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Generalized Symmetric Neutrosophic Fuzzy Matrices

M.Anandhkumar[1], G.Punithavalli[2], T.Soupramanien[3], Said Broumi[4]

[1]Assistant Professor, Department of Mathematics, IFET College of Engineering (Autonomous), Villupuram, Tamilnadu, India.
anandhkumarmm@mail.com

[2]Assistant Professor, Department of Mathematics Annamalai University (Deputed to Government Arts College, Chidambaram)
punithavarman78@gmail.com

[3] Professor, Department of Mathematics, IFET College of Engineering (Autonomous), Villupuram, Tamilnadu, India.
soupramani@gmail.com

[4]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com

Abstract –We develop the concept of range symmetric Neutrosophic Fuzzy Matrix and Kernel symmetric Neutrosophic Fuzzy Matrix analogous to that of an EP –matrix in the complex field. First we present equivalent characterizations of a range symmetric matrix and then derive equivalent conditions for a Neutrosophic Fuzzy Matrix to be kernel symmetric matrix and study the relation between range symmetric and kernel symmetric Neutrosophic Fuzzy Matrices. The idea of Kernel and k-Kernel Symmetric (k-KS) Neutrosophic Fuzzy Matrices (NFM) are introduced with an example. We present some basic results of kernel symmetric matrices. We show that k-symmetric implies k-Kernel symmetric but the converse need not be true. The equivalent relations between kernel symmetric, k-kernel symmetric and Moore-Penrose inverse of NFM are explained with numerical results.

Keywords: Range symmetric, Kernel symmetric, k-Kernel Symmetric, Moore-penrose inverse

1. Introduction

The concept of fuzzy set was introduced by Zadeh [1] in 1965. The traditional fuzzy sets are characterized by the membership value or the grade of membership value. Some- times it may be very difficult to assign the membership value for fuzzy sets. An intuitionistic fuzzy set introduced by Atanassov [2] is appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership (or simply membership) and falsity-membership(or nonmembership)values. It does not handle the indeterminate and inconsistent information which exists in belief system. Smarandache [3] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.

For a fuzzy matrix P, if P^+ exists, then it coincide with P^T, Kim and Roush [4] have studied Generalized fuzzy matrices. A Fuzzy matrix P is range symmetric if $R[P] = R[P^T]$ implies and kernel symmetric $N(P)=N(P^T)$. It is well known that for complex matrices, the concept of range symmetric and kernel symmetric is identical. For Neutrosophic Fuzzy matrix $P \in (IF)_n$, is range symmetric R[P] = R[P^T] implies N(P) = N(P^T) but the converse need not be true. Meenakshi [5] introduced the notion of fuzzy matrix. Let k-be a fixed product of disjoint transpositions in Sn = 1, 2,...,n and K be the associated permutation matrix. Hill and Waters [6] have introduced on k-real and k-hermitian matrices. Baskett and Katz [7] have studied theorems on products of EPr matrices. Schwerdtfeger [8] has studied the notion of introduction to Linear Algebra and the Theory of matrices. Meenakshi and Jayashri [9] have studied k-Kernel Symmetric Matrices. Riyaz Ahmad Padder and Murugadas [10-12] introduced on idempotent intuitionistic fuzzy Matrices of T-type, reduction of a nilpotent intuitionistic fuzzy matrix using implication operator and determinant theory for intuitionistic fuzzy matrices. Atanassov has studied [13] generalized index matrices. Meenakshi and Krishnamoorthy introduced on k-EP matrices. Ben and Greville [14] developed the concept of range symmetric fuzzy matrix and kernel symmetric fuzzy matrix analogues to that of an EP matrix in the complex field. Sumathi, Arockiarani [15] have discussed new operations on fuzzy neutrosophic soft matrices. Sumathi, Arockiarani , Jency,[16] have studied Fuzzy neutrosophic soft opological spaces. Abdel-Monem, Nabeeh and Abouhawwash [17] have studied An Integrated Neutrosophic Regional Management Ranking Method for Agricultural Water Management. Ahmed Abdelhafeez , Hoda Mohamed, Nariman Khalil [18] have discussed Rank and Analysis Several Solutions of Healthcare Waste to Achieve Cost Effectiveness and Sustainability Using Neutrosophic MCDM Model. Manas Karak, Animesh Mahata, Mahendra Rong, Supriya Mukherjee, Sankar Prasad Mondal, Said Broumi, Banamali Roy [19] have introduced A Solution Technique of Transportation Problem in Neutrosophic Environment. Meenakshi and Krishnamoorthy [20] have discussed on κ-EP matrices.

