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Value and Ambiguity Index-based Ranking Approach for Solving Neutrosophic Data Envelopment Analysis

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Abstract. The Neutrosophic set (NS), is a generalization of the fuzzy set and its extension sets, is a revolutionary type of fuzzy set that enables decision-makers (DMs) to express their level of uncertainty independently by assigned the truth, indeterminacy and falsity degrees of each element of NS. This article purposes a novel type of ranking function based on value and ambiguity index of a single value triangular neutrosophic number (SVTNN), which associated with DM's preference level and risk factor that show the attitude of the DM towards taking risk. Also, this article purposes a novel technique for solving the Neutrosophic DEA (Neu-DEA) model having multiple input-outputs are the SVTNNs. The proposed ranking function is used to converts the Neu-DEA model into a corresponding crisp DEA model which is solved to measure the efficiency of the decision making units (DMUs) in Neutrosophic environment. The efficiency scores of the DMUs are calculated based on the DM's preference level by taking a specific risk ($\lambda \in [0, 1]$). A numerical example is provided to demonstrate the proposed model's validity and existence, and to compare efficiency scores with Yang et al.'s ranking approach. Finally, How the DM's preference level and risk factor affect the efficiency score of DMUs are discussed in details.

Keywords: Efficiency Analysis; Neutrosophic Data Envelopment Analysis; Single value neutrosophic number; Value Index; Ambiguity Index; Ranking Function

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1. Introduction

Evaluating the performance of any public or commercial organization is one of the most challenging jobs for DMs to ensure progress, expansion, and sustainability. DEA is the most practical, trustworthy, and reliable way to analyze the performance/efficiency of DMUs. DEA is a data-driven, non-parametric, linear programming-based approach that evaluates piecewise linear production functions to determine the efficiency score of homogeneous DMUs with multiple inputs and outputs. Based on DEA results, DMUs are divided into efficient and inefficient group and also ranked them. This MCDM technique is extensively used in a wide range of fields to evaluate the relative efficiency of DMUs. Charnes et al. [10] introduced the concept of DEA, which is based on Farrell's earlier work [18] on measuring the efficiency of DMUs with multiple inputs and outputs. This method is generally known as CCR model which assumes that the production technology of all DMUs demonstrates constant returns to scale (CRS). Banker et al. [8] added the convexity condition in CCR Model [10] and developed a mathematical model is called the BCC model which assumes that the production technology of all DMUs demonstrates variable returns to scale (VRS). According to the best practice frontier, DMUs are either in the efficient group or inefficient group. Those DMUs are in efficient group have an efficiency score of one and are found on the frontier. Those DMUs are in inefficient group have an efficiency score ranges from 0 to 1 and are not found on the frontier. The inefficient DMUs can improve their efficiency to approach the frontier by reducing current inputs while maintaining outputs or increasing current outputs while keeping inputs unchanged. After the CCR and BCC models, several DEA models, including Additive, SBM, Super Efficiency, Undesirable, and others, have been created to evaluate the relative efficiency of DMUs. DEA has become a popular performance evaluation technique adopted by numerous industries to measure their relative efficiencies, including agriculture, insurance, operations management, banking, healthcare, education, and environmental management [11, 22, 28, 42, 44]. In real-world applications, it is not always possible to provide the clear input and output data that demands conventional/traditional DEA models. However, in practical applications, the observed values might occasionally be confusing, insufficient, inconsistent, and imprecise. This kind of uncertain data may be handled using probability or fuzzy theory. Also, the interval DEA and stochastic DEA models are frequently used to address this problem [34, 35].

The idea of a fuzzy set (FS) was established by Zadeh [46] in 1965, in which each element of the FS is associated with a membership degree (μ) that lies in [0,1], and the non-membership degree is defined as $1 - \mu$. FS theory has been widely used in practical applications of uncertainty modeling. In 1992, Sengupta [38] was the first to introduce the concept of using fuzzy numbers to represent the inputs and outputs of DMUs in the DEA model. Following this work, many authors became interested in developing various approaches for solving the fuzzy DEA model. These approaches are categorized into six types such as "the tolerance technique, the α-level-based approach, the fuzzy ranking approach, the possibility approach, the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set" [17, 19]. In 2020, Zhou and Xu [49] provided a comprehensive overview of the present state, growth prospects, practical implementations, and future research directions of fuzzy DEA studies. However, only the fuzzy set has a single-membership degree of unclear and vague information, which is often insufficient to describe evidence of support and opposition together. Atanassov [7] expanded Zadeh's fuzzy set notion to the intuitionistic fuzzy set (IFS) in 1986, and its membership and non-membership degrees are defined separately, with their sum lying between [0, 1]. Several research papers using intuitionistic fuzzy sets have been published in DEA using various techniques, such as the weighted approach, (α, β)-cut approach, optimistic-pessimistic efficiency approach, hybrid TOPSIS-DEA, parametric approach, and composition approach, and alphabetical approach [6,37]. IFS is capable of addressing missing data for a wide range of real-world problems, but it can't deal with other types of uncertainty, such as indeterminate information.

Smarandache [40] proposed the Neutrosophic Logic and Neutrosophic Set (NS) as a generalization of FSs and IFSs in 1999. Each element of the Neutrosophic set has three independent degrees, namely "truth, indeterminacy, and falsity", and their sum ranges from 0 to 3. In order to represent uncertainty in different areas, various extensions of the fuzzy set, such as Pythagorean and spherical fuzzy sets, have been developed (as shown in Figure 1). The DM always tries to increase the truth membership degree while decreasing the indeterminacy and falsity membership degrees of each NS element, which are independently assigned. Currently, the study of neutrosophic set theory is highly popular, and its applications are widespread across a range of disciplines, including mathematics, computer science, engineering, medicine, economics, social science, and environmental science [1,4,12,20,25,26,39,48]. In 2018, Edalatpanah [14] presented the initial theoretical advancement of the Neutrosophic DEA (Neu-DEA) model. As a result, many other authors became interested in developing the Neu-DEA model in various neutrosophic environments and proposing novel techniques for solving it. Kahraman et al. [23] developed a novel Neutrosophic Analytic Hierarchy Process (NAHP), which was subsequently combined with the Neu-DEA model to evaluate the efficiency of 15 private universities. Abdelfattah [2] created a Neu-DEA model with triangular neutrosophic inputs and outputs and developed a unique technique that converts the Neu-DEA model into an interval DEA model that evaluates the efficiency of the DMUs in interval form. The input-oriented Neu-DEA model is proposed by considering the inputs and outputs as simplified neutrosophic numbers, which is a nonlinear model turned into an LP model by utilizing a natural logarithm to measure the efficiency of the DMUs [16]. Subsequently, several approaches for solving Neu-DEA models were utilized to measure the efficiency of the DMUs [15, 43]. In 2020, Mao et al. [24] proposed a novel approach to solving the Neu-DEA model with undesirable outputs, where all data is considered as SVNNs, in order to assess the efficiency of the DMUs. Yang et al. [45] developed the Neutrosophic DEA (Neu-DEA) model to evaluate the efficiency of 13 hospitals affiliated with Tehran University of Medical Sciences in Iran. The proposed model considered single-value triangular neutrosophic numbers (SVTNNs) for both inputs and outputs. In 2021, Abdelfattah [3] proposed ranking and parametric approaches to solving the Neu-DEA model in order to evaluate the efficiency of 32 regional hospitals located in Tunisia. The possibility mean [29] and ranking [33] approaches are developed to solve the Neu-DEA model to measure the performance of AIIMS in India [32] and major sea ports in India [30]. Recently, there has been a growing interest among authors in developing the DEA model by incorporating various extensions of fuzzy sets. One such technique is the Fermatean fuzzy DEA (FFDEA), developed to solve the Fermatean fuzzy multi-objective transportation problem (FFMOTP) [5]. Other novel solution techniques have been developed for solving the spherical fuzzy DEA model in the presence of spherical fuzzy inputs and outputs [27, 31]. Additionally, the plithogenic set has been utilized in the DEA model to evaluate the efficiency of 20 bank branches and the performance of hotel industries [21, 36].

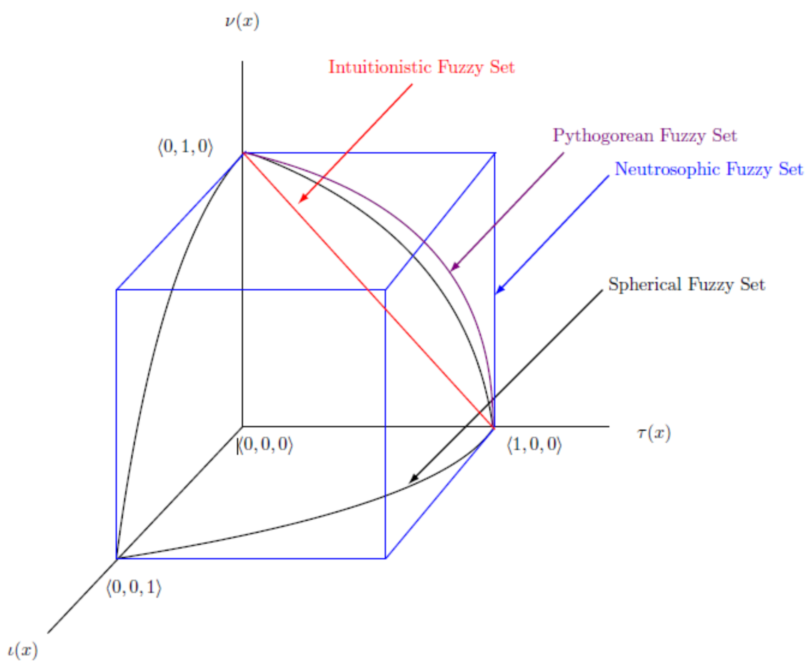

Figure 1. Representation of extended fuzzy sets

The primary contribution of this paper is the development of a novel ranking function based on the value and ambiguity index of the single-value triangular neutrosophic number (SVTNN). This ranking function is associated with risk sensitivity and DM's preferences. The suggested ranking function has been used to convert the Neu-DEA model with SVTNN inputs and outputs into the corresponding crisp DEA model. The efficiency scores of DMUs are measured using different risk factors according to the DM's specific preference level. This paper investigates how the DM's preference level affects the efficiency of the DMUs. A numerical example is presented to compare the efficiency scores to the model suggested by Yang et al. [45].

The rest of the manuscript is organized as follows: Section 2 provides some useful notation for this study and details comparison of the suggested model with other existing Nue-DEA model. Section 3 covers various aspects related to SVTNN, including Neutrosophic set, properties of SVTNN, Value and Ambiguity Index for SVTNN, as well as the proposed ranking function for SVTNN along with its properties. Section 4 discusses the Neu-DEA model based on SVTNN and how to obtain an equivalent crisp DEA model by applying the provided ranking function. Section 5 discusses step-by-step technique for solving Neu-DEA model. In Section 6, a case study is given to demonstrate the validity and applicability of the proposed model. Section 7 discusses the advantage and limitation of this study. Finally, the conclusion and recommendations for further study are presented in Section 8.

2. Notation and Comparison Study

To make it easier for readers to understand, the manuscript has been updated with additional notation that is presented in Table 1. The purpose of including this notation is to simplify the content and make it more accessible to a wider researcher.

