Neutrosophic Sets and Systems, Vol. 30, 2019
University of New Mexico

Multi-Valued Interval Neutrosophic Soft Set: Formulation and Theory

Nor Liyana Amalini Mohd Kamal [1]*, Lazim Abdullah [2], Ilyani Abdullah [3], Shawkat Alkhazaleh [4] and Faruk Karaaslan [5]

[1] Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia; liyana.nini93@gmail.com
[2] Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia; lazim_m@umt.edu.my
[3] Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia; ilyani@umt.edu.my
[4] Department of Mathematics, Faculty of science, Zarqa University, Az Zarqa, Jordan; shmk79@gmail.com
[5] Department of Mathematics, Faculty of Sciences, Çankırı Karatekin University, 18100, Çankırı, Turkey; fkaraaslan@karatekin.edu.tr

* Correspondence: liyana.nini93@gmail.com

Abstract: Neutrosophic set is a powerful general formal framework. A lot of studies on neutrosophic had been proposed and recently, in multi-valued interval values. However, sometimes there is problem involving elements of ambiguity and uncertainties in which the function of membership is difficult to be set in a particular case. Clearly, these problems can be solved by soft set since it is able to solve the lack of parameterization tool of theory. Thus, this paper introduces a concept of multi-valued interval neutrosophic soft set which amalgamates multi-valued interval neutrosophic set and soft set. The proposed set extends the notions of fuzzy set, intuitionistic fuzzy set, neutrosophic set, interval-valued neutrosophic set, multi-valued neutrosophic set, soft set and neutrosophic soft set. Further, we study some basic operations such as complement, equality, inclusion, union, intersection, "AND" and "OR" for multi-valued interval neutrosophic soft elements and discuss its associated properties. Moreover, the derivation of its properties, related examples and some proofs on the propositions are included.

Keywords: multi-valued interval neutrosophic set; multi-valued interval neutrosophic soft set; neutrosophic set, soft set

1. Introduction

Fuzzy set (FS) was firstly initiated by Zadeh [1] in order to solve the decision-making problems with fuzzy information. However, FS only considers single membership function to represent vague data. Moreover, the membership degree alone is unable to describe the information in some cases of decision-making problems. Thus, Atanassov [2] introduced intuitionistic fuzzy set (IFS) in order to measure both membership degree and non-membership degree of elements in universal set. Then, the IFSs have been extended by many researchers and have been applied in some real applications. However, the membership and non-membership degrees values in IFSs are independent with the sum of degrees of membership and non-membership is less than unity. Moreover, it is unable to cope with the indefinite and inconsistent information which exist in belief system. Both FSs and IFSs may not deal with indeterminacy in real decision-making problem. Indeterminacy is an important part in decision-making process. For example, in a survey form, there are three choices 'YES / NO/ N. A.', while for gender, Male/ Female/ Others. So, different types of uncertainty and ambiguity with indeterminacy cannot be explained by the fuzzy concept or intuitionistic fuzzy concept. Thus, Smarandache [3] proposed the theory of neutrosophic set (NS) in 1995. The concept of NS which introduced by Smarandache [4] is a mathematical tool that handles the problems with inconsistent and imprecise data. It also has been proved that the NS is a continuation of the intuitionistic fuzzy sets [5]. An NS is represented by the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively, where $]^{-}0,1^{+}[$ is the non-standard interval. Basically, it is the generalization to the standard interval in the intuitionistic fuzzy sets [2] which is $[0,1]$. The uncertainty that represented by the indeterminacy factor is independent of truth and falsity values, while the integrated ambiguity is dependent of the degree of belongingness and the degree of non-belongingness in IFS. Nowadays, the studies on the NS theory have been developed actively [6]–[13]. However, since operators necessary to be specified, there is difficulty to apply NS in some real situations. Thus, Wang et al. [14] proposed single-valued neutrosophic set (SVNS) and since then, there are many researches related to SVNS have been conducted [9–18].

Despite its success, the truth-membership, indeterminacy-membership and falsity-membership in SVNS may not be written in one specific number for some cases. Thus, interval-valued neutrosophic set (IVNS) was introduced by Wang et al. [25], so that the values of truth-membership, indeterminacy-membership and falsity-membership are determined in intervals rather than real numbers. Also, IVNS may represent the indefinite, inaccurate, inadequate and inconsistent information which is always exist in real world. Numerous real world applications of IVNS have been studied by number of researchers [20–25]. In another perspective, the value of neutrosophic elements also not always be a single real number. Thus, Wang and Li [32] generalized SVNS into multi-valued neutrosophic set (MVNS), where the values of truth-membership, indeterminacy-membership and falsity-membership are represented in several real numbers rather than one single real number [27–30]. Nevertheless, in some complicated decision problems, several decision makers can refuse to give any evaluation values if they are unfamiliar with the characteristics of decision-making. Consequently, Broumi et al. [37] proposed multi-valued interval neutrosophic set (MVINS) in order to cope with complex decision problems which involving multiple decision makers and the evaluation values of decision makers are given in form of multi-valued interval neutrosophic values. Then, it has been discussed by other scholars such as Fan and Ye [38], Yang and Pang [39] and Samuel and Narmadhagnanam [40].

Apart from NS based sets, the soft set is just another set that can be used to deal with uncertain and vague information. Molodtsov [41] who is a Russian mathematician, had solved the difficult problem involving uncertainty by proposing a new mathematical tool called "soft set theory". This theory is free from the difficulties on how to set the function of membership in a particular case and inadequacy of parameterization tool of theory. After Molodtsov's work, the soft set (SS) theory has been studied widely in numerous applications, like lattices [36–38], topology [39–41], algebraic structures [42–46], game theory [47,48], medical diagnosis [55], perron integration [56], data analysis and operations research [51–54], optimization [61] and decision-making under uncertainty [56–59]. In recent years, SS theory has been extended by embedding the ideas of other sets. For example, Maji et al. [66] firstly integrated the beneficial properties of SS and FS. Theory of fuzzy soft set (FSS) has been studied by many scholars. For instance, Cagman et al. [67] defined the theory of fuzzy soft set (FSS) and studied the related properties. Roy and Maji [68] discussed some results on the implementation of FSS in solving the problem of object recognition. Kong et al. [69] gave a comment on Roy and Maji's paper [68], by providing a counter-example to show the problem. Then, Maji [70] studied the theory of NS which proposed by Smarandache [4] and combined it with soft set to become a novel mathematical model, which is called neutrosophic soft set (NSS). After the introduction of the NSS, Karaaslan [71] redefined the NSS notion and its operations to make it become more useful. The NSS has been applied to solve decision-making problem. Mukherjee and Sarkar [72] also discussed about NSSs. They solved a medical diagnosis decision-making problem based on the NSS. Şahin and Küçük [73] introduced a novel style of NSS notion and studied some algebraic properties. Sumathi and Arockiarani [74] also studied the NSSs. Cuong et al. [75] reanalyzed the notion of NSS and discussed the basic properties of NSS, neutrosophic soft relations and neutrosophic soft compositions. Hussain and Shabir [76] investigated the algebraic operations of NSS and the properties related to the operations. Mukherjee and Sarkar [77] defined new similarity measure and weighted similarity measure between two NSSs. Maji [78] verified some operations of weighted NSSs. Chatterjee et al. [79] studied the single-valued NSSs and some uncertainty based measures. Marei [80] proposed single valued neutrosophic soft approach to rough sets based on neutrosophic right minimal structure. Then, some scholars generalized the NSS into interval form by combining the IVNS with SS. This combination is known as interval-valued neutrosophic soft set (IVNSS) and it can deal with the problem in interval form with uncertainty. Deli [81] firstly introduced the definitions and operations of IVNSS and developed decision-making approach based on level soft sets of IVNSS. Mukherjee and Sarkar [82] defined Hamming and Euclidean distance for two IVNSSs. They also studied the similarity measure based on set theoretic approach. Broumi et al. [83] introduced the relations on IVNSS and presented the several properties such as symmetry, reflexivity and transitivity of the proposed relations. Another extension of NSS set has been done by some researchers to solve the problem in several real numbers with uncertainty. The multi-valued neutrosophic soft set (MVNSS) was proposed by Alkhazaleh [84]. A theoretical study on MVNSS properties and operations have been made and an MCDM approach based on the proposed set has been provided. Alkhazaleh and Hazaymeh [85] also discussed about the MVNSS and introduced an MCDM approach based on the set. It can be seen that there are a lot of researches that integrate the NS theory with SS theory. However, the NSS need to be specified from a point of view and since very little information of MVINS combines with NS is available in literatures, thus, we fill this gap by presenting a new set which integrate two existing concepts of MVINS introduced by Broumi et al. [37] and SS introduced by Molodtsov [41]. To accompaniment the concept of MVINSS, some basic operations for MVINSS which namely complement, union, intersection, equality, inclusion, "AND" and "OR" operations the proposed. The structure of this paper is listed as follows. In section 2, the related definitions and concepts for developing MVINSS are presented. Some proving on the propositions are included. Section 3 proposes the MVINSS and its associated properties together with example. Finally, we conclude the paper in section 4.

2. Preliminaries

In this section, we present some definitions and properties which are related to neutrosophic set, single-valued neutrosophic set, interval-valued neutrosophic set, multi-valued neutrosophic set, soft set and neutrosophic soft set.

2.1. Neutrosophic Set

Definition 2.1 [3] Let U be a universe of discourse, then NS A can be defined as
$A=\{<\tau_A(y), \delta_A(y), \lambda_A(y)>/ y, y\in U\}$
where $\tau, \delta, \lambda : U \to]^-0, 1^+[$ define the degree of truth-membership $\tau_A(y)$, degree of indeterminacy $\delta_A(y)$ and degree of falsity $\lambda_A(y)$ respectively and there is no restriction on the sum of $\tau_A(y), \delta_A(y)$ and $\lambda_A(y)$, so $^-0 \le \tau_A(y)+\delta_A(y)+\lambda_A(y) \le 3^+$.
From philosophical point of view, the NS takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. Thus for technical applications, we need to take the interval $[0, 1]$ instead of $]^-0, 1^+[$ because it is hard to apply in the real applications such as problems in scientific and engineering.

2.2. Single-Valued Neutrosophic Set

Definition 2.2 [14] Let U be a universal set, with generic element of U denoted by y. An SVNS A over U is defined as $A=\{<\tau_A(y),\delta_A(y),\lambda_A(y)>/y, y\in U\}$ It is characterized by a truth-membership function $\tau_A(y)$, indeterminacy-membership function $\delta_A(y)$ and falsity-membership function $\lambda_A(y)$, with for each $y\in U, \tau_A(y), \delta_A(y), \lambda_A(y)\in[0,1]$ and $0\le \tau_A(y)+\delta_A(y)+\lambda_A(y)\le 3.$

2.3. Interval-Valued Neutrosophic Set

Definition 2.3 [25] Let U be a space of points with generic elements in U denoted by y. An IVNS $\hat{A}$ over U is characterized by truth-membership interval $\hat{\tau}_{\hat{A}}(y)$, indeterminacy-membership interval $\hat{\delta}_{\hat{A}}(y)$ and falsity-membership interval $\hat{\lambda}_{\hat{A}}(y)$. It can be defined as

$$\hat{A}=\{<\hat{\tau}_{\hat{A}}(y),\hat{\delta}_{\hat{A}}(y),\hat{\lambda}_{\hat{A}}(y)>/y, y\in U\}$$

$\hat{\tau}_{\hat{A}}(y)=[\hat{\tau}^-_{\hat{A}}(y), \hat{\tau}^+_{\hat{A}}(y)], \hat{\delta}_{\hat{A}}(y)=[\hat{\delta}^-_{\hat{A}}(y), \hat{\delta}^+_{\hat{A}}(y)], \hat{\lambda}_{\hat{A}}(y)=[\hat{\lambda}^-_{\hat{A}}(y), \hat{\lambda}^+_{\hat{A}}(y)]\subseteq[0,1]$ and $0\le[\hat{\tau}^+_{\hat{A}}(y)+\hat{\delta}^+_{\hat{A}}(y)+\hat{\lambda}^+_{\hat{A}}(y)]\le 3, y\in U$. It only considers the subunitary interval of $[0,1]$.

2.4. Multi-Valued Neutrosophic Set

Definition 2.4 [32] Let U be a space of points (objects), with a generic element in U denoted by y.

