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Calculation of shortest path on Fermatean Neutrosophic Networks

Prasanta Kumar Raut[1], Siva Prasad Behera[1*], Said Broumi[2], Debdas Mishra[1]

[1]Department of Mathematics, C.V. Raman Global University, Bhubaneswar-752054, Odisha, India

[2]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco

[1]Email:prasantaraut95@gmail.com,sivaiitkgp12@gmail.com,debmaths@gmail.com
[2]Email: broumisaid78@gmail.com
Correspondence:sivaiitkgp12@gmail.com

Abstract:

The shortest path (SP) problem (SPP) has several applications in graph theory. It can be used to calculate the distance between the provided initial and final vertex in a network. In this paper, we employed the Fermatean neutrosophic number as the appropriate edge weight of the network to estimate the SP connecting the start and end vertex. This technique is highly useful in establishing the shortest path for the decision-maker under uncertainty. We also investigated its effectiveness in comparison to several existing methods. Finally, a few numerical tests were performed to demonstrate the validity and stability of this new technique, as well as to compare different types of shortest paths with different networks.

Keywords: Fermatean neutrosophic graph; Fermatean neutrosophic number, shortest path problem, Uncertainty.

1. Introduction:

The SPP is mainly a significant and essential combinatorial network optimization decision-making problem, and we can also say it is the heart of network flows. For example, it has a broad area of applications and is used in routing [1], transportation [2], supply chain management [3], communications [4], wireless networks [5], etc. Basically, this focuses on finding the SP between a particular initial node and the final node. Nowadays, it has important applications in engineering and research. In terms of competence, efficiency, and analytical techniques, the SPP has been extremely well researched.

In a traditional problem, the length between two vertices is assumed to be a real number in a certain environment. But in case of uncertainty, fuzzy numbers can be used to obtain the best result. The SPP is an essential component of the transport management system, connecting a specific source vertex to a destination vertex. Where the variety of research papers published regarding SPP in an uncertain environment where vertex and edges are stimulated to represent transportation cost or time.

However, in a real-world situation, different types of ambiguity are usually caused like failures, deficient data, or other factors such as weather or traffic conditions. In these cases, evaluating the particular optimal path in given networks may be difficult, so here we take a fuzzy number. In real-world problems involving scheduling, transportation, vehicle green routing, etc. that use SPP, the edge weights need not be certain due to fluctuations in parameters such as weather and traffic conditions. In those circumstances, experts emphasize the use of probability concepts to handle randomness arising due to the uncertainties of the SPP. Zadeh [6] introduced the all-important uncertainty theory for dealing with imprecise data in many real-world problems. In real life, we have a tendency to find the optimum (maximum or minimum) solution to any problem. Since a large volume of the available data is imprecise and inconsistent, the results produced are inconsistent, which paves the way for the discovery of uncertainty theory. Fuzzy optimization has been a significantly popular topic among researchers in the last couple of decades due to its extensive usage in various areas involving network flow problems [7], production problems [8], Automotive Industry [9,10,11,12,13,14,15,16,17], pick-up and delivery problems [18], travelling salesman problems [19], and traffic assignment problems [20].

Over the past 20 years, numerous researchers have conducted extensive research on SPP using various fuzzy number types, such as edge and vertex weights. "Fuzzy SPP" is the name given to this type of SPP in fuzzy scenarios. In the course of time, numerous studies on the FSPP have been conducted [21,22,23,24,25,26,27].There are different research paper already done where edge weights are neutrosophic numbers (NNs), which can be single, interval, or bipolar valued, one can use a neutrosophic set (NS) to find the network's shortest path [28, 29]. In situations where the theory of fuzzy logic is not useful when handling imprecise, uncertain and indeterminate issues, Smarandache introduced neutrosophic in 1995 and proposed a tool called the neutrosophic set (NS) theory. Truth (T), indeterminacy (I), and falsity (F) are three autonomous mappings that give rise to NS and have values between [0, 1]. It is extremely challenging to use NS directly.

Fuzzy graphs can be used for SPP while there is uncertainty in the vertices and edges, but neutrosophic concepts may be better able to manage uncertainty because indeterminacy is also taken seriously [30]. Since it can able to manage uncertain, inconsistent, and indeterminate information, NS is almost indispensable when it connecting with real-world issues in science and engineering [31]. In intelligent transportation systems, maintaining routes or providing uncertain supplies is of utmost importance.