TABLE 1. Notation used in this Study

Symbol	Description
$\widehat{X}$	Single Value Triangular Neutrosophic Number (SVTNN)
$\mathbb{V}$	Value Index function
$\mathcal{A}$	Ambiguity Index function
δ, ρ, η	Preference parameter for the DM
λ	Risk parameter
$\Re$	Ranking function
A	Input Matrix
B	Output Matrix
a_{ij}	i^{th} crisp input for DMU_j
b_{ki}	k^{th} crisp output for DMU_j
$\widehat{a_{ij}}$	i^{th} SVTNN input for DMU_j $\left(\text{i.e., } \widehat{a_{ij}} = \left\langle (\overbrace{a_{ij}^L}, \overbrace{a_{ij}^M}, \overbrace{a_{ij}^U}); (a_{ij}^L, a_{ij}^M, a_{ij}^U); (\underbrace{a_{ij}^L}, \underbrace{a_{ij}^M}, \underbrace{a_{ij}^U}) \right\rangle\right)$
$\widehat{b_{ki}}$	k^{th} SVTNN output for DMU_j $\left(\text{i.e., } \widehat{b_{ki}} = \left\langle (\overbrace{b_{ki}^L}, \overbrace{b_{ki}^M}, \overbrace{b_{ki}^U}); (b_{ki}^L, b_{ki}^M, b_{ki}^U); (\underbrace{b_{ki}^L}, \underbrace{b_{ki}^M}, \underbrace{b_{ki}^U}) \right\rangle\right)$

In this part of the section, we compare our novel contribution to existing publications, as shown in Table 2. Several researchers have developed various approaches for solving Neu-DEA models without utilizing DM's preference level and risk parameter. The preference level of a DM plays a significant role in uncertainty modeling, indicating the desired decision approach: pessimistic, optimistic, or neutral. Additionally, the risk parameter reflects the DM's attitude towards risk, whether they are a risk taker, risk averse, or neutral. In this

Table 2. Comparing our proposed work to already published Nue-DEA work

Researcher	Inputs and Outputs	Concept	DEA model	Risk Factor
Edalatpanah [14]	Single Valued Triangular Neutrosophic Number (SVTNN)	Score and Accuracy function	CCR	No
Kahraman et al. [23]	Linguistic interval-valued neutrosophic number	Deneutrosophication	Hybrid AHP-CCR	No
Abdelfattah [2]	Triangular neutrosophic numbers	Ranking [1] & parametric approach	CCR	No
Edalatpanah and Smarandache [16]	Simplified neutrosophic numbers (SNN)	Logarithm approach	BCC	No
Edalatpanah [15]	Triangular neutrosophic number (TNN)	Ranking approach [1]	Dual CCR	No
Mao et al. [24]	Simplified neutrosophic numbers (SNN)	Logarithm approach	Undesirable	No
Yang et al. [45]	Single Valued Triangular Neutrosophic Number (SVTNN)	Simple Ranking approach	CCR	No
Tapia [43]	Interval-valued neutrosophic numbers	Robust tolerance approach	CCR	No
Mohanta and Sharanappa [30]	Trapezoidal Neutrosophic Number (TrNN)	Ranking Approach	CCR	No
Mohanta and Toragay [33]	Pentagonal Neutrosophic Number (PNN)	Ranking Approach	CCR	No
Proposed Work	Single Valued Triangular Neutrosophic Number (SVTNN)	Ranking approach based on Value and Ambiguity index	CCR	Yes

article, we incorporate the DM's preference level and risk factor into the value and ambiguity index to create a ranking function for SVTNN. This ranking function effectively compares SVTNNs by taking into account the DM's preference level and risk attitude. This ranking function is then used to convert the Neu-DEA model into a corresponding crisp DEA model, from which the efficiency score of the DMUs can be obtained. The preference level (δ, ρ, η) and risk parameter $(\lambda \in [0, 1])$ are important factors in the performance assessment process because they increase the DM's freedom to express their own risk while making decisions.

3. Single Value Triangular Neutrophic Number (SVTNN) and Its Properties

This section introduces the Neutrosophic set, including the mathematical features of SVTNNs, as well as the value and ambiguity index of SVTNNs, which are defined to create a new ranking function. Additionally, a new ranking algorithm is introduced that utilizes the value and ambiguity index of SVTNNs.

Definition 3.1. [40] A neutrosophic set $\widehat{A}$ in a universe of discourse Ω is given by

$$\widehat{A} = \{\langle x; \tau(x), \iota(x), \nu(x) |\ x \in \Omega\rangle\} \tag{1}$$

where $\tau(x), \iota(x)$, and $\nu(x)$ are called truth, indeterminacy and falsity membership functions, respectively. These membership functions are defined as $\tau : \Omega \to [0,1]$, $\iota : \Omega \to [0,1]$ and $\nu : \Omega \to [0,1]$ such that $0 \le \tau(x), \iota(x), \nu(x) \le 3$.

Definition 3.2. [13] The single value triangular neutrophic number (SVTNN) is defined as $\widehat{X} = \left\langle (\overbrace{x^L}, \overbrace{x^M}, \overbrace{x^U}); (x^L, x^M, x^U); (\underbrace{x^L}, \underbrace{x^M}, \underbrace{x^U}) \right\rangle$ where the truth, indeterminacy, and falsehood membership degrees of x are defined as :

$$\tau(x) = \begin{cases} \dfrac{x - \overbrace{x^L}}{\overbrace{x^M} - \overbrace{x^L}}, & x \in [\overbrace{x^L}, \overbrace{x^M}] \\ \dfrac{\overbrace{x^U} - x}{\overbrace{x^U} - \overbrace{x^M}}, & x \in [\overbrace{x^M}, \overbrace{x^U}] \\ 0, & otherwise \end{cases} \qquad \iota(x) = \begin{cases} \dfrac{x - x^L}{x^M - x^L}, & x \in [x^L, x^M] \\ \dfrac{x^U - x}{x^U - x^M} & x \in [x^M, x^U] \\ 1, & otherwise \end{cases}$$

$$\nu(x) = \begin{cases} \dfrac{x - \underbrace{x^L}}{\underbrace{x^M} - \underbrace{x^L}}, & x \in [\underbrace{x^L}, \underbrace{x^M}] \\ \dfrac{\underbrace{x^U} - x}{\underbrace{x^U} - \underbrace{x^M}} & x \in [\underbrace{x^M}, \underbrace{x^U}] \\ 1, & otherwise \end{cases}$$

where $0 \le \tau(x) + \iota(x) + \nu(x) \le 3, \ \forall \ x \in \mathbb{R}$.

Definition 3.3. [13] Suppose $\widehat{X}_1 = \left\langle (\overbrace{x_1^L}, \overbrace{x_1^M}, \overbrace{x_1^U}); (x_1^L, x_1^M, x_1^U); (\underbrace{x_1^L}, \underbrace{x_1^M}, \underbrace{x_1^U}) \right\rangle$ and $\widehat{X}_2 = \left\langle (\overbrace{x_2^L}, \overbrace{x_2^M}, \overbrace{x_2^U}); (x_2^L, x_2^M, x_2^U); (\underbrace{x_2^L}, \underbrace{x_2^M}, \underbrace{x_2^U}) \right\rangle$ two SVTNNs. The arithmetic relations in SVTNNs are defined as

(1) $\widehat{X}_1 \oplus \widehat{X}_2 = \left\langle (\overbrace{x_1^L} + \overbrace{x_2^L}, \overbrace{x_1^M} + \overbrace{x_2^M}, \overbrace{x_1^U} + \overbrace{x_2^U}); (x_1^L + x_2^L, x_1^M + x_2^M, x_1^U + x_2^U); (\underbrace{x_1^L} + \underbrace{x_2^L}, \underbrace{x_1^M} + \underbrace{x_2^M}, \underbrace{x_1^U} + \underbrace{x_2^U}) \right\rangle$.

(2) $\widehat{X}_1 - \widehat{X}_2 = \left\langle (\overbrace{x_1^L} - \overbrace{x_2^L}, \overbrace{x_1^M} - \overbrace{x_2^M}, \overbrace{x_1^U} - \overbrace{x_2^U}); (x_1^L + x_2^L, x_1^M - x_2^M, x_1^U + x_2^U); (\underbrace{x_1^L} - \underbrace{x_2^L}, \underbrace{x_1^M} - \underbrace{x_2^M}, \underbrace{x_1^U} - \underbrace{x_2^U}) \right\rangle$.

(3) $\widehat{X}_1 \otimes \widehat{X}_2 = \left\langle (\overbrace{x_1^L}\overbrace{x_2^L}, \overbrace{x_1^M}\overbrace{x_2^M}, \overbrace{x_1^U}\overbrace{x_2^U}); (x_1^L x_2^L, x_1^M x_2^M, x_1^U x_2^U); (\underbrace{x_1^L}\underbrace{x_2^L}, \underbrace{x_1^M}\underbrace{x_2^M}, \underbrace{x_1^U}\underbrace{x_2^U}) \right\rangle$.

(4) $$\alpha\widehat{X}_1 = \begin{cases} \left\langle (\alpha \overbrace{x_1^L}, \alpha \overbrace{x_1^M}, \alpha \overbrace{x_1^U}); (\alpha x_1^L, \alpha x_1^M, \alpha x_1^U); (\alpha \underbrace{x_1^L}, \alpha \underbrace{x_1^M}, \alpha \underbrace{x_1^U}) \right\rangle, & \alpha > 0 \\ \left\langle (\alpha \overbrace{x_1^U}, \alpha \overbrace{x_1^M}, \alpha \overbrace{x_1^L}); (\alpha x_1^U, \alpha x_1^M, \alpha x_1^L); (\alpha \underbrace{x_1^U}, \alpha \underbrace{x_1^M}, \alpha \underbrace{x_1^L}) \right\rangle, & \alpha < 0 \end{cases}$$

Definition 3.4. Let $\widehat{X} = \Big\langle (\overbrace{x^L}, \overbrace{x^M}, \overbrace{x^U}), (x^L, x^M, x^U), (\underbrace{x^L}, \underbrace{x^M}, \underbrace{x^U}) \Big\rangle$ be a SVTNN. The (α, β, γ)-cut of $\widehat{X}$ is defined as

$$\widehat{X}_{(\alpha,\beta,\gamma)} = \{x : \tau(x) \geq \alpha, \iota(x) \leq \beta, \nu(x) \leq \gamma\} \tag{2}$$

such that $0 \leq \alpha \leq \tau(x), \iota(x) \leq \beta \leq 1$ and $\nu(x) \leq \gamma \leq 1$.

From Definition 3.2 and equation (2), the lower and upper limit of (α, β, γ)-cut of the SVTNN $\widehat{X}$ are defined as

$$\begin{aligned}
\widehat{X}_\alpha &= \Big[L(\alpha), U(\alpha)\Big] = \Big[\overbrace{x^L} + \alpha(\overbrace{x^M} - \overbrace{x^L}), \overbrace{x^U} - \alpha(\overbrace{x^U} - \overbrace{x^M})\Big] \\
\widehat{X}_\beta &= [L(\beta), U(\beta)] = \Big[x^L + \beta(x^M - x^L), x^U - \beta(x^U - x^M)\Big] \\
\widehat{X}_\gamma &= [L(\gamma), U(\gamma)] = \Big[\underbrace{x^L} + \gamma(\underbrace{x^M} - \underbrace{x^L}), \underbrace{x^U} - \gamma(\underbrace{x^U} - \underbrace{x^M})\Big]
\end{aligned}$$

3.1. *The Proposed Ranking Function for Single Value Triangular Neutrosophic Number (SVTNN)*

The Value index and Ambiguity index play an important role to ranked the fuzzy numbers in decision making problem [9, 47]. This subsection focuses on the development of value and ambiguity index for SVTNN. Furthermore, a new ranking function is established by incorporating value and ambiguity index.

