NSS
Neutrosophic Sets and Systems, Vol. 30, 2019
University of New Mexico

Neutrosophic Generalized Pre Regular Closed Sets

I. Mohammed Ali Jaffer[1] and K. Ramesh [2,*]

[1] Department of Mathematics, Government Arts College, Udumalpet - 642126, Tamilnadu, India.
E-mail: jaffermathsgac@gmail.com
[2] Department of Mathematics, Nehru Institute of Engineering & Technology, Coimbatore - 641 105, Tamil Nadu, India.
E-mail: ramesh251989@gmail.com

* Correspondence: ramesh251989@gmail.com;

Abstract: As a generalization of fuzzy sets and intuitionistic fuzzy sets, Neutrosophic sets have been developed by Smarandache to represent imprecise, incomplete and inconsistent information existing in the real world. A neutrosophic set is characterized by a truth value, an indeterminacy value and a falsity value. In this paper, we introduce and study a new class of Neutrosophic generalized closed set, namely Neutrosophic generalized pre regular closed sets and Neutrosophic generalized pre regular open sets in Neutrosophic topological spaces. Also we study the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space and their properties are discussed.

Keywords: Neutrosophic generalized pre regular closed sets, Neutrosophic generalized pre regular open sets, $NprT_{1/2}$ space and $NprT^*_{1/2}$ space.

1. Introduction

In 1970, Levine [12] introduced the concept of g-closed sets in general topology. Generalized closed sets play a very important role in general topology and they are now the research topics of many researchers worldwide. In 1965, Zadeh [19] introduced the notion of fuzzy sets [FS]. Later, fuzzy topological space was introduced by Chang [6] in 1968 using fuzzy sets. In 1986, Atanassov [5] introduced the notion of intuitionistic fuzzy sets [IFS], where the degree of membership and degree of non-membership of an element in a set X are discussed. In 1997, Intuitionistic fuzzy topological spaces were introduced by Coker [7] using intuitionistic fuzzy sets.

Neutrality the degree of indeterminacy as an independent concept was introduced by Florentine Smarandache [8]. He also defined the Neutrosophic set on three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership) from the fuzzy sets and intuitionistic fuzzy sets. Smarandache's Neutrosophic concepts have wide range of real time applications for the fields of [1, 2, 3&4] Information systems, Computer science, Artificial Intelligence, Applied Mathematics and Decision making.

In 2012, Salama A. A and Alblowi [14] introduced the concept of Neutrosophic topological spaces by using Neutrosophic sets. Salama A. A. [15] introduced Neutrosophic closed set and Neutrosophic continuous functions in Neutrosophic topological spaces. Further the basic sets like Neutrosophic regular-open sets, Neutrosophic semi-open sets, Neutrosophic pre-open sets, Neutrosophic α-open sets and Neutrosophic generalized closed sets are introduced in Neutrosophic topological space and their properties are studied by various authors [10], [15], [17], [13]. In this direction, we introduce and analyze a new class of Neutrosophic generalized closed set called Neutrosophic generalized pre regular closed sets and Neutrosophic generalized pre regular open sets in Neutrosophic topological spaces. Also we study the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space in Neutrosophic topological spaces. Many examples are given to justify the results.

2. Preliminaries

We recall some basic definitions that are used in the sequel.

Definition 2.1: [14] Let X be a non-empty fixed set. A Neutrosophic set (NS for short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ where the functions $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

Remark 2.2: [14] A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ can be identified to an ordered triple $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ in non-standard unit interval $]^-0, 1^+[$ on X.

Remark 2.3: [14] For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$.

Example 2.4: [14] Every IFS A is a non-empty set in X is obviously on NS having the form $A = \{\langle x, \mu_A(x), 1 - (\mu_A(x) + \nu_A(x)), \nu_A(x) \rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the NS 0_N and 1_N in X as follows:

0_N may be defined as:

(0_1) $0_N = \{\langle x, 0, 0, 1 \rangle : x \in X\}$

(0_2) $0_N = \{\langle x, 0, 1, 1 \rangle : x \in X\}$

(0_3) $0_N = \{\langle x, 0, 1, 0 \rangle : x \in X\}$

(0_4) $0_N = \{\langle x, 0, 0, 0 \rangle : x \in X\}$

1_N may be defined as:

(1_1) $1_N = \{\langle x, 1, 0, 0 \rangle : x \in X\}$

(1_2) $1_N = \{\langle x, 1, 0, 1 \rangle : x \in X\}$

(1_3) $1_N = \{\langle x, 1, 1, 0 \rangle : x \in X\}$

(1_4) $1_N = \{\langle x, 1, 1, 1 \rangle : x \in X\}$

Definition 2.5: [14] Let $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ be a NS on X, then the complement of the set A [C(A) for short] may be defined as three kind of complements:

(C_1) $C(A) = \{\langle x, 1-\mu_A(x), 1-\sigma_A(x), 1-\nu_A(x) \rangle : x \in X\}$

(C_2) $C(A) = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X\}$

(C_3) $C(A) = \{\langle x, \nu_A(x), 1-\sigma_A(x), \mu_A(x) \rangle : x \in X\}$

Definition 2.6: [14] Let X be a non-empty set and Neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$. Then we may consider two possible definitions for subsets $(A \subseteq B)$.

