NSS

Neutrosophic Sets and Systems, Vol. 53, 2023

University of New Mexico

An intelligent fuzzy parameterized MADM-approach to optimal selection of electronic appliances based on neutrosophic hypersoft expert set

Muhammad Ihsan[1], Muhammad Saeed[1] and Atiqe Ur Rahman[2,*]

[1,2] University of Management and Technology Lahore, Pakistan.

mihkhb@gmail.com, muhammad.saeed@umt.edu.pk, aurkhb@gmail.com

*Correspondence: aurkhb@gmail.com

Abstract. When compared to its extension, the hypersoft set, which deals with discontinuous attribute-valued sets corresponding to different attributes, the soft set only works with a single set of attributes. Numerous scholars created models based on soft sets to address issues in a variety of domains, including decision-making and medical diagnostics. However, these models only take into account one expert, which causes numerous issues for users, particularly when creating questions. We provide a fuzzy parameterized neutrosophic hypersoft expert set to eliminate this mismatch. In addition to addressing the issue of dealing with a single expert, this approach also addresses the problem of soft sets not being adequate for discontinuous attribute-valued sets corresponding to different attributes. The notion of fuzzy parameterized neutrosophic hypersoft expert sets, which combines fuzzy parameterized neutrosophic sets and hypersoft expert sets, is first introduced in this work. Examples are provided to help illustrate some key fundamental concepts, aggregation operations and results. A decision-making application is shown at the end to demonstrate the viability of the suggested theory.

Keywords: Soft set; Soft expert set; Neutrosophic set; Hypersoft set; Fuzzy parameterized neutrosophic hypersoft expert set.

---

1. **Introduction**

For a correct description of an object in an ambiguous and uncertain environment, we sometimes consider both the truth membership and the falsity membership in professional systems, belief systems, and information systems. The neutrosophic set was defined by Smarandache [1–3] as a generalisation of classical sets, fuzzy sets, and intuitionistic fuzzy sets. Membership functions are used to define fuzzy sets [4], while membership and nonmembership functions describe intuitionistic fuzzy sets [5], which are used to solve problems involving imprecise, ambiguous, and inconsistent data. The neutrosophic set has numerous applications in a variety of disciplines, including topology, control theory, databases, and medical diagnosis.

Truth membership, indeterminacy membership, and false membership are all wholly independent in the neutrosophic set, where the indeterminacy is clearly quantified. The neutrosophic set and set-theoretic view operators need to be described from a scientific or technical perspective. If not, it will be challenging to apply in actual applications. Therefore, Wang et al. [6] described the set-theoretic operations and various properties of single-valued neutrosophic sets (SVNS). In both theories and applications, work on neutrosophic sets (NS) and their hybrid structures has made rapid progress.advanced quickly [7].

In his conceptualization of soft set theory, Molodtsov [8] described it as a brand-new family of parameterized subsets of the universe of discourse. Different structures of convexity (concavity) on an s-set were introduced by Rahman et al. [9,10]. They explored the different convexity and concavity characteristics in the context of fs-set, s-set, and hypersoft set (an extension of s-set) settings with some altered findings. As a parametrization technique to handle uncertainty, Maji et al. [11] developed fuzzy soft set. This idea has been expanded upon and used in other domains by scholars [12]. Soft expert set (SE-set) and fuzzy soft expert set (FSE-set) are concepts developed by Alkhazaleh et al. [19, 20]. They talked about how they could be used in decision-making. Convexity-cum-concavity on SE-set was conceptualised by Ihsan et al. [21], who also highlighted some of its characteristics. The convexity on the FSE-set was once more gestated and its specific qualities were elucidated by Ihsan et al. [22]. In their conceptualization of intuitionistic fuzzy soft expert sets, Broumi et al. [23] presented their use in decision-making.

Through the substitution of a multi-attribute valued function for a single attribute-valued function in 2018, Smarandache [24] extended soft set to hypersoft set. Saeed et al. [25] developed the idea and covered the principles of the hypersoft set, including its relation, sub relation, complement relation, function, matrices, and operations on hypersoft matrices, as well as its hypersoft subset, complement, and non hypersoft set. Mujahid et al. [25] discussed hypersoft points in several fuzzy-like environments. Complex hypersoft set was defined by Rahman et al. [27], who also created its hybrids with the complex fuzzy set, complex intuitionistic fuzzy set, and complex neutrosophic set. The principles, such as subset, equal sets, null set, absolute set, etc., as well as the theoretic operations, such as complement, union, intersection, etc., were also covered. Convexity and concavity were theorised on a hypersoft set by Rahman et al. [28], who also provided their pictorial representations and examples to illustrate them. Rahman et al. [33] created the preliminary HS-set structure and provided an application for the optimal chemical material choice in DM. Rahman et al. [34] developed a novel method for studying neutrosophic hypersoft graphs and discussed some of its characteristics. The aggregation operations of complex FHS-set were first employed in DMPs by Rahman et al. (AUR5). The structure of interval-valued complex FHS-set was also devised by them. The bijective HS-set was conceptualised by Rahman et al. [36] and its uses in DMPs were covered. In order to know the opinions of various experts in various models when attributed sets are further divided into disjoint attribute valued sets, Ihsan et al. [37] generalised the HS-set to hypersoft expert set (HSE-set). The fuzzy hypersoft expert set (FHSE-set) was conceptualised by Ihsan et al. [38], who also used the proposed technique to demonstrate how DMPs were used.

Çağman et al. [39] applied a significant degree to the parameters and conceptualised the fuzzy parameterized soft set (FPS-set). In order to create hybrids of the fuzzy parameterized soft expert set (FPSE-set) for use in DMPs, Bashir et al. [40] combined the structures of fuzzy parameterized with SE-set. By converting a single set of attributes into several disjoint attribute valued sets, Rahman et al. [41] enhanced the work of fuzzy parameterized soft set to fuzzy parameterized hypersoft set and examined the applications in DMPs. A novel structure for the fuzzy parameterized neutrosophic hypersoft expert set is required by the literature. New ideas on the fuzzy parameterized neutrosophic hypersoft expert set are created as a result. Figure 1 shows how the rest of the paper is organised.

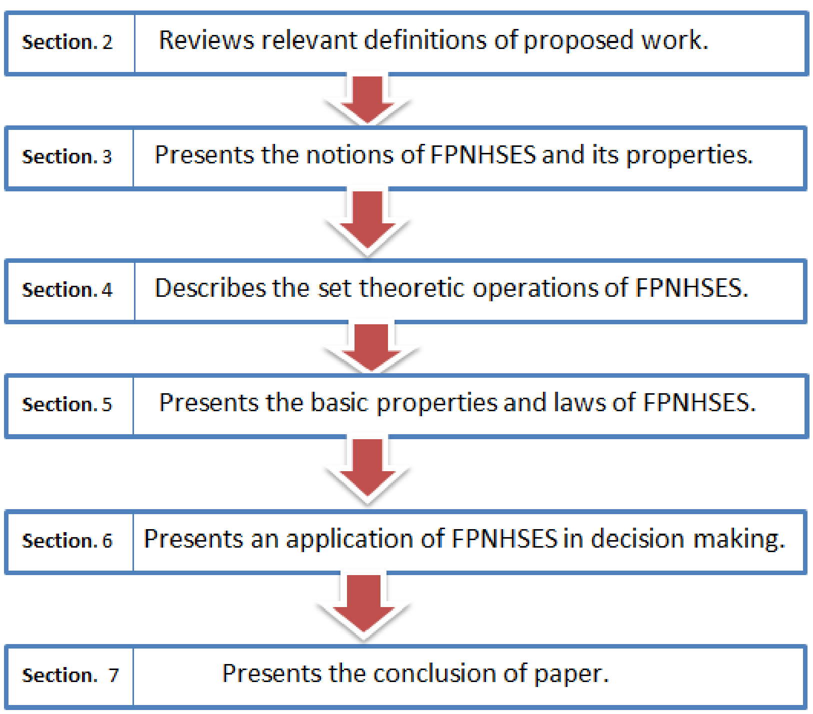

FIGURE 1. Organization of the paper

2. **Preliminaries**

This section provides definitions and explanations of several key terminology and concepts connected to the primary study.

Definition 2.1. [13] A neutrosophic set $\mathfrak{N}$ in $\triangledown$ is defined by $\mathfrak{N} = \{< \curlywedge, (\mathsf{T}_{\mathfrak{N}}(\curlywedge), \mathsf{I}_{\mathfrak{N}}(\curlywedge), F_{\mathfrak{N}}(\curlywedge)) >: \curlywedge \in \mathsf{E}, \mathsf{T}_{\mathfrak{N}}, \mathsf{I}_{\mathfrak{N}}, F_{\mathfrak{N}} \in]^{-}0, 1^{+}[\}$ where $\mathsf{T}_{\mathfrak{N}}, \mathsf{I}_{\mathfrak{N}}, F_{\mathfrak{N}}$ are truth, indeterminacy, and falsity membership function and $0^{-} \leq \mathsf{T}_{\mathfrak{N}}(\curlywedge), \mathsf{I}_{\mathfrak{N}}(\curlywedge), F_{\mathfrak{N}}(\curlywedge) \leq 3^{+}$.

Definition 2.2. [10] Let $\mathfrak{Q}$ and $\mathfrak{P}$ are two neutrosophic sets such that
$\mathfrak{Q} = \{< \curlywedge, (\mathsf{T}_{\mathfrak{Q}}(\curlywedge), \mathsf{I}_{\mathfrak{Q}}(\curlywedge), F_{\mathfrak{Q}}(\curlywedge)) >: \curlywedge \in \mathsf{E}, \mathsf{T}_{\mathfrak{Q}}, \mathsf{I}_{\mathfrak{Q}}, F_{\mathfrak{Q}} \in]^{-}0, 1^{+}[\}$,
$\mathfrak{P} = \{< \curlywedge, (\mathsf{T}_{\mathfrak{P}}(\curlywedge), \mathsf{I}_{\mathfrak{P}}(\curlywedge), F_{\mathfrak{P}}(\curlywedge)) >: \curlywedge \in \mathsf{E}, \mathsf{T}_{\mathfrak{P}}, \mathsf{I}_{\mathfrak{P}}, F_{\mathfrak{P}} \in]^{-}0, 1^{+}[\}$,
then, the following operations between two neutrosophic sets can be defined like subset, complement, union and intersection:

(1) Neutrosophic set $\mathfrak{Q}$ is a subset of another Neutrosophic set $\mathfrak{P}$ if
$\mathsf{T}_{\mathfrak{Q}}(\curlywedge) \leq \mathsf{T}_{\mathfrak{P}}(\curlywedge)$, $\mathsf{I}_{\mathfrak{Q}} \geq \mathsf{I}_{\mathfrak{P}}$, $F_{\mathfrak{Q}}(\curlywedge) \leq F_{\mathfrak{P}}(\curlywedge)$.
(2) The compliment of neutrosophic set $\mathfrak{Q}$ is defined as
$\mathfrak{Q}^{c} = \{< \curlywedge, (\mathsf{T}_{\mathfrak{Q}}(\curlywedge), 1 - \mathsf{I}_{\mathfrak{Q}}(\curlywedge), F_{\mathfrak{Q}}(\curlywedge)) >: \curlywedge \in \mathsf{E}, \mathsf{T}_{\mathfrak{Q}}, \mathsf{I}_{\mathfrak{Q}}, F_{\mathfrak{Q}} \in]^{-}0, 1^{+}[\}$,
(3) The union of neutrosophic sets between $\mathfrak{Q}$ and $\mathfrak{P}$ is defined by
$max(\mathsf{T}_{\mathfrak{Q}}(\curlywedge), \mathsf{T}_{\mathfrak{P}}(\curlywedge)$, $min(\mathsf{I}_{\mathfrak{Q}}(\curlywedge), \mathsf{I}_{\mathfrak{P}}(\curlywedge))$, $min(F_{\mathfrak{Q}}(\curlywedge), F_{\mathfrak{P}}(\curlywedge))$,
(4) The intersection of neutrosophic sets between $\mathfrak{Q}$ and $\mathfrak{P}$ is defined by
$min(\mathsf{T}_{\mathfrak{Q}}(\curlywedge), \mathsf{T}_{\mathfrak{P}}(\curlywedge)$, $max(\mathsf{I}_{\mathfrak{Q}}(\curlywedge), \mathsf{I}_{\mathfrak{P}}(\curlywedge))$, $max(F_{\mathfrak{Q}}(\curlywedge), F_{\mathfrak{P}}(\curlywedge))$.

Definition 2.3. [23] Let $\mathcal{I}$ represents set of specialists(experts) and set of parameters is denoted by $\mathcal{L}$, $\mathcal{O} = \mathcal{L} \times \mathcal{I} \times \mathcal{U}$ with $\mathcal{S} \subseteq \mathcal{O}$. While $\mathcal{U}$ represents a set of conclusions i.e, $\mathcal{U}=\{0 = disagree, 1 = agree\}$ and $\hat{\triangle}$ represents the universe with power set $\mathsf{P}(\hat{\triangle})$ and $\mathbb{I} = [0, 1]$. A $\mathcal{FPSVNSE}$-set can be described as a pair $(\mathfrak{g}_{\wedge}, \mathcal{R})$ with $\mathfrak{g}_{\wedge}$ is $\mathfrak{g}_{\wedge} : \mathcal{R} \to \mathsf{P}(\hat{\triangle})$ such that $\mathsf{P}(\hat{\triangle})$ is going to use for collection of all SVN subsets of $\hat{\triangle}$ and $\mathcal{R} \subseteq \mathcal{O}$.

Definition 2.4. [24] An agree $\mathcal{FPSVNSE}$-set can be defined as a subset of $\mathcal{FPSVNSE}$-set and shown as: $(\mathfrak{g}_{\wedge}, \mathcal{R})^{1} = \{\mathfrak{g}_{\wedge}(\ddot{v}) : \ddot{v} \in \mathcal{L} \times \mathcal{I} \times 1\}$.

Definition 2.5. [24] An disagree $\mathcal{FPSVNSE}$-set can be defined as a subset of $\mathcal{FPSVNSE}$-set and shown as: $(\mathfrak{g}_{\wedge}, \mathcal{R})^{0} = \{\mathfrak{g}_{\wedge}(\ddot{v}) : \ddot{v} \in \mathcal{L} \times \mathcal{I} \times 0\}$.

Definition 2.6. [27] Considering disjoint sets $\underline{\mathrm{H}}_1, \underline{\mathrm{H}}_2, \underline{\mathrm{H}}_3, \ldots.., \underline{\mathrm{H}}_w$ as a corresponding attribute values for w different characteristics $\underline{\mathrm{h}}_1, \underline{\mathrm{h}}_2, \underline{\mathrm{h}}_3, \ldots.., \underline{\mathrm{h}}_w$. Then hypersoft set can be considered as a pair $(\text{Æ}, \curlyvee)$, where $\curlyvee = \underline{\mathrm{H}}_1 \times \underline{\mathrm{H}}_2 \times \mathrm{H}_3 \times \ldots.. \times \underline{\mathrm{H}}_m$ and $\text{Æ} : \curlyvee \to \mathsf{P}(\Delta)$.

3. Fuzzy Parameterized Neutrosophic Hypersoft Expert Set ($\mathcal{FPNHSE}$-set)

Fuzzy parameterized single valued neutrosophic soft expert set, an existing idea, has been used to build fuzzy parameterized neutrosophic hypersoft expert set in this part. Here, several fundamental qualities are shown.

Definition 3.1. Fuzzy Parameterized Neutrosophic Hypersoft Expert Set A fuzzy parameterized neutrosophic hypersoft expert set $\Psi_{\mathcal{F}}$ over $\hat{\triangle}$ is defined as
$\Psi_{\mathcal{F}} = \left\{ \left(\left(\frac{\hat{q}}{\mu_{\mathcal{F}}(\hat{q})}, \hat{E}_i, \hat{O}_i \right), \frac{\hat{\delta}}{\psi_{\mathcal{F}}(\hat{\delta})} \right); \forall \hat{q} \in \mathcal{Q}, \hat{E}_i \in \mathcal{I}, \hat{O}_i \in \mathcal{U}, \hat{\delta} \in \hat{\triangle} \right\}$ where

(1) $\mu_{\mathcal{F}} : \check{\mathcal{J}} \to \mathsf{FP}(\hat{\triangle})$
(2) $\psi_{\mathcal{F}} : \check{\mathcal{J}} \to \mathsf{NP}(\hat{\triangle})$ is called approximate function of $\mathcal{FPNHSE}$-set
(3) $\check{\mathcal{J}} \subseteq \mathcal{H} = \mathcal{L} \times \mathcal{I} \times \mathcal{U}$ with $\mathcal{S} \subseteq \mathcal{O}$.
(4) where $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, ..., \mathcal{Q}_r$ are different sets of parameter corresponding to r different parameters $q_1, q_2, q_3, ..., q_r$.
(5) $\mathcal{I}$ be a set of specialists (operators)
(6) $\mathcal{U}$ be a set of conclusions.