As mentioned in the above introduction section, Meenakshi introduced the concept of Range symmetric and Meenakshi and Jayashri developed the notion of kernel symmetric in fuzzy matrix. Here, we have applied the concept of range symmetric and kernel symmetric in Neutrosophic fuzzy matrix (NFM). Both this concept plays a significant role in hybrid fuzzy structure and we have applied the same in NFM and studied some of the results in detail. First we present equivalent characterizations of a range and kernel symmetric matrix and then, derive equivalent conditions for an Neutrosophic fuzzy matrix to be kernel symmetric Neutrosophic fuzzy matrix and study the relation between range symmetric and kernel symmetric Neutrosophic fuzzy matrices. Equivalent condition for varies g-inverses of a kernel symmetric matrix to be kernel symmetric are determined.

2. PRELIMINARIES AND NOTATIONS

PRELIMINARIES

Let the function be defined as $\kappa(x)=(x_{k[1]}, x_{k[2]}, x_{k[3]},\ldots, x_{k[n]}) \in F_{n\times 1}$ for $x = x_1, x_2,\ldots,x_n \in F_{[1\times n]}$, where K is involuntary, the following conditions are satisfied. The associated permutation matrix, where K is a permutation matrix, $KK^T = K^TK = I_n$ then $K^T = K$.

(P_1) $K = K^T$, $K^2 = I$ and $\kappa(x) = Kx$ for all $P \in (IF)_n$,

(P_2) N(P) = N(PK) = N(KP)

(P_3) $(PK)^+ = KP^+$ and $(KP)^+ = P^+K$ exists, if P^+ exists .

(P_4) P^T is a g-inverse of P iff P^+ exist

Notations: For NFM of P ∈ $(NF)_n$,

P^T : Transpose of P, R(P) : Row space of P, C(P) : Column space of P,N(P) : Null Space of P , P^+ : Moore-Penrose inverse of P , $(NF)_n$: Square Neutrosophic Fuzzy Matrix. $F_{[1\times n]}$: The matrix one row n columns. $F_{[n\times 1]}$:

3. DEFINITIONS AND THEOREMS

Definition: 1 Let P be a NFM, if R [P] = R [P^T] then P is called as range symmetric.

Example: 1 Let us consider NFM $P=\begin{bmatrix} (0.2,0.5,0.7) & (0,0,0) & (0.6,0.4,0.2) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ <0.6,0.4,0.2> & (0,0,0) & <0.3,0.5,0.7> \end{bmatrix}$,

The following matrices are not range symmetric

$$P=\begin{bmatrix} (1,1,0) & (1,1,0) & (0,0,0) \\ (0,0,0) & (1,1,0) & (1,1,0) \\ (0,0,0) & (0,0,0) & (1,1,0) \end{bmatrix},\ P^T=\begin{bmatrix} (1,1,0) & (0,0,0) & (0,0,0) \\ (1,1,0) & (1,1,0) & (0,0,0) \\ (0,0,0) & (1,1,0) & (1,1,0) \end{bmatrix}$$

$$\left[(1,1,0)\ (1,1,0)\ (0,0,0)\right]\in R(\mathrm{P})\ ,\ \left[(1,1,0)\ (1,1,0)\ (0,0,0)\right]\in R(\mathrm{P}^T)$$

$$\left[(0,0,0)\ (1,1,0)\ (1,1,0)\right]\in R(\mathrm{P})\ ,\ \left[(0,0,0)\ (1,1,0)\ (1,1,0)\right]\in R(\mathrm{P}^T)$$

$$\left[(0,0,0)\ (0,0,0)\ (1,1,0)\right]\in R(\mathrm{P})\ ,\left[(0,0,0)\ (0,0,0)\ (1,1,0)\right]\notin R(\mathrm{P}^T)$$

$$R(\mathrm{P})\notin R(\mathrm{P}^T)$$

Definition : 2 Let P ∈ F_n be a Neutrosophic fuzzy matrix , if N(P) = N(P^T) then P is called kernel symmetric NFM where N(P)= {x/xP = (0,0,0) and x ∈ $F_{1\times n}$},

