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Calculation of Fuzzy shortest path problem using Multi-valued Neutrosophic number under fuzzy environment

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Abstract: The most well-known subject in graph theory is the shortest path problem (SPP), which has real-world applications in several different fields of study, including transportation, emergency services, network communications, fire station services, etc. The arc weights of the applicable SP problems are typically represented by fuzzy numbers in real-world applications. In this paper, we discussed the process of finding the shortest distance in a connected graph network in which the arc weights are multi-valued neutrosophic numbers (MNNs). Moreover, here we compare our method with some of the existing results and illustrate one implementation of our method with the help of one numerical example.

Keywords: Directed graph network; Multi-valued neutrosophic numbers; selection sort technique; shortest path problem.

1. Introduction

Smarandache designed the perception of a neutrosophic set first, and most importantly, it is able to manage situations that are ambiguous, insufficient, inconsistent, and unspecified by applying additional correct ways. The indication of neutrosophic sets (NS) [1] is like that of normal fuzzy sets [2], intuitionistic fuzzy associate number sets [3], and interval-valued intuitionistic fuzzy sets [4], giving basic ideas for relations and operations over sets. On a personal level, the variety-class degree of the neutrosophic sets theory is represented by an indeterminacy-category degree and a falsity-category degree. To utilize it in genuine scientific and technical areas, the idea of a neutrosophic set is proposed by Qiuping, N. [5] based on neutrosophic logic to make it more applicable to real-world circumstances. In reality, the degree of verity class, indeterminacy category, and falsity category of a handful of sure statements can't be written precisely inside the vitality items but expressed by various alternative interval values, which necessitated the use of the multi-valued neutrosophic set (MVNS). For this reason, Peng [6] suggested the idea of a multi-valued neutrosophic set (MVNS), which is superfluous exact, and more adaptable than an inter-valued neutrosophic (IVN) set. Multi-valued neutrosophic sets (MVNs) are like single-valued neutrosophic sets with three class functions, and the unit interval contains their values (0, 1).

The SPP is a fundamental and remarkable connectional optimization issue that arises in a variety of engineering and scientific fields, including road networks, transport, and other technologies. The SPP issue in a given network seeks the optimal path between two given nodes whose arc length weight is less as possible. The weight assigned to edges of a given network can reflect necessary life elements such as time, value, and others. The call maker is supposed to be confident with the parameters (length, duration, etc.) among distinct nodes in the traditional shortest route issue. However, there is always ambiguity regarding the parameters across distinct nodes in real-life conditions. Many approaches have been established for determining the shortest path (SP) in different kinds of input files with respect to fuzzy sets (FS), intuitionistic fuzzy sets (IFs), and ambiguous sets,neutrosophic and fermatean neutrosophic sets[7–16].

To find SPP in a fuzzy environment, triangular, trapezoidal, and pentagonal numbers [17, 18] are already used as the arc length in many real-world problems, and in some cases, neutrosophic numbers [19] are used to describe the uncertain behavior in the neutrosophic environment and then interval-valued neutrosophic numbers [20] are used to evaluate the path. But in this case, we used multi-valued neutrosophic numbers as the arc length to solve the SPP.
The primary purpose of this research is to identify the SPP for a given network with arc length weights determined by MVNNs. The constitution of the remaining article is as follows: In Section -2, we discuss some fundamental concepts related to neutrosophic sets, specifically single-valued, neutrosophic (SVN) sets. In Section -3 we present an approach for determining SP with connected edges with respect to neutrosophic data. Section -4 shows a realistic case solved by the suggested approach. Section -5 contains the comparison study with some of the existing methods, and Section -6 includes the conclusions and recommendations for additional research.

2. **Motivation and Contribution:**

The most important motivation of this paper is to initiate a method for SPP in an uncertain atmosphere that has a broad area of application in the real world.

The following are the contributions of this paper's

- We use MVNNs as the arc length instead of the real number.
- A new methodology is used to evaluate the (SP) problem in an uncertain environment.
- Various algorithms already exist (Table 1) to solve FSPP in an uncertain environment, but here we use a new methodology to evaluate the FSPP.

Compare our methodology with the existing methodology.

