ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(g), \ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(g)\rangle | g \in X\}$ and $N^{\varsigma}_{\kappa 2} = \{\langle \ddot{\mu}_{N^{\varsigma}_{\kappa 2}}(g), \ddot{\eta}_{N^{\varsigma}_{\kappa 2}}(g), \ddot{\nu}_{N^{\varsigma}_{\kappa 2}}(g)\rangle | g \in X\}$ given as follows:

(i) $N^{\varsigma}_{\kappa 1} \subseteq N^{\varsigma}_{\kappa 2}$ iff $\ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(g) \leq \ddot{\mu}_{N^{\varsigma}_{\kappa 2}}(g)$, $\ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(g) \leq \ddot{\eta}_{N^{\varsigma}_{\kappa 2}}(g)$ and $\ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(g) \geq \ddot{\nu}_{N^{\varsigma}_{\kappa 2}}(g)$
$N^{\varsigma}_{\kappa 1} = N^{\varsigma}_{\kappa 2}$ iff $N^{\varsigma}_{\kappa 1} \subseteq N^{\varsigma}_{\kappa 2}$ and $N^{\varsigma}_{\kappa 2} \subseteq N^{\varsigma}_{\kappa 1}$

(ii) $N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2} = \{(g, (\ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(g) \vee \ddot{\mu}_{N^{\varsigma}_{\kappa 2}}(g))), (g, (\ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(g) \wedge \ddot{\eta}_{N^{\varsigma}_{\kappa 2}}(g))), (g, (\ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(g) \wedge \ddot{\nu}_{N^{\varsigma}_{\kappa 2}}(g)))\}$

(iii) $N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} = \{(g, (\ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(g) \wedge \ddot{\mu}_{N^{\varsigma}_{\kappa 2}}(g))), (g, (\ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(g) \wedge \ddot{\eta}_{N^{\varsigma}_{\kappa 2}}(g))), (g, (\ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(g) \vee \ddot{\nu}_{N^{\varsigma}_{\kappa 2}}(g)))\}$

(iv) let $N^{\varsigma}_{\kappa 1} = \{\langle \ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(g), \ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(g), \ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(g)\rangle | g \in X\}$ then $N^{\varsigma c}_{\kappa 1} = \{\langle \ddot{\nu}_{N^{\varsigma}_{\kappa 1}}(g), \ddot{\eta}_{N^{\varsigma}_{\kappa 1}}(g), \ddot{\mu}_{N^{\varsigma}_{\kappa 1}}(g)\rangle | g \in X\}$

New scientific instrument SS, introduced by Molodtsov [1] in which parametrization helps to manage uncertainties.

Definition 2.4. [1]

A mapping $\mathcal{F} : \mathcal{A} \longrightarrow P(\mathcal{U})$

$(\mathcal{F}, \mathcal{A})$ is called a soft set over $\mathcal{U}$, where $\mathcal{A}$ is set of parameters.

In 2015 Yang et al [28] proposed PFSS which is combination of PFS and SS.

Definition 2.5. [28]

Let $\mathcal{E}$ is parametric set. Consider a function $\mathcal{F} : \mathcal{A} \longrightarrow PF(\mathcal{U})$, where $\mathcal{A} \subseteq \mathcal{E}$ and PF($\mathcal{U}$) is power set of PFS over $\mathcal{U}$ then pair $(\mathcal{F}, \mathcal{A})$ is representation of PFSS. .

In 2018 HSS defined by Smarandache [16] which is generalization of SS by transforming the mapping into a multi-attribute mapping.

Definition 2.6. [16]
Suppose n distinct attributes are $b_1, b_2, ..., b_n$, for $b \geq 1$, then for each attributes $\mathcal{Q}^{\varsigma}_{\kappa 1}, \mathcal{Q}^{\varsigma}_{\kappa 2}, ..., \mathcal{Q}^{\varsigma}_{\kappa n}$, with $\mathcal{Q}^{\varsigma}_{\kappa r} \bigcap \mathcal{Q}^{\varsigma}_{\kappa s} = \phi$, $i \neq j$, and r,s $\in \{1, 2, ..., n\}$ are corresponding attributes. The pair $(\ddot{\mathcal{H}}, \mathcal{Q}^{\varsigma}_{\kappa 1} \times \mathcal{Q}^{\varsigma}_{\kappa 2} \times ..... \times \mathcal{Q}^{\varsigma}_{\kappa n})$,
where $\ddot{\mathcal{H}} : \mathcal{Q}^{\varsigma}_{\kappa 1} \times \mathcal{Q}^{\varsigma}_{\kappa 2} \times ..... \times \mathcal{Q}^{\varsigma}_{\kappa n} \rightarrow P(\mathcal{U})$ represent Hypersoft Set over $\mathcal{U}$.

Chen and Tan [41] proposed an idea of score function which plays an important role to handle multicriteria fuzzy decision-making problems.

Definition 2.7. [41]
Let $\ddot{\mathcal{H}}(x^{\varsigma}_{\kappa i}) = \langle \mu(x^{\varsigma}_{\kappa i}), \eta(x^{\varsigma}_{\kappa i}), \nu(x^{\varsigma}_{\kappa i}) \rangle$ be PFSV. Υ and Λ denotes the score and accuracy functions respectively.

$$\Upsilon = \mu(x^{\varsigma}_{\kappa i}) - \nu(x^{\varsigma}_{\kappa i}) \quad \Upsilon \in [-1, 1]$$

$$\Lambda = \mu(x^{\varsigma}_{\kappa i}) + \eta(x^{\varsigma}_{\kappa i}) + \nu(x^{\varsigma}_{\kappa i}) \quad \Lambda \in [0, 1]$$

Jana et al [27] introduced Dombi aggregation operators in PFS sense for MADM problems.

Definition 2.8. [27]
let $\ddot{\mathcal{H}}(x^{\varsigma}_{\kappa i}) = \langle \mu(x^{\varsigma}_{\kappa i}), \eta(x^{\varsigma}_{\kappa i}), \nu(x^{\varsigma}_{\kappa i}) \rangle$ be PFSV then the function $P^n \rightarrow P$ is called PFDWA such that

$$PFDWA_{\mathcal{W}}(x^{\varsigma}_{\kappa 1}, x^{\varsigma}_{\kappa 2}, ...x^{\varsigma}_{\kappa n}) = \sum_{i=1}^{i=n} \mathcal{W}_i x^{\varsigma}_{\kappa i}$$

$$= (1 - \frac{1}{1 + \{\sum_{i=1}^{i=n} \mathcal{W}_i (\frac{\mu(x^{\varsigma}_{\kappa i})}{1-\mu(x^{\varsigma}_{\kappa i})})^k\}^k}, \frac{1}{1 + \{\sum_{i=1}^{i=n} \mathcal{W}_i (\frac{1-\eta(x^{\varsigma}_{\kappa i})}{\eta(x^{\varsigma}_{\kappa i})})^k\}^k}, \frac{1}{1 + \{\sum_{i=1}^{i=n} \mathcal{W}_i (\frac{1-\nu(x^{\varsigma}_{\kappa i})}{\nu(x^{\varsigma}_{\kappa i})})^k\}^k})$$

For each $\mathcal{W}_i \geq 0$ and $\sum_{i=1}^{i=n} \mathcal{W}_i = 1$

3. **Picture Fuzzy Hypersoft Sets**

In [28] hybrid model of PFS and SS is defined. In this section we introduce PFHSS is an extension of PFSS which helps for paying crucial role in decision making for multi attribute characteristic.

