Double neutrosophic integrals

Yaser Ahmad Alhasan[1],*, Suliman Sheen[2] and Raja Abdullah Abdulfatah[3]

[1]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, SaudiArabia.; y.alhasan@psau.edu.sa
[2]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, SaudiArabia.; S.almleh@psau.edu.sa
[3]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, SaudiArabia.; r.abdulfatah@psau.edu.sa

*Corresponding author: y.alhasan@psau.edu.sa

Abstract: Several previous papers dealt with neutrosophic integrals without introducing the idea of double neutrosophic integrals. In this article, double integrals were discussed by presenting several theories in double neutrosophic integrals over a rectangle and over a general region, the most important of which is the neutrosophic Fubini's theorem. In addition to studying the applications of double neutrosophic integrals in calculating areas.

Keywords: neutrosophic integrals, double neutrosophic integrals, neutrosophic Fubini's theorem, area.

1. Introduction

In contrast to the current logics, Smarandache suggested the Neutrosophic Logic to describe a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction. Smarandache introduced the concept of neutrosophy as a new school of philosophy [4]. He presented the definition of the standard form of neutrosophic real number [3-5], studying the concept of the Neutrosophic probability [6], the Neutrosophic statistics [5-7], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1]. A number of studies in the area of integration and differentiation were given by Y. Alhasan [9-12-15], also he presented the definition of the concept of neutrosophic complex numbers and its properties, in addition, he studied the general exponential form of a neutrosophic complex number [2-10]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [13]. The AH isometry was used to study many structures such as conic sections, real analysis concepts, and geometrical surfaces [11-16].

The calculation of area, volume, and arc length is one of the most essential uses of integration in human life. In our reality, there are things that cannot be precisely defined and contain an element of indeterminacy.

There are four sections of paper. first section, which includes a study of neutrosophic science, serves as an introduction. The second portion deals with a neutrosophic integrals theories and rules. The double neutrosophic integral and its applications are discussed in the third part. The fourth section offers the paper's conclusion.

2. Preliminaries

2.1. Neutrosophic integration by substitution method [15]

Definition 1

Let $f: D_f \subseteq R \rightarrow R_f \cup \{I\}$, to evaluate $\int f(x)dx$

put: $x = g(u) \quad \Rightarrow \quad dx = \acute{g}(u)du$

by substitution, we get:

$$\int f(x)dx = \int f(u)\acute{g}(u)du$$

Definition 3.2[16]

Let f: $R(I) \rightarrow R(I)$; $f = f(X)$ and $X = x + yI \in R(I)$ then f is called a neutrosophic real function with one neutrosophic variable. a neutrosophic real function $f(X)$ written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

3. Double neutrosophic integrals over a rectangle

Theorem 2 (neutrosophic Fubini's theorem)

Let $f(x,y,I)$ integrable over the rectangle

$R \cup I = \{(x,y,I){:}\ \dot{a} + \dot{a}_0 I \leq x \leq \dot{b} + \dot{b}_0 I \ \ and \ \ \dot{c} + \dot{c}_0 I \leq y \leq \dot{d} + \dot{d}_0 I\}$, where: $\dot{a}, \dot{a}_0, \dot{b}, \dot{b}_0, \dot{c}, \dot{c}_0, \dot{d}, \dot{d}_0$ are real numbers, while I = indeterminacy.

Then we can write the double neutrosophic integrals over a rectangle $R \cup I$ by the following formula:

$$\iint_{R\cup I} f(x,y,I) \ \ dA = \int_{\dot{a}+\dot{a}_0 I}^{\dot{b}+\dot{b}_0 I} \int_{\dot{c}+\dot{c}_0 I}^{\dot{d}+\dot{d}_0 I} f(x,y,I) \ \ dydx = \int_{\dot{c}+\dot{c}_0 I}^{\dot{b}+\dot{b}_0 I} \int_{\dot{a}+\dot{a}_0 I}^{\dot{b}+\dot{b}_0 I} f(x,y,I) \ \ dxdy$$

Integration according to the horizontal slice:

$$\int_{\dot{c}+\dot{c}_0 I}^{\dot{d}+\dot{d}_0 I} \int_{\dot{a}+\dot{a}_0 I}^{\dot{b}+\dot{b}_0 I} f(x,y,I) \ \ dxdy$$

Integration according to the vertical slice:

$$\int_{\dot{a}+\dot{a}_0 I}^{\dot{b}+\dot{b}_0 I} \int_{\dot{c}+\dot{c}_0 I}^{\dot{d}+\dot{d}_0 I} f(x,y,I) \ \ dydx$$