Definition 3.5 (Value Index)**.** The value index $\mathbb{V}_\tau(\widehat{X}), \mathbb{V}_\iota(\widehat{X})$, and $\mathbb{V}_\nu(\widehat{X})$ with respect to the truth $\tau(x)$, indeterminacy $\iota(x)$, and falsehood $\nu(x)$ membership degrees are defined as

$$\mathbb{V}_\tau(\widehat{X}) = \int_0^1 \Big(L(\alpha) + U(\alpha)\Big) f(\alpha) d\alpha,\ \mathbb{V}_\iota(\widehat{X}) = \int_0^1 \Big(L(\beta) + U(\beta)\Big) g(\beta) d\beta,$$
$$\mathbb{V}_\nu(\widehat{X}) = \int_0^1 \Big(L(\gamma) + U(\gamma)\Big) h(\gamma) d\gamma. \tag{3}$$

where $f(\alpha) = \alpha,\ g(\beta) = 1 - \beta$ and $h(\gamma) = 1 - \gamma$ can be configured to reflect the nature of decision making in real-world scenarios.

Definition 3.6 (Ambiguity Index)**.** The ambiguity index $\mathcal{A}_\tau(\widehat{X}), \mathcal{A}_\iota(\widehat{X})$, and $\mathcal{A}_\nu(\widehat{X})$, with respect to the truth $\tau(x)$, indeterminacy $\iota(x)$, and falsehood $\nu(x)$ membership degrees are defined as

$$\mathcal{A}_\tau(\widehat{X}) = \int_0^1 \Big(U(\alpha) - L(\alpha)\Big) f(\alpha) d\alpha,\ \mathcal{A}_\iota(\widehat{X}) = \int_0^1 \Big(U(\beta) - L(\beta)\Big) g(\beta) d\beta,$$
$$\mathcal{A}_\nu(\widehat{X}) = \int_1^0 \Big(U(\gamma) - L(\gamma)\Big) h(\gamma) d\gamma, \tag{4}$$

where $L(\alpha), L(\beta)$ and $L(\gamma)$ are lower and $U(\alpha), U(\beta)$ and $U(\gamma)$ are upper limits of SVTNN $\widehat{X}$.

Thus from equations (3) and (4), the value and ambiguity for the truth, indeterminacy, and falsehood membership degrees are calculated as follows:

$$\mathbb{V}_\tau(\widehat{X}) = \frac{\overbrace{x^L} + \overbrace{x^U} + 4\overbrace{x^M}}{6}, \mathbb{V}_\iota(\widehat{X}) = \frac{x^L + x^U + 4x^M}{6},\ \mathbb{V}_\nu(\widehat{X}) = \frac{\underbrace{x^L} + \underbrace{x^U} + 4\underbrace{x^M}}{6}$$

$$\mathcal{A}_\tau(\widehat{X}) = \frac{\overbrace{x^U} - \overbrace{x^L}}{6}, \mathcal{A}_\iota(\widehat{X}) = \frac{x^U - x^L}{6}, \mathcal{A}_\nu(\widehat{X}) = \frac{\underbrace{x^U} - \underbrace{x^L}}{6}$$

Definition 3.7. Suppose $\widehat{X} = \left\langle (\overbrace{x^L}, \overbrace{x^M}, \overbrace{x^U}); (x^L, x^M, x^U); (\underbrace{x^L}, \underbrace{x^M}, \underbrace{x^U}) \right\rangle$ be a SVTNN. Then, for $\widehat{X}$, the value and ambiguity index are as follows:

$$\mathbb{V}_{\delta,\rho,\eta}(\widehat{X}) = \delta\mathbb{V}_\tau(\widehat{X}) + \rho\mathbb{V}_\iota(\widehat{X}) + \eta\mathbb{V}_\nu(\widehat{X}) \tag{5}$$

$$\mathcal{A}_{\delta,\rho,\eta}(\widehat{X}) = \delta\mathcal{A}_\tau(\widehat{X}) + \rho\mathcal{A}_\iota(\widehat{X}) + \eta\mathcal{A}_\nu(\widehat{X}) \tag{6}$$

where the DMs' preference value is represented by the co-efficients δ, ρ, η of $\mathbb{V}_{\delta,\rho,\eta}$ and $\mathcal{A}_{\delta,\rho,\eta}$ with the condition $\delta + \rho + \eta = 1$. In an uncertain situation, the DM may want to make pessimistic decisions for $\delta \in [0, 1/3]$ and $\rho + \eta \in [1/3, 1]$. For $\delta \in [1/3, 1]$ and $\rho + \eta \in [0, 1/3]$, on the other hand, the DM may seek to make an optimistic decision in an uncertain situation. The impact of three membership degrees are same to the DM for $\delta = \rho = \eta = 1/3$. As a result, the value index and ambiguity index may indicate how DMs think about SVTNNs.

Lemma 3.8. *Let* $\widehat{X}_1 = \left\langle (\overbrace{x_1^L}, \overbrace{x_1^M}, \overbrace{x_1^U}); (x_1^L, x_1^M, x_1^U); (\underbrace{x_1^L}, \underbrace{x_1^M}, \underbrace{x_1^U}) \right\rangle$ *and* $\widehat{X}_2 = \left\langle (\overbrace{x_2^L}, \overbrace{x_2^M}, \overbrace{x_2^U}); (x_2^L, x_2^M, x_2^U); (\underbrace{x_2^L}, \underbrace{x_2^M}, \underbrace{x_2^U}) \right\rangle$ *be two SVTNNs in* $\mathbb{R}$. *Then for* $\delta, \rho, \eta \in [0, 1]$ *and* $\phi \in \mathbb{R}$, *the following are satisfy*

(1) $\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1} + \widehat{X_2}) = \mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1}) + \mathbb{V}_{\delta,\rho,\eta}(\widehat{X_2})$.

(2) $\mathbb{V}_{\delta,\rho,\eta}(\phi\widehat{X_1}) = \phi\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1})$.

Proof. **(1)** From Definition 3.3, the sum of $\widehat{X_1}$ and $\widehat{X_2}$ is defined as

$$\begin{aligned}\widehat{X}_1 \oplus \widehat{X}_2 = \Big\langle &(\overbrace{x_1^L} + \overbrace{x_2^L}, \overbrace{x_1^M} + \overbrace{x_2^M}, \overbrace{x_1^U} + \overbrace{x_2^U}); (x_1^L + x_2^L, x_1^M + x_2^M, x_1^U + x_2^U);\\ &(\underbrace{x_1^L} + \underbrace{x_2^L}, \underbrace{x_1^M} + \underbrace{x_2^M}, \underbrace{x_1^U} + \underbrace{x_2^U})\Big\rangle\end{aligned} \tag{7}$$

From equation (5), we have

$$\begin{aligned}\mathbb{V}_{\delta,\rho,\eta}(\widehat{X}_1 \oplus \widehat{X}_2) &= \delta\mathbb{V}_\tau(\widehat{X}_1 \oplus \widehat{X}_2) + \rho\mathbb{V}_\iota(\widehat{X}_1 \oplus \widehat{X}_2) + \eta\mathbb{V}_\nu(\widehat{X}_1 \oplus \widehat{X}_2)\\ &= \delta\Big(\frac{\overbrace{x_1^L} + \overbrace{x_2^L} + \overbrace{x_1^U} + \overbrace{x_2^U} + 4(\overbrace{x_1^M} + \overbrace{x_2^M})}{6}\Big)\\ &\quad + \rho\Big(\frac{x_1^L + x_2^L + x_1^U + x_2^U + 4(x_1^M + x_2^M)}{6}\Big)\end{aligned}$$

$$+\eta\Big(\frac{\underbrace{x_1^L}+\underbrace{x_2^L}+\underbrace{x_1^U}+\underbrace{x_2^U}+4(\underbrace{x_1^M}+\underbrace{x_2^M})}{6}\Big)$$
$$=\delta\Big(\frac{\overbrace{x_1^L}+\overbrace{x_1^U}+4\overbrace{x_1^M}}{6}\Big)+\rho\Big(\frac{x_1^L+x_1^U+4x_1^M}{6}\Big)+\eta\Big(\frac{\underbrace{x_1^L}+\underbrace{x_1^U}+4\underbrace{x_1^M}}{6}\Big)$$
$$+\delta\Big(\frac{\overbrace{x_2^L}+\overbrace{x_2^U}+4\overbrace{x_2^M}}{6}\Big)+\rho\Big(\frac{x_2^L+x_2^U+4x_2^M}{6}\Big)+\eta\Big(\frac{\underbrace{x_2^L}+\underbrace{x_2^U}+4\underbrace{x_2^M}}{6}\Big)$$
$$=\mathbb{V}_{\delta,\rho,\eta}(\widehat{X}_1)+\mathbb{V}_{\delta,\rho,\eta}(\widehat{X}_2)$$

(2) From Definition 3.3, the scalar ($\phi\in\mathbb{R}$) multiplication of $\widehat{X}_1=\Big\langle(\overbrace{x_1^L},\overbrace{x_1^M},\overbrace{x_1^U});(x_1^L,x_1^M,x_1^U);(\underbrace{x_1^L},\underbrace{x_1^M},\underbrace{x_1^U})\Big\rangle$ is defined as

$$\phi\widehat{X}_1=\Big\langle(\phi\overbrace{x_1^L},\phi\overbrace{x_1^M},\phi\overbrace{x_1^U});(\phi x_1^L,\phi x_1^M,\phi x_1^U);(\phi\underbrace{x_1^L},\phi\underbrace{x_1^M},\phi\underbrace{x_1^U})\Big\rangle$$

From equation (5), we have

$$\mathbb{V}_{\delta,\rho,\eta}(\phi\widehat{X}_1)=\delta\mathbb{V}_\tau(\phi\widehat{X}_1)+\rho\mathbb{V}_\iota(\phi\widehat{X}_1)+\eta\mathbb{V}_\nu(\phi\widehat{X}_1)$$
$$=\delta\Big(\frac{\phi\overbrace{x_1^L}+\phi\overbrace{x_1^U}+4\phi\overbrace{x_1^M}}{6}\Big)+\rho\Big(\frac{\phi x_1^L+\phi x_1^U+4\phi x_1^M}{6}\Big)+\eta\Big(\frac{\phi\underbrace{x_1^L}+\phi\underbrace{x_1^U}+4\phi\underbrace{x_1^M}}{6}\Big)$$
$$=\phi\Bigg[\delta\Big(\frac{\overbrace{x_1^L}+\overbrace{x_1^U}+4\overbrace{x_1^M}}{6}\Big)+\rho\Big(\frac{x_1^L+x_1^U+4x_1^M}{6}\Big)+\eta\Big(\frac{\underbrace{x_1^L}+\underbrace{x_1^U}+4\underbrace{x_1^M}}{6}\Big)\Bigg]$$
$$=\phi\ \mathbb{V}_{\delta,\rho,\eta}(\widehat{X}_1)$$

This complete the proof. □

Lemma 3.9. *Let* $\widehat{X}_1=\Big\langle(\overbrace{x_1^L},\overbrace{x_1^M},\overbrace{x_1^U});(x_1^L,x_1^M,x_1^U);(\underbrace{x_1^L},\underbrace{x_1^M},\underbrace{x_1^U})\Big\rangle$ *and* $\widehat{X}_2=\Big\langle(\overbrace{x_2^L},\overbrace{x_2^M},\overbrace{x_2^U});(x_2^L,x_2^M,x_2^U);(\underbrace{x_2^L},\underbrace{x_2^M},\underbrace{x_2^U})\Big\rangle$ *be two SVTNNs in* $\mathbb{R}$. *Then for* $\delta,\rho,\eta\in[0,1]$ *and* $\phi\in\mathbb{R}$ *be a real number,*

(1) $\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1}+\widehat{X_2})=\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1})+\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_2})$.