Author	Evaluation of SPP using different method	Year
Broumi, S. et al. [22]	NSPP for interval-based data was evaluated using the Dijkstra method.	2016
Broumi, S. et al. [23]	SP was found using SV-TpNNs.	2016
Broumi, S. et al. [24]	SPP was evaluated using single-valued neutrosophic graphs.	2016
Broumi, S. et al. [25]	SPP was assessed using a neutrosophic setup and the trapezoidal number.	2016
Broumi, S. et al. [26]	SPP was evaluated in a bipolar neutrosophic environment.	2017
Broumi, S. et al. [27]	SPP was evaluated using an interval-valued neutrosophic number.	2017
Broumi, S. et al. [28]	The neutrosophic version of Bellman's algorithm is introduced.	2017
Our method	Evaluating SPP under Multi-value neutrosophic number	2019

3. Preliminaries

This section contains the literature studied for the basic notions and definitions of neutrosophic sets (NSs) and MVNSs

Definition 3.1:

Assume $\tilde{X}$ is a set of space points (objects), and $\tilde{x}$ represents the associated generic elements in $\tilde{X}$; then the element in neutrosophic set $\tilde{A}$ has the form

$$\tilde{A} = \{< \tilde{x}: \tilde{T}_{\tilde{A}}(x), \tilde{I}_{\tilde{A}}(x), \tilde{F}_{\tilde{A}}(x) > \tilde{x} \epsilon \tilde{X}\}$$

Here the function takes the form $T, I, F : \tilde{X} \to [0^-, 1^+]$ where $\tilde{T}$ is called the truth-membership function, $\tilde{I}$ is called indeterminacy-membership function, and $\tilde{F}$ is called falsity membership function of the element $\tilde{x} \in \tilde{X}$.

$$0^- \leq \{\tilde{T}_{\tilde{A}}(x) + \tilde{I}_{\tilde{A}}(x) + \tilde{F}_{\tilde{A}}(x)\} \leq 3^+$$

Now $\tilde{T}_{\tilde{A}}(x), \tilde{I}_{\tilde{A}}(x), \tilde{F}_{\tilde{A}}(x)$ are representing subsets of the interval $[0^-, 1^+]$ hence it's challenging to implement NSs to real-world situations.

Definition 3.2:

If $\tilde{X}$ is a point in space and $\tilde{x}$ represents generic elements defined in $\tilde{X}$. Then truth, indeterminacy and the falsity-membership function differentiate $\tilde{A}$ in $\tilde{X}$. The multi-valued neutrosophic (MVN) set is defined as.

$$\tilde{A} = \{\tilde{x} \ , \tilde{T}_{\tilde{A}}(\tilde{x}), \tilde{I}_{\tilde{A}}(\tilde{x}), \tilde{F}_{\tilde{A}}(\tilde{x}), \tilde{x} \in \widetilde{X} \}$$

Here both $\tilde{T}_{\tilde{A}}(x), \tilde{I}_{\tilde{A}}(\tilde{x})$ and $\tilde{F}_{\tilde{A}}(\tilde{x}) \in [0,1]$ are the collection of discrete values that satisfy the criterion $0 \leq \alpha, \beta, \gamma \leq 1, 0 \leq \alpha^+, \beta^+, \gamma^+ \leq 3, \alpha \in \tilde{T}_{\tilde{A}}(x), \beta \in \tilde{I}_{\tilde{A}}(x), \gamma \in \tilde{F}_{\tilde{A}}(x)$.

$$\alpha^+ = Sup\tilde{T}_{\tilde{A}}(x), \beta^+ = Sup \ \tilde{I}_{\tilde{A}}(x), \gamma^+ = Sup\tilde{F}_{\tilde{A}}(x) - - - - - - - - -(2)$$

For the simplicity $\tilde{A} = \{\tilde{T}_{\tilde{A}}, \tilde{I}_{\tilde{A}}, \tilde{F}_{\tilde{A}}\}$ is referred to as a multi-valued neutrosophic (MVN) number.

The multi-valued neutrosophic (MVN) sets are termed as the single valued neutrosophic sets if $\tilde{A} = \{\tilde{T}_{\tilde{A}}, \tilde{I}_{\tilde{A}}, \tilde{F}_{\tilde{A}}\}$ has just one value.