(1) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)\ \forall\ x \in X$

(2) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)\ \forall\ x \in X$

Proposition 2.7: [14] For any Neutrosophic set A, the following conditions hold:

$0_N \subseteq A$, $0_N \subseteq 0_N$

$A \subseteq 1_N$, $1_N \subseteq 1_N$

Definition2.8: [14] Let X be a non-empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$ are NSs. Then $A \cap B$ may be defined as:

(I_1) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x) \rangle$

(I_2) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x) \rangle$

A∪B may be defined as:
(U_1) A∪B = ⟨x, $\mu_A(x) \vee \mu_B(x)$, $\sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)$⟩
(U_2) A∪B = ⟨x, $\mu_A(x) \vee \mu_B(x)$, $\sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)$⟩

We can easily generalize the operations of intersection and union in Definition 2.8., to arbitrary family of NSs as follows:

Definition 2.9: [14] Let {Aj: j ∈ J} be an arbitrary family of NSs in X, then
∩Aj may be defined as:

(i) $\cap A_j = \langle x, \wedge_{j\in J}\, \mu_{A_j}(x), \wedge_{j\in J}\, \sigma_{A_j}(x), \vee_{j\in J}\, \nu_{A_j}(x)\rangle$
(ii) $\cap A_j = \langle x, \wedge_{j\in J}\, \mu_{A_j}(x), \vee_{j\in J}\, \sigma_{A_j}(x), \vee_{j\in J}\, \nu_{A_j}(x)\rangle$

∪Aj may be defined as:

(i) $\cup A_j = \langle x, \vee_{j\in J}\, \mu_{A_j}(x), \vee_{j\in J}\, \sigma_{A_j}(x), \wedge_{j\in J}\, \nu_{A_j}(x)\rangle$
(ii) $\cup A_j = \langle x, \vee_{j\in J}\, \mu_{A_j}(x), \wedge_{j\in J}\, \sigma_{A_j}(x), \wedge_{j\in J}\, \nu_{A_j}(x)\rangle$

Proposition 2.10: [14] For all A and B are two Neutrosophic sets then the following conditions are true:

C(A∩B) = C(A) ∪ C(B); C(A∪B) = C(A) ∩ C(B).

***Definition 2.11**:* [14] A Neutrosophic topology [NT for short] is a non-empty set X is a family τ of Neutrosophic subsets in X satisfying the following axioms:
(NT_1) $0_N, 1_N \in \tau$,
(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(NT_3) $\cup\, G_i \in \tau$ for every $\{G_i : i \in J\} \subseteq \tau$.

Throughout this paper, the pair (X, τ) is called a Neutrosophic topological space (NTS for short). The elements of τ are called Neutrosophic open sets [NOS for short]. A complement C(A) of a NOS A in NTS (X, τ) is called a Neutrosophic closed set [NCS for short] in X.

Example 2.12: [14] Any fuzzy topological space (X, τ) in the sense of Chang is obviously a NTS in the form τ = {A: $\mu_A \in \tau$} wherever we identify a fuzzy set in X whose membership function is μ_A with its counterpart.

The following is an example of Neutrosophic topological space.

Example 2.13: [14] Let X = {x} and A = {⟨x, 0.5, 0.5, 0.4⟩: x ∈ X}, B = {⟨x, 0.4, 0.6, 0.8⟩: x ∈ X}, C = {⟨x, 0.5, 0.6, 0.4⟩: x ∈ X}, D = {⟨x, 0.4, 0.5, 0.8⟩: x ∈ X}. Then the family τ = {0_N, A, B, C, D, 1_N} of NSs in X is Neutrosophic topological space on X.

Now, we define the Neutrosophic closure and Neutrosophic interior operations in Neutrosophic topological spaces:
Definition 2.14: [14] Let (X, τ) be NTS and A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} be a NS in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by
NCl(A) = ∩{K : K is a NCS in X and A ⊆ K}
NInt(A) = ∪{G : G is a NOS in X and G ⊆ A}
It can be also shown that NCl(A) is NCS and NInt(A) is a NOS in X.

a) A is NOS if and only if A = NInt(A),
b) A is NCS if and only if A = NCl(A).