Definition 3.1. Picture Fuzzy Hypersoft Sets
Suppose m disjoint attribute-valued sets are $p^a_{b\,1}, p^a_{b\,2}, p^a_{b\,3}, ..., p^a_{b\,m}$ then their corresponding m distinct attributes are $P^a_{b\,1}, P^a_{b\,2}, P^a_{b\,3}, ..., P^a_{b\,m}$ respectively and $P^a_b = P^a_{b\,1} \times P^a_{b\,2} \times P^a_{b\,3} \times .... \times P^a_{b\,m}$. A mapping is given by $\ddot{\mathcal{H}} : P^a_b \longrightarrow PF(\mathcal{U})$

$$\ddot{\mathcal{H}}(t^a_b) = \{\langle \mu_{\ddot{\mathcal{H}}(t^a_b)}(j^a_{b\,i}), \eta_{\ddot{\mathcal{H}}(t^a_b)}(j^a_{b\,i}), \nu_{\ddot{\mathcal{H}}(t^a_b)}(j^a_{b\,i}) \rangle | (j^a_{b\,i}) \in \mathcal{U}\} \quad for \quad any \quad t^a_b \in P^a_b$$

then pair $(\ddot{\mathcal{H}}, P_b^a)$ represent PFHSS.

Example 3.2. Consider $(\ddot{\mathcal{H}}, \mathcal{P})$ be PFHSS over $\mathcal{U}$. Let $\mathcal{U} = \{j_{b\,1}^a, j_{b\,2}^a, j_{b\,3}^a, j_{b\,4}^a\}$ be four schools any where in World. let $E = \{a_1, a_2, a_3, a_4\}$ where each a_i stands for Fee Structure , facilities , faculties and labs be the attributes values respectively, $\{A_1, A_2, A_3, A_4\}$ be attribute values against each a_i. let $A_1 = \{b_{11} =$ low, $b_{12}=$ medium , $b_{13} =$ expensive $\}$

$A_2 = \{b_{21} =$ playgrounds, $b_{22}=$ library, $b_{23} =$ cafeterias, $b_{24} =$ bookshop, $b_{25} = \{b_{21}, b_{22}, b_{23}, b_{24}\}\}$

$A_3 = \{b_{31} =$ Science and Arts teacher, $b_{32} =$ Oriental teacher, $b_{33} =$ Physical education teacher, $b_{34} = \{b_{31}, b_{32}, b_{33}\}\}$

$A_4 = \{b_{41} =$ Science labs, $b_{42} =$ Computer lab, $b_{43} = \{b_{41}, b_{42}\}\}$

then

$$\tilde{L}_\kappa^\varsigma = A_1 \times A_2 \times A_3 \times A_4$$

There are one hundred eighty outcomes but for simplicity we take only four outcomes.

$$\tilde{L}_\kappa^\varsigma = \left\{ \begin{array}{l} t_{b\,1}^a = (b_{11}, b_{21}, b_{31}, b_{41}), \quad t_{b\,2}^a = (b_{12}, b_{25}, b_{34}, b_{43}), \\ t_{b\,3}^a = (b_{12}, b_{22}, b_{31}), \quad t_{b\,4}^a = (b_{11}, b_{25}, b_{34}, b_{43}), \end{array} \right\}$$

$$(\ddot{\mathcal{H}}, \tilde{L}_\kappa^\varsigma) = \left\{ \begin{array}{l} \ddot{\mathcal{H}}(t_{b\,1}^a) = \{\langle 0.7, 0.1, 0.1\rangle / j_{b\,1}^a, \langle 0.3, 0.2, 0.4\rangle / j_{b\,2}^a, \langle 0.1, 0.5, 0.3\rangle / j_{b\,3}^a, \langle 0.4, 0.1, 0.3\rangle / j_{b\,4}^a\}, \\ \ddot{\mathcal{H}}(t_{b\,2}^a) = \{\langle 0.6, 0.1, 0.2\rangle / j_{b\,1}^a, \langle 0.4, 0.1, 0.3\rangle / j_{b\,2}^a, \langle 0.7, 0.1, 0.1\rangle / j_{b\,3}^a, \langle 0.2, 0.5, 0.2\rangle / j_{b\,4}^a, \}, \\ \ddot{\mathcal{H}}(t_{b\,3}^a) = \{\langle 0.2, 0.3, 0.5\rangle / j_{b\,1}^a, \langle 0.1, 0.1, 0.6\rangle / j_{b\,2}^a, \langle 0.2, 0.1, 0.7\rangle / j_{b\,3}^a, \langle 0.8, 0.1, 0.1\rangle / j_{b\,4}^a\}, \\ \ddot{\mathcal{H}}(t_{b\,4}^a) = \{\langle 0.3, 0.2, 0.4\rangle / j_{b\,1}^a, \langle 0.5, 0.1, 0.3\rangle / j_{b\,2}^a, \langle 0.1, 0.5, 0.3\rangle / j_{b\,3}^a, \langle 0.2, 0.1, 0.7\rangle / j_{b\,4}^a\}, \end{array} \right\}$$

The PFHSS is represented by Tab 1.

Table 1. Picture Fuzzy Hyper soft Set

$\mathcal{U}$	$t_{b\,1}^a$	$t_{b\,2}^a$	$t_{b\,3}^a$	$t_{b\,4}^a$
$j_{b\,1}^a$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.2, 0.3, 0.5\rangle$	$\langle 0.3, 0.2, 0.4\rangle$
$j_{b\,2}^a$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.1, 0.1, 0.6\rangle$	$\langle 0.5, 0.1, 0.3\rangle$
$j_{b\,3}^a$	$\langle 0.1, 0.5, 0.3\rangle$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.2, 0.1, 0.7\rangle$	$\langle 0.1, 0.5, 0.3\rangle$
$j_{b\,4}^a$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.2, 0.5, 0.2\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.2, 0.1, 0.7\rangle$

Definition 3.3.

Let $(\ddot{\mathcal{H}}_1, N_{\kappa 1}^\varsigma)$ and $(\ddot{\mathcal{H}}_2, N_{\kappa 2}^\varsigma)$ be two PFHSS then $(\ddot{\mathcal{H}}_1, N_{\kappa 1}^\varsigma) \subseteq (\ddot{\mathcal{H}}_2, N_{\kappa 2}^\varsigma)$ if $N_{\kappa 1}^\varsigma \subseteq N_{\kappa 2}^\varsigma$ and $\ddot{\mathcal{H}}_1(t_b^a) \subseteq \ddot{\mathcal{H}}_2(t_b^a)$ for all $t_b^a \in N_{\kappa 1}^\varsigma$

Example 3.4.

$$(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) = \left\{ \begin{array}{c} \ddot{\mathcal{H}}_1(t^a_{b1}) = \{\langle 0.3, 0.1, 0.3\rangle/j^a_{b1}, \langle 0.4, 0.2, 0.3\rangle/j^a_{b2}, \langle 0.1, 0.4, 0.3\rangle/j^a_{b3}, \langle 0.2, 0.1, 0.3\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_1(t^a_{b3}) = \{\langle 0.3, 0.1, 0.4\rangle/j^a_{b1}, \langle 0.5, 0.1, 0.3\rangle/j^a_{b2}, \langle 0.1, 0.3, 0.5\rangle/j^a_{b3}, \langle 0.4, 0.1, 0.3\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_1(t^a_{b4}) = \{\langle 0.2, 0.3, 0.4\rangle/j^a_{b1}, \langle 0.5, 0.1, 0.3\rangle/j^a_{b2}, \langle 0.1, 0.2, 0.6\rangle/j^a_{b3}, \langle 0.4, 0.1, 0.4\rangle/j^a_{b4}\} \end{array} \right\},$$

and

$$(\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}) = \left\{ \begin{array}{c} \ddot{\mathcal{H}}_2(t^a_{b1}) = \{\langle 0.5, 0.2, 0.1\rangle/j^a_{b1}, \langle 0.5, 0.3, 0.1\rangle/j^a_{b2}, \langle 0.3, 0.5, 0.1\rangle/j^a_{b3}, \langle 0.4, 0.3, 0.2\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_2(t^a_{b3}) = \{\langle 0.5, 0.2, 0.2\rangle/j^a_{b1}, \langle 0.6, 0.2, 0.1\rangle/j^a_{b2}, \langle 0.3, 0.4, 0.1\rangle/j^a_{b3}, \langle 0.5, 0.2, 0.2\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_2(t^a_{b4}) = \{\langle 0.4, 0.4, 0.1\rangle/j^a_{b1}, \langle 0.6, 0.2, 0.1\rangle/j^a_{b2}, \langle 0.5, 0.3, 0.2\rangle/j^a_{b3}, \langle 0.5, 0.3, 0.1\rangle/j^a_{b4}\} \end{array} \right\}.$$

be two PFHSS.