Definition 3.3:

Assume that $\tilde{A}_1 = \{\tilde{T}_{\tilde{A}_1}, \tilde{I}_{\tilde{A}_1}, \tilde{F}_{\tilde{A}_1}\}$ and $\tilde{A}_2 = \{\tilde{T}_{\tilde{A}_2}, \tilde{I}_{\tilde{A}_2}, \tilde{F}_{\tilde{A}_2}\}$ are two sets of the neutrosophic numbers with multiple values. Then the functions for SVNNs are specified as follows:

(a) $\tilde{A}_1 + \ \tilde{A}_2 = \{\tilde{T}_{\tilde{A}_1} + \tilde{T}_{\tilde{A}_2} - \tilde{T}_{\tilde{A}_1}\tilde{T}_{\tilde{A}_2}, \tilde{I}_{\tilde{A}_1}\tilde{I}_{\tilde{A}_2}, \tilde{F}_{\tilde{A}_1}\tilde{F}_{\tilde{A}_2}\}$

(b) $\tilde{A}_1 \times \ \tilde{A}_2 = \{\tilde{T}_{\tilde{A}_1}\tilde{T}_{\tilde{A}_2}, \tilde{I}_{\tilde{A}_1} + \tilde{I}_{\tilde{A}_2} - \tilde{I}_{\tilde{A}_1}\tilde{I}_{\tilde{A}_2}, \tilde{F}_{\tilde{A}_1} + \tilde{F}_{\tilde{A}_2} - \tilde{F}_{\tilde{A}_1}\tilde{F}_{\tilde{A}_2}\}$

(c) $\lambda\tilde{A}_1 = \{1 - (1 - \tilde{T}_{\tilde{A}_1})^\lambda, {\tilde{I}_{\tilde{A}_1}}^\lambda, {\tilde{F}_{\tilde{A}_1}}^\lambda\}$

(d) ${\tilde{A}_1}^\lambda = \{{\tilde{T}_{\tilde{A}_1}}^\lambda, 1 - (1 - \tilde{I}_{\tilde{A}_1})^\lambda, 1 - (1 - \widetilde{\ } F_{\tilde{A}_1})^\lambda\}$

With $\lambda > 0$

Definition 3.4:

If $\tilde{A}_1 = \{\tilde{T}_{\tilde{A}_1}, \tilde{I}_{\tilde{A}_1}, \tilde{F}_{\tilde{A}_1}\}$ be a neutrosophic number of single value. Then,

The Score function are defined as the value $S\tilde{A}_1 = \frac{2 + \tilde{T}_{\tilde{A}_1} - \tilde{I}_{\tilde{A}_1} - \tilde{F}_{\tilde{A}_1}}{3}$

The Accuracy function takes the value as $a(\tilde{A}_1) = \{\tilde{T}_{\tilde{A}_1} - \tilde{F}_{\tilde{A}_1}\}$

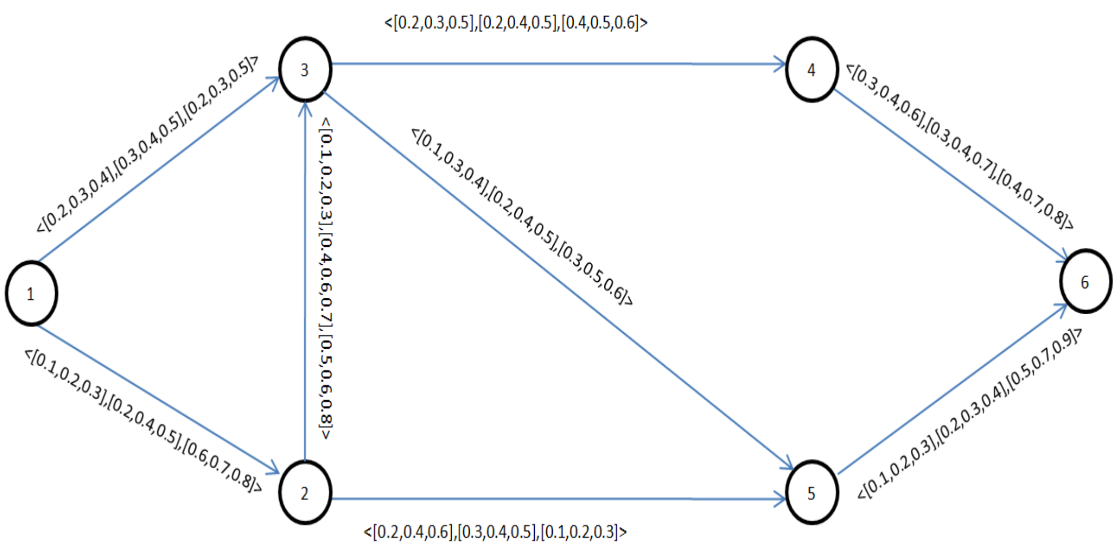
and certainty function is defined as $c(\tilde{A}_1) = \tilde{T}_{\tilde{A}_1}$