(2) $\mathcal{A}_{\delta,\rho,\eta}(\phi\widehat{X_1})=\phi\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1})$.

Proof.

(1) From Definition 3.3, the sum of two SVTNNs $\widehat{X_1}$ and $\widehat{X_2}$ is written as follows:

$$\widehat{X}_1\oplus\widehat{X}_2=\Big\langle(\overbrace{x_1^L}+\overbrace{x_2^L},\overbrace{x_1^M}+\overbrace{x_2^M},\overbrace{x_1^U}+\overbrace{x_2^U});(x_1^L+x_2^L,x_1^M+x_2^M,x_1^U+x_2^U);$$
$$(\underbrace{x_1^L}+\underbrace{x_2^L},\underbrace{x_1^M}+\underbrace{x_2^M},\underbrace{x_1^U}+\underbrace{x_2^U})\Big\rangle \tag{8}$$

From equation (6), we have

$$\mathcal{A}_{\delta,\rho,\eta}(\widehat{X}_1\oplus\widehat{X}_2)=\delta\mathcal{A}_\tau(\widehat{X}_1\oplus\widehat{X}_2)+\rho\mathcal{A}_\iota(\widehat{X}_1\oplus\widehat{X}_2)+\eta\mathcal{A}_\nu(\widehat{X}_1\oplus\widehat{X}_2)$$

$$= \delta\Big(\frac{\overbrace{x_1^U} + \overbrace{x_2^U} - (\overbrace{x_1^L} + \overbrace{x_2^L})}{6}\Big) + \rho\Big(\frac{x_1^U + x_2^U - (x_1^L + x_2^L)}{6}\Big) + \eta\Big(\frac{\underbrace{x_1^U} + \underbrace{x_2^U} - (\underbrace{x_1^L} + \underbrace{x_2^L})}{6}\Big)$$

$$= \delta\Big(\frac{\overbrace{x_1^U} - \overbrace{x_1^L}}{6}\Big) + \rho\Big(\frac{x_1^U - x_1^L}{6}\Big) + \eta\Big(\frac{\underbrace{x_1^U} - \underbrace{x_1^L}}{6}\Big) + \delta\Big(\frac{\overbrace{x_2^U} - \overbrace{x_2^L}}{6}\Big) + \rho\Big(\frac{x_2^U - x_2^L}{6}\Big) + \eta\Big(\frac{\underbrace{x_2^U} - \underbrace{x_2^L}}{6}\Big)$$

$$= \mathcal{A}_{\delta,\rho,\eta}(\widehat{X}_1) + \mathcal{A}_{\delta,\rho,\eta}(\widehat{X}_2)$$

(2) From Definition 3.3, we have

$$\phi\widehat{X}_1 = \Big\langle(\phi\overbrace{x_1^L}, \phi\overbrace{x_1^M}, \phi\overbrace{x_1^U}), (\phi x_1^L, \phi x_1^M, \phi x_1^U), (\phi\underbrace{x_1^L}, \phi\underbrace{x_1^M}, \phi\underbrace{x_1^U})\Big\rangle$$

From equation (6), we have

$$\mathcal{A}_{\delta,\rho,\eta}(\phi\widehat{X}_1) = \delta\mathcal{A}_\tau(\phi\widehat{X}_1) + \rho\mathcal{A}_\iota(\phi\widehat{X}_1) + \eta\mathcal{A}_\nu(\phi\widehat{X}_1)$$

$$= \delta\Big(\frac{\phi\overbrace{x_1^U} - \phi\overbrace{x_1^L}}{6}\Big) + \rho\Big(\frac{\phi x_1^U - \phi x_1^L}{6}\Big) + \eta\Big(\frac{\phi\underbrace{x_1^U} - \phi\underbrace{x_1^L}}{6}\Big)$$

$$= \phi\Big[\delta\Big(\frac{\overbrace{x_1^U} - \overbrace{x_1^L}}{6}\Big) + \rho\Big(\frac{x_1^U - x_1^L}{6}\Big) + \eta\Big(\frac{\underbrace{x_1^U} - \underbrace{x_1^L}}{6}\Big)\Big]$$

$$= \phi\ \mathcal{A}_{\delta,\rho,\eta}(\widehat{X}_1)$$

This complete the proof. □

Definition 3.10. Let $\widehat{X} = \Big\langle(\overbrace{x^L}, \overbrace{x^M}, \overbrace{x^U}); (x^L, x^M, x^U); (\underbrace{x^L}, \underbrace{x^M}, \underbrace{x^U})\Big\rangle$ be a SVTNN. Then, the ranking function is defined as

$$\Re(\widehat{X}) = \lambda\mathbb{V}_{\delta,\rho,\eta}(\widehat{X}) + (1-\lambda)\mathcal{A}_{\delta,\rho,\eta}(\widehat{X}). \tag{9}$$

The variable λ represents the perspective of the DM regarding risk.

(1) If λ belongs to the range [0, 0.5), then the DM is willing to take risks and prefers uncertainty.

(2) If λ equals 0.5, then the DM has a neutral stance towards risk when making parameter selections.

(3) If λ belongs to the range (0.5, 1], then the DM is sensitive to taking risks when making decisions.

Theorem 3.11. *Let $\widehat{X}_1$ and $\widehat{X}_2$ be two SVTNNs in $\mathbb{R}$. Then for $\delta, \rho, \eta \in [0,1]$ and $\phi \in \mathbb{R}$*

(1) $\Re(\widehat{X_1} + \widehat{X_2}) = \Re(\widehat{X_1}) + \Re(\widehat{X_2})$.

(2) $\Re(\phi\widehat{X_1}) = \phi\Re(\widehat{X_1})$.

Proof.

(1) From Definition 3.10, we have

$$\Re(\widehat{X_1}+\widehat{X_2}) = \lambda\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1}+\widehat{X_2}) + (1-\lambda)\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1}+\widehat{X_2})$$

From Lemma 3.8 and Lemma 3.9, we have

$$\begin{aligned}\Re(\widehat{X_1}+\widehat{X_2}) &= \lambda\Big(\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1}) + \mathbb{V}_{\delta,\rho,\eta}(\widehat{X_2})\Big) + (1-\lambda)\Big(\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1}) + \mathcal{A}_{\delta,\rho,\eta}(\widehat{X_2})\Big)\\ &= \Big[\lambda\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1}) + (1-\lambda)\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1})\Big] + \Big[\lambda\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_2}) + (1-\lambda)\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_2})\Big]\\ &= \Re(\widehat{X_1}) + \Re(\widehat{X_2})\end{aligned}$$

(2) From Definition 3.10, we have

$$\Re(\phi\widehat{X_1}) = \lambda\mathbb{V}_{\delta,\rho,\eta}(\phi\widehat{X_1}) + (1-\lambda)\mathcal{A}_{\delta,\rho,\eta}(\phi\widehat{X_1})$$

From Lemma 3.8 and Lemma 3.9, we have

$$\begin{aligned}\Re(\phi\widehat{X_1}) &= \lambda\Big(\phi\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1})\Big) + (1-\lambda)\Big(\phi\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1})\Big)\\ &= \phi\Big[\lambda\mathbb{V}_{\delta,\rho,\eta}(\widehat{X_1}) + (1-\lambda)\mathcal{A}_{\delta,\rho,\eta}(\widehat{X_1})\Big]\\ &= \phi\Re(\widehat{X_1})\end{aligned}$$

This complete the proof. □

Corollary 3.12. *Let* $\widehat{X}_i = \Big\langle(\overbrace{x_i^L}, \overbrace{x_i^M}, \overbrace{x_i^U}); (x_i^L, x_i^M, x_i^U); (\underbrace{x_i^L}, \underbrace{x_i^M}, \underbrace{x_i^U})\Big\rangle$ *be* n *SVTNNs in* $\mathbb{R}$ *and* $\phi_i \in \mathbb{R}$ *be the scalars where* $i = 1, 2, 3, \cdots, n$. *Then*

$$\begin{aligned}\Re\Big(\sum_{i=1}^{n}\phi_i\widehat{X}_i\Big) = \sum_{i=1}^{n}\Big[\lambda\Big(\delta\big(\frac{\overbrace{x_i^L}+\overbrace{x_i^U}+4\overbrace{x_i^M}}{6}\big) + \rho\big(\frac{x_i^L+x_i^U+4x_i^M}{6}\big) + \eta\big(\frac{\underbrace{x_i^L}+\underbrace{x_i^U}+4\underbrace{x_i^M},}{6}\big)\Big)\\ + (1-\lambda)\Big(\delta\big(\frac{\overbrace{x_i^U}-\overbrace{x_i^L}}{6}\big) + \rho\big(\frac{x_i^U-x_i^L}{6}\big) + \eta\big(\frac{\underbrace{x_i^U}-\underbrace{x_i^L}}{6}\big)\Big)\Big]\phi_i \qquad (10)\end{aligned}$$

Proof. This is proved by using Theorem 3.11, Lemma 3.8 and Lemma 3.9. □

Definition 3.13. Suppose $\widehat{X_1}$ and $\widehat{X_2}$ be two SVTNNs, then two SVTNNs are compared by

(1) $\widehat{X_1} \le \widehat{X_2}$ if and only if $\Re(\widehat{X_1}) \le \Re(\widehat{X_2})$,

(2) $\widehat{X_1} < \widehat{X_2}$ if and only if $\Re(\widehat{X_1}) < \Re(\widehat{X_2})$,

where $\Re(.)$ is the ranking function.

Note: If $\widehat{X} = a \in \mathbb{R}$ be a real crisp number so that it is independent of the risk factor λ, then $\Re(\widehat{X}) = a$.

4. Neutrosophic Data Envelopment Analysis

Consider $a_i = (a_{1i}, a_{2i}, \cdots, a_{mi}) \in \mathbb{R}^m$ and $b_i = (b_{1i}, b_{2i}, \cdots, b_{ri}) \in \mathbb{R}^r$ are the input and output vector of DMU_i for $i = 1, 2, \cdots, n$, respectively. The input matrix A and and the output matrix B are defined as $A = [a_1, \cdots, a_n] \in \mathbb{R}^{m\times n}$, and $B = [b_1, \cdots, b_n] \in \mathbb{R}^{r\times n}$ such that $A > 0$ and $B > 0$. Charnes et al. [10] developed the following LP model for measuring the efficiency of DMU_o

$$\begin{aligned} \max_{\omega,\mu} \theta &= \sum_{k=1}^{r} \omega_k b_{ko}, \\ \text{subject to } & \sum_{j=1}^{m} \mu_j a_{jo} = 1, \\ & \sum_{k=1}^{r} \omega_k b_{ki} \leq \sum_{j=1}^{m} \mu_j a_{ji},\ i = 1, 2, \cdots n, \\ \text{and } & \omega_k \geq 0,\ k = 1, 2, \cdots, r, \\ & \mu_j \geq 0,\ j = 1, 2, \cdots, m. \end{aligned} \tag{11}$$

This is popularly known CCR model.

In the classical DEA model, the efficiency score of the $DMUo$ will be erroneous if the input and output data of the DMUs are inaccurate, imprecise, or ambiguous. The application of Neutrosophic set theory is a powerful strategy for dealing with this type of data.