Example 3.2. Suppose that a college chain is searching for a construction company to upgrade the college building with globalisation and requires certain specialists(experts) to evaluate its working. Let $\Delta = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ be a set of companies and $\mathcal{G}_1 = \{p_{11}, p_{12}\}$, $\mathcal{G}_2 = \{p_{21}, p_{22}\}$, $\mathcal{G}_3 = \{p_{31}, p_{32}\}$ be disjoint attributive sets for distinct attributes p_1= cheap, p_2= standard, p_3= cooperative. Now $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3$

$$\mathcal{G} = \left\{ \begin{array}{c} \mho_1/0.2/0.2 = (p_{11}, p_{21}, p_{31}), \mho_2/0.3 = (p_{11}, p_{21}, p_{32}), \\ \mho_3/0.4 = (p_{11}, p_{22}, p_{31}), \mho_4/0.5 = (p_{11}, p_{22}, p_{32}), \\ \mho_5/0.6 = (p_{12}, p_{21}, p_{31}), \mho_6/0.7 = (p_{12}, p_{21}, p_{32}), \\ \mho_7/0.8 = (p_{12}, p_{22}, p_{31}), \mho_8/0.9 = (p_{12}, p_{22}, p_{32}) \end{array} \right\}$$

Now $\mathcal{H} = \mathcal{G} \times \mathcal{D} \times \mathbb{C}$

$$\mathcal{H} = \left\{ \begin{array}{c} (\mho_1/0.2, c, 0), (\mho_1/0.2, c, 1), (\mho_1/0.2, d, 0), (\mho_1/0.2, d, 1), (\mho_1/0.2, e, 0), (\mho_1/0.2, e, 1), \\ (\mho_2/0.3, c, 0), (\mho_2/0.3, c, 1), (\mho_2/0.3, d, 0), (\mho_2/0.3, d, 1), (\mho_2/0.3, e, 0), (\mho_2/0.3, e, 1), \\ (\mho_3/0.4, c, 0), (\mho_3/0.4, c, 1), (\mho_3/0.4, d, 0), (\mho_3/0.4, d, 1), (\mho_3/0.4, e, 0), (\mho_3/0.4, e, 1), \\ (\mho_4/0.5, c, 0), (\mho_4/0.5, c, 1), (\mho_4/0.5, d, 0), (\mho_4/0.5, d, 1), (\mho_4/0.5, e, 0), (\mho_4/0.5, e, 1), \\ (\mho_5/0.6, c, 0), (\mho_5/0.6, c, 1), (\mho_5/0.6, d, 0), (\mho_5/0.6, d, 1), (\mho_5/0.6, e, 0), (\mho_5/0.6, e, 1), \\ (\mho_6/0.7, c, 0), (\mho_6/0.7, c, 1), (\mho_6/0.7, d, 0), (\mho_6/0.7, d, 1), (\mho_6/0.7, e, 0), (\mho_6/0.7, e, 1), \\ (\mho_7/0.8, c, 0), (\mho_7/0.8, c, 1), (\mho_7/0.8, d, 0), (\mho_7/0.8, d, 1), (\mho_7/0.8, e, 0), (\mho_7/0.8, e, 1), \\ (\mho_8/0.9, c, 0), (\mho_8/0.9, c, 1), (\mho_8/0.9, d, 0), (\mho_8/0.9, d, 1), (\mho_8/0.9, e, 0), (\mho_8/0.9, e, 1) \end{array} \right\}$$

let

$$\mathbb{Q} = \left\{ \begin{array}{l} (\mho_1/0.2, c, 0), (\mho_1/0.2, c, 1), (\mho_1/0.2, d, 0), (\mho_1/0.2, d, 1), (\mho_1/0.2, e, 0), (\mho_1/0.2, e, 1), \\ (\mho_2/0.3, c, 0), (\mho_2/0.3, c, 1), (\mho_2/0.3, d, 0), (\mho_2/0.3, d, 1), (\mho_2/0.3, e, 0), (\mho_2/0.3, e, 1), \\ (\mho_3/0.4, c, 0), (\mho_3/0.4, c, 1), (\mho_3/0.4, d, 0), (\mho_3/0.4, d, 1), (\mho_3/0.4, e, 0), (\mho_3/0.4, e, 1), \end{array} \right\}$$

be a subset of $\mathcal{H}$ and $\mathcal{D} = \{c, d, e, \}$ be a set of specialists.

Following survey depicts choices of three specialists:

$$\hbar_1 = \hbar(\mho_1/0.2, c, 1) = \left\{ \frac{\eta_1}{<0.2,0.5,0.4>}, \frac{\eta_2}{<0.7,0.2,0.5>}, \frac{\eta_3}{<0.5,0.4,0.6>}, \frac{\eta_4}{<0.1,0.3,0.6>} \right\},$$
$$\hbar_2 = \hbar(\mho_1/0.2, d, 1) = \left\{ \frac{\eta_1}{<0.4,0.2,0.3>}, \frac{\eta_2}{<0.8,0.1,0.5>}, \frac{\eta_3}{<0.4,0.5,0.6>}, \frac{\eta_4}{<0.2,0.5,0.3>} \right\},$$
$$\hbar_3 = \hbar(\mho_1/0.2, e, 1) = \left\{ \frac{\eta_1}{<0.7,0.2,0.3>}, \frac{\eta_2}{<0.5,0.3,0.6>}, \frac{\eta_3}{<0.6,0.3,0.7>}, \frac{\eta_4}{<0.3,0.5,0.6>} \right\},$$
$$\hbar_4 = \hbar(\mho_2/0.3, c, 1) = \left\{ \frac{\eta_1}{<0.9,0.1,0.3>}, \frac{\eta_2}{<0.4,0.5,0.4>}, \frac{\eta_3}{<0.7,0.2,0.6>}, \frac{\eta_4}{<0.3,0.4,0.8>} \right\},$$
$$\hbar_5 = \hbar(\mho_2/0.3, d, 1) = \left\{ \frac{\eta_1}{<0.4,0.5,0.6>}, \frac{\eta_2}{<0.8,0.1,0.7>}, \frac{\eta_3}{<0.3,0.6,0.5>}, \frac{\eta_4}{<0.2,0.6,0.7>} \right\},$$
$$\hbar_6 = \hbar(\mho_2/0.3, e, 1) = \left\{ \frac{\eta_1}{<0.5,0.4,0.7>}, \frac{\eta_2}{<0.3,0.6,0.4>}, \frac{\eta_3}{<0.6,0.2,0.5>}, \frac{\eta_4}{<0.8,0.1,0.6>} \right\},$$
$$\hbar_7 = \hbar(\mho_3/0.4, c, 1) = \left\{ \frac{\eta_1}{<0.2,0.7,0.5>}, \frac{\eta_2}{<0.9,0.1,0.4>}, \frac{\eta_3}{<0.4,0.5,0.7>}, \frac{\eta_4}{<0.5,0.4,0.8>} \right\},$$
$$\hbar_8 = \hbar(\mho_3/0.4, d, 1) = \left\{ \frac{\eta_1}{<0.4,0.3,0.2>}, \frac{\eta_2}{<0.6,0.3,0.1>}, \frac{\eta_3}{<0.7,0.2,0.3>}, \frac{\eta_4}{<0.9,0.1,0.4>} \right\},$$
$$\hbar_9 = \hbar(\mho_3/0.4, e, 1) = \left\{ \frac{\eta_1}{<0.7,0.2,0.6>}, \frac{\eta_2}{<0.3,0.5,0.7>}, \frac{\eta_3}{<0.5,0.4,0.5>}, \frac{\eta_4}{<0.2,0.7,0.8>} \right\},$$
$$\hbar_{10} = \hbar(\mho_1/0.2, c, 0) = \left\{ \frac{\eta_1}{<0.3,0.2,0.1>}, \frac{\eta_2}{<0.2,0.4,0.5>}, \frac{\eta_3}{<0.4,0.5,0.8>}, \frac{\eta_4}{<0.1,0.8,0.3>} \right\},$$
$$\hbar_{11} = \hbar(\mho_1/0.2, d, 0) = \left\{ \frac{\eta_1}{<0.1,0.8,0.4>}, \frac{\eta_2}{<0.9,0.1,0.2>}, \frac{\eta_3}{<0.6,0.3,0.4>}, \frac{\eta_4}{<0.2,0.7,0.5>} \right\},$$
$$\hbar_{12} = \hbar(\mho_1/0.2, e, 0) = \left\{ \frac{\eta_1}{<0.2,0.7,0.5>}, \frac{\eta_2}{<0.1,0.8,0.6>}, \frac{\eta_3}{<0.3,0.5,0.7>}, \frac{\eta_4}{<0.5,0.4,0.6>} \right\},$$
$$\hbar_{13} = \hbar(\mho_2/0.3, c, 0) = \left\{ \frac{\eta_1}{<0.8,0.1,0.6>}, \frac{\eta_2}{<0.3,0.6,0.7>}, \frac{\eta_3}{<0.5,0.4,0.8>}, \frac{\eta_4}{<0.7,0.2,0.9>} \right\},$$
$$\hbar_{14} = \hbar(\mho_2/0.3, d, 0) = \left\{ \frac{\eta_1}{<0.7,0.2,0.5>}, \frac{\eta_2}{<0.2,0.6,0.4>}, \frac{\eta_3}{<0.9,0.1,0.6>}, \frac{\eta_4}{<0.4,0.5,0.7>} \right\},$$
$$\hbar_{15} = \hbar(\mho_2/0.3, e, 0) = \left\{ \frac{\eta_1}{<0.6,0.2,0.5>}, \frac{\eta_2}{<0.7,0.2,0.4>}, \frac{\eta_3}{<0.3,0.5,0.4>}, \frac{\eta_4}{<0.2,0.7,0.6>} \right\},$$
$$\hbar_{16} = \hbar(\mho_3/0.4, c, 0) = \left\{ \frac{\eta_1}{<0.1,0.7,0.5>}, \frac{\eta_2}{<0.4,0.5,0.7>}, \frac{\eta_3}{<0.7,0.2,0.9>}, \frac{\eta_4}{<0.8,0.2,0.4>} \right\},$$
$$\hbar_{17} = \hbar(\mho_3/0.4, d, 0) = \left\{ \frac{\eta_1}{<0.2,0.7,0.4>}, \frac{\eta_2}{<0.9,0.1,0.6>}, \frac{\eta_3}{<0.8,0.2,0.4>}, \frac{\eta_4}{<0.3,0.5,0.7>} \right\},$$
$$\hbar_{18} = \hbar(\mho_3/0.4, e, 0) = \left\{ \frac{\eta_1}{<0.5,0.4,0.2>}, \frac{\eta_2}{<0.3,0.6,0.1>}, \frac{\eta_3}{<0.6,0.3,0.2>}, \frac{\eta_4}{<0.1,0.8,0.3>} \right\}.$$

The FPNHSES can be described as

$(\hbar, \mathbb{Q}) =$

$$
\left\{
\begin{array}{l}
\left((\mho_1/0.2, c, 1), \left\{ \frac{\eta_1}{\langle 0.2,0.5,0.4 \rangle}, \frac{\eta_2}{\langle 0.7,0.2,0.5 \rangle}, \frac{\eta_3}{\langle 0.5,0.4,0.6 \rangle}, \frac{\eta_4}{\langle 0.1,0.3,0.6 \rangle} \right\} \right), \\
\left((\mho_1/0.2, d, 1), \left\{ \frac{\eta_1}{\langle 0.4,0.2,0.3 \rangle}, \frac{\eta_2}{\langle 0.8,0.1,0.5 \rangle}, \frac{\eta_3}{\langle 0.4,0.5,0.6 \rangle}, \frac{\eta_4}{\langle 0.2,0.5,0.3 \rangle} \right\} \right), \\
\left((\mho_1/0.2, e, 1), \left\{ \frac{\eta_1}{\langle 0.7,0.2,0.3 \rangle}, \frac{\eta_2}{\langle 0.5,0.3,0.6 \rangle}, \frac{\eta_3}{\langle 0.6,0.3,0.7 \rangle}, \frac{\eta_4}{\langle 0.3,0.5,0.6 \rangle} \right\} \right), \\
\left((\mho_2/0.3, c, 1), \left\{ \frac{\eta_1}{\langle 0.9,0.1,0.3 \rangle}, \frac{\eta_2}{\langle 0.4,0.5,0.4 \rangle}, \frac{\eta_3}{\langle 0.7,0.2,0.6 \rangle}, \frac{\eta_4}{\langle 0.3,0.4,0.8 \rangle} \right\} \right), \\
\left((\mho_2/0.3, d, 1), \left\{ \frac{\eta_1}{\langle 0.4,0.5,0.6 \rangle}, \frac{\eta_2}{\langle 0.8,0.1,0.7 \rangle}, \frac{\eta_3}{\langle 0.3,0.6,0.5 \rangle}, \frac{\eta_4}{\langle 0.2,0.6,0.7 \rangle} \right\} \right), \\
\left((\mho_2/0.3, e, 1), \left\{ \frac{\eta_1}{\langle 0.5,0.4,0.7 \rangle}, \frac{\eta_2}{\langle 0.3,0.6,0.4 \rangle}, \frac{\eta_3}{\langle 0.6,0.2,0.5 \rangle}, \frac{\eta_4}{\langle 0.8,0.1,0.6 \rangle} \right\} \right), \\
\left((\mho_3/0.4, c, 1), \left\{ \frac{\eta_1}{\langle 0.2,0.7,0.5 \rangle}, \frac{\eta_2}{\langle 0.9,0.1,0.4 \rangle}, \frac{\eta_3}{\langle 0.4,0.5,0.7 \rangle}, \frac{\eta_4}{\langle 0.5,0.4,0.8 \rangle} \right\} \right), \\
\left((\mho_3/0.4, d, 1), \left\{ \frac{\eta_1}{\langle 0.4,0.3,0.2 \rangle}, \frac{\eta_2}{\langle 0.6,0.3,0.1 \rangle}, \frac{\eta_3}{\langle 0.7,0.2,0.3 \rangle}, \frac{\eta_4}{\langle 0.9,0.1,0.4 \rangle} \right\} \right), \\
\left((\mho_3/0.4, e, 1), \left\{ \frac{\eta_1}{\langle 0.7,0.2,0.6 \rangle}, \frac{\eta_2}{\langle 0.3,0.5,0.7 \rangle}, \frac{\eta_3}{\langle 0.5,0.4,0.5 \rangle}, \frac{\eta_4}{\langle 0.2,0.7,0.8 \rangle} \right\} \right), \\
\left((\mho_1/0.2, c, 0), \left\{ \frac{\eta_1}{\langle 0.3,0.2,0.1 \rangle}, \frac{\eta_2}{\langle 0.2,0.4,0.5 \rangle}, \frac{\eta_3}{\langle 0.4,0.5,0.8 \rangle}, \frac{\eta_4}{\langle 0.1,0.8,0.3 \rangle} \right\} \right), \\
\left((\mho_1/0.2, d, 0), \left\{ \frac{\eta_1}{\langle 0.1,0.8,0.4 \rangle}, \frac{\eta_2}{\langle 0.9,0.1,0.2 \rangle}, \frac{\eta_3}{\langle 0.6,0.3,0.4 \rangle}, \frac{\eta_4}{\langle 0.2,0.7,0.5 \rangle} \right\} \right), \\
\left((\mho_1/0.2, e, 0), \left\{ \frac{\eta_1}{\langle 0.2,0.7,0.5 \rangle}, \frac{\eta_2}{\langle 0.1,0.8,0.6 \rangle}, \frac{\eta_3}{\langle 0.3,0.5,0.7 \rangle}, \frac{\eta_4}{\langle 0.5,0.4,0.6 \rangle} \right\} \right), \\
\left((\mho_2/0.3, c, 0), \left\{ \frac{\eta_1}{\langle 0.8,0.1,0.6 \rangle}, \frac{\eta_2}{\langle 0.3,0.6,0.7 \rangle}, \frac{\eta_3}{\langle 0.5,0.4,0.8 \rangle}, \frac{\eta_4}{\langle 0.7,0.2,0.9 \rangle} \right\} \right), \\
\left((\mho_2/0.3, d, 0), \left\{ \frac{\eta_1}{\langle 0.7,0.2,0.5 \rangle}, \frac{\eta_2}{\langle 0.2,0.6,0.4 \rangle}, \frac{\eta_3}{\langle 0.9,0.1,0.6 \rangle}, \frac{\eta_4}{\langle 0.4,0.5,0.7 \rangle} \right\} \right), \\
\left((\mho_2/0.3, e, 0), \left\{ \frac{\eta_1}{\langle 0.6,0.2,0.5 \rangle}, \frac{\eta_2}{\langle 0.7,0.2,0.4 \rangle}, \frac{\eta_3}{\langle 0.3,0.5,0.4 \rangle}, \frac{\eta_4}{\langle 0.2,0.7,0.6 \rangle} \right\} \right), \\
\left((\mho_3/0.4, c, 0), \left\{ \frac{\eta_1}{\langle 0.1,0.7,0.5 \rangle}, \frac{\eta_2}{\langle 0.4,0.5,0.7 \rangle}, \frac{\eta_3}{\langle 0.7,0.2,0.9 \rangle}, \frac{\eta_4}{\langle 0.8,0.2,0.4 \rangle} \right\} \right), \\
\left((\mho_3/0.4, d, 0), \left\{ \frac{\eta_1}{\langle 0.2,0.7,0.4 \rangle}, \frac{\eta_2}{\langle 0.9,0.1,0.6 \rangle}, \frac{\eta_3}{\langle 0.8,0.2,0.4 \rangle}, \frac{\eta_4}{\langle 0.3,0.5,0.7 \rangle} \right\} \right), \\
\left((\mho_3/0.4, e, 0), \left\{ \frac{\eta_1}{\langle 0.5,0.4,0.2 \rangle}, \frac{\eta_2}{\langle 0.3,0.6,0.1 \rangle}, \frac{\eta_3}{\langle 0.6,0.3,0.2 \rangle}, \frac{\eta_4}{\langle 0.1,0.8,0.3 \rangle} \right\} \right)
\end{array}
\right\}.
$$