4. Algorithm to find the shortest path with respect to a multi-valued neutrosophic number

- Step 1: Select any vertex as the source and destination point of the given network.
- Step 2: Find every path that connects to the source node to the destination node.
- Step 3: Determine all possible edge values from discrete multi-valued neutrosophic numbers to simplify the MMNN to SVN by using the fuzzy simplicity method (equation 2.0) and using the Score Function to convert SVN to a crisp number (definition 2.4).
- Step 4: After obtaining all edge values (the Crisp number), calculate the path's average.
- After getting all path values, arrange them using the selection sort technique, and finally, get the shortest path.

5. Numerical Example:

Evaluation of the shortest path (SP) using multi-valued neutrosophic numbers



Step-1:

Let us look at a multi-valued neutrosophic network with source nodes 1 and destination nodes 6, with edge weights represented by MVNNs.

Step-2:

This table shows multi-valued neutrosophic distances.

Edges	**MVN distance**
1-2	<[0.1,0.2,0.3],[0.2,0.4,0.5],[0.6,0.7,0.8]>
1-3	<[0.2,0.3,0.4],[0.3,0.4,0.5],[0.2,0.3,0.5]>
2-3	<[0.1,0.2,0.3],[0.4,0.6,0.7],[0.5,0.6,0.8]>
2-5	<[0.2,0.4,0.6],[0.3,0.4,0.5],[0.1,0.2,0.3]>
3-4	<[0.2,0.3,0.5],[0.2,0.4,0.5],[0.4,0.5,0.6]>
3-5	<[0.1,0.3,0.4],[0.2,0.4,0.5],[0.3,0.5,0.6]>
4-6	<[0.3,0.4,0.6],[0.3,0.4,0.7],[0.4,0.7,0.8]>
5-6	<[0.1,0.2,0.3],[0.2,0.3,0.4],[0.5,0.7,0.9]>

Table-1

Step-3:

Table-1: Edge information in terms of multivalued neutrosophic number. Here $\tilde{T}_{\tilde{A}}(x), \tilde{I}_{\tilde{A}}(\tilde{x})$ and $\tilde{F}_{\tilde{A}}(\tilde{x}) \in [0,1]$, are the finite discrete value that satisfy the conditions $0 \leq \alpha, \beta, \gamma \leq 1, 0 \leq \alpha^{+}, \beta^{+}, \gamma^{+} \leq 3$

So now we apply equation (2) we get the Sake of simplicity of multi-value neutrosophic number

i.e. $\tilde{A} = \{\tilde{T}_{\tilde{A}}, \tilde{I}_{\tilde{A}}, \tilde{F}_{\tilde{A}}\}$

Then the edges From 1-2 is becomes [0.3,0.5,0.8] similarly all the edges value changes. here Table -2 represents the Simplicity of Multi-value neutrosophic number.

Edges	MVN distance
1-2	[0.3,0.5,0.8]
1-3	[0.4,0.5,0.5]
2-3	[0.3,0.7,0.8]
2-5	[0.6,0.5,0.3]
3-4	[0.5,0.5,0.6]
3-5	[0.4,0.5,0.6]
4-6	[0.6,0.7,0.8]
5-6	[0.3,0.4,0.9]

Table-2

Now by using Definition-2.4 we find the score function

$$S(A_1)=\frac{2+\overline{T}_1-\overline{I}_1-\overline{F}_1}{3}$$

- $S(A_1)=\frac{2+0.3-0.5-0.8}{3}$
- *S(A₁)=0.33*

Similarly to find all the values of edge (i,j) in table-3

edges	Score function
1-2	0.33
1-3	0.46
2-3	0.26
2-5	0.60
3-4	0.46
3-5	0.43
4-6	0.36
5-6	0.33

Table-3

Step-4:

Path from source to destination is

Path $1 - 3 - 4 - 6 = \frac{0.33+0.46+0.36}{3}$

$= 0.42$

Similarly all the values of path distance in Table-2

Path	**Distance**
1-3-4-6	0.42
1-3-5-6	0.40
1-2-3-5-6	0.43
1-2-5-6	0.31

Table- 4

Step-5:

Input:

```
# Python compiler
# Find the shortest path
def selectionSort( itemsList ):
    n = len( itemsList )
    for i in range( n - 1 ):
        minValueIndex = i

        for j in range( i + 1, n ):
            if itemsList[j] < itemsList[minValueIndex] :
                minValueIndex = j

        if minValueIndex != i :
            temp = itemsList[i]
            itemsList[i] = itemsList[minValueIndex]
            itemsList[minValueIndex] = temp

    return itemsList

# Arrange the path according to the path values
p1 = [0.42,0.30,0.43,0.31]

print(selectionSort(p1))
```

Output:

```
[0.31, 0.4, 0.42, 0.43]
```

Step-6:

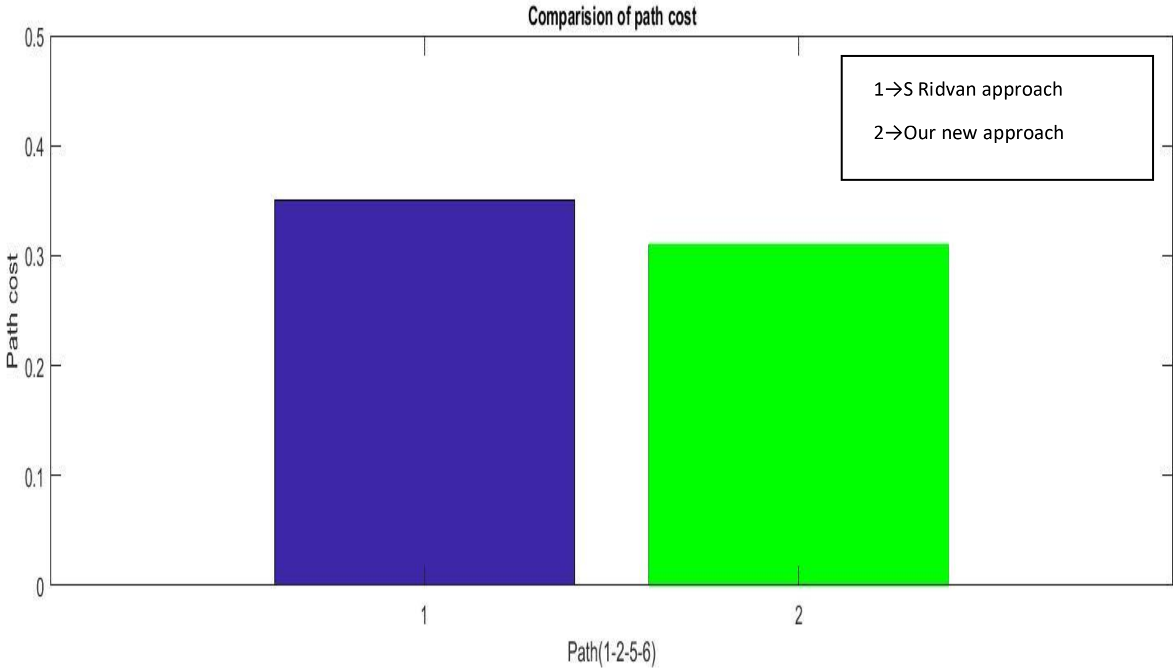
Minimum ranking value is 0.31. Hence the shortest path is 1-2-5-6