Assuming inputs and outputs of the DMUs are SVTNNs while the weights $\mu_j \in \mathbb{R}$ and $\omega_k \in \mathbb{R}$. Then, the Neutrosophic CCR (Nue-CCR) model can be defined as

$$\begin{aligned} \max_{\omega,\mu} \theta &= \sum_{k=1}^{r} \omega_k \widehat{b_{ko}}, \\ \text{subject to } & \sum_{j=1}^{m} \mu_j \widehat{a_{jo}} = 1, \\ & \sum_{k=1}^{r} \omega_k \widehat{b_{ki}} \leq \sum_{j=1}^{m} \mu_j \widehat{a_{ji}},\ i = 1, 2, \cdots n, \\ \text{and } & \omega_k \geq 0,\ k = 1, 2, \cdots, r, \\ & \mu_j \geq 0,\ j = 1, 2, \cdots, m, \end{aligned} \tag{12}$$

where $\widehat{a_{ji}} = \left\langle (\overbrace{a_{ji}^L}, \overbrace{a_{ji}^M}, \overbrace{a_{ji}^U}); (a_{ji}^L, a_{ji}^M, a_{ji}^U); (\underbrace{a_{ji}^L}, \underbrace{a_{ji}^M}, \underbrace{a_{ji}^U}) \right\rangle$ and $\widehat{b_{ki}} = \left\langle (\overbrace{b_{ki}^L}, \overbrace{b_{ki}^M}, \overbrace{b_{ki}^U}); (b_{ki}^L, b_{ki}^M, b_{ki}^U); (\underbrace{b_{ki}^L}, \underbrace{b_{ki}^M}, \underbrace{b_{ki}^U}) \right\rangle$ are the SVTNNs. The efficiency score of the Nue-CCR model is $\theta^* \in [0, 1]$.

Definition 4.1. A DMU is said to be efficient, if its efficiency score is 1; Otherwise it is consider as inefficient DMU.

Applying the ranking function ($\Re$) in the Neu-CCR model given in equation (12).

$$\begin{aligned}
\max_{\omega,\mu}\ \theta &= \Re\Big(\sum_{k=1}^{r}\omega_k \widehat{b_{ko}}\Big),\\
\text{subject to } \Re\Big(\sum_{j=1}^{m}\mu_j \widehat{a_{jo}}\Big) &= \Re\Big(1\Big),\\
\Re\Big(\sum_{k=1}^{r}\omega_k \widehat{b_{ki}}\Big) &\le \Re\Big(\sum_{j=1}^{m}\mu_j \widehat{a_{ji}}\Big),\ i=1,2,\cdots n,\\
\text{and } \omega_k \ge 0,&\ k=1,2,\cdots,r,\\
\mu_j \ge 0,&\ j=1,2,\cdots,m.
\end{aligned} \tag{13}$$

Using Definition 3.3, The equation (13) can be written as

$$\begin{aligned}
\max_{\omega,\mu}\ \theta = \Re\Bigg(\Big\langle &(\sum_{k=1}^{r}\omega_k \overbrace{b^L_{ko}}, \sum_{k=1}^{r}\omega_k \overbrace{b^M_{ko}}, \sum_{k=1}^{r}\omega_k \overbrace{b^U_{ko}}); (\sum_{k=1}^{r}\omega_k b^L_{ko}, \sum_{k=1}^{r}\omega_k b^M_{ko}, \sum_{k=1}^{r}\omega_k b^U_{ko});\\
&(\sum_{k=1}^{r}\omega_k \underbrace{b^L_{ko}}, \sum_{k=1}^{r}\omega_k \underbrace{b^M_{ko}}, \sum_{k=1}^{r}\omega_k \underbrace{b^U_{ko}})\Big\rangle\Bigg),\\
\text{s. t } \Re\Bigg(\Big\langle &(\sum_{j=1}^{m}\mu_j \overbrace{a^L_{jo}}, \sum_{j=1}^{m}\mu_j \overbrace{a^M_{jo}}, \sum_{j=1}^{m}\mu_j \overbrace{a^U_{jo}}); (\sum_{j=1}^{m}\mu_j a^L_{jo}, \sum_{j=1}^{m}\mu_j a^M_{jo}, \sum_{j=1}^{m}\mu_j a^U_{jo});\\
&(\sum_{j=1}^{m}\mu_j \underbrace{a^L_{jo}}, \sum_{j=1}^{m}\mu_j \underbrace{a^M_{jo}}, \sum_{j=1}^{m}\mu_j \underbrace{a^U_{jo}})\Big\rangle\Bigg) = 1,\\
\Re\Bigg(\Big\langle &(\sum_{k=1}^{r}\omega_k \overbrace{b^L_{ki}}, \sum_{k=1}^{r}\omega_k \overbrace{b^M_{ki}}, \sum_{k=1}^{r}\omega_k \overbrace{b^U_{ki}}); (\sum_{k=1}^{r}\omega_k b^L_{ki}, \sum_{k=1}^{r}\omega_k b^M_{ki}, \sum_{k=1}^{r}\omega_k b^U_{ki});\\
&(\sum_{k=1}^{r}\omega_k \underbrace{b^L_{ki}}, \sum_{k=1}^{r}\omega_k \underbrace{b^M_{ki}}, \sum_{k=1}^{r}\omega_k \underbrace{b^U_{ki}})\Big\rangle\Bigg) \le \Re\Bigg(\Big\langle (\sum_{j=1}^{m}\mu_j \overbrace{a^L_{ji}}, \sum_{j=1}^{m}\mu_j \overbrace{a^M_{ji}}, \sum_{j=1}^{m}\mu_j \overbrace{a^U_{ji}});\\
&(\sum_{j=1}^{m}\mu_j a^L_{ji}, \sum_{j=1}^{m}\mu_j a^M_{ji}, \sum_{j=1}^{m}\mu_j a^U_{ji}); (\sum_{j=1}^{m}\mu_j \underbrace{a^L_{ji}}, \sum_{j=1}^{m}\mu_j \underbrace{a^M_{ji}}, \sum_{j=1}^{m}\mu_j \underbrace{a^U_{ji}})\Big\rangle\Bigg), i=1,2,\cdots,n,\\
\text{and } \omega_k \ge 0,&\ k=1,2,\cdots,r,\ \mu_j \ge 0,\ j=1,2,\cdots,m.
\end{aligned}$$

Now from Theorem 3.11 and Corollary 3.12, we have

$$\begin{aligned}
\max_{\omega,\mu}\ \theta = \sum_{k=1}^{r}\Big[&\lambda\Big(\delta(\frac{\overbrace{b^L_{ko}} + \overbrace{b^U_{ko}} + 4\overbrace{b^M_{ko}}}{6}) + \rho(\frac{b^L_{ko} + b^U_{ko} + 4b^M_{ko}}{6}) + \eta(\frac{\underbrace{b^L_{ko}} + \underbrace{b^U_{ko}} + 4\underbrace{b^M_{ko}},}{6})\Big)\\
&+ (1-\lambda)\Big(\delta(\frac{\overbrace{b^U_{ko}} - \overbrace{b^L_{ko}}}{6}) + \rho(\frac{b^U_{ko} - b^L_{ko}}{6}) + \eta(\frac{\underbrace{b^U_{ko}} - \underbrace{b^L_{ko}}}{6})\Big)\Big]\omega_k\\
\text{s.t } \sum_{j=1}^{m}\Big[&\lambda\Big(\delta(\frac{\overbrace{a^L_{jo}} + \overbrace{a^U_{jo}} + 4\overbrace{a^M_{jo}}}{6}) + \rho(\frac{a^L_{jo} + a^U_{jo} + 4a^M_{jo}}{6}) + \eta(\frac{\underbrace{a^L_{jo}} + \underbrace{a^U_{jo}} + 4\underbrace{a^M_{jo}},}{6})\Big)
\end{aligned} \tag{14}$$

$$+(1-\lambda)\Big(\delta(\frac{\overbrace{a_{jo}^{U}}-\overbrace{a_{jo}^{L}}}{6})+\rho(\frac{a_{jo}^{U}-a_{jo}^{L}}{6})+\eta(\frac{\underbrace{a_{jo}^{U}}-\underbrace{a_{jo}^{L}}}{6})\Big)\Big]\mu_j=1$$

$$\sum_{k=1}^{r}\Big[\lambda\Big(\delta(\frac{\overbrace{b_{ki}^{L}}+\overbrace{b_{ki}^{U}}+4\,\overbrace{b_{ki}^{M}}}{6})+\rho(\frac{b_{ki}^{L}+b_{ki}^{U}+4b_{ki}^{M}}{6})+\eta(\frac{\underbrace{b_{ki}^{L}}+\underbrace{b_{ki}^{U}}+4\,\underbrace{b_{ki}^{M}},}{6})\Big)$$

$$+(1-\lambda)\Big(\delta(\frac{\overbrace{b_{ki}^{U}}-\overbrace{b_{ki}^{L}}}{6})+\rho(\frac{b_{ki}^{U}-b_{ki}^{L}}{6})+\eta(\frac{\underbrace{b_{ki}^{U}}-\underbrace{b_{ki}^{L}}}{6})\Big)\Big]\omega_k$$

$$\leq\sum_{j=1}^{m}\Big[\lambda\Big(\delta(\frac{\overbrace{a_{ji}^{L}}+\overbrace{a_{ji}^{U}}+4\,\overbrace{a_{ji}^{M}}}{6})+\rho(\frac{a_{ji}^{L}+a_{ji}^{U}+4a_{ji}^{M}}{6})+\eta(\frac{\underbrace{a_{ji}^{L}}+\underbrace{a_{ji}^{U}}+4\,\underbrace{a_{ji}^{M}},}{6})\Big)$$

$$+(1-\lambda)\Big(\delta(\frac{\overbrace{a_{ji}^{U}}-\overbrace{a_{ji}^{L}}}{6})+\rho(\frac{a_{ji}^{U}-a_{ji}^{L}}{6})+\eta(\frac{\underbrace{a_{ji}^{U}}-\underbrace{a_{ji}^{L}}}{6})\Big)\Big]\mu_j,\ i=1,2,\cdots,n,$$

$$\text{and } \omega_k\geq 0,\ k=1,2,\cdots,r,\ \mu_j\geq 0,\ j=1,2,\cdots,m.$$

This is the equivalent crisp LP model of the Neu-CCR model defined in equation (12).

Theorem 4.2. *The Neutrosophic CCR model presented in equation* (12) *and the corresponding crisp LP model presented in equation* (14) *of equal significance.*

Proof. By utilizing the ranking formula put forth in Definition 3.10 of the Neu-CCR model, which is illustrated in equation (13), it becomes straightforward to observe that the optimal feasible solution derived in equation (12) for every Neu-CCR model is also an optimal feasible solution of equation (14), and similarly, the converse holds true. □

5. Method for Solving Neutrosophic DEA model

Fuzzification is the process of converting precise input-output data into fuzzy input-output data by utilizing information from a knowledge base. Fuzzification is considered important and advantageous in the early stages of uncertainty theory because the fuzzifier is defined as a mapping from a crisp data space to a fuzzy data space within a particular discourse universe. This article utilizes triangular neutrosophic membership functions during the fuzzification process since they can be effectively implemented by embedded controllers in highly uncertain environments. In this particular case, a single value neutrosophic set is employed during the fuzzification process based on the observed data. The Neu-DEA model may be solved by performing the procedures listed below.

Step 1: Convert the DEA model into the Neu-DEA model by considering the input-output data are SVTNNs as shown in equation (12).

