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Number of Neutrosophic Topological Spaces on Finite Set with $\mathfrak{k} \leq 4$ Open Sets

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Abstract. In this paper, the number of neutrosophic topological spaces having two, three, and four open sets are computed for a finite set $\mathbb{X}^{NT}$ whose membership values lies in $\mathbb{M}^{NT}$. Further, the number of neutrosophic bitopological spaces and neutrosophic tritopological spaces having $\mathfrak{k}(\mathfrak{k} = 2, 3, 4)$ neutrosophic open sets on finite sets are computed.

Keywords: : Neutrosophic Set; Neutrosophic Topology; Two Open Set; Three Open Set; Four Open Set.

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1. **Introduction**

Finding the number of topologies in a set is an interesting task. Many authors have done their work in this field. Krishnamurty [1] obtained a sharper bound namely $2^{\mathfrak{n}(\mathfrak{n}-1)}$ for the number of distinct topologies. Sharp [2] shows that only discrete topology has cardinal greater than $\frac{3}{4}2^{\mathfrak{n}}$ and derived bounds for the cardinality of topologies which are connected, non-connected, non-$\mathscr{T}_0$, and some more. After obtaining all non-homeomorphic topologies with $\mathfrak{n}$ points and $> \frac{7}{16}2^{\mathfrak{n}}$ open sets, Stanley [3] also determined which of these are $\mathscr{T}_0$. The concept of partial chain topologies supported Kamel [4] to formulate a special case for computing the number of chain topologies and maximal elements with natural generalization. Ragnarsson *et al.* [5], have also studied obtainable sizes of topologies on a finite set. Benoumhani [6] computed the number of topologies having $2, 3, \ldots, 12$-open sets, and also $\mathscr{T}_0$ topologies having $\mathfrak{n}+4$, $\mathfrak{n}+5$, and $\mathfrak{n}+6$ open sets. These results are extended in [7].

Later on, Benoumhani *et al.* [8] extended their work to fuzzy topological spaces (FTS). They computed the number of FTS having $2, 3, 4$, and 5-open sets and certain cases, where the number of open sets is large. Basumatary *et al.* [9] discussed the number of fuzzy bitopological spaces and gave some formulae.

After the generalization of the fuzzy set [10] from crisp set and intuitionistic fuzzy set [11], Smarandache discovered the concept of the neutrosophic set by combining the fuzzy set and intuitionistic fuzzy set. Since the introduction of the NS (Neutrosophic set) by Smarandache [12], several authors have contributed their work in science and technology by taking NS as a tool. Wang [13] studied single-valued NSs in multiset and multistructure. Salama *et al.* [14] studied the neutrosophic topological spaces (NTS). Lupiáñez [15–18] investigated NTS. Mwchahary *et al.* [19] studied neutrosophic bitopological space (NBTS). Devi *et al.* [20] and Ozturk *et al.* [21] also discussed NBTS. Kelly [22] and Kovar [23] introduced the notion of bitopological space and tritopological space respectively. The neutrosophic crisp tri-topological spaces are studied by Al-Hamido *et al.* [24].

Ishtiaq *et al.* [25, 26] studied fixed-point results in orthogonal neutrosophic metric spaces and also certain new aspects in fuzzy fixed-point theory. Ali *et al.* [27] discussed solving nonlinear fractional differential equations for contractive and weakly compatible mappings in neutrosophic metric spaces. Hussain *et al.* [28] worked on some new aspects of the intuitionistic fuzzy and neutrosophic fixed point theory. Javed *et al.* [29] studied the fuzzy b-metric-like spaces. Hussain *et al.* [30] studied the pentagonal controlled fuzzy metric spaces with an application to dynamic market equilibrium.

From the literature survey, it is observed that generally finding the number of topologies (NoTs) for a set is not an easy task. Because of this current authors started research work in this area. This article discusses formulae for calculating the NNTSs (number of NTSs) with $2, 3$, or 4-open sets, as well as the NNBTSs (number of NBTSs) and NNTRSs (number of neutrosophic tritopological spaces) with the same number of open sets in topologies.

