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An Efficient Neutrosophic Technique for Uncertain Multi Objective Transportation Problem

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Abstract. It can be difficult to figure out how to satisfy customers' ever rising demands and keep up one's market competitiveness while containing controllable costs. Inefficiencies in the supply chain network are thus discovered by our investigation. Finding the best allocation order for products from diverse sources going to numerous destinations is the primary objective. Moreover, The information that is readily available is typically not clear-cut in real-world circumstances. So, it gives rise to the uncertain transportation problem. With the aim of helping the decision maker to have the suitable transportation plan with real suitation, in this paper, a solution procedure for multi objective transportation problem involving uncertainvariables has been studied under neutrosophic environment. A chance constraint model is constructed foruncertain multi objective transportation problem and then a neutrosophic compromise approach is used toobtain the pareto optimal solution for the problem. As neutrosophic sets are built with truth, indeterminacyand falsity membership functions, they are capable to help the decision maker in this complex transportation model. A numerical example has been reported to demonstrate the efficiency of the proposed approach towardsthe best compromise solution and a comparison study has been made with the existing methods.

Keywords: Multi objective transportation problem; Chance constraint programming; Neutrosophic set theory.

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1. Introduction

In the real world, transportation planning decision problems play a vital role in logistics and supply chain management with diverse challenges to be addressed. A transportation planning problem involves a large number of factors such as shipment, distance, delivery time; transportation cost etc and are defined on the basis of quantitative evaluation. More often than not, the market scenario keeps varying and posing challenges, because of which various objective functions are needed related to a transportation problem. For example maximizing the profit of the transportation, minimizing the transportation cost and toll tax etc. Since the cost parameters of various objectives of the transportation problem are not related to each other, these are considered as conflicting and commensurable model of the multi objective transportation problem (MOTP). In the present-day scenario, most of the transportation planning decisions is made under uncertain environment due to many unpredictable factors. Traditional methods failed to capture the decision maker's ambiguities and are non-effective to solve these complex ill-defined models. Many researchers had developed different stochastic, fuzzy and uncertain models to solve complex uncertain transportation engineering problems.

In this paper, we've proposed a solution procedure for multi-objective transportation problem whose parameters are all uncertain variables. Motivated by neutrosophic sets studied by Smarandache [19] which provides a general structure to deal with uncertainty, a compromise solution to the proposed model is obtained. The term "neutrosophy"means the knowledge of neutral thought and considers that all elements can be represented by three degrees namely-truth, falsity, indeterminacy which lie between 0 and 1. Since its establishment by Smarandache [25], some attention has been developed for optimization aspects [20]. Rizk M [21] proposed an algorithm based upon MOTP under neutrosophic environment. Since neutrosophic models effectively assist the decision-maker by incorporating satisfaction, satisfaction to some degree, and dissatisfaction of objective functions in determining the best compromise solution. we have applied the neutrosophic technique for the first time to the MOTP whose parameters are uncertain normal variables.

The rest of the paper is structured as follows. Section 2 contains the existing research papers related to the proposed work. In section 3, weve reviewed the preliminaries of uncertainty theory. In section 4, the mathematical model of uncertain multi objective transportation model is introduced. Deterministic multi objective transportation model, uncertain MOTP model and chance constraint programming model are presented in the subsections 4.1,4.2 and 4.3 respectively. In section 5, a neutrosophic compromise programming approach is introduced and we presented the preliminaries of neutrosophic set. In subsection 5.1, neutrosophic decision making is explained and in subsection 5.2, an algorithm to solve uncertain MOTP is presented. A numerical example has been given in section 6, to understand the applicability of the proposed model and compared with a existing approach. The result and discussion, Implications, and the conclusion have been presented in Section 7,8 and 9 respectively.

2. Literature Review

The basic study of the transportation problem (TP) was carried over by Hitchcock [1] and Koopmans [2] played a significant role in its development. Abdelaziz et al [3] had proposed a compromise chance constraint programming model (CCCP) for multi-objective stochastic programming portfolio models.

Aouni et al [4], for the stochastic goal programming model, explicitly introduced the decision-makers preferences adapted chance-constrained-program. A fuzzy multi-objective programming (FMOP) vendor selection model was developed by Wu et al [5]. Bit et al [6] presented an approach to multicriteria decision making transportation problem under fuzzy environment. Zimmerman [7], using fuzzy set theory, solved the multi-objective transportation problem by considering suitable membership functions. A fuzzy goal programming approach to determine an optimal compromise solution for the multi-objective transportation problem by assuming that each objective function has a fuzzy goal was proposed by Zangiabadi and Maleki [8]. Gupta et al [28] proposed a model for the probabilistic fuzzy goal multi-objective supply chain network (PFG-MOSCN) and discussed the solution procedure for the same.

Although fuzzy set theory proposed by Zadeh [9] is widely applied in many uncertain models, it could not handle human uncertainty in some contexts involving incomplete information. As an attempt to deal with such indeterminacies, Liu founded uncertainty theory [10, 11]. Nowadays, uncertainty theory is considered as a mathematical branch for modeling belief degrees and has been adopted in many mathematical models like uncertain programming, uncertain logic, uncertain graph, uncertain statistics and uncertain finance [12–14]. The belief degree of an uncertain event to happen is measured by uncertain measure. The usage of random uncertain variable and chance measure was also introduced by Liu [15]. Post that, he also presented uncertain random programming to model optimization problems containing more than one random variable. Gao [16], in his paper, newly proposed certain properties based on continuously uncertain measures. Seyyed Mojtaba Chasence [17] introduced uncertain linear fractional programming problem and also presented three methods for conversion of uncertain optimization problem into an equivalent deterministic problem. Liu [18] provided a new uncertain multi objective programming and introduced uncertain goal programming as a compromised method to solve multi-objective programming with the uncertain variables, considering the operational law of uncertain variables through inverse uncertainty distribution. Gupta et al [29] formulated the model of an Uncertain multi-objective capacitated transportation problem with mixed constraints. Latter, Srikant Gupta et al [30] proposed the procedure for solving multi-objective capacitated transportation problem under an uncertain environment. S Das et al [39] presented a solution procedure for solving fully fuzzy linear programming problems whose parameters are considered as the trapezoidal fuzzy number. Utilising the aggregate ranking function, Sapan Kumar Das [40] constructed a new framework for neutrosophic integer programming problems involving triangular neutrosophic numbers. SK Das's [41] studied a transportation problem involving pentagonal Neutrosophic numbers where in the supply, demand, and cost of transportation were all ambiguous . Constraints under neutrosophic environment Das et al [42] proposed the solution procedure for solving the Linear Programming Problems with Mixed . Motivated by the above said works, we have proposed the solution procedure for solving the uncertain MOTP by using the neutosophic techniques.

