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MAGDM Model Using the Aczel-Alsina Aggregation Operators of Neutrosophic Entropy Elements in the Case of Neutrosophic Multi-Valued Sets

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Abstract: To overcome the limitations of both the conversion method based on the standard deviation and the decision flexibility in existing neutrosophic multi-valued decision-making models, this study aims to propose various new techniques including a conversion method, Aczel-Alsina aggregation operations, and a multi-attribute group decision making (MAGDM) model in the case of neutrosophic multi-valued sets (MVNSs). First, we propose a conversion method to convert neutrosophic multi-valued elements (MVNEs) into neutrosophic entropy elements (NEEs) based on the mean and normalized Shannon/probability entropy of truth, falsity, and indeterminacy sequences. Second, the score and accuracy functions of NEEs are defined for the ranking of NEEs. Third, the Aczel-Alsina t-norm and t-conorm operations of NEEs and the NEE Aczel-Alsina weighted arithmetic averaging (NEEAAWAA) and NEE Aczel-Alsina weighted geometric averaging (NEEAAWGA) operators are presented to reach the advantage of flexible operations by an adjustable parameter. Fourth, we propose a MAGDM model in light of the NEEAAWAA and NEEAAWGA operators and the score and accuracy functions in the case of NMVSs to solve flexible MAGDM problems with an adjustable parameter subject to decision makers' preference. Finally, an illustrative example is given to verify the impact of different parameter values on the decision results of the proposed MAGDM model. Compared with existing techniques, the new techniques not only overcome the defects of existing techniques but also be broader and more versatile than existing techniques when dealing with MAGDM problems in the case of NMVSs.

Keywords: neutrosophic multi-valued set; neutrosophic entropy element; Aczel-Alsina aggregation operator; group decision making

1. Introduction

In indeterminate and inconsistent situations, multi-valued neutrosophic sets (MVNSs) or neutrosophic hesitant fuzzy sets (NHFSs) can be depicted by the multi-valued sequences of the truth, falsity, and indeterminacy membership degrees, which were the extension of neutrosophic sets [1]. Then, relation operations, aggregation algorithms, and measure methods of MVNSs/NHFSs are critical research topics and play important roles in the fuzzy decision-making issues. Therefore, MVNSs/NHFSs have been used in medical diagnosis, decision making, engineering experiments, measurements, etc. Under the environment of NHFSs, some aggregation operators of single and interval valued NHFSs were presented and utilized in multi-attribute decision making (MADM) problems [2-4]. Then, MADM models based on the extended grey relation analysis [5] and the TOPSIS method [6] were introduced in the setting of NHFSs. Under the environment of MVNSs, some aggregation operators of MVNSs were proposed for multi-valued neutrosophic MADM problems [7, 8]. The Dice similarity measure of single-valued neutrosophic multisets (SVNMs) was introduced and used for medical diagnosis [9]. Furthermore, the correlation coefficient of dynamic SVNMs was presented for MADM problems [10]. The TODIM methods were introduced for MADM problems with MVNSs [11, 12]. However, there are the operational difficulty and complexity between different sequence lengths/cardinalities in multi-valued/hesitant sequences. To solve these issues, Fan et al. [13] introduced a conversion method from SVNMs to single-valued neutrosophic sets (SVNSs) by the average aggregation values of truth, indeterminacy, and falsity sequences, and then proposed the cosine similarity measure of SVNSs for MADM problems in the case of SVNMs. But this conversion method in [13] may result in some loss/distortion of information. To solve this problem, Ye et al. [14] further proposed a reasonable conversion method of neutrosophic multi-valued sets (NMVSs) (including MVNSs, NHFSs, and SVNMs) in light of the average values and consistency degrees (complement of standard deviation) of truth, indeterminacy, and falsity sequences to realize the reasonable information expression and operations of consistency neutrosophic sets/elements (CNS/CNEs), and then developed a multi-attribute group decision making (MAGDM) method using correlation coefficients of CNSs in the case of NMVSs. Then, the conversion method based on the average value and standard deviation [14] is only suitable for normal distribution, which indicates its limitation. Moreover, the existing MAGDM method based on two correlation coefficients of CNSs [14] lacks decision flexibility in the case of NMVSs. Therefore, it is difficult to satisfy the preference of decision makers and/or application needs. Under a probabilistic MVNS environment, Liu and Cheng [15] proposed a three-phase MAGDM method based on the multi-attributive border approximation area comparison (MABAC) method. Since the probability method needs a large number of evaluation values to reasonably give their probabilistic values in MAGDM problems, it is difficult to apply it in actual MAGDM problems. According to the theory of probability and statistics, it is seen that the probability value yielded from a few of the evaluation values (small-scale sample data) is unreasonable and may cause the probability distortion. Moreover, the three-phase MAGDM method also lacks its flexible decision-making feature in the setting of probabilistic MVNSs.

Recently, many researchers have proposed various Aczel-Alsina aggregation operators and their decision-making approaches in various fuzzy circumstances because the operations based on the Aczel-Alsina t-norm and t-conorm [16, 17] reflect the advantage of changeability by an adjustable parameter. For example, Fu et al. [18] proposed the Aczel-Alsina aggregation operators of entropy fuzzy elements and their MAGDM model for renal cancer surgery options in the case of fuzzy multi-sets. Yong et al. [19] introduced the Aczel-Alsina aggregation operators of simplified neutrosophic elements and their MADM approach. Senapati [20] proposed the Aczel-Alsina average aggregation operators of fuzzy picture elements and their MADM approach. Hussain et al. [21] presented the Aczel-Alsina aggregation operators of T-spherical fuzzy elements and their decision-making problems. Then, Senapati et al. [22-24] developed the Aczel-Alsina aggregation operators of (interval-valued) intuitionistic fuzzy elements and their MADM approach. Senapati et al. [25] introduced hesitant fuzzy aggregation operators and applied them to the assessment of cyclone disasters. However, these Aczel-Alsina aggregation operators cannot deal with the aggregation operations and MAGDM issues of NMVSs.

To solve the aforementioned limitations/deflects of the existing methods in the case of NMVSs, the purposes of this research are: (1) to propose a conversion method from a neutrosophic multi-valued element (NMVE) to a neutrosophic entropy element (NEE) in light of the average values and Shannon/probability entropy of truth, falsity, and indeterminacy sequences, (2) to define score and accuracy functions of NEE and ranking laws of NEEs, (3) to propose the Aczel-Alsina t-norm and t-conorm operations of NEEs and the NEE Aczel-Alsina weighted arithmetic averaging (NEEAAWAA) and NEE Aczel-Alsina weighted geometric averaging (NEEAAWGA) operators, and

(4) to develop a MAGDM method by the proposed NEEAAWAA and NEEAAWGA operators and score and accuracy functions to be effectively used for flexible decision-making issues with the information of NMVSs.

In order to verify the impact of different parameter values on the decision results of the proposed MAGDM model, an illustrative example indicates the efficiency and rationality of the proposed MAGDM model. Then, comparative analysis shows that our new techniques not only overcome the defects of the existing techniques, but also are broader and more versatile than the existing techniques when dealing with MAGDM problems in the setting of NMVSs.

However, the conversion method, the NEEAAWAA and NEEAAWGA operators, and the MAGDM model proposed in this research show new contributions and outstanding advantages of these new techniques.

The remainder of this paper contains the following sections. Section 2 proposes a conversion method from NMVE to NEE in terms of the mean and Shannon entropy of the truth, indeterminacy and falsity sequences in NMVEs, and then defines score and accuracy functions of NEE, ranking laws of NEEs, and the Aczel-Alsina t-norm and t-conorm operations of NEEs. Section 3 presents the NEEAAWAA and NEEAAWGA operators and their properties. In Section 4, a MAGDM model is established by the NEEAAWAA and NEEAAWGA operators and the score and accuracy functions of NEEs in the NMVS setting. Section 5 introduces an illustrative example and comparison with existing techniques to show the efficiency and rationality of the new techniques. The last section contains conclusions and further work.

2. NEEs Based on the Mean and Normalized Shannon Entropy in the Case of NMVSs

In the setting of NMVSs, this section first presents a NEE concept by a conversion method based on the Shannon entropy and average values of truth, falsity and indeterminacy sequences, and then defines the score and accuracy functions and ranking laws of NEEs and the Aczel-Alsina t-norm and t-conorm operations of NEEs.

Definition 1 [14]. Set $Y = \{y_k \mid k = 1, 2, \ldots, m\}$ as a finite universe set. A NMVS M on Y is defined as

$$M = \left\{ \left\langle y_k, M_T(y_k), M_I(y_k), M_F(y_k) \right\rangle \mid y_k \in Y \right\},$$

where $M_T(y_k)$, $M_I(y_k)$ and $M_F(y_k)$ are the truth, indeterminacy, and falsity sequences with the same and/or different fuzzy values, which are denoted by $M_T(y_k) = (\alpha_T^1(y_k), \alpha_T^2(y_k), \ldots, \alpha_T^{r_k}(y_k))$, $M_I(y_k) = (\alpha_I^1(y_k), \alpha_I^2(y_k), \ldots, \alpha_I^{r_k}(y_k))$ and $M_F(y_k) = (\alpha_F^1(y_k), \alpha_F^2(y_k), \ldots, \alpha_F^{r_k}(y_k))$ for $y_k \in Y$, along with the length of their sequence r_k and $0 \le \sup M_T(y_k) + \sup M_I(y_k) + \sup M_F(y_k) \le 3$ ($k = 1, 2, \ldots, m$).