An MVNS $\tilde{A}$ over U is characterized by $\tilde{A}=\{<\tilde{\tau}^l_{\tilde{A}}(y),\tilde{\delta}^m_{\tilde{A}}(y),\tilde{\lambda}^n_{\tilde{A}}(y)>/y, y\in U\}$

where $\tilde{\tau}^l_{\tilde{A}}(y)=\tilde{\tau}^1_{\tilde{A}}(y), \tilde{\tau}^2_{\tilde{A}}(y), \ldots, \tilde{\tau}^q_{\tilde{A}}(y)$, $\tilde{\delta}^m_{\tilde{A}}(y)=\tilde{\delta}^1_{\tilde{A}}(y), \tilde{\delta}^2_{\tilde{A}}(y), \ldots, \tilde{\delta}^r_{\tilde{A}}(y)$ and $\tilde{\lambda}^n_{\tilde{A}}(y)=\tilde{\lambda}^1_{\tilde{A}}(y), \tilde{\lambda}^2_{\tilde{A}}(y), \ldots, \tilde{\lambda}^s_{\tilde{A}}(y)$ are three sets in the form of subset of $[0,1]$, denoting the truth-membership sequence $\tilde{\tau}^l_{\tilde{A}}(y)$, indeterminacy-membership sequence $\tilde{\delta}^m_{\tilde{A}}(y)$ and falsity-membership sequence $\tilde{\lambda}^n_{\tilde{A}}(y)$ respectively, satisfying $0\le\tilde{\tau}^l_{\tilde{A}}(y), \tilde{\delta}^m_{\tilde{A}}(y), \tilde{\lambda}^n_{\tilde{A}}(y)\le 1$ and $0\le\tilde{\tau}^l_{\tilde{A}}(y), \tilde{\delta}^m_{\tilde{A}}(y), \tilde{\lambda}^n_{\tilde{A}}(y)\le 3$ for $l=1,2,\ldots,q,\ m=1,2,\ldots,r,\ n=1,2,\ldots,s$ for all $y\in U$. Also, l, m, n are called as the dimension of MVNS.
If U has only one element, then $\tilde{A}$ is called a multi-valued neutrosophic number (MVNN), denoted by $\tilde{A}=\left\langle\tilde{\tau}^l_{\tilde{A}}(y),\tilde{\delta}^m_{\tilde{A}}(y),\tilde{\lambda}^n_{\tilde{A}}(y)\right\rangle$. For convenience, an MVNN can be denoted by $\tilde{A}=\left\langle\tilde{\tau}^l_{\tilde{A}},\tilde{\delta}^m_{\tilde{A}},\tilde{\lambda}^n_{\tilde{A}}\right\rangle$.
The set of all MVNNs is represented as MVNS.

2.5. Multi-Valued Interval Neutrosophic Set

Definition 2.5 [37] Let U be a space of points (objects), with a generic element in U denoted by $y.$ An MVINS $\dddot{A}$ over U can be defined as

$$\dddot{A}=\{<\dddot{\tau}_{\dddot{A}}^{l}\leq(y),\dddot{\delta}_{\dddot{A}}^{m}(y),\dddot{\lambda}_{\dddot{A}}^{n}(y)>/y,\ y\in U\}$$

where

$\dddot{\tau}_{\dddot{A}}^{l}(y)$=$[\dddot{\tau}_{\dddot{A}}^{1\ -}(y),\ \dddot{\tau}_{\dddot{A}}^{1\ +}(y)],\ [\dddot{\tau}_{\dddot{A}}^{2\ -}(y),\ \dddot{\tau}_{\dddot{A}}^{2\ +}(y)],\ \ldots,\ [\dddot{\tau}_{\dddot{A}}^{q\ -}(y),\ \dddot{\tau}_{\dddot{A}}^{q\ +}(y)]$, $\dddot{\delta}_{\dddot{A}}^{m}(y)=[\dddot{\delta}_{\dddot{A}}^{1\ -}(y),\ \dddot{\delta}_{\dddot{A}}^{1\ +}(y)],\ [\dddot{\delta}_{\dddot{A}}^{2\ -}(y),\ \dddot{\delta}_{\dddot{A}}^{2\ +}(y)],\ \ldots,\ [\dddot{\delta}_{\dddot{A}}^{r\ -}(y),\ \dddot{\delta}_{\dddot{A}}^{r\ +}(y)]$, $\dddot{\lambda}_{\dddot{A}}^{n}(y)=[\dddot{\lambda}_{\dddot{A}}^{1\ -}(y),\ \dddot{\lambda}_{\dddot{A}}^{1\ +}(y)],\ [\dddot{\lambda}_{\dddot{A}}^{2\ -}(y),\ \dddot{\lambda}_{\dddot{A}}^{2\ +}(y)],\ldots,\ [\dddot{\lambda}_{\dddot{A}}^{s\ -}(y),\ \dddot{\lambda}_{\dddot{A}}^{s\ +}(y)]\ \in U\}$ such that $0\leq\tilde{\tau}_{\tilde{A}}^{l+}(y),\tilde{\delta}_{\tilde{A}}^{m+}(y),\tilde{\lambda}_{\tilde{A}}^{n+}(y)\leq 3,$ for all $l=1,2,\ldots,q,\ m=1,2,\ldots,r,\ n=1,2,\ldots,s.$

In this research, dimension of the interval truth-membership sequence $\dddot{\tau}_{\dddot{A}}^{l}(y),$ interval indeterminacy-membership sequence $\dddot{\delta}_{\dddot{A}}^{m}(y)$ and interval falsity-membership sequence $\dddot{\lambda}_{\dddot{A}}^{n}(y)$ of the element y are considered as equal that is $q=r=s,$ respectively. Also, l,m,n are called the dimension of MVINS $A.$ Obviously, when the values of upper and lower of $\dddot{\tau}_{\dddot{A}}^{l}(y),\dddot{\delta}_{\dddot{A}}^{m}(y),\dddot{\lambda}_{\dddot{A}}^{n}(y)$ are equal, then the MVINS is reduced to MVNS.

2.6. Soft Set

Definition 2.6 [41] Let U be an initial universe set and E be a set of parameters. Consider $A\subset E.$ Let $P(U)$ denotes the power SS of $U.$ A pair (L,A) is called a SS over U and the function L is a mapping defined by $L:A\rightarrow P(U)$ such that $L(\varepsilon)(y)=\phi$ if $y\notin U.$

Here, $L(\varepsilon)$ is called approximate function of the soft set $(L,A),$ and the value $L(\varepsilon)(y)$ is a set called x-element of the soft set for all $y\in U.$ The sets may be arbitrary, empty, or have non-empty intersection.

2.7. Neutrosophic Soft Set

Definition 2.7 [70]

Let U be an initial universe set and E be a set of parameters. Consider $A\subset E.$ Let $P(U)$ denotes the set of all NSS of $U.$ The collection (L,A) is called an NSS over U and the function $L(\varepsilon)$ is a mapping defined by $L:A\rightarrow P(U)$ such that $L(\varepsilon)(y)=\phi$ if $y\notin U.$

(L,A) is characterized by $\tau_{L(\varepsilon)}(y),\ \delta_{L(\varepsilon)}(y)$ and $\lambda_{(\varepsilon)}(y).$ in the form of subset of $[0,1]$ and here, $L(\varepsilon)$ is called approximate function of the NSS $(L,A),$ such that

$$(L,\ A)=\{<\tau_{L(\varepsilon)}(y),\ \delta_{L(\varepsilon)}(y),\ \lambda_{L(\varepsilon)}(y)>/y;\ \forall\varepsilon\in A,\ y\in U\}$$

where $\tau_{L(\varepsilon)}(y),\ \delta_{L(\varepsilon)}(y)$ and $\lambda_{L(\varepsilon)}(y)$ are the truth-membership, indeterminacy-membership and falsity-membership values of object y respectively that object y holds on parameter $\varepsilon.$

2.8. Interval-Valued Neutrosophic Soft Set

Definition 2.8 [81]

Let U be an initial universe set and E be a set of parameters. Consider $A\subset E.$ Let $P(U)$ denotes the set of all IVNSS of $U.$ The collection $(\hat{L},A)$ is called an IVNSS over U and the function $\hat{L}(\varepsilon)$ is a mapping defined by $\hat{L}:A\rightarrow P(U)$ such that $\hat{L}(\varepsilon)(y)=\phi$ if $y\notin U.$

$(\hat{L},A)$ is characterized by $\hat{\tau}_{\hat{L}(\varepsilon)}(y),\ \hat{\delta}_{\hat{L}(\varepsilon)}(y)$ and $\hat{\lambda}_{\hat{L}(\varepsilon)}(y)$ in the interval form of subset of $[0,1]$ and here, $\hat{L}(\varepsilon)$ is called approximate function of the IVNSS $(\hat{L},A),$ such that

$$(\hat{L},\ A)=\{<\hat{\tau}_{\hat{L}(\varepsilon)}(y),\ \hat{\delta}_{\hat{L}(\varepsilon)}(y),\ \hat{\lambda}_{\hat{L}(\varepsilon)}(y)>/y;\ \forall\varepsilon\in A,\ y\in U\}$$

where $\hat{\tau}_{\hat{L}(\varepsilon)}(y) = [\hat{\tau}_{\hat{L}(\varepsilon)}^{-}(y), \hat{\tau}_{\hat{L}(\varepsilon)}^{+}(y)]$, $\hat{\delta}_{\hat{L}(\varepsilon)}(y) = [\hat{\delta}_{\hat{L}(\varepsilon)}^{-}(y), \hat{\delta}_{\hat{L}(\varepsilon)}^{+}(y)]$ and $\hat{\lambda}_{\hat{L}(\varepsilon)}(y) = [\hat{\lambda}_{\hat{L}(\varepsilon)}^{-}(y), \hat{\lambda}_{\hat{L}(\varepsilon)}^{+}(y)]$ are the interval truth-membership, interval indeterminacy-membership and interval falsity-membership respectively that object y holds on parameter ε. s

2.9. Multi-Valued Neutrosophic Soft Sets

Definition 2.9 [86] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$. Let $P(U)$ denotes the set of all MVNSS of U. The collection $(\tilde{L}, A)$ is called an MVNSS over U and the function $\tilde{L}(\varepsilon)$ is a mapping defined by $\tilde{L} : A \to P(U)$ such that $\tilde{L}(\varepsilon)(y) = \phi$ if $y \notin U$.
$(\tilde{L}, A)$ is characterized by $\tilde{\tau}_{\tilde{L}(\varepsilon)}(y), \tilde{\delta}_{\tilde{L}(\varepsilon)}(y)$ and $\tilde{\lambda}_{\tilde{L}(\varepsilon)}(y)$ in the form of subset of $[0,1]$ and here, $\tilde{L}(\varepsilon)$ is called approximate function of the MVNSS $(\tilde{L}, A)$, such that

$$(\tilde{L}, A) = \{< \tilde{\tau}^{l}_{\tilde{L}(\varepsilon)}(y), \tilde{\delta}^{m}_{\tilde{L}(\varepsilon)}(y), \tilde{\lambda}^{n}_{\tilde{L}(\varepsilon)}(y) > / y; \forall \varepsilon \in A, y \in U\}$$

where $\tilde{\tau}^{l}_{\tilde{L}(\varepsilon)}(y) = \tilde{\tau}^{1}_{\tilde{L}(\varepsilon)}(y), \tilde{\tau}^{2}_{\tilde{L}(\varepsilon)}(y), \ldots, \tilde{\tau}^{q}_{\tilde{L}(\varepsilon)}(y)$, $\tilde{\delta}^{m}_{\tilde{L}(\varepsilon)}(y) = \tilde{\delta}^{1}_{\tilde{L}(\varepsilon)}(y), \tilde{\delta}^{2}_{\tilde{L}(\varepsilon)}(y), \ldots, \tilde{\delta}^{r}_{\tilde{L}(\varepsilon)}(y)$ and $\tilde{\lambda}^{n}_{\tilde{L}(\varepsilon)}(y) = \tilde{\lambda}^{1}_{\tilde{L}(\varepsilon)}(y), \tilde{\lambda}^{2}_{\tilde{L}(\varepsilon)}(y), \ldots, \tilde{\lambda}^{s}_{\tilde{L}(\varepsilon)}(y)$ are the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence respectively that object y holds on parameter ε.

3. Proposed Multi-Valued Interval Neutrosophic Soft Set

In this section, we propose the definition of a multi-valued interval neutrosophic soft set (MVINSS) and its basic operations such as complement, inclusion, equality, union, intersection, "AND" and "OR" are defined as follows.