TABLE 1. Comparision between existing transportation models with proposed model

Author	Nature of the objective		Environment	Methodology Used
	Single	Multiple		
Lakhveer et al [31]	×	✓	Crisp	Using the weighted approach
Subhakantra Dash et al [32]	✓	×	Rough	Using the uncertainity distribution
Bharati et al [33]	×	✓	Interval valued intuitionistic fuzzy sets	Based on extended Yager's function Interval valued intuitionistic fuzzy sets
Haiying Guo et al [34]	✓	×	Uncertainty theory	Using the simplex method
Thamaraiselvi [35]	✓	×	Neutrosophic	The arithmetic operations on single valued neutrosophic trapezoidal numbers areemployed
RizkM.Rizk Al-lah [36]	×	✓	Neutrosophic	Using Neutrosophic compromise programming approach
Somnath maity [37]	✓	×	Type-2 fuzzy	Using fuzzy number approximation
Deshabrata Roy Mahapatra [38]	×	✓	stochastic	Using fuzzy goal programming
Proposed Model	✓	✓	Uncertainty theory	Using **BOTH** uncertainty theory and Neutrosophic method

The current research on the transportation issue is presented in Table 1. We compared the transportation problems on the basis of the numbers of objectives and the various types of environments. To the best of our knowledge, no one has investigated a multi-objective transportation problem with the simultaneous goals of maximization of profit, minimization of toll tax, and minimization of transportation cost in both neutrosophic and uncertain environment. We have used both the methods to bring the level of indeterminancy down to the maximum.

3. Preliminaries

The concepts and definitions which will be used in the subsequent discussions has been presented in the section.

Definition 3.1. [13] [10] Let $\mathcal{L}$ be a σ - algebra of collection of events Λ of a universal set Γ. A set function $\mathcal{M}$ is said to be uncertain measure defined on the σ - algebra where $\mathcal{M}\{\Lambda\}$ indicate the belief degree with which we believe that the event will happen; It satisfies the following axioms:

(1) Normality Axiom: For the universal set Γ, we have $\mathcal{M}\{\Gamma\} = 1$.
(2) Duality Axiom: For any event Γ, we have $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^C\} = 1$.
(3) Subadditivity Axiom: For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have $\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$.
(4) Product Axiom: Let $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i)$ be uncertainty
spaces for $i = 1, 2, 3, \cdots$ The product uncertain measure is an uncertain measure holds $\mathcal{M}\{\Pi_{i=1}^{\infty} \wedge_i\} = \wedge_{i=1}^{\infty} \mathcal{M}\{\wedge_i\}$ where $\wedge_i \in \mathcal{L}_i$ for $i = 1, 2, 3, \cdots \infty$.

Definition 3.2. [10] A function $\xi : (\Gamma, \mathcal{L}, \mathcal{M}) \to \mathcal{R}$ is said to be an uncertain variable such that $\{\xi \in B\} = \{\gamma \in \Gamma / \xi(\gamma) \in B\}$ is an event for any Borel set B of real numbers.

Definition 3.3. [10] An uncertain variable ξ defined on the uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ is said to be non- negative if $\mathcal{M}\{\xi < 0\} = 0$ and positive if $\mathcal{M}\{\xi \leq 0\} = 0$.

Definition 3.4. [10] The uncertainty distribution $\phi(x)$ of an uncertain variable ξ for any real number x is defined by $\phi(x) = \mathcal{M}\{\xi \leq x\}$.

Definition 3.5. Let $\phi(x)$ be the regular uncertainty distribution of an uncertain variable ξ. Then $\phi^{-1}(\alpha)$ is called inverse uncertainty distribution of ξ and it exists on (0, 1).

Definition 3.6. [10] The uncertain variable
ξ_i $(i = 1, 2, 3, \cdots n)$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n} (\xi_i \in B_i)\right\} = \wedge_{i=1}^{n} \mathcal{M}(\xi_i \in B_i) \tag{1}$$

where $B_i (i = 1, 2, 3, \cdots n)$ are called Borel sets of real numbers.

Theorem 3.7. *Let ξ be an uncertain variable with regular uncertain distribution function ψ. Then its α - optimistic value and α - pessimistic values are*

$$\xi_{\sup}(\alpha) = \psi^{-1}(1-\alpha),\ \xi_{\inf}(\alpha) = \psi^{-1}(\alpha) \tag{2}$$

Theorem 3.8. *[11] The regular uncertainty distributions of independent uncertain variables $\xi_i(i = 1, 2, 3, \cdots n)$ are $\phi_i(i = 1, 2, 3, \cdots n)$ respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing and strictly decreasing with respect to $x_1, x_2, \cdots, x_m$ and $x_{m+1}, x_{m+2}, \cdots, x_n$ respectively then the uncertain variable $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an inverse uncertainty distribution*

$$\begin{aligned}\psi^{-1}(\alpha) =& f(\phi_1^{-1}(\alpha), \phi_2^{-1}(\alpha), \cdots, \phi_m^{-1}(\alpha),\\ &\phi_{m+1}^{-1}(1-\alpha), \phi_{m+2}^{-1}(1-\alpha), \cdots, \phi_n^{-1}(1-\alpha))\end{aligned} \tag{3}$$

Definition 3.9. [10] The expected value of uncertain variable ξ is given by

$$E(\xi) = \int_0^\infty \mathcal{M}\{\xi \geq x\}dx - \int_{-\infty}^0 \mathcal{M}\{\xi \leq x\}dx \tag{4}$$

This is valid only if at least one of the integral is finite.

Theorem 3.10. *[22] Let $\phi_i(i = 1, 2, 3, \cdots n)$ be regular uncertainty distributions of independent $\xi_i(i = 1, 2, 3, \cdots n)$ with respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing and strictly decreasing w.r.to*
$x_1, x_2, \cdots, x_m$ and $x_{m+1}, x_{m+2}, \cdots, x_n$ respectively, then

$$\begin{aligned}E(\xi) = \int_0^1 & f(\phi_1^{-1}(\alpha), \cdots, \phi_m^{-1}(\alpha),\\ &\phi_{m+1}^{-1}(1-\alpha), \cdots, \phi_n^{-1}(1-\alpha))d\alpha\end{aligned} \tag{5}$$

From the above theorem, we know that

$$E(\xi) = \int_0^1 \phi^{-1}(\alpha)d\alpha \tag{6}$$

where ξ is an uncertain variable with regular uncertainty distribution Φ.

Definition 3.11. [10] A linear uncertain variable ξ is defined as

$$\phi(x) = \begin{cases} 0 & \text{if}\, x \leq 1 \\ \dfrac{x-l}{m-l} & \text{if}\, l \leq x \leq m \\ 1 & \text{if}\, x \geq m \end{cases} \tag{7}$$

represented by $L(l, m)$, where l and $m \in R$ with $l < m$.