Definition 3.3. A FPNHSES $(\hbar_1, \mathbb{Q})$ is said to be FPNHSE subset of $(\hbar_2, \mathbb{P})$ over Δ, if

(i) $\mathbb{Q} \subseteq \mathbb{P}$,

(ii) $\forall\ \gamma \in \mathbb{Q}, \hbar_1(\gamma) \subseteq \hbar_2(\gamma)$ and shown by $(\hbar_1, \mathbb{Q}) \subseteq (\hbar_2, \mathbb{P})$.

Whereas $(\hbar_2, \mathbb{P})$ is said to be FPNHSE-superset of $(\hbar_1, \mathbb{Q})$.

Example 3.4. Considering Example 3.2, suppose

$$\mathbb{Q}_1 = \left\{ \begin{array}{l} (\mho_1/0.2, c, 1), (\mho_3/0.4, c, 0), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1), \\ (\mho_3/0.4, d, 0), (\mho_1/0.2, e, 0), (\mho_3/0.4, e, 1) \end{array} \right\}$$

$$\mathbb{Q}_2 = \left\{ \begin{array}{l} (\mho_1/0.2, c, 1), (\mho_3/0.4, c, 0), (\mho_3/0.4, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1), \\ (\mho_1/0.2, d, 0), (\mho_3/0.4, d, 0), (\mho_1/0.2, e, 0), (\mho_3/0.4, e, 1), (\mho_1/0.2, e, 1) \end{array} \right\}$$

It is clear that $\mathbb{Q}_1 \subset \mathbb{Q}_2$.

Suppose $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ be defined as following

$$(\hbar_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{\frac{\eta_1}{<0.1,0.6,0.7>}, \frac{\eta_2}{<0.6,0.5,0.8>}, \frac{\eta_3}{<0.4,0.6,0.9>}, \frac{\eta_4}{<0.1,0.8,0.6>}\right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{\frac{\eta_1}{<0.3,0.4,0.5>}, \frac{\eta_2}{<0.6,0.4,0.6>}, \frac{\eta_3}{<0.2,0.5,0.7>}, \frac{\eta_4}{<0.1,0.5,0.6>}\right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{\frac{\eta_1}{<0.2,0.6,0.4>}, \frac{\eta_2}{<0.5,0.4,0.7>}, \frac{\eta_3}{<0.6,0.5,0.8>}, \frac{\eta_4}{<0.8,0.6,0.4>}\right\}\right), \\ \left((\mho_3/0.4, e, 1), \left\{\frac{\eta_1}{<0.6,0.4,0.3>}, \frac{\eta_2}{<0.2,0.7,0.6>}, \frac{\eta_3}{<0.4,0.5,0.3>}, \frac{\eta_4}{<0.1,0.7,0.4>}\right\}\right), \\ \left((\mho_1/0.2, e, 0), \left\{\frac{\eta_1}{<0.1,0.6,0.3>}, \frac{\eta_2}{<0.1,0.7,0.4>}, \frac{\eta_3}{<0.2,0.7,0.6>}, \frac{\eta_4}{<0.1,0.6,0.7>}\right\}\right), \\ \left((\mho_3/0.4, c, 0), \left\{\frac{\eta_1}{<0.1,0.8,0.6>}, \frac{\eta_2}{<0.3,0.6,0.5>}, \frac{\eta_3}{<0.6,0.3,0.4>}, \frac{\eta_4}{<0.7,0.2,0.6>}\right\}\right), \\ \left((\mho_3/0.4, d, 0), \left\{\frac{\eta_1}{<0.1,0.7,0.4>}, \frac{\eta_2}{<0.6,0.3,0.6>}, \frac{\eta_3}{<0.7,0.2,0.5>}, \frac{\eta_4}{<0.2,0.7,0.4>}\right\}\right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{\frac{\eta_1}{<0.2,0.3,0.6>}, \frac{\eta_2}{<0.7,0.4,0.7>}, \frac{\eta_3}{<0.5,0.4,0.8>}, \frac{\eta_4}{<0.2,0.4,0.5>}\right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{\frac{\eta_1}{<0.4,0.3,0.4>}, \frac{\eta_2}{<0.8,0.3,0.5>}, \frac{\eta_3}{<0.4,0.3,0.6>}, \frac{\eta_4}{<0.2,0.6,0.5>}\right\}\right), \\ \left((\mho_3/0.4, c, 1), \left\{\frac{\eta_1}{<0.1,0.3,0.4>}, \frac{\eta_2}{<0.9,0.1,0.3>}, \frac{\eta_3}{<0.4,0.5,0.4>}, \frac{\eta_4}{<0.5,0.3,0.4>}\right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{\frac{\eta_1}{<0.4,0.2,0.3>}, \frac{\eta_2}{<0.6,0.3,0.6>}, \frac{\eta_3}{<0.7,0.4,0.5>}, \frac{\eta_4}{<0.9,0.5,0.2}\right\}\right), \\ \left((\mho_1/0.2, e, 1), \left\{\frac{\eta_1}{<0.7,0.2,0.4>}, \frac{\eta_2}{<0.5,0.2,0.6>}, \frac{\eta_3}{<0.6,0.2,0.7>}, \frac{\eta_4}{<0.3,0.5,0.8}\right\}\right), \\ \left((\mho_3/0.4, e, 1), \left\{\frac{\eta_1}{<0.7,0.3,0.1>}, \frac{\eta_2}{<0.3,0.5,0.4>}, \frac{\eta_3}{<0.5,0.4,0.2>}, \frac{\eta_4}{<0.2,0.6,0.3>}\right\}\right), \\ \left((\mho_1/0.2, e, 0), \left\{\frac{\eta_1}{<0.2,0.5,0.1>}, \frac{\eta_2}{<0.2,0.6,0.3>}, \frac{\eta_3}{<0.3,0.5,0.4>}, \frac{\eta_4}{<0.5,0.3,0.5>}\right\}\right), \\ \left((\mho_1/0.2, d, 0), \left\{\frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.9,0.1,0.6>}, \frac{\eta_3}{<0.6,0.3,0.8>}, \frac{\eta_4}{<0.2,0.6,0.8>}\right\}\right), \\ \left((\mho_3/0.4, c, 0), \left\{\frac{\eta_1}{<0.2,0.7,0.4>}, \frac{\eta_2}{<0.4,0.5,0.3>}, \frac{\eta_3}{<0.7,0.2,0.1>}, \frac{\eta_4}{<0.8,0.1,0.5>}\right\}\right), \\ \left((\mho_3/0.4, d, 0), \left\{\frac{\eta_1}{<0.2,0.5,0.1>}, \frac{\eta_2}{<0.7,0.2,0.3>}, \frac{\eta_3}{<0.8,0.2,0.4>}, \frac{\eta_4}{<0.3,0.5,0.2>}\right\}\right) \end{array} \right\}$$

which shows that $(\hbar_1, \mathbb{Q}_1) \subseteq (\hbar_2, \mathbb{Q}_2)$.

Definition 3.5. Two $\mathcal{FPNHSE}$-sets $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ over Δ are said to be equal if $(\hbar_1, \mathbb{Q}_1)$ is a $\mathcal{FPNHSE}$-subset of $(\hbar_2, \mathbb{Q}_2)$ and $(\hbar_2, \mathbb{Q}_2)$ is a $\mathcal{FPNHSE}$-subset of $(\hbar_1, \mathbb{Q}_1)$.

Definition 3.6. The complement of a $\mathcal{FPNHSE}$-set $(\hbar, \mathbb{Q})$, denoted by $(\hbar, \mathbb{Q})^c$, is defined by $(\hbar, \mathbb{Q})^c = \tilde{c}(\hbar(\sigma))\ \forall\ \sigma \in \Delta$ while $\tilde{c}$ is a NF complement.

Example 3.7. Taking complement of $\mathcal{FPNHSE}$-set determined in 3.2, we have

$$(\hbar,\mathbb{Q})^c=\left\{\begin{array}{l}
\left\{(\zeta_1/0.2,c,1),\left\{\frac{\eta_1}{<0.4,0.5,0.2>},\frac{\eta_2}{<0.5,0.8,0.7>},\frac{\eta_3}{<0.6,0.6,0.5>},\frac{\eta_4}{<0.6,0.7,0.1>}\right\}\right\},\\
\left\{(\mho_1/0.2,d,1),\left\{\frac{\eta_1}{<0.3,0.8,0.6>},\frac{\eta_2}{<0.5,0.9,0.8>},\frac{\eta_3}{<0.6,0.5,0.4>},\frac{\eta_4}{<0.3,0.5,0.2>}\right\}\right\},\\
\left((\mho_1/0.2,e,1),\left\{\frac{\eta_1}{<0.3,0.8,0.7>},\frac{\eta_2}{<0.6,0.7,0.5>},\frac{\eta_3}{<0.7,0.7,0.6>},\frac{\eta_4}{<0.6,0.5,0.3>}\right\}\right),\\
\left((\mho_2/0.3,c,1),\left\{\frac{\eta_1}{<0.3,0.9,0.9>},\frac{\eta_2}{<0.4,0.5,0.4>},\frac{\eta_3}{<0.6,0.8,0.7>},\frac{\eta_4}{<0.8,0.6,0.3>}\right\}\right),\\
\left((\mho_2/0.3,d,1),\left\{\frac{\eta_1}{<0.6,0.5,0.4>},\frac{\eta_2}{<0.7,0.9,0.8>},\frac{\eta_3}{<0.5,0.4,0.3>},\frac{\eta_4}{<0.7,0.4,0.2>}\right\}\right),\\
\left((\mho_2/0.3,e,1),\left\{\frac{\eta_1}{<0.7,0.6,0.5>},\frac{\eta_2}{<0.4,0.4,0.3>},\frac{\eta_3}{<0.5,0.8,0.6>},\frac{\eta_4}{<0.6,0.9,0.8>}\right\}\right),\\
\left((\mho_3/0.4,c,1),\left\{\frac{\eta_1}{<0.5,0.3,0.2>},\frac{\eta_2}{<0.4,0.9,0.9>},\frac{\eta_3}{<0.7,0.5,0.4>},\frac{\eta_4}{<0.8,0.6,0.5>}\right\}\right),\\
\left((\mho_3/0.4,d,1),\left\{\frac{\eta_1}{<0.2,0.7,0.4>},\frac{\eta_2}{<0.1,0.7,0.6>},\frac{\eta_3}{<0.3,0.8,0.7>},\frac{\eta_4}{<0.4,0.9,0.9>}\right\}\right),\\
\left((\mho_3/0.4,e,1),\left\{\frac{\eta_1}{<0.6,0.8,0.7>},\frac{\eta_2}{<0.7,0.5,0.3>},\frac{\eta_3}{<0.5,0.6,0.5>},\frac{\eta_4}{<0.8,0.3,0.2>}\right\}\right),\\
\left((\mho_1/0.2,c,0),\left\{\frac{\eta_1}{<0.1,0.8,0.3>},\frac{\eta_2}{<0.5,0.6,0.2>},\frac{\eta_3}{<0.8,0.5,0.4>},\frac{\eta_4}{<0.3,0.2,0.1>}\right\}\right),\\
\left((\mho_1/0.2,d,0),\left\{\frac{\eta_1}{<0.4,0.2,0.1>},\frac{\eta_2}{<0.2,0.9,0.9>},\frac{\eta_3}{<0.4,0.7,0.6>},\frac{\eta_4}{<0.5,0.3,0.2>}\right\}\right),\\
\left((\mho_1/0.2,e,0),\left\{\frac{\eta_1}{<0.5,0.3,0.2>},\frac{\eta_2}{<0.6,0.2,0.1>},\frac{\eta_3}{<0.7,0.5,0.3>},\frac{\eta_4}{<0.6,0.6,0.5>}\right\}\right),\\
\left((\mho_2/0.3,c,0),\left\{\frac{\eta_1}{<0.6,0.9,0.8>},\frac{\eta_2}{<0.7,0.4,0.3>},\frac{\eta_3}{<0.8,0.6,0.5>},\frac{\eta_4}{<0.9,0.8,0.7>}\right\}\right),\\
\left((\mho_2/0.3,d,0),\left\{\frac{\eta_1}{<0.5,0.8,0.7>},\frac{\eta_2}{<0.4,0.4,0.2>},\frac{\eta_3}{<0.6,0.9,0.9>},\frac{\eta_4}{<0.7,0.5,0.4>}\right\}\right),\\
\left((\mho_2/0.3,e,0),\left\{\frac{\eta_1}{<0.5,0.8,0.6>},\frac{\eta_2}{<0.4,0.8,0.7>},\frac{\eta_3}{<0.4,0.5,0.3>},\frac{\eta_4}{<0.6,0.3,0.2>}\right\}\right),\\
\left((\mho_3/0.4,c,0),\left\{\frac{\eta_1}{<0.5,0.3,0.1>},\frac{\eta_2}{<0.7,0.5,0.3>},\frac{\eta_3}{<0.9,0.8,0.8>},\frac{\eta_4}{<0.4,0.8,0.8>}\right\}\right),\\
\left((\mho_3/0.4,d,0),\left\{\frac{\eta_1}{<0.4,0.3,0.2>},\frac{\eta_2}{<0.6,0.9,0.9>},\frac{\eta_3}{<0.4,0.8,0.8>},\frac{\eta_4}{<0.7,0.5,0.3>}\right\}\right),\\
\left((\mho_3/0.4,e,0),\left\{\frac{\eta_1}{<0.2,0.6,0.5>},\frac{\eta_2}{<0.1,0.4,0.3>},\frac{\eta_3}{<0.2,0.7,0.6>},\frac{\eta_4}{<0.3,0.2,0.1>}\right\}\right)
\end{array}\right\}.$$

Definition 3.8. An agree-$\mathcal{FPNHSE}$-set $(\hbar,\mathbb{Q})_{ag}$ over Δ, is a $\mathcal{FPNHSE}$-subset of $(\hbar,\mathbb{Q})$ and is characterized as $(\hbar,\mathbb{Q})_{ag}=\{\hbar_{ag}(\sigma):\sigma\in G\times D\times\{1\}\}$.