Let $\mathbb{X}^{NT}$ be a non-empty finite set, $\mathbb{M}^{NT}$ be the finite totally ordered set with $|\mathbb{M}^{NT}| = \mathfrak{m} \geq 2$ and $\mathcal{N}_{\mathbb{X}}^{\mathcal{T}}$ be a set that contains all the neutrosophic subsets (NSubs) of $\mathbb{X}^{NT}$ with membership values in $\mathbb{M}^{NT}$.

Note that in this paper $\mathcal{T}_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, \mathfrak{k})$ denotes NNTSs on $\mathbb{X}^{NT}$ with $|\mathbb{X}^{NT}| = \mathfrak{n}$ and $\mathfrak{k}$-open sets, $(\mathcal{T}_{\imath}^{NT}, \mathcal{T}_{\jmath}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, \mathfrak{k})$ and $(\mathcal{T}_{\imath}^{NT}, \mathcal{T}_{\jmath}^{NT}, \mathcal{T}_{\hbar}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, \mathfrak{k})$ denotes NNBTSs and NNTRSs respectively on $\mathbb{X}^{NT}$ consisting $\mathfrak{k}$-open sets in topologies at a time where $\mathfrak{n}, \mathfrak{m}, \mathfrak{k} \in \mathbb{N}$, $\mathfrak{n} \geq 1, \mathfrak{m} \geq 2$ and $\mathfrak{k} \geq 2$.

2. Preliminaries

Definition 2.1. [14] On a universe of discourse $\mathbb{X}^{NT}$ a NS $\mathfrak{U}^{NT}$ is defined as $\mathfrak{U}^{NT} = \langle \frac{u}{(T_{\mathfrak{U}}^{NT}(u), I_{\mathfrak{U}}^{NT}(u), F_{\mathfrak{U}}^{NT}(u))} : u \in \mathbb{X}^{NT} \rangle$, where $T_{\mathfrak{U}}^{NT}, I_{\mathfrak{U}}^{NT}, F_{\mathfrak{U}}^{NT} : \mathbb{X}^{NT} \rightarrow]^{-}0, 1^{+}[$. Here $^{-}0 \leq T_{\mathfrak{U}}^{NT}(u) + I_{\mathfrak{U}}^{NT}(u) + F_{\mathfrak{U}}^{NT}(u) \leq 3^{+}$; $T_{\mathfrak{U}}^{NT}(u)$ represents degree of membership function, $I_{\mathfrak{U}}^{NT}(u)$ degree of indeterminacy and $F_{\mathfrak{U}}^{NT}(u)$ degree of non-membership function.

Definition 2.2. [14,15] Let $\mathscr{T}^{NT} \subseteq \mathscr{N}_{\mathbb{X}}^{NT}$ then $\mathscr{T}^{NT}$ is called a neutrosophic topology (NT) on $\mathbb{X}^{NT}$ if

- $0^{NT}, 1^{NT} \in \mathscr{T}^{NT}$
- $\mathfrak{U}_1^{NT} \cap \mathfrak{U}_2^{NT} \in \mathscr{T}^{NT}$ for any $\mathfrak{U}_1^{NT}, \mathfrak{U}_2^{NT} \in \mathscr{T}^{NT}$.
- $\cup \mathfrak{U}_i^{NT} \in \mathscr{T}^{NT}$, for arbitrary family $\{\mathfrak{U}_i^{NT} : i \in \mathbb{I}\} \in \mathscr{T}^{NT}$.

The pair $(\mathbb{X}^{NT}, \mathscr{T}^{NT})$ is called NTS and any NS in $\mathscr{T}^{NT}$ is called NOS (neutrosophic open set) in $\mathbb{X}^{NT}$.