Example 1

Let $R \cup I = \{(x,y,I){:}\ 0 \leq x \leq 2 + 2I \ \ and \ \ 1 + I \leq y \leq 4 + 4I\}$, then let find:

$$\iint_{R\cup I} (x^2 + 2Ixy) \ \ dA$$

Solution:

Integration according to the vertical slice:

$$\int_{0}^{2+2I}\int_{1+I}^{4+4I}(x^2+2Ixy)dydx=\int_{0}^{2+2I}(x^2y+Ixy^2)|_{1+I}^{4+4I}dx$$

$$=\int_{0}^{2+2I}([(4+4I)x^2+Ix(4+4I)^2]-[(1+I)x^2+Ix(1+I)^2])dx$$

$$=\int_{0}^{2+2I}[(3+3I)x^2+60Ix]dx=\left((1+I)x^3+30Ix^2\right)\Big|_{0}^{2+2I}$$

$$=(1+I)(2+2I)^3+30I(2+2I)^2$$

$$=(1+I)(8+56I)+480I$$

$$=8+56I+8I+56I+480I=\boxed{8+600I}$$

Integration according to the horizontal slice:

$$\int_{1+I}^{4+4I}\int_{0}^{2+2I}(x^2+2Ixy)dxdy=\int_{1+I}^{4+4I}\left(\frac{x^3}{3}+Ix^2y\right)\Bigg|_{0}^{2+2I}dy$$

$$=\int_{1+I}^{4+4I}\left(\left[\left(\frac{(2+2I)^3}{3}+I(2+2I)^2y\right)\right]-[0]\right)dy$$

$$=\int_{1+I}^{4+4I}\left(\frac{(8+56I)}{3}+16Iy\right)dy$$

$$=\left(\frac{(8+56I)}{3}y+8Iy^2\right)\Bigg|_{1+I}^{4+4I}$$

$$=\left[\frac{(8+56I)}{3}(4+4I)+8I(4+4I)^2\right]-\left[\frac{(8+56I)}{3}(1+I)+8Iy(1+I)^2\right]$$

$$=(1+I)[(8+56I)+296I]$$

$$=(1+I)(8+296I)$$

$$=8+296I+296I+8I=\boxed{8+600I}$$

note that we got the same result.

3.1 Double neutrosophic integrals over a general region

Theorem 3

Let $f(x,y,I)$ is continuous on the region

$R \cup I = \{(x, y, I): \dot{a} + \dot{a}_0 I \leq x \leq \dot{b} + \dot{b}_0 I \ \ and \ \ g_1(x, I) \leq y \leq g_2(x, I)\}$, where: $\dot{a}, \dot{a}_0, \dot{b}, \dot{b}_0$ are real numbers, while I = indeterminacy, and $g_1(x, I)$, $g_2(x, I)$ continuous neutrosophic functions, where $g_1(x, I) \leq g_2(x, I)$, for all $x \in [\dot{a} + \dot{a}_0 I \ , \dot{b} + \dot{b}_0 I]$.

Then we can write the double neutrosophic integrals a general region $R \cup I$ by the following formula:

$$\iint_{R\cup I} f(x, y, I) \ dA = \int_{\dot{a}+\dot{a}_0 I}^{\dot{b}+\dot{b}_0 I} \int_{g_1(x,I)}^{g_2(x,I)} f(x, y, I) \ dydx$$

Example 2

Let $R \cup I = \{(x, y, I): 0 \leq x \leq 4 + 4I \ \ and \ \ 0 \leq y \leq (1 + I)x\}$, then let find:

$$\iint_{R\cup I} \left(2e^{x^2} - siny\right) \ dA$$

Solution:

$$\int_0^{4+4I} \int_0^{(1+I)x} \left(2e^{x^2} - sin\, y\right) dydx = \int_0^{4+4I} \left(2ye^{x^2} + cos\, y\right)\Big|_0^{(1+I)x} dx$$

$$= \int_0^{4+4I} \left(\left[2(1 + I)xe^{x^2} + cos(1 + I)x\right] - [0 + cos\, 0]\right)dx$$

$$= \int_0^{4+4I} \left(\left[2(1 + I)xe^{x^2} + cos(1 + I)x\right] - [1]\right)dx$$

$$= \left((1 + I)e^{x^2} + \frac{1}{1 + I} sin(1 + I)x - x\right)\Big|_0^{4+4I}$$

