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# Single-Valued Neutrosophic Covering-Based Rough Set Model Over Two Universes and Its Application in MCDM

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**Abstract:** This article aims to propose a new type of single-valued neutrosophic(SVN) covering-based rough sets over two universes by using Wang's single-valued neutrosophic covering rough sets. Wang's model is based on one universe but the proposed model is based on two universes and thus the new model gives a new perspective for decision-making on uncertain problems. First, we define SVN β-neighborhood, which is considered as a mapping from the universe to the set of SVN sets in another universe and study its properties. Then we investigate the properties of the new type of SVN covering-based rough set model over two universes. Also, we give a necessary and sufficient condition under which two SVN β-coverings generate the same SVN covering lower and the upper approximation. In addition, we also present the matrix representation of SVN covering lower and upper approximation operators over two universes for solving real-life-based multi-criteria decision-making problems.

**Keywords:** SVN sets; SVN β-neighborhood; SVN covering-based rough set; MCDM

## 1. Introduction

The discovery of fuzzy set (FS), introduced by Zadeh[1] has been regarded as the finest discovery that is utilized to solve vague and uncertain information with an aid of a membership function. The FS concept provides a new perspective for the decision-makers to address the issues that cannot be tackled by using traditional mathematical tools. Due to the novelty of FS, it can be employed in various practical applications given in [2-6]. To realize the importance of the non-membership value along with the membership value of an attribute in a universe, Atanassov[7] introduced the intuitionistic fuzzy set(IFS). To handle more complexity that arises in various real uncertain decision-making problems; the FS concept has been further extended by introducing interval-valued fuzzy set [8], interval-valued intuitionistic fuzzy set [9], picture fuzzy set [10], spherical fuzzy set [11], hesitant fuzzy set [12], Pythagorean fuzzy set [13], etc.

In our previous discussion, we were mainly concerned with fuzzy sets and their various extensions to address uncertain and vague information. But, all these types of fuzzy sets are not capable to model the indeterminate information present in human cognition. To fill up this gap, Smarandache[14] introduced the notion of neutrosophy as a new branch of philosophy. Later on, he introduced the neutrosophic set (NS) [15] as an extension of IFS. In NS, every object in the universe is characterized by the three membership functions called the truth-membership function ($T_A$), indeterminate-membership function ($I_A$), and the falsity membership function ($F_A$) with a restriction $T_A, I_A, F_A : X \to \left]^{-}0, 1^{+}\right[$ and $^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}$. Later on, Wang et al.[16] introduced the single-valued neutrosophic set(SVNS) as an instance of NS. For handling decision-making problems under a neutrosophic environment, the decision-makers face problems while deciding due to the involvement of non-standard unit intervals. So, SVNS is introduced where the non-standard unit interval is replaced by a standard unit interval. The concept of SVNS has been extensively used in numerous decision-making problems (see the references [17-21]). Also, we would like to discuss some other important topics that are useful for the further development of the proposed study as follows: fixed point results in orthogonal neutrosophic metric spaces [22]; contractive and weakly compatible mappings in neutrosophic metric spaces are utilized in solving nonlinear differential equations [23]; some new aspects of fixed point theory under the intuitionistic fuzzy set and neutrosophic set [24]; fuzzy b-metric like spaces [25]; new aspects in fuzzy fixed point theory [26]; pentagonal controlled fuzzy metric spaces and its application [27].

In 1982, the Polish mathematician Pawlak [28] proposed another useful mathematical tool known as the rough set(RS) theory. Like FS, RS is another kind of generalization of a classical set. In RS, every subset of the universe is characterized by lower and upper approximations (see [29]). Also, in RS, the concept of equivalence classes is the key issue to form two approximations. It can be useful in discovering the hidden data, modeling information systems, eliminating the redundant data, and applied in data analysis, pattern recognition, data mining, intelligent systems, medical diagnosis, machine learning, and many more (see the references [30-34]). RS deals with crisp approximation space. But, we encounter some information system that contains fuzzy characteristics. To cope with such an issue, a rough set is combined with different types of fuzzy sets and obtain new hybrid structures and their associated applications are as follows: rough fuzzy sets and soft rough sets [35], intuitionistic fuzzy rough sets and their topological properties [36], interval-valued intuitionistic rough set [37], generalized interval-valued fuzzy rough set and its decision-making approach [38], etc. Furthermore, with the combination of rough set and neutrosophic set, many theories and their practical implications are proposed in [39-46].

Pawlak's rough set model is based on partition or equivalence relation. There exist many applications in real life where the notion of an equivalence relation is restrictive. To overcome such difficulties, Yiyu et al.[47] introduced the covering-based rough set model as an extension of classical RS. In [48], Kong et al. proposed the covering-based fuzzy rough sets and their properties. By introducing the fuzzy $\beta$-covering and fuzzy $\beta$-neighborhood, Ma [49] presented two types of fuzzy covering RS models. Zhang et al.[50] introduced fuzzy $\beta$-covering $(I,T)$ fuzzy rough set model and its application in MADM problem. In [51], Zhang et al. defined the TOPSIS-WAA method built upon a covering-based fuzzy rough set. Zhou et al.[52]defined three types of fuzzy covering-based RS models. Furthermore, Yang et al. introduced some types of covering based rough sets [53]; Deer et al. investigated the properties and interrelationships of fuzzy covering-based rough set models [54]; fuzzy information system based covering based rough sets are prosed in [55]; fuzzy covering-based rough set on two different universes and its application is successfully executed by Yang [56]; Zhan et al. [57] initiated the PROMETHEE EDAS method via covering-based variable precision fuzzy rough sets [58]; MADM method under the hesitant fuzzy $\beta$ -covering rough sets setting is successfully applied in [59]; TOPSIS method for MADM via covering-based spherical fuzzy rough set model is given in [60]. For further extension of the hybrid covering-based rough set model, Zhan et al.[61] defined covering-based intuitionistic fuzzy rough sets and their application in the MADM problem; two types of intuitionistic fuzzy covering rough sets in the MCGDM problem are defined by Wang et al.[62]; two types of single-valued neutrosophic rough sets and their decision-making approach are proposed in [63]; in [64], Wang et al. introduced a new type of single-valued neutrosophic covering rough set model. Some more recent works are based on neutrosophic covering rough set model proposed in [65-68].

The objectives of this paper are furnished below:

- The purpose of this article is to propose a single-valued neutrosophic covering-based rough set model over two universes by using Wang's approach given in **[63].**
- Construction of SVN $\beta$ -neighborhood operators on two universes and study their properties.
- Construction of SVN $\beta$ -covering lower and upper approximation operators over two universes.
- Matrix representation of SVN $\beta$ -neighborhood and SVN $\beta$ -covering lower and upper approximation operators over two universes and studied some propositions on them.
- A new type of MCDM problem is solved under the proposed study with the help of an algorithm.

To visualize the effectiveness of the proposed study over the existing theories, see the following Fig 1.

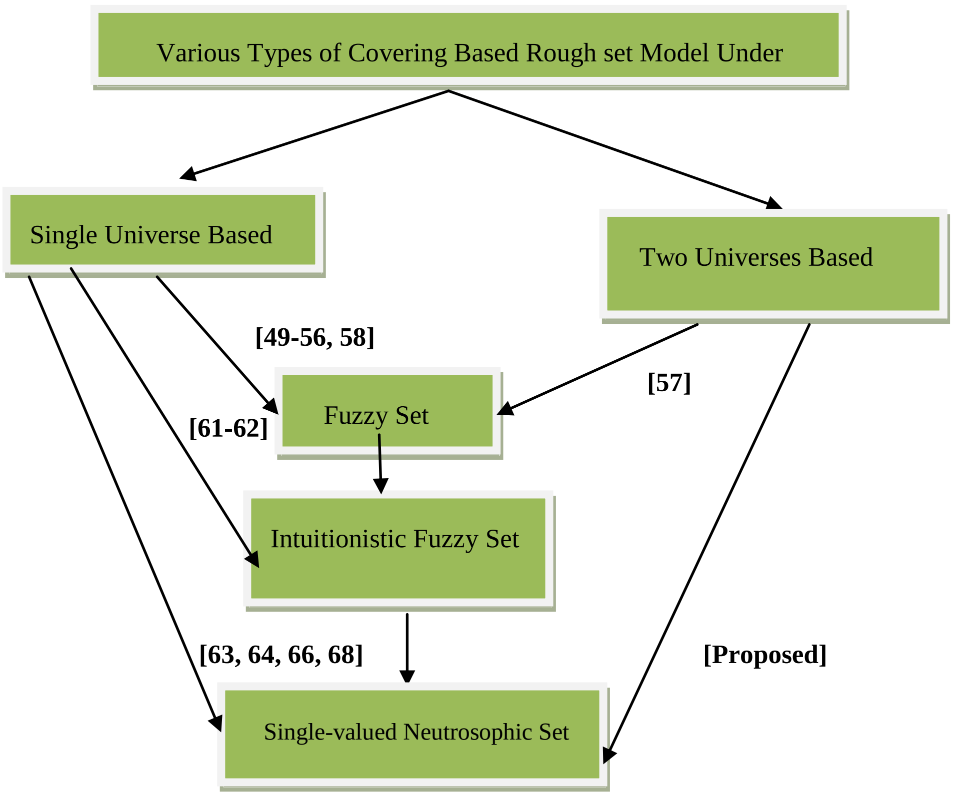


**Fig 1.** A brief diagrammatic presentation of the proposed study

**2. Preliminaries**

In this section, we give some basic concepts that are useful for the proposed study.

**Definition 2.1** [29] Let ύ be a universal set and R be an equivalence relation onύ. Then the pair (ύ, R) is called a Pawlak approximation space. Let $\Upsilon_1, \Upsilon_2, ..., \Upsilon_n$ are the equivalence classes generated by R. Therefore, R generates a partition ύ/ R $=\{\Upsilon_1, \Upsilon_2, ..., \Upsilon_n\}$ onύ.

**Definition 2.2** [69] Let ύ be a universal set and C be a family of non-empty subsets ofύ. If $\bigcup$ C = ύ, then C is known as a covering of ύ. Also, the pair (ύ, C) is called a covering approximation space.

**Definition 2.3** [16] A single-valued neutrosophic set (SVNS) $\mathrm{A}$ defined on $\mathrm{X}$ is an object of the form given below:

$\mathrm{A}=\left\{\left\langle \varepsilon, T_{\mathrm{A}}(\varepsilon), I_{\mathrm{A}}(\varepsilon), F_{\mathrm{A}}(\varepsilon)\right\rangle : \varepsilon \in \mathrm{X}\right\}$, where $T_{\mathrm{A}}(\varepsilon)$ is the degree of truth-membership, $I_{\mathrm{A}}(\varepsilon)$ is the degree of indeterminacy-membership, and $F_{\mathrm{A}}(\varepsilon)$ is the degree of falsity-membership such that $T_{\mathrm{A}}(\varepsilon), I_{\mathrm{A}}(\varepsilon), F_{\mathrm{A}}(\varepsilon) \in [0,1]$ and $0 \le T_{\mathrm{A}}(\varepsilon) + I_{\mathrm{A}}(\varepsilon) + F_{\mathrm{A}}(\varepsilon) \le 3$ for all $\varepsilon \in \mathrm{X}$. The family of SVNS over $\mathrm{X}$ is denoted by $\mathrm{I}^{SVNS(\mathrm{X})}$.

**Definition 2.4** [63] Let $\mathrm{I}^{SVNS(\mathrm{X})}$ denotes the family of SVNS in $\mathrm{X}$ and $\beta = \langle p, q, r \rangle$ be a SVN number. Then, for $\tilde{\mathrm{M}} = \{M_1, M_2, ...., M_k\}$ with $M_j \in \mathrm{I}^{SVNS(\mathrm{X})}$ $(j = 1, 2, .., k)$, a SVN $\beta$-covering of $\mathrm{X}$, if for all $\varepsilon \in \mathrm{X}$, there exists $M_j \in \tilde{\mathrm{M}}$ such that $M_j(\varepsilon) \ge \beta$. The pair ( $\mathrm{X}$, $\tilde{\mathrm{M}}$ ) is called a SVN $\beta$-covering approximation space.

**Definition 2.5** [63] Let $\tilde{\mathrm{M}}$ be a SVN $\beta$-covering of $\mathrm{X}$, where $\tilde{\mathrm{M}} = \{M_1, M_2, ...., M_k\}$. For any $\varepsilon \in \mathrm{X}$, the SVN $\beta$-neighborhood $\overset{\approx}{\mathrm{N}}{}^{\beta}_{\varepsilon}$ of $\varepsilon$ induced by $\tilde{\mathrm{M}}$ can be defined as

$$\overset{\approx}{\mathrm{N}}{}^{\beta}_{\varepsilon} = \bigcap\left\{M_j \in \tilde{\mathrm{M}} : M_j(\varepsilon) \ge \beta\right\}$$

It is to be noted that $M_j(\varepsilon) \ge \beta \Rightarrow T_{M_j}(\varepsilon) \ge p, I_{M_j}(\varepsilon) \le q$ and $F_{M_j}(\varepsilon) \le r$, where $\beta = \langle p, q, r \rangle$ is a SVN number.