Definition 3.1
The pair $(\dddot{L}, A)$ is called an MVINSS over $\ddot{P}(U)$, where $\dddot{L}$ is a mapping given by $\dddot{L} : A \to \ddot{P}(U)$. $\ddot{P}(U)$ denotes the set of all MVINSS of U with parameters from A and the function $\dddot{L}(\varepsilon)$ is a mapping defined by

$$\dddot{L} : A \to \ddot{P}(U) \text{ such that } \dddot{L}(\varepsilon)(y) = \phi \text{ if } y \notin U.$$

$(\dddot{L}, A)$ is characterized by $\dddot{\tau}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}_{\dddot{L}(\varepsilon)}(y)$ and $\dddot{\lambda}_{\dddot{L}(\varepsilon)}(y)$ in the form of subset of $[0,1]$ and can be defined as follows:

$$(\dddot{L}, A) = \left\{< \dddot{\tau}^{l}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}^{m}_{\dddot{L}(\varepsilon)}(y), \dddot{\lambda}^{n}_{\dddot{L}(\varepsilon)}(y) > / y; \forall \varepsilon \in A, y \in U\right\}$$

where

$\dddot{\tau}^{l}_{\dddot{L}(\varepsilon)}(y) = [\dddot{\tau}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\tau}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y)], [\dddot{\tau}^{2}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\tau}^{2}_{\dddot{L}(\varepsilon)}{}^{+}(y)], \ldots, [\dddot{\tau}^{q}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\tau}^{q}_{\dddot{L}(\varepsilon)}{}^{+}(y)]$, $\dddot{\delta}^{m}_{\dddot{L}(\varepsilon)}(y) = [\dddot{\delta}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\delta}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y)], [\dddot{\delta}^{2}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\delta}^{2}_{\dddot{L}(\varepsilon)}{}^{+}(y)], \ldots, [\dddot{\delta}^{r}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\delta}^{r}_{\dddot{L}(\varepsilon)}{}^{+}(y)]$ and $\dddot{\lambda}^{n}_{\dddot{L}(\varepsilon)}(y) = [\dddot{\lambda}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\lambda}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y)], [\dddot{\lambda}^{2}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\lambda}^{2}_{\dddot{L}(\varepsilon)}{}^{+}(y)], \ldots, [\dddot{\lambda}^{s}_{\dddot{L}(\varepsilon)}{}^{-}(y), \dddot{\lambda}^{s}_{\dddot{L}(\varepsilon)}{}^{+}(y)]$ are the interval truth-membership sequence, interval indeterminacy-membership sequence and interval falsity-membership sequence respectively that object y holds on parameter ε.

An example of an MVINSS is given as follows.

Example 3.1 Let $U = \{y_1, y_2, y_3\}$ be the set of laptops under consideration and A is a set of parameters which describes the attractiveness of the laptop. Consider $A = \{\varepsilon_1 = thin, \varepsilon_2 = light, \varepsilon_3 = cheap, \varepsilon_4 = large\}$. Define a mapping $\dddot{L} : A \to \ddot{P}(U)$ as

$$\ddot{L}(\varepsilon_1)=\left\{\frac{\langle([0.2,0.6],[0.1,0.3]),([0.3,0.5],[0.1,0.4]),([0.2,0.6],[0.4,0.8])\rangle}{y_1},\frac{\langle([0.1,0.3],[0.2,0.4]),([0.3,0.6],[0.4,0.8]),([0.3,0.5],[0.2,0.7])\rangle}{y_2},\frac{\langle([0.1,0.6],[0.2,0.7]),([0.2,0.5],[0.3,0.5]),([0.5,0.8],[0.3,0.8])\rangle}{y_3}\right\},$$

$$\ddot{L}(\varepsilon_2)=\left\{\frac{\langle([0.4,0.6],[0.2,0.5]),([0.2,0.6],[0.4,0.7]),([0.6,0.9],[0.5,0.8])\rangle}{y_1},\frac{\langle([0.3,0.6],[0.3,0.5]),([0.5,0.8],[0.5,0.7]),([0.4,0.8],[0.6,0.9])\rangle}{y_2},\frac{\langle([0.6,0.9],[0.3,0.6]),([0.1,0.4],[0.4,0.8]),([0.2,0.5],[0.7,0.9])\rangle}{y_3}\right\},$$

$$\ddot{L}(\varepsilon_3)=\left\{\frac{\langle([0.5,0.9],[0.1,0.4]),([0.2,0.4],[0.6,0.7]),([0.3,0.7],[0.2,0.5])\rangle}{y_1},\frac{\langle([0.6,0.9],[0.1,0.5]),([0.3,0.8],[0.5,0.8]),([0.2,0.6],[0.1,0.5])\rangle}{y_2},\frac{\langle([0.1,0.4],[0.1,0.5]),([0.6,0.8],[0.2,0.5]),([0.6,0.9],[0.6,0.8])\rangle}{y_3}\right\},$$

$$\ddot{L}(\varepsilon_4)=\left\{\frac{\langle([0.1,0.5],[0.2,0.5]),([0.2,0.5],[0.7,0.9]),([0.3,0.5],[0.1,0.5])\rangle}{y_1},\frac{\langle([0.2,0.6],[0.3,0.7]),([0.7,0.8],[0.2,0.5]),([0.1,0.6],[0.4,0.7])\rangle}{y_2},\frac{\langle([0.6,0.8],[0.6,0.7]),([0.3,0.6],[0.4,0.5]),([0.6,0.9],[0.2,0.4])\rangle}{y_3}\right\}.$$

Then, the multi-valued interval neutrosophic soft set $(\ddot{L},A)$ can be written as the following collection of approximations:

$$(\ddot{L}, A) = \left\{ \left(\varepsilon_1, \left\{ \frac{\langle ([0.2, 0.6],[0.1, 0.3]), ([0.3, 0.5],[0.1, 0.4]), ([0.2, 0.6],[0.4, 0.8]) \rangle}{y_1}, \frac{\langle ([0.1, 0.3],[0.2, 0.4]), ([0.3, 0.6],[0.4, 0.8]), ([0.3, 0.5],[0.2, 0.7]) \rangle}{y_2}, \frac{\langle ([0.1, 0.6],[0.2, 0.7]), ([0.2, 0.5],[0.3, 0.5]), ([0.5, 0.8],[0.3, 0.8]) \rangle}{y_3} \right\} \right), \right.$$

$$\left(\varepsilon_2, \left\{ \frac{\langle ([0.4, 0.6],[0.2, 0.5]), ([0.2, 0.6],[0.4, 0.7]), ([0.6, 0.9],[0.5, 0.8]) \rangle}{y_1}, \frac{\langle ([0.3, 0.6],[0.3, 0.5]), ([0.5, 0.8],[0.5, 0.7]), ([0.4, 0.8],[0.6, 0.9]) \rangle}{y_2}, \frac{\langle ([0.6, 0.9],[0.3, 0.6]), ([0.1, 0.4],[0.4, 0.8]), ([0.2, 0.5],[0.7, 0.9]) \rangle}{y_3} \right\} \right),$$

$$\left(\varepsilon_3, \left\{ \frac{\langle ([0.5, 0.9],[0.1, 0.4]), ([0.2, 0.4],[0.6, 0.7]), ([0.3, 0.7],[0.2, 0.5]) \rangle}{y_1}, \frac{\langle ([0.6, 0.9],[0.1, 0.5]), ([0.3, 0.8],[0.5, 0.8]), ([0.2, 0.6],[0.1, 0.5]) \rangle}{y_2}, \frac{\langle ([0.1, 0.4],[0.1, 0.5]), ([0.6, 0.8],[0.2, 0.5]), ([0.6, 0.9],[0.6, 0.8]) \rangle}{y_3} \right\} \right),$$

$$\left. \left(\varepsilon_4, \left\{ \frac{\langle ([0.1, 0.5],[0.2, 0.5]), ([0.2, 0.5],[0.7, 0.9]), ([0.3, 0.5],[0.1, 0.5]) \rangle}{y_1}, \frac{\langle ([0.2, 0.6],[0.3, 0.7]), ([0.7, 0.8],[0.2, 0.5]), ([0.1, 0.6],[0.4, 0.7]) \rangle}{y_2}, \frac{\langle ([0.6, 0.8],[0.6, 0.7]), ([0.3, 0.6],[0.4, 0.5]), ([0.6, 0.9],[0.2, 0.4]) \rangle}{y_3} \right\} \right) \right\}.$$

The MVINSS can be represented in tabular form. The entries are c_{ij} corresponding to the laptop y_i and the parameter ε_j where c_{ij} refers to interval truth-membership sequence of y_i interval The MVINSS can be represented in tabular form. The entries are indeterminacy-membership sequence of y_i, and interval falsity-membership sequence of y_i, in $\ddot{L}(\varepsilon_j)$.

The tabular representation of multi-valued interval neutrosophic soft set $(\ddot{L}, A)$ is as follow:

Table 1. The tabular representation of $(\ddot{L}, A)$

U	$\varepsilon_1 = thin$	$\varepsilon_2 = light$
y_1	$\langle ([0.2, 0.6],[0.1, 0.3]), ([0.3, 0.5],[0.1, 0.4]), ([0.2, 0.6],[0.4, 0.8]) \rangle$	$\langle ([0.4, 0.6],[0.2, 0.5]), ([0.2, 0.6],[0.4, 0.7]), ([0.6, 0.9],[0.5, 0.8]) \rangle$
y_2	$\langle ([0.1, 0.3],[0.2, 0.4]), ([0.3, 0.6],[0.4, 0.8]), ([0.3, 0.5],[0.2, 0.7]) \rangle$	$\langle ([0.3, 0.6],[0.3, 0.5]), ([0.5, 0.8],[0.5, 0.7]), ([0.4, 0.8],[0.6, 0.9]) \rangle$
y_3	$\langle ([0.1, 0.6],[0.2, 0.7]), ([0.2, 0.5],[0.3, 0.5]), ([0.5, 0.8],[0.3, 0.8]) \rangle$	$\langle ([0.6, 0.9],[0.3, 0.6]), ([0.1, 0.4],[0.4, 0.8]), ([0.2, 0.5],[0.7, 0.9]) \rangle$

U	$\varepsilon_3 = cheap$	$\varepsilon_4 = large$
y_1	$\langle ([0.5, 0.9],[0.1, 0.4]), ([0.2, 0.4],[0.6, 0.7]), ([0.3, 0.7],[0.2, 0.5]) \rangle$	$\langle ([0.6, 0.9],[0.1, 0.5]), ([0.3, 0.8],[0.5, 0.8]), ([0.2, 0.6],[0.1, 0.5]) \rangle$
y_2	$\langle ([0.1, 0.5],[0.2, 0.5]), ([0.2, 0.5],[0.7, 0.9]), ([0.3, 0.5],[0.1, 0.5]) \rangle$	$\langle ([0.2, 0.6],[0.3, 0.7]), ([0.7, 0.8],[0.2, 0.5]), ([0.1, 0.6],[0.4, 0.7]) \rangle$
y_3	$\langle ([0.1, 0.4],[0.1, 0.5]), ([0.6, 0.8],[0.2, 0.5]), ([0.6, 0.9],[0.6, 0.8]) \rangle$	$\langle ([0.6, 0.8],[0.6, 0.7]), ([0.3, 0.6],[0.4, 0.5]), ([0.6, 0.9],[0.2, 0.4]) \rangle$

Suppose $(\ddot{L},A)$ is a multi-valued interval neutrosophic soft set in $MVINSS(U)$ where $U=\{y_1, y_2, y_3\}$. The basic operations on MVINSS are given as follows:

We also define the complement operation for MVINSS and give an illustrative example.

Definition 3.2 The complement of a multi-valued interval neutrosophic soft set $(\ddot{L},A)$ is denoted by $(\ddot{L},A)^C$ and is defined as $(\ddot{L},A)^C=(\ddot{L}^C,A)$ where $\ddot{L}^C: A \rightarrow MVINSS(U)$ is a mapping given by $\ddot{L}^C(\varepsilon)=c(\ddot{L}(\varepsilon))$, so that $(\ddot{L},A)^c=\{<\ddot{\lambda}_{\ddot{L}(\varepsilon)}(y), 1-\ddot{\delta}_{\ddot{L}(\varepsilon)}(y), \ddot{\delta}(y)>/y;\ \forall\varepsilon\in A;\ y\in U\}.$

Example 3.2 Consider Example 3.1, then $(\ddot{L},A)^C$ is given by

$$(\ddot{\mathrm{L}},A)^C=\left\{\left(\varepsilon_1,\left\{\frac{\langle([0.2,0.6],[0.4,0.8]),([0.5,0.7],[0.6,0.9]),([0.2,0.6],[0.1,0.3])\rangle}{y_1},\frac{\langle([0.3,0.5],[0.2,0.7]),([0.4,0.7],[0.2,0.6]),([0.1,0.3],[0.2,0.4])\rangle}{y_2},\frac{\langle([0.5,0.8],[0.3,0.8]),([0.5,0.8],[0.5,0.7]),([0.1,0.6],[0.2,0.7])\rangle}{y_3}\right\}\right),\right.$$

$$\left(\varepsilon_2,\left\{\frac{\langle([0.6,0.9],[0.5,0.8]),([0.4,0.8],[0.3,0.6]),([0.4,0.6],[0.2,0.5])\rangle}{y_1},\frac{\langle([0.4,0.8],[0.6,0.9]),([0.2,0.5],[0.3,0.5]),([0.3,0.6],[0.3,0.5])\rangle}{y_2},\frac{\langle([0.2,0.5],[0.7,0.9]),([0.6,0.9],[0.2,0.6]),([0.6,0.9],[0.3,0.6])\rangle}{y_3}\right\}\right),$$

$$\left(\varepsilon_3,\left\{\frac{\langle([0.3,0.7],[0.2,0.5]),([0.6,0.8],[0.3,0.4]),([0.5,0.9],[0.1,0.4])\rangle}{y_1},\frac{\langle([0.2,0.6],[0.1,0.5]),([0.2,0.7],[0.2,0.5]),([0.6,0.9],[0.1,0.5])\rangle}{y_2},\frac{\langle([0.6,0.9],[0.6,0.8]),([0.2,0.4],[0.5,0.8]),([0.1,0.4],[0.1,0.5])\rangle}{y_3}\right\}\right),$$

$$\left.\left(\varepsilon_4,\left\{\frac{\langle([0.3,0.5],[0.1,0.5]),([0.5,0.8],[0.1,0.3]),([0.1,0.5],[0.2,0.5])\rangle}{y_1},\frac{\langle([0.1,0.6],[0.4,0.7]),([0.2,0.3],[0.5,0.8]),([0.2,0.6],[0.3,0.7])\rangle}{y_2},\frac{\langle([0.6,0.9],[0.2,0.4]),([0.4,0.7],[0.5,0.6]),([0.6,0.8],[0.6,0.7])\rangle}{y_3}\right\}\right)\right\}.$$

We will next define the subset hood of two MVINSS and give an illustrative example.