This implies that $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \subseteq (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$.

Definition 3.5.

The extended union of two PFHSS $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})$ and $(\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$ is defined as $(\ddot{\mathcal{H}}_3, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$, where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2}$ and for all $t^a_b \in N^{\varsigma}_{\kappa 3}$,

$$\ddot{\mathcal{H}}_3(t^a_b) = \left\{ \begin{array}{ll} \ddot{\mathcal{H}}_1(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \ddot{\mathcal{H}}_2(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ \ddot{\mathcal{H}}_1(t^a_b) \cup \ddot{\mathcal{H}}_2(t^a_b) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

Example 3.6. Considering example 3.4, we have

$$(\ddot{\mathcal{H}}_3, C) = \left\{ \begin{array}{c} \ddot{\mathcal{H}}_3(t^a_{b1}) = \{\langle 0.5, 0.1, 0.1\rangle/j^a_{b1}, \langle 0.5, 0.2, 0.1\rangle/j^a_{b2}, \langle 0.3, 0.4, 0.1\rangle/j^a_{b3}, \langle 0.4, 0.1, 0.2\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_3(t^a_{b3}) = \{\langle 0.5, 0.1, 0.2\rangle/j^a_{b1}, \langle 0.6, 0.1, 0.1\rangle/j^a_{b2}, \langle 0.3, 0.3, 0.1\rangle/j^a_{b3}, \langle 0.5, 0.1, 0.2\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_3(t^a_{b4}) = \{\langle 0.4, 0.3, 0.1\rangle/j^a_{b1}, \langle 0.6, 0.1, 0.1\rangle/j^a_{b2}, \langle 0.5, 0.2, 0.2\rangle/j^a_{b3}, \langle 0.5, 0.1, 0.1\rangle/j^a_{b4}\} \end{array} \right\}.$$

Definition 3.7.

The extended intersection of $(\ddot{\mathcal{H}}_1, A)$ and $(\ddot{\mathcal{H}}_2, B)$ is defined as $(\ddot{\mathcal{H}}_4, C) = (\ddot{\mathcal{H}}_1, A) \cap_e (\ddot{\mathcal{H}}_2, B)$, where $C = A \cup B$ and for all $t^a_b \in C$,

$$\ddot{\mathcal{H}}_4(t^a_b) = \left\{ \begin{array}{ll} \ddot{\mathcal{H}}_1(t^a_b), & if\ t^a_b \in A \setminus B \\ \ddot{\mathcal{H}}_2(t^a_b), & if\ t^a_b \in B \setminus A \\ \ddot{\mathcal{H}}_1(t^a_b) \cap \ddot{\mathcal{H}}_2(t^a_b) & if\ t^a_b \in A \cap B \end{array} \right\}$$

Example 3.8. In example 3.4, we get

$$(\ddot{\mathcal{H}}_4, C) = \left\{ \begin{array}{c} \ddot{\mathcal{H}}_4(t^a_{b1}) = \{\langle 0.3, 0.1, 0.3\rangle/j^a_{b1}, \langle 0.4, 0.2, 0.3\rangle/j^a_{b2}, \langle 0.1, 0.4, 0.3\rangle/j^a_{b3}, \langle 0.2, 0.1, 0.3\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_4(t^a_{b3}) = \{\langle 0.3, 0.1, 0.4\rangle/j^a_{b1}, \langle 0.5, 0.1, 0.3\rangle/j^a_{b2}, \langle 0.1, 0.3, 0.5\rangle/j^a_{b3}, \langle 0.4, 0.1, 0.3\rangle/j^a_{b4}\}, \\ \ddot{\mathcal{H}}_4(t^a_{b4}) = \{\langle 0.2, 0.3, 0.4\rangle/j^a_{b1}, \langle 0.5, 0.1, 0.3\rangle/j^a_{b2}, \langle 0.1, 0.2, 0.6\rangle/j^a_{b3}, \langle 0.4, 0.1, 0.4\rangle/j^a_{b4}\} \end{array} \right\}.$$

Definition 3.9. The restricted union of $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})$ and $(\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$ is defined as $(\ddot{\mathcal{H}}_5, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$, where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \neq \emptyset$ and for all $t^a_b \in N^{\varsigma}_{\kappa 3}$,

Example 3.10. From example 3.4, we get

$$(\ddot{\mathcal{H}}_3, C) = \left\{ \begin{array}{c} \ddot{\mathcal{H}}_3(t^a_{b\,1}) = \{\langle 0.5, 0.1, 0.1\rangle/j^a_{b\,1}, \langle 0.5, 0.2, 0.1\rangle/j^a_{b\,2}, \langle 0.3, 0.4, 0.1\rangle/j^a_{b\,3}, \langle 0.4, 0.1, 0.2\rangle/j^a_{b\,4}\}, \\ \ddot{\mathcal{H}}_3(t^a_{b\,3}) = \{\langle 0.5, 0.1, 0.2\rangle/j^a_{b\,1}, \langle 0.6, 0.1, 0.1\rangle/j^a_{b\,2}, \langle 0.3, 0.3, 0.1\rangle/j^a_{b\,3}, \langle 0.5, 0.1, 0.2\rangle/j^a_{b\,4}\}, \\ \ddot{\mathcal{H}}_3(t^a_{b\,4}) = \{\langle 0.4, 0.3, 0.1\rangle/j^a_{b\,1}, \langle 0.6, 0.1, 0.1\rangle/j^a_{b\,2}, \langle 0.5, 0.2, 0.2\rangle/j^a_{b\,3}, \langle 0.5, 0.1, 0.1\rangle/j^a_{b\,4}\} \end{array} \right\}.$$

Definition 3.11. The restricted intersection of $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})$ and $(\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$ is defined as $(\ddot{\mathcal{H}}_5, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$, where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \neq \emptyset$ and for all $t^a_b \in N^{\varsigma}_{\kappa 3}$,

Example 3.12. Example 3.4, implies that

$$(\ddot{\mathcal{H}}_4, C) = \left\{ \begin{array}{c} \ddot{\mathcal{H}}_4(t^a_{b\,1}) = \{\langle 0.3, 0.1, 0.3\rangle/j^a_{b\,1}, \langle 0.4, 0.2, 0.3\rangle/j^a_{b\,2}, \langle 0.1, 0.4, 0.3\rangle/j^a_{b\,3}, \langle 0.2, 0.1, 0.3\rangle/j^a_{b\,4}\}, \\ \ddot{\mathcal{H}}_4(t^a_{b\,3}) = \{\langle 0.3, 0.1, 0.4\rangle/j^a_{b\,1}, \langle 0.5, 0.1, 0.3\rangle/j^a_{b\,2}, \langle 0.1, 0.3, 0.5\rangle/j^a_{b\,3}, \langle 0.4, 0.1, 0.3\rangle/j^a_{b\,4}\}, \\ \ddot{\mathcal{H}}_4(t^a_{b\,4}) = \{\langle 0.2, 0.3, 0.4\rangle/j^a_{b\,1}, \langle 0.5, 0.1, 0.3\rangle/j^a_{b\,2}, \langle 0.1, 0.2, 0.6\rangle/j^a_{b\,3}, \langle 0.4, 0.1, 0.4\rangle/j^a_{b\,4}\} \end{array} \right\}.$$