For convenience, the kth element $\left\langle y_k, M_T(y_k), M_I(y_k), M_F(y_k) \right\rangle$ in M is denoted as the NMVE $Ms_k = \left\langle M_{Tk}, M_{Ik}, M_{Fk} \right\rangle = \left\langle (\alpha_{Tk}^1, \alpha_{Tk}^2, \ldots, \alpha_{Tk}^{r_k}), (\alpha_{Ik}^1, \alpha_{Ik}^2, \ldots, \alpha_{Ik}^{r_k}), (\alpha_{Fk}^1, \alpha_{Fk}^2, \ldots, \alpha_{Fk}^{r_k}) \right\rangle$ in decreasing sequences.

First, the concept of the Shannon/probability entropy [26] is introduced below.

Set $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ as a probability distribution on a set of random variables. Then, the Shannon entropy of the probability distribution α is expressed as

$$P(\alpha) = -\sum_{j=1}^{n} \alpha_j \ln(\alpha_j). \tag{1}$$

where $\alpha_j \in [0, 1]$ and $\sum_{j=1}^{n} \alpha_j = 1$.

If all values of α_j ($j = 1, 2, \ldots, n$) are the same, then the entropy $P(\alpha)$ reaches the maximum value, which means perfect consistency of α_j. Generally, there is an approximately linear relationship between entropy and standard deviation: the larger the standard deviation, the smaller the entropy.

In the following, we present the definition of NEE by a conversion method in light of the normalized Shannon entropy and average values of truth, falsity, and indeterminacy sequences in NMVE.

Definition 2. Set $Ms_k = \langle M_{Tk}, M_{Ik}, M_{Fk} \rangle = \langle (\alpha_{Tk}^1, \alpha_{Tk}^2, ..., \alpha_{Tk}^{r_k}), (\alpha_{Ik}^1, \alpha_{Ik}^2, ..., \alpha_{Ik}^{r_k}), (\alpha_{Fk}^1, \alpha_{Fk}^2, ..., \alpha_{Fk}^{r_k}) \rangle$ as the kth NMVE. Then, its NEE is represented as follows:

$$N_{Ek} = \langle (\alpha_{Tk}, e_{Tk}), (\alpha_{Ik}, e_{Ik}), (\alpha_{Fk}, e_{Fk}) \rangle,$$

where α_{Tk}, α_{Ik}, $\alpha_{Fk} \in [0, 1]$ are the average values of the truth, indeterminacy, and falsity sequences and e_{Tk}, e_{Ik}, $e_{Fk} \in [0, 1]$ are the normalized entropy values of the truth, indeterminacy, and falsity sequences, which are yielded by the following formulae:

(1) $$\alpha_{Tk} = \frac{1}{r_k} \sum_{j=1}^{r_k} \alpha_{Tk}^j \text{ and } e_{Tk} = -\frac{1}{\ln r_k} \sum_{j=1}^{r_k} \left(\frac{\alpha_{Tk}^j}{\sum_{j=1}^{r_k} \alpha_{Tk}^j} \ln \frac{\alpha_{Tk}^j}{\sum_{j=1}^{r_k} \alpha_{Tk}^j} \right);$$

(2) $$\alpha_{Ik} = \frac{1}{r_k} \sum_{j=1}^{r_k} \alpha_{Ik}^j \text{ and } e_{Ik} = -\frac{1}{\ln r_k} \sum_{j=1}^{r_k} \left(\frac{\alpha_{Ik}^j}{\sum_{j=1}^{r_k} \alpha_{Ik}^j} \ln \frac{\alpha_{Ik}^j}{\sum_{j=1}^{r_k} \alpha_{Ik}^j} \right);$$

(3) $$\alpha_{Fk} = \frac{1}{r_k} \sum_{j=1}^{r_k} \alpha_{Fk}^j \text{ and } e_{Fk} = -\frac{1}{\ln r_k} \sum_{j=1}^{r_k} \left(\frac{\alpha_{Fk}^j}{\sum_{j=1}^{r_k} \alpha_{Fk}^j} \ln \frac{\alpha_{Fk}^j}{\sum_{j=1}^{r_k} \alpha_{Fk}^j} \right).$$

Remark 1. Since the entropy of r_k components cannot exceed $\ln r_k$ ($r_k > 1$), the defined normalized Shannon entropy measures satisfy e_{Tk}, e_{Ik}, $e_{Fk} \in [0, 1]$, and also there exist the following results:

$$\sum_{j=1}^{r_k} \frac{\alpha_{Tk}^j}{\sum_{j=1}^{r_k} \alpha_{Tk}^j} = 1, \quad \sum_{j=1}^{r_k} \frac{\alpha_{Ik}^j}{\sum_{j=1}^{r_k} \alpha_{Ik}^j} = 1, \quad \sum_{j=1}^{r_k} \frac{\alpha_{Fk}^j}{\sum_{j=1}^{r_k} \alpha_{Fk}^j} = 1,$$

which can satisfy the Shannon entropy conditions. When all components in a multi-valued sequence are the same value, the normalized Shannon entropy is equal to one (the maximum value).

Example 1. Let Ms = <(0.8, 0.7, 0.5), (0.3, 0.2, 0.1), (0.2, 0.2, 0.2)> be NMVE. Using the formulae (1)-(3) in Definition 2, we obtain the following NEE:

N_E = <(0.6667, 0.9835), (0.2, 0.9206), (0.2, 1)>.

Then, we can give the definition of some relations of NEEs below.

Definition 3. Set N_{E1} = <(α_{T1}, e_{T1}), (α_{I1}, e_{I1}), (α_{F1}, e_{F1})> and N_{E2} = <(α_{T2}, e_{T2}), (α_{I2}, e_{I2}), (α_{F2}, e_{F2})> as two NEEs. Then, their relations are defined as follows:

(1) $N_{E1} \supseteq N_{E2} \Leftrightarrow \alpha_{T1} \geq \alpha_{T2}$, $e_{T1} \geq e_{T2}$, $\alpha_{I2} \geq \alpha_{I1}$, $e_{I2} \geq e_{I1}$, $\alpha_{F2} \geq \alpha_{F1}$, and $e_{F2} \geq e_{F1}$;

(2) $N_{E1} = N_{E2} \Leftrightarrow N_{E1} \supseteq N_{E2}$ and $N_{E2} \supseteq N_{E1}$;

(3) $N_{E1} \cup N_{E2} = \langle (\alpha_{T1} \vee \alpha_{T2}, e_{T1} \vee e_{T2}), (\alpha_{I1} \wedge \alpha_{I2}, e_{I1} \wedge e_{I2}), (\alpha_{F1} \wedge \alpha_{F2}, e_{F1} \wedge e_{F2}) \rangle$;

(4) $N_{E1} \cap N_{E2} = \langle (\alpha_{T1} \wedge \alpha_{T2}, e_{T1} \wedge e_{T2}), (\alpha_{I1} \vee \alpha_{I2}, e_{I1} \vee e_{I2}), (\alpha_{F1} \vee \alpha_{F2}, e_{F1} \vee e_{F2}) \rangle$;

(5) $(N_{E1})^c = \langle (\alpha_{F1}, e_{F1}), (1-\alpha_{I1}, 1-e_{I1}), (\alpha_{T1}, e_{T1}) \rangle$ (Complement of N_{E1}).

To sort NEEs, we define the score and accuracy functions and ranking laws of NEEs below.

Definition 4. Let N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> for k = 1, 2 be two NEEs. Then, the score and accuracy functions of NEEs are defined as follows:

$$R(N_{Ek}) = (2 + \alpha_{Tk} \times e_{Tk} - \alpha_{Ik} \times e_{Ik} - \alpha_{Fk} \times e_{Fk}) / 3 \;\text{ for }\; R(N_{Ek}) \in [0,1], \tag{2}$$

$$Q(N_{Ek}) = \alpha_{Tk} \times e_{Tk} - \alpha_{Fk} \times e_{Fk} \;\text{ for }\; Q(N_{Ek}) \in [-1,1]. \tag{3}$$

Thus, the two NEEs N_{E1} and N_{E2} are ranked by the following laws:

(1) If $R(N_{E1}) > R(N_{E2})$, then $N_{E1} > N_{E2}$;

(2) If $R(N_{E1}) = R(N_{E2})$ and $Q(N_{E1}) > Q(N_{E2})$, then $N_{E1} > N_{E2}$;

(3) If $R(N_{E1}) = R(N_{E2})$ and $Q(N_{E1}) = Q(N_{E2})$, then $N_{E1} = N_{E2}$

Example 2. There are two NEEs N_{E1} = <(0.6333, 0.6376), (0.1333, 0.6534), (0.3, 0.6783)> and N_{E2} = <(0.4667, 0.6464), (0.2, 0.6338), (0.2333, 0.7346)>. By Eq. (2), the score values and ranking of the two NEEs are given as follows:

$R(N_{E1})$ = (2+0.6333×0.6376 – 0.1333×0.6534 – 0.3×0.6783)/3 = 0.7044,

$R(N_{E2})$ = (2+0.4667×0.6464 – 0.2×0.6338 – 0.2333×0.7346)/3 = 0.6678.