The inverse distribution function of a linear uncertain variable $L(l, m)$ is given by

$$\phi^{-1}(\alpha) = (1-\alpha)l + \alpha m \tag{8}$$

and its expected value is given by

$$E(\xi) = \frac{l+m}{2} \tag{9}$$

Definition 3.12. [10] The distribution function of a normal uncertain variable is

$$\phi(x) = \left(1 + exp^{\left(\frac{\pi(\mu - x)}{\sigma\sqrt{3}}\right)}\right)^{-1}, x \geq 0 \tag{10}$$

and it is denoted as $N(\mu, \sigma); \mu, \sigma \in R$ with $\sigma > 0$.
The inverse uncertainty distribution and the expected value of $N(\mu, \sigma)$ is defined as follows

$$\phi^{-1}(\alpha) = \mu + \frac{\sigma\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha} \tag{11}$$

$$E(\xi) = \mu \tag{12}$$

4. Uncertain Multi objective transportation model

In this section, we introduce the mathematical formulation of uncertain multi objective transportation problem (UMOTP). For the formulation of UMOTP, the following assumptions such as indexes, decision variables and parameters are considered as follows.

i index for origins

j index for destinations

k index for objective function

x_{ij} quantity transported from i^{th} origin to j^{th} destination

$\mathcal{Z}_k$ k^{th} objective function

c_{ij}^k the unit cost of transportation from i^{th} origin to j^{th} destination for the k^{th} objective function

a_i	total amount of product available at orgin i
b_j	total demand of the product at destination j
$Z_k(\boldsymbol{x} : \boldsymbol{\xi})$	k^{th} objective function with uncertain variable
ξ_{ij}^k	uncertain cost coefficient of the k^{th} objective
γ_i	uncertain availability at origin i
η_j	uncertain capacity of destination j
α	confidence level for objective function, $\alpha \in (0,1)$
α_i	confidence level for availability constraint, $\alpha_i \in (0,1)$
β_j	confidence level for destination constraint, $\beta_j \in (0,1)$
ψ^k	regular uncertainty distribution for the independent uncertain variable ξ^k
ψ_{ij}^k	regular uncertainty distribution for the independent uncertain variable ξ_{ij}^k
ϕ_i	regular uncertainty distribution for the independent uncertain variable γ_i
θ_j	regular uncertainty distribution for the independent uncertain variable η_j
N	neutrosophic set
X	space of objects
T_N	truth membership function
I_N	indeterminacy membership function
F_N	falsity membership function
t_k, s_k	predetermined numbers in (0,1).
U_k	upper bound of the k^{th} objective
L_k	lower bound of the k^{th} objective
D_N	neutrosophic decision set
G_k	neutrosophic goal
C_i	neutrosophic constraint
$\lambda_T, \lambda_I, \lambda_F$	auxiliary parameters

4.1. *Deterministic model of Multi objective transportation problem*

The mathematical formulation of deterministic multi objective transportation problem is

$$\begin{aligned} \text{Min } Z_k(x) &= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \; (k = 1, 2, \cdots, K) \\ \text{subject to} &\sum_{j=1}^{n} x_{ij} \leq a_i, \; i = 1, 2, \cdots, m \\ &\sum_{i=1}^{m} x_{ij} \geq b_j, \; j = 1, 2, \cdots, n \\ &x_{ij} \geq 0, \forall i, j \end{aligned} \tag{13}$$

Here c_{ij}^{k}, a_i, $(i = 1, 2, \cdots, m)$ and b_j, $(j = 1, 2, \cdots, n)$ are the cost, supply and demand parameters of multi objective transportation problem respectively which are represented by crisp numbers. Without loss of generality, it may be considered that $a_i \geq 0, \forall i, b_j \geq 0, \forall j$ and $c_{ij}^{k} \geq 0, \forall k$ and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

4.2. *Mathematical model for uncertain multi objective transportation problem*

In real life scenario, planning is made in prior before the transportation process. But many uncertain factors like road conditions, climate changes, changes in sales due to attitude of customers, operate parallelly, making demand, supply and transportation cost remain uncertain. Hence, cost, supply and demand parameters c_{ij}^{k}, a_i and b_j respectively are considered as uncertain variables and are represented by ξ_{ij}^{k}, γ_i and η_j.
Then the mathematical model for uncertain multi objective transportation problem is defined as

$$\begin{aligned} \text{Min } Z_k(x; \xi) &= \sum_{i=1}^{m}\sum_{j=1}^{n} \xi_{ij}^{k} x_{ij} \; (k = 1, 2, \cdots, K) \\ \text{subject to} &\sum_{j=1}^{n} x_{ij} \leq \gamma_i, \; i = 1, 2, \cdots, m \\ &\sum_{i=1}^{m} x_{ij} \geq \eta_j, \; j = 1, 2, \cdots, n \\ &x_{ij} \geq 0, \forall i, j \end{aligned} \tag{14}$$

As we cannot deal with uncertain environment directly, we have to convert(14) into an equivalent deterministic model by using expected value model or chance constrained model or taking confidence level on the constraint functions and expected value on the objective function. As chance constraint programming model provides most suitable solutions [23], we make use of the chance constraint model for uncertain multi objective transportation problem as shown below.

4.3. *Chance constraint model of UMOTP*

Let α be the predetermined confidence level with $\alpha \in (0,1)$. The decision maker aims to get a smallest value $\tilde{f}$ such that uncertain variable $Z_k(\boldsymbol{x} : \boldsymbol{\xi}) \leq \tilde{f}$ with the predetermined confidence level α.

Definition 4.1. The solution vector $\boldsymbol{x} = (x_{ij}) \geq 0$ is a feasible solution of the model (14), if it holds the below constraints.

$$\mathcal{M}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\xi_{ij}^{k}x_{ij} \leq \tilde{f}\right\} \geq \alpha,\ k = 1,2,\cdots,K \tag{15}$$

$$\mathcal{M}\left\{\sum_{j=1}^{n} x_{ij} \leq \gamma_i\right\} \geq \alpha_i,\ i = 1,2,\cdots,m \tag{16}$$

$$\mathcal{M}\left\{\sum_{i=1}^{m} x_{ij} \geq \eta_j\right\} \geq \beta_j,\ j = 1,2,\cdots,n \tag{17}$$

Definition 4.2. A feasible solution x^* is said to be pareto optimal solution of the model (14) if there exists no other feasible solution $\boldsymbol{x}$ such that

$$\begin{gathered}\text{Min}\left\{\tilde{f}/\mathcal{M}\left\{Z_k(\boldsymbol{x} : \boldsymbol{\xi}) \leq \tilde{f}\right\} \geq \alpha\right\} \leq \text{Min}\left\{\tilde{f}/\mathcal{M}\left\{Z_k(\boldsymbol{x^*} : \boldsymbol{\xi}) \leq \tilde{f}\right\} \geq \alpha\right\} \\ \forall k = 1,2,\cdots,K\end{gathered} \tag{18}$$

Definition 4.3.

$$\begin{gathered}\text{Min}\left\{\tilde{f}/\mathcal{M}\left\{Z_k(\boldsymbol{x} : \boldsymbol{\xi}) \leq \tilde{f}\right\} \geq \alpha\right\} < \text{Min}\left\{\tilde{f}/\mathcal{M}\left\{Z_k(\boldsymbol{x^*} : \boldsymbol{\xi}) \leq \tilde{f}\right\} \geq \alpha\right\} \\ \text{for atleast one}\, k = 1,2,\cdots,K\end{gathered} \tag{19}$$

The chance constraint programming model of UMOTP can be constructed as follows

$$\begin{aligned}&\text{Min}\tilde{f}\\ &\text{subject to}\\ &\mathcal{M}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\xi_{ij}^{k}\boldsymbol{x}_{ij} \leq \tilde{f}\right\} \geq \alpha,\ k = 1,2,\cdots,K\\ &\mathcal{M}\left\{\sum_{j=1}^{n}\boldsymbol{x}_{ij} \leq \gamma_i\right\} \geq \alpha_i\\ &\mathcal{M}\left\{\sum_{i=1}^{m}\boldsymbol{x}_{ij} \geq \eta_j\right\} \geq \beta_j\\ &\boldsymbol{x}_{ij} \geq 0, i = 1,2,\cdots,m, j = 1,2,\cdots,n\end{aligned} \tag{20}$$

Here, the confidence levels $\alpha, \alpha_i, \beta_j$ are predetermined from the interval (0,1).