Definition 3.3 Let $(\ddot{L},A)$ and $(\ddot{M},B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. $(\ddot{L},A)$ is a multi-valued interval neutrosophic soft subset of $(\ddot{M},B)$ denoted by $(\ddot{L},A)\subseteq(\ddot{M},B)$ if and only if $A\subseteq B$ and $\forall\varepsilon\in A$, $\ddot{L}(\varepsilon)$ is a multi-valued interval neutrosophic soft subset of $\ddot{M}(\varepsilon)$.

Example 3.3 Consider Table 1 and $(\ddot{M},B)$ is another MVINSS over the common universe U. Let B be a set of parameters which describes the size of the laptops. Consider $B=\{\varepsilon_4=large,\ \varepsilon_5=small\}$ and given $(\ddot{M},B)$ is represented in tabular form as follows.

Table 2. The tabular representation of $(\ddot{\mathrm{M}}, B)$

U	$\varepsilon_4 = large$	$\varepsilon_5 = small$
y_1	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.5, 0.8], [0.5, 0.7]), ([0.4, 0.8], [0.6, 0.9])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.3, 0.5], [0.1, 0.4]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.5, 0.9], [0.1, 0.4]), ([0.2, 0.4], [0.6, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.7, 0.9]), ([0.3, 0.5], [0.1, 0.5])\rangle$

It is clear that $(\dddot{M}, B) \subseteq (\dddot{L}, A)$.

Definition 3.4 Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. $(\dddot{L}, A)$ is equal to $(\dddot{M}, B)$ denoted by $(\dddot{L}, A) = (\dddot{M}, B)$ if and only if $(\dddot{L}, A) \subseteq (\dddot{M}, B)$ and $(\dddot{M}, B) \subseteq (\dddot{L}, A)$.

In the following, we define the union of two NVSSs and give an illustrative example.

Definition 3.5 Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued neutrosophic soft sets over the common universe U. Then the union of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ is denoted by '$(\dddot{L}, A) \cup (\dddot{M}, B)$' and is defined by $(\dddot{L}, A) \cup (\dddot{M}, B) = (\dddot{N}, C)$ where $C = A \cup B$ and $(\dddot{N}, C) = \{< \dddot{\tau}^{l}_{\dddot{N}(\varepsilon)}(y), \dddot{\delta}^{m}_{\dddot{N}(\varepsilon)}(y), \dddot{\lambda}^{n}_{\dddot{N}(\varepsilon)}(y) > / y;\ y \in U\}$ such that

$$
\begin{array}{lll}
\dddot{\tau}^{l}_{\dddot{N}(\varepsilon)}(y) & = [\dddot{\tau}^{1-}_{\dddot{L}(\varepsilon)}(y), \dddot{\tau}^{1+}_{\dddot{L}(\varepsilon)}(y)], [\dddot{\tau}^{2-}_{\dddot{L}(\varepsilon)}(y), \dddot{\tau}^{2+}_{\dddot{L}(\varepsilon)}(y)], \ldots, [\dddot{\tau}^{q-}_{\dddot{L}(\varepsilon)}(y), \dddot{\tau}^{q+}_{\dddot{L}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A - B; \\
 & = [\dddot{\tau}^{1-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{1+}_{\dddot{M}(\varepsilon)}(y)], [\dddot{\tau}^{2-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{2+}_{\dddot{M}(\varepsilon)}(y)], \ldots, [\dddot{\tau}^{q-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{q+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in B - A; \\
 & = [\dddot{\tau}^{1-}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{1-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{1+}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{1+}_{\dddot{M}(\varepsilon)}(y)],, \ldots, [\dddot{\tau}^{q-}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{q-}_{\dddot{M}(\varepsilon)}(y), \dddot{\tau}^{q+}_{\dddot{L}(\varepsilon)}(y) \vee \dddot{\tau}^{q+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A \cap B; \\
\dddot{\delta}^{m}_{\dddot{N}(\varepsilon)}(y) & = [\dddot{\delta}^{1-}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}^{1+}_{\dddot{L}(\varepsilon)}(y)], [\dddot{\delta}^{2-}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}^{2+}_{\dddot{L}(\varepsilon)}(y)], \ldots, [\dddot{\delta}^{r-}_{\dddot{L}(\varepsilon)}(y), \dddot{\delta}^{r+}_{\dddot{L}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A - B; \\
 & = [\dddot{\delta}^{1-}_{\dddot{M}(\varepsilon)}(y), \dddot{\delta}^{1+}_{\dddot{M}(\varepsilon)}(y)], [\dddot{\delta}^{2-}_{\dddot{M}(\varepsilon)}(y), \dddot{\delta}^{2+}_{\dddot{M}(\varepsilon)}(y)], \ldots, [\dddot{\delta}^{r-}_{\dddot{M}(\varepsilon)}(y), \dddot{\delta}^{r+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in B - A; \\
 & = \dfrac{[\dddot{\delta}^{1-}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{1-}_{\dddot{M}(\varepsilon)}(y),}{2}, \dfrac{\dddot{\delta}^{1+}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{1+}_{\dddot{M}(\varepsilon)}(y)]}{2}, \ldots, \dfrac{[\dddot{\delta}^{r-}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{r-}_{\dddot{M}(\varepsilon)}(y),}{2}, \dfrac{\dddot{\delta}^{r+}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}^{r+}_{\dddot{M}(\varepsilon)}(y)]}{2}; & \text{if } \varepsilon \in A \cap B; \\
\dddot{\lambda}^{n}_{\dddot{N}(\varepsilon)}(y) & = [\dddot{\lambda}^{1-}_{\dddot{L}(\varepsilon)}(y), \dddot{\lambda}^{1+}_{\dddot{L}(\varepsilon)}(y)], [\dddot{\lambda}^{2-}_{\dddot{L}(\varepsilon)}(y), \dddot{\lambda}^{2+}_{\dddot{L}(\varepsilon)}(y)], \ldots, [\dddot{\lambda}^{s-}_{\dddot{L}(\varepsilon)}(y), \dddot{\lambda}^{s+}_{\dddot{L}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A - B; \\
 & = [\dddot{\lambda}^{1-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{1+}_{\dddot{M}(\varepsilon)}(y)], [\dddot{\lambda}^{2-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{2+}_{\dddot{M}(\varepsilon)}(y)], \ldots, [\dddot{\lambda}^{s-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{s+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in B - A; \\
 & = [\dddot{\lambda}^{1-}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{1-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{1+}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{1+}_{\dddot{M}(\varepsilon)}(y)],, \ldots, [\dddot{\lambda}^{s-}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{s-}_{\dddot{M}(\varepsilon)}(y), \dddot{\lambda}^{s+}_{\dddot{L}(\varepsilon)}(y) \wedge \dddot{\lambda}^{s+}_{\dddot{M}(\varepsilon)}(y)]; & \text{if } \varepsilon \in A \cap B;
\end{array}
$$

It can be simplified as:

$$
(\dddot{N}, C)(\varepsilon) = \begin{cases} \dddot{L}(\varepsilon) & \text{if } \varepsilon \in A - B; \\ \dddot{M}(\varepsilon) & \text{if } \varepsilon \in B - A; \\ \max(\dddot{\tau}_{\dddot{L}(\varepsilon)}(\mathrm{y}), \dddot{\tau}_{\dddot{M}(\varepsilon)}),\ \dfrac{\dddot{\delta}_{\dddot{L}(\varepsilon)}(y) + \dddot{\delta}_{\dddot{M}(\varepsilon)}(y)}{2},\ \min(\dddot{\lambda}_{\dddot{L}(\varepsilon)}(\mathrm{y}), \dddot{\lambda}_{\dddot{M}(\varepsilon)}) & \text{if } \varepsilon \in A \cap B. \end{cases}
$$

Refer to Example 3.3, the union of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ can be represented as follows.

Table 3. The union of $(\dddot{L}, A)$ and $(\dddot{M}, B)$

U	$\varepsilon_1 = thin$	$\varepsilon_2 = light$
y_1	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.3, 0.5], [0.1, 0.4]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.4, 0.6], [0.2, 0.5]), ([0.2, 0.6], [0.4, 0.7]), ([0.6, 0.9], [0.5, 0.8])\rangle$
y_2	$\langle([0.1, 0.3], [0.2, 0.4]), ([0.3, 0.6], [0.4, 0.8]), ([0.3, 0.5], [0.2, 0.7])\rangle$	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.5, 0.8], [0.5, 0.7]), ([0.4, 0.8], [0.6, 0.9])\rangle$
y_3	$\langle([0.1, 0.6], [0.2, 0.7]), ([0.2, 0.5], [0.3, 0.5]), ([0.5, 0.8], [0.3, 0.8])\rangle$	$\langle([0.6, 0.9], [0.3, 0.6]), ([0.1, 0.4], [0.4, 0.8]), ([0.2, 0.5], [0.7, 0.9])\rangle$

U	$\varepsilon_3 = cheap$	$\varepsilon_4 = large$
y_1	$\langle([0.5, 0.9], [0.1, 0.4]), ([0.2, 0.4], [0.6, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.6, 0.9], [0.3, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.7, 0.9]), ([0.3, 0.5], [0.1, 0.5])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.5, 0.65], [0.15, 0.45]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.1, 0.4], [0.1, 0.5]), ([0.6, 0.8], [0.2, 0.5]), ([0.6, 0.9], [0.6, 0.8])\rangle$	$\langle([0.6, 0.9], [0.6, 0.7]), ([0.25, 0.5], [0.5, 0.6]), ([0.3, 0.7], [0.2, 0.4])\rangle$

U	$\varepsilon_5 = small$
y_1	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.7, 0.9]), ([0.3, 0.5], [0.1, 0.5])\rangle$

Then, we present the definition of intersection operation and give an illustrative example.

Let $(\dddot{L}, A)$ and $(\dddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. Then the intersection of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ is denoted by '$(\dddot{L}, A) \cap (\dddot{M}, B)$' and is defined by $(\dddot{L}, A) \cap (\dddot{M}, B) = (\dddot{N}, C)$ where $C = A \cap B$ and $(\dddot{N}, C) = \{< \dddot{\tau}^{l}_{\dddot{N}(\varepsilon)}(y), \dddot{\delta}^{m}_{\dddot{N}(\varepsilon)}(y), \dddot{\lambda}^{n}_{\dddot{N}(\varepsilon)}(y) > / y;\ y \in U\}$ such that for every $\varepsilon \in C$,

$$\dddot{\tau}^{l}_{\dddot{N}(\varepsilon)}(y) = [\dddot{\tau}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y) \wedge \dddot{\tau}^{1}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\tau}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y) \wedge \dddot{\tau}^{1}_{\dddot{M}(\varepsilon)}{}^{+}(y)], \ldots, [\dddot{\tau}^{q}_{\dddot{L}(\varepsilon)}{}^{-}(y) \wedge \dddot{\tau}^{q}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\tau}^{q}_{\dddot{L}(\varepsilon)}{}^{+}(y) \wedge \dddot{\tau}^{q}_{\dddot{M}(\varepsilon)}{}^{+}(y)];$$

$$\dddot{\delta}^{m}_{\dddot{N}(\varepsilon)}(y) = \frac{[\dddot{\delta}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y) + \dddot{\delta}^{1}_{\dddot{M}(\varepsilon)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y) + \dddot{\delta}^{1}_{\dddot{M}(\varepsilon)}{}^{+}(y)]}{2}, \ldots, \frac{[\dddot{\delta}^{r}_{\dddot{L}(\varepsilon)}{}^{-}(y) + \dddot{\delta}^{r}_{\dddot{M}(\varepsilon)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{r}_{\dddot{L}(\varepsilon)}{}^{+}(y) + \dddot{\delta}^{r}_{\dddot{M}(\varepsilon)}{}^{+}(y)]}{2};$$

$$\dddot{\lambda}^{l}_{\dddot{N}(\varepsilon)}(y) = [\dddot{\lambda}^{1}_{\dddot{L}(\varepsilon)}{}^{-}(y) \vee \dddot{\lambda}^{1}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\lambda}^{1}_{\dddot{L}(\varepsilon)}{}^{+}(y) \vee \dddot{\lambda}^{1}_{\dddot{M}(\varepsilon)}{}^{+}(y)], \ldots, [\dddot{\lambda}^{s}_{\dddot{L}(\varepsilon)}{}^{-}(y) \vee \dddot{\lambda}^{s}_{\dddot{M}(\varepsilon)}{}^{-}(y),\ \dddot{\lambda}^{s}_{\dddot{L}(\varepsilon)}{}^{+}(y) \vee \dddot{\lambda}^{s}_{\dddot{M}(\varepsilon)}{}^{+}(y)];$$

Refer to Example 3.3, the intersection of $(\dddot{L}, A)$ and $(\dddot{M}, B)$ can be represented as follows.