## 3. Construction of SVN β-covering Approximation Space over Two Universes

In this section, we first introduce the notion of SVN β-neighborhood, and then we define a type of SVN covering-based rough set model over two universes. Here $\Gamma(X, Y)$ denotes the family of all mappings from $X$ to $Y$.

**Definition 3.1** Let $\overset{\prec}{G}$ be a SVN β-covering of the universe $X$ where $\overset{\prec}{G}=\left\{G_1,G_2,G_3,......,G_p\right\}$. For any $x\in X$, the SVN β-neighborhood $\overset{\approx}{\mathrm{N}}_x^{\beta}$ of $x$ induced by $\overset{\prec}{G}$ can be defined as:

$$\overset{\approx}{\mathrm{N}}_x^{\beta}=\bigcap\left\{G_i\in\overset{\prec}{G}:G_i\left(x\right)\geq\beta,\, i=1,2,..,p\right\}.$$

By introducing $\overset{\approx}{\mathrm{N}}_x^{\beta}:X\to SVN(Y)$, we define a new type of SVN covering-based rough set model over two universes.

By Wang's **[63]** approach, If $\overset{\prec}{G}$ be a SVN β-covering of the universe $X$ for some $\beta=\left(\mu,\nu,\gamma\right)$ where $\mu,\nu,\gamma\in\left[0,1\right]$ such that $\mu+\nu+\gamma\leq 3$, we do not sure that $f\left(\overset{\prec}{G}\right)$ is a SVN β-covering over $Y$. To support this claim we give an example in the following:

**Example 3.2** Let $X=\left\{x_1,x_2,x_3,x_4\right\}$ and $Y=\left\{y_1,y_2,y_3\right\}$ be two universal sets and $f\in\Gamma\left(X,Y\right)$ such that $f\left(x_1\right)=f\left(x_2\right)=y_1$, and $f\left(x_3\right)=f\left(x_4\right)=y_2$. Let $\overset{\prec}{G}=\left\{G_1,G_2\right\}$, where

$$G_1=\left(\frac{\langle 0.3,0.2,0.2\rangle}{x_1},\frac{\langle 0.4,0.1,0.3\rangle}{x_2},\frac{\langle 0.3,0.1,0.3\rangle}{x_3},\frac{\langle 0.3,0.3,0.4\rangle}{x_4}\right)$$

$$G_2=\left(\frac{\langle 0.4,0.3,0.4\rangle}{x_1},\frac{\langle 0.3,0.4,0.3\rangle}{x_2},\frac{\langle 0.4,0.2,0.4\rangle}{x_3},\frac{\langle 0.3,0.3,0.4\rangle}{x_4}\right)$$

It is to be noted that $\overset{\prec}{G}$ is a SVN β-covering of $X$, where $\beta=\left(0.3,0.3,0.4\right)$.

Now, $f\left(G_1\right)\left(y_1\right)=\bigcup_{x\in f^{-1}(y_1)}G_1\left(x\right)=G_1\left(x_1\right)\vee G_1\left(x_2\right)=\langle 0.4,0.1,0.2\rangle$

$$f\left(G_1\right)\left(y_2\right)=\bigcup_{x\in f^{-1}(y_2)}G_1\left(x\right)=G_1\left(x_3\right)\vee G_1\left(x_4\right)=\langle 0.3,0.1,0.3\rangle$$

$$f\left(G_1\right)\left(y_3\right)=\bigcup_{x\in f^{-1}(y_3)}G_1\left(x\right)=0$$

$$f\left(G_1\right)=\left(\frac{\langle 0.4,0.1,0.2\rangle}{y_1},\frac{\langle 0.3,0.1,0.3\rangle}{y_2}\right)$$

Similarly, $f\left(G_2\right)\left(y_1\right)=\bigcup_{x\in f^{-1}(y_1)}G_2\left(x\right)=G_2\left(x_1\right)\vee G_2\left(x_2\right)=\langle 0.4,0.3,0.3\rangle$

$$f\left(G_2\right)\left(y_2\right)=\bigcup_{x\in f^{-1}(y_2)} G_2\left(x\right)=G_2\left(x_3\right)\vee G_2\left(x_4\right)=\langle 0.4,0.2,0.4\rangle$$

$$f\left(G_2\right)\left(y_3\right)=\bigcup_{x\in f^{-1}(y_3)} G_2\left(x\right)=0$$

$$f\left(G_2\right)=\left(\frac{\langle 0.4,0.3,0.3\rangle}{y_1},\frac{\langle 0.4,0.2,0.4\rangle}{y_2}\right)$$

Therefore, $\left\{f\left(G_1\right), f\left(G_2\right)\right\}$ is not a SVN $\beta$ -covering of $Y$ .

Based on the above example, a natural question arises that under what condition $f\left(\overset{\prec}{G}\right)$ is a SVN $\beta$ -covering of $Y$ for which $\overset{\prec}{G}$ is a SVN $\beta$ -covering of $X$ . For further investigation we discuss the following:

**Proposition 3.3** Let $X$ and $Y$ be two universes and $\beta=\left(\mu,\nu,\gamma\right)$ where $\mu,\nu,\gamma\in\left[0,1\right]$ such that $\mu+\nu+\gamma\leq 3$ and the family of all surjective mappings from $X$ to $Y$ be denoted by $Sur(X,Y)$ , where $f\in Sur(X,Y)$ . Then we consider the following:

(1) If $\overset{\prec}{G}$ is a SVN $\beta$ -covering of $X$ , then $f\left(\overset{\prec}{G}\right)$ is a SVN $\beta$ -covering of $Y$ .

(2) If $\overset{\prec}{H}$ is a SVN $\beta$ -covering of $Y$ if and only if $f^{-1}\left(\overset{\prec}{H}\right)$ is a SVN $\beta$ -covering of $X$ .

The converse of the Proposition (1) does not hold. To hold the converse of Proposition (1), we give the following necessary condition:

**Theorem 3.4** Let $f:X\rightarrow Y$ be a bijection from $X$ to $Y$ , $\beta=\left(\mu,\nu,\gamma\right)$ where $\mu,\nu,\gamma\in\left[0,1\right]$ such that $\mu+\nu+\gamma\leq 3$ and $\overset{\prec}{G}$ be a family of SVN sets on $X$ . Then $\overset{\prec}{G}$ is a SVN $\beta$ -covering of $X$ iff $f\left(\overset{\prec}{G}\right)$ is also a SVN $\beta$ -covering of $Y$ .

**Definition 3.5** Let $X$ and $Y$ be two non-empty finite universal sets and $f\in Sur(X,Y)$ . Let $\overset{\prec}{G}=\left\{G_1,G_2,....,G_m\right\}$ be a family of SVN $\beta$ -covering for some $\beta=\left(\mu,\nu,\gamma\right)$. For all $x\in X$ , we define the SVN $\beta$ -neighborhood $\overset{\approx}{\mathrm{N}}_x^{\beta}$ as:

$$\overset{\approx}{\mathrm{N}}_x^{\beta}=\bigcap\left\{f\left(G_i\right):G_i\left(x\right)\geq\beta\right\}$$

In [50], the SVN $\beta$-neighborhood of $x \in X$ was defined in $\left(X, \overset{\prec}{G}\right)$ but in the proposed study, we define this in $\left(X, Y, \overset{\prec}{G}\right)$. For better understanding, we consider the following example:

**Example 3.6** Let $f: X \to Y$ be a surjection, where $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $Y = \{y_1, y_2, y_3, y_4\}$ and

$$f(x) = \begin{cases} y_1, x \in \{x_1, x_3\} \\ y_2, x \in \{x_2, x_5\} \\ y_3, x = x_4 \\ y_4, x = x_6 \end{cases}$$

Let, $\overset{\prec}{G} = \{G_1, G_2, G_3, G_4\}$ be a SVN set over $X$, where

$$G_1 = \left\{ \frac{\langle 0.3, 0.5, 0.4\rangle}{x_1}, \frac{\langle 0.4, 0.2, 0.6\rangle}{x_2}, \frac{\langle 0.2, 0.4, 0.5\rangle}{x_3}, \frac{\langle 0.3, 0.5, 0.6\rangle}{x_4}, \frac{\langle 0.5, 0.6, 0.3\rangle}{x_5}, \frac{\langle 0.3, 0.4, 0.2\rangle}{x_6} \right\}$$

$$G_2 = \left\{ \frac{\langle 0.4, 0.3, 0.2\rangle}{x_1}, \frac{\langle 0.5, 0.3, 0.4\rangle}{x_2}, \frac{\langle 0.3, 0.3, 0.5\rangle}{x_3}, \frac{\langle 0.3, 0.4, 0.5\rangle}{x_4}, \frac{\langle 0.6, 0.3, 0.2\rangle}{x_5}, \frac{\langle 0.5, 0.3, 0.3\rangle}{x_6} \right\}$$

$$G_3 = \left\{ \frac{\langle 0.5, 0.2, 0.3\rangle}{x_1}, \frac{\langle 0.4, 0.3, 0.4\rangle}{x_2}, \frac{\langle 0.6, 0.5, 0.3\rangle}{x_3}, \frac{\langle 0.4, 0.2, 0.1\rangle}{x_4}, \frac{\langle 0.4, 0.5, 0.6\rangle}{x_5}, \frac{\langle 0.4, 0.5, 0.3\rangle}{x_6} \right\}$$

$$G_4 = \left\{ \frac{\langle 0.4, 0.3, 0.5\rangle}{x_1}, \frac{\langle 0.5, 0.3, 0.4\rangle}{x_2}, \frac{\langle 0.6, 0.4, 0.4\rangle}{x_3}, \frac{\langle 0.3, 0.4, 0.5\rangle}{x_4}, \frac{\langle 0.5, 0.5, 0.4\rangle}{x_5}, \frac{\langle 0.3, 0.4, 0.2\rangle}{x_6} \right\}$$

$$f(G_1) = \left\{ \frac{\langle 0.3, 0.4, 0.4\rangle}{y_1}, \frac{\langle 0.5, 0.2, 0.3\rangle}{y_2}, \frac{\langle 0.3, 0.5, 0.6\rangle}{y_3}, \frac{\langle 0.3, 0.4, 0.2\rangle}{y_4} \right\}$$

$$f(G_2) = \left\{ \frac{\langle 0.4, 0.3, 0.2\rangle}{y_1}, \frac{\langle 0.6, 0.3, 0.2\rangle}{y_2}, \frac{\langle 0.3, 0.4, 0.5\rangle}{y_3}, \frac{\langle 0.5, 0.3, 0.3\rangle}{y_4} \right\}$$

$$f(G_3) = \left\{ \frac{\langle 0.6, 0.2, 0.3\rangle}{y_1}, \frac{\langle 0.4, 0.3, 0.4\rangle}{y_2}, \frac{\langle 0.4, 0.2, 0.1\rangle}{y_3}, \frac{\langle 0.4, 0.5, 0.3\rangle}{y_4} \right\}$$

$$f(G_4)=\left\{\frac{\langle 0.6,0.3,0.4\rangle}{y_1},\frac{\langle 0.5,0.3,0.4\rangle}{y_2},\frac{\langle 0.3,0.4,0.5\rangle}{y_3},\frac{\langle 0.3,0.4,0.2\rangle}{y_4}\right\}$$

For $\beta=(0.2,0.6,0.6)$, $\overset{\prec}{G}$ and $f\left(\overset{\prec}{G}\right)$ are SVN $\beta$-coverings over $X$ and $Y$ respectively.