Definition 4.4 (Pareto optimal solution)**.** Pareto optimal solution is defined as a set of 'non-inferior' solutions in the objective space defining a boundary beyond which none of the objectives can be improved without sacrificing at least one of the other objectives.

Theorem 4.5. *Suppose that $\xi_{ij}^k, \gamma_i, \eta_j$ are independent uncertain variables with regular uncertainty distribution $\psi_{ij}^k, \phi_i, \theta_j$ respectively. The equivalent deterministic model of chance constraint model is*

$$\begin{aligned} &Min\, Z_k^* = \sum_{i=1}^{m}\sum_{j=1}^{n} (\psi_{ij}^k)^{-1}(\alpha)\; x_{ij}\, (k = 1, 2, \cdots, K) \\ &subject\ to \\ &\sum_{j=1}^{n} x_{ij} \leq (\phi_i)^{-1}(1-\alpha_i),\, i = 1, 2, \cdots, m \\ &\sum_{i=1}^{m} x_{ij} \geq (\theta_j)^{-1}(\beta_j),\, j = 1, 2, \cdots, n \\ &x_{ij} \geq 0, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \end{aligned} \tag{21}$$

Proof:

Assume that uncertainty variable $\xi_k = \sum_{i=1}^m \sum_{j=1}^n (\xi_{ij}^k) x_{ij}$ has distribution function ψ_k.
Let $f(y_{11}, y_{12}, \cdots, y_{mn}) = y_{11}x_{11} + y_{12}x_{12} + \cdots + y_{mn}x_{mn}$
It is clear that this function is strictly increasing with respect to $y_{11}, y_{12}, \cdots, y_{mn}$ then by the theorem (3.8), the uncertain variable $\boldsymbol{\xi}_k$ has an inverse uncertainty distribution.

$$(\psi_k)^{-1}(\alpha) = \sum_{j=1}^{n}\sum_{i=1}^{m} (\psi_{ij}^k)^{-1}(\alpha) x_{ij}$$

So, we have

$$\begin{aligned} &\mathcal{M}\left\{ \sum_{j=1}^{n}\sum_{i=1}^{m} (\xi_{ij})^k x_{ij} \leq \tilde{f} \right\} \geq \alpha \\ &\Leftrightarrow \psi^k(\tilde{f}) \geq \alpha \\ &\Leftrightarrow (\psi^k)^{-1}(\alpha) \leq \tilde{f} \\ &(i.e.) \sum_{j=1}^{n}\sum_{i=1}^{m} (\psi_{ij}^k)^{-1}(\alpha) x_{ij} \leq \tilde{f} \end{aligned}$$

For the constraints, we have

$$\begin{aligned}
&\mathcal{M}\left\{\sum_{j=1}^{n} x_{ij} \leq \gamma_i\right\} \geq \alpha_i \\
&\Leftrightarrow \mathcal{M}\left\{\sum_{j=1}^{n} x_{ij} - \gamma_i \leq 0\right\} \geq \alpha_i \\
&\Leftrightarrow \sum_{j=1}^{n} x_{ij} - (\varphi_i)^{-1}(1-\alpha) \leq 0 \\
&\Leftrightarrow \sum_{j=1}^{n} x_{ij} \leq (\varphi_i)^{-1}(1-\alpha_i)
\end{aligned}$$

Similarly $\mathcal{M}\left\{\sum_{i=1}^{m} x_{ij} \geq \eta_j\right\} \geq \beta_j$ is equivalent to
$\sum_{i=1}^{m} x_{ij} \geq (\theta_j)^{-1}(\beta_j)$, $j = 1, 2, \cdots, n$.
Hence the theorem is proved.

Corollary 4.6. *Let $x_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$ be the non negative decision variable and $\xi_k, k = 1, 2, \cdots, K$ are independently uncertain variables with expected values $e_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$ and the variances $\sigma_{ij}^2, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$ respectively. If ξ be a normal uncertain variable $N(e, \sigma)$, then for any $\alpha \in (0, 1)$, the model (21) can be converted into the following model.*

$$\begin{aligned}
&\text{Min}\,(e_{ij})_k + \frac{(\sigma_{ij})_k\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha},\ k = 1, 2, \cdots, K \\
&\text{subject to } \sum_{j=1}^{n} x_{ij} \leq e_i + \frac{\sigma_i\sqrt{3}}{\pi}\ln\frac{1-\alpha_i}{\alpha_i}\ i = 1, 2, \cdots, m \\
&\sum_{i=1}^{m} x_{ij} \geq e_j^* + \frac{\sigma_j^*\sqrt{3}}{\pi}\ln\frac{\beta_j}{1-\beta_j},\ j = 1, 2, \cdots, n \\
&x_{ij} \geq 0, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n
\end{aligned} \tag{22}$$

5. **Neutrosophic compromise programming approach**

In this section first we introduce some basic definitions of neutrosophic set theory and then we will discuss about neutrosophic compromise programming approach.

Definition 5.1. A neutrosophic set N defined in the universal set X is characterized by truth membership function $T_N(x)$, indeterminacy membership function
$I_N(x)$ and a falsity membership function $F_N(x)$ and is denoted by

$$N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \,|x \in X\} \tag{23}$$

where $T_N(x), I_N(x), F_N(x)$ are real standard or non standard subsets belonging to]0- ,1+[. Also the membership grades of truth, indeterminacy and falsity are the functions from X to]0- ,1+[. Also we have $0^- \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3^+$ as there is no restriction on the sum of $T_N(x), I_N(x) \,\&\, F_N(x)$.
Wang [24] introduced Single valued Neutrosophic set (SVNS) in engineering problem as it is computationally more comfortable.

Definition 5.2. [24] A single valued neutrosophic set N defined on X is expressed as
$N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \,|x \in X\}$ where
$T_N(x), I_N(x), F_N(x) \in [0,1], \forall x \in X$ and
$0 \leq T_N(x), I_N(x), F_N(x) \leq 3$. Clearly, SVNS is subset of neutrosophic set.

Definition 5.3. [25] Let P and Q are the two Single Valued Netuosophic Sets (SVNSs). Then their union also a SVNS and their membership functions are given by

$$T_{P\cup Q}(x) = \text{Max}\{T_P(x), T_Q(x)\};$$
$$I_{P\cup Q}(x) = \text{Max}\{I_P(x), I_Q(x)\};$$
$$F_{P\cup Q}(x) = \text{Min}\{F_P(x), F_Q(x)\}$$

Definition 5.4. [25] Let P and Q are SVNS, then their intersection also a SVNS with the following membership functions

$$T_{P\cap Q}(x) = \text{Min}\{T_P(x), T_Q(x)\};$$
$$I_{P\cap Q}(x) = \text{Min}\{I_P(x), I_Q(x)\};$$
$$F_{P\cap Q}(x) = \text{Max}\{F_P(x), F_Q(x)\}$$

Definition 5.5. The complement of the neutrosophic set N is denoted by $c(N)$ and is defined by $T_{c(N)}(x) = F_N(x), I_{c(N)}(x) = 1 - I_N(x),\ F_{c(N)}(x) = T_N(x),\ \forall x \in X$