Example: 2 Let us consider NFM $P=\begin{bmatrix} (0.4,0.5,0.6) & (0,0,0) & (0.6,0.4,0.8) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.4,0.5,0.7) & (0,0,0) & (0.4,0.3,0.6) \end{bmatrix}$,

$$N(\mathrm{P})=N(\mathrm{P}^T)=(0,0,0)$$

Definition 3. Unit Neutrosophic fuzzy matrix (UNFM)

If $(NF)_n$ is said to be UNFM if a_{ii} = (1,1,0) and a_{ij} = (0,1,1) $i \neq j$, for all $i = j$. It is denoted by I.

Example: 3 Let us consider NFM, $I=\begin{bmatrix} (1,1,0) & (0,1,1) & (0,1,1) \\ (0,1,1) & (1,1,0) & (0,1,1) \\ (0,1,1) & (0,1,1) & (1,1,0) \end{bmatrix}$

Definition 4. Symmetric Neutrosophic fuzzy matrix

If $(NF)_n$ is said to be symmetric Neutrosophic fuzzy matrix if $a_{ij} = a_{ji}$

Example: 4 Let us consider NFM $P = \begin{bmatrix} (0.3, 0.5, 0.8) & (0,0,0) & (0.5, 0.3, 0.1) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ < 0.5, 0.3, 0.1 > & (0,0,0) & < 0.3, 0.5, 0.7 > \end{bmatrix}$,

Definition 5. Permutation neutrosophic fuzzy matrix (PNFM)

Every row single (1,1,0) with (0,0,1) 's everywhere else is called PNFM.

Example: 5 Let us consider NFPM, $K = \begin{bmatrix} (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (1,1,0) & (0,0,1) & (0,0,1) \end{bmatrix}$

Definition 6 (Null neutrosophic fuzzy matrix) Neutrosophic fuzzy matrix is said to be Null if all its entries are zero, i.e., all elements are (0,0,0).

Example: 6 Let us consider NFM $P = \begin{bmatrix} (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \end{bmatrix}$,

Note:1 For Neutrosophic fuzzy matrix $P \in F_n$ with det P > <0,0,0>, has non- zero rows and non-columns, hereafter N(P) = <0,0,0> = N(P^T). Furthermore, a symmetric matrix P = P^T, that is N(P)= N(P^T).

Theorem:1 For P,Q∈$(NF)_n$ and K be a Neutrosophic fuzzy permutation matrix , N(P) = N(Q) ⇔ N(KPK^T) = N(KQK^T)

Proof: Let $w \in N(KPK^T)$

$\Rightarrow w(KPK^T) = (0,0,0)$

$\Rightarrow zK^T = (0,0,0)$ where $z = wKP$

$\Rightarrow z \in N(K^T)$

Since, $\det K = \det K^T > (0,0,0)$ (By Note:1)

Therefore, $N(K^T) = (0,0,0)$

Hence, $z = (0,0,0)$

$\Rightarrow wKP = (0,0,0)$

$\Rightarrow wK \in N(\mathrm{P}) = N(\mathrm{Q})$

$\Rightarrow wKQK^T = (0,0,0)$

$\Rightarrow w \in N(KQK^T)$

$N(KPK^T) \subseteq N(KQK^T)$

Similarly, $N(KQK^T) \subseteq N(KPK^T)$

Therefore, N (KPK^T) = N (KQK^T)

Conversely, if N (KPK^T) = N (KQK^T), then by the above proof,

N (P) = N (K^T (KPK^T) K)

= N (K^T (KQK^T) K)

N (P) = N(Q).