Since R(N_{E1}) > R(N_{E2}), the ranking of both is $N_{E1} > N_{E2}$.

Regarding the t-norm and t-conorm operations, Aczel and Alsina [16] and Alsina et al. [17] defined the Aczel-Alsina t-norms $G_\rho(c,d)$: $[0, 1]^2 \to [0,1]$ and the Aczel-Alsina t-conorms $H_\rho(c,d)$: $[0, 1]^2 \to [0,1]$ for all $c, d \in [0, 1]$ and $\rho \geq 0$ as follows:

(a) The Aczel-Alsina t-norms are defined as

$$G_\rho(c,d) = \begin{cases} G_D(c,d), & if\, \rho = 0 \\ \min(c,d), & if\, \rho = \infty \\ e^{-((-\ln c)^\rho + (-\ln d)^\rho)^{1/\rho}}, & \text{otherwise} \end{cases}.$$

(b) The Aczel-Alsina t-conorms are defined as

$$H_\rho(c,d) = \begin{cases} H_D(c,d), & if\, \rho = 0 \\ \max(c,d), & if\, \rho = \infty \\ 1 - e^{-((-\ln(1-c))^\rho + (-\ln(1-d))^\rho)^{1/\rho}}, & \text{otherwise} \end{cases},$$

where $G_D(c, d)$ and $H_D(c, d)$ are the drastic t-norm and the drastic t-conorm, respectively, which are denoted as

$$G_D(c,d) = \begin{cases} c, & \text{if } d = 1 \\ d, & \text{if } c = 1 \\ 0, & \text{otherwise} \end{cases} \text{ and } H_D(c,d) = \begin{cases} c, & \text{if } d = 1 \\ d, & \text{if } c = 1 \\ 1, & \text{otherwise} \end{cases}.$$

Since the operations based on the Aczel-Alsina t-norm and t-conorm [16, 17] reflect the advantage of changeability by an adjustable parameter ρ, we can give the definition of the Aczel-Alsina t-norm and t-conorm operations of NEEs.

Definition 5. Let N_{E1} = <(α_{T1}, e_{T1}), (α_{I1}, e_{I1}), (α_{F1}, e_{F1})> and N_{E2} = <(α_{T2}, e_{T2}), (α_{I2}, e_{I2}), (α_{F2}, e_{F2})> be two NEEs, $\rho \geq 1$, and $\gamma > 0$. Then, their operations are defined below:

(1) $$N_{E1} \oplus N_{E2} = \left\langle \begin{array}{l} \left(1-e^{-((-\ln(1-\alpha_{T1}))^{\rho}+(-\ln(1-\alpha_{T2}))^{\rho})^{1/\rho}}, 1-e^{-((-\ln(1-e_{T1}))^{\rho}+(-\ln(1-e_{T2}))^{\rho})^{1/\rho}}\right), \\ \left(e^{-((-\ln\alpha_{I1})^{\rho}+(-\ln\alpha_{I2})^{\rho})^{1/\rho}}, e^{-((-\ln e_{I1})^{\alpha}+(-\ln e_{I2})^{\rho})^{1/\rho}}\right), \\ \left(e^{-((-\ln\alpha_{F1})^{\rho}+(-\ln\alpha_{F2})^{\rho})^{1/\rho}}, e^{-((-\ln e_{F1})^{\rho}+(-\ln e_{F2})^{\rho})^{1/\rho}}\right) \end{array} \right\rangle;$$

(2) $$N_{E1} \otimes N_{E2} = \left\langle \begin{array}{l} \left(e^{-((-\ln\alpha_{T1})^{\rho}+(-\ln\alpha_{T2})^{\rho})^{1/\rho}}, e^{-((-\ln e_{T1})^{\rho}+(-\ln e_{T2})^{\rho})^{1/\rho}}\right), \\ \left(1-e^{-((-\ln(1-\alpha_{I1}))^{\rho}+(-\ln(1-\alpha_{I2}))^{\rho})^{1/\rho}}, 1-e^{-((-\ln(1-e_{I1}))^{\rho}+(-\ln(1-e_{I2}))^{\rho})^{1/\rho}}\right), \\ \left(1-e^{-((-\ln(1-\alpha_{F1}))^{\rho}+(-\ln(1-\alpha_{F2}))^{\rho})^{1/\rho}}, 1-e^{-((-\ln(1-e_{F1}))^{\rho}+(-\ln(1-e_{F2}))^{\rho})^{1/\rho}}\right) \end{array} \right\rangle;$$

(3) $$\gamma N_{E1} = \left\langle \begin{array}{l} \left(1-e^{-(\gamma(-\ln(1-\alpha_{T1}))^{\rho})^{1/\rho}}, 1-e^{-(\gamma(-\ln(1-e_{T1}))^{\rho})^{1/\rho}}\right), \\ \left(e^{-(\gamma(-\ln(\alpha_{I1}))^{\rho})^{1/\rho}}, e^{-(\gamma(-\ln e_{I1})^{\rho})^{1/\rho}}\right), \left(e^{-(\gamma(-\ln\alpha_{F1})^{\rho})^{1/\rho}}, e^{-(\gamma(-\ln e_{F1})^{\rho})^{1/\rho}}\right) \end{array} \right\rangle;$$

(4) $$(N_{E1})^{\gamma} = \left\langle \begin{array}{l} \left(e^{-(\gamma(-\ln\alpha_{T1})^{\rho})^{1/\rho}}, e^{-(\gamma(-\ln e_{T1})^{\rho})^{1/\rho}}\right), \\ \left(1-e^{-(\gamma(-\ln(1-\alpha_{I1}))^{\rho})^{1/\rho}}, 1-e^{-(\gamma(-\ln(1-e_{I1}))^{\rho})^{1/\rho}}\right), \\ \left(1-e^{-(\gamma(-\ln(1-\alpha_{F1}))^{\rho})^{1/\rho}}, 1-e^{-(\gamma(-\ln(1-e_{F1}))^{\rho})^{1/\rho}}\right) \end{array} \right\rangle.$$

Example 3. Let N_{E1} = <(0.6333, 0.6376), (0.1333, 0.6534), (0.3, 0.6783)> and N_{E2} = <(0.4667, 0.6464), (0.2, 0.6338), (0.2333, 0.7346)> be two NEEs, ρ = 3, and γ = 0.6. Using the operations (1)-(4) in Definition 5, we obtain the following operational results:

(1) $$N_{E1} \oplus N_{E2} = \left\langle \begin{array}{l} \left(1-e^{-((-\ln(1-0.6333))^{3}+(-\ln(1-0.4667))^{3})^{1/3}}, 1-e^{-((-\ln(1-0.6376))^{3}+(-\ln(1-0.6464))^{3})^{1/3}}\right), \\ \left(e^{-((-\ln 0.1333)^{3}+(-\ln 0.2)^{3})^{1/3}}, e^{-((-\ln 0.6534)^{3}+(-\ln 0.6338)^{3})^{1/3}}\right), \\ \left(e^{-((-\ln 0.3)^{3}+(-\ln 0.2333)^{3})^{1/3}}, e^{-((-\ln 0.6783)^{3}+(-\ln 0.7346)^{3})^{1/3}}\right) \end{array} \right\rangle;$$

$$= \left\langle (0.6603,\ 0.7260),\ (0.0991,\ 0.5735),\ (0.1845,\ 0.6411) \right\rangle$$

(2) $$N_{E1} \otimes N_{E2} = \left\langle \begin{array}{l} \left(e^{-((-\ln 0.6333)^{3}+(-\ln 0.4667)^{3})^{1/3}}, e^{-((-\ln 0.6376)^{3}+(-\ln 0.6464)^{3})^{1/3}}\right), \\ \left(1-e^{-((-\ln(1-0.1333))^{3}+(-\ln(1-0.2))^{3})^{1/3}}, 1-e^{-((-\ln(1-0.6534))^{3}+(-\ln(1-0.6338))^{3})^{1/3}}\right), \\ \left(1-e^{-((-\ln(1-0.3))^{3}+(-\ln(1-0.2333))^{3})^{1/3}}, 1-e^{-((-\ln(1-0.6783))^{3}+(-\ln(1-0.7346))^{3})^{1/3}}\right) \end{array} \right\rangle;$$

$$= \left\langle (0.4434,\ 0.5721),\ (0.2143,\ 0.7278),\ (0.3299,\ 0.7898) \right\rangle$$