5.1. *Neutrosophic Decision making*

In this section, a neutrosophic approach to solve a deterministic model (21) is presented. Indeterminacy part present in the optimization problem considered, is handled by neutrosophic programming approach as it simultaneously maximizes the degree of satisfaction(truth) and the degree of dissatisfaction(falsity) and minimizes the degree of satisfaction to some extent(Indeterminacy) of neutrosophic decis ion [21, 26]. A conjunction of neutrosophic goal G_k and neutrosophic constraint C_i is the neutrosophic decision set D_N, that is,

$$D_N = \left(\bigcap_{k=1}^{K} G_k\right)\left(\bigcap_{i=1}^{m} C_i\right)$$
$$= \{\langle x, T_D(x), I_D(x), F_D(x)\rangle \,|x \in X\}$$

where

$T_D(x) = \min \left\{ \begin{array}{l} T_{G_1}(x), T_{G_2}(x), \cdots, T_{G_k}(x); \\ T_{C_1}(x), T_{C_2}(x), \cdots, T_{C_m}(x); \end{array} \right\}, x \in X$

$I_D(x) = \min \left\{ \begin{array}{l} I_{G_1}(x), I_{G_2}(x), \cdots, I_{G_k}(x); \\ I_{C_1}(x), I_{C_2}(x), \cdots, I_{C_m}(x); \end{array} \right\}, x \in X$

$F_D(x) = \max \left\{ \begin{array}{l} F_{G_1}(x), F_{G_2}(x), \cdots, F_{G_k}(x); \\ F_{C_1}(x), F_{C_2}(x), \cdots, F_{C_m}(x); \end{array} \right\}, x \in X$

where $T_D(x), I_D(x), F_D(x)$ are truth, indeterminacy and falsity membership functions respectively of neutrosophic decision set D_N. To formulate the membership function for the deterministic model (21) for the uncertain MOTP, the upper bound U_k and lower bound L_k for each objective function is calculated. By solving K objective function individually subject to the constraints we obtained k solutions $\boldsymbol{x_1, x_2, \cdots, x_K}$.

To find the bounds for each objective function, these K solutions are substituted in each objective function.

$$\begin{aligned} &(i.e.)\, U_k = \max\{F_k(\boldsymbol{x_1}), F_k(\boldsymbol{x_2}), \cdots, F_k(\boldsymbol{x_K})\} \\ &\text{and } L_k = \min\{F_k(\boldsymbol{x_1}), F_k(\boldsymbol{x_2}), \cdots, F_k(\boldsymbol{x_K})\} \end{aligned} \tag{24}$$

Hence, the upper and lower bounds for truth, falsity and indeterminacy membership function are given by

$$\left. \begin{aligned} &U_k^T = U_k, L_k^T = L_k \\ &U_k^F = U_k^T, L_k^F = L_k^T + t_k(U_k^T - L_k^T) \\ &U_k^I = L_k^T + s_k(U_k^T - L_k^T), L_k^I = L_k^T \end{aligned} \right\} \tag{25}$$

where t_k, s_k are predetermined real numbers in (0,1).

Using the above upper and lower bounds, the membership functions of truth, indeterminacy and falsity of model (21) can be interpreted as follows:

$$T_k(Z_k^*(x)) = \begin{cases} 1 & \text{if } Z_k^*(x) < L_k^T \\ \dfrac{U_k^T - Z_k^*(x)}{U_k^T - L_k^T} & \text{if } L_k^T \le Z_k^*(x) \le U_k^T \\ 0 & \text{if } Z_k^*(x) > U_k^T \end{cases} \tag{26}$$

$$I_k(Z_k^*(x)) = \begin{cases} 1 & \text{if } Z_k^*(x) < L_k^I \\ \dfrac{U_k^I - Z_k^*(x)}{U_k^I - L_k^I} & \text{if } L_k^I \le Z_k^*(x) \le U_k^I \\ 0 & \text{if } Z_k^*(x) > U_k^I \end{cases} \tag{27}$$

$$F_k(Z_k^*(x)) = \begin{cases} 1 & \text{if } Z_k^*(x) > U_k^F \\ \dfrac{Z_k^F - L_k^*(x)}{U_k^F - L_k^F} & \text{if } L_k^F \le Z_k^*(x) \le U_k^F \\ 0 & \text{if } Z_k^*(x) < L_k^F \end{cases} \tag{28}$$

where $U_k^{(.)} \neq L_k^{(.)}$ for all objectives. The value of this membership function is set to one, if $U_k^{(.)} = L_k^{(.)}$. Following the Bellman and Zadeh [26], the neutroshopic optimization model of (21) can be stated as follows

$$\begin{aligned}
&\text{Max} \min_k \{T_k(Z_k^*(x))\} : k = 1, 2, \cdots, K\\
&\text{Min} \max_k \{F_k(Z_k^*(x))\} : k = 1, 2, \cdots, K\\
&\text{Max} \min_k \{I_k(Z_k^*(x))\} : k = 1, 2, \cdots, K
\end{aligned}$$

where

$$\begin{aligned}
&\text{Min } Z_k^*(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}(\psi_{ij}^k)^{-1}(\alpha)x_{ij},\, k = 1, 2, \cdots, K\\
&\text{subject to}\\
&\sum_{j=1}^{n} x_{ij} \leq (\varphi_i)^{-1}(1-\alpha_i)\, i = 1, 2, \cdots, m\\
&\sum_{i=1}^{m} x_j \geq (\theta_j)^{-1}(\beta_j),\, j = 1, 2, \cdots, n\\
&x_{ij} \geq 0, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n
\end{aligned} \tag{29}$$

By using the auxiliary parameters, the above problem can be transformed as

$$\begin{aligned}
&\text{Max}\, \lambda_T\\
&\text{Max}\, \lambda_I\\
&\text{Min}\, \lambda_F\\
&\text{subject to}\\
&T_{z_k}(x) \geq \lambda_T,\, I_{z_k}(x) \geq \lambda_I,\, F_{z_k}(x) \leq \lambda_F\\
&\sum_{j=1}^{n} x_{ij} \leq (\varphi_i)^{-1}(1-\alpha_i)\, i = 1, 2, \cdots, m\\
&\sum_{i=1}^{m} x_{ij} \geq (\theta_j)^{-1}(\beta_j),\, j = 1, 2, \cdots, n\\
&x_{ij} \geq 0, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n\\
&\lambda_T \geq \lambda_I, \lambda_T \geq \lambda_F, \lambda_T + \lambda_I + \lambda_F \leq 3, \lambda_T, \lambda_I, \lambda_F \in [0, 1]
\end{aligned} \tag{30}$$

The simplified model of uncertain MOTP (21) can be represented as follows:

$$\begin{aligned}
&\text{Max}\, \lambda_T - \lambda_F + \lambda_I \\
&\text{subject to} \\
&\sum_{j=1}^{n} x_{ij} \leq (\varphi_i)^{-1}(1-\alpha_i),\ i = 1, 2, \cdots, m \\
&\sum_{i=1}^{m} x_{ij} \geq (\theta_j)^{-1}(\beta_j),\ j = 1, 2, \cdots, n \\
&x_{ij} \geq 0, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \\
&Z_k^*(x) + (U_k^T - L_k^T)\lambda_T \leq U_k^T \\
&Z_k^*(x) + (U_k^I - L_k^I)\lambda_I \leq U_k^I \\
&Z_k^*(x) - (U_k^F - L_k^F)\lambda_F \leq L_k^F \\
&\lambda_T \geq \lambda_I, \lambda_T \geq \lambda_F, \lambda_T + \lambda_I + \lambda_F \leq 3, \\
&\lambda_T, \lambda_I, \lambda_F \in [0, 1]
\end{aligned} \tag{31}$$

5.2. *Algorithm for solving uncertain MOTP under Neutrosophic environment*

In this section, the algorithm for solving uncertain MOTP under neutosophic environment to obtain the pareto optimal solution is presented.