Suppose $\beta=(0.3,0.4,0.3)$, then we calculate the SVN $\beta$-neighborhood $\overset{\approx}{N}_x^{\beta}$ for each $x\in X$ as follows:

$$\overset{\approx}{N}_{x_1}^{(0.3,0.4,0.3)}=\frac{\langle 0.4,0.3,0.2\rangle}{y_1}\cap\frac{\langle 0.6,0.2,0.3\rangle}{y_1}=\frac{\langle 0.4,0.3,0.3\rangle}{y_1}$$

$$\overset{\approx}{N}_{x_2}^{(0.3,0.4,0.3)}=\varnothing,\ \overset{\approx}{N}_{x_3}^{(0.3,0.4,0.3)}=\varnothing,\ \overset{\approx}{N}_{x_4}^{(0.3,0.4,0.3)}=\frac{\langle 0.4,0.2,0.1\rangle}{y_3},\ \overset{\approx}{N}_{x_5}^{(0.3,0.4,0.3)}=\varnothing$$

$$\overset{\approx}{N}_{x_6}^{(0.3,0.4,0.3)}=\frac{\langle 0.3,0.4,0.2\rangle}{y_4}\cap\frac{\langle 0.5,0.3,0.3\rangle}{y_4}\cap\frac{\langle 0.3,0.4,0.2\rangle}{y_4}=\frac{\langle 0.3,0.4,0.3\rangle}{y_4}$$

**Proposition 3.7** Let $f\in Sur(X,Y)$ and $\overset{\prec}{G}=\{G_1,G_2,G_3,\ldots..,G_m\}$ be a SVN $\beta$-covering for some $\beta=(\mu,\nu,\gamma)$. Then we consider the following properties:

(1) $\overset{\approx}{N}_x^{\beta}(f(x))\geq\beta$ for each $x\in X$.

(2) Let $f$ be injective and for all $x,y,z\in X$, if $\overset{\approx}{N}_x^{\beta}(f(y))\geq\beta$ and $\overset{\approx}{N}_y^{\beta}(f(z))\geq\beta$, then $\overset{\approx}{N}_x^{\beta}(f(z))\geq\beta$.

(3) For each $\beta=(\mu,\nu,\gamma)$, where $\mu,\nu,\gamma\in[0,1]$ and $\mu+\nu+\gamma\leq 3$, we can write the following:

$$f(G_i)\supseteq\bigcup\left\{\overset{\approx}{N}_x^{\beta}:G_i(x)\geq\beta,\ x\in X\text{ and i=1,2,..,m}\right\}$$

(4) If $0<\beta_1\leq\beta_2\leq\beta$, then $\overset{\approx}{N}_x^{\beta_1}\subseteq\overset{\approx}{N}_x^{\beta_2}$ for all $x\in X$.

**Proof.** (1) For each $x\in X$,

$$\overset{\approx}{N}_x^{\beta}(f(x))=\left(\bigcap_{G_i(x)\geq\beta}f(G_i)\right)(f(x))=\bigwedge_{G_i(x)\geq\beta}f(G_i)(f(x))=\bigwedge_{G_i(x)\geq\beta}\left(\bigvee_{x^*\in f^{-1}(f(x))}G_i(x^*)\right)$$

$$\geq\bigwedge_{G_i(x)\geq\beta}G_i(x)\geq\beta$$

(2) We have $\overset{\approx}{\mathrm{N}}_x^{\beta}\left(f\left(y\right)\right) \geq \underset{G_i(x)\geq\beta}{\wedge} G_i\left(y\right)\geq\beta$ and $\overset{\approx}{\mathrm{N}}_y^{\beta}\left(f\left(z\right)\right) \geq \underset{G_i(y)\geq\beta}{\wedge} G_i\left(z\right)\geq\beta$. Then for each $i=1,2,..,m$, $G_i(x)\geq\beta \Rightarrow G_i(y)\geq\beta$ and $G_i(y)\geq\beta\Rightarrow G_i(z)\geq\beta$. Thus, $\overset{\approx}{\mathrm{N}}_x^{\beta}\left(f\left(z\right)\right)=\underset{G_i(x)\geq\beta}{\wedge} G_i\left(z\right)\geq\beta$.

(3) By definition **3.5,** $G_i(x)\geq\beta\Rightarrow \overset{\approx}{\mathrm{N}}_x^{\beta}\subseteq f\left(G_i\right)$.

Then $f\left(G_i\right)\supseteq\bigcup\left\{\overset{\approx}{\mathrm{N}}_x^{\beta} : G_i\left(x\right)\geq\beta, x\in X \text{ and i=1,2,..,m}\right\}$

(4) For each $x\in X$, $\beta_1\leq\beta_2\Rightarrow\left\{f\left(G_i\right):G_i\left(x\right)\geq\beta_1\right\}\supseteq\left\{f\left(G_i\right):G_i\left(x\right)\geq\beta_2\right\}$.

Then, $\overset{\approx}{\mathrm{N}}_x^{\beta_1}=\bigcap\left\{f\left(G_i\right):G_i\left(x\right)\geq\beta_1\right\}\subseteq\bigcap\left\{f\left(G_i\right):G_i\left(x\right)\geq\beta_2\right\}=\overset{\approx}{\mathrm{N}}_x^{\beta_2}$ for all $x\in X$.

**Proposition 3.8** Let $X$ and $Y$ be two finite universes, $f\in Sur\left(X,Y\right)$, $f$ be injective and $\overset{\prec}{G}=\left\{G_1,G_2,G_3,.....,G_m\right\}$ be a SVN $\beta$ -covering for some $\beta=\left(\mu,\nu,\gamma\right)$. For all $x,y\in X$, $\overset{\approx}{\mathrm{N}}_x^{\beta}\left(f\left(y\right)\right)\geq\beta$ iff $\overset{\approx}{\mathrm{N}}_y^{\beta}\subseteq\overset{\approx}{\mathrm{N}}_x^{\beta}$. So $\overset{\approx}{\mathrm{N}}_x^{\beta}\left(f\left(y\right)\right)\geq\beta$ and $\overset{\approx}{\mathrm{N}}_y^{\beta}\left(f\left(x\right)\right)\geq\beta$ if and only if $\overset{\approx}{\mathrm{N}}_y^{\beta}=\overset{\approx}{\mathrm{N}}_x^{\beta}$.

**Proof.** $(\Rightarrow)$ $\overset{\approx}{\mathrm{N}}_x^{\beta}\left(f\left(y\right)\right)=\left(\bigcap_{G_i(x)\geq\beta} f\left(G_i\right)\right)\left(y\right)=\underset{G_i(x)\geq\beta}{\wedge} f\left(G_i\right)\left(f(y)\right)\geq\beta$.

We have, $\left\{f\left(G_i\right)\in f\left(\overset{\prec}{G}\right):G_i\left(x\right)\geq\beta\right\}\subseteq\left\{f\left(G_i\right)\in f\left(\overset{\prec}{G}\right):G_i\left(y\right)\geq\beta\right\}$.

Since, $\overset{\approx}{\mathrm{N}}_x^{\beta}\left(f\left(y\right)\right)\geq\beta$,

$$T_{\overset{\approx}{\mathrm{N}}_x^{\beta}(f(y))}=T\bigcap_{\substack{T_{G_i(x)\geq\mu}\\ I_{G_i(x)\leq\nu}\\ F_{G_i(x)\leq\gamma}}} f\left(G_i\right)f\left(y\right)=\underset{\substack{T_{G_i(x)\geq\mu}\\ I_{G_i(x)\leq\nu}\\ F_{G_i(x)\leq\gamma}}}{\wedge} T_{f(G_i)f(y)}\geq\mu ,$$

$$I_{\overset{\approx}{\mathrm{N}}_x^{\beta}(f(y))}=I\bigcap_{\substack{T_{G_i(x)\geq\mu}\\ I_{G_i(x)\leq\nu}\\ F_{G_i(x)\leq\gamma}}} f\left(G_i\right)f\left(y\right)=\underset{\substack{T_{G_i(x)\geq\mu}\\ I_{G_i(x)\leq\nu}\\ F_{G_i(x)\leq\gamma}}}{\vee} I_{f(G_i)f(y)}\leq\nu ,$$

and $$F_{\overset{\approx}{\mathrm{N}}_x^{\beta}(f(y))}=F\bigcap_{\substack{T_{G_i(x)\geq\mu}\\ I_{G_i(x)\leq\nu}\\ F_{G_i(x)\leq\gamma}}} f\left(G_i\right)f\left(y\right)=\underset{\substack{T_{G_i(x)\geq\mu}\\ I_{G_i(x)\leq\nu}\\ F_{G_i(x)\leq\gamma}}}{\vee} F_{f(G_i)f(y)}\leq\gamma$$

Again, for $z\in X$,

$$T_{\overset{\approx}{N}_x^{\beta}(f(z))} = \bigwedge_{\substack{T_{G_i}(x)\geq\mu \\ I_{G_i}(x)\leq\nu \\ F_{G_i}(x)\leq\gamma}} T_{f(G_i)f(z)} \geq \bigwedge_{\substack{T_{G_i}(y)\geq\mu \\ I_{G_i}(y)\leq\nu \\ F_{G_i}(y)\leq\gamma}} T_{f(G_i)f(z)} = T_{\overset{\approx}{N}_y^{\beta}(f(z))}$$

$$I_{\overset{\approx}{N}_x^{\beta}(f(z))} = \bigvee_{\substack{T_{G_i}(x)\geq\mu \\ I_{G_i}(x)\leq\nu \\ F_{G_i}(x)\leq\gamma}} I_{f(G_i)f(z)} \leq \bigvee_{\substack{T_{G_i}(y)\geq\mu \\ I_{G_i}(y)\leq\nu \\ F_{G_i}(y)\leq\gamma}} I_{f(G_i)f(z)} = I_{\overset{\approx}{N}_y^{\beta}(f(z))}$$

$$F_{\overset{\approx}{N}_x^{\beta}(f(z))} = \bigvee_{\substack{T_{G_i}(x)\geq\mu \\ I_{G_i}(x)\leq\nu \\ F_{G_i}(x)\leq\gamma}} F_{f(G_i)f(z)} \leq \bigvee_{\substack{T_{G_i}(y)\geq\mu \\ I_{G_i}(y)\leq\nu \\ F_{G_i}(y)\leq\gamma}} F_{f(G_i)f(z)} = F_{\overset{\approx}{N}_y^{\beta}(f(z))}$$

Therefore, for $z \in X$, $\overset{\approx}{N}_y^{\beta} \subseteq \overset{\approx}{N}_x^{\beta}$.

$(\Leftarrow)$: For any $x, y \in X$, we have $\overset{\approx}{N}_y^{\beta} \subseteq \overset{\approx}{N}_x^{\beta}$,

$T_{\overset{\approx}{N}_x^{\beta}(f(y))} \geq T_{\overset{\approx}{N}_y^{\beta}(f(y))} \geq \mu$, $I_{\overset{\approx}{N}_x^{\beta}(f(z))} \leq I_{\overset{\approx}{N}_y^{\beta}(f(y))} \leq \nu$, $F_{\overset{\approx}{N}_x^{\beta}(f(y))} \leq F_{\overset{\approx}{N}_y^{\beta}(f(y))} \leq \gamma$

Therefore, $\overset{\approx}{N}_x^{\beta}\left(f(y)\right) \geq \beta$.

**Proposition 3.9** Let $f \in Sur(X,Y)$, $f$ be injective and $\overset{\prec}{G} = \{G_1, G_2, G_3, \ldots, G_m\}$ be a SVN $\beta$-covering for some $\beta = (\mu, \nu, \gamma)$. For all $x, y, z \in X$, if $f(x) \in \overset{\approx}{N}_y^{\beta}$ and $f(y) \in \overset{\approx}{N}_z^{\beta}$, then $f(x) \in \overset{\approx}{N}_z^{\beta}$.

**Proof.** For all $x, y, z \in X$,

$f(x) \in \overset{\approx}{N}_y^{\beta} \Leftrightarrow \overset{\approx}{N}_y^{\beta}\left(f(x)\right) \geq \beta \Leftrightarrow \overset{\approx}{N}_x^{\beta} \subseteq \overset{\approx}{N}_y^{\beta}$ and $f(y) \in \overset{\approx}{N}_z^{\beta} \Leftrightarrow \overset{\approx}{N}_z^{\beta}\left(f(y)\right) \geq \beta \Leftrightarrow \overset{\approx}{N}_y^{\beta} \subseteq \overset{\approx}{N}_z^{\beta}$.

Then $\overset{\approx}{N}_x^{\beta} \subseteq \overset{\approx}{N}_y^{\beta} \subseteq \overset{\approx}{N}_z^{\beta}$ and $\beta \leq \overset{\approx}{N}_x^{\beta}\left(f(x)\right) \leq \overset{\approx}{N}_z^{\beta}\left(f(x)\right)$. Therefore, $f(x) \in \overset{\approx}{N}_z^{\beta}$. This completes the proof.