Definition 2.3. [19] Let $\mathscr{T}_1^{NT}$ and $\mathscr{T}_2^{NT}$ be the two NTs on $\mathbb{X}^{NT}$. Then $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_2^{NT})$ is called a NBTS.

Example 2.4. If $\mathbb{X}^{NT} = \{u, v, w\}$ and if $\mathscr{T}_1^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}\}$ and $\mathscr{T}_2^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}\}$, where

$$\mathfrak{U}_1^{NT} = \langle \frac{u}{(0.7,0.1,0.5)}, \frac{v}{(0.5,0.2,0.3)}, \frac{w}{(0.3,0.4,0.4)} \rangle, \mathfrak{U}_2^{NT} = \langle \frac{u}{(0.2,0.5,0.1)}, \frac{v}{(0.1,0.2,0.3)}, \frac{w}{(0.6,0.3,0.5)} \rangle.$$

Then $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT})$ and $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT})$ form NTS. Therefore, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_2^{NT})$ is a NBTS.

Definition 2.5. [31] Let $\mathscr{T}_1^{NT}, \mathscr{T}_2^{NT}$ and $\mathscr{T}_3^{NT}$ be the three NTs on $\mathbb{X}^{NT}$. Then $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_3^{NT})$ is called a neutrosophic tritopological space (NTRS).

Example 2.6. If $\mathbb{X}^{NT} = \{u, v, w\}$ and consider $\mathscr{T}_1^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}\}$, $\mathscr{T}_2^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}\}$ and $\mathscr{T}_3^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}\}$.

Here, $\mathfrak{U}_1^{NT} = \langle \frac{u}{(0.7,0.1,0.5)}, \frac{v}{(0.5,0.2,0.3)}, \frac{w}{(0.3,0.6,0.2)} \rangle$, $\mathfrak{U}_2^{NT} = \langle \frac{u}{(0.6,0.5,0.3)}, \frac{v}{(0.7,0,0.2)}, \frac{w}{(0.8,0.1,0.1)} \rangle$,

$$\mathfrak{U}_3^{NT} = \langle \frac{u}{(0.5,0.2,0.3)}, \frac{v}{(0.2,0.1,0.2)}, \frac{w}{(0.1,0,0.1)} \rangle.$$

Then $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT})$ and $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT})$ form NTS.

Therefore $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_3^{NT})$ is a NTRS. In this case, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_3^{NT})$ is a NTRS having 3-NOS in each of the topologies.

3. Results on NNTS

Proposition 3.1. *The NNTs (Number of Neutrosophic Topologies) on $\mathbb{X}^{NT}$, whose membership values lies in $\mathbb{M}^{NT}$, is finite if and only if both $\mathbb{X}^{NT}$ and $\mathbb{M}^{NT}$ are finite.*

Result 3.2. *The NNTSs having 2-NOS is one i.e., $\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 2) = 1$.*

The NT having 2-open set is the indiscrete NT which is $\mathscr{T}_1^{NT} = \{0^{NT}, 1^{NT}\}$.

Result 3.3. *The NNTs having 3-NOS is $\mathfrak{m}^{\mathfrak{n}} - 2$ i.e., $\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 3) = \mathfrak{m}^{\mathfrak{n}} - 2$.*

These NTs necessarily consists of a chain containing $0^{NT}, 1^{NT}$ and any one NSub of $\mathbb{X}^{NT}$. In this case NTs are in the chain, of the form $0^{NT} \subseteq \mathfrak{U}_1^{NT} \subseteq 1^{NT}, \mathfrak{U}_1^{NT}$ is any NSub of $\mathbb{X}^{NT}$.

Example 3.4. Let $\mathbb{X}^{NT} = \{u, v\}$ and $\mathbb{M}^{NT} = \{(0,1,1), (0.6,0.1,0.2), (1,0,0)\}$. It is seen that, $|\mathbb{X}^{NT}| = \mathfrak{n} = 2$, $|\mathbb{M}^{NT}| = \mathfrak{m} = 3$.