Step 2: Applying the ranking function ($\Re$) in the Neu-DEA model given in equation (12).

Step 3: Convert the Nue-DEA model into equivalent crisp LP model as shown in equation (14).

Step 4: Solve the given crisp LP model effectively and determine the optimal solution θ^* with different DM's preference parameters (δ, ρ, η) for each risk factor $\lambda \in [0, 1]$ which represents the risk taking attitude of the DM, given in Definition 3.10.

Step 5: The DMUs are ranked according to the average of each DMU's efficiency scores for each DM's preference level.

The flowchart depicted in Figure 2 illustrates the step-by-step approach utilized to solve the Neu-DEA model. This technique serves as a visual representation of the process employed to address the model's complexities and arrive at a solution.

FIGURE 2. Flowchart for solving Neu-DEA model

6. Numerical Example

In order to demonstrate the validity and applicability of the suggested model, this section consider a real case study of hospital performance assessment provided by Yang et al. [45] in a neutrosophic environment. The input parameters, which consist of "the number of doctors and the number of nurses", are displayed in Table 3. Correspondingly, the output parameters, including "days of hospitalization, patient satisfaction, and the number of outpatients", are shown in Table 4.

Table 3. The SVTNNs input data

DMU	Number of Doctors	Number of Nurses
D1	$\langle(404, 540, 674); (350, 440, 560); (420, 645, 700)\rangle$	$\langle(520, 530, 535); (520, 525, 530); (532, 534, 540)\rangle$
D2	$\langle(119, 136, 182); (122, 125, 137); (125, 178, 200)\rangle$	$\langle(177, 180, 188); (173, 175, 179); (185, 189, 195)\rangle$
D3	$\langle(139, 145, 158); (139, 140, 147); (146, 155, 167)\rangle$	$\langle(208, 214, 218); (195, 209, 215); (210, 217, 230)\rangle$
D4	$\langle(86, 93, 151); (83, 85, 87); (89, 138, 160)\rangle$	$\langle(114, 116, 118); (114, 115, 117); (116, 118, 125)\rangle$
D5	$\langle(84, 93, 143); (84, 89, 120); (90, 140, 155)\rangle$	$\langle(110, 117, 121); (105, 112, 120); (113, 119, 128)\rangle$
D6	$\langle(101, 113, 170); (110, 112, 115); (112, 120, 177)\rangle$	$\langle(101, 107, 111); (95, 100, 104); (108, 112, 115)\rangle$
D7	$\langle(561, 694, 864); (510, 640, 750), (582, 857, 930)\rangle$	$\langle(492, 495, 508); (492, 494, 500); (493, 506, 520)\rangle$
D8	$\langle(123, 179, 199); (122, 125, 130); (195, 200, 205)\rangle$	$\langle(66, 68, 73); (63, 67, 69); (68, 70, 78)\rangle$
D9	$\langle(101, 153, 155); (140, 145, 150); (145, 149, 167)\rangle$	$\langle(192, 195, 198); (185, 193, 197); (194, 196, 205)\rangle$
D10	$\langle(147, 164, 170); (147, 160, 167); (165, 169, 180)\rangle$	$\langle(333, 340, 357); (335, 338, 350); (338, 347, 364)\rangle$
D11	$\langle(130, 158, 192); (110, 144, 173); (146, 177, 205)\rangle$	$\langle(96, 100, 114); (97, 99, 103); (99, 110, 129)\rangle$
D12	$\langle(128, 137, 187); (128, 133, 164); (134, 184, 199)\rangle$	$\langle(213, 220, 224); (208, 215, 223); (216, 222, 231)\rangle$
D13	$\langle(151, 160, 210); (151, 156, 187); (157, 207, 222)\rangle$	$\langle(320, 327, 331); (315, 322, 330); (323, 329, 338)\rangle$

Table 4. The SVTNNs output data

DMU	Days of Hospitalization	Patient satisfaction	Numbers of Outpatient
D1	$\langle(121.13, 139.24, 140.04); (138.64, 139.14, 139.81); (139.14, 140.02, 141.17)\rangle$	$\langle(38, 41, 45); (38, 40, 43); (41, 44, 49)\rangle$	$\langle(104.23, 114.04, 278.51); (102.37, 109.15, 235.72); (104.81, 275.25, 279.88)\rangle$
D2	$\langle(31.54, 34.15, 38.27); (31.54, 34.93, 38.89); (34.86, 38.15, 39.83)\rangle$	$\langle(40, 44, 47); (35, 42, 45); (41, 46, 50)\rangle$	$\langle(34.54, 36.98, 54.82); (36.45, 36.80, 41.57); (47.61, 54.25, 55.35)\rangle$
D3	$\langle(81.62, 82.07, 85.51); (81.41, 81.94, 83.35); (81.78, 85.49, 88.16)\rangle$	$\langle(18, 20, 29); (19, 21, 23); (28, 30, 35)\rangle$	$\langle(157.75, 177.57, 264.52); (157.75, 176.68, 250.75); (180.29, 263.98, 272.16)\rangle$
D4	$\langle(19.54, 20.41, 20.59); (20.15, 20.25, 20.32); (20.54, 20.58, 20.70)\rangle$	$\langle(18, 21, 25); (15, 19, 23); (20, 24, 30)\rangle$	$\langle(32.89, 35.56, 87.74); (35.25, 35.50, 35.61); (87.50, 87.94, 88.30)\rangle$

D5	$\langle(23.89, 24.60, 26.09);$ $(23.56, 23.60, 23.68);$ $(25.97, 26.35, 26.72)\rangle$	$\langle(30, 36, 41); (34, 35, 37);$ $(35, 40, 57)\rangle$	$\langle(63.23, 69.58, 120.73);$ $(63, 65.17, 94.93);$ $(64.47, 118.75, 124.75)\rangle$
D6	$\langle(21.33, 21.49, 23.31);$ $(20.94, 24.25, 22.68);$ $(21.38, 23.14, 23.94)\rangle$	$\langle(50, 55, 60); (50, 53, 57);$ $(56, 59, 70)\rangle$	$\langle(72.84, 82.84, 94.18);$ $(82.15, 82.68, 84.89);$ $(85.75, 93.50, 97.18)\rangle$
D7	$\langle(145.77, 148.28, 169.01);$ $(145.77, 147.16, 168.31);$ $(150.69, 168.95, 175.18)\rangle$	$\langle(40, 44, 46); (42, 43, 45);$ $(43, 44, 55)\rangle$	$\langle(147.59, 150.37, 227.12);$ $(147.30, 147.45, 148.25);$ $(218.24, 224.61, 229.63)\rangle$
D8	$\langle(11.56, 11.74, 12.96);$ $(11.42, 11.61, 11.98);$ $(11.58, 12.64, 13.16)\rangle$	$\langle(60, 75, 80); (55, 60, 62);$ $(78, 83, 85)\rangle$	$\langle(189.37, 202.08, 284.99);$ $(189.37, 200.52, 281.63);$ $(270.16, 284.55, 289.12)\rangle$
D9	$\langle(57.55, 62.67, 63.03);$ $(62.15, 62.50, 62.93);$ $(62.50, 62.97, 63.61)\rangle$	$\langle(32, 35, 38); (32, 33, 35);$ $(34, 36, 45)\rangle$	$\langle(14.63, 14.85, 29.40);$ $(14.70, 14.75, 15.25);$ $(24.75, 28.36, 32.64)\rangle$
D10	$\langle(73.21, 76.03, 81.90);$ $(75.76, 76.05, 76.25);$ $(81.67, 82.27, 82.64)\rangle$	$\langle(22, 25, 40); (20, 24, 27);$ $(23, 25, 29)\rangle$	$\langle(96.77, 97.27, 110.39);$ $(96.77, 96.89, 105.14);$ $(99.76, 108.62, 115.27)\rangle$
D11	$\langle(22.90, 27.71, 35.56);$ $(22.90, 26.45, 31.28);$ $(27.92, 34.62, 39.41)\rangle$	$\langle(20, 23, 26); (21, 22, 24);$ $(22, 25, 30)\rangle$	$\langle(171.53, 182.46, 384.99);$ $(171.12, 178.65, 210.34);$ $(175.59, 270.65, 400.12)\rangle$
D12	$\langle(58.41, 59.12, 60.61);$ $(58.08, 58.12, 58.20);$ $(60.49, 60.87, 61.24)\rangle$	$\langle(25, 31, 37); (29, 30, 32);$ $(30, 35, 52)\rangle$	$\langle(59.87, 66.22, 117.37);$ $(59.64, 61.81, 91.57);$ $(61.11, 115.39, 121.39)\rangle$
D13	$\langle(66.97, 67.68, 69.17);$ $(66.64, 66.68, 66.76);$ $(69.05, 69.43, 69.80)\rangle$	$\langle(20, 27, 31); (23, 26, 28);$ $(24, 30, 46)\rangle$	$\langle(96.97, 103.32, 154.47);$ $(96.74, 98.91, 128.67);$ $(98.21, 152.49, 158.50)\rangle$

TABLE 5. Efficiency Score of the DMUs

DM's Preference	DMUs	Efficiency Score						Ranking
		$\lambda = 0$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$\lambda = 1$	Mean	
$(\delta, \rho, \eta) = (1, 0, 0)$	D1	1	0.703	0.6827	0.6763	0.6731	0.74702	10
	D2	0.8808	0.8957	0.916	0.9126	0.9095	0.90292	5
	D3	1	1	1	1	1	1	1
	D4	1	0.6057	0.6456	0.6532	0.6566	0.71222	11
	D5	0.582	0.9747	1	1	1	0.91134	4
	D6	0.4806	0.948	1	1	1	0.88572	6
	D7	1	0.8354	0.8061	0.7978	0.7937	0.8466	8
	D8	1	1	1	1	1	1	1
	D9	1	0.936	0.9892	1	1	0.98504	2