Example: 5 Let us consider NFM

$$P = \begin{bmatrix} (0,0,0.5) & (0,0,0) & (0.3,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.7,0,0) & (0,0,0) & (0.3,0.2,0) \end{bmatrix}, \mathrm{Q} = \begin{bmatrix} (0.3,0.4,0.2) & (0,0,0) & (0.4,0.2,0.6) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.5,0.3,0.4) & (0,0,0) & (0.5,0.3,0.6) \end{bmatrix},$$

$$K = \begin{bmatrix} (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (1,1,0) & (0,0,1) & (0,0,1) \end{bmatrix}$$

Theorem: 2 For Neutrosophic P ∈ $(\mathrm{NF})_n$, the following statements are equivalent

(i) N(P) = N(P^T)

(ii) N (KPK^T) = N ($KP^T K^T$) for some permutation NFM K.

(iii) Neutrosophic fuzzy permutation matrices K such that $KPK^T = \begin{bmatrix} D & (0,0,0) \\ (0,0,0) & (0,0,0) \end{bmatrix}$

with det D > $(0,0,0)$

Proof: (i) iff (ii).This equivalence follows from the theorem (1)

(i) iff (iii): Let N(P) = N(P^T)

If det P > $(0,0,0)$ then P has no zero row and columns,

Hence (iii) holds by taking K = I and D = P itself.

Suppose det P = (0, 0, 0) then N (P) = N (P^T) then N (P) = N (P^T) $\neq (0,0,0)$

For $x \neq (0,0,0)$, $x \in N(P)$ equivalent to each non-zero co efficient x_i of x, the fuzzy sums

$\sum x_i a_{ik} = (0,0,0)$ and $\sum x_i a_{ki} = (0,0,0)$ for all k.

As a result, P's i^{th} column and i^{th} row are both filled with zeros.

Now, by appropriately permuting the rows and columns,

All of the zero rows and zero columns can be shifted to the bottom and right, respectively.

Therefore, P is of the form $KPK^T = \begin{bmatrix} D & (0,0,0) \\ (0,0,0) & (0,0,0) \end{bmatrix}$

Where D - square matrix, D has non- zero rows and non-zero columns.

Therefore, det D > (0,0,0)

Thus (iii) holds

(iii) implies (ii) : If det P > (0,0,0) by remark ,D is kernel symmetric,

$\begin{bmatrix} D & <0,0,0> \\ <0,0,0> & <0,0,0> \end{bmatrix}$ is also kernel symmetric (ii) holds.

Example : 6 Let us Consider NFM ,

$$P = \begin{bmatrix} (0,0,0.5) & (0,0,0) & (0.3,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.7,0,0) & (0,0,0) & (0.3,0.2,0) \end{bmatrix}, K = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix}$$

$$PK^T = \begin{bmatrix} (0,0,0.5) & (0,0,0) & (0.3,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.7,0,0) & (0,0,0) & (0.3,0.2,0) \end{bmatrix} \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix} = \begin{bmatrix} (0,1,0.5) & (0.3,1,0) & (0,1,0) \\ (0,1,0) & (0,1,0) & (0,1,0) \\ (0.7,1,0) & (0.3,1,0) & (0,1,0) \end{bmatrix}$$

$$KPK^T = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (1,1,0) & (0,0,1) \end{bmatrix} \begin{bmatrix} (0,1,0.5) & (0.3,1,0) & (0,1,0) \\ (0,1,0) & (0,1,0) & (0,1,0) \\ (0.7,1,0) & (0.3,1,0) & (0,1,0) \end{bmatrix} = \begin{bmatrix} (0,0,0.5) & (0.3,0,0) & (0,0,0) \\ (0.7,0,0) & (0.3,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \end{bmatrix}$$

$$\det(\mathrm{P}) = (0,0,0),\ N(\mathrm{P}) = N(\mathrm{P}^T) = (0,0,0)$$

$KPK^T = \begin{bmatrix} D & (0,0,0) \\ (0,0,0) & (0,0,0) \end{bmatrix}$, Where $D = \begin{bmatrix} <0,0,0.5> & <0.3,0,0> \\ <0.7,0,0> & <0.3,0,0> \end{bmatrix}$, determined of D > (0,0,0)