Some days before, Antony Crispin Sweety and Jansi R [32] introduced a new idea called the Fermatean neutrosophic set (FNS) by mixing the concept of two sets i.e Fermatean fuzzy sets and neutrosophic sets. After FNs, some new concepts were introduced by Said Broumi et al. [33], such as regular and Fermatean neutrosophic graphs, Cartesian, composition, and lexicographic products of FNG.

The primary goal of this paper is to present an efficient algorithmic strategy for SPP that can be adapted to an uncertain environment arising in practical situations. The following are the main contributions of this paper:

- We used the Fermatean neutrosophic number FNSP as a vertex and then calculated the arc length of a Fermatean neutrosophic network. Because a graph or network may contain ambiguity or imperfection in its relationships or connections in some cases.
- Fermatean neutrosophic number (FNSP) is an extension of the neutrosophic number (NS) that addresses uncertainty by allowing the representation and analysis of uncertain or ambiguous information.
- Here we introduce a new algorithm to handle the SPP in an uncertain environment that calculates the length of the shortest path connecting two given nodes.

The remaining portion of the paper is prearranged in the following way: In Section 1, the literature analysis has been compiled. In Section 2, an overview of a Fermatean neutrosophic set is available. In Section 3, with the help of the suggested score function, novel algorithms are proposed. Section 4 provides a numerical example of finding the FNSP in Fermatean neutrosophic environments. Section 5 discussed the comparison of the shortest path with different networks and with different parameters, along with the benefits of the suggested approach. Section 6 provides the presented work's conclusion.

2. Preliminaries:

In this part of the article, the fundamental ideas and definitions of the neutrosophic set, Fermatean neutrosophic set, Fermatean neutrosophic relation, and score function of the Fermatean neutrosophic number are presented.

Definition 2.1 [35]

A neutrosophic set (NS) A in a universal set X is defined as $A = \langle x, \left(\check{T}_A(x), \check{I}_A(x), \check{F}_A(x) \right) : x \in X$. where $\check{T}_A(x)$,$\check{I}_A(x)$,$\check{F}_A(x)$ is truth, Indeterminacy and falsity membership degree. and sum of these three degrees of membership is written as $0^- \leq \breve{T}_A(x) + \breve{I}_A(x) + \breve{F}_A(x) \leq 3^+$.

Definition 2.2 [31]

A Fermatean neutrosophic set (FNS) A in universal set X is characterized by $= \{x, \left(\breve{T}_A(x), \breve{I}_A(x), \breve{F}_A(x)\right) : x \in X\}$.

Where $\breve{T}_A(x)$ shows the membership degree, $\breve{I}_A(x)$ indicates the indeterminacy-membership degree, and $\breve{F}_A(x)$ shows the non-membership degree.

Where sum cube of membership and falsity membership degree lies in between [0,1] i.e. $0 \leq (\breve{T}_A(x))^3 + (\breve{F}_A(x))^3 \leq 1$ and cube of indeterminacy degree lies in between[0,1] i.e. $0 \leq (\breve{I}_A(x))^3 \leq 1$.

Finally, the sum cube of this three membership degree lies in between [0, 2] i.e. $0 \leq ((\breve{T}_A(x))^3 + (\breve{I}_A(x))^3 + \left(\breve{F}_A(x)\right)^3 \leq 2$.

Definition 2.3 [31]

Let X is a universal set and a mapping $S = ((\breve{T}_S, \breve{I}_S, \breve{F}_S) : X \times X \to [0,1])$ is called a Fermatean Neutrosophic relation on X such that $(\breve{T}_S(u,v), \breve{I}_S(u,v), \breve{F}_S(u,v) \in [0,1]$ for all $u, v \in X$.

Definition 2.4[31]

Let $S = (\breve{T}_S, \breve{I}_S, \breve{F}_S)$ and $R = (\breve{T}_R, \breve{I}_R, \breve{F}_R)$ be Fermatean Neutrosophic number on a vertices

.then the edge length from R to S is defined as

$$\breve{T}_R(u,v) = min\{\breve{T}_S(u), \breve{T}_S(v)\}$$

$$\breve{I}_R(u,v) = max\{\breve{I}_S(u), \breve{I}_S(v)\}$$

$$\breve{F}_R(u,v) = max\{\breve{F}_S(u), \breve{F}_S(v)\}$$

if $\breve{T}_R(u,v), \breve{I}_R(u,v), \breve{F}_R(u,v) \in [0,1]$

Definition 2.5[31]

Let the vertices $T_S(u,v), I_S(u,v), F_S(u,v)$ be Fermatean neutrosophic number then the score function is defined as $S = \frac{\check{T}_R(u,v)+\check{I}_R(u,v)+1-\check{F}_R(u,v)}{3}$

3. Proposed Shortest Path algorithm based on Fermatean neutrosophic number

In this section, the proposed Shortest Path algorithm based on Fermatean neutrosophic number aims to address the limitations of existing path finding algorithms by incorporating the concept of Fermatean neutrosophic numbers

- Step-1: Choose one vertex as the initial and one vertex as the final point of the associated network.
- Step-2: Find the total possible path from initial node to final node of the associated network.
- Step-3: By using definition -2.4 to find the edge weights of a network from nodes.
- Step-4: After getting all edge value now convert it to crisp number by using score function (Definition-2.5).
- Step-5: Calculate the average value of a path by adding all the edges. Arrange the path in ascending order from lowest to highest path.