Proposition 2.15: [14] For any Neutrosophic set A is (X, τ) we have

a) NCl(C(A)) = C(NInt(A)),
b) NInt(C(A)) = C(NCl(A)).

Proposition 2.16: [14] Let (X, τ) be NTS and A, B be two Neutrosophic sets in X. Then the following properties are holds:

a) NInt(A) ⊆ A,
b) A ⊆ NCl(A),
c) A ⊆ B ⇒ NInt(A) ⊆ NInt(B),
d) A ⊆ B ⇒ NCl(A) ⊆ NCl(B),
e) NInt(NInt(A)) = NInt(A),
f) NCl(NCl(A)) = NCl(A),
g) NInt(A ∩ B) = NInt(A) ∩ NInt(B),
h) NCl(A ∪ B) = NCl(A) ∪ NCl(B),
i) NInt(0_N) = 0_N,
j) NInt(1_N) = 1_N,
k) NCl(0_N) = 0_N,
l) NCl(1_N) = 1_N,
m) A ⊆ B ⇒ C(A) ⊆ C(B),
n) NCl(A ∩ B) ⊆ NCl(A) ∩ NCl(B),
o) NInt(A ∪ B) ⊇ NInt(A) ∪ NInt(B).

Definition 2.17: [9] A NS A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} in a NTS (X, τ) is said to be

(i) Neutrosophic regular closed set (NRCS for short) if A = NCl(NInt(A)),
(ii) Neutrosophic regular open set (NROS for short) if A = NInt(NCl(A)),
(iii) Neutrosophic semi closed set (NSCS for short) if NInt(NCl(A)) ⊆ A,
(iv) Neutrosophic semi open set (NSOS for short) if A ⊆ NCl(NInt(A)),
(v) Neutrosophic pre closed set (NPCS for short) if NCl(NInt(A)) ⊆ A,
(vi) Neutrosophic pre open set (NPOS for short) if A ⊆ NInt(NCl(A)),
(vii) Neutrosophic α- closed set (NSCS for short) if NCl(NInt(NCl(A))) ⊆ A,
(viii) Neutrosophic α- open set (NSOS for short) if A ⊆ NInt(NCl(NInt(A))).

Definition 2.18: [18] Let (X, τ) be NTS and A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} be a NS in X. Then the Neutrosophic pre closure and Neutrosophic pre interior of A are defined by

NPCl(A) = ∩{K : K is a NPCS in X and A ⊆ K},
NPInt(A) = ∪{G : G is a NPOS in X and G ⊆ A}.

Definition 2.18: [13] A NS A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized closed set (NGCS for short) if NCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic generalized open set (NGOS for short) if C(A) is a NGCS in (X, τ).

Definition 2.20: [11] A NS A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic α- generalized closed set (NαGCS for short) if NαCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic α- generalized open set (NαGOS for short) if C(A) is a NαGCS in (X, τ).

Definition 2.21: [16] A NS A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic ω closed set (NωCS for short) if NCl(A) ⊆ U whenever A ⊆ U and U is a NSOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic ω open set (NωOS for short) if C(A) is a NωCS in (X, τ).

Definition 2.22: [9] A NS A = {⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic regular generalized closed set (NRGCS for short) if NCl(A) ⊆ U whenever A ⊆ U and U is a NROS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic regular generalized open set (NRGOS for short) if C(A) is a NRGCS in (X, τ).

Definition 2.23: [18] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic generalized pre closed set (NGPCS for short) if NPCl(A) ⊆ U whenever A ⊆ U and U is a NOS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic generalized pre open set (NGPOS for short) if C(A) is a NGPCS in (X, τ).

Definition 2.24: [9] A NS A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} in a NTS (X, τ) is said to be a Neutrosophic regular α generalized closed set (NRαGCS for short) if NαCl(A) ⊆ U whenever A ⊆ U and U is a NROS in (X, τ). A NS A of a NTS (X, τ) is called a Neutrosophic regular α generalized open set (NRαGOS for short) if C(A) is a NRGCS in (X, τ).

3. Neutrosophic Generalized Pre Regular Closed Sets

In this section we introduce Neutrosophic generalized pre regular closed sets in the Neutrosophic topological space and study some of their properties.

Definition 3.1: A NS A in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular closed set (NGPRCS for short) if NPCl(A) ⊆ U whenever A ⊆ U and U is a NROS in (X, τ). The family of all NGPRCSs of a NTS(X, τ) is denoted by NGPRC(X).