## 4. Construction of Single-valued neutrosophic covering based approximation operators over Two Universes

In this section, a new type of SVN covering based rough set model over two universes for neutrosophic subsets is defined and its properties are explored:

**Definition 4.1** Let $X$ and $Y$ be two non-empty finite universes, $f \in sur(X,Y)$ and $\overset{\prec}{G}$ be a SVN $\beta$-covering on $X$ for some $\beta = (\mu, \nu, \gamma)$. For each $A \in f(Y)$, we define the SVN covering lower approximation $\overset{\prec}{G}_*(A)$ and SVN covering upper approximation $\overset{\prec}{G}^*(A)$ as follows:

$$\overset{\prec}{G}_*(A)(x)=\left\{\left\langle x,\underset{y\in Y}{\wedge}\left[\left(1-T_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee T_{A(y)}\right],\underset{y\in Y}{\vee}\left[I_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge I_{A(y)}\right],\underset{y\in Y}{\vee}\left[F_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge F_{A(y)}\right]\right\rangle:x\in X\right\}$$

$$\overset{\prec}{G}^*(A)(x)=\left\{\left\langle x,\underset{y\in Y}{\vee}\left[T_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge T_{A(y)}\right],\underset{y\in Y}{\wedge}\left[\left(1-I_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee I_{A(y)}\right],\underset{y\in Y}{\wedge}\left[\left(1-F_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee F_{A(y)}\right]\right\rangle:x\in X\right\}$$

If $\overset{\prec}{G}_*(A)\neq\overset{\prec}{G}^*(A)$, then $A$ is called the SVN covering based rough set.

**Example 4.1.1** Let us consider the two finite universes as $X=\{x_1,x_2,x_3,x_4,x_5\}$ ,

$Y=\{y_1,y_2,y_3\}$ and $f:X\to Y,\ f(x)=\begin{cases} y_1, x\in\{x_1,x_2\}\\ y_2, x\in\{x_3,x_4\}\\ y_3, x=x_5\end{cases}$.

Let $\overset{\prec}{G}=\{G_1,G_2,G_3,G_4\}$, where

$$G_1=\left\{\frac{\langle 0.3,0.4,0.5\rangle}{x_1},\frac{\langle 0.6,0.7,0.4\rangle}{x_2},\frac{\langle 0.4,0.5,0.6\rangle}{x_3},\frac{\langle 0.8,0.4,0.7\rangle}{x_4},\frac{\langle 0.5,0.5,0.6\rangle}{x_5}\right\}$$

$$G_2=\left\{\frac{\langle 0.2,0.6,0.4\rangle}{x_1},\frac{\langle 0.5,0.4,0.4\rangle}{x_2},\frac{\langle 0.6,0.3,0.7\rangle}{x_3},\frac{\langle 0.5,0.6,0.4\rangle}{x_4},\frac{\langle 0.3,0.7,0.5\rangle}{x_5}\right\}$$

$$G_3=\left\{\frac{\langle 0.7,0.6,0.4\rangle}{x_1},\frac{\langle 0.5,0.4,0.6\rangle}{x_2},\frac{\langle 0.6,0.4,0.5\rangle}{x_3},\frac{\langle 0.7,0.5,0.6\rangle}{x_4},\frac{\langle 0.3,0.6,0.5\rangle}{x_5}\right\}$$

$$G_4=\left\{\frac{\langle 0.6,0.5,0.6\rangle}{x_1},\frac{\langle 0.5,0.6,0.7\rangle}{x_2},\frac{\langle 0.6,0.4,0.5\rangle}{x_3},\frac{\langle 0.4,0.6,0.5\rangle}{x_4},\frac{\langle 0.5,0.6,0.4\rangle}{x_5}\right\}$$

$$f(G_1)=\left\{\frac{\langle 0.6,0.4,0.4\rangle}{y_1},\frac{\langle 0.8,0.4,0.6\rangle}{y_2},\frac{\langle 0.5,0.5,0.6\rangle}{y_3}\right\}$$

$$f(G_2)=\left\{\frac{\langle 0.5,0.4,0.4\rangle}{y_1},\frac{\langle 0.6,0.3,0.4\rangle}{y_2},\frac{\langle 0.3,0.7,0.5\rangle}{y_3}\right\}$$

$$f(G_3)=\left\{\frac{\langle 0.7,0.4,0.4\rangle}{y_1},\frac{\langle 0.7,0.4,0.5\rangle}{y_2},\frac{\langle 0.3,0.6,0.5\rangle}{y_3}\right\}$$

$$f(G_4)=\left\{\frac{\langle 0.6,0.5,0.6\rangle}{y_1},\frac{\langle 0.6,0.4,0.5\rangle}{y_2},\frac{\langle 0.5,0.6,0.4\rangle}{y_3}\right\}$$

Clearly, for $\beta=\langle 0.2,0.7,0.7\rangle$, $\overset{\prec}{G}$ and $f\left(\overset{\prec}{G}\right)$ are SVN $\beta$-coverings of $X$ and $Y$ respectively.

$$\overset{\approx}{N}_{x_1}^{(0.2,0.7,0.7)}=\frac{\langle 0.6,0.4,0.4\rangle}{y_1}\cap\frac{\langle 0.5,0.4,0.4\rangle}{y_1}\cap\frac{\langle 0.7,0.4,0.4\rangle}{y_1}\cap\frac{\langle 0.6,0.5,0.6\rangle}{y_1}=\frac{\langle 0.5,0.5,0.6\rangle}{y_1}$$

$$\overset{\approx}{N}_{x_2}^{(0.2,0.7,0.7)}=\frac{\langle 0.6,0.4,0.4\rangle}{y_1}\cap\frac{\langle 0.5,0.4,0.4\rangle}{y_1}\cap\frac{\langle 0.7,0.4,0.4\rangle}{y_1}\cap\frac{\langle 0.6,0.5,0.6\rangle}{y_1}=\frac{\langle 0.5,0.5,0.6\rangle}{y_1}$$

$$\overset{\approx}{N}_{x_3}^{(0.2,0.7,0.7)}=\frac{\langle 0.8,0.4,0.6\rangle}{y_2}\cap\frac{\langle 0.6,0.3,0.4\rangle}{y_2}\cap\frac{\langle 0.7,0.4,0.5\rangle}{y_2}\cap\frac{\langle 0.6,0.4,0.5\rangle}{y_2}=\frac{\langle 0.6,0.4,0.6\rangle}{y_2}$$

$$\overset{\approx}{N}_{x_4}^{(0.2,0.7,0.7)}=\frac{\langle 0.8,0.4,0.6\rangle}{y_2}\cap\frac{\langle 0.6,0.3,0.4\rangle}{y_2}\cap\frac{\langle 0.7,0.4,0.5\rangle}{y_2}\cap\frac{\langle 0.6,0.4,0.5\rangle}{y_2}=\frac{\langle 0.6,0.4,0.6\rangle}{y_2}$$

$$\overset{\approx}{N}_{x_5}^{(0.2,0.7,0.7)}=\frac{\langle 0.5,0.5,0.6\rangle}{y_3}\cap\frac{\langle 0.3,0.7,0.5\rangle}{y_3}\cap\frac{\langle 0.3,0.6,0.5\rangle}{y_3}\cap\frac{\langle 0.5,0.6,0.4\rangle}{y_3}=\frac{\langle 0.3,0.7,0.6\rangle}{y_3}$$

For $A=\left\{\frac{\langle 0.3,0.5,0.6\rangle}{y_1},\frac{\langle 0.4,0.5,0.6\rangle}{y_2},\frac{\langle 0.6,0.4,0.3\rangle}{y_3}\right\}$

$$\overset{\prec}{G}_*(A)(x)=\left\{\frac{\langle 0.5,0.5,0.6\rangle}{x_1},\frac{\langle 0.5,0.5,0.6\rangle}{x_2},\frac{\langle 0.4,0.4,0.6\rangle}{x_3},\frac{\langle 0.4,0.4,0.6\rangle}{x_4},\frac{\langle 0.7,0.5,0.6\rangle}{x_5}\right\}$$

$$\overset{\prec}{G}^*(A)(x)=\left\{\frac{\langle 0.5,0.5,0.4\rangle}{x_1},\frac{\langle 0.5,0.5,0.4\rangle}{x_2},\frac{\langle 0.6,0.6,0.4\rangle}{x_3},\frac{\langle 0.6,0.6,0.4\rangle}{x_4},\frac{\langle 0.3,0.4,0.4\rangle}{x_5}\right\}$$

Thus, $\overset{\prec}{G}_*(A)\neq\overset{\prec}{G}^*(A)$

Some properties of SVN covering-based rough set model over two universes can be presented through the following proposition:

**Proposition 4.2** Let $X$ and $Y$ be two non-empty finite universes and $f\in Sur(X,Y)$. Let $\overset{\prec}{G}=\{G_1,G_2,G_3,.....,G_m\}$ be a SVN $\beta$-covering on $X$ for some $\beta=(\mu,\nu,\gamma)$, where $\mu,\nu,\gamma\in[0,1]$ and $\mu+\nu+\gamma\leq 3$. For each $A,B\in\Gamma(X,Y)$, we have the following statements:

(1) $\overset{\prec}{G}_*(Y)=X,\ \overset{\prec}{G}^*(\varnothing)=\varnothing$

$$(2)\overset{\prec}{G}_*\left(A^c\right)=\left(\overset{\prec}{G}^*\left(A\right)\right)^c,\overset{\prec}{G}^*\left(A^c\right)=\left(\overset{\prec}{G}_*\left(A\right)\right)^c$$

$$(3)\overset{\prec}{G}_*\left(A\cap B\right)=\overset{\prec}{G}_*\left(A\right)\cap\overset{\prec}{G}_*\left(B\right),\overset{\prec}{G}^*\left(A\cup B\right)=\overset{\prec}{G}^*\left(A\right)\cup\overset{\prec}{G}^*\left(B\right)$$

$$(4)\textit{If } A\subseteq B, \textit{then } \overset{\prec}{G}_*\left(A\right)\subseteq\overset{\prec}{G}_*\left(B\right),\overset{\prec}{G}^*\left(A\right)\subseteq\overset{\prec}{G}^*\left(B\right)$$

$$(5)\overset{\prec}{G}_*\left(A\cup B\right)\supseteq\overset{\prec}{G}_*\left(A\right)\cup\overset{\prec}{G}_*\left(B\right),\overset{\prec}{G}^*\left(A\cap B\right)\subseteq\overset{\prec}{G}^*\left(A\right)\cap\overset{\prec}{G}^*\left(B\right)$$

$$(6)\textit{For each } x\in X, \textit{if } 1-T_{\overset{\approx}{N}_x^{\beta}(f(y))}\leq T_{A(y)}\leq T_{\overset{\approx}{N}_x^{\beta}(f(y))}, I_{\overset{\approx}{N}_x^{\beta}(f(y))}\leq I_{A(y)}\leq 1-I_{\overset{\approx}{N}_x^{\beta}(f(y))} \textit{ and }$$

$$F_{\overset{\approx}{N}_x^{\beta}(f(y))}\leq F_{A(y)}\leq 1-F_{\overset{\approx}{N}_x^{\beta}(f(y))} \textit{ for all } y\in Y, \textit{then } \overset{\prec}{G}_*\left(A\right)\subseteq\overset{\prec}{G}^*\left(A\right)$$

**Proof.**

(1) For each $x\in X$ ,

$$\overset{\prec}{G}_*\left(Y\right)(x)=$$

$$\left\{\left\langle x, \underset{y\in Y}{\wedge}\left[\left(1-T_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee T_{Y(y)}\right], \underset{y\in Y}{\vee}\left[I_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge I_{Y(y)}\right], \underset{y\in Y}{\vee}\left[F_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge F_{Y(y)}\right]\right\rangle : x\in X\right\}=X\left(x\right)$$

$$\overset{\prec}{G}^*\left(\varnothing\right)(x)=\left\{\left\langle x, \underset{y\in Y}{\vee}\left[T_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge T_{\varnothing(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-I_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee I_{\varnothing(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-F_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee F_{\varnothing(y)}\right]\right\rangle : x\in X\right\}=\varnothing\left(x\right)$$

Hence

$$\overset{\prec}{G}_*\left(Y\right)=X, \overset{\prec}{G}^*\left(\varnothing\right)=\varnothing$$

(2) For each $x\in X$ ,

$$\begin{aligned}\overset{\prec}{G}_*\left(A^c\right)(x)&=\left\{\left\langle x, \underset{y\in Y}{\wedge}\left[\left(1-T_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee T_{A^c(y)}\right], \underset{y\in Y}{\vee}\left[I_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge I_{A^c(y)}\right], \underset{y\in Y}{\vee}\left[F_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge F_{A^c(y)}\right]\right\rangle : x\in X\right\}\\ &=1-\left\{\left\langle x, \underset{y\in Y}{\vee}\left[T_{\overset{\approx}{N}_x^{\beta}(f(y))}\wedge T_{A(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-I_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee I_{A(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-F_{\overset{\approx}{N}_x^{\beta}(f(y))}\right)\vee F_{A(y)}\right]\right\rangle : x\in X\right\}\\ &=1-\overset{\prec}{G}^*\left(A\right)(x)\\ &=\left(\overset{\prec}{G}^*\left(A\right)\right)^c(x)\end{aligned}$$