Table 4. The intersection of $(\dddot{L}, A)$ and $(\dddot{M}, B)$

U	$\varepsilon_4 = large$
y_1	$\langle([0.3, 0.6], [0.1, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.4, 0.8], [0.6, 0.9])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.5, 0.65], [0.15, 0.45]), ([0.2, 0.6], [0.4, 0.8])\rangle$
y_3	$\langle([0.5, 0.8], [0.1, 0.4]), ([0.25, 0.5], [0.5, 0.6]), ([0.6, 0.9], [0.2, 0.5])\rangle$

Some properties of union and intersection are derived as follows.

Proposition 3.1

Idempotency Laws:

(1) $(\dddot{L}, A) \cup (\dddot{L}, A) = (\dddot{L}, A)$

(2) $(\tilde{F}, A) \cap (\tilde{F}, A) = (\tilde{F}, A)$.

Commutative Laws:

(3) $(\dddot{L}, A) \cup (\dddot{M}, B) = (\dddot{M}, B) \cup (\dddot{L}, A)$

(4) $(\dddot{L}, A) \cap (\dddot{M}, B) = (\dddot{M}, B) \cap (\dddot{L}, A)$

Proof 1

Let ε be an arbitrary element of $(\dddot{L}, A) \cup (\dddot{L}, A)$. Then, $\varepsilon \in (\dddot{L}, A)$ or $\varepsilon \in (\dddot{L}, A)$. Hence $\varepsilon \in (\dddot{L}, A)$. Thus, $(\dddot{L}, A) \cup (\dddot{L}, A) \subseteq (\dddot{L}, A)$. Conversely, if ε is an arbitrary element of $(\dddot{L}, A)$, then $\varepsilon \in (\dddot{L}, A) \cup (\dddot{L}, A)$ since it is in $(\dddot{L}, A)$. Therefore $(\dddot{L}, A) \subseteq (\dddot{L}, A) \cup (\dddot{L}, A)$.

$\therefore (\dddot{L}, A) \cup (\dddot{L}, A) = (\dddot{L}, A)$ □

Proof 2

Let ε be an arbitrary element of $(\ddot{L},A)\cap(\ddot{L},A)$. Then, $\varepsilon\in(\ddot{L},A)$ and $\varepsilon\in(\ddot{L},A)$. Hence $\varepsilon\in(\ddot{L},A)$. Thus, $(\ddot{L},A)\cap(\ddot{L},A)\subseteq(\ddot{L},A)$. Conversely, if $\varepsilon\in(\ddot{L},A)$ is arbitrary, then $\varepsilon\in(\ddot{L},A)$ and $\varepsilon\in(\ddot{L},A)$. Therefore $(\ddot{L},A)\subseteq(\ddot{L},A)\cap(\ddot{L},A)$. .

$\therefore(\ddot{L},A)\cap(\ddot{L},A)=(\ddot{L},A)$ □

Proof 3

Let ε is any element in $(\ddot{L},A)\cup(\ddot{M},B)$. Then, by definition of union, $\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in(\ddot{M},B)$. But, if ε is in $(\ddot{L},A)$ or $(\ddot{M},B)$, then it is in $(\ddot{M},B)$, or $(\ddot{L},A)$ and by definition of union, this means $\varepsilon\in(\ddot{L},A)\cup(\ddot{M},B)$. Therefore, $(\ddot{L},A)\cup(\ddot{M},B)\subseteq(\ddot{M},B)\cup(\ddot{L},A)$.

The other inclusion is identical. If ε is any element of $(\ddot{M},B)\cup(\ddot{L},A)$. Then, $\varepsilon\in(\ddot{M},B)$ or $\varepsilon\in(\ddot{L},A)$. But, $\varepsilon\in(\ddot{M},B)$ or $\varepsilon\in(\ddot{L},A)$. implies that ε is in $(\ddot{L},A)$ or $(\ddot{M},B)$. Hence, $\varepsilon\in(\ddot{M},\mathrm{B})\cup(\ddot{L},B)$. Therefore $(\ddot{M},B)\cup(\ddot{L},A)\subseteq(\ddot{L},A)\cup(\ddot{M},B)$.

$\therefore(\ddot{L},A)\cup(\ddot{M},B)=(\ddot{M},B)\cup(\ddot{L},A)$ □

Proof 4

Let ε is any element in $(\ddot{L},A)\cap(\ddot{M},B)$. Then, by definition of intersection, $\varepsilon\in(\ddot{L},A)$ and $\varepsilon\in(\ddot{M},B)$. Hence, $\varepsilon\in(\ddot{M},B)$. and $\varepsilon\in(\ddot{L},A)$. So, $\varepsilon\in(\ddot{M},B)\cap(\ddot{L},A)$. Therefore, $(\ddot{L},A)\cap(\ddot{M},B)\subseteq(\ddot{M},B)\cap(\ddot{L},A)$.
The reverse inclusion is again identical. If ε is any element of $(\ddot{M},B)\cap(\ddot{L},A)$. Then, $\varepsilon\in(\ddot{M},B)$. and $\varepsilon\in(\ddot{L},A)$. Hence, $\varepsilon\in(\ddot{L},A)$. and $\varepsilon\in(\ddot{M},B)$. This implies $\varepsilon\in(\ddot{L},A)\cap(\ddot{M},B)$. Therefore $(\ddot{M},B)\cap(\ddot{L},A)\subseteq(\ddot{L},A)\cap(\ddot{M},B)$.

$\therefore(\ddot{L},A)\cap(\ddot{M},B)=(\ddot{M},B)\cap(\ddot{L},A)$ □

For three multi-valued neutrosophic soft sets $(\ddot{L},A),(\ddot{M},B)$ and $(\ddot{N},C)$ over the common universe U, we have the following propositions:

Proposition 3.2

Associative Laws:

1. $(\ddot{L},A)\cup[(\ddot{M},B)\cup(\ddot{N},C)]=[(\ddot{L},A)\cup(\ddot{M},B)]\cup(\ddot{N},C)$.
2. $(\ddot{L},A)\cap[(\ddot{M},B)\cap(\ddot{N},C)]=[(\ddot{L},A)\cap(\ddot{M},B)]\cap(\ddot{N},C)$.

Distributive Laws:

3. $(\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]=[(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$.
4. $(\ddot{L},A)\cap[(\ddot{M},B)\cup(\ddot{N},C)]=[(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$.

Proof 1

Let $\varepsilon\in(\ddot{L},A)\cup[(\ddot{M},B)\cup(\ddot{N},C)]$. If $\varepsilon\in(\ddot{L},A)\cup[(\ddot{M},B)\cup(\ddot{N},C)]$,
then ε is either in $(\ddot{L},A)$ or in $[(\ddot{M},B)$ or $(\ddot{N},C)]$.

$\Rightarrow\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in[(\ddot{M},B)$ or $(\ddot{N},C)]$

$\Rightarrow\varepsilon\in(\ddot{L},A)$ or $\{\varepsilon\in(\ddot{M},B)$ or $\varepsilon\in(\ddot{N},C)\}$

$\Rightarrow\{\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in(\ddot{M},B)\}$ or $\{\varepsilon\in(\ddot{L},A)$ or $\varepsilon\in(\ddot{N},C)\}$

$\Rightarrow\varepsilon\in[(\ddot{L},A)$ or $(\ddot{M},B)]$ or $\varepsilon\in[(\ddot{L},A)$ or $(\ddot{N},C)]$

$\Rightarrow\varepsilon\in[(\ddot{L},A)\cup(\ddot{M},B)]\cup\ \varepsilon\in[(\ddot{L},A)\cup(\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$

Since $\exists \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$,

therefore $(\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)] \subseteq [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$,

then ε is in $[(\ddot{L},A)$ or $(\ddot{M},B)]$ or ε is in $[(\ddot{L},A)$ or $(\ddot{N},C)]$.

$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $(\ddot{M},B)]$ or $\varepsilon \in (\ddot{L},A)$ or $(\ddot{N},C)]$

$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{M},B)\}$ or $\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\{\varepsilon \in (\ddot{M},B)$ or $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\{\varepsilon \in [(\ddot{M},B)$ or $(\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cup \{\varepsilon \in [(\ddot{M},B) \cup (\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$

Since $\exists \varepsilon \in [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$ such that $\varepsilon \in (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$,

therefore $[(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)] \subseteq (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)]$.

$\therefore (\ddot{L},A) \cup [(\ddot{M},B) \cup (\ddot{N},C)] = [(\ddot{L},A) \cup (\ddot{M},B)] \cup [(\ddot{L},A) \cup (\ddot{N},C)]$ □

Proof 2

Let $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$. If $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$,

then ε is either in $(\ddot{L},A)$ and in $[(\ddot{M},B)$ and $(\ddot{N},C)]$.

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in [(\ddot{L},A)$ and $(\ddot{M},B)]$ and $\varepsilon \in [(\ddot{L},A)$ and $(\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap \varepsilon \in [(\ddot{L},A) \cap (\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$

$\Rightarrow \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$

Since $\exists \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$,

therefore $(\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)] \subseteq [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$,

then ε is in $[(\ddot{L},A)$ and $(\ddot{M},B)]$ and ε is in $[(\ddot{L},A)$ and $(\ddot{N},C)]$.

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $(\ddot{M},B)]$ and $\varepsilon \in (\ddot{L},A)$ and $(\ddot{N},C)]$

$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$

$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cap \{\varepsilon \in [(\ddot{M},B) \cap (\ddot{N},C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$

Since $\exists \varepsilon \in [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$ such that $\varepsilon \in (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$,

therefore $[(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)] \subseteq (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)]$.

$\therefore (\ddot{L},A) \cap [(\ddot{M},B) \cap (\ddot{N},C)] = [(\ddot{L},A) \cap (\ddot{M},B)] \cap [(\ddot{L},A) \cap (\ddot{N},C)]$ □

Proof 3

Let $\varepsilon \in (\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]$. If $\varepsilon \in (\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]$,
then ε is either in $(\ddot{L},A)$ or in $[(\ddot{M},B)$ and $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A)$ or $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $(\ddot{M},B)]$ and $\varepsilon \in [(\ddot{L},A)$ or $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A)\cup(\ddot{M},B)]\cap \varepsilon \in [(\ddot{L},A)\cup(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$
Since $\exists \varepsilon \in (\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$,
therefore $(\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]\subseteq[(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$,
then ε is in $[(\ddot{L},A)$ or $(\ddot{M},B)]$ and ε is in $[(\ddot{L},A)$ or $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $(\ddot{M},B)]$ and $\varepsilon \in [(\ddot{L},A)$ or $(\ddot{N},C)]$
$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{M},B)\}$ and $\{\varepsilon \in (\ddot{L},A)$ or $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $\{\varepsilon \in (\ddot{M},B)$ and $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ or $\{\varepsilon \in [(\ddot{M},B)$ and $(\ddot{N},C)]\}$
$\Rightarrow \varepsilon \in (\ddot{L},A)\cup\{\varepsilon \in [(\ddot{M},B)\cap(\ddot{N},C)]\}$
$\Rightarrow \varepsilon \in (\ddot{L},A)\cup(\ddot{M},B)\cap(\ddot{N},C)]$
Since $\exists \varepsilon \in [(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$ such that $\varepsilon \in (\ddot{L},A)\cup(\ddot{M},B)\cap(\ddot{N},C)]$,
therefore $[(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]\subseteq(\ddot{L},A)\cup(\ddot{M},B)\cap(\ddot{N},C)]$.