6. Comparison with existing Algorithm

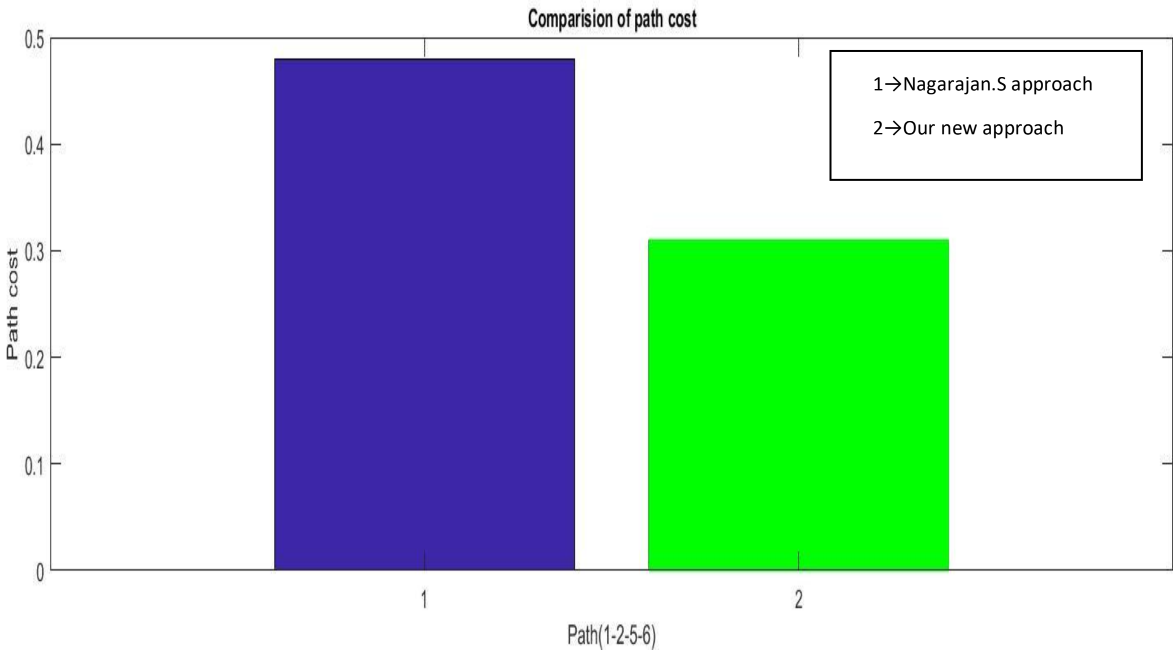
Here in this section we compare our proposed method with some of the existing method of for neutrosophic shortest path problems

Authors	Path sequence	Path length(Crisp)
Ridvan.S [29]	1-2-5-6	0.35
Nagarajan.S[30]	1-2-5-6	0.48
Broumi.S[31]	1-2-5-6	[0.35,0.60][0.01,0.04][0.008,0.075]
Our new approach	1-2-5-6	0.31

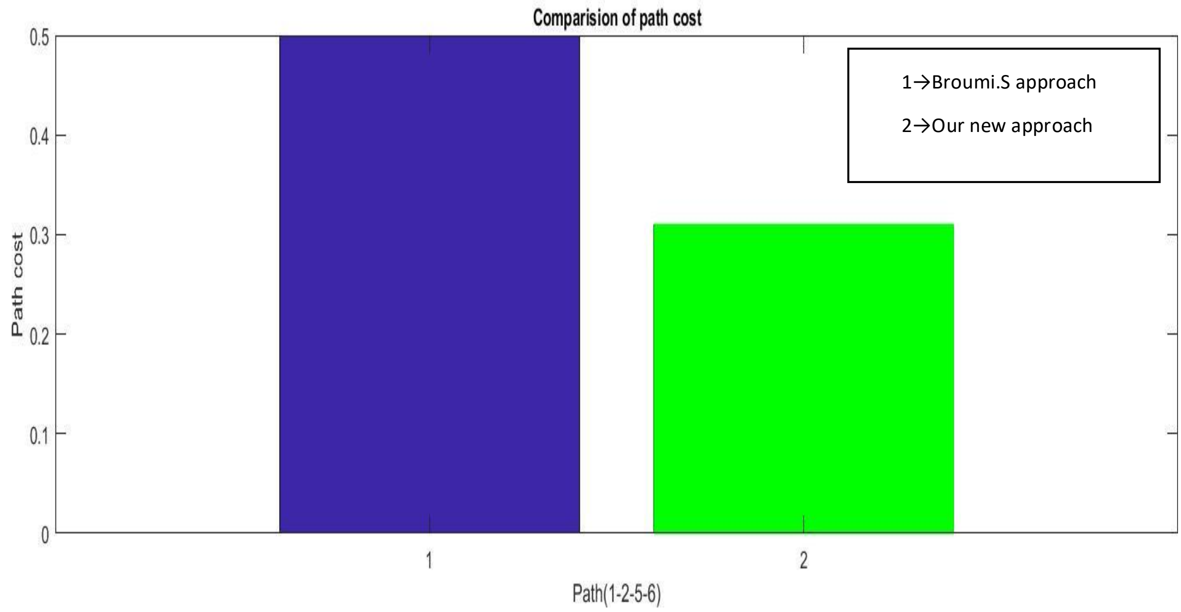
The results shows that our proposed algorithm is giving the crisp path length