Similarly,

$$\overset{\prec}{G}^{*}\left(A^{c}\right)(x)=\left(\overset{\prec}{G}_{*}(A)\right)^{c}(x)$$

Then, $\overset{\prec}{G}_{*}\left(A^{c}\right)=\left(\overset{\prec}{G}^{*}(A)\right)^{c}, \overset{\prec}{G}^{*}\left(A^{c}\right)=\left(\overset{\prec}{G}_{*}(A)\right)^{c}$

(3) For each $x \in X$ ,

$$\overset{\prec}{G}_{*}(A\cap B)(x)=$$

$$\left\{\left\langle x, \bigwedge_{y\in Y}\left[\left(1-T_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\right)\vee T_{A\cap B(y)}\right], \bigvee_{y\in Y}\left[I_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge I_{A\cap B(y)}\right], \bigvee_{y\in Y}\left[F_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge F_{A\cap B(y)}\right]\right\rangle : x\in X\right\}$$

$$=\left\{\left\langle \begin{array}{l} x, \bigwedge_{y\in Y}\left[\left(\left(1-T_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\right)\vee T_{A(y)}\right)\wedge\left(\left(1-T_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\right)\vee T_{B(y)}\right)\right], \\ \bigvee_{y\in Y}\left[\left(I_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge I_{A(y)}\right)\wedge\left(I_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge I_{B(y)}\right)\right], \bigvee_{y\in Y}\left[\left(F_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge F_{A(y)}\right)\wedge\left(F_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge F_{B(y)}\right)\right] \end{array}\right\rangle : x\in X\right\}$$

$$=\overset{\prec}{G}_{*}(A)(x)\cap\overset{\prec}{G}_{*}(B)(x)$$

and

$$\overset{\prec}{G}^{*}(A\cup B)(x)=\left\{\left\langle \begin{array}{l} x, \bigvee_{y\in Y}\left[T_{\tilde{\tilde{N}}x^{\beta}(f(y))}\wedge T_{A\cup B(y)}\right], \bigwedge_{y\in Y}\left[\left(1-I_{\tilde{\tilde{N}}x^{\beta}(f(y))}\right)\vee I_{A\cup B(y)}\right], \\ \bigwedge_{y\in Y}\left[\left(1-F_{\tilde{\tilde{N}}x^{\beta}(f(y))}\right)\vee F_{A\cup B(y)}\right] \end{array}\right\rangle : x\in X\right\}$$

$$=\left\{\left\langle \begin{array}{l} x, \bigvee_{y\in Y}\left[\left(T_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge T_{A(y)}\right)\vee\left(T_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\wedge T_{B(y)}\right)\right], \\ \bigwedge_{y\in Y}\left[\left(\left(1-I_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\right)\vee I_{A(y)}\right)\vee\left(\left(1-I_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\right)\vee I_{B(y)}\right)\right], \\ \bigwedge_{y\in Y}\left[\left(\left(1-F_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\right)\vee F_{A(y)}\right)\vee\left(\left(1-F_{\tilde{\tilde{N}}_x^{\beta}(f(y))}\right)\vee F_{B(y)}\right)\right] \end{array}\right\rangle : x\in X\right\}$$

$$=\overset{\prec}{G}^{*}(A)(x)\cup\overset{\prec}{G}^{*}(B)(x)$$

*Then,* $\overset{\prec}{G}_{*}(A\cap B)=\overset{\prec}{G}_{*}(A)\cap\overset{\prec}{G}_{*}(B), \overset{\prec}{G}^{*}(A\cup B)=\overset{\prec}{G}^{*}(A)\cup\overset{\prec}{G}^{*}(B)$

$(4)$ *If* $A\subseteq B$, *then* $A(Y)\leq B(Y)$ for each $y\in Y$ . For every $x\in X$ , we have

$$\overset{\prec}{G}_*(A)(x)=\left\{\left\langle x, \underset{y\in Y}{\wedge}\left[\left(1-T_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\right)\vee T_{A(y)}\right], \underset{y\in Y}{\vee}\left[I_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\wedge I_{A(y)}\right], \underset{y\in Y}{\vee}\left[F_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\wedge F_{A(y)}\right]\right\rangle : x\in X\right\}$$

$$\leq\left\{\left\langle x, \underset{y\in Y}{\wedge}\left[\left(1-T_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\right)\vee T_{B(y)}\right], \underset{y\in Y}{\vee}\left[I_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\wedge I_{B(y)}\right], \underset{y\in Y}{\vee}\left[F_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\wedge F_{B(y)}\right]\right\rangle : x\in X\right\}$$

$$=\overset{\prec}{G}_*(B)(x)$$

and

$$\overset{\prec}{G}{}^*(A)(x)=\left\{\left\langle x, \underset{y\in Y}{\vee}\left[T_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\wedge T_{A(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-I_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\right)\vee I_{A(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-F_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\right)\vee F_{A(y)}\right]\right\rangle : x\in X\right\}$$

$$\leq\left\{\left\langle x, \underset{y\in Y}{\vee}\left[T_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\wedge T_{B(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-I_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\right)\vee I_{B(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-F_{\overset{\approx}{N}{}^{\beta}_{x}(f(y))}\right)\vee F_{B(y)}\right]\right\rangle : x\in X\right\}$$

$$=\overset{\prec}{G}{}^*(B)(x)$$

Then, $\overset{\prec}{G}_*(A)\subseteq\overset{\prec}{G}_*(B), \overset{\prec}{G}{}^*(A)\subseteq\overset{\prec}{G}{}^*(B)$.

(5) Since $A\subseteq A\bigcup B, B\subseteq A\bigcup B,\ A\bigcap B\subseteq A$ and $A\bigcap B\subseteq B$, then from (4) we can write

$\overset{\prec}{G}_*(A)\subseteq\overset{\prec}{G}_*(A\bigcup B)$, $\overset{\prec}{G}_*(B)\subseteq\overset{\prec}{G}_*(A\bigcup B)$, $\overset{\prec}{G}{}^*(A\bigcap B)\subseteq\overset{\prec}{G}{}^*(A)$ and $\overset{\prec}{G}{}^*(A\bigcap B)\subseteq\overset{\prec}{G}{}^*(B)$.

Therefore, $\overset{\prec}{G}_*(A\bigcup B)\supseteq\overset{\prec}{G}_*(A)\bigcup\overset{\prec}{G}_*(B), \overset{\prec}{G}{}^*(A\bigcap B)\subseteq\overset{\prec}{G}{}^*(A)\bigcap\overset{\prec}{G}{}^*(B)$.

(6) This proof is obvious.

**Proposition 4.3** Let $X$ and $Y$ be two non-empty finite universes and $f\in Sur(X,Y)$. Again, let $\overset{\prec}{G}=\{G_1,G_2,...,G_p\}$ be a SVN $\beta$ -covering on $X$ for some $\beta=(\mu,\nu,\gamma)$, where $\mu,\nu,\gamma\in[0,1]$ and $\mu+\nu+\gamma\leq 3$. For $M\in\mathcal{F}(Y)$, where $\mathcal{F}(Y)$ denotes the family of all subsets of $Y$ and $\lambda\in[0,1]$, we have the following results:

(1) $\overset{\prec}{G}{}^*(M\cap\lambda_Y)=\overset{\prec}{G}{}^*(M)\cap\lambda_X$

(2) $\overset{\prec}{G}_*(M\cup\lambda_Y)=\overset{\prec}{G}_*(M)\cup\lambda_X$

**Proof.** (1) For any $x\in X$,

$$\overset{\prec}{G}^{*}\left(M\cap\lambda_Y\right)(x) =$$

$$\left\langle \underset{y\in Y}{\vee}\left[T_{\tilde{N}_x^{\beta}(y)}\wedge T_{M\cap\lambda_Y(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-I_{\tilde{N}_x^{\beta}(y)}\right)\vee I_{M\cap\lambda_Y(y)}\right], \underset{y\in Y}{\wedge}\left[\left(1-F_{\tilde{N}_x^{\beta}(y)}\right)\vee F_{M\cap\lambda_Y(y)}\right]\right\rangle$$

$$=\left\langle \underset{y\in Y}{\vee}\left[T_{\tilde{N}_x^{\beta}(y)}\wedge T_{M(y)}\wedge\lambda\right], \underset{y\in Y}{\wedge}\left[\left(1-I_{\tilde{N}_x^{\beta}(y)}\right)\vee I_{M(y)}\wedge\lambda\right], \underset{y\in Y}{\wedge}\left[\left(1-F_{\tilde{N}_x^{\beta}(y)}\right)\vee F_{M(y)}\wedge\lambda\right]\right\rangle$$

$$=\left\langle \begin{array}{l}\left(\left[\underset{y\in Y}{\vee}\left(T_{\tilde{N}_x^{\beta}(y)}\wedge T_{M(y)}\right)\right]\wedge\left[\underset{y\in Y}{\vee}\left(T_{\tilde{N}_x^{\beta}(y)}\wedge\lambda\right)\right]\right),\left(\left[\underset{y\in Y}{\wedge}\left(\left(1-I_{\tilde{N}_x^{\beta}(y)}\right)\vee I_{M(y)}\right)\right]\wedge\left[\underset{y\in Y}{\wedge}\left(\left(1-I_{\tilde{N}_x^{\beta}(y)}\right)\vee\lambda\right)\right]\right),\\ \left(\left[\underset{y\in Y}{\wedge}\left(\left(1-F_{\tilde{N}_x^{\beta}(y)}\right)\vee F_{M(y)}\right)\right]\wedge\left[\underset{y\in Y}{\wedge}\left(\left(1-F_{\tilde{N}_x^{\beta}(y)}\right)\vee\lambda\right)\right]\right)\end{array}\right\rangle$$

$$=\overset{\prec}{G}^{*}\left(M\right)(x)\cap\lambda$$

Thus, $\overset{\prec}{G}^{*}\left(M\cap\lambda_Y\right)=\overset{\prec}{G}^{*}\left(M\right)\cap\lambda_X$

(2) For every $z\in X$ ,

$$\overset{\prec}{G}_{*}\left(M\cup\lambda_Y\right)(z)=$$

$$\left\langle \underset{z\in Y}{\wedge}\left[\left(1-T_{\tilde{N}_x^{\beta}(z)}\right)\vee T_{M\cup\lambda_Y(z)}\right], \underset{z\in Y}{\vee}\left[I_{\tilde{N}_x^{\beta}(z)}\wedge I_{M\cup\lambda_Y(z)}\right], \underset{z\in Y}{\vee}\left[F_{\tilde{N}_x^{\beta}(z)}\wedge F_{M\cup\lambda_Y(z)}\right]\right\rangle$$

$$=\left\langle \underset{z\in Y}{\wedge}\left[\left(1-T_{\tilde{N}_x^{\beta}(z)}\right)\vee T_{M(z)}\vee\lambda\right], \underset{z\in Y}{\vee}\left[I_{\tilde{N}_x^{\beta}(z)}\wedge I_{M(z)}\vee\lambda\right], \underset{z\in Y}{\vee}\left[F_{\tilde{N}_x^{\beta}(z)}\wedge F_{M(z)}\vee\lambda\right]\right\rangle$$

$$=\left\langle \begin{array}{l}\left(\left[\underset{z\in Y}{\wedge}\left(\left(1-T_{\tilde{N}_x^{\beta}(z)}\right)\vee T_{M(z)}\right)\right]\vee\left[\underset{z\in Y}{\wedge}\left(\left(1-T_{\tilde{N}_x^{\beta}(z)}\right)\vee\lambda\right)\right]\right),\left(\left[\underset{z\in Y}{\vee}\left(I_{\tilde{N}_x^{\beta}(z)}\wedge I_{M(z)}\right)\right]\vee\left[\underset{z\in Y}{\vee}\left(I_{\tilde{N}_x^{\beta}(z)}\wedge\lambda\right)\right]\right),\\ \left(\left[\underset{z\in Y}{\vee}\left(F_{\tilde{N}_x^{\beta}(z)}\wedge F_{M(z)}\right)\right]\vee\left[\underset{z\in Y}{\vee}\left(F_{\tilde{N}_x^{\beta}(z)}\wedge\lambda\right)\right]\right)\end{array}\right\rangle$$

$$=\overset{\prec}{G}_{*}\left(M\right)(z)\cup\lambda$$

Hence, $\overset{\prec}{G}_{*}\left(M\cup\lambda_Y\right)=\overset{\prec}{G}_{*}\left(M\right)\cup\lambda_X$

**5. Matrix Representation of SVN Covering-Based Approximation Operators**

In this section, we have investigated the matrix representations of SVN covering-based lower and upper approximation operators and performed some matrix operations on them. Also, the algorithmic representation helps to calculate the matrix operations through the computer.