$\therefore (\ddot{L},A)\cup[(\ddot{M},B)\cap(\ddot{N},C)]=[(\ddot{L},A)\cup(\ddot{M},B)]\cap[(\ddot{L},A)\cup(\ddot{N},C)]$. □

Proof 4

Let $\varepsilon \in (\ddot{L},A)\cap[(\ddot{M},B)\cup(\ddot{N},C)]$. If $\varepsilon \in (\ddot{L},A)\cap[(\ddot{M},B)\cup(\ddot{N},C)]$,
then ε is in $(\ddot{L},A)$ and $[(\ddot{M},B)$ or $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\varepsilon \in [(\ddot{M},B)$ or $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A)$ and $\{\varepsilon \in (\ddot{M},B)$ or $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{M},B)\}$ or $\{\varepsilon \in (\ddot{L},A)$ and $\varepsilon \in (\ddot{N},C)\}$
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ and $(\ddot{M},B)]$ or $\varepsilon \in [(\ddot{L},A)$ and $(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A)\cap(\ddot{M},B)]\cup \varepsilon \in [(\ddot{L},A)\cap(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$
$\Rightarrow \varepsilon \in (\ddot{L},A)\cap[(\ddot{M},B)\cup(\ddot{N},C)]$
$\Rightarrow \varepsilon \in [(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$
Since $\exists \varepsilon \in (\ddot{L},A)\cap[(\ddot{M},B)\cup(\ddot{N},C)]$ such that $\varepsilon \in [(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$,
therefore $(\ddot{L},A)\cap[(\ddot{M},B)\cup(\ddot{N},C)]\subseteq[(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$.

Let $\varepsilon \in [(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$. If $\varepsilon \in [(\ddot{L},A)\cap(\ddot{M},B)]\cup[(\ddot{L},A)\cap(\ddot{N},C)]$,
then ε is in $[(\ddot{L},A)$ and $(\ddot{M},B)]$ or ε is in $[(\ddot{L},A)$ and $(\ddot{N},C)]$.
$\Rightarrow \varepsilon \in [(\ddot{L},A)$ and $(\ddot{M},B)]$ or $\varepsilon \in [(\ddot{L},A)$ and $(\ddot{N},C)]$

$\Rightarrow \{\varepsilon \in (\ddot{L}, A)$ and $\varepsilon \in (\ddot{M}, B)\}$ or $\{\varepsilon \in (\ddot{L}, A)$ and $\varepsilon \in (\ddot{N}, C)\}$

$\Rightarrow \varepsilon \in [(\ddot{L}, A)$ and $\{\varepsilon \in (\ddot{M}, B)$ or $\varepsilon \in (\ddot{N}, C)\}$

$\Rightarrow \varepsilon \in [(\ddot{L}, A)$ and $\{\varepsilon \in [(\ddot{M}, B)$ or $(\ddot{N}, C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L}, A) \cap \{\varepsilon \in [(\ddot{M}, B) \cup (\ddot{N}, C)]\}$

$\Rightarrow \varepsilon \in (\ddot{L}, A) \cap (\ddot{M}, B) \cup (\ddot{N}, C)]$

Since $\exists \varepsilon \in [(\ddot{L}, A) \cap (\ddot{M}, B)] \cup [(\ddot{L}, A) \cap (\ddot{N}, C)]$ such that $\varepsilon \in (\ddot{L}, A) \cap (\ddot{M}, B) \cup (\ddot{N}, C)]$,
therefore $[(\ddot{L}, A) \cap (\ddot{M}, B)] \cup [(\ddot{L}, A) \cap (\ddot{N}, C)] \subseteq (\ddot{L}, A) \cap (\ddot{M}, B) \cup (\ddot{N}, C)]$.

$\therefore (\ddot{L}, A) \cap [(\ddot{M}, B) \cup (\ddot{N}, C)] = [(\ddot{L}, A) \cap (\ddot{M}, B)] \cup [(\ddot{L}, A) \cap (\ddot{N}, C)]$ □

Then, we introduce the definition of 'AND' and 'OR' operations and give the illustrative example.

Definition 3.6

Let $(\ddot{L}, A)$ and $(\ddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. Then the 'AND' operation between $(\ddot{L}, A)$ and $(\ddot{M}, B)$ is denoted by $'(\ddot{L}, A) \wedge (\ddot{M}, B)'$ and is defined by $'(\ddot{L}, A) \wedge (\ddot{M}, B)' = (\ddot{N}, A \times B)$ where $(\ddot{N}, A \times B) = \{< \dddot{\tau}^{l}_{\ddot{N}(\alpha,\beta)}(y), \dddot{\delta}^{m}_{\ddot{N}(\alpha,\beta)}(y), \dddot{\lambda}^{n}_{\ddot{N}(\alpha,\beta)}(y) > / y; y \in U\}$ such that for every $\alpha \in A, \beta \in B, y \in U$.

$$\dddot{\tau}^{l}_{\ddot{N}(\alpha,\beta)}(y) = [\dddot{\tau}^{1}_{\ddot{L}(\alpha)}{}^{-}(y) \wedge \dddot{\tau}^{1}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\tau}^{1}_{\ddot{L}(\alpha)}{}^{+}(y) \wedge \dddot{\tau}^{1}_{\ddot{M}(\beta)}{}^{+}(y)], \ldots, [\dddot{\tau}^{q}_{\ddot{L}(\alpha)}{}^{-}(y) \wedge \dddot{\tau}^{q}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\tau}^{q}_{\ddot{L}(\alpha)}{}^{+}(y) \wedge \dddot{\tau}^{q}_{\ddot{M}(\beta)}{}^{+}(y)];$$

$$\dddot{\delta}^{m}_{\ddot{N}(\alpha,\beta)}(y) = \frac{[\dddot{\delta}^{1}_{\ddot{L}(\alpha)}{}^{-}(y) + \dddot{\delta}^{1}_{\ddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{1}_{\ddot{L}(\alpha)}{}^{+}(y) + \dddot{\delta}^{1}_{\ddot{M}(\beta)}{}^{+}(y)]}{2}, \ldots, \frac{[\dddot{\delta}^{r}_{\ddot{L}(\alpha)}{}^{-}(y) + \dddot{\delta}^{r}_{\ddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\dddot{\delta}^{r}_{\ddot{L}(\alpha)}{}^{+}(y) + \dddot{\delta}^{r}_{\ddot{M}(\beta)}{}^{+}(y)]}{2};$$

$$\dddot{\lambda}^{n}_{\ddot{N}(\alpha,\beta)}(y) = [\dddot{\lambda}^{1}_{\ddot{L}(\alpha)}{}^{-}(y) \vee \dddot{\lambda}^{1}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\lambda}^{1}_{\ddot{L}(\alpha)}{}^{+}(y) \vee \dddot{\lambda}^{1}_{\ddot{M}(\beta)}{}^{+}(y)], \ldots, [\dddot{\lambda}^{s}_{\ddot{L}(\alpha)}{}^{-}(y) \vee \dddot{\lambda}^{s}_{\ddot{M}(\beta)}{}^{-}(y), \dddot{\lambda}^{s}_{\ddot{L}(\alpha)}{}^{+}(y) \vee \dddot{\lambda}^{s}_{\ddot{M}(\beta)}{}^{+}(y)];$$

Refer to Example 3.3, the 'AND' operation of $(\ddot{L}, A)$ and $(\ddot{M}, B)$ can be represented as follows.

Table 5. The 'AND' operation of $(\ddot{L}, A)$ and $(\ddot{M}, B)$

U	(*thin*, *large*)	(*thin*, *small*)
y_1	$\langle$([0.3, 0.6], [0.1, 0.5]), ([0.4, 0.73], [0.4, 0.65]), ([0.4, 0.8], [0.6, 0.9])$\rangle$	$\langle$([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.7], [0.4, 0.7]), ([0.2, 0.6], [0.1, 0.5])$\rangle$
y_2	$\langle$([0.2, 0.6], [0.1, 0.3]), ([0.4, 0.6], [0.2, 0.53]), ([0.2, 0.6], [0.4, 0.8])$\rangle$	$\langle$([0.2, 0.6], [0.3, 0.7]), ([0.6, 0.75], [0.25, 0.58]), ([0.1, 0.6], [0.4, 0.7])$\rangle$
y_3	$\langle$([0.1, 0.6], [0.1, 0.4]), ([0.2, 0.45], [0.55, 0.7]), ([0.3, 0.7], [0.2, 0.5])$\rangle$	$\langle$([0.1, 0.5], [0.2, 0.5]), ([0.2, 0.5], [0.6, 0.8]), ([0.3, 0.5], [0.1, 0.5])$\rangle$

U	(*light*, *large*)	(*light*, *small*)
y_1	$\langle$([0.3, 0.6], [0.2, 0.5]), ([0.35, 0.7], [0.45, 0.7]), ([0.6, 0.9], [0.6, 0.9])$\rangle$	$\langle$([0.4, 0.6], [0.1, 0.5]), ([0.25, 0.7], [0.45, 0.75]), ([0.6, 0.9], [0.5, 0.8])$\rangle$
y_2	$\langle$([0.2, 0.6], [0.1, 0.3]), ([0.4, 0.65], [0.3, 0.55]), ([0.4, 0.8], [0.6, 0.9])$\rangle$	$\langle$([0.2, 0.6], [0.3, 0.5]), ([0.6, 0.8], [0.35, 0.6]), ([0.4, 0.8], [0.6, 0.9])$\rangle$
y_3	$\langle$([0.5, 0.9], [0.1, 0.4]), ([0.15, 0.4], [0.5, 0.75]), ([0.3, 0.7], [0.7, 0.9])$\rangle$	$\langle$([0.1, 0.5], [0.2, 0.5]), ([0.15, 0.45], [0.55, 0.85]), ([0.3, 0.5], [0.7, 0.9])$\rangle$

U	(*cheap*, *large*)	(*cheap*, *small*)
y_1	$\langle$([0.3, 0.6], [0.1, 0.4]), ([0.35, 0.6], [0.55, 0.7]), ([0.4, 0.8], [0.6, 0.9])$\rangle$	$\langle$([0.5, 0.9], [0.1, 0.4]), ([0.25, 0.6], [0.55, 0.75]), ([0.3, 0.7], [0.2, 0.5])$\rangle$
y_2	$\langle$([0.1, 0.5], [0.1, 0.3]), ([0.25, 0.5], [0.4, 0.65]), ([0.3, 0.6], [0.4, 0.8])$\rangle$	$\langle$([0.1, 0.5], [0.2, 0.5]), ([0.45, 0.65], [0.45, 0.7]), ([0.3, 0.6], [0.4, 0.7])$\rangle$
y_3	$\langle$([0.1, 0.4], [0.1, 0.4]), ([0.4, 0.6], [0.4, 0.6]), ([0.6, 0.9], [0.6, 0.8])$\rangle$	$\langle$([0.1, 0.4], [0.1, 0.5]), ([0.4, 0.65], [0.45, 0.7]), ([0.6, 0.9], [0.6, 0.8])$\rangle$

U	(*large*, *large*)	(*large*, *small*)
y_1	$\langle([0.3, 0.6], [0.1, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.4, 0.8], [0.6, 0.9])\rangle$	$\langle([0.6, 0.9], [0.6, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.1, 0.3]), ([0.5, 0.65], [0.15, 0.45]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.5, 0.8], [0.1, 0.4]), ([0.25, 0.5], [0.5, 0.6]), ([0.6, 0.9], [0.2, 0.5])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.25, 0.55], [0.55, 0.7]), ([0.6, 0.9], [0.2, 0.5])\rangle$