Then number of elements in $N_{\mathbb{X}}^{\mathscr{T}}$ i.e., $|N_{\mathbb{X}}^{\mathscr{T}}| = 3^2 = 9$. These are

0^{NT}, 1^{NT}, $\mathfrak{U}_1^{NT} = \langle \frac{u}{(0,1,1)}, \frac{v}{(0.6,0.1,0.2)} \rangle$, $\mathfrak{U}_2^{NT} = \langle \frac{u}{(0,1,1)}, \frac{v}{(1,0,0)} \rangle$, $\mathfrak{U}_3^{NT} = \langle \frac{u}{(0.6,0.1,0.2)}, \frac{v}{(0,1,1)} \rangle$, $\mathfrak{U}_4^{NT} \langle \frac{u}{(0.6,0.1,0.2)}, \frac{v}{(0.6,0.1,0.2)} \rangle$, $\mathfrak{U}_5^{NT} = \langle \frac{u}{(0.6,0.1,0.2)}, \frac{v}{(1,0,0)} \rangle$, $\mathfrak{U}_6^{NT} = \langle \frac{u}{(1,0,0)}, \frac{v}{(0,1,1)} \rangle$, $\mathfrak{U}_7^{NT} = \langle \frac{u}{((1,0,0)}, \frac{v}{(0.6,0.1,0.2)} \rangle$.

So, $\mathscr{T}_{\mathbb{X}}^{NT}(2,3,3) = 3^2 - 2 = 7$.

The NTs having 3-open sets are:

$$\mathscr{T}_1^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}\},\ \mathscr{T}_2^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}\},\ \mathscr{T}_3^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}\},$$
$$\mathscr{T}_4^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_4^{NT}\},\ \mathscr{T}_5^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_5^{NT}\},\ \mathscr{T}_6^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_6^{NT}\},$$
$$\mathscr{T}_7^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_7^{NT}\}.$$

Result 3.5. *An arbitrary NT with 4-NOSs is an NT consisting of $1^{NT}, 0^{NT}$ and other two NSubs. These NSubs are either chain of 2-elements or anti-chain of 2-elements having 1^{NT} and 0^{NT} as union and intersection respectively.*

Theorem 3.6. *In $\hat{\mathscr{N}}_{\mathbb{X}}^{\mathscr{T}} = \mathscr{N}_{\mathbb{X}}^{\mathscr{T}} - \{0^{NT}, 1^{NT}\}$, the number of chains (NCs) of length 2 is obtained by*

$$c_2(\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}) = \binom{\mathfrak{m}+1}{2}^{\mathfrak{n}} - 3\mathfrak{m}^{\mathfrak{n}} + 3.$$

Corollary 3.7. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, the NCs of length 4 having both 0^{NT} and 1^{NT} is same as $c_2(\mathscr{N}_{\mathbb{X}}^{\mathscr{T}})$.*

Lemma 3.8. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, the number of anti-chains (NACs) of size 2 (having 2-elements) with 1^{NT} as union and 0^{NT} as intersection is $2^{\mathfrak{n}-1} - 1$.*

Corollary 3.9. *The NAC NTs of $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$ consisting of 4-open set is $2^{\mathfrak{n}-1} - 1$.*

Theorem 3.10. *The NNTs in $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$ with 4-NOSss is*

$$\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 4) = \left(\frac{\mathfrak{m}(\mathfrak{m}+1)}{2}\right)^{\mathfrak{n}} - 3\mathfrak{m}^{\mathfrak{n}} + 2^{\mathfrak{n}-1} + 2.$$

Follow Cor. 3.7 and Cor. 3.9 for the prove of theorem.