Example 3.9. Finding agree-$\mathcal{FPNHSE}$-set determined in 3.2, we get

$$(\hbar,\mathbb{Q})=\left\{\begin{array}{l}
\left((\mho_1/0.2,c,1),\left\{\frac{\eta_1}{<0.2,0.5,0.4>},\frac{\eta_2}{<0.7,0.2,0.5>},\frac{\eta_3}{<0.5,0.4,0.6>},\frac{\eta_4}{<0.1,0.3,0.6>}\right\}\right),\\
\left((\mho_1/0.2,d,1),\left\{\frac{\eta_1}{<0.4,0.2,0.3>},\frac{\eta_2}{<0.8,0.1,0.5>},\frac{\eta_3}{<0.4,0.5,0.6>},\frac{\eta_4}{<0.2,0.5,0.3>}\right\}\right),\\
\left((\mho_1/0.2,e,1),\left\{\frac{\eta_1}{<0.7,0.2,0.3>},\frac{\eta_2}{<0.5,0.3,0.6>},\frac{\eta_3}{<0.6,0.3,0.7>},\frac{\eta_4}{<0.3,0.5,0.6>}\right\}\right),\\
\left((\mho_2/0.3,c,1),\left\{\frac{\eta_1}{<0.9,0.1,0.3>},\frac{\eta_2}{<0.4,0.5,0.4>},\frac{\eta_3}{<0.7,0.2,0.6>},\frac{\eta_4}{<0.3,0.4,0.8>}\right\}\right),\\
\left((\mho_2/0.3,d,1),\left\{\frac{\eta_1}{<0.4,0.5,0.6>},\frac{\eta_2}{<0.8,0.1,0.7>},\frac{\eta_3}{<0.3,0.6,0.5>},\frac{\eta_4}{<0.2,0.6,0.7>}\right\}\right),\\
\left((\mho_2/0.3,e,1),\left\{\frac{\eta_1}{<0.5,0.4,0.7>},\frac{\eta_2}{<0.3,0.6,0.4>},\frac{\eta_3}{<0.6,0.2,0.5>},\frac{\eta_4}{<0.8,0.1,0.6>}\right\}\right),\\
\left((\mho_3/0.4,c,1),\left\{\frac{\eta_1}{<0.2,0.7,0.5>},\frac{\eta_2}{<0.9,0.1,0.4>},\frac{\eta_3}{<0.4,0.5,0.7>},\frac{\eta_4}{<0.5,0.4,0.8>}\right\}\right),\\
\left((\mho_3/0.4,d,1),\left\{\frac{\eta_1}{<0.4,0.3,0.2>},\frac{\eta_2}{<0.6,0.3,0.1>},\frac{\eta_3}{<0.7,0.2,0.3>},\frac{\eta_4}{<0.9,0.1,0.4>}\right\}\right),\\
\left((\mho_3/0.4,e,1),\left\{\frac{\eta_1}{<0.7,0.2,0.6>},\frac{\eta_2}{<0.3,0.5,0.7>},\frac{\eta_3}{<0.5,0.4,0.5>},\frac{\eta_4}{<0.2,0.7,0.8>}\right\}\right)
\end{array}\right\}.$$

Definition 3.10. A disagree-$\mathcal{FPNHSE}$-set $(\hbar,\mathbb{Q})_{dag}$ over Δ, is a $\mathcal{FPNHSE}$-subset of $(\hbar,\mathbb{Q})$ and is characterized as $(\hbar,\mathbb{Q})_{dag}=\{\hbar_{dag}(\sigma):\sigma\in\mathbb{G}\times\mathcal{D}\times\{0\}\}$.

Example 3.11. Getting disagree-$\mathcal{FPNHSE}$-set determined in 3.2,

$$(\hbar,\mathbb{Q})=\left\{\begin{array}{l}
((\mho_1/0.2,c,0),\{\frac{\eta_1}{<0.3,0.2,0.1>},\frac{\eta_2}{<0.2,0.4,0.5>},\frac{\eta_3}{<0.4,0.5,0.8>},\frac{\eta_4}{<0.1,0.8,0.3>}\}),\\
((\mho_1/0.2,d,0),\{\frac{\eta_1}{<0.1,0.8,0.4>},\frac{\eta_2}{<0.9,0.1,0.2>},\frac{\eta_3}{<0.6,0.3,0.4>},\frac{\eta_4}{<0.2,0.7,0.5>}\}),\\
((\mho_1/0.2,e,0),\{\frac{\eta_1}{<0.2,0.7,0.5>},\frac{\eta_2}{<0.1,0.8,0.6>},\frac{\eta_3}{<0.3,0.5,0.7>},\frac{\eta_4}{<0.5,0.4,0.6>}\}),\\
((\mho_2/0.3,c,0),\{\frac{\eta_1}{<0.8,0.1,0.6>},\frac{\eta_2}{<0.3,0.6,0.7>},\frac{\eta_3}{<0.5,0.4,0.8>},\frac{\eta_4}{<0.7,0.2,0.9>}\}),\\
((\mho_2/0.3,d,0),\{\frac{\eta_1}{<0.7,0.2,0.5>},\frac{\eta_2}{<0.2,0.6,0.4>},\frac{\eta_3}{<0.9,0.1,0.6>},\frac{\eta_4}{<0.4,0.5,0.7>}\}),\\
((\mho_2/0.3,e,0),\{\frac{\eta_1}{<0.6,0.2,0.5>},\frac{\eta_2}{<0.7,0.2,0.4>},\frac{\eta_3}{<0.3,0.5,0.4>},\frac{\eta_4}{<0.2,0.7,0.6>}\}),\\
((\mho_3/0.4,c,0),\{\frac{\eta_1}{<0.1,0.7,0.5>},\frac{\eta_2}{<0.4,0.5,0.7>},\frac{\eta_3}{<0.7,0.2,0.9>},\frac{\eta_4}{<0.8,0.2,0.4>}\}),\\
((\mho_3/0.4,d,0),\{\frac{\eta_1}{<0.2,0.7,0.4>},\frac{\eta_2}{<0.9,0.1,0.6>},\frac{\eta_3}{<0.8,0.2,0.4>},\frac{\eta_4}{<0.3,0.5,0.7>}\}),\\
((\mho_3/0.4,e,0),\{\frac{\eta_1}{<0.5,0.4,0.2>},\frac{\eta_2}{<0.3,0.6,0.1>},\frac{\eta_3}{<0.6,0.3,0.2>},\frac{\eta_4}{<0.1,0.8,0.3>}\})
\end{array}\right\}.$$

Definition 3.12. A $\mathcal{FPNHSE}$-set $(\hbar_1,\mathbb{Q}_1)$ is called a relative null $\mathcal{FPNHSE}$-set w.r.t $\mathbb{Q}_1\subset\mathbb{Q}$, denoted by $(\hbar_1,\mathbb{Q}_1)$, if $\hbar_1(g)=\emptyset,\forall g\in\mathbb{Q}_1$.

Example 3.13. Taking the concept of Example 3.2, if

$$(\hbar_1,\mathbb{Q}_1)=\{((\eta_1,c,1),\emptyset),((\eta_2,d,1),\emptyset),((\eta_3,e,1),\emptyset)\}$$

.

Definition 3.14. A $\mathcal{FPNHSE}$-set $(\hbar_2,\mathbb{Q}_2)$ is called a relative whole $\mathcal{FPNHSE}$-set w.r.t $\mathbb{Q}_2\subset\mathbb{Q}$, denoted by $(\hbar_2,\mathbb{Q}_2)_\Delta$, if $\hbar_1(g)=\Delta$, $\forall$ g $\in\mathbb{Q}_2$.

Example 3.15. Taking the concept of Example 3.2, if

$$(\hbar_2,\mathbb{Q}_2)_\Delta=\{((\eta_1,c,1),\Delta),((\eta_2,d,1),\Delta),((\eta_3,e,1),\Delta)\}$$

where $\mathbb{Q}_2\subseteq\mathbb{Q}$.

Definition 3.16. A $\mathcal{FPNHSE}$-set $(\hbar,\mathbb{Q})$ is called absolute whole $\mathcal{FPNHSE}$-set denoted by $(\hbar,\mathbb{Q})_\Delta$, if $\hbar(g)=\Delta$, $\forall$ g $\in\mathbb{Q}$.

Example 3.17. Using Example 3.2, if

$$(\Psi,\mathbb{S})_\Delta=\left\{\begin{array}{l}
(\mho_1/0.2,c,1),\Delta),(\mho_1/0.2,d,1),\Delta),(\mho_1/0.2,e,1),\Delta),(\mho_3/0.4,c,1),\Delta),\\
(\mho_3/0.4,d,1),\Delta),(\mho_3/0.4,e,1),\Delta),(\mho_5/0.6,c,1),\Delta),(\mho_5/0.6,d,1),\Delta),\\
(\mho_5/0.6,e,1),\Delta),(\mho_1/0.2,c,0),\Delta),(\mho_1/0.2,d,0),\Delta),(\mho_1/0.2,e,0),\Delta),\\
(\mho_3/0.4,c,0),\Delta),(\mho_3/0.4,d,0),\Delta),(\mho_3/0.4,e,0),\Delta),(\mho_5/0.6,c,0),\Delta),\\
(\mho_5/0.6,d,0),\Delta),(\mho_5/0.6,e,0),\Delta)
\end{array}\right\}$$

Proposition 3.18. *Suppose* $(\hbar_1,\mathbb{Q}_1)_\Delta$, $(\hbar_2,\mathbb{Q}_2)_\Delta$, $(\hbar_3,\mathbb{Q}_3)_\Delta$, *be three* $\mathcal{FPNHSE}$*-sets over* Δ*, then*

(1) $(\hbar_1,\mathbb{Q}_1)\subset(\hbar_2,\mathbb{Q}_2)_\Delta$,
(2) $(\hbar_1,\mathbb{Q}_1)_\hbar\subset(\hbar_1,\mathbb{Q}_1)$,

(3) $(\hbar_1, \mathbb{Q}_1) \subset (\hbar_1, \mathbb{Q}_1)$,

(4) *If* $(\hbar_1, \mathbb{Q}_1) \subset (\hbar_2, \mathbb{Q}_2)$, $(\hbar_2, \mathbb{Q}_2) \subset (\hbar_3, \mathbb{Q}_3)$, *then* $(\hbar_1, \mathbb{Q}_1) \subset (\hbar_3, \mathbb{S}_3)$,

(5) *If* $(\hbar_1, \mathbb{Q}_1) = (\hbar_2, \mathbb{Q}_2)$, $(\hbar_2, \mathbb{Q}_2) = (\hbar_3, \mathbb{Q}_3)$, *then* $(\hbar_1, \mathbb{Q}_1) = (\hbar_3, \mathbb{Q}_3)$.

Proposition 3.19. *If* $(\hbar, \mathbb{Q})$ *is a* $\mathcal{FPNHSE}$*-set over* Δ*, then*

(1) $((\hbar, \mathbb{Q})^c)^c = (\hbar, \mathbb{Q})$

(2) $(\hbar, \mathbb{Q})^c_{ag} = (\hbar, \mathbb{Q})_{dag}$

(3) $(\hbar, \mathbb{Q})^c_{dag} = (\hbar, \mathbb{Q})_{ag}$.

4. Set Theoretic Operations of $\mathcal{FPNHSE}$-set

In this portion, some set theoretic operations are presented with detailed examples.

Definition 4.1. The union of $\mathcal{FPNHSE}$-sets $(\hbar_1, \mathbb{Q})$ and $(\hbar_2, \mathbb{R})$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{Q} \cup \mathbb{R}$, defined as

$$\hbar_3(\sigma) = \begin{cases} \hbar_1(\sigma) & ;\ \sigma \in \mathbb{Q} - \mathbb{R} \\ \hbar_2(\sigma) & ;\ \sigma \in \mathbb{R} - \mathbb{Q} \\ \cup(\hbar_1(\sigma), \hbar_2(\sigma)) & ;\ \sigma \in \mathbb{Q} \cap \mathbb{R} \end{cases}$$

where

$$\cup(\hbar_1(\sigma), \hbar_2(\sigma)) = \{< u, max(\hbar_1(\sigma), \hbar_2(\sigma)), min(\hbar_1(\sigma), (\hbar_2(\sigma)), min(\hbar_1(\sigma), (\hbar_2(\sigma))\} >: u \in \Delta\}.$$

Example 4.2. Using Example 3.2, with two sets

$$\mathbb{Q}_1 = \Big\{ (\mho_1/0.2, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1) \Big\}$$

$$\mathbb{Q}_2 = \Big\{ (\mho_1/0.2, c, 1), (\mho_3/0.4, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1) \Big\}.$$

Suppose $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ over Δ are two $\mathcal{FPNHSE}$-sets such that

$$(\hbar_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} ((\mho_1/0.2, c, 1), \{ \frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.6,0.3,0.2>}, \frac{\eta_3}{<0.4,0.5,0.1>}, \frac{\eta_4}{<0.1,0.8,0.5>} \}), \\ ((\mho_1/0.2, d, 1), \{ \frac{\eta_1}{<0.3,0.4,0.5>}, \frac{\eta_2}{<0.6,0.2,0.3>}, \frac{\eta_3}{<0.2,0.5,0.6>}, \frac{\eta_4}{<0.1,0.5,0.3>} \}), \\ ((\mho_3/0.4, d, 1), \{ \frac{\eta_1}{<0.2,0.6,0.7>}, \frac{\eta_2}{<0.5,0.2,0.3>}, \frac{\eta_3}{<0.6,0.3,0.5>}, \frac{\eta_4}{<0.8,0.1,0.9>} \}) \end{array} \right\}$$

$$(\hbar_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} ((\mho_1/0.2, c, 1), \{ \frac{\eta_1}{<0.2,0.3,0.4>}, \frac{\eta_2}{<0.7,0.4,0.5>}, \frac{\eta_3}{<0.5,0.4,0.6>}, \frac{\eta_4}{<0.2,0.4,0.7>} \}), \\ ((\mho_1/0.2, d, 1), \{ \frac{\eta_1}{<0.4,0.3,0.8>}, \frac{\eta_2}{<0.8,0.3,0.5>}, \frac{\eta_3}{<0.4,0.3,0.5>}, \frac{\eta_4}{<0.2,0.6,0.7>} \}), \\ ((\mho_3/0.4, c, 1), \{ \frac{\eta_1}{<0.1,0.3,0.6>}, \frac{\eta_2}{<0.9,0.1,0.7>}, \frac{\eta_3}{<0.4,0.5,0.8>}, \frac{\eta_4}{<0.5,0.3,0.5>} \}), \\ ((\mho_3/0.4, d, 1), \{ \frac{\eta_1}{<0.4,0.2,0.3>}, \frac{\eta_2}{<0.6,0.3,0.5>}, \frac{\eta_3}{<0.7,0.4,0.5>}, \frac{\eta_4}{<0.9,0.5,0.7} \}), \end{array} \right\}$$

Then $(\hbar_1, \mathbb{Q}_1) \cup (\hbar_2, \mathbb{Q}_2) = (\hbar_3, \mathbb{Q}_3)$

$$(\hbar_3, \mathbb{Q}_3) = \left\{ \begin{array}{l} ((\mho_1/0.2, c, 1), \{ \frac{\eta_1}{<0.2,0.3,0.4>}, \frac{\eta_2}{<0.7,0.3,0.5>}, \frac{\eta_3}{<0.5,0.4,0.1>}, \frac{\eta_4}{<0.2,0.4,0.5>} \}), \\ ((\mho_1/0.2, d, 1), \{ \frac{\eta_1}{<0.4,0.3,0.5>}, \frac{\eta_2}{<0.8,0.2,0.3>}, \frac{\eta_3}{<0.4,0.3,0.5>}, \frac{\eta_4}{<0.2,0.4,0.3>} \}), \\ ((\mho_3/0.4, c, 1), \{ \frac{\eta_1}{<0.1,0.30,0.6>}, \frac{\eta_2}{<0.9,0.10,0.7>}, \frac{\eta_3}{<0.4,0.5,0.8>}, \frac{\eta_4}{<0.5,0.3,0.5>} \}), \\ ((\mho_3/0.4, d, 1), \{ \frac{\eta_1}{<0.4,0.2,0.3>}, \frac{\eta_2}{<0.6,0.2,0.3>}, \frac{\eta_3}{<0.7,0.3,0.5>}, \frac{\eta_4}{<0.9,0.1,0.7} \}) \end{array} \right\}.$$

Definition 4.3. Restricted Union of two fuzzy parameterized neutrosophic hypersoft expert sets $(\hbar_1, \mathbb{Q}_1)$, $(\hbar_2, \mathbb{Q}_2)$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{Q}_1 \cap \mathbb{Q}_2$, defined as
$\hbar_3(\sigma) = \hbar_1(\sigma) \cup_{\mathbb{R}} \hbar_2(\sigma)$ for $\sigma \in \mathbb{Q}_1 \cap \mathbb{Q}_2$.