Theorem:3 For P∈$(NF)_n$ is kernel symmetric Neutrosophic fuzzy matrix and K being a permutation matrix if and only if $N(KPK^T) = N(K P^T K^T)$

Proof: Let $x \in N(KPK^T)$

$\Rightarrow x(KAK^T) = (0,0,0)$

$\Rightarrow yK^T = (0,0,0)$ where $y = xKP$

$\Rightarrow y \in N(K^T)$

Since, $\det K = \det K^T >$ $(0,0,0)$

Therefore, $N(K^T) =$ $(0,0,0)$

Hence, $y = (0,0,0)$

$\Rightarrow xKP =$ $(0,0,0)$

$\Rightarrow xK \in N(\mathrm{P}) = N(\mathrm{P}^T)$

$\Rightarrow xKA^T K^T = (0,0,0)$

$\Rightarrow x \in N(K\left(P^T\right)K^T)$

$N(KPK^T) \subseteq N(KP^T K^T)$

Similarly, $N(KP^T K^T) \subseteq N(KPK^T)$

Therefore, $N(KPK^T) = N(KP^T K^T)$

Conversely, if $N(KPK^T) = N(K P^T K^T)$, then by the above proof ,

$N(P) = N(K^T(KPK^T)K)$

$= N(K^T(K P^T K^T)K)$

$N(P) = N(P^T)$.

Example:7 Let us consider NFM

$$P=\begin{bmatrix}(0,0.2,0.5) & (0,0,0) & (0.3,0.4,0)\\ (0,0,0) & (0,0,0) & (0,0,0)\\ (0.7,0.2,0) & (0,0,0) & (0.3,0.2,0)\end{bmatrix},\ K=\begin{bmatrix}(0,0,1) & (0,0,1) & (1,1,0)\\ (0,0,1) & (1,1,0) & (0,0,1)\\ (1,1,0) & (0,0,1) & (0,0,1)\end{bmatrix}$$

Theorem: 4 For $P\in(NF)_n$, is kernel symmetric Neutrosophic fuzzy matrix, then $N(PP^T) = N(P) = N(P^TP)$

Proof: Let, $x \in N(\mathrm{P})$

$\Leftrightarrow$ xP = $(0,0,0)$

$\Leftrightarrow$ xPP^T = $(0,0,0)$

$\Leftrightarrow\ x \in N(PP^T)$

$\Leftrightarrow\ N(\mathrm{P}) \subseteq N(\mathrm{PP}^T)$

Similarly, $N(\mathrm{PP}^T) \subseteq N(\mathrm{P})$

Therefore, N (P) = N (PPT)

Similarly, $N(\mathrm{P}) = N(\mathrm{P}^T\mathrm{P})$

Therefore, N (PPT) = N (P) = N (PTP)

Example:8 Let us consider NFM

$$P=\begin{bmatrix}(0.4,0.5,0) & (0,0,0) & (0.6,0.4,0.2)\\ (0,0,0) & (0,0,0) & (0,0,0)\\ (0.4,0.5,0.3) & (0,0,0) & (0.4,0.3,0.5)\end{bmatrix},$$

Theorem: 5 Let P, Q be the NFM and K NFPM, R (P) = R (Q) $\Leftrightarrow$ R (KPK^T) = R (KQK^T)

Proof: Let R (P) = R (Q)

Then, R (PK^T) = R (P) K^T

= R (P) K^T

= R (PK^T)