Example 3.11. Let, $\mathbb{X}^{NT} = \{u, v\}$ and $\mathbb{M}^{NT} = \{(0,1,1), (0.1,0.3,0.8), (1,0,0)\}$. Therefore $|\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}| = 3^2 = 9$. These NSubs are

$0^{NT} = \langle \frac{u}{(0,1,1)}, \frac{v}{(0,1,1)} \rangle$, $1^{NT} = \langle \frac{u}{(1,0,0)}, \frac{v}{(1,0,0)} \rangle$, $\mathfrak{U}_1^{NT} = \langle \frac{u}{(0,1,1)}, \frac{v}{(0.1,0.3,0.8)} \rangle$,
$\mathfrak{U}_2^{NT} = \langle \frac{u}{(0,1,1)}, \frac{v}{(1,0,0)} \rangle$, $\mathfrak{U}_3^{NT} = \langle \frac{u}{(0.1,0.3,0.8)}, \frac{v}{(0,1,1)} \rangle$, $\mathfrak{U}_4^{NT} = \langle \frac{u}{(0.1,0.3,0.8)}, \frac{v}{(0.1,0.3,0.8)} \rangle$,
$\mathfrak{U}_5^{NT} = \langle \frac{u}{(0.1,0.3,0.8)}, \frac{v}{(1,0,0)} \rangle$, $\mathfrak{U}_6^{NT} = \langle \frac{u}{(1,0,0)}, \frac{v}{(0,1,1)} \rangle$, $\mathfrak{U}_7^{NT} = \langle \frac{u}{(1,0,0)}, \frac{v}{(0.1,0.3,0.8)} \rangle$.

In this case, $\mathfrak{n} = 2$, $\mathfrak{m} = 3$,

Therefore, $\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4) = \left(\frac{3(3+1)}{2}\right)^2 - 3.3^2 + 2^{2-1} + 2 = 6^2 - 23 = 13$.

These NTs with 4-NOSs are

$$\mathscr{T}_1^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_2^{NT}\},\quad \mathscr{T}_2^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_4^{NT}\},$$

$\mathscr{T}_3^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_5^{NT}\}$, $\mathscr{T}_4^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_7^{NT}\}$,
$\mathscr{T}_5^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}, \mathfrak{U}_5^{NT}\}$, $\mathscr{T}_6^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}, \mathfrak{U}_6^{NT}\}$,
$\mathscr{T}_7^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_4^{NT}\}$, $\mathscr{T}_8^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_5^{NT}\}$,
$\mathscr{T}_9^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_6^{NT}\}$, $\mathscr{T}_{10}^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_7^{NT}\}$,
$\mathscr{T}_{11}^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_4^{NT}, \mathfrak{U}_5^{NT}\}$, $\mathscr{T}_{12}^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_4^{NT}, \mathfrak{U}_7^{NT}\}$,
$\mathscr{T}_{13}^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_6^{NT}, \mathfrak{U}_7^{NT}\}$.

Here, the only anti-chain NTs in $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$ is $\mathscr{T}_6^{NT}$ with 0^{NT} and 1^{NT} as intersection and union respectively.

4. Results on NNBTS

In this section, the NBTS having 3-NOSs in both NTs and the NBTS having 3-NOSs in both NTs without repetition means NBTS of the form $(\mathbb{X}^{NT}, \mathscr{T}_i^{NT}, \mathscr{T}_j^{NT})$, where $\mathscr{T}_i^{NT}, \mathscr{T}_j^{NT}$ are identical or non-identical topologies, and non-identical topologies having 3-NOSs respectively. A similar meaning is used for 4-NOSs.

Result 4.1. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, the NNBTS with two NOSs in both the NTs is*

$$(\mathscr{T}_i^{NT}, \mathscr{T}_j^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 2) = 1.$$

From Result 3.2, $\mathscr{T}_{\mathbb{X}}^{\mathscr{T}}(\mathfrak{n}, \mathfrak{m}, 2) = 1$, which is the indiscrete topology $\mathscr{T}_1^{NT} = \{0^{NT}, 1^{NT}\}$. Hence, NBTS with 2-NOSs is only one i.e., $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_1^{NT})$.