Definition 3.7 Let $(\ddot{L}, A)$ and $(\ddot{M}, B)$ be two multi-valued interval neutrosophic soft sets over the common universe U. Then, the 'OR' operation between $(\ddot{L}, A)$ and $(\ddot{M}, B)$ is denoted by $'(\ddot{L}, A)\vee(\ddot{M}, B)'$ and is defined by $(\ddot{L}, A)\vee(\ddot{M}, B) = (\ddot{N}, A\times B)$ where $(\ddot{N}, A\times B) = \{<\ddot{\tau}^{l}_{\ddot{N}(\alpha,\beta)}(y), \ddot{\delta}^{m}_{\ddot{N}(\alpha,\beta)}(y), \ddot{\lambda}^{n}_{\ddot{N}(\alpha,\beta)}(y) > / y; y\in U\}$ such that for every $\alpha\in A, \beta\in B, y\in Y$,

$$\ddot{\tau}^{l}_{\ddot{N}(\alpha,\beta)}(y) = [\ddot{\tau}^{1}_{\ddot{L}(\alpha)}{}^{-}(y)\vee\ddot{\tau}^{1}_{\ddot{M}(\beta)}{}^{-}(y), \ddot{\tau}^{1}_{\ddot{L}(\alpha)}{}^{+}(y)\vee\ddot{\tau}^{1}_{\ddot{M}(\beta)}{}^{+}(y)], \ldots, [\ddot{\tau}^{q}_{\ddot{L}(\alpha)}{}^{-}(y)\vee\ddot{\tau}^{q}_{\ddot{M}(\beta)}{}^{-}(y), \ddot{\tau}^{q}_{\ddot{L}(\alpha)}{}^{+}(y)\vee\ddot{\tau}^{q}_{\ddot{M}(\beta)}{}^{+}(y)];$$

$$\ddot{\delta}^{m}_{\ddot{N}(\alpha,\beta)}(y) = \frac{[\ddot{\delta}^{1}_{\ddot{L}(\alpha)}{}^{-}(y)+\ddot{\delta}^{1}_{\ddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\ddot{\delta}^{1}_{\ddot{L}(\alpha)}{}^{+}(y)+\ddot{\delta}^{1}_{\ddot{M}(\beta)}{}^{+}(y)]}{2}, \ldots, \frac{[\ddot{\delta}^{r}_{\ddot{L}(\alpha)}{}^{-}(y)+\ddot{\delta}^{r}_{\ddot{M}(\beta)}{}^{-}(y),}{2}, \frac{\ddot{\delta}^{r}_{\ddot{L}(\alpha)}{}^{+}(y)+\ddot{\delta}^{r}_{\ddot{M}(\beta)}{}^{+}(y)]}{2};$$

$$\ddot{\lambda}^{n}_{\ddot{N}(\alpha,\beta)}(y) = [\ddot{\lambda}^{1}_{\ddot{L}(\alpha)}{}^{-}(y)\wedge\ddot{\lambda}^{1}_{\ddot{M}(\beta)}{}^{-}(y), \ddot{\lambda}^{1}_{\ddot{L}(\alpha)}{}^{+}(y)\wedge\ddot{\lambda}^{1}_{\ddot{M}(\beta)}{}^{+}(y)], \ldots, [\ddot{\lambda}^{s}_{\ddot{L}(\alpha)}{}^{-}(y)\wedge\ddot{\lambda}^{s}_{\ddot{M}(\beta)}{}^{-}(y), \ddot{\lambda}^{s}_{\ddot{L}(\alpha)}{}^{+}(y)\wedge\ddot{\lambda}^{s}_{\ddot{M}(\beta)}{}^{+}(y)];$$

Refer to Example 3.3, the 'OR' operation of $(\ddot{L}, A)$ and $(\ddot{M}, B)$ can be represented as follows.

Table 6. The 'OR' operation of $(\ddot{L}, A)$ and $(\ddot{M}, B)$

U	(*thin*, *large*)	(*thin*, *small*)
y_1	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.4, 0.73], [0.4, 0.65]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.7], [0.4, 0.7]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.2, 0.4]), ([0.4, 0.6], [0.2, 0.53]), ([0.2, 0.5], [0.2, 0.7])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.6, 0.75], [0.25, 0.58]), ([0.1, 0.5], [0.2, 0.7])\rangle$
y_3	$\langle([0.5, 0.9], [0.2, 0.7]), ([0.2, 0.45], [0.55, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.1, 0.6], [0.2, 0.7]), ([0.2, 0.5], [0.6, 0.8]), ([0.3, 0.5], [0.1, 0.5])\rangle$

U	(*light*, *large*)	(*light*, *small*)
y_1	$\langle([0.4, 0.6], [0.3, 0.5]), ([0.35, 0.7], [0.45, 0.7]), ([0.4, 0.8], [0.5, 0.8])\rangle$	$\langle([0.6, 0.9], [0.2, 0.5]), ([0.25, 0.7], [0.45, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.3, 0.6], [0.3, 0.5]), ([0.4, 0.65], [0.3, 0.55]), ([0.2, 0.6], [0.4, 0.8])\rangle$	$\langle([0.3, 0.6], [0.3, 0.7]), ([0.6, 0.8], [0.35, 0.6]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.6, 0.9], [0.3, 0.6]), ([0.15, 0.4], [0.5, 0.75]), ([0.2, 0.5], [0.2, 0.5])\rangle$	$\langle([0.6, 0.9], [0.3, 0.6]), ([0.15, 0.45], [0.55, 0.85]), ([0.2, 0.5], [0.1, 0.5])\rangle$

U	(*cheap*, *large*)	(*cheap*, *small*)
y_1	$\langle([0.5, 0.9], [0.3, 0.5]), ([0.35, 0.6], [0.55, 0.7]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.25, 0.6], [0.55, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.2, 0.5]), ([0.25, 0.5], [0.4, 0.65]), ([0.2, 0.5], [0.1, 0.5])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.45, 0.65], [0.45, 0.7]), ([0.1, 0.5], [0.1, 0.5])\rangle$
y_3	$\langle([0.5, 0.9], [0.1, 0.5]), ([0.4, 0.6], [0.4, 0.6]), ([0.3, 0.7], [0.2, 0.5])\rangle$	$\langle([0.1, 0.5], [0.2, 0.5]), ([0.4, 0.65], [0.45, 0.7]), ([0.3, 0.5], [0.1, 0.5])\rangle$

U	(*large*, *large*)	(*large*, *small*)
y_1	$\langle([0.6, 0.9], [0.3, 0.5]), ([0.4, 0.8], [0.5, 0.75]), ([0.2, 0.6], [0.1, 0.5])\rangle$	$\langle([0.6, 0.9], [0.1, 0.5]), ([0.3, 0.8], [0.5, 0.8]), ([0.2, 0.6], [0.1, 0.5])\rangle$
y_2	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.5, 0.65], [0.15, 0.45]), ([0.1, 0.6], [0.4, 0.7])\rangle$	$\langle([0.2, 0.6], [0.3, 0.7]), ([0.7, 0.8], [0.2, 0.5]), ([0.1, 0.6], [0.4, 0.7])\rangle$
y_3	$\langle([0.6, 0.9], [0.6, 0.7]), ([0.25, 0.5], [0.5, 0.6]), ([0.3, 0.7], [0.2, 0.4])\rangle$	$\langle([0.6, 0.8], [0.6, 0.7]), ([0.25, 0.55], [0.55, 0.7]), ([0.3, 0.5], [0.1, 0.4])\rangle$

For three multi-valued interval neutrosophic soft sets $(\ddot{L}, A)$, $(\ddot{M}, B)$ and $(\ddot{N}, C)$ over the common universe, then De Morgan's Law are given as follows.

Preposition 3

(1) $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$

(2) $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C = [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$

(3) $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C$

(4) $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C = [(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C$

Proof 1

Let $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \wedge (\ddot{M}, B)$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$ such that $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$,

Therefore $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C \subseteq [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \wedge (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$,

Therefore $[(\ddot{L}, A) \wedge (\ddot{M}, B)]^C \subseteq (\ddot{L}, A)^C \vee (\ddot{M}, B)^C$.

$\therefore (\ddot{L}, A)^C \vee (\ddot{M}, B)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B)]^C$ □

Proof 2

Let $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \vee (\ddot{M}, B)$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$ such that $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$,

Therefore $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \subseteq [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A) \vee (\ddot{M}, B)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$,

Therefore $[(\ddot{L}, A) \vee (\ddot{M}, B)]^C \subseteq (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C$.

$\therefore (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C = [(\ddot{L}, A) \vee (\ddot{M}, B)]^C$ □

Proof 3

Let $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$ or $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A)]$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$ such that $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$,

Therefore $(\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C \sqsubseteq [(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \wedge (\ddot{M}, A)]$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ or $\varepsilon \notin (\ddot{M}, B)$ or $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ or $\varepsilon \in (\ddot{M}, B)^C$ or $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \wedge (\ddot{M}, A) \wedge (\ddot{N}, C)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$,

Therefore $[(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C \sqsubseteq (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C$.

$\therefore (\ddot{L}, A)^C \vee (\ddot{M}, B)^C \vee (\ddot{N}, C)^C = [(\ddot{L}, A) \wedge (\ddot{M}, B) \wedge (\ddot{N}, C)]^C$ □

Proof 4

Let $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$ and $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A)]$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]$

$\Rightarrow \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$

Since $\exists \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$ such that $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$,

Therefore $(\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C \sqsubseteq [(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C$.

Then consider $\varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]$

$\Rightarrow \varepsilon \notin [(\ddot{L}, A) \vee (\ddot{M}, A)]$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \notin (\ddot{L}, A)$ and $\varepsilon \notin (\ddot{M}, B)$ and $\varepsilon \notin (\ddot{N}, C)$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C$ and $\varepsilon \in (\ddot{M}, B)^C$ and $\varepsilon \in (\ddot{N}, C)^C$

$\Rightarrow \varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$

Since $\exists \varepsilon \in [(\ddot{L}, A) \vee (\ddot{M}, A) \vee (\ddot{N}, C)]^C$ such that $\varepsilon \in (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$,

Therefore $[(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C \sqsubseteq (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C$.

$\therefore (\ddot{L}, A)^C \wedge (\ddot{M}, B)^C \wedge (\ddot{N}, C)^C = [(\ddot{L}, A) \vee (\ddot{M}, B) \vee (\ddot{N}, C)]^C$ □

The definition of MVINSS, its arithmetic operations and properties would provide a good insight in mining a new knowledge of NS.

4. Conclusions

In this paper, the concept of multi-valued interval neutrosophic soft set (MVINSS) has been successfully proposed by integrating the multi-valued interval neutrosophic set and soft set. It is already known that neutrosophic soft set considers the indeterminate and inconsistent information. But the proposed set was introduced to improve the result in decision-making problem with multi-valued interval neutrosophic soft elements. The proposed set has several significant features. Firstly, it emphasized the hesitant, indeterminate and uncertainty and can be used more practical to solve decision-making problem. Secondly, some basic properties of MVINSS such as complement, equality, inclusion, union, intersection, "AND" and "OR" were well defined. The propositions related to the proposed properties were mathematically proven and some examples were provided. For future work, this novel proposed set can be applied and utilized in solving supply chain, time series forecasting and decision-making problem such as partner selection, wastewater treatment selection and renewable energy selection.

Funding: This research was funded by Fundamental Research Grant Scheme (FRGS), Malaysian Ministry of Higher Education, grant number FRGS/2018/59522.

Acknowledgments: The authors would like to extend a deep appreciation to the Universiti Malaysia Terengganu for providing financial support under the Fundamental Research Grant Scheme (FRGS), Malaysian Ministry of Higher Education with vote number FRGS/2018/59522.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, pp. 338–353, 1965.
2. K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, pp. 87–96, 1986.
3. F. Smarandache, "Neutrosophic set – A generalization of the intuitionistic fuzzy set," *Neutrosophic Probab. Set, Logic. Amer. Res.*, pp. 1–15, 1995.
4. F. Smarandache, *A unifying field in logics: Neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability*. 1999.
5. F. Smarandache, "Neutrosophic set - A generalization of the intuitionistic fuzzy set," *Int. J. Pure Appl. Math.*, vol. 24, no. 3, pp. 287–297, 2005.
6. F. Smarandache and S. K. Samanta, "On similarity and entropy of neutrosophic sets," *J. Intell. Fuzzy Syst.*, vol. 26, pp. 1245–1252, 2013.
7. W. Jiang, Y. Zhong, and X. Deng, "A neutrosophic set based fault diagnosis method based on multi-stage fault template data," *Symmetry (Basel).*, vol. 10, no. 346, pp. 1–16, 2018.
8. P. Majumdar and S. K. Samanta, "On similarity and entropy of neutrosophic sets," *J. Intell. Fuzzy Syst.*, pp. 1245–1252, 2014.
9. Abdel-Basset, M., Manogaran, G., Gamal, A., & Chang, V. (2019). A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT. IEEE Internet of Things Journal.
10. M. Abdel-Basset, N. A. Nabeeh, H. A. El-Ghareeb, and A. Aboelfetouh, "Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises," *Enterp. Inf. Syst.*, pp. 1–21, 2019.
11. N. A. Nabeeh, M. Abdel-Basset, H. A. El-Ghareeb, and A. Aboelfetouh, "Neutrosophic Multi-Criteria Decision Making Approach for IoT-Based Enterprises," *IEEE Access*, pp. 1–19, 2019.
12. M. Abdel-Basset, G. Manogaran, A. Gamal, and F. Smarandache, "A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection," *J. Med. Syst.*, vol. 43, no. 2, 2019.
13. M. Abdel-Basset, M. Saleh, A. Gamal, and F. Smarandache, "An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number," *Appl. Soft Comput. J.*, vol. 77, pp. 438–452, 2019.
14. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Tech.*

Sci. Appl. Math., 1995.