**Definition 5.1** Let $P=\left(p_{ij}\right)_{m\times n}=\left(T_{p_{ij}},I_{p_{ij}},F_{p_{ij}}\right)_{m\times n}$ and $Q=\left(q_{jk}\right)=\left(T_{q_{jk}},I_{q_{jk}},F_{q_{jk}}\right)_{n\times l}$ be two SVN matrices. Then, we perform the following two operations on $P=\left(p_{ij}\right)_{m\times n}$ and $Q=\left(q_{jk}\right)_{n\times l}$ as follows:

$$P\Delta Q=\left(r_{ik}\right)_{m\times l}=\left\langle \overset{n}{\underset{j=1}{\vee}}\left(T_{p_{ij}}\wedge T_{q_{jk}}\right), \overset{n}{\underset{j=1}{\wedge}}\left(\left(1-I_{p_{ij}}\right)\vee I_{q_{jk}}\right), \overset{n}{\underset{j=1}{\wedge}}\left(\left(1-F_{p_{ij}}\right)\vee F_{q_{jk}}\right)\right\rangle \quad , \quad i=1,2,..,m \quad ; \quad j=1,2,..,1$$

$$P\nabla Q=\left(s_{ik}\right)_{m\times l}=\left\langle \overset{n}{\underset{j=1}{\wedge}}\left(\left(1-T_{p_{ij}}\right)\vee T_{q_{jk}}\right), \overset{n}{\underset{j=1}{\vee}}\left(I_{p_{ij}}\wedge I_{q_{jk}}\right), \overset{n}{\underset{j=1}{\vee}}\left(F_{p_{ij}}\wedge F_{q_{jk}}\right)\right\rangle \quad , \quad i=1,2,..,m \quad ; \quad j=1,2,..,1$$

**Definition 5.2** Let $X=\left\{x_1,x_2,...,x_m\right\}$ and $Y=\left\{y_1,\mathrm{y}_2,...,\mathrm{y}_n\right\}$ be two finite universal sets and $f\in Sur(X,Y)$. Then the Boolean matrix under SVN environment is denoted by $Z_f=\left(z_{ij}\right)_{m\times n}$, where

$$z_{ij}=\begin{cases}\langle 1,0,0\rangle, \; when \; f\left(x_i\right)=y_j\\ \left(0,1,1\right), \; when \; f\left(x_i\right)\neq y_j\end{cases}$$

**Definition 5.3** Let $X=\left\{x_1,x_2,...,x_m\right\}$ be a non-empty finite universe and $\overset{\prec}{G}=\left\{G_1,G_2,...,G_n\right\}$ be a SVN $\beta$ -covering on $X$ for some $\beta=\left(\mu,\nu,\gamma\right)$ , where $\mu,\nu,\gamma\in\left[0,1\right]$ and $\mu+\nu+\gamma\leq 3.$ Then $Z_{\overset{\prec}{G}}=\left(G_j\left(x_i\right)\right)_{m\times n}$ is a matrix representation of $\overset{\prec}{G}$. Also, the Boolean matrix $Z_\beta=\left(t_{ij}\right)_{m\times n}$ is called a SVN covering based $\beta$ -matrix representation of $\overset{\prec}{G}$, where

$$t_{ij}=\begin{cases}\langle 1,0,0\rangle, \; when \, G_j\left(x_i\right)\geq\beta\\ \quad \left(0,1,1\right), \quad otherwise\end{cases}$$

**Example 5.3.1** Let $X=\left\{x_1,x_2,x_3,x_4,x_5\right\}$ and $Y=\left\{y_1,\mathrm{y}_2,\mathrm{y}_3\right\}$ be two non-empty finite universes and $f:X\rightarrow Y$, where $f\left(x\right)=\begin{cases}y_1, \, x\in\left\{x_1,x_4\right\}\\ y_2, \, x\in\left\{x_2\right\}\\ y_3, \, x\in\left\{x_3,x_5\right\}\end{cases}$.

Let $\overset{\prec}{G}=\left\{G_1,G_2,G_3\right\}$, where

$$G_1=\left\{\frac{\langle 0.3,0.6,0.5\rangle}{x_1},\frac{\langle 0.4,0.6,0.4\rangle}{x_2},\frac{\langle 0.6,0.3,0.5\rangle}{x_3},\frac{\langle 0.7,0.5,0.6\rangle}{x_4},\frac{\langle 0.4,0.3,0.2\rangle}{x_5}\right\}$$

$$G_2=\left\{\frac{\langle 0.4,0.6,0.3\rangle}{x_1},\frac{\langle 0.5,0.8,0.5\rangle}{x_2},\frac{\langle 0.6,0.4,0.3\rangle}{x_3},\frac{\langle 0.7,0.5,0.6\rangle}{x_4},\frac{\langle 0.7,0.3,0.4\rangle}{x_5}\right\}$$

$$G_3=\left\{\frac{\langle 0.2,0.3,0.4\rangle}{x_1},\frac{\langle 0.5,0.3,0.6\rangle}{x_2},\frac{\langle 0.6,0.4,0.5\rangle}{x_3},\frac{\langle 0.4,0.6,0.5\rangle}{x_4},\frac{\langle 0.3,0.4,0.5\rangle}{x_5}\right\}$$

For $\beta=(0.2,0.9,0.7)$, $\overset{\prec}{G}$ is a SVN $\beta$-covering of $X$.

Now,

$$Z_f=\begin{matrix} x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{matrix}\begin{bmatrix} \overset{y_1}{(1,0,0)} & \overset{y_2}{(0,1,1)} & \overset{y_3}{(0,1,1)}\\ (0,1,1) & (1,0,0) & (0,1,1)\\ (0,1,1) & (0,1,1) & (1,0,0)\\ (1,0,0) & (0,1,1) & (0,1,1)\\ (0,1,1) & (0,1,1) & (1,0,0)\end{bmatrix},\ Z_{\overset{\prec}{G}}=\begin{matrix} x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{matrix}\begin{bmatrix} \overset{G_1}{(0.3,0.6,0.5)} & \overset{G_2}{(0.4,0.6,0.3)} & \overset{G_3}{(0.2,0.3,0.4)}\\ (0.4,0.6,0.4) & (0.5,0.8,0.5) & (0.5,0.3,0.6)\\ (0.6,0.3,0.5) & (0.6,0.4,0.3) & (0.6,0.4,0.5)\\ (0.7,0.5,0.6) & (0.7,0.5,0.6) & (0.4,0.6,0.5)\\ (0.4,0.3,0.2) & (0.7,0.3,0.4) & (0.3,0.4,0.5)\end{bmatrix}$$

For $\beta=(0.4,0.5,0.6)$

$$Z_{(0.4,0.5,0.6)}=\begin{matrix} x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{matrix}\begin{bmatrix} \overset{G_1}{(0,1,1)} & \overset{G_2}{(0,1,1)} & \overset{G_3}{(0,1,1)}\\ (0,1,1) & (0,1,1) & (1,0,0)\\ (1,0,0) & (1,0,0) & (1,0,0)\\ (1,0,0) & (1,0,0) & (0,1,1)\\ (1,0,0) & (1,0,0) & (0,1,1)\end{bmatrix}$$

**Proposition 5.4** Let $X$ and $Y$ be two non-empty finite universes, and $f\in Sur(X,Y)$. Let $\overset{\prec}{G}$ be a SVN $\beta$ -covering on $X$ for some $\beta=(\mu,\nu,\gamma)$, where $\mu,\nu,\gamma\in[0,1]$ and $\mu+\nu+\gamma\le 3$. Then $(Z_f)^T\,\Delta\,Z_{\overset{\prec}{G}}=Z_{f\left(\overset{\prec}{G}\right)}$.

**Proof.** The proof is simple and straight forward.

**Example 5.4.1** Considering the example **5.3.1**, we have

$$Z_{f\left(\overset{\prec}{G}\right)} = \begin{bmatrix} (1,0,0) & (0,1,1) & (0,1,1) & (1,0,0) & (0,1,1) \\ (0,1,1) & (1,0,0) & (0,1,1) & (0,1,1) & (0,1,1) \\ (0,1,1) & (0,1,1) & (1,0,0) & (0,1,1) & (1,0,0) \end{bmatrix} \Delta \begin{matrix} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} \begin{bmatrix} G_1 & G_2 & G_3 \\ (0.3,0.6,0.5) & (0.4,0.6,0.3) & (0.2,0.3,0.4) \\ (0.4,0.6,0.4) & (0.5,0.8,0.5) & (0.5,0.3,0.6) \\ (0.6,0.3,0.5) & (0.6,0.4,0.3) & (0.6,0.4,0.5) \\ (0.7,0.5,0.6) & (0.7,0.5,0.6) & (0.4,0.6,0.5) \\ (0.4,0.3,0.2) & (0.7,0.3,0.4) & (0.3,0.4,0.5) \end{bmatrix}$$

$$= \begin{bmatrix} \langle \vee(0.3,0,0,0.7,0), \wedge(1,0.6,0.3,1,0.3), \wedge(1,0.4,0.5,1,0.2) \rangle & \langle \vee(0.4,0,0,0.7,0), \wedge(1,0.8,0.4,1,0.3), \wedge(1,0.5,0.3,1,0.4) \rangle & \langle \vee(0.2,0,0,0.4,0), \wedge(1,0.3,0.4,1,0.4), \wedge(1,0.4,0.5,1,0.5) \rangle \\ \langle \vee(0,0.4,0,0,0), \wedge(0.6,1,0.3,0.5,0.3), \wedge(0.5,1,0.5,0.6,0.2) \rangle & \langle \vee(0,0.5,0,0,0), \wedge(0.6,1,0.4,0.5,0.3), \wedge(0.3,1,0.3,0.6,0.4) \rangle & \langle \vee(0,0.5,0,0,0), \wedge(0.3,1,0.4,0.6,0.4), \wedge(0.4,1,0.5,0.5,0.5) \rangle \\ \langle \vee(0,0,0.6,0,0.4), \wedge(0.6,0.6,1,0.5,0.1), \wedge(0.5,0.4,1,0.6,1) \rangle & \langle \vee(0,0,0.6,0,0.7), \wedge(0.6,0.8,1,0.5,1), \wedge(0.3,0.5,1,0.6,1) \rangle & \langle \vee(0,0,0.6,0,0.3), \wedge(0.3,0.3,1,0.6,1), \wedge(0.4,0.6,1,0.5,1) \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle 0.7,0.3,0.2 \rangle & \langle 0.7,0.3,0.4 \rangle & \langle 0.4,0.3,0.4 \rangle \\ \langle 0.4,0.3,0.2 \rangle & \langle 0.5,0.3,0.3 \rangle & \langle 0.5,0.3,0.4 \rangle \\ \langle 0.6,0.5,0.4 \rangle & \langle 0.7,0.5,0.3 \rangle & \langle 0.6,0.3,0.4 \rangle \end{bmatrix}$$

**Proposition 5.5** Let $X = \{x_1, x_2, ..., x_m\}$ and $Y = \{y_1, y_2, ..., y_n\}$ be two non-empty finite universal sets $f \in Sur(X,Y)$ and $\overset{\prec}{G} = \{G_1, G_2, ..., G_l\}$ be a SVN $\beta$-covering on $X$ for some $\beta = (\mu, \nu, \gamma)$, where $\mu, \nu, \gamma \in [0,1]$ and $\mu + \nu + \gamma \leq 3$. If $Z_\beta$ be a $\beta$-matrix representation of $\overset{\prec}{G}$, $Z_f$ be a matrix representation of $f$, and $Z_{\overset{\prec}{G}}$ be a matrix representation of $\overset{\prec}{G}$, then

$$Z_\beta \nabla \left( \left( Z_f \right)^T \Delta\, Z_{\overset{\prec}{G}} \right)^T = \left( \overset{\approx}{\mathrm{N}}{}_{x_i}^{\beta} \left( y_j \right) \right)_{m \times n}.$$

**Proof.** This proof is simple and obvious.

**Example 5.5.1** With reference to example **5.3.1** and the continuation of example **5.4.1,** we have

$$\left( \overset{\approx}{\mathrm{N}}{}_{x_i}^{\beta} \left( y_j \right) \right) = \begin{bmatrix} (0,1,1) & (0,1,1) & (0,1,1) \\ (0,1,1) & (0,1,1) & (1,0,0) \\ (1,0,0) & (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) & (0,1,1) \\ (1,0,0) & (1,0,0) & (0,1,1) \end{bmatrix} \nabla \begin{bmatrix} \langle 0.7,0.3,0.2 \rangle & \langle 0.4,0.3,0.2 \rangle & \langle 0.6,0.5,0.4 \rangle \\ \langle 0.7,0.3,0.4 \rangle & \langle 0.5,0.3,0.3 \rangle & \langle 0.7,0.5,0.3 \rangle \\ \langle 0.4,0.3,0.4 \rangle & \langle 0.5,0.3,0.4 \rangle & \langle 0.6,0.3,0.4 \rangle \end{bmatrix}$$

$$=\begin{bmatrix} \langle 1,0.3,0.4\rangle & \langle 1,0.5,0.4\rangle & \langle 1,0.5,0.4\rangle \\ \langle 0.4,0.3,0.4\rangle & \langle 0.5,0.3,0.4\rangle & \langle 0.6,0.5,0.4\rangle \\ \langle 0.4,0,0\rangle & \langle 0.4,0,0\rangle & \langle 0.6,0,0\rangle \\ \langle 0.7,0.3,0.4\rangle & \langle 0.4,0.3,0.4\rangle & \langle 0.6,0.3,0.4\rangle \\ \langle 0.7,0.3,0.4\rangle & \langle 0.4,0.3,0.4\rangle & \langle 0.6,0.3,0.4\rangle \end{bmatrix}$$