Definition 3.13. If $(\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa})$ be PFHSS then

$$(\ddot{\mathcal{H}}, P)' = \{\langle \nu_{\ddot{\mathcal{H}}(t^a_b)}(j^a_{b\,i}), \eta_{\ddot{\mathcal{H}}(t^a_b)}(j^a_{b\,i}), \mu_{\ddot{\mathcal{H}}(t^a_b)}(j^a_{b\,i})\rangle | (j^a_{b\,i}) \in \mathcal{U}\} \quad for \quad any \quad t^a_b \in \tilde{L}^{\varsigma}_{\kappa}$$

Example 3.14. If

$$(\ddot{\mathcal{H}}, N^{\varsigma}_{\kappa 1}) = \left\{ \ddot{\mathcal{H}}(t^a_b) = \{\langle 0.7, 0.1, 0.1\rangle/j^a_{b\,1}, \langle 0.2, 0.5, 0.2\rangle/j^a_{b\,3}, \langle 0.6, 0.1, 0.2\rangle/j^a_{b\,6}, \langle 0.4, 0.1, 0.3\rangle/j^a_{b\,8}\}, \right\}.$$

then

$$(\ddot{\mathcal{H}}, N^{\varsigma}_{\kappa 1})' = \left\{ \ddot{\mathcal{H}}(t^a_b) = \{\langle 0.1, 0.1, 0.7\rangle/j^a_{b\,1}, \langle 0.2, 0.5, 0.2\rangle/j^a_{b\,3}, \langle 0.2, 0.1, 0.6\rangle/j^a_{b\,6}, \langle 0.3, 0.1, 0.4\rangle/j^a_{b\,8}\}, \right\}.$$

Remark 3.15.

(i) $(\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) \cup_e (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) = (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) \cup_r (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) = (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa})$

(ii) $(\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) \cap_e (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) = (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) \cap_r (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa}) = (\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa})$

Next, we check validity of the De Morgans laws in PFHSS with respect to extended, union and intersection.

Theorem 3.16. *If $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})$ and $(\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$ be two PFHSS over $\mathcal{U}$. Then*

(i) $((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))' = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$

(ii) $((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))' = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$

Proof. (i) Since
$(\ddot{\mathcal{H}}_3, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$, where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2}$

Then $(\ddot{\mathcal{H}}_3, N^{\varsigma}_{\kappa 3})' = ((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))'$

$$\ddot{\mathcal{H}}_3(t^a_b) = \left\{ \begin{array}{ll} \ddot{\mathcal{H}}_1(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \ddot{\mathcal{H}}_2(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ \ddot{\mathcal{H}}_1(t^a_b) \cup \ddot{\mathcal{H}}_2(t^a_b) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

for all $t^a_b \in N^{\varsigma}_{\kappa 3}$ then

$$(\ddot{\mathcal{H}}_3(t^a_b))' = \left\{ \begin{array}{ll} (\ddot{\mathcal{H}}_1(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ (\ddot{\mathcal{H}}_2(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ (\ddot{\mathcal{H}}_1(t^a_b) \cup \ddot{\mathcal{H}}_2(t^a_b))' & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

Since De Morgans laws hold in Picture Fuzzy Soft set

$$(\ddot{\mathcal{H}}_3(t^a_b))' = \left\{ \begin{array}{ll} (\ddot{\mathcal{H}}_1(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ (\ddot{\mathcal{H}}_2(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ (\ddot{\mathcal{H}}_1(t^a_b))' \cap (\ddot{\mathcal{H}}_2(t^a_b))' & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\} \in (\ddot{\mathcal{H}}_3, N^{\varsigma}_{\kappa 3})'$$

Suppose $(\ddot{\mathcal{H}}_a, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$, where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2}$ for all $t^a_b \in N^{\varsigma}_{\kappa 3}$

$$(\ddot{\mathcal{H}}_a(t^a_b)) = \left\{ \begin{array}{ll} (\ddot{\mathcal{H}}_1(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ (\ddot{\mathcal{H}}_2(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ (\ddot{\mathcal{H}}_1(t^a_b))' \cap (\ddot{\mathcal{H}}_2(t^a_b))' & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\} \in (\ddot{\mathcal{H}}_3, N^{\varsigma}_{\kappa 3})'$$

This Implies

$$((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))' = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$$

(ii) Since

$(\ddot{\mathcal{H}}_4, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$, where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2}$
then $((\ddot{\mathcal{H}}_4, N^{\varsigma}_{\kappa 3}))' = ((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))'$,

$$\ddot{\mathcal{H}}_4(t^a_b) = \left\{ \begin{array}{ll} \ddot{\mathcal{H}}_1(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \ddot{\mathcal{H}}_2(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ \ddot{\mathcal{H}}_1(t^a_b) \cap \ddot{\mathcal{H}}_2(t^a_b) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

for all $t^a_b \in N^{\varsigma}_{\kappa 3}$

$$(\ddot{\mathcal{H}}_4(t^a_b))' = \left\{ \begin{array}{ll} (\ddot{\mathcal{H}}_1(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ (\ddot{\mathcal{H}}_2(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ (\ddot{\mathcal{H}}_1(t^a_b) \cap \ddot{\mathcal{H}}_2(t^a_b))' & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

As we know that

$$(\ddot{\mathcal{H}}_4(t^a_b))' = \left\{ \begin{array}{ll} (\ddot{\mathcal{H}}_1(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ (\ddot{\mathcal{H}}_2(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ (\ddot{\mathcal{H}}_1(t^a_b))' \cup (\ddot{\mathcal{H}}_2(t^a_b))' & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\} \in (\ddot{\mathcal{H}}_4, N^{\varsigma}_{\kappa 3})'$$

Suppose $(\ddot{\mathcal{H}}_b, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$, where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2}$ for all $t^a_b \in N^{\varsigma}_{\kappa 3}$

$$(\ddot{\mathcal{H}}_b(t^a_b)) = \left\{ \begin{array}{ll} (\ddot{\mathcal{H}}_1(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ (\ddot{\mathcal{H}}_2(t^a_b))', & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ (\ddot{\mathcal{H}}_1(t^a_b))' \cup (\ddot{\mathcal{H}}_2(t^a_b))' & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\} \in (\ddot{\mathcal{H}}_4, N^{\varsigma}_{\kappa 3})'$$

Hence

$$((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))' = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$$

□

We want to prove De Morgans laws for restricted union and restricted intersection in PFHSS.

Theorem 3.17.

(i) $((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))' = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cap_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$

(ii) $((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))' = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cup_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$

Proof. (i) Since $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \neq \emptyset$ and
$(\ddot{\mathcal{H}}_5, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$ then
$(\ddot{\mathcal{H}}_5, N^{\varsigma}_{\kappa 3})' = ((\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}))'$
for all $t^a_b \in N^{\varsigma}_{\kappa 3}$ $\ddot{\mathcal{H}}_5(t^a_b) = \ddot{\mathcal{H}}_1(t^a_b) \cup_r \ddot{\mathcal{H}}_2(t^a_b)$ since De morgan law hold in PFSS
Therefore $(\ddot{\mathcal{H}}_5(t^a_b))' = (\ddot{\mathcal{H}}_1(t^a_b))' \cap_r (\ddot{\mathcal{H}}_2(t^a_b))' \in (\ddot{\mathcal{H}}_5, N^{\varsigma}_{\kappa 3})'$
Suppose $(\ddot{\mathcal{H}}_c, N^{\varsigma}_{\kappa 3}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})' \cap_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})'$
$\ddot{\mathcal{H}}_c(t^a_b) = (\ddot{\mathcal{H}}_1(t^a_b))' \cap_r (\ddot{\mathcal{H}}_2(t^a_b))'$ for all $t^a_b \in N^{\varsigma}_{\kappa 3}$ where $N^{\varsigma}_{\kappa 3} = N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2}$
hence

$$((\ddot{\mathcal{H}}_1, A) \cup_r (\ddot{\mathcal{H}}_2, B))' = (\ddot{\mathcal{H}}_1, A)' \cap_r (\ddot{\mathcal{H}}_2, B)'$$

(ii) Straightforward □

4. Generalized Picture Fuzzy Hypersoft Sets

In this section, we describe an extension of PFHSS. It is a hybrid modal of PHSS and PFS known as generalized picture fuzzy hypersoft set (GPFHSS). GPFHSS has a character in decision-making exertion, when taking an important decision according to the given attributes it will minimize evaluation, and output will be in the form of PFS.