Step 1: Convert the Uncertain MOTP (14) into a deterministic model by using chance constraint model (21).

Step 2: Solve each objective function individually subject to the constraints.

Let $\boldsymbol{x_1, x_2, \cdots, x_K}$ represent the respective ideal solutions for k objective transportation problems. If all k objectives have same solutions $\boldsymbol{x_1 = x_2 = \cdots = x_K} = \{x_{ij}\}_{i,j=1}^{m,n}$ choose one of them as optimal compromise solution, otherwise go to step 3.

Step 3: Calculate the lower and upper bounds for all objectives functions

$$\begin{aligned}
&U_1 = \text{Max}\,\{F_1(x_1), \ldots, F_1(x_k)\} \\
&U_2 = \text{Max}\,\{F_2(x_1), \ldots, F_2(x_k)\} \\
&\vdots \\
&U_k = \text{Max}\,\{F_k(x_1), \ldots, F_k(x_k)\} \\
&L_1 = \text{Min}\,\{F_1(x_1), \ldots, F_1(x_k)\} \\
&\vdots \\
&L_k = \text{Min}\,\{F_k(x_1), \ldots, F_k(x_k)\}
\end{aligned} \tag{32}$$

Step 4: Define the truth, indeterminacy and falsity membership functions of the objective functions and constraints using equations (26), (27), (28).

Step 5: Formulate the neutrosophic compromise programming model for given the uncertain MOTP using the model (31) and solve it for Pareto optimal solution.

6. **Illustrative example**

Illustrative example from Gurupada et al [27] is considered to demonstrate the proposed approach where all the multi objective functions parameters are considered to be uncertain. The decision maker aims to distribute the product from three sources namely M_1, M_2, M_3 to 4 destinations namely C_1, C_2, C_3 and C_4 in the planning process he likes to optimize the following objective function as

* Minimize the transportation cost (Z_1)
* Minimize the toll tax (Z_2)
* Maximize the profit (Z_3)

TABLE 2. Transportation cost C^1_{ij} (in \$) and loss of time (in week)

	C_1	C_2	C_3	C_4
M_1	(20, .1)	(18, .1)	(22, .1)	(24 , .1)
M_2	(10, 0)	(12, .2)	(15, 0)	(13 , 0)
M_3	(22, 0)	(20, .1)	(24, 1)	(23, .15)

TABLE 3. Toll tax cost C^2_{ij} (in \$) for transportation goods

	C_1	C_2	C_3	C_4
M_1	5	6	4	3
M_2	6	5	5	4
M_3	9	8	8	10

TABLE 4. Cost parameters C^3_{ij} related to profit (in \$) and loss of time (in week).

	C_1	C_2	C_3	C_4
M_1	(3, 0.1)	(3.5, 0.1)	(2.5, 0.1)	(5 ,0.1)
M_2	(3, 0)	(6, 0.2)	(4, 0)	(4 ,0)
M_3	(4, 0)	(3, 0.1)	(4, 1)	(5, 0.15)

The supply parameters a_1, a_2 and a_3 of mines M_1, M_2 and M_3 the demand parameters b_1, b_2, b_3 and b_4 of cities C_1, C_2, C_3 and C_4 follow normal distribution $N(e_i^1, \sigma_i^1)$, for $i = 1, 2, 3$ and $N(e_j^2, \sigma_j^2)$, for $j = 1, 2, 3, 4$ respectively. The data for supply a_i and demand $b_j, \forall i, j$ are presented in table 4 and 5.

TABLE 5. Uncertain supply parameters a_i.

M_1	M_2	M_3
(55, 4)	(60, 5)	(70, 4)

TABLE 6. Uncertain demand parameters b_j.

C_1	C_2	C_3	C_4
(40, 3)	(36, 4)	(35, 5)	(40, 3)

Step 1:

Assume the confidence level as $\alpha = 0.9, \alpha_i = 0.9$ and $\beta_j = 0.9$ for all $i = 1, 2, 3$ and $j = 1, 2, 3, 4$. By using the theorem (4.5), the equivalent deterministic model of the problem is

Min Z_1=Min $Z_1^* = 20.1x_{11} + 18.1x_{12} + 22.1x_{13} + 24.1x_{14} + 10x_{21}$
$+12.2x_{22} + 15x_{23} + 13x_{24} + 22x_{31} + 20.1x_{32} + 25.2x_{33} + 23.2x_{34}$

Min Z_2=Min $Z_2^* = 5x_{11} + 6x_{12} + 4x_{13} + 3x_{14} + 6x_{21} + 5x_{22}$
$+5x_{23} + 4x_{24} + 9x_{31} + 8x_{32} + 8x_{33} + 10x_{34}$

Max Z_3=Min $Z_3^* = -3.1x_{11} - 3.6x_{12} - 2.6x_{13} - 5.1x_{14} - 3x_{21}$
$-6.2x_{22} - 4x_{23} - 4x_{24} - 4x_{31} - 3.1x_{32} - 5.2x_{33} - 5.2x_{34}$

Subject to

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 50.2$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 53.9$

$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 65.2$

$x_{11} + x_{21} + x_{31} = 43.6$

$x_{12} + x_{22} + x_{32} = 40.8$

$x_{13} + x_{23} + x_{33} = 41.1$

$x_{14} + x_{24} + x_{34} = 43.6$

$x_{15} + x_{25} + x_{35} = 0.2$

Step 2: Solving the above objective functions individually, we get

$\boldsymbol{x_1} = (0, 9.1, 41.1, 0, 0, 43.6, 0, 0, 10.3, 0, 0, 31.7, 0, 33.3, 0.2)$

$\boldsymbol{x_2} = (0, 0, 6.6, 43.6, 0, 0, 19.4, 34.5, 0, 0, 43.6, 21.4, 0, 0, 0, 2)$

$\boldsymbol{x_3} = (6.6, 0, 0, 43.6, 0, 12.9, 40.8, 0, 0, 0.2, 24.1, 0, 41.1, 0, 0)$

Clearly $\boldsymbol{x_1 \neq x_2 \neq x_3}$.

Step 3: By using the above solutions, we have

$\boldsymbol{Z_1^*(x_1)} = 3052.65, \boldsymbol{Z_1^*(x_2)} = 3340.14, \boldsymbol{Z_1^*(x_3)} = 3376.1$

$\boldsymbol{Z_2^*(x_1)} = 1108.4, \boldsymbol{Z_2^*(x_2)} = 990.3, \boldsymbol{Z_2^*(x_3)} = 990.9$

$\boldsymbol{Z_3^*(x_1)} = -583.05, \boldsymbol{Z_3^*(x_2)} = -738.54, \boldsymbol{Z_3^*(x_3)} = -844.6$

The upper and lower bounds of each objective functions are as follows:

$U_{Z_1^*} = 3376.1, L_{Z_1^*} = 3052.65, U_{Z_2^*} = 1108.4,$

$L_{Z_2^*} = 990.3, U_{Z_3^*} = -583.05, L_{Z_3^*} = -844.6$

Step 4: Formulate the membership functions of the given objectives using the equations (26), (27) and (28).