Example 3.2: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ is a NGPRCS in (X, τ). Since A ⊆ U and U is a NROS, we have NPCl(A) = A ⊆ U.

Theorem 3.3: Every NCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NCS in (X, τ), we have NCl (A) = A. Therefore NPCl(A) ⊆ NCl (A) = A ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.4: In Example 3.2., the NS A= A= ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ is a NGPRCS but not NCS in (X, τ).

Theorem 3.5: Every NαCS in (X, τ) is an NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NαCS in (X, τ), we have NCl(NInt(NCl(A))) ⊆ A, now A ⊆ NCl(A), NCl(NInt(A)) ⊆ NCl(NInt(NCl(A))) ⊆ A. Therefore NPCl(A) = A∪ NCl(NInt(A)) ⊆ A∪A = A ⊆ U. Hence A is a NGPRCS in (X, τ).

Example 3.6: In Example 3.2., the NS A= A= ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ is a NGPRCS but not NαCS in (X, τ).

Theorem 3.7: Every NωCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NωCS in (X, τ), we have NCl (A) ⊆ U because every NROS is NSOS in (X, τ). Therefore NPCl(A) ⊆ NCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.8: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.6, 0.5, 0.2), (0.7, 0.5, 0.1)⟩ and V = ⟨(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.4, 0.3, 0.7), (0.3, 0.2, 0.6)⟩ is a NGPRCS in (X, τ). Since A ⊆ V and V is a NROS, we have NPCl(A) = A ⊆ V. But A is not NωCS in (X, τ). Since A ⊆ V and V is a NSOS, we have NCl(A) = C(V) ⊈ V.

Theorem 3.9: Every NPCS in (X, τ) is an NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NPCS in (X, τ), we have NCl(NInt(A)) ⊆ A. Therefore NPCl(A) = A∪ NCl(NInt(A)) ⊆ A∪A = A ⊆ U. Hence A is a NGPRCS in (X, τ).

Example 3.10: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.2, 0.6), (0.1, 0.2, 0.7)⟩ and V = ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩ is a NGPRCS in (X, τ). Since A ⊆ 1_N, we have NPCl(A) = 1_N ⊆ 1_N. But A is not NPCS in (X, τ). Since NCl(NInt(A)) = 1_N ⊈ A.

Theorem 3.11: Every NGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NPCl(A) ⊆ NCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.12: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.3, 0.5, 0.7), (0.4, 0.5, 0.6)⟩ and V = ⟨(0.8, 0.5, 0.2), (0.7, 0.5, 0.3)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.3, 0.5, 0.7), (0.3, 0.5, 0.7)⟩ is a NGPRCS in (X, τ). Since A ⊆ U and U is a NROS, we have NPCl(A) = A ⊆ U. But A is not NGCS in (X, τ). Since A ⊆ U and U is a NOS, we have NCl(A) = C(U) ⊈ U.

Theorem 3.13: Every NαGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NαGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NPCl(A) ⊆ NαCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.14: Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.4, 0.3, 0.6), (0.3, 0.4, 0.7)⟩ is a NGPRCS in (X, τ). Since A ⊆ U and U is a NROS, we have NPCl(A) = A ⊆ U. But A is not NαGCS in (X, τ). Since A ⊆ U and U is a NOS, we have NαCl(A) = C(U) ⊈ U.

Theorem 3.15: Every NRαGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NRαGCS in (X, τ). Therefore NPCl(A) ⊆ NαCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.16: In Example 3.14., the NS A= ⟨(0.4, 0.3, 0.6), (0.3, 0.4, 0.7)⟩ is a NGPRCS but not NRαGCS in (X, τ).

Theorem 3.17: Every NGPCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NGPCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NPCl(A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.18: In Example 3.10., the NS A= ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩ is a NGPRCS in (X, τ). Since A ⊆ 1_N, we have NPCl(A) = 1_N ⊆ 1_N. But A is not NGPCS in (X, τ). Since A ⊆ V and V is a NOS, we have NPCl(A) = 1_N ⊈ V.

Theorem 3.19: Every NRGCS in (X, τ) is a NGPRCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NRGCS in (X, τ). Therefore NPCl(A) ⊆ NCl (A) ⊆ U, by hypothesis. Hence A is a NGPRCS in (X, τ).

Example 3.20: In Example 3.8., the NS A= ⟨(0.4, 0.3, 0.7), (0.3, 0.2, 0.6)⟩ is a NGPRCS but not NRGCS in (X, τ).

Theorem 3.21: Every NαGCS in (X, τ) is a NRαGCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NαGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NαCl (A) ⊆ U, by hypothesis. Hence A is a NRαGCS in (X, τ).