Example 4.4. Taking Example 3.2, with two sets
$\mathbb{Q}_1 = \left\{ (\mho_1/0.2, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1) \right\}$
$\mathbb{Q}_2 = \left\{ (\mho_1/0.2, c, 1), (\mho_3/0.4, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1) \right\}$
Suppose $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ over Δ are two $\mathcal{FPNHSE}$-sets such that

$$(\hbar_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{\frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.6,0.3,0.2>}, \frac{\eta_3}{<0.4,0.5,0.1>}, \frac{\eta_4}{<0.1,0.8,0.5>}\right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{\frac{\eta_1}{<0.3,0.4,0.5>}, \frac{\eta_2}{<0.6,0.2,0.3>}, \frac{\eta_3}{<0.2,0.5,0.6>}, \frac{\eta_4}{<0.1,0.5,0.3>}\right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{\frac{\eta_1}{<0.2,0.6,0.7>}, \frac{\eta_2}{<0.5,0.2,0.3>}, \frac{\eta_3}{<0.6,0.3,0.5>}, \frac{\eta_4}{<0.8,0.1,0.9>}\right\}\right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{\frac{\eta_1}{<0.2,0.3,0.4>}, \frac{\eta_2}{<0.7,0.4,0.5>}, \frac{\eta_3}{<0.5,0.4,0.6>}, \frac{\eta_4}{<0.2,0.4,0.7>}\right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{\frac{\eta_1}{<0.4,0.3,0.8>}, \frac{\eta_2}{<0.8,0.3,0.5>}, \frac{\eta_3}{<0.4,0.3,0.5>}, \frac{\eta_4}{<0.2,0.6,0.7>}\right\}\right), \\ \left((\mho_3/0.4, c, 1), \left\{\frac{\eta_1}{<0.1,0.3,0.6>}, \frac{\eta_2}{<0.9,0.1,0.7>}, \frac{\eta_3}{<0.4,0.5,0.8>}, \frac{\eta_4}{<0.5,0.3,0.5>}\right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{\frac{\eta_1}{<0.4,0.2,0.3>}, \frac{\eta_2}{<0.6,0.3,0.5>}, \frac{\eta_3}{<0.7,0.4,0.5>}, \frac{\eta_4}{<0.9,0.5,0.7}\right\}\right), \end{array} \right\}$$

Then $(\hbar_1, \mathbb{Q}_1) \cup_{\mathbb{R}} (\hbar_2, \mathbb{Q}_2) = (\hbar_3, \mathbb{L})$

$$(\hbar_3, \mathbb{L}) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{\frac{\eta_1}{<0.2,0.3,0.4>}, \frac{\eta_2}{<0.7,0.3,0.5>}, \frac{\eta_3}{<0.5,0.4,0.1>}, \frac{\eta_4}{<0.2,0.4,0.5>}\right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{\frac{\eta_1}{<0.4,0.3,0.5>}, \frac{\eta_2}{<0.8,0.2,0.3>}, \frac{\eta_3}{<0.4,0.3,0.5>}, \frac{\eta_4}{<0.2,0.4,0.3>}\right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{\frac{\eta_1}{<0.4,0.2,0.3>}, \frac{\eta_2}{<0.6,0.2,0.3>}, \frac{\eta_3}{<0.7,0.3,0.5>}, \frac{\eta_4}{<0.9,0.1,0.7}\right\}\right) \end{array} \right\}.$$

Proposition 4.5. *If* $(\hbar_1, \mathbb{Q}_1)$,$(\hbar_2, \mathbb{Q}_2)$ *and* $(\hbar_3, \mathbb{Q}_3)$ *are three* $\mathcal{FPNHSE}$*-sets over* Δ*, then*

(1) $(\hbar_1, \mathbb{Q}_1) \cup (\hbar_2, \mathbb{Q}_2) = (\hbar_2, \mathbb{Q}_2) \cup (\hbar_1, \mathbb{Q}_1)$,
(2) $((\hbar_1, \mathbb{Q}_1) \cup (\hbar_2, \mathbb{Q}_2)) \cup (\hbar_3, \mathbb{Q}_3) = (\hbar_1, \mathbb{Q}_1) \cup ((\hbar_2, \mathbb{Q}_2) \cup (\hbar_3, N_3))$,
(3) $(\hbar, \mathbb{Q}) \cup \Phi = (\hbar, \mathbb{Q})$.

Definition 4.6. The intersection of $\mathcal{FPNHSE}$-sets $(\hbar_1, \mathbb{Q})$ and $(\hbar_2, \mathbb{R})$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{Q} \cap \mathbb{R}$, defined as

$$\hbar_3(\sigma) = \begin{cases} \hbar_1(\sigma) & ; \sigma \in \mathbb{Q} - \mathbb{R} \\ \hbar_2(\sigma) & ; \sigma \in \mathbb{R} - \mathbb{Q} \\ \cap(\hbar_1(\sigma), \hbar_2(\sigma)) & ; \sigma \in \mathbb{Q} \cap \mathbb{R} \end{cases}$$

where

$$\cap(\hbar_1(\sigma), \hbar_2(\sigma)) = \{< u, min(\hbar_1(\sigma), \hbar_2(\sigma)), max(\hbar_1(\sigma), (\hbar_2(\sigma)), max(\hbar_1(\sigma), (\hbar_2(\sigma))\} >: u \in \Delta\}.$$

Example 4.7. Using Example 3.2, with two sets
$\mathbb{Q}_1 = \left\{ (\mho_1/0.2, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1) \right\}$

$\mathbb{Q}_2 = \Big\{ (\mho_1/0.2, c, 1), (\mho_3/0.4, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1) \Big\}$

Suppose $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ over Δ are two $\mathcal{FPNHSE}$-sets such that

$$(\hbar_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{ \frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.6,0.3,0.2>}, \frac{\eta_3}{<0.4,0.5,0.1>}, \frac{\eta_4}{<0.1,0.8,0.5>} \right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{ \frac{\eta_1}{<0.3,0.4,0.5>}, \frac{\eta_2}{<0.6,0.2,0.3>}, \frac{\eta_3}{<0.2,0.5,0.6>}, \frac{\eta_4}{<0.1,0.5,0.3>} \right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{ \frac{\eta_1}{<0.2,0.6,0.7>}, \frac{\eta_2}{<0.5,0.2,0.3>}, \frac{\eta_3}{<0.6,0.3,0.5>}, \frac{\eta_4}{<0.8,0.1,0.9>} \right\}\right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{ \frac{\eta_1}{<0.2,0.3,0.4>}, \frac{\eta_2}{<0.7,0.4,0.5>}, \frac{\eta_3}{<0.5,0.4,0.6>}, \frac{\eta_4}{<0.2,0.4,0.7>} \right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{ \frac{\eta_1}{<0.4,0.3,0.8>}, \frac{\eta_2}{<0.8,0.3,0.5>}, \frac{\eta_3}{<0.4,0.3,0.5>}, \frac{\eta_4}{<0.2,0.6,0.7>} \right\}\right), \\ \left((\mho_3/0.4, c, 1), \left\{ \frac{\eta_1}{<0.1,0.3,0.6>}, \frac{\eta_2}{<0.9,0.1,0.7>}, \frac{\eta_3}{<0.4,0.5,0.8>}, \frac{\eta_4}{<0.5,0.3,0.5>} \right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{ \frac{\eta_1}{<0.4,0.2,0.3>}, \frac{\eta_2}{<0.6,0.3,0.5>}, \frac{\eta_3}{<0.7,0.4,0.5>}, \frac{\eta_4}{<0.9,0.5,0.7} \right\}\right), \end{array} \right\}$$

Then $(\hbar_1, \mathbb{Q}_1) \cap (\hbar_2, \mathbb{Q}_2) = (\hbar_3, \mathbb{Q}_3)$

$$(\hbar_3, \mathbb{Q}_3) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{ \frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.6,0.3,0.2>}, \frac{\eta_3}{<0.4,0.5,0.6>}, \frac{\eta_4}{<0.2,0.8,0.7>} \right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{ \frac{\eta_1}{<0.4,0.4,0.8>}, \frac{\eta_2}{<0.6,0.3,0.5>}, \frac{\eta_3}{<0.2,0.5,0.6>}, \frac{\eta_4}{<0.1,0.6,0.7>} \right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{ \frac{\eta_1}{<0.2,0.6,0.7>}, \frac{\eta_2}{<0.5,0.3,0.5>}, \frac{\eta_3}{<0.6,0.4,0.5>}, \frac{\eta_4}{<0.8,0.5,0.9} \right\}\right) \end{array} \right\}.$$

Definition 4.8. Extended intersection of $(\hbar_1, \mathbb{S})$ and $(\hbar_2, \mathbb{R})$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{S} \cup \mathbb{R}$, defined as

$$\hbar_3(\sigma) = \begin{cases} \hbar_1(\sigma) & ; \sigma \in \mathbb{S} - \mathbb{R} \\ \hbar_2(\sigma) & ; \sigma \in \mathbb{R} - \mathbb{S} \\ \hbar_1(\sigma) \cap \hbar_2(\sigma) & ; \sigma \in \mathbb{S} \cap \mathbb{R}. \end{cases}$$

Example 4.9. Reconsidering Example 3.2, consider the following two sets

$$\mathbb{Q}_1 = \left\{ \begin{array}{l} (\mho_1/0.2, c, 1), (\mho_3/0.4, c, 0), (\mho_1/0.2, d, 1), (\mho_3/0.4, d, 1), \\ (\mho_3/0.4, d, 0), (\mho_1/0.2, e, 0), (\mho_3/0.4, e, 1) \end{array} \right\}$$

$$\mathbb{Q}_2 = \left\{ \begin{array}{l} (\mho_1/0.2, c, 1), (\mho_3/0.4, c, 0), (\mho_3/0.4, c, 1), (\mho_1/0.2, d, 1), \\ (\mho_3/0.4, d, 1), (\mho_1/0.2, d, 0), (\mho_3/0.4, d, 0), (\mho_1/0.2, e, 0), \\ (\mho_3/0.4, e, 1), (\mho_1/0.2, e, 1) \end{array} \right\}$$

Suppose $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ over Δ are two $\mathcal{FPNHSE}$-sets such that

$$(\hbar_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} \left((\mho_1/0.2, c, 1), \left\{ \frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.6,0.3,0.2>}, \frac{\eta_3}{<0.4,0.5,0.1>}, \frac{\eta_4}{<0.1,0.8,0.5>} \right\}\right), \\ \left((\mho_1/0.2, d, 1), \left\{ \frac{\eta_1}{<0.3,0.4,0.5>}, \frac{\eta_2}{<0.6,0.2,0.3>}, \frac{\eta_3}{<0.2,0.5,0.6>}, \frac{\eta_4}{<0.1,0.5,0.3>} \right\}\right), \\ \left((\mho_3/0.4, d, 1), \left\{ \frac{\eta_1}{<0.2,0.6,0.7>}, \frac{\eta_2}{<0.5,0.2,0.3>}, \frac{\eta_3}{<0.6,0.3,0.5>}, \frac{\eta_4}{<0.8,0.1,0.9>} \right\}\right), \\ \left((\mho_3/0.4, e, 1), \left\{ \frac{\eta_1}{<0.6,0.2,0.4>}, \frac{\eta_2}{<0.2,0.7,0.6>}, \frac{\eta_3}{<0.4,0.3,0.5>}, \frac{\eta_4}{<0.1,0.5,0.4>} \right\}\right), \\ \left((\mho_1/0.2, e, 0), \left\{ \frac{\eta_1}{<0.1,0.3,0.5>}, \frac{\eta_2}{<0.1,0.7,0.6>}, \frac{\eta_3}{<0.2,0.7,0.4>}, \frac{\eta_4}{<0.4,0.6,0.8>} \right\}\right), \\ \left((\mho_3/0.4, c, 0), \left\{ \frac{\eta_1}{<0.1,0.6,0.9>}, \frac{\eta_2}{<0.3,0.6,0.7>}, \frac{\eta_3}{<0.6,0.1,0.2>}, \frac{\eta_4}{<0.7,0.2,0.3>} \right\}\right), \\ \left((\mho_3/0.4, d, 0), \left\{ \frac{\eta_1}{<0.1,0.7,0.3>}, \frac{\eta_2}{<0.8,0.1,0.2>}, \frac{\eta_3}{<0.7,0.2,0.4>}, \frac{\eta_4}{<0.2,0.7,0.6>} \right\}\right) \end{array} \right\}$$

$$(\hbar_2,\mathbb{Q}_2)=\left\{\begin{array}{l}\left((\mho_1/0.2,c,1),\left\{\frac{\eta_1}{<0.2,0.3,0.4>},\frac{\eta_2}{<0.7,0.4,0.5>},\frac{\eta_3}{<0.5,0.4,0.6>},\frac{\eta_4}{<0.2,0.4,0.7>}\right\}\right),\\ \left((\mho_1/0.2,d,1),\left\{\frac{\eta_1}{<0.4,0.3,0.8>},\frac{\eta_2}{<0.8,0.3,0.5>},\frac{\eta_3}{<0.4,0.3,0.5>},\frac{\eta_4}{<0.2,0.6,0.7>}\right\}\right),\\ \left((\mho_3/0.4,c,1),\left\{\frac{\eta_1}{<0.1,0.3,0.6>},\frac{\eta_2}{<0.9,0.1,0.7>},\frac{\eta_3}{<0.4,0.5,0.8>},\frac{\eta_4}{<0.5,0.3,0.5>}\right\}\right),\\ \left((\mho_3/0.4,d,1),\left\{\frac{\eta_1}{<0.4,0.2,0.3>},\frac{\eta_2}{<0.6,0.3,0.5>},\frac{\eta_3}{<0.7,0.4,0.5>},\frac{\eta_4}{<0.9,0.5,0.7}\right\}\right),\\ \left((\mho_1/0.2,e,1),\left\{\frac{\eta_1}{<0.7,0.2,0.3>},\frac{\eta_2}{<0.5,0.2,0.4>},\frac{\eta_3}{<0.6,0.2,0.4>},\frac{\eta_4}{<0.3,0.5,0.6}\right\}\right),\\ \left((\mho_3/0.4,e,1),\left\{\frac{\eta_1}{<0.7,0.3,0.7>},\frac{\eta_2}{<0.3,0.5,0.6>},\frac{\eta_3}{<0.5,0.4,0.3>},\frac{\eta_4}{<0.2,0.6,0.4>}\right\}\right),\\ \left((\mho_1/0.2,e,0),\left\{\frac{\eta_1}{<0.2,0.5,0.4>},\frac{\eta_2}{<0.2,0.6,0.3>},\frac{\eta_3}{<0.3,0.5,0.6>},\frac{\eta_4}{<0.5,0.3,0.7>}\right\}\right),\\ \left((\mho_1/0.2,d,0),\left\{\frac{\eta_1}{<0.1,0.6,0.3>},\frac{\eta_2}{<0.9,0.1,0.2>},\frac{\eta_3}{<0.6,0.3,0.4>},\frac{\eta_4}{<0.2,0.6,0.3>}\right\}\right),\\ \left((\mho_3/0.4,c,0),\left\{\frac{\eta_1}{<0.2,0.7,0.5>},\frac{\eta_2}{<0.4,0.5,0.6>},\frac{\eta_3}{<0.7,0.2,0.3>},\frac{\eta_4}{<0.8,0.1,0.4>}\right\}\right),\\ \left((\mho_3/0.4,d,0),\left\{\frac{\eta_1}{<0.2,0.5,0.4>},\frac{\eta_2}{<0.7,0.2,0.3>},\frac{\eta_3}{<0.8,0.2,0.6>},\frac{\eta_4}{<0.3,0.5,0.7>}\right\}\right)\end{array}\right\}$$