Result 4.2. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, the NNBTSs having 3-NOSs in both NTs is*

$$(\mathscr{T}_i^{NT}, \mathscr{T}_j^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 3) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},3)+1}{2} = \frac{\mathfrak{m}^{2\mathfrak{n}} - 3\mathfrak{m}^{\mathfrak{n}} + 2}{2}.$$

Example 4.3. Example 3.4 gives $\mathscr{T}_{\mathbb{X}}^{NT}(2, 3, 3) = 7$.
Therefore,$(\mathscr{T}_i^{NT}, \mathscr{T}_j^{NT})_{\mathbb{X}}^{NT}(2, 3, 3) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(2,3,3)+1}{2} = 28$.
Then, these NBTSs are
$(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_1^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_2^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_3^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_4^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_7^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_2^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_3^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_4^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_5^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_7^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_3^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_4^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_6^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_7^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_4^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_7^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_7^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_7^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_7^{NT})$.

Result 4.4. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, the NNBTSs having 3-NOSs in both NTs without repetition is*

$$(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 3) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},3)}{2}.$$

Example 4.5. Following Example 3.4 and Result 4.4., the number of NBTSs without repetition is

$$21 = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(2,3,3)}{2} = \binom{7}{2}.$$

Result 4.6. *The NNBTSs in $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, consisting 4-NOSs in both the NT is*

$$(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 4) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)+1}{2}.$$

Example 4.7. Let $\mathbb{X}^{NT} = \{u, v\}$ and $\mathbb{M}^{NT} = \{(0,1,1), (0.1, 0.3, 0.8), (1,0,0)\}$.
Then, $\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4) = 13$.
and the NNBTSs is

$$(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT})_{\mathbb{X}}^{NT}(2,3,4) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4)+1}{2} = 91.$$

These NBTSs are
$(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_1^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_2^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_3^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_4^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_7^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_8^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_9^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_{12}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_1^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_2^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_3^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_4^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_5^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_7^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_8^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_9^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_2^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_3^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_4^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_6^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_7^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_8^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_9^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_{10}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_3^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_4^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_7^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_8^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_9^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_{11}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_4^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_5^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_7^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_8^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_9^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_{12}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_5^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_6^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_7^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_8^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_9^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_6^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_7^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_8^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_9^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_{10}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_7^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_8^{NT}, \mathscr{T}_8^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_8^{NT}, \mathscr{T}_9^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_8^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_8^{NT}, \mathscr{T}_{11}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_8^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_8^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_9^{NT}, \mathscr{T}_9^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_9^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_9^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_9^{NT}, \mathscr{T}_{12}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_9^{NT}, \mathscr{T}_{13}^{NT})$,

$(\mathbb{X}^{NT}, \mathscr{T}_{10}^{NT}, \mathscr{T}_{10}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{10}^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{10}^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{10}^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{11}^{NT}, \mathscr{T}_{11}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{11}^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{11}^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{12}^{NT}, \mathscr{T}_{12}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{12}^{NT}, \mathscr{T}_{13}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{13}^{NT}, \mathscr{T}_{13}^{NT})$.

Result 4.8. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, the NNBTSs having 4-NOSs in both NTs without repetition is*

$$(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 4) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)}{2}.$$

Example 4.9. Following Example 3.11 and result 4.8, the number of NBTSs without repetition is $78 = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4)}{2} = \binom{13}{2}$.

5. Results on NNTRS

In this section, the NTRS having 3-NOS in three NTs and the NTRS having 3-NOS in three NTs without repetition means NTRS of the form $(\mathbb{X}^{NT}, \mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT}, \mathscr{T}_{k}^{NT})$ where $\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT}, \mathscr{T}_{k}^{NT}$ are identical or non-identical topologies and non-identical topologies having 3-NOS respectively. A similar meaning is used for 4-NOS.