Example 3.22: In Example 3.10., the NS A= ⟨(0.7, 0.2, 0.3), (0.8, 0.2, 0.2)⟩ is a NRαGCS but not NαGCS in (X, τ).

Theorem 3.23: Every NGCS in (X, τ) is a NαGCS in (X, τ) but not conversely.

Proof: Let U be a NOS in (X, τ) such that A ⊆ U. Since A is NGCS in (X, τ). Therefore NαCl(A) ⊆ NCl (A) ⊆ U, by hypothesis. Hence A is a NαGCS in (X, τ).

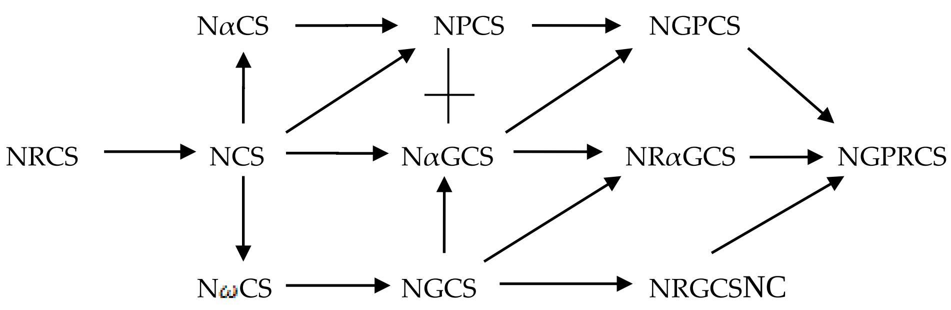
Example 3.24: Let X= {a} and τ = {0_N, U, V, 1_N} where U= ⟨0.5, 0.4, 0.7⟩ and V = ⟨0.8, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨0.2, 0.2, 0.8⟩ is a NαGCS in (X, τ). Since A ⊆ U and U is a NOS, we have NαCl(A) = A ⊆ U. But A is not NGCS in (X, τ). Since A ⊆ U, we have NCl(A) = C(V) ⊈ U.

Theorem 3.25: Every NGCS in (X, τ) is a NRGCS in (X, τ) but not conversely.

Proof: Let U be a NROS in (X, τ) such that A ⊆ U. Since A is NGCS in (X, τ) and every NROS in (X, τ) is a NOS in (X, τ). Therefore NCl (A) ⊆ U, by hypothesis. Hence A is a NRGCS in (X, τ).

Example 3.26: Let X= {a, b, c} and τ = {0_N, U, 1_N} where U= ⟨(0.6, 0.4, 0.3), (0.8, 0.5, 0.2), (0.7, 0.4, 0.8)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.5, 0.6, 0.6), (0.3, 0.5, 0.3), (0.5, 0.4, 0.3)⟩ is a NRGCS in (X, τ). Since A ⊆ 1_N, we have NCl(A) = 1_N ⊆ 1_N. but A is not NGCS in (X, τ). Since A ⊆ U and U is a NOS, we have NCl(A) = 1_N ⊈ U.

The following diagram, we have provided the relation between NGPRCS and the other existed NSs.



In this diagram by A ⟶ B means A implies B but not conversely and A ┼ B means A & B are independent.

Remark 3.27: The union of any two NGPRCSs in (X, τ) is not an NGPRCS in (X, τ) in general as seen from the following example.

Example 3.28: Let X = {a, b} and τ = {0_N, U, V, 1_N} where U = ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then the NSs A = ⟨(0.2, 0.1, 0.7), (0.4, 0.4, 0.7)⟩ and B=⟨(0.5, 0.3, 0.6), (0.2, 0.2, 0.8)⟩ are NGPRCSs in (X, τ) but A⋃B=⟨(0.5,0.3,0.6), (0.4,0.4,0.7)⟩ is not a NGPRCS in (X, τ). Since A⋃B ⊆ U but NPCl(A⋃B) = C(U) ⊈ U.

Remark 3.29: The intersection of any two NGPRCSs in (X, τ) is not an NGPRCS in (X, τ) in general as seen from the following example.

Example 3.30: Let X = {a, b} and τ = {0_N, U, V, 1_N} where U = ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then the NSs A = ⟨(0.5, 0.5, 0.4), (0.7, 0.6, 0.7)⟩ and B = ⟨(0.6, 0.3, 0.6), (0.4, 0.4, 0.3)⟩ are NGPRCSs in (X, τ) but A∩B = ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ is not a NGPRCS in (X, τ). Since A∩B ⊆ U but NPCl(A∩B) = C(U) ⊈ U.