4. Illustrative Example

Consider a Fermatean neutrosophic network, where the vertex weights are defined by Fermatean numbers. The source vertex is 1 and the destination vertex is 6. By utilizing Fermatean neutrosophic values, we can explore complex systems, decision-making processes, or social networks with incomplete or uncertain information. Neutrosophic graph theory and related techniques aim to handle indeterminacy and provide a more accurate representation of uncertain realities.

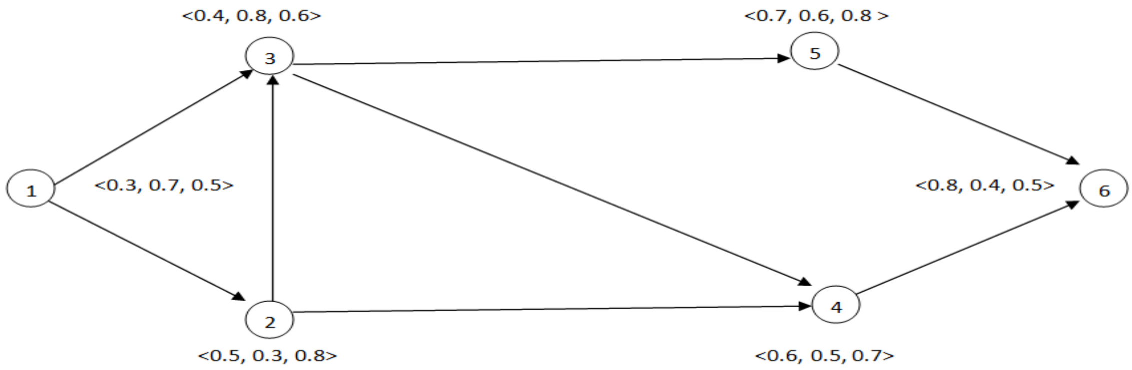

Figure1.Fermatean neutrosophic network

Here, Fermatean neutrosophic values on reality represent the recognition and acknowledgment of the existence of multiple perspectives and uncertainties in any given situation.

Vertices	Fermatean neutrosophic number
1	<0.3.0.7,0.5 >
2	<0.5,0.3.0.8>
3	<0.4,0.8,0.6>
4	<0.6,0.5,0.7>
5	<0.7,0.6,0.8>
6	<0.8,0.4,0.5>

Table-1: Vertices Weights

Implementation of Algorithm

In this section, it involves the conversion of a theoretical algorithm, which is a series of logical instructions, into a practical and executable solution that solves a specific problem.

Step-1:

From Figure1. Source vertex weight is 1 and destination vertex weight is 6.

Step-2:

The consequent of all probable paths connecting from source vertex to destination vertex are shown in Table-2.

Sl.no	Path
1	1-2-4-6
2	1-3-5-6
3	1-3 -4-6
4	1-2-3-4-6

Table-2: Path of a Network

Step-3:

In this given network, the nodes' weights are given. We can find the edge length of a given network by using definition 5.

From node 1 nodes weight is <0.3, 0.7, 0.5 >

From node 2 nodes weight is <0.5, 0.3, 0.8>

Then, the edge length from node 1 to node 2 is

$$\check{T}_R(u,v) = min\{\check{T}_S(u), \check{T}_S(v)\}$$

$$\check{I}_R(u,v) = max\{\check{I}_S(u), \check{I}_S(v)\}$$

$$\check{F}_R(u,v) = max\{\check{F}_S(u), \check{F}_S(v)\}$$

i.e

$\check{T}_R(u,v) = min\{0.3,0.5\} = 0.3$

$\check{I}_R(u,v) = max\{0.7,0.3\} = 0.7$

$\check{F}_R(u,v) = max\{0.5,0.8\} = 0.8$

So, the edge length from node-1 to node-2 is <0.3, 0.7, 0.8>

Similarly, we can find the edge lengths of all the nodes in the network given in the table.