For Z_1^*:

$U_{Z_1^*}^T = 3376.1, L_{Z_1^*}^T = 3052.65$

$U_{Z_1^*}^F = 3376.1, L_{Z_1^*}^F = 3052.65 + 323.45t_1$

$U_{Z_1^*}^I = 3052.65 + 323.45s_1, L_{Z_1^*}^I = 3052.65$

$$T_1(Z_1^*(x)) = \begin{cases} 1 & \text{if } Z_1^*(x) < 3052.65 \\ \dfrac{3376.1 - Z_1^*(x)}{3376.1 - 3052.65} & \text{if } 3052.65 \leq Z_1^*(x) \leq 3376.1 \\ 0 & \text{if } Z_1^*(x) > 3376.1 \end{cases}$$

$$I_1(Z_1^*(x)) = \begin{cases} 1 & \text{if } Z_1^*(x) < 3052.65 \\ \dfrac{3052.65 + 323.45s_1 - Z_1^*(x)}{323.45s_1} & \text{if } 3052.65 \leq Z_1^*(x) \leq 3052.65 + 323.45s_1 \\ 0 & \text{if } Z_1^*(x) > 3052.65 + 323.45s_1 \end{cases}$$

$$F_1(Z_1^*(x)) = \begin{cases} 1 & \text{if } Z_1^*(x) > 3376.1 \\ \dfrac{Z_1^*(x) - 3052.65 - 323.45t_1}{323.45 - 323.45t_1} & \text{if } 3052.65 + 323.45t_1 \leq Z_1^*(x) \leq 3376.1 \\ 0 & \text{if } Z_1^*(x) < 3052.65 + t_1(323.45) \end{cases}$$

For Z_2^*:

$U_{Z_2^*}^T = 1108.4, L_{Z_2^*}^T = 990.3$

$U_{Z_2^*}^F = 1108.4, L_{Z_2^*}^F = 990.3 + 118.1t_2$

$U_{Z_2^*}^I = 990.3 + 118.1s_2, L_{Z_2^*}^I = 990.3$

$$T_2(Z_2^*(x)) = \begin{cases} 1 & \text{if } Z_2^*(x) < 990.3 \\ \dfrac{1108.4 - Z_2^*(x)}{118.1} & \text{if } 990.3 \leq Z_2^*(x) \leq 1108.4 \\ 0 & \text{if } Z_2^*(x) > 1108.4 \end{cases}$$

$$I_2(Z_2^*(x)) = \begin{cases} 1 & \text{if } Z_2^*(x) < 990.3 \\ \dfrac{990.3 + 118.1s_2 - Z_2^*(x)}{118.1s_2} & \text{if } 990.3 \leq Z_2^*(x) \leq 990.3 + 118.1s_2 \\ 0 & \text{if } Z_2^*(x) > 990.3 + 118.1s_2 \end{cases}$$

$$F_2(Z_2^*(x)) = \begin{cases} 1 & \text{if } Z_2^*(x) > 1108.4 \\ \dfrac{Z_2^*(x) - 990.3 - 118.1t_2}{118.1 - 118.1t_2} & \text{if } 990.3 + 118.1t_2 \leq Z_2^*(x) \leq 1108.4 \\ 0 & \text{if } Z_2^*(x) < 990.3 + 118.1t_2 \end{cases}$$

For Z_3^*:

$U^T_{Z_3^*} = -583.05, L^T_{Z_3^*} = -844.6$

$U^F_{Z_3^*} = -583.05, L^F_{Z_3^*} = -844.6 + 261.55t_3$

$U^I_{Z_3^*} = -844.6 + 261.55s_3, L^I_{Z_3^*} = -844.6$

$$T_3(Z_3^*(x)) = \begin{cases} 1 & \text{if } Z_3^*(x) < -844.6 \\ \dfrac{-583.05 - Z_3^*(x)}{261.55} & \text{if } -844.6 \leq Z_3^*(x) \leq -583.05 \\ 0 & \text{if } Z_3^*(x) > -583.05 \end{cases}$$

$$I_3(Z_3^*(x)) = \begin{cases} 1 & \text{if } Z_3^*(x) < -844.6 \\ \dfrac{-844.6 + 261.55s_3 - Z_3^*(x)}{261.55s_3} & \text{if } -844.6 \leq Z_3^*(x) \leq -844.6 + 261.55s_3 \\ 0 & \text{if } Z_3^*(x) > -844.6 + 261.55s_3 \end{cases}$$

$$F_3(Z_3^*(x)) = \begin{cases} 1 & \text{if } Z_3^*(x) > -583.05 \\ \dfrac{Z_3^*(x) + 844.6 - 261.55t_3}{261.55 - 261.55t_3} & \text{if } -844.6 + 261.55t_3 \leq Z_3^*(x) \leq -583.05 \\ 0 & \text{if } Z_3^*(x) < -844.6 + 261.55t_3 \end{cases}$$

Step 5: The neutrosophic compromise programming model for given the uncertain MOTP using the model (31) is

$$\text{Max } \lambda_T - \lambda_F + \lambda_I$$

subject to

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 50.2$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 53.9$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 65.2$$

$$x_{11} + x_{21} + x_{31} = 43.6$$

$$x_{12} + x_{22} + x_{32} = 40.8$$

$$x_{13} + x_{23} + x_{33} = 41.1$$

$$x_{14} + x_{24} + x_{34} = 43.6$$

$$x_{15} + x_{25} + x_{35} = 0.2$$

$20.1x_{11} + 18.1x_{12} + 22.1x_{13} + 24.1x_{14} + 10x_{21} + 12.2x_{22} + 15x_{23} + 13x_{24} + 22x_{31} + 20.1x_{32} + 25.2x_{33} + 23.2x_{34} + 233.45\lambda_T \leq 3376.1$

$5x_{11}+6x_{12}+4x_{13}+3x_{14}+6x_{21}+5x_{22}+5x_{23}+4x_{24}+9x_{31}+8x_{32}+8x_{33}+10x_{34}+118.1\lambda_T \leq 1108.4$

$-3.1x_{11} - 3.6x_{12} - 2.6x_{13} - 5.1x_{14} - 3x_{21} - 6.2x_{22} - 4x_{23} - 4x_{24} - 4x_{31} - 3.1x_{32} - 5.2x_{33} - 5.2x_{34}$

$+261.55\lambda_T \leq -583.05$

$20.1x_{11} + 18.1x_{12} + 22.1x_{13} + 24.1x_{14} + 10x_{21} + 12.2x_{22} + 15x_{23} + 13x_{24} + 22x_{31} + 20.1x_{32} + 25.2x_{33} + 23.2x_{34} + 323.45t_1(\lambda_T - 1) \leq 3052.65$