Theorem 3.31: Let (X, τ) be a NTS. Then for every A ∈ NGPRC(X) and for every NS B ∈ NS(X), A ⊆ B ⊆ NPCl(A) implies B ∈ NGPRC(X).

Proof: Let B ⊆ U and U is a NROS in (X, τ). Since A ⊆ B, then A ⊆ U. Given A is a NGPRCS, it follows that NPCl(A) ⊆ U. Now B ⊆ NPCl(A) implies NPCl(B) ⊆ NPCl(NPCl(A)) = NPCl(A). Thus, NPCl(B) ⊆ U. This proves that B ∈ NGPRC(X).

Theorem 3.32: If A is a NROS and a NGPRCS in (X, τ), then A is a NPCS in (X, τ).

Proof: Since A ⊆ A and A is a NROS in (X, τ), by hypothesis, NPCl(A) ⊆ A. But since A ⊆ NPCl(A). Therefore NPCl(A)= A. Hence A is a NPCS in (X, τ).

Theorem 3.33: Let (X, τ) be a NTS and NPC(X) (resp. NRO(X)) be the family of all NPCSs (resp. NROSs) of X. If NPC(X) = IRO(X) then every Neutrosophic subset of X is NGPRCS in (X, τ).

Proof: If NPC(X) = IRO(X) and A is any Neutrosophic subset of X such that A ⊆ U where U is NROS in X. Then by hypothesis, U is NPCS in X which implies that NPCl(U) = U. Then NPCl(U) ⊆ NPCl(U) = U. Therefore A is NGPRCS in (X, τ).

Definition 3.34: Let (X, τ) be a NTS and A = {⟨x, $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$⟩: x ∈ X} be the subset of X. Then

NGPRCl(A) = ⋂{K : K is a NGPRCS in X and A ⊆ K} and

NGPRInt(A) = ⋃{G : G is a NGPROS in X and G ⊆ A}.

Lemma 3.35: Let A and B be subsets of (X, τ). Then the following results are obvious.

a) NGPRCl(0_N) = 0_N.
b) NGPRCl(1_N) = 1_N.
c) A ⊆ NGPRCl(A).
d) A ⊆ B ⇒ *NGPRCl(A)* ⊆ *NGPRCl(B).*

4. Neutrosophic Generalized Pre Regular Open Sets

In this section we introduce Neutrosophic generalized pre regular open sets in Neutrosophic topological space.

Definition 4.1: A NS A in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular open set (NGPROS for short) if NPInt(A) ⊇ U whenever A ⊇ U and U is a NRCS in (X, τ). Alternatively, A NS A is said to be a Neutrosophic generalized pre regular open set (NGPROS for short) if the complement of C(A) is a NGPRCS in (X, τ).

The family of all NGPROSs of a NTS(X, τ) is denoted by NGPRO(X).

Example 4.2*:* Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩ and V = ⟨(0.7, 0.5, 0.3), (0.7, 0.5, 0.2)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)⟩ is a NGPROS in (X, τ). Since A ⊇ C(U) and C(U) is a NRCS, we have NPInt(A) = A ⊇ C(U).

Theorem 4.3: Every NOS is a NGPROS in (X, τ) but the converses may not be true in general.

Proof: Let U be a NRCS in (X, τ) such that A ⊇ U. Since A is NOS, NInt(A) = A. By hypothesis, NPInt(A) = A ∩ NInt(NCl(A)) = A ∩ NCl(A) ⊇ A ∩ A = A ⊇ U. Therefore A is a NGPROS in (X, τ).

Example 4.4: In Example 4.2., the NS A= ⟨(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)⟩ is an NGPROS in (X, τ) but not a NOS in (X, τ).

Theorem 4.5*:* Every NαOS, NWOS, NPOS, NGOS, NαGOS, NGPOS, NRGOS, NRαGOS is a NGPROS in (X, τ) but the converses are not true in general.

Example 4.6: Let X= {a, b} and τ = {0_N, U, 1_N} where U = ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A = ⟨(0.2, 0.8, 0.6), (0.6, 0.4, 0.9)⟩ is a NGPROS in (X, τ). Since A ⊇ 0_N, we have NPInt(A) = 0_N ⊇ 0_N. but A is not a NαOS, NWOS, NPOS in (X, τ).

Example 4.7: Let X= {a, b} and τ = {0_N, U, 1_N} where U = ⟨(0.4, 0.2, 0.3), (0.8, 0.6, 0.7)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A = ⟨(0.3, 0.8, 0.4), (0.7, 0.4, 0.8)⟩ is a NGPROS in (X, τ). Since A ⊇ 0_N, we have NPInt(A) = 0_N ⊇ 0_N. but A is not a NGOS, NαGOS, NGPOS in (X, τ).