(3) $$0.6 N_{E1} = \left\langle \begin{array}{l} \left(1-e^{-(0.6(-\ln(1-0.6333))^{3})^{1/3}}, 1-e^{-(0.6(-\ln(1-0.6376))^{3})^{1/3}}\right), \\ \left(e^{-(0.6(-\ln 0.1333)^{3})^{1/3}}, e^{-(0.6(-\ln 0.6534)^{3})^{1/3}}\right), \left(e^{-(0.6(-\ln 0.3)^{3})^{1/3}}, e^{-(0.6(-\ln 0.6783)^{3})^{1/3}}\right) \end{array} \right\rangle;$$

$$= \left\langle (0.5709,\ 0.5752),\ (0.1827,\ 0.6984),\ (0.3622,\ 0.7208) \right\rangle$$

(4) $$(N_{E1})^{0.6}=\left\langle \begin{array}{l} \left(e^{-(0.6(-\ln 0.6333)^3)^{1/3}}, e^{-(0.6(-\ln 0.6376)^3)^{1/3}}\right), \\ \left(1-e^{-(0.6(-\ln(1-0.1333))^3)^{1/3}}, 1-e^{-(0.6(-\ln(1-0.6534))^3)^{1/3}}\right), \\ \left(1-e^{-(0.6(-\ln(1-0.3))^3)^{1/3}}, 1-e^{-(0.6(-\ln(1-0.6783))^3)^{1/3}}\right) \end{array} \right\rangle .$$

$$=\langle (0.6803,\ 0.6841),\ (0.1137,\ 0.5909),\ (0.2598,\ 0.6158)\rangle$$

3. Aczel-Alsina Aggregation Operators of NEEs

3.1 NEEAAWAA Operator

This part proposes the NEEAAWAA operator according to the operations in Definition 5.

Definition 6. Set N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> (k = 1, 2, …, m) as a group of NEEs with the weight vector of N_{Ek} $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_m)$ for $\gamma_k \in [0, 1]$ and $\sum_{k=1}^{m}\gamma_k = 1$. Then, the definition of a NEEAAWAA operator is given by the following form:

$$NEEAAWAA(N_{E1}, N_{E2}, \ldots, N_{Em}) = \mathop{\oplus}_{k=1}^{m} \gamma_k N_{Ek} . \tag{4}$$

Thus, the NEEAAWAA operator has the following theorem.

Theorem 1. Set N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> (k = 1, 2, …, m) as a group of NEEs with the weight vector of N_{Ek} $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_m)$ for $\gamma_k \in [0, 1]$ and $\sum_{k=1}^{m}\gamma_k = 1$. Then, the collected value of the NEEAAWAA operator is till NEE, which is given by the formula:

$$NEEAAWAA(N_{E1}, N_{E2}, \ldots, N_{Em}) = \mathop{\oplus}_{k=1}^{m} \gamma_k N_{Ek} = \left\langle \begin{array}{l} \left(1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\alpha_{Tk}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-e_{Tk}))^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln\alpha_{Ik})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln e_{Ik})^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln\alpha_{Fk})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln e_{Fk})^{\rho}\right)^{1/\rho}}\right) \end{array} \right\rangle . \tag{5}$$

Proof. Theorem 1 is proved by mathematical induction below.

(1) Let k = 2. According to Definition 5 and Eq. (4), the operational results are given as

$$
\begin{aligned}
&NEEAAWAA(N_{E1}, N_{E2}) = \gamma_1 N_{E1} \oplus \gamma_2 N_{E2} \\
&= \left\langle \begin{array}{l} \left(1-e^{-\left(\gamma_1(-\ln(1-\alpha_{T1}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\gamma_1(-\ln(1-e_{T1}))^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\gamma_1(-\ln\alpha_{I1})^{\rho}\right)^{1/\rho}}, e^{-\left(\gamma_1(-\ln e_{I1})^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\gamma_1(-\ln\alpha_{F1})^{\rho}\right)^{1/\rho}}, e^{-\left(\gamma_1(-\ln e_{F1})^{\rho}\right)^{1/\rho}}\right) \end{array} \right\rangle \oplus \left\langle \begin{array}{l} \left(1-e^{-\left(\gamma_2(-\ln(1-\alpha_{T2}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\gamma_2(-\ln(1-e_{T2}))^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\gamma_2(-\ln\alpha_{I2})^{\rho}\right)^{1/\rho}}, e^{-\left(\gamma_2(-\ln e_{I2})^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\gamma_2(-\ln\alpha_{F2})^{\rho}\right)^{1/\rho}}, e^{-\left(\gamma_2(-\ln e_{F2})^{\rho}\right)^{1/\rho}}\right) \end{array} \right\rangle \\
&= \left\langle \begin{array}{l} \left(1-e^{-\left(\sum_{k=1}^{2}\gamma_k(-\ln(1-\alpha_{Tk}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{2}\gamma_k(-\ln(1-e_{Tk}))^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\sum_{k=1}^{2}\gamma_k(-\ln\alpha_{Ik})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{2}\gamma_k(-\ln e_{Ik})^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\sum_{k=1}^{2}\gamma_k(-\ln\alpha_{Fk})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{2}\gamma_k(-\ln e_{Fk})^{\rho}\right)^{1/\rho}}\right) \end{array} \right\rangle. \qquad (6)
\end{aligned}
$$

(2) Assume Eq. (5) for $k = s$ exists. Then, there exists the following result:

$$
NEEAAWAA(N_{E1}, Nz_{E2},..., N_{Es}) = \mathop{\oplus}_{k=1}^{s}\gamma_k N_{Ek} = \left\langle \begin{array}{l} \left(1-e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln(1-\alpha_{Tk}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln(1-e_{Tk}))^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln\alpha_{Ik})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln e_{Ik})^{\rho}\right)^{1/\rho}}\right), \\ \left(e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln\alpha_{Fk})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln e_{Fk})^{\rho}\right)^{1/\rho}}\right) \end{array} \right\rangle. \qquad (7)
$$

(3) Let $k = s$+1. By Eqs. (6) and (7), there is the following result:

$$NEEAAWAA(N_1, N_2, ..., N_s, N_{s+1}) = \mathop{\oplus}_{k=1}^{s+1} \gamma_k N_{Ek}$$

$$= \left\langle \begin{array}{l} \left(1-e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln(1-\alpha_{Tk}))^\rho\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln(1-e_{Tk}))^\rho\right)^{1/\rho}} \right), \\ \left(e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln\alpha_{Ik})^\rho\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln e_{Ik})^\rho\right)^{1/\rho}} \right), \\ \left(e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln\alpha_{Fk})^\rho\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln e_{Fk})^\rho\right)^{1/\rho}} \right) \end{array} \right\rangle \oplus \left\langle \begin{array}{l} \left(1-e^{-\left(\gamma_{m+1}(-\ln(1-\alpha_{Tm+1}))^\rho\right)^{1/\rho}}, 1-e^{-\left(\gamma_{m+1}(-\ln(1-e_{Tm+1}))^\rho\right)^{1/\rho}} \right), \\ \left(e^{-\left(\gamma_{m+1}(-\ln\alpha_{Im+1})^\rho\right)^{1/\rho}}, e^{-\left(\gamma_{m+1}(-\ln e_{Im+1})^\rho\right)^{1/\rho}} \right), \\ \left(e^{-\left(\gamma_{m+1}(-\ln\alpha_{Fm+1})^\rho\right)^{1/\rho}}, e^{-\left(\gamma_{m+1}(-\ln e_{Fm+1})^\rho\right)^{1/\rho}} \right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left(1-e^{-\left(\sum_{k=1}^{s+1}\gamma_k(-\ln(1-\alpha_{Tk}))^\rho\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{s+1}\gamma_k(-\ln(1-e_{Tk}))^\rho\right)^{1/\rho}} \right), \\ \left(e^{-\left(\sum_{k=1}^{s+1}\gamma_k(-\ln\alpha_{Ik})^\alpha\right)^{1/\alpha}}, e^{-\left(\sum_{k=1}^{s+1}\gamma_k(-\ln e_{Ik})^\rho\right)^{1/\rho}} \right), \\ \left(e^{-\left(\sum_{k=1}^{s+1}\gamma_k(-\ln\alpha_{Fk})^\rho\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{s+1}\gamma_k(-\ln e_{Fk})^\rho\right)^{1/\rho}} \right) \end{array} \right\rangle.$$

Based on the above (1)-(3), Eq. (5) can hold for any k. □

Moreover, the NEEAAWAA operator of Eq. (5) implies the following properties.

Theorem 2. The NEEAAWAA operator contains the properties (P1)-(P4):

(P1) *Idempotency:* Set N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> (k = 1, 2, …, m) as a group of NEEs. If $N_{Ek} = N_E$ (k = 1, 2, …, m), there is NEEAAWAA(N_{E1}, N_{E2}, …, N_{Em}) = N_E.