**Proposition 5.6** Let $X=\{x_1,x_2,...,x_m\}$ and $Y=\{y_1,\mathrm{y}_2,...,\mathrm{y}_n\}$ be two non-empty finite universal sets $f\in Sur(X,Y)$ and $\overset{\prec}{G}=\{G_1,G_2,...,G_l\}$ be a SVN $\beta$-covering on $X$ for some $\beta=(\mu,\nu,\gamma)$, where $\mu,\nu,\gamma\in[0,1]$ and $\mu+\nu+\gamma\le 3$. If $Z_\beta$ be a $\beta$-matrix representation of $\overset{\prec}{G}$, $Z_f$ be a matrix representation of $f$, and $Z_{\overset{\prec}{G}}$ be a matrix representation of $\overset{\prec}{G}$, then for each $X\in \acute{F}(Y)$, we have

$$\overset{\prec}{G}_*(X)=\left(Z_\beta\nabla\left(\left(Z_f\right)^T\Delta Z_{\overset{\prec}{G}}\right)^T\right)\nabla Z_X \quad , \quad \overset{\prec}{G}^*(X)=\left(Z_\beta\nabla\left(\left(Z_f\right)^T\Delta Z_{\overset{\prec}{G}}\right)^T\right)\Delta Z_X,$$

where $Z_X=\left(X\left(y_i\right)\right)_{1\times n}$.

**Proof.** It is obvious.

**Example 5.6.1** In a continuation of example **5.5.1**, we can obtain $\overset{\prec}{G}_*(X)$ and $\overset{\prec}{G}^*(X)$ as follows:

$$\overset{\prec}{G}_*(X)=\begin{bmatrix} \langle 1,0.3,0.4\rangle & \langle 1,0.5,0.4\rangle & \langle 1,0.5,0.4\rangle \\ \langle 0.4,0.3,0.4\rangle & \langle 0.5,0.3,0.4\rangle & \langle 0.6,0.5,0.4\rangle \\ \langle 0.4,0,0\rangle & \langle 0.4,0,0\rangle & \langle 0.6,0,0\rangle \\ \langle 0.7,0.3,0.4\rangle & \langle 0.4,0.3,0.4\rangle & \langle 0.6,0.3,0.4\rangle \\ \langle 0.7,0.3,0.4\rangle & \langle 0.4,0.3,0.4\rangle & \langle 0.6,0.3,0.4\rangle \end{bmatrix}\nabla\begin{bmatrix} \langle 0.3,0.4,0.5\rangle \\ \langle 0.2,0.5,0.6\rangle \\ \langle 0.5,0.7,0.4\rangle \end{bmatrix}$$

$$=\begin{bmatrix} \langle 0.2,0.5,0.4\rangle \\ \langle 0.5,0.5,0.4\rangle \\ \langle 0.5,0,0\rangle \\ \langle 0.3,0.3,0.4\rangle \\ \langle 0.3,0.3,0.4\rangle \end{bmatrix}$$

$$\overset{\prec}{G}^{*}(X)=\begin{bmatrix}\langle 1,0.3,0.4\rangle & \langle 1,0.5,0.4\rangle & \langle 1,0.5,0.4\rangle\\ \langle 0.4,0.3,0.4\rangle & \langle 0.5,0.3,0.4\rangle & \langle 0.6,0.5,0.4\rangle\\ \langle 0.4,0,0\rangle & \langle 0.4,0,0\rangle & \langle 0.6,0,0\rangle\\ \langle 0.7,0.3,0.4\rangle & \langle 0.4,0.3,0.4\rangle & \langle 0.6,0.3,0.4\rangle\\ \langle 0.7,0.3,0.4\rangle & \langle 0.4,0.3,0.4\rangle & \langle 0.6,0.3,0.4\rangle\end{bmatrix}\Delta\begin{bmatrix}\langle 0.3,0.4,0.5\rangle\\ \langle 0.2,0.5,0.6\rangle\\ \langle 0.5,0.7,0.4\rangle\end{bmatrix}$$

$$=\begin{bmatrix}\langle 0.5,0.5,0.6\rangle\\ \langle 0.5,0.7,0.6\rangle\\ \langle 0.5,1,1\rangle\\ \langle 0.5,0.7,0.6\rangle\\ \langle 0.5,0.7,0.6\rangle\end{bmatrix}$$

Clearly, $\overset{\prec}{G}_{*}(X)\neq\overset{\prec}{G}^{*}(X)$

**6. Application of SVN Covering Based Rough Set Model over Two Universes in MCDM Problem**

Multiple-criteria decision-making (MCDM) is a scientific approach that is useful to evaluate an optimal alternative under certain criteria or attributes. It is taken care of while evaluating the multiple conflicting criteria. Due to the uncertainty involved in many decision-making problems, makes the decision model more complex, and to overcome such type and reach a better decision, we need to consider a multiple-criteria model that provides a better option for the decision-makers to select the best option. Over the years, a variety of methods and approaches are developed to implement MCDM in many fields to enhance the decision-making approach. According to the traditional approach to MCDM, we select the best alternative according to the attribute values. But in modern MCDM methods, the selection of the best alternative is done according to the profit/loss type attribute values. So, the modern MCDM approaches are more flexible and powerful than the traditional approaches. The MCDM methods include TOPSIS, DEA, AHP, ANP, MULTIMOORA, etc.

In this section, we put forward an attempt to initiate a new approach to MCDM problems based on SVN covering-based rough set over two universes. For this, we describe the following MCDM problem:

Let $X=\{x_1,x_2,...,x_m\}$ be the set of $m$ patients and $Y=\{y_1,y_2,....,y_n\}$ be the set of $n$ diseases. Again, let $\overset{\prec}{G}=\{G_1,G_2,...,G_l\}$ be the set of diagnosis set, it is also known as SVN $\beta$ -covering on $X$ for some $\beta=(\mu,\nu,\gamma)$ , where $\mu,\nu,\gamma\in[0,1]$ and $\mu+\nu+\gamma\leq 3.$ Let $f\in sur(X,Y)$ such that $f(x_i)=y_j, where\ i=1,2,...,m\ and\ j=1,2,...,n$. We claim that $f$ partitions $X$ into $n$ classes. Therefore, to identify the disease of patients through diagnosis, the set of doctors (experts) specifies a suitable diagnosis scores line according to the symptoms of all the patients. For this, we set a suitable $\beta=\left(\vee_{i=1}^{m}\left(\wedge_{j=1}^{n}T_{\overset{\prec}{G}_j(x_i)}\right),\wedge_{i=1}^{m}\left(\vee_{j=1}^{n}I_{\overset{\prec}{G}_j(x_i)}\right),\wedge_{i=1}^{m}\left(\vee_{j=1}^{n}F_{\overset{\prec}{G}_j(x_i)}\right)\right)$. It can be easily verified that

$\overset{\prec}{G}$ is a SVN $\beta$ -covering on $X$ . Afterward, we obtain $f\left(\overset{\prec}{G}_k\right)\left(y_j\right)=\left(\bigvee_{(x_i\in X)\wedge\left(x_i\in f^{-1}\left(y_j\right)\right)} T_{\overset{\prec}{G}_k(x_i)}, \bigwedge_{(x_i\in X)\vee\left(x_i\in f^{-1}\left(y_j\right)\right)} I_{\overset{\prec}{G}_k(x_i)}, \bigwedge_{(x_i\in X)\vee\left(x_i\in f^{-1}\left(y_j\right)\right)} F_{\overset{\prec}{G}_k(x_i)}\right)$ which denotes the degree of criteria $\overset{\prec}{G}_k$ to the diseases $y_j$ . Also, $\overset{\approx}{N}^{\beta}_{x_i}\left(y_j\right)=\bigwedge_{\overset{\prec}{G}_k(x)\geq\beta} f\left(\overset{\prec}{G}_k\right)\left(y_j\right)$ denotes the possibility of the patient $x_i$ having a disease $y_j$ .

Moreover, for a given criterion $M$ over a SVN set of the universe $Y$ , the SVN covering-based lower approximation $\overset{\prec}{G}_*\left(M\right)$ of $M$ denotes the neighborhood degree of $M$ and $\overset{\approx}{N}^{\beta}_{x_i}$ . And the SVN covering-based upper approximation $\overset{\prec}{G}^*\left(M\right)$ of $M$ denotes the degree of intersection of $M$ and $\overset{\approx}{N}^{\beta}_{x_i}$ . If $\overset{\prec}{G}_*\left(M\right)\left(x_i\right)<\beta$ and $\overset{\prec}{G}^*\left(M\right)\left(x_i\right)<\beta$ , then the patient $x_i$ does not satisfied with the attribute $M$ . Otherwise, if $\overset{\prec}{G}_*\left(M\right)\left(x_i\right)\geq\beta$ and $\overset{\prec}{G}^*\left(M\right)\left(x_i\right)\geq\beta$ , then the patient $x_i$ satisfies the criteria.

To implement the MCDM process, we consider the following steps:

**<u>Input:</u>** Assuming the SVN information system $\left(X,Y,\overset{\prec}{G}\right)$ over two universes for MCDM problem , $f\in Sur(X,Y)$ and a criteria value $\beta=\left(\mu,\nu,\gamma\right)$ , where $\mu,\nu,\gamma\in\left[0,1\right]$ and $\mu+\nu+\gamma\leq 3$.

**<u>Computations:</u>**

**Step 1:** Construct a SVN covering-based rough set model over two universes.

**Step2:** Calculate the SVN covering-based lower approximation $\overset{\prec}{G}_*\left(M\right)$ and the SVN covering-based upper approximation $\overset{\prec}{G}^*\left(M\right)$ for the criterion $M$ (defined by the SVN set of the universe $Y$ ) provided by the hospital.

**Step 3:** If $\overset{\prec}{G}_*\left(M\right)\left(x_i\right)\vee\overset{\prec}{G}^*\left(M\right)\left(x_i\right)<\beta$ , then the patient $x_i$ cannot be diagnosed to detect the disease $y_j$ under the critical value $\beta$ .

**Step 4:** If $\overset{\prec}{G}_*\left(M\right)\left(x_i\right)\vee\overset{\prec}{G}^*\left(M\right)\left(x_i\right)\geq\beta$ , then the patient $x_i$ be diagnosed to detect the disease $y_j$ under the critical value $\beta$ .

**Step 5:** Rank the alternatives to select the patient who needs a diagnosis to detect a certain disease.

**Output:** Ranking orders of all the alternatives.

## 7. Conclusions and Future Scope

The notion of a single-valued neutrosophic $\beta$-covering set is introduced by Wang et al.[63], which makes a connection between a single-valued neutrosophic set and a covering-based rough set. Using this concept, in this paper, a new type of SVN covering-based rough set model over two universes is developed. We also introduce SVN $\beta$-covering rough set model over two universes with an aid of SVN $\beta$-neighborhood and studied some of its properties. Furthermore, we have presented the matrix representations of the SVN covering-based lower and upper approximation operators. Finally, we give a method for MCDM under the SVN $\beta$-covering-based lower and upper approximation operators over two universes.

In the future, to handle more critical decision-making problems, we can extend the proposed model by replacing the SVN covering information with the refined single-valued neutrosophic(RSVN) and quadripartitioned single-valued neutrosophic(QSVN) covering information and use them to develop TOPSIS, AHP, MULTIMOORA method in the MADM, MCDM, MCGDM, MAGDM problems. Topology and Entropy-based study in the same setting can also be possible to develop soon. To handle the parametric information, we can add the flavor of the soft set and hypersoft set in the present study to make it more flexible to encounter the uncertain information in a sophisticated way.

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## References

[1] L.A. Zadeh (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353. doi:10.1016/s0019-9958(65)90241-x

[2] Gupta, M.M.; Ragade, R.K. (1977). *Fuzzy set theory and its applications: A survey. IFAC Proceedings Volumes, 10(6), 247–259.* doi:10.1016/b978-0-08-022010-9.50038-4

[3] Biswas, R. (1995). An application of fuzzy sets in students' evaluation. *Fuzzy sets and systems*, *74*(2), 187-194.