Result 5.1. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$ the NNTRS consisting 2-NOSs in three NT is*

$$(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT}, \mathscr{T}_{k}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 2) = 1.$$

In this case NT with 2-NOSs is the indiscrete one i.e., $\mathscr{T}_{1}^{NT} = \{0^{NT}, 1^{NT}\}$. Therefore, NNTRS with 2-NOSs is exactly one, namely $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{1}^{NT})$.

Result 5.2. *The NNTRSs consisting 3-NOSs in all three NT in $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$ is*

$$(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT}, \mathscr{T}_{k}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 3) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},3)+2}{3}.$$

Example 5.3. Example 3.4 implies $(\mathscr{T}_{\mathbb{X}}^{NT}(2,3,3) = 7$.
Therefore, $(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT}, \mathscr{T}_{k}^{NT})_{\mathbb{X}}^{NT}(2,3,3) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(2,3,3)+2}{3} = \frac{9\times8\times7}{6} = 84$.

Result 5.4. *The NNTRSs consisting 3-NOSs in all three NT without repetition in $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$ is*

$$(\mathscr{T}_{i}^{NT}, \mathscr{T}_{j}^{NT}, \mathscr{T}_{k}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n}, \mathfrak{m}, 3) = \binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},3)}{3}.$$

Example 5.5. From Example 3.4, $\mathscr{T}_{\mathbb{X}}^{NT}(2,3,3) = 7$. In this case, the NTRSs having 3-NOSs in three NTs without repetition are

$(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{2}^{NT}, \mathscr{T}_{3}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{2}^{NT}, \mathscr{T}_{4}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{2}^{NT}, \mathscr{T}_{5}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{2}^{NT}, \mathscr{T}_{6}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{2}^{NT}, \mathscr{T}_{7}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{3}^{NT}, \mathscr{T}_{4}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{3}^{NT}, \mathscr{T}_{5}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{3}^{NT}, \mathscr{T}_{6}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{3}^{NT}, \mathscr{T}_{7}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{4}^{NT}, \mathscr{T}_{5}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{4}^{NT}, \mathscr{T}_{6}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{4}^{NT}, \mathscr{T}_{7}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{5}^{NT}, \mathscr{T}_{6}^{NT})$, $(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{5}^{NT}, \mathscr{T}_{7}^{NT})$,
$(\mathbb{X}^{NT}, \mathscr{T}_{1}^{NT}, \mathscr{T}_{6}^{NT}, \mathscr{T}_{7}^{NT})$,

$$(\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_3^{NT},\mathscr{T}_4^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_3^{NT},\mathscr{T}_5^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_3^{NT},\mathscr{T}_6^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_3^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_4^{NT},\mathscr{T}_5^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_4^{NT},\mathscr{T}_6^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_4^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_5^{NT},\mathscr{T}_6^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_5^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_2^{NT},\mathscr{T}_6^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_3^{NT},\mathscr{T}_4^{NT},\mathscr{T}_5^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_3^{NT},\mathscr{T}_4^{NT},\mathscr{T}_6^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_3^{NT},\mathscr{T}_4^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_3^{NT},\mathscr{T}_5^{NT},\mathscr{T}_6^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_3^{NT},\mathscr{T}_5^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_3^{NT},\mathscr{T}_6^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_4^{NT},\mathscr{T}_5^{NT},\mathscr{T}_6^{NT}),\ (\mathbb{X}^{NT},\mathscr{T}_4^{NT},\mathscr{T}_5^{NT},\mathscr{T}_7^{NT}),$$
$$(\mathbb{X}^{NT},\mathscr{T}_4^{NT},\mathscr{T}_6^{NT},\mathscr{T}_7^{NT}).$$
$$(\mathbb{X}^{NT},\mathscr{T}_5^{NT},\mathscr{T}_6^{NT},\mathscr{T}_7^{NT}).$$