Let $z \in \left\{R(KPK^T)\right\}$

$z = w(KPK^T)$ for some $w \in V^n$

$z = rPK^T, \ r = wK$

$z \in R\left(PK^T\right) = R\left(Q\left(K^T\right)\right)$

$z = uQK^T$ for some $u \in V^n$

$z = \left(uK^T\right)KQK^T$

$z = vKQK^T$ for some $v \in V^n$

$z \in R(KQK^T)$

Therefore, $R(KPK^T) \subseteq R(KQK^T)$

Similarly, $R(KQK^T) \subseteq R(KPK^T)$

Therefore, $R(KPK^T) = R(KQK^T)$

Conversely, Let $R(KPK^T) = R(KQK^T)$. Then by above proof

$R(\mathrm{P}) = R[K^T(KPK^T)K]$

$= R[K^T(KQK^T)K]$

$= R(\mathrm{Q})$

$R(\mathrm{P}) = R(\mathrm{Q})$

Example:9 Let us consider NFM

$$P = \begin{bmatrix} (0.2,0.5,0.4) & (0,0,0) & (0.7,0.2,0.6) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.7,0.2,0.6) & (0,0,0) & (0.3,0.2,0.4) \end{bmatrix}, \mathrm{Q} = \begin{bmatrix} (0.7,0.2,0.6) & (0,0,0) & (0.3,0.2,0.4) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.2,0.5,0.4) & (0,0,0) & (0.7,0.2,0.6) \end{bmatrix}$$

$$K = \begin{bmatrix} (1,1,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

$R(P) = R(Q) \Leftrightarrow R(KPK^T) = R(KQK^T)$

Theorem:6 For $P \in (NF)_n$ be the NFM and K NFPM, $R(P) = R(P^T) \Leftrightarrow R(KPK^T) = R(K\,P^T\,K^T)$

Example: 10 $P=\begin{bmatrix}(0.4,0.5,0.6) & (0,0,0) & (0.3,0.5,0.6)\\ (0,0,0) & (0,0,0) & (0,0,0)\\ (0.3,0.5,0.6) & (0,0,0) & (0.3,0.2,0.4)\end{bmatrix}, K=\begin{bmatrix}(1,1,0) & (0,0,1) & (0,0,1)\\ (0,0,1) & (1,1,0) & (0,0,1)\\ (0,0,1) & (0,0,1) & (1,1,0)\end{bmatrix}$

Theorem:7 Let P,Q be the Neutrosophic fuzzy matrix and K being a permutation matrix, C(P) = C(Q) ⇔ C(KPK^T) = C(KQK^T)

Example:11 Let us consider NFM

$$P=\begin{bmatrix}(0.2,0.5,0.6) & (0,0,0) & (0.7,0.2,0.8)\\ (0,0,0) & (0,0,0) & (0,0,0)\\ (0.7,0.2,0.4) & (0,0,0) & (0.3,0.2,0.5)\end{bmatrix}, Q=\begin{bmatrix}(0.7,0.2,0.8) & (0,0,0) & (0.2,0.5,0.6)\\ (0,0,0) & (0,0,0) & (0,0,0)\\ (0.3,0.2,0.5) & (0,0,0) & (0.7,0.2,0.4)\end{bmatrix}$$

C(P)= C(Q) ⇔ C(KPK^T) = C(KQK^T)

k-KERNEL SYMMETRIC NFM

Definition: 3 Let P be a NFM .If P belongs to $(NF)_n$ is called k-Kernel symmetric Neutrosophic fuzzy if N(P) = N(KP^{T}K)

Note:2 Let P is k-Symmetric NFM implies it is k-kernel symmetric NFM, for P = K(P^T)K spontaneously implies N(P) = N(KP^T K) .Example 12. shows that the if and only if need not be true.

Example: 12 Let us Consider NFM

$$P=\begin{bmatrix}(0,0,0.5) & (0,0,0.4) & (0.3,0.4,0.5)\\ (0.5,0.4,0.6) & (0.1,0.4,0.6) & (0,0,0.4)\\ (0.4,0.5,0.3) & (0.3,0.4,0.5) & (0,0,0.3)\end{bmatrix}, K=\begin{bmatrix}(0,0,1) & (0,0,1) & (1,1,0)\\ (0,0,1) & (1,1,0) & (0,0,1)\\ (1,1,0) & (0,0,1) & (0,0,1)\end{bmatrix}$$

$$KP^TK=\begin{bmatrix}(0,0,0.3) & (0,0,0.4) & (0.3,0.4,0.5)\\ (0.3,0,0.5) & (0.1,0,0.6) & (0,0.4,0.4)\\ (0.4,0,0.3) & (0.5,0,0.6) & (0,0.4,0.5)\end{bmatrix}$$