Then $(\hbar_1,\mathbb{Q}_1)\cap_E(\hbar_2,\mathbb{Q}_2)=(\hbar_3,\mathbb{L})$

$$(\hbar_3,\mathbb{L})=\left\{\begin{array}{l}\left((\mho_1/0.2,c,1),\left\{\frac{\eta_1}{<0.1,0.6,0.4>},\frac{\eta_2}{<0.6,0.4,0.5>},\frac{\eta_3}{<0.4,0.5,0.6>},\frac{\eta_4}{<0.1,0.8,0.7>}\right\}\right),\\ \left((\mho_1/0.2,d,1),\left\{\frac{\eta_1}{<0.3,0.4,0.8>},\frac{\eta_2}{<0.6,0.3,0.5>},\frac{\eta_3}{<0.2,0.5,0.6>},\frac{\eta_4}{<0.1,0.6,0.7>}\right\}\right),\\ \left((\mho_3/0.4,d,1),\left\{\frac{\eta_1}{<0.2,0.6,0.7>},\frac{\eta_2}{<0.5,0.4,0.5>},\frac{\eta_3}{<0.6,0.4,0.5>},\frac{\eta_4}{<0.8,0.1,0.7>}\right\}\right),\\ \left((\mho_3/0.4,e,1),\left\{\frac{\eta_1}{<0.6,0.3,0.7>},\frac{\eta_2}{<0.2,0.7,0.6>},\frac{\eta_3}{<0.4,0.4,0.5>},\frac{\eta_4}{<0.1,0.6,0.4>}\right\}\right),\\ \left((\mho_1/0.2,e,0),\left\{\frac{\eta_1}{<0.1,0.5,0.5>},\frac{\eta_2}{<0.1,0.6,0.6>},\frac{\eta_3}{<0.2,0.7,0.6>},\frac{\eta_4}{<0.4,0.6,0.8>}\right\}\right),\\ \left((\mho_3/0.4,c,0),\left\{\frac{\eta_1}{<0.1,0.7,0.9>},\frac{\eta_2}{<0.3,0.6,0.7>},\frac{\eta_3}{<0.6,0.2,0.3>},\frac{\eta_4}{<0.7,0.2,0.4>}\right\}\right),\\ \left((\mho_3/0.4,d,0),\left\{\frac{\eta_1}{<0.1,0.7,0.4>},\frac{\eta_2}{<0.8,0.2,0.3>},\frac{\eta_3}{<0.7,0.2,0.6>},\frac{\eta_4}{<0.2,0.7,0.7>}\right\}\right)\\ \left((\mho_1/0.2,d,0),\left\{\frac{\eta_1}{<0.1,0.6,0.3>},\frac{\eta_2}{<0.9,0.1,0.2>},\frac{\eta_3}{<0.6,0.3,0.4>},\frac{\eta_4}{<0.2,0.6,0.3>}\right\}\right),\\ \left((\mho_1/0.2,e,0),\left\{\frac{\eta_1}{<0.2,0.5,0.4>},\frac{\eta_2}{<0.2,0.6,0.3>},\frac{\eta_3}{<0.3,0.5,0.6>},\frac{\eta_4}{<0.5,0.3,0.7>}\right\}\right),\\ \left((\mho_3/0.4,c,1),\left\{\frac{\eta_1}{<0.1,0.3,0.6>},\frac{\eta_2}{<0.9,0.1,0.7>},\frac{\eta_3}{<0.4,0.5,0.8>},\frac{\eta_4}{<0.5,0.3,0.5>}\right\}\right)\end{array}\right\}.$$

Proposition 4.10. *If* $(\hbar_1,\mathbb{Q}_1)$,$(\hbar_2,\mathbb{Q}_2)$ *and* $(\hbar_3,\mathbb{Q}_3)$ *are three* $\mathcal{FPNHSE}$*-sets over* Δ*, then*

(1) $(\hbar_1,\mathbb{Q}_1)\cap(\hbar_2,\mathbb{Q}_2)=(\hbar_2,\mathbb{Q}_2)\cap(\hbar_1,\mathbb{Q}_1)$,
(2) $((\hbar_1,\mathbb{Q}_1)\cap(\hbar_2,\mathbb{Q}_2))\cap(\hbar_3,\mathbb{Q}_3)=(\hbar_1,\mathbb{Q}_1)\cap((\hbar_2,\mathbb{Q}_2)\cap(\hbar_3,\mathbb{Q}_3))$,
(3) $(\hbar,\mathbb{Q})\cap\phi=\phi$.

Proposition 4.11. *If* $(\hbar_1,\mathbb{Q}_1)$,$(\hbar_2,\mathbb{Q}_2)$ *and* $(\hbar_3,\mathbb{Q}_3)$ *are three* $\mathcal{FPNHSE}$*-sets over* Δ*, then*

(1) $(\hbar_1,\mathbb{Q}_1)\cup((\hbar_2,\mathbb{Q}_2)\cap(\hbar_3,\mathbb{Q}_3))=((\hbar_1,\mathbb{Q}_1)\cup((\hbar_2,\mathbb{Q}_2))\cap((\hbar_1,\mathbb{Q}_1)\cup(\hbar_3,\mathbb{Q}_3))$,
(2) $(\hbar_1,\mathbb{Q}_1)\cap((\hbar_2,\mathbb{Q}_2)\cup(\hbar_3,\mathbb{Q}_3))=((\hbar_1,\mathbb{Q}_1)\cap((\hbar_2,\mathbb{Q}_2))\cup((\hbar_1,\mathbb{Q}_1)\cap(\hbar_3,\mathbb{Q}_3))$.

Definition 4.12. If $(\hbar_1,\mathbb{Q}_1)$ and $(\hbar_2,\mathbb{Q}_2)$ are two $\mathcal{FPNHSE}$-sets over Δ then $(\hbar_1,\mathbb{Q}_1)$ AND $(\hbar_2,\mathbb{Q}_2)$ denoted by $(\hbar_1,\mathbb{Q}_1)\wedge(\hbar_2,\mathbb{Q}_2)$ is defined by $(\hbar_1,\mathbb{Q}_1)\wedge(\hbar_2,\mathbb{Q}_2)=(\hbar_3,\mathbb{Q}_1\times\mathbb{Q}_2)$, while $\hbar_3(\sigma,\gamma)=\hbar_1(\sigma)\cap\hbar_2(\gamma),\forall(\sigma,\gamma)\in\mathbb{Q}_1\times\mathbb{Q}_2$.

Example 4.13. Retaking Example 3.2, let two sets
$\mathbb{Q}_1=\left\{(\mho_1/0.2,c,1),(\mho_1/0.2,d,1),(\mho_3/0.4,c,0)\right\}$
$\mathbb{Q}_2=\left\{(\mho_1/0.2,c,0),(\mho_3/0.4,c,1)\right\}$

Suppose $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ over Δ are two $\mathcal{FPNHSE}$-sets such that

$$(\hbar_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} ((\mho_1/0.2, c, 1), \{\frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.6,0.4,0.5>}, \frac{\eta_3}{<0.4,0.5,0.6>}, \frac{\eta_4}{<0.1,0.8,0.7>}\}), \\ ((\mho_1/0.2, d, 1), \{\frac{\eta_1}{<0.3,0.4,0.8>}, \frac{\eta_2}{<0.6,0.3,0.5>}, \frac{\eta_3}{<0.2,0.5,0.6>}, \frac{\eta_4}{<0.1,0.6,0.7>}\}), \\ ((\mho_3/0.4, c, 0), \{\frac{\eta_1}{<0.1,0.6,0.9>}, \frac{\eta_2}{<0.3,0.6,0.7>}, \frac{\eta_3}{<0.6,0.1,0.2>}, \frac{\eta_4}{<0.7,0.2,0.3>}\}), \end{array} \right\}$$

$$(\hbar_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} ((\mho_1/0.2, c, 0), \{\frac{\eta_1}{<0.2,0.1,0.3>}, \frac{\eta_2}{<0.7,0.2,0.4>}, \frac{\eta_3}{<0.5,0.2,0.5>}, \frac{\eta_4}{<0.2,0.3,0.6>}\}), \\ ((\mho_3/0.4, c, 1), \{\frac{\eta_1}{<0.1,0.5,0.6>}, \frac{\eta_2}{<0.4,0.2,0.5>}, \frac{\eta_3}{<0.7,0.1,0.2>}, \frac{\eta_4}{<0.8,0.1,0.4>}\}), \end{array} \right\}$$

Then $(\hbar_1, \mathbb{Q}_1) \wedge (\hbar_2, \mathbb{Q}_2) = (\hbar_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$

$$\left\{ \begin{array}{l} (((\mho_1/0.2, c, 1), (\mho_1/0.2, c, 0)), \{\frac{\eta_1}{<0.1,0.35,0.4>}, \frac{\eta_2}{<0.6,0.30,0.5>}, \frac{\eta_3}{<0.4,0.35,0.6>}, \frac{\eta_4}{<0.1,0.55,0.7>}\}), \\ (((\mho_1/0.2, d, 1), (\mho_1/0.2, c, 0)), \{\frac{\eta_1}{<0.2,0.25,0.8>}, \frac{\eta_2}{<0.6,0.25,0.5>}, \frac{\eta_3}{<0.2,0.35,0.6>}, \frac{\eta_4}{<0.1,0.45,0.7>}\}), \\ (((\mho_1/0.2, d, 1), (\mho_3/0.4, c, 1)), \{\frac{\eta_1}{<0.1,0.45,0.8>}, \frac{\eta_2}{<0.4,0.25,0.5>}, \frac{\eta_3}{<0.2,0.30,0.6>}, \frac{\eta_4}{<0.1,0.35,0.7>}\}), \\ (((\mho_1/0.2, c, 1), (\mho_3/0.4, c, 1)), \{\frac{\eta_1}{<0.1,0.55,0.6>}, \frac{\eta_2}{<0.4,0.30,0.5>}, \frac{\eta_3}{<0.4,0.30,0.6>}, \frac{\eta_4}{<0.1,0.45,0.7>}\}), \\ (((\mho_3/0.4, c, 0), (\mho_1/0.2, c, 0)), \{\frac{\eta_1}{<0.1,0.35,0.9>}, \frac{\eta_2}{<0.3,0.40,0.7>}, \frac{\eta_3}{<0.5,0.15,0.5>}, \frac{\eta_4}{<0.2,0.25,0.6>}\}), \\ (((\mho_3/0.4, c, 0), (\mho_3/0.4, c, 1)), \{\frac{\eta_1}{<0.1,0.55,0.9>}, \frac{\eta_2}{<0.3,0.40,0.7>}, \frac{\eta_3}{<0.6,0.10,0.2>}, \frac{\eta_4}{<0.7,0.15,0.4>}\}). \end{array} \right\}$$

Definition 4.14. If $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ are two $\mathcal{FPNHSE}$-sets over Δ, then $(\hbar_1, \mathbb{Q}_1)$ OR $(\hbar_2, \mathbb{Q}_2)$ denoted by $(\hbar_1, \mathbb{Q}_1) \vee (\hbar_2, \mathbb{Q}_2)$ is defined by $(\hbar_1, \mathbb{Q}_1) \vee (\hbar_2, \mathbb{Q}_2) = (\hbar_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$, while $\hbar_3(\delta, \gamma) = \hbar_1(\delta) \cup \hbar_2(\gamma), \forall (\delta, \gamma) \in \mathbb{Q}_1 \times \mathbb{Q}_2$.

Example 4.15. Reconsidering Example 3.2, suppose the following sets

$\mathbb{Q}_1 = \left\{ (\mho_1/0.2, c, 1), (\mho_1/0.2, d, 1), (\mho_3/0.4, c, 0) \right\}$

$\mathbb{Q}_2 = \left\{ (\mho_1/0.2, c, 0), (\mho_3/0.4, c, 1) \right\}$

Suppose $(\hbar_1, \mathbb{Q}_1)$ and $(\hbar_2, \mathbb{Q}_2)$ over Δ are two $\mathcal{FPNHSE}$-sets such that

$$(\hbar_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} ((\mho_1/0.2, c, 1), \{\frac{\eta_1}{<0.1,0.6,0.4>}, \frac{\eta_2}{<0.6,0.4,0.5>}, \frac{\eta_3}{<0.4,0.5,0.6>}, \frac{\eta_4}{<0.1,0.8,0.7>}\}), \\ ((\mho_1/0.2, d, 1), \{\frac{\eta_1}{<0.3,0.4,0.8>}, \frac{\eta_2}{<0.6,0.3,0.5>}, \frac{\eta_3}{<0.2,0.5,0.6>}, \frac{\eta_4}{<0.1,0.6,0.7>}\}), \\ ((\mho_3/0.4, c, 0), \{\frac{\eta_1}{<0.1,0.6,0.9>}, \frac{\eta_2}{<0.3,0.6,0.7>}, \frac{\eta_3}{<0.6,0.1,0.2>}, \frac{\eta_4}{<0.7,0.2,0.3>}\}) \end{array} \right\}$$

$$(\hbar_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} ((\mho_1/0.2, c, 0), \{\frac{\eta_1}{<0.2,0.1,0.3>}, \frac{\eta_2}{<0.7,0.2,0.4>}, \frac{\eta_3}{<0.5,0.2,0.5>}, \frac{\eta_4}{<0.2,0.3,0.6>}\}), \\ ((\mho_3/0.4, c, 1), \{\frac{\eta_1}{<0.1,0.5,0.6>}, \frac{\eta_2}{<0.4,0.2,0.5>}, \frac{\eta_3}{<0.7,0.1,0.2>}, \frac{\eta_4}{<0.8,0.1,0.4>}\}). \end{array} \right\}$$

Then $(\hbar_3, \mathbb{Q}_3) \vee (\hbar_2, \mathbb{Q}_2) = (\hbar_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$

$$\left\{ \begin{array}{l} (((\mho_1/0.2, c, 1), (\mho_1/0.2, c, 0)), \{\frac{\eta_1}{<0.2,0.35,0.3>}, \frac{\eta_2}{0<0.7,0.30,0.4>}, \frac{\eta_3}{<0.5,0.35,0.5>}, \frac{\eta_4}{<0.2,0.55,0.6>}\}), \\ (((\mho_1/0.2, d, 1), (\mho_1/0.2, c, 0)), \{\frac{\eta_1}{<0.3,0.25,0.3>}, \frac{\eta_2}{<0.7,0.25,0.4>}, \frac{\eta_3}{<0.5,0.35,0.5>}, \frac{\eta_4}{<0.2,0.45,0.6>}\}), \\ (((\mho_1/0.2, d, 1), (\mho_3/0.4, c, 1)), \{\frac{\eta_1}{<0.3,0.45,0.6>}, \frac{\eta_2}{<0.6,0.25,0.5>}, \frac{\eta_3}{<0.7,0.30,0.2>}, \frac{\eta_4}{<0.8,0.35,0.4>}\}), \\ (((\mho_1/0.2, c, 1), (\mho_3/0.4, c, 1)), \{\frac{\eta_1}{<0.1,0.55,0.4>}, \frac{\eta_2}{<0.6,0.30,0.5>}, \frac{\eta_3}{<0.7,0.30,0.2>}, \frac{\eta_4}{<0.8,0.45,0.4>}\}), \\ (((\mho_3/0.4, c, 0), (\mho_1/0.2, c, 0)), \{\frac{\eta_1}{<0.2,0.35,0.3>}, \frac{\eta_2}{<0.7,0.40,0.4>}, \frac{\eta_3}{<0.6,0.15,0.2>}, \frac{\eta_4}{<0.7,0.25,0.3>}\}), \\ (((\mho_3/0.4, c, 0), (\mho_3/0.4, c, 1)), \{\frac{\eta_1}{<0.1,0.55,0.6>}, \frac{\eta_2}{<0.4,0.40,0.5>}, \frac{\eta_3}{<0.7,0.10,0.2>}, \frac{\eta_4}{<0.8,0.15,0.4>}\}). \end{array} \right\}$$