[4] Pal, S. K., & King, R. A. (1980). Image enhancement using fuzzy set. *Electronics letters*, *16*(10), 376-378.

[5] Jiang, H., & Eastman, J. R. (2000). Application of fuzzy measures in multi-criteria evaluation in GIS. *International Journal of Geographical Information Science*, *14*(2), 173-184.

[6] Yager, R. R. (1982). Measuring tranquility and anxiety in decision making: an application of fuzzy sets. *International Journal of General Systems, 8(3),*139-146.

[7] Krassimir T. Atanassov (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems , 20(1), 87–96.* doi:10.1016/s0165-0114(86)80034-3

[8] Gorzałczany, M. B. (1987). A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy sets and systems*, *21*(1), 1-17.

[9] K. Atanassov; G. Gargov (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems , 31(3), 343–349.* doi:10.1016/0165-0114(89)90205-4

[10] Cuong, B. C., & Kreinovich, V. (2013). Picture Fuzzy Sets-a new concept for computational intelligence problems. In *2013 third world congress on information and communication technologies (WICT 2013)* (pp. 1-6). IEEE.

[11] Ashraf, S., Abdullah, S., Aslam, M., Qiyas, M., & Kutbi, M. A. (2019). Spherical fuzzy sets and its representation of spherical fuzzy t-norms and t-conorms. *Journal of Intelligent & Fuzzy Systems, 36(6), 6089–6102.* doi:10.3233/JIFS-181941

[12] Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems*, *25*(6), 529-539.

[13] Yager, R. R. (2013, June). Pythagorean fuzzy subsets. In *2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)* (pp. 57-61). IEEE.

[14] Smarandache, F. (1998). Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. *American Research Press, ISBN 1879585634.*

[15] Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International journal of pure and applied mathematics*, *24*(3), 287-297.

[16] Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Technical Sciences and Applied Mathematics,*10-14.

[17] Pramanik, S., Dalapati, S., Alam, S., Smarandache, F., & Roy, T. K. (2018). NS-cross entropy-based MAGDM under single-valued neutrosophic set environment. *Information*, *9*(2), 37. https://doi.org/10.3390/info9020037

[18] Ye, J. (2014). Single valued neutrosophic cross-entropy for multicriteria decision making problems. *Applied Mathematical Modelling*, *38*(3), 1170-1175.

[19] Biswas, P., Pramanik, S., & Giri, B. C. (2016). TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural computing and Applications*, *27*(3), 727-737.

[20] Kazimieras Zavadskas, E., Baušys, R., & Lazauskas, M. (2015). Sustainable assessment of alternative sites for the construction of a waste incineration plant by applying WASPAS method with single-valued neutrosophic set. *Sustainability*, *7*(12), 15923-15936.

[21] Jiang, W., & Shou, Y. (2017). A novel single-valued neutrosophic set similarity measure and its application in multicriteria decision-making. *Symmetry*, *9*(8), 127.

[22] Ishtiaq, U., Javed, K., Uddin, F., Sen, M. D. L., Ahmed, K., & Ali, M. U. (2021). Fixed point results in orthogonal neutrosophic metric spaces. *Complexity*, *2021*.

[23] Ali, U., Alyousef, H. A., Ishtiaq, U., Ahmed, K., & Ali, S. (2022). Solving Nonlinear Fractional Differential Equations for Contractive and Weakly Compatible Mappings in Neutrosophic Metric Spaces. *Journal of Function Spaces*, *2022*.

[24] Hussain, A., Al Sulami, H., & Ishtiaq, U. (2022). Some New Aspects in the Intuitionistic Fuzzy and Neutrosophic Fixed Point Theory. *Journal of Function Spaces*, *2022*.

[25] Javed, K., Uddin, F., Aydi, H., Arshad, M., Ishtiaq, U., & Alsamir, H. (2021). On fuzzy b-metric-like spaces. *Journal of Function Spaces*, *2021*.

[26] Ishtiaq, U., Hussain, A., & Al Sulami, H. (2022). Certain new aspects in fuzzy fixed point theory. *AIMS Mathematics*, *7*(5), 8558-8573.

[27] Hussain, A., Ishtiaq, U., Khalil, A., & Al-Sulami, H. (2022). On pentagonal controlled fuzzy metric spaces with an application to dynamic market equilibrium. *Journal of function spaces*, *2022*.

[28] Pawlak, Z. Rough sets. *International Journal of Computer and Information Sciences 11***,** 341–356 (1982). https://doi.org/10.1007/BF01001956

[29] Degang, C., Wenxiu, Z., Yeung, D., & Tsang, E. C. (2006). Rough approximations on a complete completely distributive lattice with applications to generalized rough sets. *Information Sciences*, *176*(13), 1829-1848.

[30] Pawlak, Z. (1998). Rough set theory and its applications to data analysis. *Cybernetics & Systems*, *29*(7), 661-688.

[31] Mitra, S., & Banka, H. (2007). Application of Rough Sets in Pattern Recognition. *Lecture Notes in Computer Science*, *4400*, 151–169. doi:10.1007/978-3-540-71663-1_10

[32] Slimani, T. (2013). Application of rough set theory in data mining. *International Journal of Computer Science and Network Solutions, 1(3),* 1-10.

[33] Pan, W., Yi, J., & San, Y. (2008). Rough set theory and its application in the intelligent systems," *2008 7th World Congress on Intelligent Control and Automation*, 2008, pp. 3706-3711, doi: 10.1109/WCICA.2008.4593519.

[34] Paszek, P., & Wakulicz–Deja, A. (2007). Applying Rough Set Theory to Medical Diagnosing. *Lecture Notes in Computer Science*, *4585*, 427–435. doi:10.1007/978-3-540-73451-2_45

[35] J. Hua. (2008). Study on the application of rough sets theory in machine learning. *Second International Symposium on Intelligent Information Technology Application*, 2008, pp. 192-196, doi: 10.1109/IITA.2008.154.

[36] Dubois, D., & Prade, H. (1990). Rough fuzzy sets and fuzzy rough sets. *International Journal of General System*, *17*(2-3), 191-209.

[37] Zhou, Lei; Wu, Wei-Zhi; Zhang, Wen-Xiu (2009). On intuitionistic fuzzy rough sets and their topological structures. *International Journal of General Systems, 38(6), 589–616.* doi:10.1080/03081070802187723

[38] Zhang, Z. (2009). An interval-valued intuitionistic fuzzy rough set model. *Fundamenta Informaticae,* 97 , 471–498.

[39] Zhang, H., & Shu, L. (2015). Generalized interval-valued fuzzy rough set and its application in decision making. *International Journal of Fuzzy Systems, 17(2), 279–291.* doi:10.1007/s40815-015-0012-9

[40] Yang, H. L., Zhang, C. L., Guo, Z. L., Liu, Y. L., & Liao, X. (2017). A hybrid model of single valued neutrosophic sets and rough sets: single valued neutrosophic rough set model. *Soft Computing*, *21*(21), 6253-6267.

[41] Zhao, H., & Zhang, H. Y. (2020). On hesitant neutrosophic rough set over two universes and its application. *Artificial Intelligence Review*, *53*(6), 4387-4406.

[42] Akram, M., Ishfaq, N., Sayed, S., & Smarandache, F. (2018). Decision-making approach based on neutrosophic rough information. *Algorithms*, *11*(5), 59. https://doi.org/10.3390/a11050059

[43] Bo, C., Zhang, X., Shao, S., & Smarandache, F. (2018). Multi-granulation neutrosophic rough sets on a single domain and dual domains with applications. *Symmetry*, *10*(7), 296. https://doi.org/10.3390/sym10070296

[44] Abdel-Basset, M., & Mohamed, M. (2018). The role of single valued neutrosophic sets and rough sets in smart city: Imperfect and incomplete information systems. *Measurement*, *124*, 47-55.

[45] Mondal, K., & Pramanik, S. (2015). Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. *Neutrosophic Sets and Systems*, *8*, 14-21.

[46] Pramanik, S., & Mondal, K. (2015). Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. *Journal of New Theory*, (4), 90-102. https://dergipark.org.tr/en/pub/jnt/issue/34490/381119

[47] Mondal, K., & Pramanik, S. (2015). Rough neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems*, *7*(2015), 8-17.

[48] Yiyu Y., & Bingxue Y. (2012). Covering based rough set approximations. *Information Sciences, 200, 91–107.* doi:10.1016/j.ins.2012.02.065

[49] Kong, Q. Z., Wei, Z. X., Batyrshin, I., Pamučar, D. S., Crippa, P., & Liu, F. (2015). Covering-based fuzzy rough sets. *Journal of Intelligent & Fuzzy Systems, 29(6), 2405–2411.* doi:10.3233/IFS-151940

[50] Ma, L. (2016). Two fuzzy covering rough set models and their generalizations over fuzzy lattices. *Fuzzy Sets and Systems, 294,* 1-17. S0165011415002171–. doi:10.1016/j.fss.2015.05.002

[51] Zhang, K., Zhan, J., Wu, W., & Alcantud, J. C. R. (2019). Fuzzy β-covering based (I, T)-fuzzy rough set models and applications to multi-attribute decision-making. *Computers & Industrial Engineering*, *128*, 605-621.

[52] Zhang, K., Zhan, J., & Wang, X. (2020). TOPSIS-WAA method based on a covering-based fuzzy rough set: an application to rating problem. *Information Sciences*, *539*, 397-421.

[53] Zhou, J., Xu, F., Guan, Y., & Wang, H. (2021). Three types of fuzzy covering-based rough set models. *Fuzzy Sets and Systems*, *423*, 122-148.

[54] Yang, B., & Hu, B. Q. (2017). On some types of fuzzy covering-based rough sets. *Fuzzy sets and Systems*, *312*, 36-65.

[55] D'eer, L., & Cornelis, C. (2018). A comprehensive study of fuzzy covering-based rough set models: Definitions, properties and interrelationships. *Fuzzy Sets and Systems*, *336*, 1-26.

[56] Yang, B., & Hu, B. Q. (2018). Communication between fuzzy information systems using fuzzy covering-based rough sets. *International Journal of Approximate Reasoning*, *103*, 414-436.

[57] Yang, B. (2022). Fuzzy covering-based rough set on two different universes and its application. *Artificial Intelligence Review*, 1-37.

[58] Zhan, J., Jiang, H., & Yao, Y. (2020). Covering-based variable precision fuzzy rough sets with PROMETHEE-EDAS methods. *Information Sciences*, *538*, 314-336.

[59] Zhou, J. J., & Li, X. Y. (2021). Hesitant fuzzy β covering rough sets and applications in multi-attribute decision making. *Journal of Intelligent & Fuzzy Systems*, *41*, 2387-2402.

[60] Zeng, S., Hussain, A., Mahmood, T., Irfan Ali, M., Ashraf, S., & Munir, M. (2019). Covering-based spherical fuzzy rough set model hybrid with TOPSIS for multi-attribute decision-making. *Symmetry*, *11*(4), 547.

[61] Zhan, J., & Sun, B. (2020). Covering-based intuitionistic fuzzy rough sets and applications in multi-attribute decision-making. *Artificial Intelligence Review*, *53*(1), 671-701.

[62] Wang, J., & Zhang, X. (2018). Two types of intuitionistic fuzzy covering rough sets and an application to multiple criteria group decision making. *Symmetry*, *10*(10), 462.

[63] Wang, J., & Zhang, X. (2018). Two types of single valued neutrosophic covering rough sets and an application to decision making. *Symmetry*, *10*(12), 710.

[64] Wang, J., & Zhang, X. (2019). A new type of single valued neutrosophic covering rough set model. *Symmetry*, *11*(9), 1074.

[65] Mao, L. (2020, December). Reducts in single valued neutrosophic ß-covering approximation spaces. In *Journal of Physics: Conference Series* (Vol. 1693, No. 1, p. 012024). IOP Publishing.

[66] Zhang, X., Atef, M., & Khalil, A. M. (2021). On different types of single-valued neutrosophic covering rough set with application in decision-making. *Mathematical Problem in Engineering*, *2021*, 1-16.

[67] Xu, D., Xian, H., & Lu, X. (2021). Interval neutrosophic covering rough sets based on neighborhoods. *AIMS Mathematics, 6(4),* 3772-3787.

[68] Wang, J. Q., & Zhang, X. H. (2020). Multigranulation single valued neutrosophic covering-based rough sets and their applications to multi-criteria group decision making. *Iranian Journal of Fuzzy Systems, 17(5),* 109-126.

[69] Pomykala, J. A. (1987). Approximation operations in approximation space. *Bull. Pol. Acad. Sci*, *35,* 653-662.

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