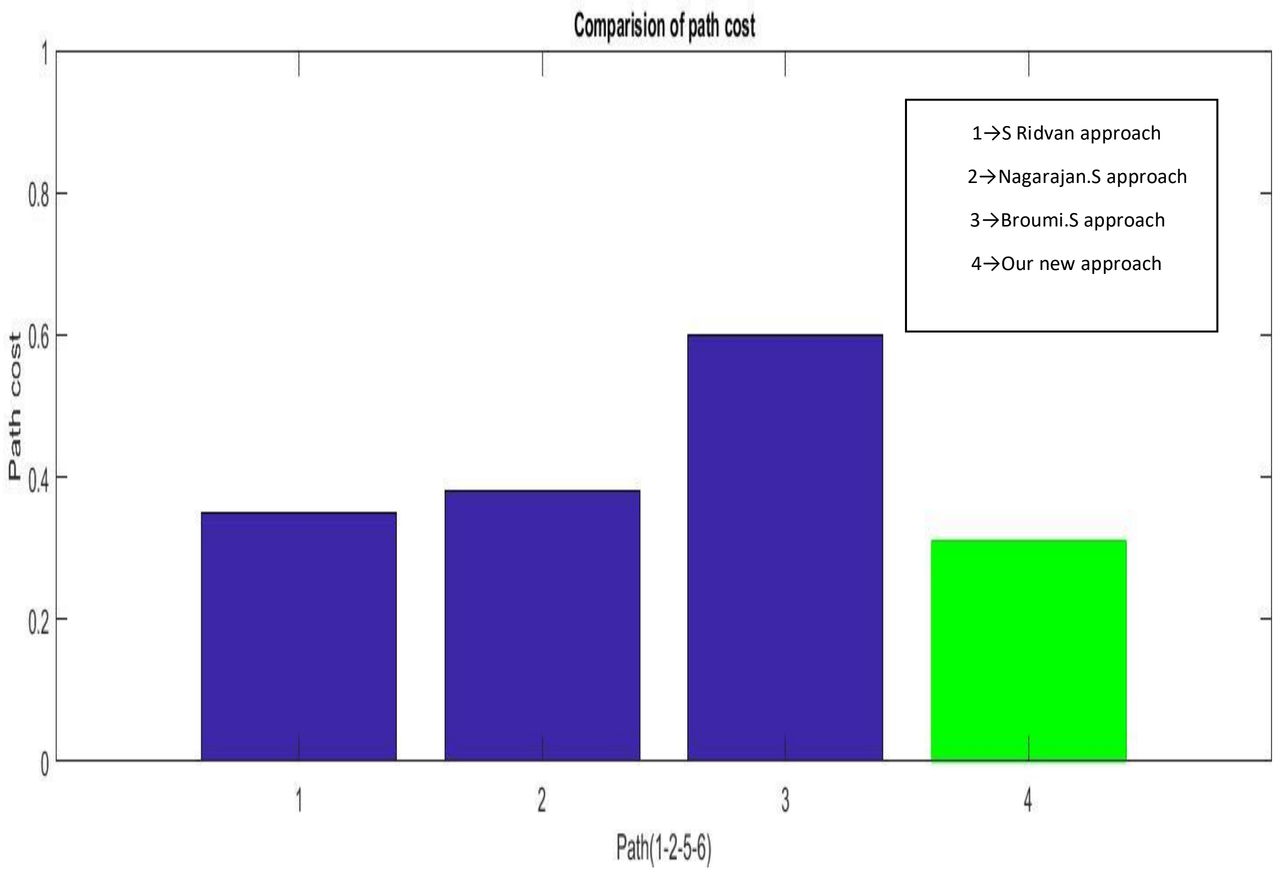
Comparison of our method with S Ridvan [21]



Comparison of our method with S.Nagarajan [22]



Comparison of our method with S broumi [23]



Final graph

Where the shortest neutrosophic path remains the same namely 1-2-5-6.

7. Conclusion:

This paper describes the NSP using edge weights represented by MVNS and the benefits of using MVNS with the NSP. The traditional new method is used in MVNS to integrate uncertainty between the destination and source nodes. To express the effectiveness of the suggested approach, we use numerical examples. The primary purpose of this study is to explain the NSP algorithm in a neutrosophic environment using MVNS as edge weights. For real-world issues, the suggested technique is quite successful. In future studies, it will be important to investigate a large-scale and realistic shortest-path issue using the suggested method.

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