Definition 4.1. Generalized Picture Fuzzy Hypersoft Sets
Suppose $N^{\varsigma}_{\kappa 1}, N^{\varsigma}_{\kappa 2}, N^{\varsigma}_{\kappa 3}, ..., N^{\varsigma}_{\kappa m}$ be disjoint attribute-valued sets corresponding to m distinct attributes $p^a_{b1}, p^a_{b2}, p^a_{b3}, ..., p^a_{bm}$ respectively and $P = N^{\varsigma}_{\kappa 1} \times N^{\varsigma}_{\kappa 2} \times N^{\varsigma}_{\kappa 3} \times .... \times N^{\varsigma}_{\kappa m}$. A triplet $(\ddot{\mathcal{H}}, P, \Phi)$ is called a generalized picture fuzzy hypersoft set (GPFHSS), where Φ is a mapping given by $\Phi : P \longrightarrow \pounds(P)$. where $\pounds(P)$ is the set of all picture fuzzy hypersoft subsets of P and is called parametric picture fuzzy hypersoftset of GPFHSS.

Example 4.2. Considering example 3.4, $(\ddot{\mathcal{H}}, \tilde{L}^{\varsigma}_{\kappa})$ is PHSS and $\Phi = \{\langle 0.2, 0.1, 0.1\rangle/t^a_{b\,1}, \langle 0.5, 0.2, 0.1\rangle/t^a_{b\,2}, \langle 0.7, 0.2, 0.0\rangle/t^a_{b\,3}, \langle 0.6, 0.2, 0.1\rangle/t^a_{b\,4}\}$
where Φ is an extra viewpoint of a arbitrator on the general standard of work done to check out alternatives on the basis of given multi-attributes.

Table 2. GPFHSS

$\mathcal{U}$	$t^a_{b\,1}$	$t^a_{b\,2}$	$t^a_{b\,3}$	$t^a_{b\,4}$
$j^a_{b\,1}$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.2, 0.3, 0.5\rangle$	$\langle 0.6, 0.2, 0.1\rangle$
$j^a_{b\,2}$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.3, 0.2, 0.4\rangle$
$j^a_{b\,3}$	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.2, 0.1, 0.7\rangle$	$\langle 0.6, 0.2, 0.1\rangle$
$j^a_{b\,4}$	$\langle 0.6, 0.1, 0.2$	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.4, 0.1, 0.1\rangle$	$\langle 0.5, 0.1, 0.2\rangle$
Φ	$\langle 0.2, 0.1, 0.1\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.7, 0.2, 0.0\rangle$	$\langle 0.6, 0.2, 0.1\rangle$

Definition 4.3. let $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ and $\nabla_2 = (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ be two GPFHSS over $\mathcal{U}$. Then $\nabla_1 \subseteq \nabla_2$ if

(i) $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \subseteq (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$

(ii) $\langle \mu_{\Phi_1(t^a_b)}(j^a_{b\,i}) \leq \langle \mu_{\Phi_2(t^a_b)}(j^a_{b\,i})$, $\langle \eta_{\Phi_1(t^a_b)}(j^a_{b\,i}) \leq \langle \eta_{\Phi_2(t^a_b)}(j^a_{b\,i})$, $\langle \nu_{\Phi_1(t^a_b)}(j^a_{b\,i}) \geq \langle\langle \nu_{\Phi_2(t^a_b)}(j^a_{b\,i})$

Definition 4.4. Two GPFHSS $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ and $\nabla_2 = (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ over $\mathcal{U}$ are said to be equal if $\ddot{\mathcal{H}}_1 = \ddot{\mathcal{H}}_2$, $N^{\varsigma}_{\kappa 1} = N^{\varsigma}_{\kappa 2}$ and $\Phi_1 = \Phi_2$.

Definition 4.5. let $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ be GPFHSS then the complement of $\nabla_1 =$ is denoted as $(\nabla_1)^c$ and defined as
$(\nabla_1)^c = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)^c = (\ddot{\mathcal{H}}_5, N^{\varsigma}_{\kappa 5}, \Phi_3)$ where $(\ddot{\mathcal{H}}_5, N^{\varsigma}_{\kappa 5})$ is compliment of $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})$ and Φ_3 is compliment of Φ_1.

5. Basic Operation of Generalized Picture Fuzzy Hypersoft Sets

In this section, we introduce some basic operations for GPFHSS.

Definition 5.1. The extended union of two GPFHSS $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ and $\nabla_2 = (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ over $\mathcal{U}$ is denoted by
$\nabla_3 = (\ddot{\mathcal{H}}_6, N^{\varsigma}_{\kappa 6}, \Phi_4) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1) \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ and defined as

(i) $(\ddot{\mathcal{H}}_6, N^{\varsigma}_{\kappa 6}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$ where $N^{\varsigma}_{\kappa 6} = N^{\varsigma}_{\kappa 1} \cup N^{\varsigma}_{\kappa 2}$

(ii)

$$\mu_{\Phi_4}(t^a_b) = \left\{ \begin{array}{ll} \mu_{\Phi_1}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \mu_{\Phi_2}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ max(\mu_{\Phi_1}(t^a_b)\mu_{\Phi_2}(t^a_b)) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

(iii)

$$\eta_{\Phi_4}(t^a_b) = \left\{ \begin{array}{ll} \eta_{\Phi_1}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \eta_{\Phi_2}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ min(\eta_{\Phi_1}(t^a_b)\eta_{\Phi_2}(t^a_b)) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

(iv)

$$\nu_{\Phi_4}(t^a_b) = \left\{ \begin{array}{ll} \nu_{\Phi_1}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \nu_{\Phi_2}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ min(\nu_{\Phi_1}(t^a_b)\nu_{\Phi_2}(t^a_b)) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

Definition 5.2. The extended intersection of two GPFHSS $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ and $\nabla_2 = (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ over $\mathcal{U}$ is denoted by
$\nabla_3 = (\ddot{\mathcal{H}}_6, N^{\varsigma}_{\kappa 6}, \Phi_4) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1) \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ and defined as

(i) $(\ddot{\mathcal{H}}_6, N^{\varsigma}_{\kappa 6}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_e (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2})$ where $N^{\varsigma}_{\kappa 6} = N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2}$

(ii)

$$\mu_{\Phi_4}(t^a_b) = \left\{ \begin{array}{ll} \mu_{\Phi_1}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \mu_{\Phi_2}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ min(\mu_{\Phi_1}(t^a_b)\mu_{\Phi_2}(t^a_b)) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

(iii)

$$\eta_{\Phi_4}(t^a_b) = \left\{ \begin{array}{ll} \eta_{\Phi_1}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \eta_{\Phi_2}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ min(\eta_{\Phi_1}(t^a_b)\eta_{\Phi_2}(t^a_b)) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