Proposition 4.16. *If* $(\hbar_1, \mathbb{Q}_1)$,$(\hbar_2, \mathbb{Q}_2)$ *and* $(\hbar_3, \mathbb{Q}_3)$ *are three* $\mathcal{FPNHSE}$*-sets over* Δ*, then*

(1) $((\hbar_1, \mathbb{Q}_1) \wedge (\hbar_2, \mathbb{Q}_2))^c = ((\hbar_1, \mathbb{Q}_1))^c \vee ((\hbar_2, \mathbb{Q}_2))^c$

(2) $((\hbar_1, \mathbb{Q}_1) \vee (\hbar_2, \mathbb{Q}_2))^c = ((\hbar_1, \mathbb{Q}_1))^c \wedge ((\hbar_2, \mathbb{Q}_2))^c$

Proposition 4.17. *If* $(\hbar_1, \mathbb{Q}_1)$,$(\hbar_2, \mathbb{Q}_2)$ *and* $(\hbar_3, \mathbb{Q}_3)$ *are three* $\mathcal{FPNHSE}$*-sets over* Δ*, then*

(1) $((\hbar_1, \mathbb{Q}_1) \wedge (\hbar_2, \mathbb{Q}_2)) \wedge (\hbar_3, \mathbb{Q}_3) = (\hbar_1, \mathbb{Q}_1) \wedge ((\hbar_2, \mathbb{Q}_2) \wedge (\hbar_3, \mathbb{Q}_3))$
(2) $((\hbar_1, \mathbb{Q}_1) \vee (\hbar_2, \mathbb{Q}_2)) \vee (\hbar_3, \mathbb{Q}_3) = (\hbar_1, \mathbb{Q}_1) \vee ((\hbar_2, \mathbb{Q}_2) \vee (\hbar_3, \mathbb{Q}_3))$
(3) $(\hbar_1, \mathbb{Q}_1) \vee ((\hbar_2, \mathbb{Q}_2) \wedge (\hbar_3, \mathbb{Q}_3) = ((\hbar_1, \mathbb{Q}_1) \vee ((\hbar_2, \mathbb{Q}_2)) \wedge ((\hbar_1, \mathbb{Q}_1) \vee (\hbar_3, \mathbb{Q}_3))$
(4) $(\hbar_1, \mathbb{Q}_1) \wedge ((\hbar_2, \mathbb{Q}_2) \vee (\hbar_3, \mathbb{Q}_3)) = ((\hbar_1, \mathbb{Q}_1) \wedge ((\hbar_2, \mathbb{Q}_2)) \vee ((\hbar_1, \mathbb{Q}_1) \wedge (\hbar_3, \mathbb{Q}_3))$.

5. An Application to Fuzzy Parameterized Neutrosophic Hypersoft Expert Set

In this section, an application of $\mathcal{FPNHSE}$-set theory with a proposed algorithm in a decision-making problem, is presented.

Statement of the problem

The procurement of an electronic equipment has evolved in the product selection scenario into a difficult issue for a person and an organisation. For the usage of his family, Mr. Bay is looking for an LED TV. He has never purchased it before. He solicits assistance from his buddies who may have knowledge on where to buy such a device. Consider the following while buying this device in light of their friends' experiences:

(1) **Screen Resolution**: The screen goal of a LED TV is the number of pixels in each aspect that the TV can show locally. Higher-goal screens permit you to see all the more fine subtleties in your beloved substance.
(2) **Refresh Rate**: Assuming you're on the lookout for a LED TV, you've likely heard a great deal about "speed." When promotions and audits talk about how quick a LED TV is, they allude to the showcase's invigorate rate or how regularly it changes the image. TV and motion pictures don't show natural movement, even handfuls, and many casings each second, similar to a reel of film or a colossal flipbook. The quicker the LED TV, the more casings it shows each second.
(3) **Warranty**: Service agreement for your TV or TV covering all assembling absconds, programming issues, and electrical glitches or breakdowns. The maintenance agreement for TV or TV begins following your producer or OEM guarantee lapses.
(4) **Ports**: Somewhere around four ports ought to be accessible, including USB, HDMI, sound/video, and VGA. Additionally, ensure it upholds every hard circle and pen drive to play recordings.
(5) **Screen Size**: There is a broad scope of Tv sizes accessible in the market to choose from. The right TV size gives you a vivid review experience. Looking for an ideal space from room, lounge, and nearness to the TV screen. Contrast TV stands and divider mounted set up to track down a suitable spot in the space to put the TV set. The right screen size and distance give you immersive survey encounters.

Proposed Algorithm : Selection of LED TV

▷ **Start**:

▷ **Construction**:

———1. Construct $\mathcal{FPNHSE}$-set (ξ, K)

▷ **Computation**:

———2. Determine Agree-$\mathcal{FPNHSE}$-set and Disagree-$\mathcal{FPNHSE}$-set.

———3. Calculation of Values of $T_(0_i) - I_(0_i) - F_(0_i)$ for each $0_i \in \Omega$.

———4. Calculate the the highest numerical grade for Agree and Disagree-$\mathcal{FPNHSE}$-sets.

———5. Determine the score of each element $0_i \in \Omega$ for Agree and Disagree-$\mathcal{FPNHSE}$-sets.

———-6. Determine the score difference for each element $c_i \in \Omega$.

▷ **Output**:

———7. Compute n, for which $M=$ max j_i to decide the best solution of the problem.

▷ **End**:

Step-1

Let four categories of LED TV forming the universe of discourse $\Omega = \{t_1, t_2, t_3, t_4\}$ and X $= \{E_1 = Henry, E_2 = John, E_3 = Watson\}$ be a set of experts for this purchase. The following are the attribute-valued sets for prescribed attributes:

$W_1 = ScreenResolution = \{w_1 = 1280 \times 720pixels, w_2 = 1920 \times 1080pixels\}$

$W_2 = RefreshRate = \{w_3 = 60Hz, w_4 = 120Hz\}$, $W_3 = Warranty = \{w_5 = 4years, w_6 = 5years\}$, $W_4 = Ports = \{w_7 = 4, w_8 = 5\}$, $W_5 = ScreenSize = \{w_9 = 24inch, w_{10} = 32inch\}$ and then $W =_1 \times W_2 \times W_3 \times W_4 \times W_5$

$$W = \left\{ \begin{array}{l} (w_1, w_3, w_5, w_7, w_9), (w_1, w_3, w_5, w_7, w_{10}), (w_1, w_3, w_5, w_8, w_9), (w_1, w_3, w_5, w_8, w_{10}), \\ (w_1, w_3, w_6, w_7, w_9), (w_1, w_3, w_6, w_7, w_{10}), (w_1, w_3, w_6, w_8, w_9), (w_1, w_3, w_6, w_8, w_{10}), \\ (w_1, w_4, w_5, w_7, w_9), (w_1, w_4, w_5, w_7, w_{10}), (w_1, w_4, w_5, w_8, w_9), (w_1, w_4, w_5, w_8, w_{10}), \\ (w_1, w_4, w_6, w_7, w_9), (w_1, w_4, w_6, w_7, w_{10}), (w_1, w_4, w_6, w_8, w_9), (w_1, w_4, w_6, w_8, w_{10}), \\ (w_2, w_3, w_5, w_7, w_9), (w_2, w_3, w_5, w_7, w_{10}), (w_2, w_3, w_5, w_8, w_9), (w_2, w_3, w_5, w_8, w_{10}), \\ (w_2, w_3, w_6, w_7, w_9), (w_2, w_3, w_6, w_7, w_{10}), (w_2, w_3, w_6, w_8, w_9), (w_2, w_3, w_6, w_8, w_{10}), \\ (w_2, w_4, w_5, w_7, w_9), (w_2, w_4, w_5, w_7, w_{10}), (w_2, w_4, w_5, w_8, w_9), (w_2, w_4, w_5, w_8, w_{10}), \\ (w_2, w_4, w_6, w_7, w_9), (w_2, w_4, w_6, w_7, w_{10}), (w_2, w_4, w_6, w_8, w_9), (w_2, w_4, w_6, w_8, w_{10}) \end{array} \right\}$$

Now take $K \subseteq N$ as

$K = \{k_1/0.2 = (w_1, w_3, w_5, w_7, w_9), k_2/0.3 = (w_1, w_3, w_6, w_7, w_{10}), k_3/0.4 = (w_1, w_4, w_6, w_8, w_9), k_4/0.5 = (w_2, w_3, w_6, w_8, w_9), k_5/0.6 = (w_2, w_4, w_6, w_7, w_{10})\}$

$$(\xi, K)_1 = \left\{ \begin{array}{l}
((k_1, E_1, 1), \{ \frac{c_1}{<0.9,0.1,0.7>}, \frac{c_2}{<0.5,0.2,0.1>}, \frac{c_3}{<0.6,0.2,0.4>}, \frac{c_4}{<0.7,0.1,0.2>} \}), \\
((k_1, E_2, 1), \{ \frac{c_1}{<0.8,0.2,0.4>}, \frac{c_2}{<0.9,0.2,0.3>}, \frac{c_3}{<0.6,0.2,0.3>}, \frac{c_4}{<0.9,0.1,0.5>} \}), \\
((k_1, E_3, 1), \{ \frac{c_1}{<0.7,0.3,0.1>}, \frac{c_2}{<0.4,0.1,0.2>}, \frac{c_3}{<0.6,0.1,0.4>}, \frac{c_4}{<0.8,0.6,0.1>} \}), \\
((k_2, E_1, 1), \{ \frac{c_1}{<0.8,0.4,0.2>}, \frac{c_2}{<0.4,0.2,0.1>}, \frac{c_3}{<0.7,0.1,0.5>}, \frac{c_4}{<0.5,0.2,0.1>} \}), \\
((k_2, E_2, 1), \{ \frac{c_1}{<0.5,0.1,0.3>}, \frac{c_2}{<0.9,0.4,0.4>}, \frac{c_3}{<0.7,0.4,0.1>}, \frac{c_4}{<0.9,0.2,0.3>} \}), \\
((k_2, E_3, 1), \{ \frac{c_1}{<0.7,0.3,0.1>}, \frac{c_2}{<0.8,0.2,0.4>}, \frac{c_3}{<0.9,0.2,0.5>}, \frac{c_4}{<0.8,0.2,0.1>} \}), \\
((k_3, E_1, 1), \{ \frac{c_1}{<0.9,0.4,0.2>}, \frac{c_2}{<0.7,0.2,0.3>}, \frac{c_3}{<0.8,0.2,0.5>}, \frac{c_4}{<0.7,0.2,0.4>} \}), \\
((k_3, E_2, 1), \{ \frac{c_1}{<0.8,0.3,0.4>}, \frac{c_2}{<0.9,0.2,0.5>}, \frac{c_3}{<0.6,0.2,0.3>}, \frac{c_4}{<0.9,0.2,0.3>} \}), \\
((k_3, E_3, 1), \{ \frac{c_1}{<0.7,0.4,0.1>}, \frac{c_2}{<0.6,0.3,0.2>}, \frac{c_3}{<0.8,0.2,0.3>}, \frac{c_4}{<0.7,0.1,0.4>} \}), \\
((k_4, E_1, 1), \{ \frac{c_1}{<0.9,0.1,0.7>}, \frac{c_2}{<0.8,0.3,0.1>}, \frac{c_3}{<0.7,0.1,0.4>}, \frac{c_4}{<0.6,0.3,0.2>} \}), \\
((k_4, E_2, 1), \{ \frac{c_1}{<0.8,0.1,0.4>}, \frac{c_2}{<0.4,0.3,0.1>}, \frac{c_3}{<0.8,0.3,0.1>}, \frac{c_4}{<0.7,0.2,0.1>} \}), \\
((k_4, E_3, 1), \{ \frac{c_1}{<0.6,0.2,0.4>}, \frac{c_2}{<0.1,0.3,0.2>}, \frac{c_3}{<0.9,0.5,0.3>}, \frac{c_4}{<0.6,0.1,0.3>} \}), \\
((k_5, E_1, 1), \{ \frac{c_1}{<0.6,0.3,0.2>}, \frac{c_2}{<0.8,0.2,0.4>}, \frac{c_3}{<0.3,0.2,.01>}, \frac{c_4}{<0.7,0.1,0.4>} \}), \\
((k_5, E_2, 1), \{ \frac{c_1}{<0.5,0.3,0.2>}, \frac{c_2}{<0.6,0.2,0.3>}, \frac{c_3}{<0.4,0.3,0.1>}, \frac{c_4}{<0.6,0.1,0.2>} \}), \\
((k_5, E_3, 1), \{ \frac{c_1}{<0.4,0.3,0.1>}, \frac{c_2}{<0.6,0.1,0.2>}, \frac{c_3}{<0.3,0.1,0.2>}, \frac{c_4}{<0.9,0.4,0.2>} \})
\end{array} \right\}.$$

and

$$(\xi, K)_0 = \left\{ \begin{array}{l}
((k_1, E_1, 0), \{ \frac{c_1}{<0.6,0.3,0.1>}, \frac{c_2}{<0.9,0.1,0.2>}, \frac{c_3}{<0.8,0.2,0.3>}, \frac{c_4}{<0.6,0.1,0.4>} \}), \\
((k_1, E_2, 0) \{ \frac{c_1}{<0.7,0.2,0.3>}, \frac{c_2}{<0.6,0.3,0.2>}, \frac{c_3}{<0.7,0.2,0.1>}, \frac{c_4}{<0.8,0.2,0.1>} \}), \\
((k_1, E_3, 0), \{ \frac{c_1}{<0.8,0.3,0.4>}, \frac{c_2}{<0.6,0.2,0.1>}, \frac{c_3}{<0.7,0.4,0.2>}, \frac{c_4}{<0.9,0.4,0.4>} \}), \\
((k_2, E_1, 0), \{ \frac{c_1}{<0.9,0.4,0.2>}, \frac{c_2}{<0.9,0.3,0.3>}, \frac{c_3}{<0.8,0.1,0.4>}, \frac{c_4}{<0.8,0.2,0.2>} \}), \\
((k_2, E_2, 0), \{ \frac{c_1}{<0.9,0.5,0.1>}, \frac{c_2}{<0.7,0.2,0.3>}, \frac{c_3}{<0.6,0.3,0.2>}, \frac{c_4}{<0.5,0.3,0.2>} \}), \\
((k_2, E_3, 0), \{ \frac{c_1}{<0.7,0.2,0.1>}, \frac{c_2}{<0.8,0.5,0.2>}, \frac{c_3}{<0.7,0.1,0.2>}, \frac{c_4}{<0.8,0.4,0.3>} \}), \\
((k_3, E_1, 0), \{ \frac{c_1}{<0.9,0.1,0.6>}, \frac{c_2}{<0.7,0.2,0.1>}, \frac{c_3}{<0.9,0.1,0.5>}, \frac{c_4}{<0.8,0.2,0.1>} \}), \\
((k_3, E_2, 0), \{ \frac{c_1}{<0.8,0.2,0.1>}, \frac{c_2}{<0.9,0.2,0.1>}, \frac{c_3}{<0.9,0.6,0.1>}, \frac{c_4}{<0.9,0.2,0.1>} \}), \\
((k_3, E_3, 0), \{ \frac{c_1}{<0.6,0.2,0.4>}, \frac{c_2}{<0.6,0.1,0.3>}, \frac{c_3}{<0.9,0.4,0.1>}, \frac{c_4}{<0.7,0.4,0.2>} \}), \\
((k_4, E_1, 0), \{ \frac{c_1}{<0.6,0.3,0.2>}, \frac{c_2}{<0.7,0.5,0.1>}, \frac{c_3}{<0.8,0.2,0.1>}, \frac{c_4}{<0.9,0.4,0.1>} \}), \\
((k_4, E_2, 0), \{ \frac{c_1}{<0.5,0.2,0.1>}, \frac{c_2}{<0.5,0.2,0.1>}, \frac{c_3}{<0.3,0.1,0.0>}, \frac{c_4}{<0.6,0.2,0.1>} \}), \\
((k_4, E_3, 0), \{ \frac{c_1}{<0.7,0.1,0.3>}, \frac{c_2}{<0.7,0.2,0.1>}, \frac{c_3}{<0.6,0.1,0.4>}, \frac{c_4}{<0.9,0.1,0.6>} \}), \\
((k_5, E_1, 0), \{ \frac{c_1}{<0.9,0.5,0.1>}, \frac{c_2}{<0.6,0.2,0.1>}, \frac{c_3}{<0.9,0.1,0.3>}, \frac{c_4}{<0.9,0.2,0.5>} \}), \\
((k_5, E_2, 0), \{ \frac{c_1}{<0.8,0.6,0.1>}, \frac{c_2}{<0.9,0.2,0.1>}, \frac{c_3}{<0.8,0.5,0.2>}, \frac{c_4}{<0.7,0.3,0.2>} \}), \\
((k_5, E_3, 0), \{ \frac{c_1}{<0.8,0.1,0.3>}, \frac{c_2}{<0.5,0.2,0.4>}, \frac{c_3}{<0.8,0.1,0.7>}, \frac{c_4}{<0.6,0.1,0.3>} \})
\end{array} \right\}$$

are $\mathcal{FPNHSE}$-sets.