15. P. Biswas, S. Pramanik, and B. C. Giri, "TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment," *Neural Comput. Appl.*, pp. 727–737, 2016.
16. X. Peng and J. Dai, "Approaches to single-valued neutrosophic MADM based on MABAC , TOPSIS and new similarity measure with score function," *Neural Comput. Appl.*, 2016.
17. A. Aydo, "On similarity and entropy of single valued neutrosophic sets," vol. 29, no. 1, pp. 67–74, 2015.
18. S. Zhao, D. Wang, C. Liang, Y. Leng, and J. Xu, "Some single-valued neutrosophic power Heronian aggregation operators and their application to multiple-attribute," *Symm*, vol. 11, p. 653, 2019.
19. P. Biswas, S. Pramanik, and B. C. Giri, "Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments," *Neutrosophic Sets Syst.*, vol. 2, pp. 105–113, 2014.
20. W. Jiang and Y. Shou, "A novel single-valued neutrosophic set similarity measure and its application in multicriteria decision-making," *Symmetry (Basel).*, vol. 9, no. 127, pp. 1–14, 2017.
21. A. Awang, A. T. A. Ghani, L. Abdullah, and M. F. Ahmad, "A DEMATEL method with single valued neutrosophic set (SVNS) in identifying the key contribution factors of Setiu Wetland's coastal erosion," *AIP Conf. Proc.*, vol. 1974, 2018.
22. R. Şahin and A. Küçük, "Subsethood measure for single valued neutrosophic sets," *J. Intell. Fuzzy Syst.*, vol. 29, no. 2, pp. 525–530, 2015.
23. R. Tan, W. Zhang, and S. Chen, "Some generalized single valued neutrosophic linguistic operators and their application to multiple attribute group decision making," *J. Syst. Sci. Inf.*, vol. 5, no. 2, pp. 148–162, 2017.
24. D.-S. Xu, C. Wei, and G.-W. Wei, "TODIM method for single-valued neutrosophic multiple attribute decision making," *Information*, vol. 8, no. 4, p. 125, 2017.
25. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Interval neutrosophic sets and logic : Theory and applications in computing," no. January, 2005.
26. R. Bausys and E. K. Zavadskas, "Multicriteria decision making approach by VIKOR under interval neutrosophic set environment," *Econ. Comput. Econ. Cybern. Stud. Res.*, vol. 49, no. 4, pp. 33–48, 2015.
27. Y. H. Huang, G. W. Wei, and C. Wei, "VIKOR method for interval neutrosophic multiple attribute group decision-making," *Inf.*, vol. 8, no. 4, pp. 1–10, 2017.
28. A. Aydoğdu, "On entropy and similarity measure of interval valued neutrosophic sets," *Neutrosophic Sets Syst.*, vol. 9, no. September 2015, pp. 47–49, 2015.
29. Z. Tian, H. Zhang, J. Wang, J. Wang, and X. Chen, "Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets," *Int. J. Syst. Sci. ISSN*, vol. 47, no. 15, pp. 3598–3608, 2016.
30. J. Ye, "Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making," *J. Intell. Fuzzy Syst.*, vol. 26, no. 1, pp. 165–172, 2014.
31. H. Yang, X. Wang, and K. Qin, "New similarity and entropy measures of interval neutrosophic sets with applications in multi-attribute decision-making," *Symmetry (Basel).*, vol. 11, no. 370, 2019.
32. J. Q. Wang and X. E. Li, "TODIM method with multi-valued neutrosophic sets," *Control Decis.*, vol. 30, no. 6, 2015.
33. P. Liu, L. Zhang, X. Liu, and P. Wang, "Multi-valued neutrosophic number Bonferroni mean operators with their applications in multiple attribute group decision-making," *Int. J. Inf. Technol. Decis. Mak.*, vol. 15, no. 5, pp. 1181–1210, 2016.
34. H. Peng, H. Zhang, and J. Wang, "Probability multi-valued neutrosophic sets and its application in multi-criteria group decision-making problems," *Neural Comput. Appl.*, 2016.
35. I. Deli, S. Broumi, and F. Smarandache, "On neutrosophic refined sets and their applications in medical diagnosis," *J. New Theory*, no. September, pp. 88–98, 2015.
36. P. Ji, H. Zhang, and J. Wang, "A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection," *Neural Comput. Appl.*, 2016.
37. S. Broumi, I. Deli, and F. Smarandache, "N-valued interval neutrosophic sets and their application in medical diagnosis," *Crit. Rev.*, vol. 10, pp. 45–69, 2015.
38. C. Fan and J. Ye, "The cosine measure of refined-single valued neutrosophic sets and refined-interval neutrosophic sets for multiple attribute decision-making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 4, pp. 2281–2289, 2017.
39. W. Yang and Y. Pang, "New multiple attribute decision making method based on DEMATEL and TOPSIS for multi-valued interval neutrosophic sets," *Symmetry (Basel).*, vol. 10, no. 115, 2018.
40. A. E. Samuel and R. Narmadhagnanam, "Execution of n-valued interval neutrosophic sets in medical

diagnosis," *Int. J. Math. Trends Technol.*, vol. 58, no. 1, pp. 66–70, 2018.
41. D. Molodtsov, "Soft set theory first results," *An Int. J. - Comput. Math. with Appl.*, vol. 37, pp. 19–31, 1999.
42. Ş. Yilmaz and O. Kazanci, "Soft lattices(ideals, filters) related to fuzzy point," *UPB Sci. Bull. Ser. A Appl. Math. Phys.*, vol. 75, no. 3, pp. 75–90, 2013.
43. E. K. R. Nagarajan and G. Meenambigai, "An application of soft sets to lattices," *Kragujev. J. Math.*, vol. 35, no. 1, pp. 75–87, 2011.
44. F. Karaaslan, N. Çağman, and S. Enginoğlu, "Soft lattices," *J. New Results Sci.*, vol. 1, no. January 2014, pp. 5–17, 2012.
45. N. Cagman, S. Karatas, and S. Enginoglu, "Soft topology," *Comput. Math. with Appl.*, vol. 62, pp. 351–358, 2011.
46. M. Shabir and M. Naz, "On soft topological spaces," *Comput. Math. with Appl.*, vol. 61, pp. 1786–1799, 2011.
47. W. K. Min, "A note on soft topological spaces," *Comput. Math. with Appl.*, vol. 62, pp. 3524–3528, 2011.
48. U. Acar, F. Koyuncu, and B. Tanay, "Soft sets and soft rings," *Comput. Math. with Appl.*, vol. 59, no. 11, pp. 3458–3463, 2010.
49. H. Aktas and N. Cagman, "Soft sets and soft groups," *Inf. Sci. (Ny).*, vol. 177, pp. 2726–2735, 2007.
50. A. Aygünoğlu and H. Aygün, "Introduction to fuzzy soft groups," *Comput. Math. with Appl.*, vol. 58, no. 6, pp. 1279–1286, 2009.
51. Y. B. Jun, "Soft BCK/BCI-Algebras," *Comput. Math. with Appl.*, vol. 178, no. 11, pp. 2466–2475, 2008.
52. Y. B. Jun and S. S. Ahn, "Applications of soft sets in BE-Algebras," *Algebra*, vol. 2013, pp. 1–8, 2013.
53. I. Deli and N. Cagman, "Application of soft sets in decision making based on game theory," *Act. Strateg. Using Drama Theory*, pp. 65–93, 2015.
54. I. Deli and N. Çağman, "Fuzzy soft games," *Filomat*, vol. 29, no. 9, pp. 1901–1917, 2015.
55. S. Yuksel, T. Dizman, G. Yildizdan, and U. Sert, "Application of soft sets to diagnose the prostate cancer risk," *J. Inequalities Appl.*, vol. 2013, no. 229, pp. 1–11, 2013.
56. E. J. McShane, "On Perron integration," *Bull. Am. Math. Soc.*, vol. 48, no. 10, pp. 718–727, 1942.
57. X. Ge and S. Yang, "Investigations on some operations of soft sets," *World Acad. Sci. Eng. Technol.*, vol. 51, no. 3, pp. 1112–1115, 2011.
58. P. Zhu and Q. Wen, "Operations on soft sets revisited," *J. Appl. Math.*, vol. 2013, pp. 1–7, 2013.
59. M. I. Ali, F. Feng, X. Liu, W. Keun, and M. Shabir, "On some new operations in soft set theory," *Comput. Math. with Appl.*, vol. 57, no. 9, pp. 1547–1553, 2009.
60. D. Molodtsov, "The theory of soft sets," *URSS Publ. Moscow*, no. in Russian, 2004.
61. D. V. Kovkov, V. M. Kolbanov, and D. A. Molodtsov, "Soft sets theory-based optimization," *J. Comput. Syst. Sci. Int.*, vol. 46, no. 6, pp. 872–880, 2007.
62. A. Kharal, "Distance and similarity measures for soft sets," *New Math. Nat. Comput.*, vol. 06, no. 03, pp. 321–334, 2010.
63. P. K. Maji and A. R. Roy, "An application of soft sets in a decision making problem," *Comput. Math. with Appl.*, vol. 1221, no. 02, pp. 1077–1083, 2002.
64. N. Cagman and S. Enginog, "Soft set theory and uni–int decision making," *Eur. J. Oper. Res.*, vol. 207, pp. 848–855, 2010.
65. N. Çağman and I. Deli, "Means of FP-soft sets and their applications," *Hacettepe J. Math. Stat.*, vol. 41, no. 5, pp. 615–625, 2012.
66. P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 589–602, 2001.
67. N. Cagman, S. Enginoglu, and F. Citak, "Fuzzy soft set theory and its applications," *Iran. J. Fuzzy Syst.*, vol. 8, no. 3, pp. 137–147, 2011.
68. A. R. Roy and P. K. Maji, "A fuzzy soft set theoretic approach to decision making problems," *J. Comput. Appl. Math.*, vol. 203, pp. 412–418, 2007.
69. Z. Kong, L. Gao, and L. Wang, "Comment on 'A fuzzy soft set theoretic approach to decision making problems,'" *J. Appl. Math.*, vol. 223, no. 2, pp. 540–542, 2009.
70. P. K. Maji, "Neutrosophic soft set," *Ann. Fuzzy Math. Informatics*, vol. 5, no. 1, pp. 157–168, 2013.
71. F. Karaaslan, "Neutrosophic soft sets with applications in decision making," *Int. J. Inf. Sci. Intell. Syst.*, vol. 4, no. 2, pp. 1–20, 2015.
72. A. Mukherjee and S. Sarkar, "Several similarity measures of neutrosophic soft sets and its application in real life problems," *Ann. Pure Appl. Math.*, vol. 7, no. 1, pp. 1–6, 2014.
73. R. Şahin and A. Küçük, "On similarity and entropy of neutrosophic soft sets," *J. Intell. Fuzzy Syst.*, vol. 27, no. 5, pp. 2417–2430, 2014.

74. I. R. Sumathi and I. Arockiarani, "Cosine similarity measures of neutrosophic soft set," *Ann. Fuzzy Math. Informatics*, pp. 1–10, 2016.
75. B. C. Cuong, P. H. Phong, and F. Smarandache, "Standard neutrosophic soft theory: Some first results," *Neutrosophic Sets Syst.*, vol. 12, no. February, pp. 80–91, 2016.
76. A. Hussain and M. Shabir, "Algebraic structures of neutrosophic soft sets," *Neutrosophic Sets Syst.*, vol. 7, pp. 53–61, 2015.
77. A. Mukherjee and S. Sadhan, "A new method of measuring similarity between two neutrosophic soft sets and its application in pattern recognition problems," *Neutrosophic Sets Syst.*, vol. 8, no. 3, pp. 63–68, 2015.
78. P. K. Maji, "Weighted Neutrosophic Soft Sets," *Neutrosophic Sets Syst.*, vol. 6, pp. 6–11, 2014.
79. R. Chatterjee, P. Majumdar, and S. K. Samanta, "Interval-valued possibility Quadripartitioned single valued neutrosophic soft sets and some uncertainty based measures on them," *Neutrosophic Sets Syst.*, vol. 14, pp. 35–43, 2016.
80. E. Marei, "Single valued neutrosophic soft approach to rough sets, theory and application," *Neutrosophic Sets Syst.*, vol. 20, pp. 76–85, 2018.
81. I. Deli, "Interval-valued neutrosophic soft sets and its decision making," *Int. J. Mach. Learn. Cybern.*, vol. 8, no. 2, pp. 665–676, Apr. 2014.
82. A. Mukherjee and S. Sarkar, "Several similarity measures of interval valued neutrosophic soft sets and their application in pattern recognition problems," *Neutrosophic Sets Systms*, vol. 6, pp. 54–60, 2014.
83. S. Broumi, I. Deli, and F. Smarandache, "Relations on interval valued neutrosophic soft sets," *J. New Results Sci.*, no. 5, pp. 1–20, 2014.
84. S. Alkhazaleh, "N-valued refined neutrosophic soft set theory," *J. Intell. Fuzzy Syst.*, vol. 32, no. 6, pp. 4311–4318, 2016.
85. S. Alkhazaleh and A. A. Hazaymeh, "N-valued refined neutrosophic soft sets and their applications in decision-making problems and medical diagnosis," *JAISCR*, vol. 8, no. 1, pp. 79–86, 2017.
86. I. Deli, S. Broumi, and M. Ali, "Neutrosophic soft multiset theory," *Ital. J. Pure Appl. Math.*, vol. 32, pp. 503–514, 2014.

Received: Sep 28, 2019. Accepted: Dec 06, 2019