Example 4.8*:* Let X= {a, b} and τ = {0_N, U, V, 1_N} where U= ⟨(0.6, 0.5, 0.2), (0.7, 0.5, 0.1)⟩ and V = ⟨(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)⟩. Then (X, τ) is a Neutrosophic topological space. Here the NS A= ⟨(0.8, 0.8, 0.2), (0.7, 0.9, 0.3)⟩ is a NGPROS in (X, τ). Since A ⊇ C(V) and C(V) is a NRCS, we have NPInt(A) = A ⊇ C(V). but A is not NRGOS, NRαGOS in (X, τ).

Theorem 4.9: Let (X, τ) be a NTS. Then for every A ∈ NGPRO(X) and for every B ∈ NP(X), NPInt(A) ⊆ B ⊆ A implies B ∈ NGPRO(X).

Proof: Let A be any NGPROS of (X, τ) and B be any NS of X. By hypothesis NPInt(A) ⊆ B ⊆ A. Then C(A) is an NGPRCS in (X, τ) and C(A) ⊆ C(B) ⊆ NPCl(C(A)). By Theorem 3.31., C(B) is an NGPRCS in (X, τ). Therefore B is an NGPROS in (X, τ). Hence B ∈ NGPRO(X).

Theorem 4.10: A NS A of a NTS (X, τ) is a NGPROS in (X, τ) if and only if F ⊆ Npint(A) whenever F is a NRCS in (X, τ) and F ⊆ A.

Proof: Necessity*:* Suppose A is a NGPROS in (X, τ). Let F be a NRCS in (X, τ) such that F ⊆ A. Then C(F) is a NROS and C(A) ⊆ C(F). By hypothesis C(A) is a NGPRCS in (X, τ), we have NPCl(C(A)) ⊆ C(F). Therefore F ⊆ Npint(A).

Sufficiency: Let U be a NROS in (X, τ) such that C(A) ⊆ U. By hypothesis, C(U) ⊆ Npint(A). Therefore NPCl(C(A)) ⊆ U and C(A) is a NGPRCS in (X, τ). Hence A is a NGPROS in (X, τ).

Theorem 4.11*:* Let (X, τ) be a NTS and NPO(X) (resp. NGPRO(X)) be the family of all NPOSs (resp. NGPROSs) of X. Then NPO(X) ⊆ NGPRO(X).

Proof*:* Let A ∈ NPO(X). Then C(A) is NPCS and so NGPRCS in (X, τ). This implies that A is NGPROS in (X, τ). Hence A ∈ NGPRO(X). Therefore NPO(X) ⊆ NGPRO(X).

5. Separation Axioms of Neutrosophic Generalized Pre Regular Closed Sets

In this section we have provide some applications of Neutrosophic generalized pre regular closed sets in Neutrosophic topological spaces.

Definition 5.1: If every NGPRCS in (X, τ) is a NPCS in (X, τ), then the space (X, τ) can be called a Neutrosophic pre regular $T_{1/2}$ ($NPRT_{1/2}$ for short) space.

Theorem 5.2: An NTS (X, τ) is a $NPRT_{1/2}$ space if and only if NPOS(X) = NGPRO(X).

Proof: Necessity: Let (X, τ) be a $NPRT_{1/2}$ space. Let A be a NGPROS in (X, τ). By hypothesis, C(A) is a NGPRCS in (X, τ) and therefore A is a NPOS in (X, τ). Hence NPO(X) = NGPRO(X).
Sufficiency: Let NPO(X, τ) = NGPRO(X, τ). Let A be a NGPRCS in (X, τ). Then C(A) is a NGPROS in (X, τ). By hypothesis, C(A) is a NPOS in (X, τ) and therefore A is a NPCS in (X, τ). Hence (X, τ) is a $NPRT_{1/2}$ space.

Definition 5.3: A NTS (X, τ) is said to be a Neutrosophic pre regular $T^*_{1/2}$ space ($NPRT^*_{1/2}$ space for short) if every NGPRCS is a NCS in (X, τ).

Remark 5.4: Every $NPRT^*_{1/2}$ space is a $NPRT_{1/2}$ space but not conversely.

Proof: Assume be a $NPRT^*_{1/2}$ space. Let A be a NGPRCS in (X, τ). By hypothesis, A is an NCS. Since every NCS is a NPCS, A is a NPCS in (X, τ). Hence (X, τ) is a $NPRT_{1/2}$ space.