(P2) *Commutativity:* Assume that a group of NEEs $(N'_{E1}, N'_{E2}, ..., N'_{Em})$ is any permutation of (N_{E1}, N_{E2}, …, N_{Em}). Then, $NEEAAWAA\,(N'_{E1}, N'_{E2}, ..., N'_{Em}) = NEEAAWAA\,\left(N_{E1}, N_{E2}, \ldots, N_{Em}\right)$ can exist.

(P3) *Boundedness:* If the maximum and minimum NEEs are specified as follows:

$$N_{E\max} = \left\langle \left(\max_k(\alpha_{Tk}), \max_k(e_{Tk})\right), \left(\min_k(\alpha_{Ik}), \min_k(e_{Ik})\right), \left(\min_k(\alpha_{Fk}), \min_k(e_{Fk})\right) \right\rangle,$$

$$N_{E\min} = \left\langle \left(\min_k(\alpha_{Tk}), \min_k(e_{Tk})\right), \left(\max_k(\alpha_{Ik}), \max_k(e_{Ik})\right), \left(\max_k(\alpha_{Fk}), \max_k(e_{Fk})\right) \right\rangle,$$

then $N_{E\min}$ ≤ NEEAAWAA(N_{E1}, N_{E2}, …, N_{Em}) ≤ $N_{E\max}$ can exist.

(P4) *Monotonicity:* If $N_{Ek} \le N^*_{Ek}$ (k = 1, 2, …, m), there is NEEAAWAA(N_{E1}, N_{E2}, …, N_{Em}) ≤ NEEAAWAA($N^*_{E1}, N^*_{E2}, ..., N^*_{Em}$).

Proof. (P1) If N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> = N_E (k = 1, 2, …, m), by Eq. (4) we yield the result:

$$NEEAAWAA(N_{E1},N_{E2},...,N_{Em})=\mathop{\oplus}_{k=1}^{m}\gamma_k N_{Ek}$$

$$=\left\langle\begin{array}{l}\left(1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\alpha_{Tk}))^{\rho}\right)^{1/\rho}},1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-e_{Tk}))^{\rho}\right)^{1/\rho}}\right),\\ \left(e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln\alpha_{Ik})^{\rho}\right)^{1/\rho}},e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln e_{Ik})^{\rho}\right)^{1/\rho}}\right),\\ \left(e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln\alpha_{Fk})^{\rho}\right)^{1/\rho}},e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln e_{Fk})^{\rho}\right)^{1/\rho}}\right)\end{array}\right\rangle=\left\langle\begin{array}{l}\left(1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\alpha_{T}))^{\rho}\right)^{1/\rho}},1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-e_{T}))^{\rho}\right)^{1/\rho}}\right),\\ \left(e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln\alpha_{I})^{\rho}\right)^{1/\rho}},e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln e_{I})^{\rho}\right)^{1/\rho}}\right),\\ \left(e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln\alpha_{F})^{\rho}\right)^{1/\rho}},e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln e_{F})^{\rho}\right)^{1/\rho}}\right)\end{array}\right\rangle$$

$$=\left\langle\begin{array}{l}\left(1-e^{-\left((-\ln(1-\alpha_T))^{\rho}\right)^{1/\rho}},1-e^{-\left((-\ln(1-e_T))^{\rho}\right)^{1/\rho}}\right),\\ \left(e^{-\left((-\ln\alpha_I)^{\rho}\right)^{1/\rho}},e^{-\left((-\ln e_I)^{\rho}\right)^{1/\rho}}\right),\\ \left(e^{-\left((-\ln\alpha_F)^{\rho}\right)^{1/\rho}},e^{-\left((-\ln e_F)^{\rho}\right)^{1/\rho}}\right)\end{array}\right\rangle=\left\langle\begin{array}{l}\left(1-e^{\ln(1-\alpha_T)},1-e^{\ln(1-e_T)}\right),\\ \left(e^{\ln\alpha_I},e^{\ln e_I}\right),\\ \left(e^{\ln\alpha_F},e^{\ln e_F}\right)\end{array}\right\rangle=\langle(\alpha_T,e_T),(\alpha_I,e_I),(\alpha_F,e_F)\rangle=N_E.$$

(P2) The property (P2) is straightforward.

(P3) Since the inequalities $\min_k(\alpha_{Tk})\le\alpha_{Tk}\le\max_k(\alpha_{Tk})$, $\min_k(e_{Tk})\le e_{Tk}\le\max_k(e_{Tk})$, $\min_k(\alpha_{Ik})\le \alpha_{Ik}\le\max_k(\alpha_{Ik})$, $\min_k(e_{Ik})\le e_{Ik}\le\max_k(e_{Ik})$, $\min_k(\alpha_{Fk})\le \alpha_{Fk}\le\max_k(\alpha_{Fk})$, and $\min_k(e_{Fk})\le e_{Fk}\le\max_k(e_{Fk})$ exist based on the maximum and minimum NEEs, there are the following inequalities:

$$1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\min_k(\alpha_{Tk})))^{\rho}\right)^{1/\rho}}\le 1-e^{-\left(\sum_{k=1}^{s}\gamma_k(-\ln(1-\alpha_{Tk}))^{\rho}\right)^{1/\rho}}\le 1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\max_k(\alpha_{Tk})))^{\rho}\right)^{1/\rho}},$$

$$1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\min_k(e_{Tk})))^{\alpha}\right)^{1/\alpha}}\le 1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-e_{Tk}))^{\rho}\right)^{1/\rho}}\le 1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\max_k(e_{Tk})))^{\rho}\right)^{1/\rho}},$$

$$e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\max_k(\alpha_{Ik})))^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln\alpha_{Ik})^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\min_k(\alpha_{Ik})))^{\rho}\right)^{1/\rho}},$$

$$e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\max_k(e_{Ik})))^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln e_{Ik})^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\min_k(e_{Ik})))^{\rho}\right)^{1/\rho}},$$

$$e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\max_k(\alpha_{Fk})))^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-ln\alpha_{Fk})^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\min_k(\alpha_{Fk})))^{\rho}\right)^{1/\rho}},$$

$$e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\max_k(e_{Fk})))^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-ln e_{Fk})^{\rho}\right)^{1/\rho}}\le e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(\min_k(e_{Fk})))^{\rho}\right)^{1/\rho}}.$$

Regarding the property (P1) and the score value of Eq. (2), we can obtain $N_{E\min}\le\mathop{\oplus}_{k=1}^{m}\gamma_k N_{Ek}\le N_{E\max}$, then there is $N_{E\min}\le$ NEEAAWAA(N_{E1}, N_{E2}, …, N_{Em}) $\le N_{E\max}$.

(P4) When $N_{Ek}\le N_{Ek}^{*}$ (k = 1, 2, …, m), there exists $\mathop{\oplus}_{k=1}^{m}\gamma_k N_{Ek}\le\mathop{\oplus}_{k=1}^{m}\gamma_k N_{Ek}^{*}$. Thus, NEEAAWAA($N_{E1}$, N_{E2}, …, N_{Em}) ≤ NEEAAWAA($N_{E1}^{*},N_{E2}^{*},...,N_{Em}^{*}$) can exist. □

Especially when ρ = 1, the NEEAAWAA operator of Eq. (5) is reduced to the NEE weighted arithmetic averaging (NEEWAA) operator:

$$NEEWAA(N_{E1}, Nz_{E2}, ..., N_{Em}) = \overset{m}{\underset{k=1}{\oplus}} \gamma_k N_{Ek} = \left\langle \begin{array}{l} \left(1-\prod_{k=1}^{m}(1-\alpha_{Tk})^{\gamma_k}, 1-\prod_{k=1}^{m}(1-e_{Tk})^{\gamma_k}\right), \\ \left(\prod_{k=1}^{m}(\alpha_{Ik})^{\gamma_k}, \prod_{k=1}^{m}(e_{Ik})^{\gamma_k}\right), \left(\prod_{k=1}^{m}(\alpha_{Fk})^{\gamma_k}, \prod_{k=1}^{m}(e_{Fk})^{\gamma_k}\right) \end{array} \right\rangle. \quad (8)$$

3.2 NEEAAWGA Operator

This part presents the NEEAAWGA operator according to the operations in Definition 5.

Definition 7. Set N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> (k = 1, 2, …, m) as a group of NEEs with the weight vector of N_{Ek} $\boldsymbol{\gamma}$= (γ_1, γ_2, …, γ_m) for $\gamma_k \in$ [0, 1] and $\sum_{k=1}^{m}\gamma_k = 1$. Thus, a NEEAAWGA operator is defined as

$$NEEAAWGA(N_{E1}, N_{E2}, ..., N_{Em}) = \overset{m}{\underset{k=1}{\otimes}} (N_{Ek})^{\gamma_k}. \quad (9)$$

Then, the NEEAAWGA operator shows the following theorem.