(iv)

$$\nu_{\Phi_4}(t^a_b) = \left\{ \begin{array}{ll} \nu_{\Phi_1}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \setminus N^{\varsigma}_{\kappa 2} \\ \nu_{\Phi_2}(t^a_b), & if\ t^a_b \in N^{\varsigma}_{\kappa 2} \setminus N^{\varsigma}_{\kappa 1} \\ max(\nu_{\Phi_1}(t^a_b)\nu_{\Phi_2}(t^a_b)) & if\ t^a_b \in N^{\varsigma}_{\kappa 1} \cap N^{\varsigma}_{\kappa 2} \end{array} \right\}$$

Definition 5.3. The restricted union of Two GPFHSS $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ and $\nabla_2 = (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ over $\mathcal{U}$ is defined as
$\nabla_5 = (\ddot{\mathcal{H}}_8, N^{\varsigma}_{\kappa 8}, \Phi_6) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1) \cup_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ such that

(i) $(\ddot{\mathcal{H}}_8, N^{\varsigma}_{\kappa 8}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cup_r (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})$

(ii) $\mu_{\Phi_8}(t^a_b) = max(\mu_{\Phi_1}(t^a_b), \mu_{\Phi_2}(t^a_b))$, $\eta_{\Phi_8}(t^a_b) = min(\eta_{\Phi_1}(t^a_b), \eta_{\Phi_2}(t^a_b))$ and $\nu_{\Phi_8}(t^a_b) = min(\nu_{\Phi_1}(t^a_b), \nu_{\Phi_2}(t^a_b))$

Definition 5.4. The restricted intersection of two GPFHSS $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ and $\nabla_2 = (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ over $\mathcal{U}$ is defined as
$\nabla_6 = (\ddot{\mathcal{H}}_9, N^{\varsigma}_{\kappa 9}, \Phi_7) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1) \cap_r (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$ such that

(i) $(\ddot{\mathcal{H}}_9, N^{\varsigma}_{\kappa 9}) = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}) \cap_r (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1})$

(ii) $\mu_{\Phi_9}(t^a_b) = min(\mu_{\Phi_1}(t^a_b), \mu_{\Phi_2}(t^a_b))$, $\eta_{\Phi_9}(t^a_b) = min(\eta_{\Phi_1}(t^a_b), \eta_{\Phi_2}(t^a_b))$ and $\nu_{\Phi_9}(t^a_b) = max(\nu_{\Phi_1}(t^a_b), \nu_{\Phi_2}(t^a_b))$

Example 5.5. Suppose $\mathcal{U} = \{c^{\varsigma}_{\kappa 1}, c^{\varsigma}_{\kappa 2}, c^{\varsigma}_{\kappa 3}, c^{\varsigma}_{\kappa 4}\}$ be four hospital. . Let $E = \{e_1, e_2, e_3, e_4\}$ stand for organ transplant services, medical and surgical specialties and support services whose corresponding attribute values are $\{A_1, A_2, A_3\}$ respectively. let $A_1 = \{b_{11}=$ liver transplant,$b_{12} =$ kidney transplant,$b_{13}=$ corneal transplant,$b_{14} = \{b_{11}, b_{12}, b_{13}\}\}$
$A_2 = \{b_{21} =$ medical and surgical clinics, $b_{22} =$ emergency services, $b_{23} =$ diagnostic services, $b_{24} = \{b_{21}, b_{22}, b_{23}\}\}$
$A_3 = \{b_{31} =$ Pharmecy $\}$
then

$$\tilde{L}^{\varsigma}_{\kappa} = A_1 \times A_2 \times A_3$$

$$\tilde{L}^{\varsigma}_{\kappa} = \left\{ \begin{array}{ll} t^a_{b1} = (b_{11}, b_{21}, b_{31},), & t^a_{b2} = (b_{14}, b_{24}, b_{31}), \\ t^a_{b3} = (b_{12}, b_{22}), & t^a_{b4} = (b_{12}, b_{31}), \end{array} \right\}$$

$$(\ddot{\mathcal{H}}_1, \tilde{L}^{\varsigma}_{\kappa}) = \left\{ \begin{array}{l} \ddot{\mathcal{H}}(t^a_{b1}) = \{\langle 0.7, 0.1, 0.1\rangle/c^{\varsigma}_{\kappa 1}, \langle 0.3, 0.2, 0.4\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.1, 0.5, 0.3\rangle/c^{\varsigma}_{\kappa 3}, \langle 0.4, 0.1, 0.3\rangle/c^{\varsigma}_{\kappa 4}\}, \\ \ddot{\mathcal{H}}(t^a_{b2}) = \{\langle 0.7, 0.1, 0.1\rangle/c^{\varsigma}_{\kappa 4}, \langle 0.2, 0.5, 0.2\rangle/c^{\varsigma}_{\kappa 1}, \langle 0.6, 0.1, 0.2\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.4, 0.1, 0.3\rangle/c^{\varsigma}_{\kappa 3}\}, \\ \ddot{\mathcal{H}}(t^a_{b3}) = \{\langle 0.2, 0.3, 0.5\rangle/c^{\varsigma}_{\kappa 1}, \langle 0.2, 0.1, 0.7\rangle/c^{\varsigma}_{\kappa 4}, \langle 0.1, 0.1, 0.6\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.8, 0.1, 0.1\rangle/c^{\varsigma}_{\kappa 3}\}, \\ \ddot{\mathcal{H}}(t^a_{b4}) = \{\langle 0.3, 0.2, 0.4\rangle/c^{\varsigma}_{\kappa 3}, \langle 0.5, 0.1, 0.3\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.1, 0.5, 0.3\rangle/c^{\varsigma}_{\kappa 4}, \langle 0.2, 0.1, 0.7\rangle/c^{\varsigma}_{\kappa 1}\}, \end{array} \right\}$$

In addition, Φ_1 is the PPFS which is given by
$\Phi_1 = \{\langle 0.5, 0.1, 0.2\rangle$, $\langle 0.8, 0.2, 0.0\rangle$, $\langle 0.5, 0.2, 0.2\rangle$, $\langle 0.9, 0.0, 0.0\rangle\}$ Tabular representation of $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$ is given in Table

TABLE 3. $\nabla_1 = (\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_1)$

$\mathcal{U}$	t^a_{b1}	t^a_{b2}	t^a_{b3}	t^a_{b4}
$c^{\varsigma}_{\kappa 1}$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.2, 0.5, 0.2\rangle$	$\langle 0.2, 0.3, 0.5\rangle$	$\langle 0.2, 0.1, 0.7\rangle$
$c^{\varsigma}_{\kappa 2}$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.1, 0.1, 0.6\rangle$	$\langle 0.5, 0.1, 0.3\rangle$
$c^{\varsigma}_{\kappa 3}$	$\langle 0.1, 0.5, 0.3\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.3, 0.2, 0.4\rangle$
$c^{\varsigma}_{\kappa 4}$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.2, 0.1, 0.7\rangle$	$\langle 0.1, 0.5, 0.3\rangle$
Φ_1	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.8, 0.2, 0.0\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.9, 0.0, 0.0\rangle$

$$(\ddot{\mathcal{H}}_2, \tilde{L}^{\varsigma}_{\kappa}) = \left\{ \begin{array}{l} \ddot{\mathcal{H}}(t^a_{b1}) = \{\langle 0.6, 0.2, 0.1\rangle/c^{\varsigma}_{\kappa 1}, \langle 0.3, 0.3, 0.4\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.1, 0.2, 0.4\rangle/c^{\varsigma}_{\kappa 3}, \langle 0.5, 0.1, 0.3\rangle/c^{\varsigma}_{\kappa 4}\}, \\ \ddot{\mathcal{H}}(t^a_{b2}) = \{\langle 0.7, 0.0, 0.2\rangle/c^{\varsigma}_{\kappa 4}, \langle 0.2, 0.1, 0.3\rangle/c^{\varsigma}_{\kappa 1}, \langle 0.8, 0.1, 0.1\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.6, 0.1, 0.3\rangle/c^{\varsigma}_{\kappa 3}\}, \\ \ddot{\mathcal{H}}(t^a_{b3}) = \{\langle 0.5, 0.1, 0.2\rangle/c^{\varsigma}_{\kappa 1}, \langle 0.2, 0.1, 0.1\rangle/c^{\varsigma}_{\kappa 4}, \langle 0.4, 0.1, 0.1\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.9, 0.1, 0.0\rangle/c^{\varsigma}_{\kappa 3}\}, \\ \ddot{\mathcal{H}}(t^a_{b4}) = \{\langle 0.7, 0.2, 0.1\rangle/c^{\varsigma}_{\kappa 3}, \langle 0.6, 0.1, 0.3\rangle/c^{\varsigma}_{\kappa 2}, \langle 0.5, 0.3, 0.2\rangle/c^{\varsigma}_{\kappa 4}, \langle 0.2, 0.1, 0.6\rangle/c^{\varsigma}_{\kappa 1}\}, \end{array} \right\}$$