	D10	1	1	0.9501	0.9306	0.919	0.95994	3
	D11	1	1	1	1	1	1	1
	D12	0.6187	0.8863	0.9223	0.9316	0.9336	0.8585	7
	D13	0.582	0.7698	0.8055	0.8165	0.8206	0.75888	9
$(\delta,\rho,\eta)=(\frac{1}{2},\frac{1}{4},\frac{1}{4})$	D1	1	0.6929	0.6781	0.6736	0.6713	0.74318	10
	D2	1	0.8486	0.8483	0.8474	0.8467	0.8782	5
	D3	1	1	1	1	1	1	1
	D4	0.8889	0.5962	0.6006	0.6047	0.607	0.65948	11
	D5	0.6375	0.9041	0.915	0.9185	0.9202	0.85906	6
	D6	0.6327	1	1	1	1	0.92654	3
	D7	1	0.8525	0.816	0.8037	0.7979	0.85402	7
	D8	1	1	1	1	1	1	1
	D9	0.8183	0.9418	0.9585	0.9639	0.9666	0.92982	2
	D10	1	0.9175	0.8957	0.8888	0.8854	0.91748	4
	D11	1	1	1	1	1	1	1
	D12	0.6646	0.8427	0.8657	0.8754	0.8808	0.82584	8
	D13	0.6646	0.7452	0.7656	0.7741	0.7788	0.74566	9
$(\delta,\rho,\eta)=(\frac{1}{3},\frac{1}{3},\frac{1}{3})$	D1	1	0.6895	0.6766	0.6726	0.6707	0.74188	10
	D2	1	0.8219	0.8221	0.8217	0.8214	0.85742	6
	D3	1	1	1	1	1	1	1
	D4	1	0.5869	0.5865	0.5876	0.5893	0.67006	12
	D5	0.6182	0.8696	0.8834	0.8882	0.8905	0.82998	8
	D6	0.7943	1	1	1	1	0.95886	3
	D7	1	0.8583	0.8199	0.807	0.8005	0.85714	7
	D8	1	1	1	1	1	1	1
	D9	0.7916	0.9381	0.9466	0.9493	0.9507	0.91526	4
	D10	1	0.8912	0.8794	0.8757	0.874	0.90406	5
	D11	1	1	1	1	0.9955	0.9991	2
	D12	0.6353	0.8223	0.8455	0.8553	0.8606	0.8038	9
	D13	0.6525	0.7335	0.7518	0.7596	0.7639	0.73226	11
$(\delta,\rho,\eta)=(0,0,1)$	D1	1	0.6747	0.6712	0.67	0.6694	0.73706	9
	D2	0.9612	0.6921	0.7016	0.7063	0.7088	0.754	7
	D3	1	1	1	1	1	1	1
	D4	0.6007	0.5211	0.5332	0.5386	0.5412	0.54696	10
	D5	1	0.8298	0.8077	0.8036	0.8017	0.84856	4
	D6	1	1	1	1	1	1	1
	D7	1	0.8883	0.8581	0.8479	0.8428	0.88742	3
	D8	1	1	1	1	1	1	1
	D9	1	0.9575	0.9315	0.9231	0.9189	0.9462	2
	D10	0.5714	0.8732	0.8783	0.88	0.8808	0.81674	6
	D11	1	1	1	1	1	1	1
	D12	1	0.7787	0.7705	0.7681	0.767	0.81686	5

	D13	1	0.695	0.6862	0.6853	0.6854	0.75038	8
$(\delta, \rho, \eta) = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$	D1	0.8799	0.6959	0.6829	0.6785	0.6763	0.7227	7
	D2	1	0.8793	0.8663	0.8616	0.8592	0.89328	3
	D3	1	1	1	1	1	1	1
	D4	1	0.6968	0.671	0.6617	0.657	0.7373	6
	D5	0.1528	0.7717	0.857	0.891	0.9102	0.71654	9
	D6	1	1	1	1	1	1	1
	D7	1	0.8499	0.8002	0.7838	0.7757	0.84192	4
	D8	1	1	1	1	1	1	1
	D9	0.2363	0.901	0.9133	0.9185	0.9211	0.77804	5
	D10	0.2158	0.8411	0.8559	0.8609	0.8634	0.72742	6
	D11	1	0.9837	0.9435	0.9314	0.9257	0.95686	2
	D12	0.1528	0.7988	0.8613	0.8846	0.8968	0.71886	8
	D13	0.2026	0.7333	0.7748	0.79	0.7978	0.6597	10
$(\delta, \rho, \eta) = (0, 1, 0)$	D1	1	0.6899	0.678	0.674	0.672	0.74278	10
	D2	1	0.836	0.8324	0.8309	0.8301	0.86588	6
	D3	1	1	1	1	1	1	1
	D4	1	0.6015	0.6017	0.6025	0.6032	0.68178	12
	D5	0.4948	0.8477	0.8775	0.8884	0.8941	0.8005	8
	D6	0.743	1	1	1	1	0.9486	3
	D7	1	0.8562	0.814	0.7998	0.7931	0.85262	7
	D8	1	1	1	1	1	1	1
	D9	0.699	0.9294	0.9389	0.9421	0.9436	0.8906	5
	D10	1	0.8803	0.8743	0.8726	0.8718	0.8998	4
	D11	1	1	1	0.9912	0.9791	0.99406	2
	D12	0.5075	0.8157	0.849	0.8621	0.8691	0.78068	9
	D13	0.5456	0.7328	0.7572	0.7669	0.772	0.7149	11

The relative efficiencies of the 13 DMUs (or hospitals) are evaluated within a neutrosophic environment by solving the crisp LP model, as described in equation (14), which corresponds to the Neu-DEA model. Table 5 Table 5 shows the outcomes obtained by solving the suggested Neu-DEA model in MATLAB R2021a., which determines the relative efficiency of all DMUs across all degrees of risk.

The performance of DMUs is calculated by varying the risk parameter according to each DM's preference level. Table 5 demonstrates how the DMUs' performance is influenced by the DM's preference level and risk factor. The DMUs are ranked according to their efficiency by calculating the average efficiency for different risk factors corresponding to each DM's preference level. When the mean efficiency of a DMU is 1, it implies that the DMU is performing efficiently across all risk factors and is marked in blue color. Such DMUs are referred to as "fully efficient". On the other hand, when a DMU performs inefficiently across all risk factors, it is labeled in red color and called "fully inefficient". However, some DMUs may perform efficiently under some risk factors but not all. These DMUs are considered "partially efficient" or "partially inefficient." When considering optimistic, neutral, or pessimistic decisions, it is observed that DMUs D3 and D8 consistently maintain their efficiency across different risk factors.

As a result, $D3$ and $D8$ can be considered the top-performing DMUs among all the others. This finding holds true even when analyzing the neutral DM's preference level.

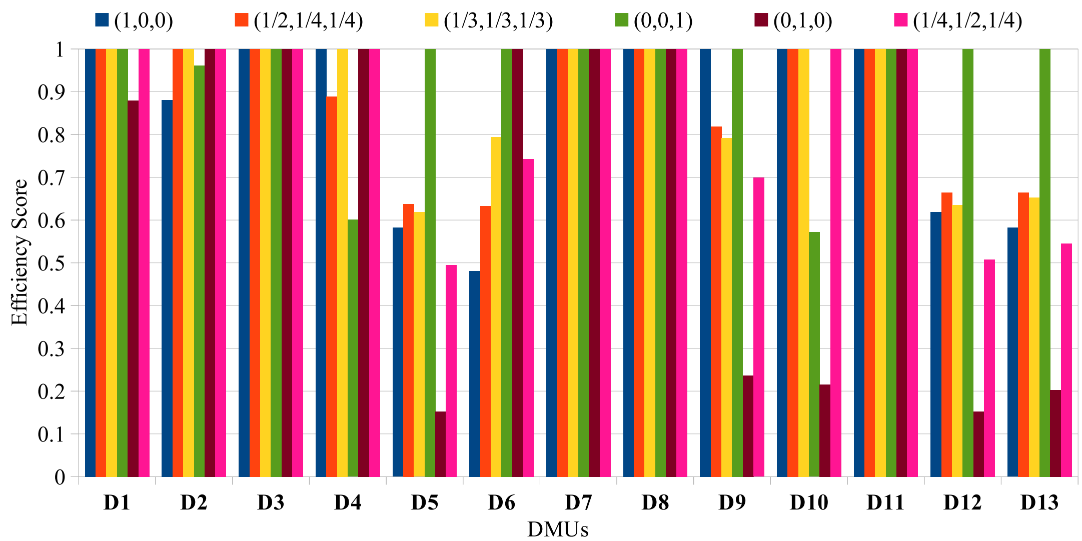

Figure 3. Efficiency Score of different preference level with risk factor $\lambda = 0$

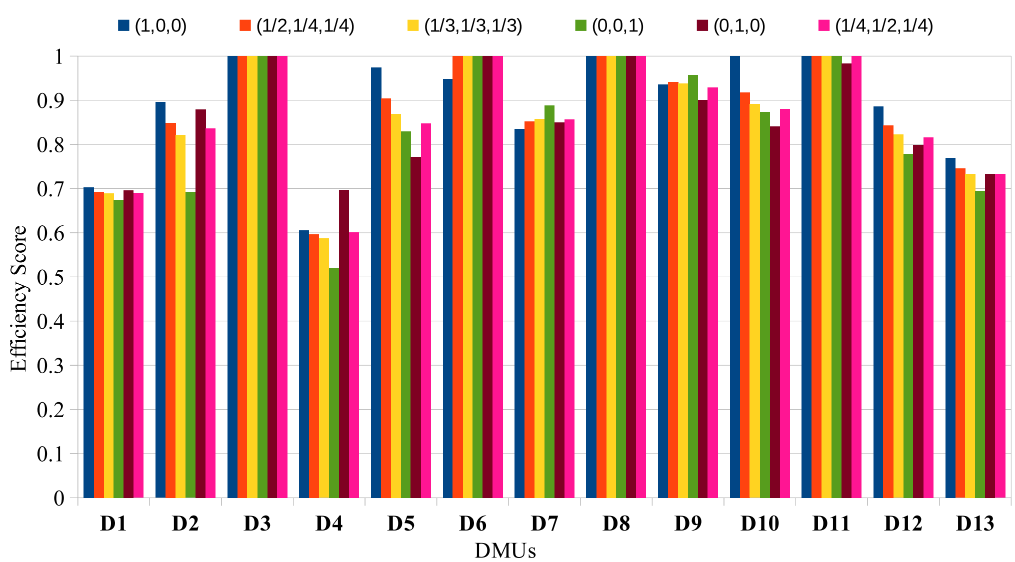

Figure 4. Efficiency Score of different preference level with risk factor $\lambda = 0.25$

In Figure 3, the efficiency of DMUs is compared across different preference levels, with a fixed risk factor of $\lambda = 0$. Approximately 60% of the DMUs are found to be efficient. Figure 4 examines the efficiency of DMUs across different preference levels, using a fixed risk factor of $\lambda = 0.25$. It is observed that approximately 29% of the DMUs are efficient. Similarly, Figure 5 analyzes the efficiency of DMUs with various preference levels, keeping the risk factor fixed at $\lambda = 0.5$. Approximately 30% of the DMUs

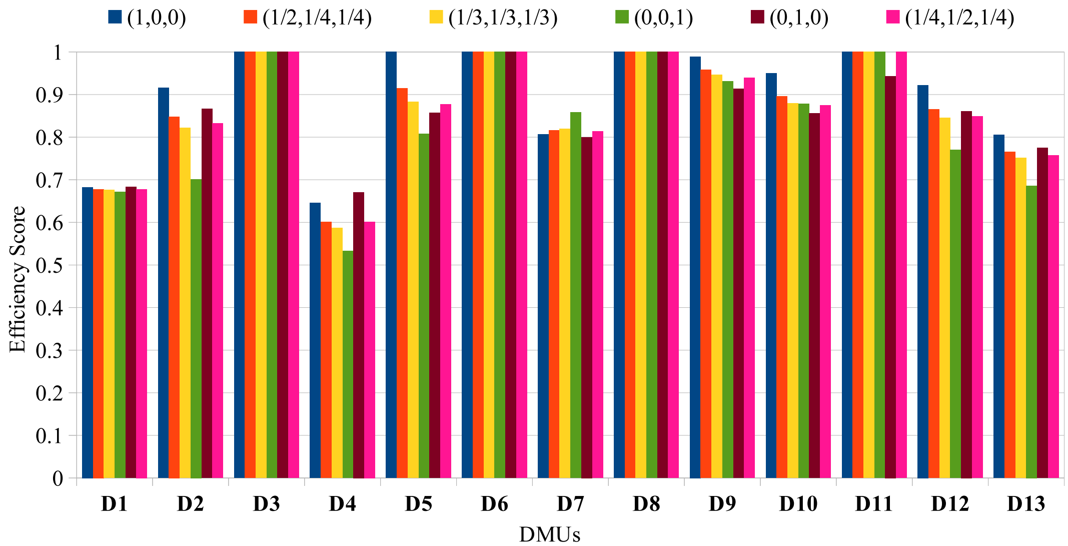

FIGURE 5. Efficiency Score of different preference level with risk factor $\lambda = 0.5$

are determined to be fully efficient. Figure 6 compares the efficiency of DMUs at various preference