Example 5.8: Let X= {a, b} and let τ = {0N, U, 1N} where U= ⟨(0.5, 0.4, 0.7), (0.4, 0.5, 0.6)⟩. Then (X, τ) is a NPRT1/2 space, but it is not $NPRT^*_{1/2}$ space. Here the NS A= ⟨(0.2, 0.3, 0.8), (0.3, 0.4, 0.8)⟩ is a NGPRCS but not a NCS in (X, τ).

Theorem 5.9: Let (X, τ) be a $NPRT^*_{1/2}$ space then,

(i) the union of NGPRCSs is NGPRCS in (X, τ)
(ii) the intersection of NGPROSs is NGPROS in (X, τ)

Proof: (i) Let $\{Ai\}_{i\in J}$ be a collection of NGPRCSs in a $NPRT^*_{1/2}$ space (X, τ). Thus, every NGPRCSs is a NCS. However, the union of NCSs is a NCS in (X, τ). Therefore the union of NGPRCSs is NGPRCS in (X, τ). (ii) Proved by taking the complement in (i).

6. Conclusion

In this paper, we have defined new class of Neutrosophic generalized closed sets called Neutrosophic generalized pre regular closed sets; Neutrosophic generalized pre regular open sets and studied some of their properties in Neutrosophic topological spaces. Furthermore, the work was extended as the separation axioms of Neutrosophic generalized pre regular closed sets, namely Neutrosophic pre regular $T_{1/2}$ space and Neutrosophic pre regular $T^*_{1/2}$ space and discussed their properties. Further, the relation between Neutrosophic generalized pre regular closed set and existing Neutrosophic closed sets in Neutrosophic topological spaces were established. Many examples are given to justify the results.

Acknowledgements

The authors would like to thank the referees for their valuable suggestions to improve the paper.

References

1. Abdel-Basset, M., El-hoseny, M., Gamal, A., & Smarandache, F. (2019). A novel model for evaluation Hospital medical care systems based on plithogenic sets. *Artificial intelligence in medicine*, **2019**, 100, 101710.
2. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F., A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics, *Symmetry*, **2019**, *11(7)*, 903.
3. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77, 438-452.
4. Abdel-Baset, M., Chang, V., & Gamal, A., Evaluation of the green supply chain management practices: A novel neutrosophic approach, *Computers in Industry*, **2019**, *108*, 210-220.
5. Atanassov K. T., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **1986**, *20*, 87-96.
6. Chang C. L., Fuzzy topological spaces, *J.Math.Anal.Appl.*, **1968**, *24*, 182- 190.
7. Dogan Coker, An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and Systems*, **1997**, *88(1)*, 81-89.
8. Floretin Smarandache, Neutrosophic Set:- A Generalization of Intuitionistiic Fuzzy set, *Journal of Defense Resourses Management*, **2010**, *1*,107–116.
9. Harshitha A. and Jayanthi D., Regular α Generalized closed sets in neutrosophic topological spaces, *IOSR Journal of Mathematics*, **2019**, *15(02)*, 11-18.
10. Ishwarya P. and Bageerathi K., On Neutrosophic semi-open sets in neutrosophic topological spaces, *International Jour. of Math. Trends and Tech*, **2016**, 214-223.
11. Jayanthi D., On α Generalized closed sets in neutrosophic topological spaces, *International Conference on Recent Trends in Mathematics and Information Technology*, **2018**, *March*, 88-91.
12. Levine N., Generalized closed set in topology, *Rend.Circ.Mat Palermo*, **1970**, 19, 89-96.
13. Pushpalatha A. and Nandhini T., Generalized closed sets via neutrosophic topological Spaces, *Malaya Journal of Matematik*, **2019**, *7(1)*, 50-54.
14. Salama A. A. and Alblowi S. A., Neutrosophic set and Neutrosophic topological spaces, *IOSR Jour. of Mathematics,* **2012**, 31-35.
15. Salama A. A., Florentin Smarandache and Valeri Kroumov, Neutrosophic Closed set and Neutrosophic Continuous Function, *Neutrosophic Sets and Systems*, **2014**, 4, 4–8.
16. Santhi R. and Udhayarani N., Nω –Closed sets in Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems*, **2016**, 12, 114-117.
17. Venkateswara Rao V. and Srinivasa Rao Y., Neutrosophic Pre-open sets and Pre-closed sets in Neutrosphic Topology, *International Jour. ChemTech Research*, **2017**, 10(10), 449-458.
18. Wadei Al-Omeri and Saeid Jafari, On Generalized Closed Sets and Generalized Pre-Closed in Neutrosophic Topological Spaces, *Mathematics MDPI*, **2018**, 7(1), 01-12.
19. Zadeh L. A., Fuzzy sets, *Information and control*, **1965**, 8, 338-353.

Received: Sep 21, 2019. Accepted: Nov 29, 2019.