In addition, Φ_2 is the PPFS which is given by
$\Phi_2 = \{\langle 0.7, 0.1, 0.2\rangle, \langle 0.5, 0.2, 0.1\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.8, 0.1, 0.0\rangle\}$ Whose tabular representation is given in Table 4

TABLE 4. $\nabla_2 = (\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_2)$

$\mathcal{U}$	t^a_{b1}	t^a_{b2}	t^a_{b3}	t^a_{b4}
$c^{\varsigma}_{\kappa 1}$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.2, 0.1, 0.3\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.2, 0.1, 0.6\rangle$
$c^{\varsigma}_{\kappa 2}$	$\langle 0.3, 0.3, 0.4\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.1\rangle$	$\langle 0.6, 0.1, 0.3\rangle$
$c^{\varsigma}_{\kappa 3}$	$\langle 0.1, 0.2, 0.4\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.9, 0.1, 0.0\rangle$	$\langle 0.7, 0.2, 0.1\rangle$
$c^{\varsigma}_{\kappa 4}$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.7, 0.0, 0.2\rangle$	$\langle 0.2, 0.1, 0.1\rangle$	$\langle 0.5, 0.3, 0.2\rangle$
Φ_2	$\langle 0.7, 0.1, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.8, 0.1, 0.0\rangle$

TABLE 5. Intersection of GPFHSSs

$\mathcal{U}$	t^a_{b1}	t^a_{b2}	t^a_{b3}	t^a_{b4}
$c^{\varsigma}_{\kappa 1}$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.2, 0.1, 0.2\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.2, 0.1, 0.6\rangle$
$c^{\varsigma}_{\kappa 2}$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.1\rangle$	$\langle 0.6, 0.1, 0.3\rangle$
$c^{\varsigma}_{\kappa 3}$	$\langle 0.1, 0.2, 0.3\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.9, 0.1, 0.0\rangle$	$\langle 0.7, 0.2, 0.1\rangle$
$c^{\varsigma}_{\kappa 4}$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.7, 0.0, 0.1\rangle$	$\langle 0.2, 0.1, 0.1\rangle$	$\langle 0.5, 0.3, 0.2\rangle$
Φ_3	$\langle 0.7, 0.1, 0.2\rangle$	$\langle 0.5, 0.2, 0.0\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.9, 0.0, 0.0\rangle$

In this section of paper we introduce, an expectation score function and an algorithm for interpreting MADM problems.

Definition 5.6. let $\ddot{\mathcal{H}}(t^a_b) = \langle \mu_{\ddot{\mathcal{H}}(t^a_b)}(x^{\varsigma}_{\kappa i}), \eta_{\ddot{\mathcal{H}}(t^a_b)}(x^{\varsigma}_{\kappa i}), \nu_{\ddot{\mathcal{H}}(t^a_b)}(x^{\varsigma}_{\kappa i})\rangle$ be PFHSV then the expectation score function is define as
$\mathcal{S}(\ddot{\mathcal{H}}(t^a_b)) = \frac{2+\mu_{\ddot{\mathcal{H}}(t^a_b)}(x^{\varsigma}_{\kappa i})+\eta_{\ddot{\mathcal{H}}(t^a_b)}(x^{\varsigma}_{\kappa i})-\nu_{\ddot{\mathcal{H}}(t^a_b)}(x^{\varsigma}_{\kappa i})}{3} \qquad \mathcal{S}(\ddot{\mathcal{H}}(t^a_b)) \quad \in [0,1]$

Definition 5.7. The Weight vector $\mathcal{W}(\ddot{\mathcal{H}}(t^a_b))$ is defined as
$\mathcal{W}(\ddot{\mathcal{H}}(t^a_b)) = \frac{\mathcal{S}(\ddot{\mathcal{H}}(t^a_b))}{m}$
where $m = \sum \mathcal{S}(\ddot{\mathcal{H}}(t^a_b)$

6. **Algorithim**

$\nabla_1 \leftarrow$ First GPFHSS

$\nabla_2 \leftarrow$ Second GPFHSS

$\nabla_1 \cap_e \nabla_2 \leftarrow$ Extended intersection of First and Second GPFHSS

$\mathcal{S}(\ddot{\mathcal{H}}(t^a_b)) \leftarrow$ Compute expected sore function

$\mathcal{W}(\ddot{\mathcal{H}}(t^a_b)) \leftarrow$ Compute weight vector

DAPFDV $\leftarrow$Compute Dombi aggregated picture fuzzy decision values

$\mathcal{SF} \leftarrow$ Compute Score Function

Rank $\leftarrow$ Maximum value of score function is greater.

7. Case Study: Construction Company Problem

A firm want to construct a labor colony for their worker, major qualities to look for completion of their project are qualities, services, skilled team and equipments. Consider $\mathcal{U} = \{c^{\varsigma}_{\kappa 1}, c^{\varsigma}_{\kappa 2}, c^{\varsigma}_{\kappa 3}, c^{\varsigma}_{\kappa 4}, c^{\varsigma}_{\kappa 5}\}$ be be five construction company.

Let $E = \{e_1, e_2, e_3, e_4\}$ stand for qualities, services, skilled team and equipments whose corresponding attribute values are $\{A_1, A_2, A_3, A_4\}$ respectively. let $A_1 = \{b_{11}$ = Credentials, b_{12} = Experience, b_{13} = Goodwill and Reputation, $b_{14} = \{b_{11}, b_{12}, b_{13}\}\}$

$A_2 = \{b_{21}$ = New construction, b_{22} = Repair, b_{23} = demolition, $b_{24} = \{b_{21}, b_{22}, b_{23}\}\}$

$A_3 = \{b_{31}$ = builders and architects, b_{31} = civil engineers $\}$

$A_4 = \{b_{41}$ = Modern equipments $\}$

$$\tilde{L}^{\varsigma}_{\kappa} = \left\{ \begin{array}{ll} t^a_{b\,1} = (b_{11}, b_{21}, b_{31}), & t^a_{b\,2} = (b_{14}, b_{24}, b_{31}, b_{41}), \\ t^a_{b\,3} = (b_{12}, b_{22}), & t^a_{b\,4} = (b_{12}, b_{31}), \\ t^a_{b\,5} = (b_{11}, b_{22}, b_{31}, b_{42}), & t^a_{b\,6} = (b_{11}, b_{23}), \end{array} \right\}$$

The CEO makes two groups of members of administration of firm to do the evaluation.. The set of attributes $N^{\varsigma}_{\kappa 1} = \{t^a_{b\,1}, t^a_{b\,2}, t^a_{b\,3}, t^a_{b\,4}\}$ observed by first group and second group monitoring attributes value $N^{\varsigma}_{\kappa 2} = \{t^a_{b\,2}, t^a_{b\,4}, t^a_{b\,5}, t^a_{b\,6}\}$. Two GPFHSS $(\ddot{\mathcal{H}}_1, N^{\varsigma}_{\kappa 1}, \Phi_3)$ and $(\ddot{\mathcal{H}}_2, N^{\varsigma}_{\kappa 2}, \Phi_4)$ are given in table.