Therefore, the NNTRSs consisting 3-NOSs in all three NTs without repetition is

$$(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(2,3,3)=35=\binom{\mathscr{T}_{\mathbb{X}}^{NT}(2,3,3)}{3}=\binom{7}{3}.$$

Result 5.6. $(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},3)=\frac{\mathfrak{m}^{\mathfrak{n}}}{3}(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},3).$

Example 5.7. From Example 4.3 and 5.3, we have,

$$(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT})_{\mathbb{X}}^{NT}(2,3,3)=28 \text{ and } (\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(2,3,3)=84.$$

Therefore $\frac{3^2}{3}\times(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT})_{\mathbb{X}}^{NT}(2,3,3)=\frac{3^2}{3}\times 28=84=(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(2,3,3).$

Result 5.8. *In $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$, the NNTRSs consisting 4-NOSs in three NTs is*

$$(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)=\binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)+2}{3}.$$

Example 5.9. Example 3.11 implies,

$$\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4)=13.$$

Then the NNTRS having 4-NOSs is

$$(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(2,3,4)=\binom{\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4)+2}{3}=\frac{13(13+1)(13+2)}{6}=455.$$

Result 5.10. *The NNTRSs consisting 4-NOSs in all three NT without repetition in $\mathscr{N}_{\mathbb{X}}^{\mathscr{T}}$ is*

$$(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)=\binom{\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)}{3}.$$

Example 5.11. From Example 3.11, $\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4)=13$. Following Example 5.5 and result 5.10, the NNTRSs consisting 4-NOSs in all three NT without repetition is $(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(2,3,4)=286.$

Result 5.12. $(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)=\frac{(\mathscr{T}_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4)+2)}{3}(\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT})_{\mathbb{X}}^{NT}(\mathfrak{n},\mathfrak{m},4).$

Example 5.13. From Examples 3.11, 4.7 and 5.9, we have

$$\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4)=13,\ (\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT})_{\mathbb{X}}^{NT}(2,3,4)=91 \text{ and } (\mathscr{T}_{\imath}^{NT},\mathscr{T}_{\jmath}^{NT},\mathscr{T}_{\Bbbk}^{NT})_{\mathbb{X}}^{NT}(2,3,4)=455.$$

Therefore,

$$\frac{(\mathscr{T}_{\mathbb{X}}^{NT}(2,3,4)+2)}{3}(\mathscr{T}_{i}^{NT},\mathscr{T}_{j}^{NT})_{\mathbb{X}}^{NT}(2,3,4)=\frac{13+2}{3}\times 91=455=(\mathscr{T}_{i}^{NT},\mathscr{T}_{j}^{NT},\mathscr{T}_{k}^{NT})_{\mathbb{X}}^{NT}(2,3,4).$$

6. Effective of the proposed method

The formula for giving the number of topologies $T(\mathfrak{n})$ is still not obtained for a finite set $\mathbb{X}$ having $\mathfrak{n}$ elements. If $\mathfrak{n}$ is small, then we can compute it by hand. But the difficulty increases when $\mathfrak{n}$ becomes large. Studying this particular area is also a highly valued part of the topology, and this is one of the fascinating and challenging research areas. Note that the explicit formula for finding the number of topologies is undetermined till now. This paper is towards the formulae for finding the number of neutrosophic topological spaces having $2,3,4$-open sets, the number of neutrosophic bitopological spaces, and tritopological spaces having the same number of open sets in topologies.

7. Conclusions

In this paper, the NNTSs consisting of small NOSs i.e., $2,3$, and 4-open sets are computed. Moreover, the NNBTSs and NNTRSs are computed. It is also observed that formulae for finding NNTSs, NNBTSs, and NNTRSs are interrelated. Hope this work will help in further study of NNTSs with greater open sets. In the future, the NNBTSs having k,l-open sets and the NNTRSs having k,l,m-open sets can be found where $k\neq l\neq m$. Moreover, we aim to extend our work to study the existence of NNTSs in the topological group.

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