Theorem 3. Set N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> (k = 1, 2, …, m) as a group of NEEs with the weight vector of N_{Ek} $\boldsymbol{\gamma}$ = (γ_1, γ_2, …, γ_m) for $\gamma_k \in$ [0, 1] and $\sum_{k=1}^{m}\gamma_k = 1$. Then, the collected value of the NEEAAWGA operator is also NEE, which is obtained by the following formula:

$$NEEAAWGA(N_{E1}, N_{E2}, ..., N_{Em}) = \overset{m}{\underset{k=1}{\otimes}} (N_{Ek})^{\gamma_k} = \left\langle \begin{array}{l} \left(e^{-\left(\sum_{k=1}^{m}\beta_k(-\ln\alpha_{Tk})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln e_{Tk})^{\rho}\right)^{1/\rho}} \right), \\ \left(1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\alpha_{Ik}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-e_{Ik}))^{\rho}\right)^{1/\rho}} \right), \\ \left(1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-\alpha_{Fk}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{m}\gamma_k(-\ln(1-e_{Fk}))^{\rho}\right)^{1/\rho}} \right) \end{array} \right\rangle. \quad (10)$$

By the similar proof way of Theorem 1, we can easily verify Theorem 3, which is omitted.

Similarly, the NEEAAWGA operator also contains some properties.

Theorem 4. The NEEAAWGA operator includes these properties (P1)-(P4):

(P1) *Idempotency:* Set N_{Ek} = <(α_{Tk}, e_{Tk}), (α_{Ik}, e_{Ik}), (α_{Fk}, e_{Fk})> (k = 1, 2, …, m) as a group of NEEs. When N_{Ek} = N_E (k = 1, 2, …, m), NEEAAWGA(N_{E1}, N_{E2}, …, N_{Em}) = N_E exists.

(P2) *Commutativity:* Assume that a group of NEEs $(N_{E1}^{'}, N_{E2}^{'}, ..., N_{Em}^{'})$ is any permutation of (N_{E1}, N_{E2}, …, N_{Em}). Then, $NEEAAWGA(N_{E1}^{'}, N_{E2}^{'}, ..., N_{Em}^{'}) = NEEAAWGA\left(N_{E1}, N_{E2}, \ldots, N_{Em}\right)$ can exist.

(P3) *Boundedness:* If the maximum and minimum NEEs are specified below:

$N_{E\max} = \left\langle \left(\max_k(\alpha_{Tk}), \max_k(e_{Tk})\right), \left(\min_k(\alpha_{Ik}), \min_k(e_{Ik})\right), \left(\min_k(\alpha_{Fk}), \min_k(e_{Fk})\right) \right\rangle$,

$N_{E\min} = \left\langle \left(\min_k(\alpha_{Tk}), \min_k(e_{Tk})\right), \left(\max_k(\alpha_{Ik}), \max_k(e_{Ik})\right), \left(\max_k(\alpha_{Fk}), \max_k(e_{Fk})\right) \right\rangle$,

then $N_{E\min}$ ≤ NEEAAWGA(N_{E1}, N_{E2}, …, N_{Em}) ≤ $N_{E\max}$ can hold.

(P4) *Monotonicity:* Set $N_{Ek} \le N_{Ek}^{*}$ (k = 1, 2, …, m). Then, NEEAAWGA(N_{E1}, N_{E2}, …, N_{Em}) ≤ NEEAAWGA($N_{E1}^{*}, N_{E2}^{*}, ..., N_{Em}^{*}$) exists.

By the similar proof method of Theorem 2, we can easily verify Theorem 4, which is not repeated here.

Especially when $\rho = 1$, the NEEAAWGA operator is reduced to the NEE weighted geometric averaging (NEEWGA) operator:

$$NEEWGA(N_{E1}, Nz_{E2}, ..., N_{Em}) = \overset{m}{\underset{k=1}{\otimes}} (N_{Ek})^{\gamma_k}$$
$$= \left\langle \begin{array}{l} \left(\prod_{k=1}^{m} (\alpha_{Tk})^{\gamma_k}, \prod_{k=1}^{m} (e_{Tk})^{\gamma_k} \right), \left(1 - \prod_{k=1}^{m} (1-\alpha_{Ik})^{\gamma_k}, 1 - \prod_{k=1}^{m} (1-e_{Ik})^{\gamma_k} \right), \\ \left(1 - \prod_{k=1}^{m} (1-\alpha_{Fk})^{\gamma_k}, 1 - \prod_{k=1}^{m} (1-e_{Fk})^{\gamma_k} \right) \end{array} \right\rangle. \quad (11)$$

4. MAGDM Model Based on the NEEAAWAA and NEEAAWGA Operators and the Score and Accuracy Functions

In this section, a MAGDM model is established by the proposed NEEAAWAA and NEEAAWGA operators and score and accuracy functions to solve MAGDM problems in the NMVS setting.

Regarding a MAGDM problem, a set of s alternatives $L = \{L_1, L_2, …, L_s\}$ is preliminarily provided and satisfactorily evaluated by a set of m attributes $B = \{b_1, b_2, …, b_m\}$. Then, the importance of various attributes b_k ($k = 1, 2, …, m$) is assigned by a weight vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, …, \gamma_m)$ with $\gamma_k \in [0, 1]$ and $\sum_{k=1}^{m} \gamma_k = 1$. The satisfactory evaluation values of each alternative L_i ($i = 1, 2, …, s$) over each attribute b_k ($k = 1, 2, …, m$) are assigned by a group of experts/decision makers, then the evaluated truth, indeterminacy, and falsity sequences are denoted as the NMVE $Ms_{ik} = \langle M_{Tik}, M_{Iik}, M_{Fik} \rangle = \langle (\alpha_{Tik}^1, \alpha_{Tik}^2, ..., \alpha_{Tik}^{r_k}), (\alpha_{Iik}^1, \alpha_{Iik}^2, ..., \alpha_{Iik}^{r_k}), (\alpha_{Fik}^1, \alpha_{Fik}^2, ..., \alpha_{Fik}^{r_k}) \rangle$ for $0 \le \sup M_{Tik} + \sup M_{Iik} + \sup M_{Fik} \le 3$ and α_{Tik}^j, $\alpha_{Iik}^j, \alpha_{Fik}^j \in [0, 1]$ ($j = 1, 2, …, r_k$; $i = 1, 2, …, s$; $k = 1, 2, …, m$). Then, the evaluated NMVEs are represented as the decision matrix of NMVEs $M_D = (Ms_{ik})_{s\times m}$.

Regarding this MAGDM problem, we give the decision steps below.

Step 1: By the formulae (1)-(3) in Definition 2, all NMVEs in the decision matrix M_D are conversed into the NEEs $N_{Eik} = \langle(\alpha_{Tik}, e_{Tik}), (\alpha_{Iik}, e_{Iik}), (\alpha_{Fik}, e_{Fik})\rangle$ for $\alpha_{Tik}, \alpha_{Iik}, \alpha_{Fik} \in [0, 1]$ and $e_{Tik}, e_{Iik}, e_{Fik} \in [0, 1]$ ($i = 1, 2, …, s$; $k = 1, 2, …, m$), which are constructed as the decision matrix of NEEs $N_D = (N_{Eik})_{s\times m}$.

Step 2: Using one of Eq. (5) and Eq. (10), the aggregated NEE N_{Ei} ($i = 1, 2, …, s$) for L_i is given by one of two formulae:

$$N_{Ei} = NEEAAWAA(N_{Ei1}, N_{Ei2}, ..., N_{Eim}) = \overset{m}{\underset{k=1}{\oplus}} \gamma_k N_{Eik} = \left\langle \begin{array}{l} \left(1 - e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln(1-\alpha_{Tik}))^{\rho}\right)^{1/\rho}}, 1 - e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln(1-e_{Tik}))^{\rho}\right)^{1/\rho}} \right), \\ \left(e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln \alpha_{Iik})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln e_{Iik})^{\rho}\right)^{1/\rho}} \right), \\ \left(e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln \alpha_{Fik})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln e_{Fik})^{\rho}\right)^{1/\rho}} \right) \end{array} \right\rangle, \quad (12)$$

$$N_{Ei} = NEEAAWGA(N_{Ei1}, N_{Ei2}, ..., N_{Eim}) = \overset{m}{\underset{k=1}{\otimes}} (N_{Eik})^{\gamma_k} = \left\langle \begin{array}{l} \left(e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln \alpha_{Tik})^{\rho}\right)^{1/\rho}}, e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln e_{Tik})^{\rho}\right)^{1/\rho}} \right), \\ \left(1-e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln(1-\alpha_{Iik}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln(1-e_{Iik}))^{\rho}\right)^{1/\rho}} \right), \\ \left(1-e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln(1-\alpha_{Fik}))^{\rho}\right)^{1/\rho}}, 1-e^{-\left(\sum_{k=1}^{m} \gamma_k (-\ln(1-e_{Fik}))^{\rho}\right)^{1/\rho}} \right) \end{array} \right\rangle. \quad (13)$$

Step 3: The score values of $R(N_{Ei})$ (the accuracy values of $Q(N_{Ei})$ subject to necessary) (i = 1, 2, …, s) are obtained by Eq. (2) (Eq. (3)).