$$= \left[(1 + I)e^{(4+4I)^2} + \frac{1}{1 + I} sin(1 + I)(4 + 4I) - 4 - 4I\right] - \left[(1 + I)e^0 + \frac{1}{1 + I} sin(0) - 0\right]$$

$$= (1 + I)e^{(4+4I)^2} + \frac{1}{1 + I} sin(1 + I)(4 + 4I) - 4 - 4I - 1 - I$$

$$= (1 + I)e^{16+48I} + \left(1 - \frac{1}{2}I\right) sin(4 + 12I) - 5 - 5I$$

$$= (1 + I)(e^{16} + I[e^{64} - e^{16}]) + \left(1 - \frac{1}{2}I\right) (sin(4) + I\, [sin(16) - sin(4)]) - 5 - 5I$$

Example 2

Let $R \cup I = \{(x, y, I): 0 \leq x \leq 1 + I \ \ and \ \ 0 \leq y \leq x\}$, then let find:

$$\iint_{R\cup I} \frac{sin\left(\frac{\pi}{3} + \frac{\pi}{4}I\right)x}{x} \ dA$$

Solution:

$$\iint\limits_{R\cup I} \frac{\sin\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)x}{x} \quad dA = \int_{0}^{1+I}\int_{0}^{x} \frac{\sin\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)x}{x} dydx$$

$$= \int_{0}^{1+I} \frac{\sin\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)x}{x} \quad y\Big|_{0}^{x} dx$$

$$= \int_{0}^{1+I} \left(\left[\frac{\sin\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)x}{x} \quad x\right] - \left[\frac{\sin\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)x}{x} \quad (0)\right]\right) dx$$

$$= \int_{0}^{1+I} \sin\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)x\,dx$$

$$= \frac{-1}{\frac{\pi}{3}+\frac{\pi}{4}} \cos\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)x\Big|_{0}^{1+I}$$

$$= \left[\frac{-1}{\frac{\pi}{3}+\frac{\pi}{4}} \cos\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)(1+I)\right] - \left[\frac{-1}{\frac{\pi}{3}+\frac{\pi}{4}} \cos\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)(0)\right]$$

$$= \left[\frac{-1}{\frac{\pi}{3}+\frac{\pi}{4}} \cos\left(\frac{\pi}{3}+\frac{\pi}{4}I\right)(1+I)\right] - \left[\frac{-1}{\frac{\pi}{3}+\frac{\pi}{4}}\right]$$

$$= \left[\left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right)\cos\left(\frac{\pi}{3}+\frac{5\pi}{6}I\right)\right] - \left[\frac{-3}{\pi}+\frac{9}{7\pi}I\right]$$

$$= \left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right)\left[\cos\left(\frac{\pi}{3}\right) + I\left[\cos\left(\frac{7\pi}{6}\right) - \cos\left(\frac{\pi}{3}\right)\right]\right] - \left[\frac{-3}{\pi}+\frac{9}{7\pi}I\right]$$

$$= \left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right)\left[\frac{1}{2} + I\left[\frac{-\sqrt{3}}{2}-\frac{1}{2}\right]\right] - \left[\frac{-3}{\pi}+\frac{9}{7\pi}I\right]$$

$$= \frac{1}{2}\left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right) + I\left(\frac{-\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right) - \left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right)$$

$$= \frac{-1}{2}\left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right) + I\left(\frac{-\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{-3}{\pi}+\frac{9}{7\pi}I\right)$$

$$= \frac{3}{2\pi} - \frac{9}{14\pi}I + \frac{3\sqrt{3}}{2\pi}I - \frac{9\sqrt{3}}{14\pi}I + \frac{3}{2\pi}I - \frac{9}{14\pi}I$$

$$= \frac{3}{2\pi} + \left(\frac{3+12\sqrt{3}}{14\pi}\right)I$$

Example 3

$$\int_{0}^{1+\frac{1}{2}I}\int_{0}^{x} x\sin\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)y\,dydx$$

$$=\int_{0}^{1+\frac{1}{2}I} \frac{-1}{\frac{\pi}{3}+\frac{\pi}{6}I}x\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)y\bigg|_{0}^{x} dx$$

$$=\int_{0}^{1+\frac{1}{2}I}\left(\left[\frac{-1}{\frac{\pi}{3}+\frac{\pi}{6}I}x\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)y\right]-\left[\frac{-1}{\frac{\pi}{3}+\frac{\pi}{6}I}x\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)(0)\right]\right)dx$$