Step-2

Table 1 and Table 2 represent the values of $T_{(}c_i)$-$I_{(}c_i)$-$F_{(}c_i)$.

Step-(2)

Grade values of agree and disagree $\mathcal{FPNHSE}$-sets have been represented in Table 3 and Table 4 respectively.

Step-(3-5)

Table 1. Agree-$\mathcal{FPNHSE}$-set

C	c_1	c_2	c_3	c_4
$(k_1, E_1, 1)$	0.1	0.3	0.0	0.4
$(k_1, E_2, 1)$	0.2	0.4	0.1	0.3
$(k_1, E_3, 1)$	0.4	0.1	0.1	0.1
$(k_2, E_1, 1)$	0.2	0.1	0.1	0.3
$(k_2, E_2, 1)$	0.2	0.1	0.2	0.4
$(k_2, E_3, 1)$	0.3	0.2	0.2	0.2
$(k_3, E_1, 1)$	0.3	0.2	0.1	0.1
$(k_3, E_2, 1)$	0.1	0.2	0.3	0.2
$(k_3, E_3, 1)$	0.2	0.1	0.3	0.2
$(k_4, E_1, 1)$	0.1	0.4	0.2	0.1
$(k_4, E_2, 1)$	0.3	0.0	0.2	0.4
$(k_4, E_3, 1)$	0.0	0.0	0.1	0.2
$(k_5, E_1, 1)$	0.1	0.2	0.0	0.3
$(k_5, E_2, 1)$	0.0	0.1	0.0	0.3
$(k_5, E_3, 1)$	0.0	0.4	0.0	0.3

Table 2. Disagree-$\mathcal{FPNHSE}$-set

C	c_1	c_2	c_3	c_4
$(k_1, E_1, 0)$	0.4	0.6	0.3	0.2
$(k_1, E_2, 0)$	0.2	0.1	0.4	0.5
$(k_1, E_3, 0)$	0.1	0.1	0.1	0.4
$(k_2, E_1, 0)$	0.3	0.3	0.3	0.4
$(k_2, E_2, 0)$	0.3	0.2	0.1	0.0
$(k_2, E_3, 0)$	0.3	0.1	0.4	0.1
$(k_3, E_1, 0)$	0.2	0.4	0.3	0.5
$(k_3, E_2, 0)$	0.5	0.4	0.2	0.6
$(k_3, E_3, 0)$	0.0	0.4	0.2	0.3
$(k_4, E_1, 0)$	0.1	0.1	0.5	0.4
$(k_4, E_2, 0)$	0.1	0.2	0.0	0.3
$(k_4, E_3, 0)$	0.4	0.1	0.1	0.2
$(k_5, E_1, 0)$	0.1	0.3	0.5	0.2
$(k_5, E_2, 0)$	0.1	0.6	0.1	0.2
$(k_5, E_3, 0)$	0.5	0.1	0.0	0.4

The difference of scores of agree and disagree-$\mathcal{FPNHSE}$-sets have been shown in Table 5. The scores for agree-$\mathcal{FPNHSE}$-set are :

$S(c_1) = 0.6$, $S(c_2) = 1.3$, $S(c_3) = 0.6$ and $S(c_4) = 2.0$

whereas scores for disagree-$\mathcal{FPNHSE}$-set are:

TABLE 3. Numerical Grades of agree $\mathcal{FPNHSE}$-set

Pairs	c_i	Highest Numerical Grade
$(k_1, E_1, 1)$	c_4	0.4
$(k_1, E_2, 1)$	c_2	0.4
$(k_1, E_3, 1)$	c_1	0.4
$(k_2, E_1, 1)$	c_2	0.1
$(k_2, E_2, 1)$	c_4	0.4
$(k_2, E_3, 1)$	c_1	0.3
$(k_3, E_1, 1)$	c_1	0.3
$(k_3, E_2, 1)$	c_3	0.3
$(k_3, E_3, 1)$	c_3	0.3
$(k_4, E_1, 1)$	c_2	0.4
$(k_4, E_2, 1)$	c_4	0.4
$(k_4, E_3, 1)$	c_4	0.2
$(k_5, E_1, 1)$	c_4	0.3
$(k_5, E_2, 1)$	c_4	0.3
$(k_5, E_3, 1)$	c_2	0.4

TABLE 4. Numerical Grades of disagree $\mathcal{FPNHSE}$-set

Pairs	c_i	Highest Numerical Grade
$(k_1, E_1, 0)$	c_2	0.6
$(k_1, E_2, 0)$	c_3	0.4
$(k_1, E_3, 0)$	c_4	0.4
$(k_2, E_1, 0)$	c_4	0.4
$(k_2, E_2, 0)$	c_1	0.4
$(k_2, E_3, 0)$	c_3	0.4
$(k_3, E_1, 0)$	c_4	0.5
$(k_3, E_2, 0)$	c_4	0.6
$(k_3, E_3, 0)$	c_2	0.4
$(k_4, E_1, 0)$	c_3	0.5
$(k_4, E_2, 0)$	c_4	0.3
$(k_4, E_3, 0)$	c_1	0.4
$(k_5, E_1, 0)$	c_3	0.5
$(k_5, E_2, 0)$	c_2	0.6
$(k_5, E_3, 0)$	c_1	0.5

$S(c_1) = 1.3$, $S(c_2) = 1.6$, $S(c_3) = 1.8$ and $S(c_4) = 2.2$.

Step-6; Decision

As from above result, c_4 is preferred to be best and have been mentioned in Figure 2.

TABLE 5. Numerical values of $j_i = G_i - H_i$

G_i	H_i	$j_i = G_i - H_i$
$S(c_1) = 0.6$	$S(c_1) = 1.3$	-0.7
$S(c_2) = 1.3$	$S(c_2) = 1.6$	-0.3
$S(c_3) = 0.6$	$S(c_3) = 1.8$	-1.2
$S(c_4) = 2.0$	$S(c_4) = 2.2$	0.2

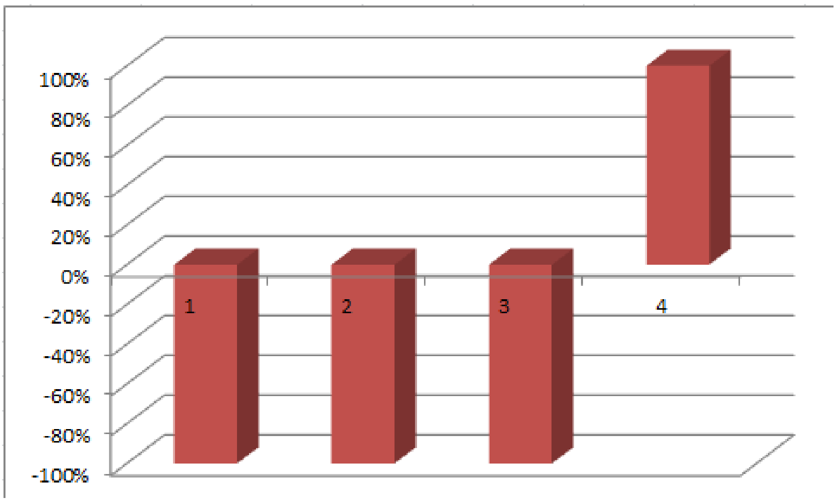

FIGURE 2. Ranking of Alternatives for Algorithm

6. Conclusions

The foundations of the fuzzy parameterized neutrosophic hypersoft expert set are developed in this study, along with certain generalisations of theoretical operations like union, intersection, complement, AND, and OR. With specific instances, some fundamental concepts like exclusion, contradiction, and laws are explored. These concepts include idempotent, absorption, domination, identity, associative, and distributive laws. In the end, an algorithm is created to describe how the decision-making problem is solved. This new work inspires further advancements of related research and practical applications while providing an exceptional expansion to existing theories for handling ambiguity, untruth, and truthness.

References

1. Smarandache. F. (2005). A unifying field in logics. neutrosophy: neutrosophic probability, set and logic. Rehoboth: American Research Press.
2. Smarandache. F. (2005). Neutrosophic set, a generialization of the intuituionistics fuzzy sets. Inter. J. Pure Appl. Math., 24, 287-297.
3. Smarandache. F. (2013). Introduction to neutrosophic measure, neutrosophic measure neutrosophic integral, and neutrosophic propability.

4. Zadeh, L. A. (1965). Fuzzy sets, Information and Control, 8, 338-353.
5. Atanassov. K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87-96.
6. Wang. H., Smarandache, F., Zhang. Y., & Sunderraman, R. (2010). Single valued neutrosophic sets. Multisspace and Multistructure, 4, 410-413.
7. Kharal, A. A. (2013). Neutrosophic multicriteria decision making method, New Mathematics and Natural Computation, Creighton University, USA.
8. Molodtsov, D. (1999). Soft set theoryfirst results. Computers and Mathematics with Applications, 37(4-5), 19-31.
9. Rahman, A. U., Saeed, M., Arshad, M., Ihsan, M., & Ahmad, M. R. (2021). (m; n)-convexity-cum-concavity on fuzzy soft set with applications in first and second sense. Punjab University Journal of Mathematics. 53(1), 19-33.
10. Rahman, A. U., Saeed, M., Arshad, M., Ihsan, M., & Ayaz, S. (2021). A conceptual framework of m-convex and m-concave sets under soft set environment with properties. Transactions in Mathematical and Computational Sciences, 1(1), 40-60.
11. Maji, P. K., Roy, A. R., & Biswas, R. (2001). Fuzzy Soft Sets. Journal of Fuzzy Mathematics, 9, 589-602.
12. Chen, D., Tsang, E. C. C. D., Yeung, S., & Wang, X. (2005). The parameterization reduction of soft sets and its applications. Computers and Mathematics with Applications, 49(5-6), 757-763.
13. Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. Computers and Mathematics with Applications, 45(4-5), 555-562.
14. Maji, P. K., Roy, A. R., & Biswas, R. (2002). An application of soft sets in a decision making problem. Computers and Mathematics with Applications, 44(8-9), 1077-1083.
15. Alkhazaleh, S., Salleh, A. R., & Hassan, N. (2011). Soft multisets theory. Applied Mathematical Sciences, 5(72), 3561-3573.
16. Alkhazaleh, S., Salleh, A. R., & Hassan, N. (2011). Possibility fuzzy soft set. Advances in Decision Sciences, 2011.
17. Alkhazaleh, S., Salleh, A. R., & Hassan, N. (2011). Fuzzy parameterized interval-valued fuzzy soft set. Applied Mathematical Sciences, 5(67), 3335-3346.
18. Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. Computers and Mathematics with Applications, 57(9), 1547-1553.
19. Alkhazaleh, S., & Salleh, A. R. (2001). Soft expert sets. Advances in Decision Sciences, 2001, 757868-1.
20. Alkhazaleh, S., & Salleh, A. R. (2014). Fuzzy soft expert set and its application. Applied Mathematics, 5, 1349-1368.
21. Ihsan, M., Saeed, M., & Rahman, A. U. (2021). A rudimentary approach to develop context for convexity cum concavity on soft expert set with some generalized results. Punjab University Journal of Mathematics, 53(9), 621-629.
22. Ihsan, M., Rahman, A. U., Saeed, M., & Khalifa, H. A. E. W. (2021). Convexity-cum-concavity on fuzzy soft expert set with certain properties. International Journal of Fuzzy Logic and Intelligent Systems, 21(3), 233-242.
23. Broumi, S., & Smarandache, F. (2015). Intuitionistic fuzzy soft expert sets and its application in decision making. Journal of new theory, 1, 89-105.
24. Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets and Systems, 22, 168-170.
25. Saeed, M., Rahman, A. U., Ahsan, M., & Smarandache, F. (2022). Theory of hypersoft sets: axiomatic properties, aggregation operations, relations, functions and matrices. Neutrosophic Sets and Systems, 51, 744-765.

26. Abbas, M., Murtaza, G., & Smarandache, F. (2020). Basic operations on hypersoft sets and hypersoft point. Neutrosophic Sets and Systems, 35, 407-421.
27. Rahman, A. U., Saeed, M., Smarandache, F., & Ahmad, M. R. (2020). Development of hybrids of hypersoft set with complex fuzzy set, complex intuitionistic fuzzy set and complex neutrosophic set. Neutrosophic Sets and Systems, 38, 335-354.
28. Rahman, A. U., Saeed, M., & Smarandache, F. (2020). Convex and concave hypersoft sets with some properties. Neutrosophic Sets and Systems, 38, 497-508.
29. Al-Quran, A., & Hassan, N. (2016). Neutrosophic vague soft expert set theory. Journal of Intelligent, Fuzzy Systems, 30(6), 3691-3702
30. Bashir, M., & Salleh, A. R. (2012). Fuzzy parameterized soft expert set. Abstract and Applied Analysis, 2012.
31. Bashir, M., & Salleh, A. R. (2012). Possibility fuzzy soft expert set. Open Journal of Applied Sciences, 12, 208-211.
32. Al-Quran, A., & Hassan, N. (2016). Fuzzy parameterised single valued neutrosophic soft expert set theory and its application in decision making. Int. J. Appl. Decis. Sci., 9, 212-227.
33. Rahman, A. U., Hafeez, A., Saeed, M., Ahmad, M. R., & Farwa, U. (2021). Development of rough hypersoft set with application in decision making for the best choice of chemical material. In Theory and application of hypersoft set, Pons Publication House, Brussel, 192-202.
34. Rahman, A. U., Saeed, M., Arshad, M., & Dhital, A. (2021). A novel approach to neutrosophic hypersoft graphs with properties. Neutrosophic Sets and Systems, 46, 336-355.
35. Rahman, A. U., Saeed, M., Khalid, A., Ahmad, M. R., & Ayaz, S. (2021). Decision-making application based on aggregations of complex fuzzy hypersoft set and development of interval-valued complex fuzzy hypersoft set. Neutrosophic Sets and Systems, 46, 300-317.
36. Rahman, A. U., Saeed, M., & Hafeez, A. (2021). Theory of bijective hypersoft set with application in decision making. Punjab University Journal of Mathematics, 53(7), 511-527.
37. Ihsan, M., Rahman, A. U., & Saeed, M. (2021). Hypersoft expert set with application in decision making for recruitment process. Neutrosophic Sets and Systems, 42, 191-207.
38. Ihsan, M., Rahman, A. U., & Saeed, M. (2021). Fuzzy hypersoft expert set with application in decision making for the best selection of product. Neutrosophic Sets and Systems, 46, 318-335.
39. Çağman, N., Citak, F., & Enginoglu, S. (2011). FP-soft set theory and its applications. Annals of Fuzzy Mathematics and Informatics, 2(2), 219-226.
40. Bashir, M., & Salleh, A. R. (2012). Fuzzy parameterized soft expert set. Abstract and Applied Analysis, 2012, 1-15.
41. Rahman, A. U., Saeed, M., & Khalifa, H. A. E. W. (2022). Decision making application based on parameterization of fuzzy hypersoft set with fuzzy setting. Italian Journal of Pure and Applied Mathematics, 48, 1033-1048.

Received: Sep 10, 2022. Accepted: Dec 22, 2022