Step 4: All alternatives are sorted based on the ranking laws and the best one is chosen.

Step 5: End.

5. Illustrative Example and Comparison with Existing Techniques

5.1 Example on the Performance Assessment of Service Robots

Service robotics contain many application fields, such as industrial service robots, home service robots, and medical service robots. They are improving our daily lives in various ways. Then, most of them have unique designs and different degrees of automation (from full teleoperation to fully autonomous operation) to affect the quality of our work and lives. However, the performance evaluation of the service robots is an important issue for users. To indicate the applicability of the developed MAGDM model under the environment of NMVSs, this subsection applies the developed MAGDM model to the performance assessment of service robots.

Suppose that there are four kinds of service robots/alternatives, which are denoted as their set $L = \{L_1, L_2, L_3, L_4\}$. Then, they must satisfy the requirements of the four performance indices/attributes: mobility (b_1), dexterity (b_2), working ability (b_3), and communication and control capability (b_4). The weight vector of the four attributes is given as $\boldsymbol{\gamma}$ = (0.25, 0.24, 0.26, 0.25) by experts/decision makers. The assessment of four types of service robots over the four attributes is performed by three experts/decision makers, where their evaluation values are assigned by the NMVEs $Ms_{ik} = \langle M_{Tik}, M_{Iik}, M_{Fik} \rangle = \langle (\alpha_{Tik}^1, \alpha_{Tik}^2, ..., \alpha_{Tik}^{r_k}), (\alpha_{Iik}^1, \alpha_{Iik}^2, ..., \alpha_{Iik}^{r_k}), (\alpha_{Fik}^1, \alpha_{Fik}^2, ..., \alpha_{Fik}^{r_k}) \rangle$ (consisting of the truth, indeterminacy, and falsity sequences) for $0 \le \sup M_{Tik} + \sup M_{Iik} + \sup M_{Fik} \le 3$ and α_{Tik}^j, $\alpha_{Iik}^j, \alpha_{Fik}^j \in [0, 1]$ (j = 1, 2, 3; i, k = 1, 2, 3, 4; r_k = 3). Thus, all assessed NMVEs can be expressed as the following decision matrix of NMVEs $M_D = (Ms_{ik})_{4\times4}$:

$$M_D = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} \left\langle \begin{matrix} (0.8,0.7,0.7), \\ (0.3,0.2,0.1), \\ (0.2,0.2,0.1) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.8,0.7,0.6), \\ (0.3,0.1,0.1), \\ (0.4,0.3,0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.7,0.7,0.6), \\ (0.3,0.3,0.3), \\ (0.3,0.2,0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.7,0.7,0.7), \\ (0.3,0.1,0.1), \\ (0.3,0.3,0.3) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (0.7,0.7,0.6), \\ (0.2,0.2,0.1), \\ (0.2,0.2,0.1) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.7,0.7,0.7), \\ (0.3,0.3,0.2), \\ (0.3,0.1,0.1) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.8,0.8,0.7), \\ (0.1,0.1,0.1), \\ (0.3,0.2,0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.8,0.8,0.8), \\ (0.2,0.1,0.1), \\ (0.4,0.3,0.3) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (0.8,0.7,0.6), \\ (0.3,0.3,0.2), \\ (0.3,0.2,0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.7,0.7,0.6), \\ (0.2,0.1,0.1), \\ (0.3,0.3,0.1) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.8,0.7,0.7), \\ (0.1,0.1,0.1), \\ (0.2,0.1,0.1) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.8,0.8,0.7), \\ (0.2,0.1,0.1), \\ (0.4,0.4,0.3) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (0.8,0.8,0.6), \\ (0.2,0.2,0.1), \\ (0.3,0.2,0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.9,0.8,0.8), \\ (0.1,0.1,0.1), \\ (0.3,0.2,0.1) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.8,0.7,0.7), \\ (0.4,0.4,0.2), \\ (0.5,0.3,0.3) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.7,0.7,0.7), \\ (0.2,0.1,0.1), \\ (0.2,0.2,0.1) \end{matrix} \right\rangle \end{bmatrix}.$$

In the MAGDM example, the proposed MAGDM model is given by the following decision process.

First, using the formulae (1)-(3) in Definition 2 for the decision matrix $M_D = (M_{Sik})_{4\times4}$, we obtain the NEE decision matrix N_D:

$$N_D = \begin{bmatrix} \langle(0.7333, 0.9981), (0.2000, 0.9206), (0.1667, 0.9602)\rangle & \langle(0.7000, 0.9938), (0.1667, 0.8650), (0.3000, 0.9656)\rangle & \langle(0.6667, 0.9977), (0.3000, 1.0000), (0.2333, 0.9821)\rangle & \langle(0.7000, 1.0000), (0.1667, 0.8650), (0.3000, 1.0000)\rangle \\ \langle(0.6667, 0.9977), (0.1667, 0.9602), (0.1667, 0.9602)\rangle & \langle(0.7000, 1.0000), (0.2667, 0.9851), (0.1667, 0.8650)\rangle & \langle(0.7667, 0.9983), (0.1000, 1.0000), (0.2333, 0.9821)\rangle & \langle(0.8000, 1.0000), (0.1333, 0.9464), (0.3333, 0.9461)\rangle \\ \langle(0.7000, 0.9938), (0.2667, 0.9851), (0.2333, 0.9821)\rangle & \langle(0.6667, 0.9977), (0.1333, 0.9464), (0.2333, 0.9141)\rangle & \langle(0.7333, 0.9981), (0.1000, 1.0000), (0.1333, 0.9464)\rangle & \langle(0.7667, 0.9983), (0.1333, 0.9464), (0.3667, 0.9922)\rangle \\ \langle(0.7333, 0.9922), (0.1667, 0.9602), (0.2333, 0.9414)\rangle & \langle(0.8333, 0.9986), (0.1000, 1.0000), (0.2000, 0.9206)\rangle & \langle(0.7333, 0.9981), (0.3333, 0.9602), (0.3667, 0.9713)\rangle & \langle(0.7000, 1.0000), (0.1333, 0.9464), (0.1667, 0.9602)\rangle \end{bmatrix}.$$

Then using one of Eqs. (12) and (13), the aggregated NEEs N_{Ei} (i = 1, 2, …, s) are calculated corresponding to various values of ρ, and then score values of N_{Ei} (i = 1, 2, …, s) for L_i and ranking orders of the four alternatives are given by Eq. (2) and the ranking laws, which are shown in Tables 1 and 2.

Table 1. Decision results based on Eq. (12) and Eq. (2)

ρ	Score value	Ranking	The best one
1	0.7594, 0.7953, 0.7863, 0.7909	$L_2 > L_4 > L_3 > L_1$	L_2
3	0.7673, 0.8067, 0.7981, 0.8038	$L_2 > L_4 > L_3 > L_1$	L_2
5	0.7730, 0.8148, 0.8066, 0.8133	$L_2 > L_4 > L_3 > L_1$	L_2
7	0.7777, 0.8209, 0.8130, 0.8206	$L_2 > L_4 > L_3 > L_1$	L_2
9	0.7815, 0.8255, 0.8178, 0.8264	$L_4 > L_2 > L_3 > L_1$	L_4
11	0.7846, 0.8290, 0.8217, 0.8309	$L_4 > L_2 > L_3 > L_1$	L_4

Table 2. Decision results based on Eq. (13) and Eq. (2)

ρ	Score value	Ranking	The best one
1	0.7448, 0.7820, 0.7717, 0.7728	$L_2 > L_4 > L_3 > L_1$	L_2
3	0.7340, 0.7617, 0.7496, 0.7446	$L_2 > L_3 > L_4 > L_1$	L_2
5	0.7250, 0.7455, 0.7326, 0.7247	$L_2 > L_3 > L_4 > L_1$	L_2
7	0.7183, 0.7341, 0.7212, 0.7123	$L_2 > L_3 > L_1 > L_4$	L_2
9	0.7135, 0.7262, 0.7134, 0.7043	$L_2 > L_1 > L_3 > L_4$	L_2
11	0.7100, 0.7207, 0.7079, 0.6988	$L_2 > L_1 > L_3 > L_4$	L_2

According to the decision results in Tables 1 and 2, the ranking orders produced by Eq. (12) and Eq. (13) show their difference, then the best alternative L_2 is the same by taking ρ = 1, 3. Moreover, in the proposed MAGDM model, using different values of ρ and different aggregation operators can affect the ranking orders of alternatives and show its decision flexibility, then the change of the parameter ρ is sensitive to the ranking impact of alternatives. However, the best alternative of the example is L_2 or L_4 depending on a preference selection of decision makers.

5.2 *Comparison with existing techniques in the setting of NMVSs*

In this part, we compare our new techniques with existing techniques [14] in the setting of NMVSs.