Edges	Edge weights
1-2	<0.3,0.7,0.8>
1-3	<0.3,0.8,0.6>
2-3	<0.4,0.8,0.8>

2-4	<0.5,0.5,0.8>
3-4	<0.3,0.7,0.8>
3-5	<0.4,0.8,0.8>
4-6	<0.6,0.5,0.7>
5-6	<0.7,0.6,0.8>

Table-3:Edge weights

Step-4:

Now we convert Fermatean neutrosophic edge weight to crisp edge weight by using score function $S = \frac{\check{T}_R(u,v)+\check{I}_R(u,v)+1-\check{F}_R(u,v)}{3}$.

Edge weight from node-1 to node-2 is

$$S = \frac{0.3+0.7+1-0.8}{3} = \frac{1.2}{3} = 0.40$$

Similarly to convert all Fermatean neutrosophic edge weight to crisp edge in Table-4

Edges	Edge weights in crisp number
1-2	0.40
1-3	0.53
2-3	0.46
2-4	0.30
3-4	0.30
3-5	0.46
4-6	0.46
5-6	0.50

Table-4:Edge weights in crisp number

Step-5:

$$\text{Average weights of a Path} = \frac{\text{Sum of edges of path}}{\text{number of edges}}$$

$$\text{Average weights of a Path(1-2-4-6)} = \frac{(1-2)+(2-4)+(4-6)}{3}$$

$$\text{Average weights of a Path(1-2-4-6)} = \frac{0.40+0.30+0.46}{3} = 0.38$$

Similarly, to calculate average weights of all possible paths from source to destination.

Possible paths	Average weights of a path
1-2-4-6	0.38
1-3-5-6	0.49
1-3 -4-6	0.43
1-2-3-4-6	0.40

Table-5: average weights of all possible paths

Here, the shortest path 1-2-4-6 and shortest path value is 0.3

5. Comparison study:

In this segment, we evaluate our algorithm with Fermatean neutrosophic environment and with some existing methodology [30] and [31].

Shortest path with different network	**Path**	**Shortest path Length**
shortest path with IVNNs [34]	$1 \rightarrow 2 \rightarrow 4 \rightarrow 6$	[0.35,0.60],[0.01,0.04],[0.008,0.75]
shortest path with trapezoidal and triangular neutrosophic numbers [35]	$1 \rightarrow 2 \rightarrow 4 \rightarrow 6$	0.485
Our proposed algorithm	$1 \rightarrow 2 \rightarrow 4 \rightarrow 6$	0.38

Table-6: Shortest path with different network

and also we do a comparisons study for evaluation of SPP with different parameter as shown in Table-7

Evaluating SPP with Different parameter	**Arc lengths/vertices**	**Indeterminacy**	**Ambiguity**	**Uncertainty**	**Advantages**	**Limitations**
Crisp parameters	crisp Number	insufficient to handle	insufficient to handle	insufficient to handle	Able to determine easily information	unable to fully express the uncertain information
Fuzzy parameters	Fuzzy Number	Not able to manage	Not able to manage	capable to manage with uncertainty	Able to determine easily uncertain in formation	Able determine only membership information but not for non membership
Intuitionistic fuzzy parameters	Intuitionistic Fuzzy Number	insufficient to handle	able to manage	able to manage	Able determine both membership and non membership information	Not able to determine sum of membership and non membership value greater than one or not
Neutrosophic parameters	Neutrosophic Number	sufficient to handle	able to manage	May or may not be able to manage	Able determine both Truth, Indeterminacy and Falsity membership information	Not able to determine the cube sum of Truth, Indeterminacy and falsity membership value > 2
Fermatean neutrosophic parameters	Fermatean neutrosophic number	sufficient to handle	able to manage	able to manage	Able to determine the cube sum of Truth, Indeterminacy and falsity membership value in between 0 to 3	Not able to determine the interval data

Table-7: Shortest path with different parameter

6. Conclusion:

Uncertainty is essential to all scientific and engineering concerns. Fuzzy theory, intuitionistic fuzzy theory, and neutrosophic theory are the most valuable tools for determining the best answer to multi-criteria decision-making situations like the shortest path problem in a network.

In this paper, we study the advantages of employing the fermatean neutrosophic number in NSP. It incorporates uncertainty with the help of the Fermatean neutrosophic number edge weight between the source and destination vertex in a Fermatean neutrosophic environment. We are also expanding our research on this novel idea to include Interval-valued fermatean neutrosophic numbers, Interval-valued fermatean triangles, and trapezoidal neutrosophic numbers, as well as their applications, in future work.

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