Therefore, P $\neq$ KP^T K

But, N (P) = N(KP^T K) = $(0,0,0)$

Theorem: 8 For Neutrosophic fuzzy matrix P∈ $(NF)_n$, the given statements are equivalent:

(i) N(P) = N(KP^{T}K)

(ii) N(KP) = N((KP)^T)

(iii) N(PK) = N((PK)^T)

(iv) N(P^T) = N(KP),

(v) $N(P) = N((PK)^T)$

(vi) P^+ is k-KSNFM

(vii) $N(P) = N(P^+K)$

(viii) $K P^+P = PP^+K$

(ix) $P^+PK = KPP^+$

Proof: (i) implies (ii)

$\Leftrightarrow$ $N(P) = N(KP^TK)$

$\Leftrightarrow$ $N(KP) = N(P^TK)$ (By P_2) ($K^2 = I$)

$\Leftrightarrow$ $N(KP) = N((KP)^T)$ (Because , $(KP)^T = P^TK^T = P^TK$)

$\Leftrightarrow$ KP is Kernel symmetric,

Therefore, (ii) holds

(i) Implies (iii)

$\Leftrightarrow$ $N(P) = N(KP^TK)$

$\Leftrightarrow$ $N(PK) = N(KP^T)$ (By P_2) ($K^2 = I$)

$\Leftrightarrow$ $N(PK) = N((PK)^T)$ (Because , $(PK)^T = K^TP^T = KP^T$)

PK is Kernel symmetric,

Therefore, (iii) holds

(ii) Implies (iv)

$\Leftrightarrow$ $N(KP) = N(KP)^T = N(P^TK)$

$\Leftrightarrow$ $N(KP) = N(P^T)$ (By P_2)

Therefore, (iv) holds

(iii) Implies (v)

$\Leftrightarrow$ $N(PK) = N((PK)^T)$

$\Leftrightarrow$ $N(P) = N((PK)^T)$ (By P_2)

(ii) Implies (vi)

$\Leftrightarrow$ $N(KP) = N(KP)^T$

$\Leftrightarrow$ $N(KP) = N(P^TK)$ (By P_2)

$\Leftrightarrow$ $N(KP) = N(P^+K)$ Since $N(KP^+K) = N(P^+K)$

$\Leftrightarrow$ $N(KP) = N(P^+)$

P^+ is k-Kernel symmetric IFM

(i) implies (vii)

$\Leftrightarrow$ $N(P) = N(KP^TK)$

$\Leftrightarrow$ $N(P) = N(KP^TK) = N(P^TK)$

$\Leftrightarrow$ $N(P) = N(KP)^T$

$\Leftrightarrow$ $N(P) = N(P^+K)$ (By P_2)

(i) Implies (viii)

PK is Kernel symmetric NFM

$\Leftrightarrow$ $(PK)(PK)^+ = (PK)^+(PK)$

$\Leftrightarrow$ $(PK)(KP^+) = (KP^+)(PK)$

$\Leftrightarrow$ $PP^+ = KP^+PK$

$\Leftrightarrow$ $PP^+K = KP^+P$

Thus equivalence of (iii) and (viii) is proved.

(viii) $\Leftrightarrow$ (ix): Since, by the property (P_1),K^2=I, this uniformity follows by pre- and post multiplying by K.

$\Leftrightarrow$ $KP^+P = PP^+ K$

$\Leftrightarrow$ $K^2 P^+ AK = KPP^+ K^2$

$\Leftrightarrow$ $P^+ PK = KPP^+$.

4. CONCLUSION

Here some Theorem is described regarding the properties of kernel and range symmetric Neutrosophic Fuzzy Matrices. We introduced the concept of Kernel and k-Kernel Symmetric Neutrosophic Fuzzy Matrices with suitable examples. In addition, we have investigated some results of κ − kernel symmetric Neutrosophic Fuzzy Matrices with examples. In future, we shall prove some related properties of g-inverse of k-Kernel Symmetric NFM.

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