Step: 01

Table 6. GPFHSS 1

$\mathcal{U}$	$t^a_{b\,1}$	$t^a_{b\,2}$	$t^a_{b\,3}$	$t^a_{b\,4}$
$c^{\varsigma}_{\kappa 1}$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.7, 0.1, 0.2\rangle$	$\langle 0.3, 0.1, 0.3\rangle$	$\langle 0.3, 0.1, 0.2\rangle$
$c^{\varsigma}_{\kappa 2}$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.6, 0.1, 0.1\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.8, 0.0, 0.2\rangle$
$c^{\varsigma}_{\kappa 3}$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.9, 0.1, 0.0\rangle$	$\langle 0.5, 0.2, 0.1\rangle$
$c^{\varsigma}_{\kappa 4}$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.1\rangle$	$\langle 0.4, 0.3, 0.3\rangle$
$c^{\varsigma}_{\kappa 5}$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.3, 0.1, 0.5\rangle$	$\langle 0.3, 0.1, 0.4\rangle$	$\langle 0.7, 0.1, 0.2\rangle$
Φ_3	$\langle 0.8, 0.0, 0.1\rangle$	$\langle 0.6, 0.1, 0.1\rangle$	$\langle 0.1, 0.2, 0.7\rangle$	$\langle 0.3, 0.1, 0.5\rangle$

Step: 02 Calculate intersection of GPFHSS 1 and GPFHSS 2.

Table 7. GPFHSS 2

$\mathcal{U}$	t^a_{b2}	t^a_{b4}	t^a_{b5}	t^a_{b6}
$c^s_{\kappa1}$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.3, 0.1, 0.3\rangle$
$c^s_{\kappa2}$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.5, 0.0, 0.3\rangle$	$\langle 0.2, 0.1, 0.3\rangle$	$\langle 0.7, 0.1, 0.2\rangle$
$c^s_{\kappa3}$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.8, 0.1, 0.0\rangle$	$\langle 0.9, 0.0, 0.1\rangle$	$\langle 0.5, 0.1, 0.1\rangle$
$c^s_{\kappa4}$	$\langle 0.3, 0.3, 0.1\rangle$	$\langle 0.4, 0.2, 0.0\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.6, 0.2, 0.1\rangle$
$c^s_{\kappa5}$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.7, 0.0, 0.1\rangle$	$\langle 0.2, 0.1, 0.1\rangle$	$\langle 0.5, 0.3, 0.2\rangle$
Φ_4	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.1, 0.2, 0.7\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.3, 0.1, 0.5\rangle$

Table 8. Intersection of GPFHSS 1 and GPFHSS 2

$\mathcal{U}$	t^a_{b1}	t^a_{b2}	t^a_{b3}	t^a_{b4}	t^a_{b5}	t^a_{b6}
$c^s_{\kappa1}$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.7, 0.1, 0.2\rangle$	$\langle 0.3, 0.1, 0.3\rangle$	$\langle 0.3, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.3, 0.1, 0.3\rangle$
$c^s_{\kappa2}$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.5, 0.0, 0.3\rangle$	$\langle 0.2, 0.1, 0.3\rangle$	$\langle 0.7, 0.1, 0.2\rangle$
$c^s_{\kappa3}$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.9, 0.1, 0.0\rangle$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.9, 0.0, 0.1\rangle$	$\langle 0.5, 0.1, 0.1\rangle$
$c^s_{\kappa4}$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.3, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.1\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.6, 0.2, 0.1\rangle$
$c^s_{\kappa5}$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.3, 0.1, 0.5\rangle$	$\langle 0.3, 0.1, 0.4\rangle$	$\langle 0.7, 0.0, 0.2\rangle$	$\langle 0.2, 0.1, 0.1\rangle$	$\langle 0.5, 0.3, 0.2\rangle$
Φ_4	$\langle 0.8, 0.0, 0.1\rangle$	$\langle 0.1, 0.2, 0.7\rangle$	$\langle 0.1, 0.2, 0.7\rangle$	$\langle 0.1, 0.1, 0.7\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.3, 0.1, 0.5\rangle$

Step:03 Calculate value of expectation score function using 5.6 and weight vector by 5.7 shown in Tab 9

Table 9. Expectation Score Function and Weight vector

$\mathcal{U}$	t^a_{b1}	t^a_{b2}	t^a_{b3}	t^a_{b4}	t^a_{b5}	t^a_{b6}
Φ_4	$\langle 0.8, 0.0, 0.1\rangle$	$\langle 0.1, 0.2, 0.7\rangle$	$\langle 0.1, 0.2, 0.7\rangle$	$\langle 0.1, 0.1, 0.7\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.3, 0.1, 0.5\rangle$
$\mathcal{S}(\ddot{\mathcal{H}}(t^a_b))$	0.9333	0.5333	0.5333	0.5	0.8666	0.6333
$\mathcal{W}(\ddot{\mathcal{H}}(t^a_b))$	0.2333	0.1333	0.1333	0.125	0.217	0.158

Step: 04 Calculate Dombi aggregated picture fuzzy decision values (DAPFDVs) for k = 1 by 2.8 and score function using 2.7. The DAPFDVs can be calculated as:

From above table

$c^s_{\kappa2} \prec c^s_{\kappa5} \prec c^s_{\kappa1} \prec c^s_{\kappa4} \prec c^s_{\kappa3}$

$c^s_{\kappa3}$ is suitable construction company for labor colony construction.

8. **Comparison**

The algorithm proposed by Jaber et al. [25] face challenges to deal with the MADM problem where attributes of the alternates have their corresponding sub-attributes. To overcome such

Table 10. DAPFDVs and Score Function

$\mathcal{U}$	$DAPFDVs$	$\mathcal{SF}$
$c^{\varsigma}_{\kappa 1}$	$\langle 0.4645, 0.1000, 0.1760\rangle$	$\langle 0.2885\rangle$
$c^{\varsigma}_{\kappa 2}$	$\langle 0.4952, 0.0000, 0.2599\rangle$	$\langle 0.2353\rangle$
$c^{\varsigma}_{\kappa 3}$	$\langle 0.8027, 0.0000, 0.0000\rangle$	$\langle 0.8027\rangle$
$c^{\varsigma}_{\kappa 4}$	$\langle 0.5081, 0.1333, 0.1313\rangle$	$\langle 0.3768\rangle$
$c^{\varsigma}_{\kappa 5}$	$\langle 0.4362, 0.0000, 0.1868\rangle$	$\langle 0.2494\rangle$

difficulties we developed an extension of GPFSS by changing the mapping to a multi-attribute mapping. With the help of GPFHSS, we define an algorithm, which plays an important role to study the picture fuzzy and hypersoft environment.

9. **Conclusions**

In this article, we introduce PFHSS and GPFHSS. We defined some operations of PFHSS and GPFHSS, also proved De Morgans laws for these operations. For the solution of MADM problems, we construct an algorithm by using the extended intersection of GPFHSS information and also we introduced a new expectation score function to find the value of the weight vector. With the help of the weight vector and new expectation score function, we calculate DAPFDVs and score function. Then we rank the construction companies which are given in the example of a case study of the construction of labor colonies by using ascending values of the score function. After comparison with prior proposed techniques and arise it to be more generalized and productive to deal with multi-attribute classifications.

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Received: Sep 10, 2022. Accepted: Dec 20, 2022