$5x_{11}+6x_{12}+4x_{13}+3x_{14}+6x_{21}+5x_{22}+5x_{23}+4x_{24} +9x_{31}+8x_{32}+8x_{33}+10x_{34}+118.1t_2(\lambda_T-1) \leq 990.3$

$-3.1x_{11} - 3.6x_{12} - 2.6x_{13} - 5.1x_{14} - 3x_{21} - 6.2x_{22} - 4x_{23} - 4x_{24} - 4x_{31} - 3.1x_{32} - 5.2x_{33} - 5.2x_{34} + 261.55t_3(\lambda_T - 1) \leq -844.6$

$20.1x_{11} + 18.1x_{12} + 22.1x_{13} + 24.1x_{14} + 10x_{21} + 12.2x_{22} + 15x_{23} + 13x_{24} + 22x_{31} + 20.1x_{32} + 25.2x_{33} + 23.2x_{34} + (\lambda_F - 1)(3052.65 + 323.45s_1) - 3376.1\lambda_F \leq 0$

$5x_{11} + 6x_{12} + 4x_{13} + 3x_{14} + 6x_{21} + 5x_{22} + 5x_{23} + 4x_{24} + 9x_{31} + 8x_{32} + 8x_{33} + 10x_{34} + (\lambda_F - 1)(990.3 + 118.1s_2) - 1108.4\lambda_F \leq 0$

$-3.1x_{11} - 3.6x_{12} - 2.6x_{13} - 5.1x_{14} - 3x_{21} - 6.2x_{22} - 4x_{23} - 4x_{24} - 4x_{31} - 3.1x_{32} - 5.2x_{33} - 5.2x_{34} + (\lambda_F - 1)(-844.6 + 261.55s_3) + 583.05\lambda_F \leq 0$

$\lambda_T \geq \lambda_I, \lambda_T \geq \lambda_F, \lambda_T + \lambda_F + \lambda_I \leq 3, \lambda_T \leq 1, \lambda_I \leq 1, \lambda_F \leq 1$

$0 \leq t_1, s_1 \leq 323.5, 0 \leq t_2, s_2 \leq 118.1, 0 \leq t_3, s_3 \leq 261.55, \lambda_T, \lambda_F, \lambda_I \in [0, 1]$

solving the above model by using the LINGO (17.0) software, we get

$\lambda_T = 0.523, \lambda_F = 0, \lambda_I = 0.52,$

$x_{11} = 21.1, x_{12} = 28.1, x_{14} = 0.9, x_{22} = 11.2,$

$x_{24} = 42.6, x_{31} = 22, x_{32} = 1.4, x_{33} = 41.1, x_{35} = 0.2$

$t_1 = 1, t_2 = 1.2, t_3 = 0.9,$

$s_1 = 1.2, s_2 = 1.2, s_3 = 0.47,$

$Z_1 = 3192.71, Z_2 = 1041.2, Z_3 = 717.06.$

Table 7 illustrates the comparison between the results obtained from Fuzzy Multi Choice goal programming method and the proposed method. Table 8 provides the comparison study of solution obtained by fuzzy goal programming method and proposed method.

In Gurupada et al [27] work, wherein he proved that Fuzzy multi choice goal programming was more efficient in providing an optimal solution than by employing goal programming and revised multi choice goal programming approach. Contrasting to his work in the proposed method, the decision maker need not fix the goals of the objective function using any of the existing techniques, to get a better optimal value for the objective function. In short, we have overcome the difficulty of the decision maker to fix the objective value goal.

Clearly it can be seen that by using neutrosophic compromise programming approach, we obtained an improvised pareto optimal solution. As in table 8, we can observe that the proposed method yields a more minimal value for transportation cost and a considerable increase in profit. As neutrosophic programming explores the indeterminacy part of a optimization problem, it helps the decision maker to get better results.

Table 7. Comparison between the pareto optimal solution of the existing and the proposed method.

Method	Pareto-optimal solution
Fuzzy Multi Choice goal programming method [27]	$x_{11} = 3.12$, $x_{12} = 0$, $x_{13} = 18.95$, $x_{14} = 29.10$, $x_{21} = 11.26$, $x_{22} = 25.07$, $x_{23} = 4.36$, $x_{24} = 14.54$, $x_{31} = 29.26$, $x_{32} = 15.42$, $x_{33} = 17.74$, $x_{34} = 0$
Proposed method	$x_{11} = 21.1$, $x_{12} = 28.1$, $x_{14} = 0.9$, $x_{22} = 11.2$, $x_{24} = 42.6$, $x_{31} = 22$, $x_{32} = 1.4$, $x_{33} = 41.1$, $x_{35} = 0.2$

Table 8. The comparison between the existing and the proposed method.

Method	Min Z_1	Min Z_2	Max Z_3
Fuzzy Multi Choice goal programming method [27]	3400	980.13	650
Proposed method	3192.71	1041.2	717.06

7. Result and Discussion

In our work, we have obtained the compromise solution of the Uncertain MOTP using the neutrosphic technique.

Table 7 illustrates the comparison between the results obtained from Fuzzy Multi Choice goalprogramming method and the proposed method. Table 8 provides the comparison study ofsolution obtained by fuzzy goal programming method and proposed method.In Gurupada et al [27] work, wherein he proved that Fuzzy multi choice goal programmingwas more efficient in providing an optimal solution than by employing goal programming andrevised multi choice goal programming approach. Contrasting to his work in the proposedmethod, the decision maker need not fix the goals of the objective function using any of theexisting techniques, to get a better optimal value for the objective function. In short, we haveovercome the difficulty of the decision maker to fix the objective value goal.Clearly it can be seen that by using neutrosophic compromise programming approach, weobtained an improvised pareto optimal solution. As in Table 8, we can observe that the proposed method yields a more minimal value for transportation cost and a considerable increasein profit. As neutrosophic programming explores the indeterminacy part of a optimizationproblem, it helps the decision maker to get better results.

8. Implications

This paper used the neutrosophic approach to discuss the uncertain MOTP. The literature review section includes studies that are comparable to these ones. According to the author's knowledge, no research has been done on applying the neutrosophic method to solve the uncertain MOTP. The method for solving uncertain MOTP utilizing the neutrosophic technique has been provided in the suggested work to close the aforementioned research gap. The efficiency of the proposed work has been demonstrated by comparing Gurupata's [27]'s work. It has been explained that the suggested work will assist the decision maker to have the suitable and desired transportation plan.

9. Conclusion

In this work, a procedure to solve multi objective transportation problem with uncertainvariables is studied under neutrosophic environment. The uncertain MOTP is converted into an equivalent chance constraint deterministic model with the use of operational law of uncertain variables. Then using neutrosophic compromise programming approach the best compromise solution is obtained. Since the solution searches of UMOTP based on different membership function such as truth, indeterminacy and falsity, it allows the decision maker to know about the various functions and provides more practicable and reasonable compromise solution. More info It has been established that, in order to obtain a better optimal value for the objective function, the decision maker does not need to fix the goals of the objective function using any of the available strategies. In other words, we have succeeded in fixing the decision-difficulty maker's with regard to the objective value aim. A numerical example had been considered and obtained the compromise solution and is tabulated in Table 8. It is evident that we were able to achieve an improvised pareto optimum solution by applying the neutrosophic compromise programming approach.

Conflicts of Interest: The authors confirm that there are no known conflicts of interest associated with this publication.

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