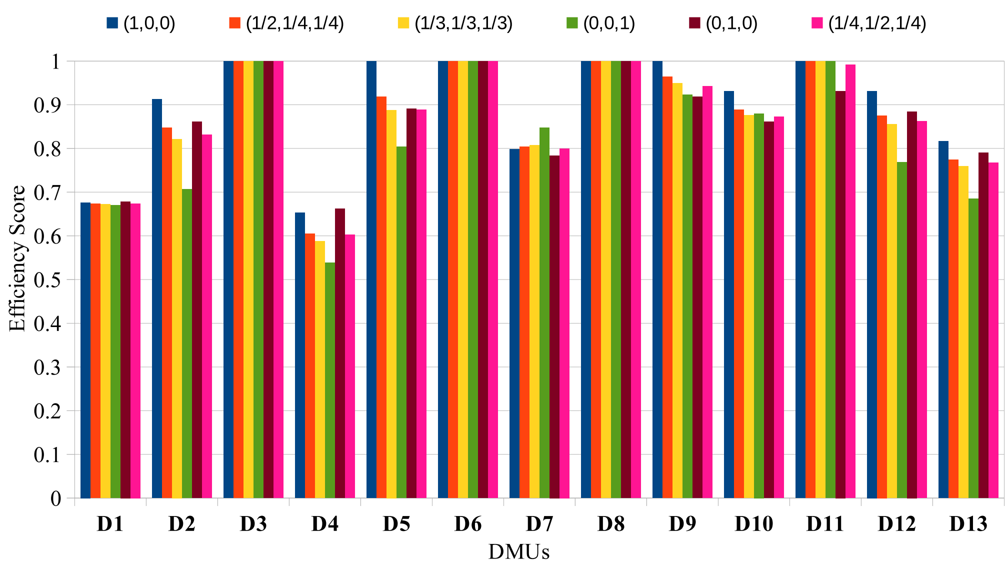

FIGURE 6. Efficiency Score of different preference level with risk factor $\lambda = 0.75$

levels while keeping a constant risk factor of $lambda = 0.75$. It has been found that around 30% of the DMUs are fully efficient. Figure 7 compares the efficiency of DMUs across various preference levels with a constant risk factor of $lambda = 1$. It has been shown that around 29% of DMUs are fully efficient. When considering the mean efficiency of the DMUs, it is found that approximately 22% of the DMUs are fully efficient. These results demonstrate the impact of both the risk factor and preference level on efficiency of DMUs. Notably, as the risk factor increases, the number of efficient DMUs tends to decrease.

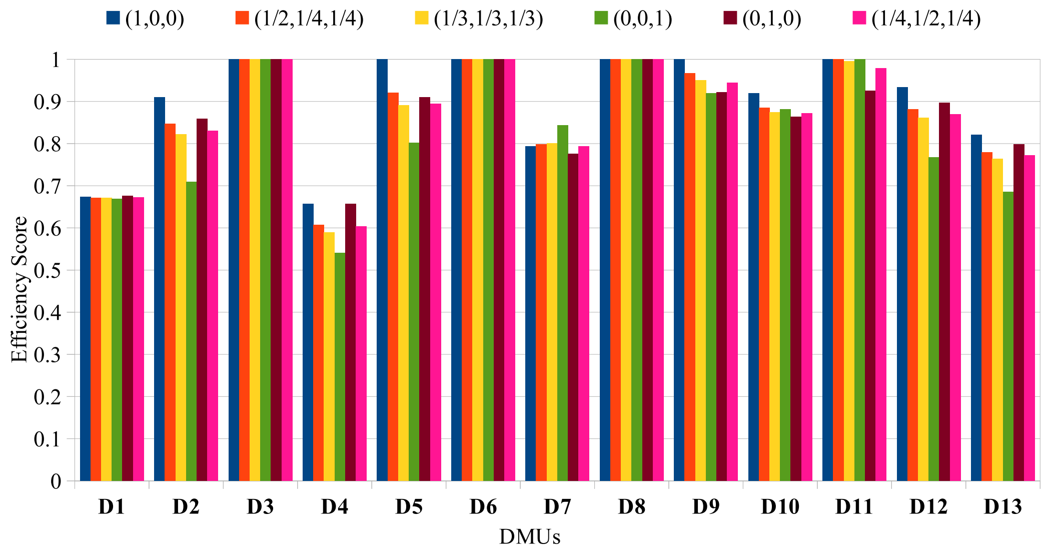

FIGURE 7. Efficiency Score of different preference level with risk factor $\lambda = 1$

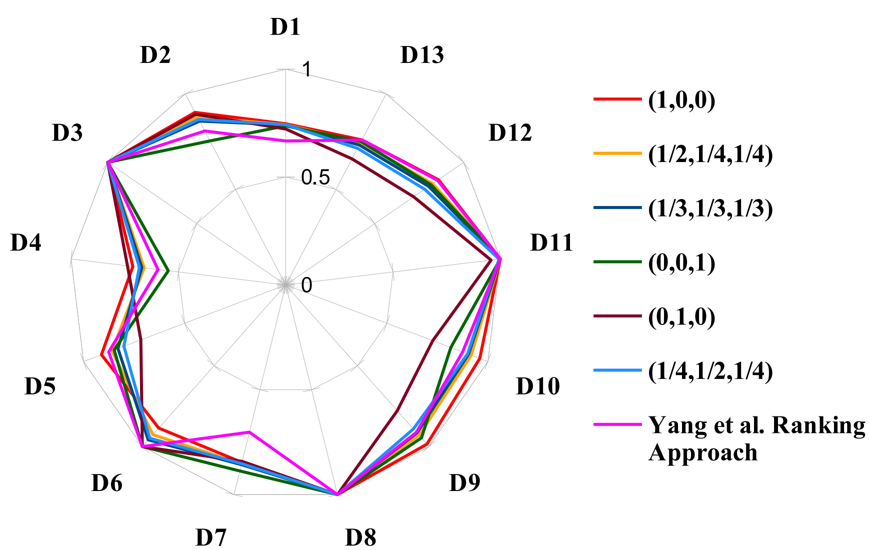

FIGURE 8. Compare the mean efficiency scores of DMUs in different preference level with Yang et al. [45] ranking approach

The DMUs are evaluated and ranked based on their mean efficiency scores while considering a fixed DM's preference level. The DMUs that achieve an efficiency score of 1 are identified as the most efficient and are assigned a rank of 1, indicating their top performance. Conversely, DMUs with efficiency scores below 1 are deemed inefficient, and their rankings are determined by their relative efficiency scores.

DMUs with higher relative efficiency scores receive higher rankings, whereas those with lower relative efficiency scores are ranked lower (See Table 5).

Figure 8 presents a comparison between the mean efficiency of DMUs obtained through the proposed solution technique and the efficiency scores derived from the method developed by Yang et al. [45]. The results show that DMUs D3 and D8 have the same efficiency score in both methods. However, for other DMUs, their efficiency scores differ based on the DM's preference level. The proposed solution technique is deemed more effective than the existing ranking approach since it allows the DM to obtain the efficiency of the DMUs according to their own preference level and risk factor. This gives the DM greater freedom to set their own preferences level and risk parameter while making a decision.

7. Advantages and Limitations of this Study

The key advantage of the suggested ranking function is that it enables DMs to evaluate their own degree of risk while making a decision. The ranking function is established by incorporating the value and ambiguity index, and it is related with the preference level of the DM, which indicates whether the DM prefers a pessimistic, optimistic, or neutral a decision. Also, the ranking function is associated with the risk factor which show the risk taking attitude of the DM. The suggested ranking function is utilised to solve the Neu-DEA model by converting it into an equivalent crisp DEA model that can be solved using existing LP approaches. The efficiency score of the DMUs is defined by a specific preference and risk level. This preference level and risk factor provide DMs with more freedom when analysing performance and making decisions. As a result, the suggested approach to solving the Neu-DEA model gives DMs greater freedom to consider their degree of risk and preferences when making decisions, allowing them to make better informed choices that are consistent with their objectives and values.

The present study is a case analysis that aims to measure the performance of hospitals. The investigation focuses on a small number of input and output variables, specifically two inputs and three outputs. However, the overall performance of hospitals is affected by a variety of variables that relate to either their inputs or their outputs. Therefore, the efficiency score may be changed by increasing or decreasing the number of inputs or outputs. The DMUs are ranked according to the mean efficiency scores of the DMUs with various risk factors for each DM's preference level, but this approach does not completely rank the DMUs, which is shown in the Table 5. Furthermore, the inputs of the DMUs may be interconnected with various internal structures rather than having a direct relationship with the final output. This suggested model only depends on the initial inputs and ultimate outputs, disregarding the internal structure of the DMUs. Therefore, while evaluating the performance of the DMUs, it is essential to take into account every relevant factor and internal DMU structures since these factors may have a big influence on the DMUs' rankings and efficiency evaluations. The suggested model is valuable, but it has limits in terms of its abilities to rank DMUs completely and accurately as well as the model does not consider the complex internal structures of DMUs.

8. Conclusions and Future Directions

The Neutrosophic Set is a generalized version of ordinary fuzzy sets, Intuitionistic fuzzy sets, Pythagorean fuzzy sets, and Spherical fuzzy sets [41]. It is becoming a famous scholarly topic because it is used to solve many problems in decision-making, data analysis, and artificial intelligence. The neutrosophic set has developed as a valuable technique for dealing with imprecise, indeterminate and inconsistent data. This set gives a more flexible way to analyse data, where data can be described using multiple combinations of truth, indeterminacy, and falsity membership functions. The neutrosophic set may reflect the complexities of real-world situations and give a more realistic representation of the underlying data by including these three variables. This research focuses on the construction of a ranking function for a SVTNN that integrates both the value and ambiguity indexes, as well as the DM's preference level and risk variables. Additionally, a novel solution technique is developed for the Neutrosophic DEA models with SVTNN inputs and outputs. The proposed ranking function converts the Neu-DEA model into an equivalent crisp LP model which is solved with different preference levels and risk parameters to measure the relative efficiency of the DMUs. The study examines how the DM's preference level and risk factor together affect the efficiency of the DMUs. Finally, to demonstrate the applicability and validity of the proposed model, a numerical example is presented and the efficiency scores are compared with Yang et al.'s ranking approach [45], as shown in Figure 8. It has been observed that DMU D3 and D8 are the most efficient DMUs, whereas D12 and D13 are the least efficient DMUs, and the number of efficient DMUs decreases as the risk factor increases. For a fixed DM's preference level, DMUs are ranked according to the mean efficiency score with different risk factors.

Future research directions can focus on addressing the limitations identified in the previous section. One potential area of investigation is the increase of the number of inputs and outputs to improve the accuracy of efficiency measurement. Additionally, there is a need to develop complete ranking techniques that can be provide a complete ranking of DMUs in the proposed work. Furthermore, the application of the network DEA model in a neutrosophic setting can be explored to calculate the efficiency of DMUs while considering the internal structure of the system. This novel technique gives encouraging outcomes and can be utilized to solve various additional DEA models, including "BCC, Super Efficiency, and Undesirable DEA, and Dynamic DEA models", by incorporating SVTNN inputs and outputs. Also as an application of this proposed approach employed for addressing MCDM, LP, agricultural economic, assignment problem, banking and finance, manufacturing and production, and transportation problems in neutrosophic environments.

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