On the one hand, the characteristic comparison between our new techniques and the existing techniques is indicated in Table 3.

Table 3. Characteristic comparison between our new techniques and the existing techniques

Method	**Evaluation information**	**Conversion form**	**Decision-making model with an adjustable parameter**	**Using condition**
Existing techniques [14]	NMVS/NMVE	CNE based on the mean and consistency degree (complement of standard deviation)	No	Normal distribution
Our new techniques	NMVS/NMVE	NEE based on the mean and Shannon entropy	Yes	No limitation

Regarding the comparative results of Table 3, our new techniques are often broader and more versatile than the existing techniques when dealing with MAGDM problems in the setting of NMVSs.

On the other hand, we can apply the existing MAGDM model using two consistency neutrosophic correlation coefficients [14] to the above example. By the existing MAGDM model using two consistency neutrosophic correlation coefficients, we give all decision results, which are shown in Table 4.

Table 4. Decision results of the existing MAGDM model using two correlation coefficients

Existing decision-making model	**Ranking**	**The best one**
Correlation coefficient 1 [14]	$L_2 > L_3 > L_4 > L_1$	L_2
Correlation coefficient 2 [14]	$L_1 > L_3 > L_4 > L_2$	L_1

Although there is the same ranking order between the existing MAGDM model using the correlation coefficient 1 [14] and our proposed MAGDM model using the NEEAAWGA operator for ρ = 3, 5, the existing MAGDM model lacks its decision flexibility. Furthermore, in the existing MAGDM model, the conversion technique based on the mean and standard deviation only is suitable for the normal distribution of multi-valued sequences in NMVEs. Therefore, our proposed model can not only overcome the limitation and insufficiency of the existing model [14], but also show its outstanding advantage of diversified decision results to satisfy the preference order of decision makers in actual applications. However, our new conversion method and decision-making model are superior to the existing ones in the setting of NMVSs.

6. Conclusions

To overcome the shortcomings of existing MAGDM method under the environment of NMVSs, this study proposed a NEE concept based on the normalized Shannon extropy and average values of the truth, falsity, and indeterminacy sequences in NMVSs to overcome the limitation of the existing conversion method based on the mean and standard deviation of the truth, falsity, and indeterminacy sequences. Then, the proposed ranking laws based on the score and accuracy functions of NEEs and the proposed Aczel-Alsina t-norm and t-conorm operations and NEEAAWAA and NEEAAWGA operators provided important mathematical tools for solving flexible decision-making issues in the case of NMVSs. The developed MAGDM model can effectively carry out flexible decision-making issues with the information of NMVSs, where various parameter values can affect ranking orders of alternatives to satisfy decision makers' preference requirements. Finally, an illustrative example was given to verify the efficiency and rationality of the developed MAGDM model. Compared with the existing techniques, our proposed techniques are broader and more versatile than the existing techniques when dealing with MAGDM problems in the case of NMVSs. However, in this study, the proposed information expression, operations, and aggregation operators of NEEs and the established MAGDM method show the highlighting advantages of these new techniques.

Regarding these new techniques, we have many future researches to be performed in various areas, such as image processing, medical diagnosis, and information fusion. Meanwhile, the proposed Aczel-Alsina t-norm and t-conorm operations and aggregation operators of NEEs are also extended to cubic neutrosophic sets, refined neutrosophic sets, consistency neutrosophic sets, neutrosophic rough sets, etc. Then, they can be applied in engineering management, slope risk/stability evaluation, as well as clustering analysis, information retrieval, data mining, and so on in the case of NMVSs.

Data Availability: All data generated or analyzed during this study are included in this article.

Conflict of Interest: The authors declare no conflict of interest.

References

1. Smarandache, F. Neutrosophy: neutrosophic probability, set, and logic. American Research Press, Rehoboth, USA, **1998**.
2. Liu, P.; Shi, L. The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making. *Neural Computing and Applications* **2015**, 26, 457–471.
3. Şahin, R.; Liu, P. Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making. *Neural Computing and Applications* **2017**, 28, 1387–1395.
4. Li, X.; Zhang, X. Single-valued neutrosophic hesitant fuzzy Choquet aggregation operators for multi-attribute decision making. *Symmetry* **2018**, 10(2), 50.
5. Biswas, P.; Pramanik, S.; Giri, B.C. NH-MADM strategy in neutrosophic hesitant fuzzy set environment based on extended GRA. *Informatica* **2019**, 30(2), 213–242.
6. Giri, B.C.; Molla, M.U.; Biswas, P. TOPSIS method for neutrosophic hesitant fuzzy multi-attribute decision making. *Informatica* **2020**, 31(1), 35–63.
7. Peng, J.J.; Wang, J.Q.; Wu, X.H.; Wang, J.; Chen, X.H. Multivalued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. *International Journal of Computational Intelligence Systems* **2015**, 8(2), 345–363.
8. Liu, P.; Zhang, L.; Liu, X.; Wang, P. Multi-valued neutrosophic number Bonferroni mean operators with their applications in multiple attribute group decision making. *International Journal of Information Technology & Decision Making* **2016**, 15(05), 1181–1210.
9. Ye, S.; Ye, J. Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis. *Neutrosophic Sets and Systems* **2014**, 6, 48–53.
10. Ye, J. Correlation coefficient between dynamic single valued neutrosophic multisets and its multiple attribute decision-making method. *Information* **2017**, 8, 41.
11. Wang, J.Q.; Li, X.E. TODIM method with multi-valued neutrosophic sets. *Control and Decision* **2015**, 30(6), 1139–1142.

12. Ji, P.; Zhang, H.Y.; Wang, J.Q. A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection. *Neural Computing and Applications* **2018**, 29(1), 221–234.
13. Fa, C.X.; Fan, E.; Ye, J. The cosine measure of single-valued neutrosophic multisets for multiple attribute decision-making. *Symmetry* **2018**, 10, 154.
14. Ye, J.; Song, J.M.; Du, S.G. Correlation coefficients of consistency neutrosophic sets regarding neutrosophic multi-valued sets and their multi-attribute decision-making method. *International Journal of Fuzzy Systems* **2022**, 24(2), 925–932.
15. Liu, P.; Cheng, S. An improved MABAC group decision-making method using regret theory and likelihood in probability multi-valued neutrosophic sets. *International Journal of Information Technology & Decision Making* **2020**, 19(05), 1353–1387.
16. Aczel, J.; Alsina, C. Characterization of some classes of quasilinear functions with applications to triangular norms and to synthesizing judgements. *Aequationes mathematicae* **1982**, 25(1), 313–315.
17. Alsina, C.; Frank, M.J.; Schweizer, B. *Associative functions: Triangular norms and copulas*. World Scientific Publishing, Danvers, MA, **2006**.
18. Fu, J.; Ye, J.; Xie, L. Group decision-making model of renal cancer surgery options using entropy fuzzy element Aczel-Alsina weighted aggregation operators under the environment of fuzzy multi-sets. *CMES-Computer Modeling in Engineering & Sciences* **2022**, 130(3), 1751–1769.
19. Yong, R.; Ye, J.; Du, S.G.; Zhu, A.; Zhang, Y.Y. Aczel-Alsina weighted aggregation operators of simplified neutrosophic numbers and its application in multiple attribute decision making. *CMES-Computer Modeling in Engineering & Sciences* **2022**, 132(2), 569–583.
20. Senapati, T. Approaches to multi-attribute decision-making based on picture fuzzy Aczel–Alsina average aggregation operators. *Computational and Applied Mathematics* **2022**, 41(1), 1–19.
21. Hussain, A.; Ullah, K.; Yang, M.S.; Pamucar, D. Aczel-Alsina Aggregation Operators on T-spherical fuzzy (TSF) information with application to TSF multi-attribute decision making. *IEEE Access* **2022**, 10, 26011–26023.
22. Senapati, T.; Chen, G.; Mesiar, R.; Yager, R.R. Novel Aczel–Alsina operations-based interval-valued intuitionistic fuzzy aggregation operators and their applications in multiple attribute decision-making process. *International Journal of Intelligent Systems* **2022**, 37(8), 5059–5081.
23. Senapati, T.; Chen, G.; Mesiar, R.; Yager, R.R. Intuitionistic fuzzy geometric aggregation operators in the framework of Aczel-Alsina triangular norms and their application to multiple attribute decision making. *Expert Systems with Applications* **2022**, 118832.
24. Senapati, T.; Chen, G.; Yager, R.R. Aczel–Alsina aggregation operators and their application to intuitionistic fuzzy multiple attribute decision making. *International Journal of Intelligent Systems* **2022**, 37(2), 1529–1551.
25. Senapati, T.; Chen, G.; Mesiar, R.; Yager, R.R.; Saha, A. Novel Aczel–Alsina operations-based hesitant fuzzy aggregation operators and their applications in cyclone disaster assessment. *International Journal of General Systems* **2022**, 51(5), 511–546.
26. Shannon, C.E. A mathematical theory of communication. *The Bell System Technical Journal* **1948**, 27, 379–423.

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