$$=\int_{0}^{1+\frac{1}{2}I}\left(\left[\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)x\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)x\right]-\left[\frac{-3}{\pi}-\frac{1}{\pi}I\right]x\right)dx$$

derivation	integration
$(+)\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)\ x$	$\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)x$
$(-)\quad\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)$	$\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)\sin\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)x$
0	$\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)^2\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)x$

$$=\left(\left[\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)^2 x\sin\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)x-\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)^3\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)x\right]-\left[\frac{-3}{\pi}-\frac{1}{\pi}I\right]\frac{x^2}{2}\right)\bigg|_{0}^{1+\frac{1}{2}I}$$

$$=\left(\left[\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)^2\left(1+\frac{1}{2}I\right)\sin\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)\left(1+\frac{1}{2}I\right)-\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)^3\cos\left(\frac{\pi}{3}+\frac{\pi}{6}I\right)\left(1+\frac{1}{2}I\right)\right]\right.$$
$$\left.-\left[\frac{-3}{\pi}-\frac{1}{\pi}I\right]\frac{\left(1+\frac{1}{2}I\right)^2}{2}\right)-\left(\frac{-3}{\pi}-\frac{1}{\pi}I\right)^3$$

$$=\left(\left[\left(\frac{9}{\pi^2}-\frac{3}{\pi^2}I\right)\sin\left(\frac{\pi}{3}+\frac{7\pi}{12}I\right)-\left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)\cos\left(\frac{\pi}{3}+\frac{7\pi}{12}I\right)\right]-\left[\frac{-3}{2\pi}-\frac{3}{4\pi}I\right]\right)-\left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)$$

$$=\left(\frac{9}{\pi^2}-\frac{3}{\pi^2}I\right)\left(\sin\left(\frac{\pi}{3}\right)+I\left[\sin\left(\frac{11\pi}{12}\right)-\sin\left(\frac{\pi}{3}\right)\right]\right)-\left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)\left(\cos\left(\frac{\pi}{3}\right)+I\left[\cos\left(\frac{11\pi}{12}\right)-\cos\left(\frac{\pi}{3}\right)\right]\right)$$
$$+\frac{3}{2\pi}+\frac{3}{4\pi}I-\left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)$$

$$= \left(\frac{9}{\pi^2}-\frac{3}{\pi^2}I\right)\left(\frac{\sqrt{3}}{2}+I\left[\frac{\sqrt{6}-\sqrt{2}}{4}-\frac{\sqrt{3}}{2}\right]\right)-\left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)\left(\frac{1}{2}+I\left[\frac{-\sqrt{6}-\sqrt{2}}{4}-\frac{1}{2}\right]\right)+\frac{3}{2\pi}+\frac{3}{4\pi}I - \left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)$$

$$= \left(\frac{9}{\pi^2}-\frac{3}{\pi^2}I\right)\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{6}-\sqrt{2}-2\sqrt{3}}{4}I\right)-\left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)\left(\frac{1}{2}+\frac{-\sqrt{6}-\sqrt{2}-2}{4}I\right)+\frac{3}{2\pi}+\frac{3}{4\pi}I - \left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)$$

$$= \left(\frac{9}{\pi^2}-\frac{3}{\pi^2}I\right)\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{6}-\sqrt{2}-2\sqrt{3}}{4}I\right)-\left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)\left(\frac{1}{2}+\frac{-\sqrt{6}-\sqrt{2}-2}{4}I\right)+\frac{3}{2\pi}+\frac{3}{4\pi}I - \left(\frac{27}{\pi^3}+\frac{29}{\pi^3}I\right)$$

Theorem 4

Let $f(x,y,I)$ is continuous on the region

$R \cup I = \{(x,y,I):\ \dot{c}+\dot{c}_0 I \le y \le \dot{d}+\dot{d}_0 I \ \ and \ \ h_1(y,I) \le y \le h_2(y,I)\}$, where: $\dot{c},\dot{c}_0,\dot{d},\dot{d}_0$ are real numbers, while I = indeterminacy, and $h_1(y,I)$, $h_2(y,I)$ continuous neutrosophic functions, where $h_1(y,I) \le h_2(y,I)$, for all $y \in [\dot{c}+\dot{c}_0 I\ ,\dot{d}+\dot{d}_0 I]$.

Then we can write the double neutrosophic integrals a general region $R \cup I$ by the following formula:

$$\iint_{R\cup I} f(x,y,I)\ dA = \int_{\dot{c}+\dot{c}_0 I}^{\dot{d}+\dot{d}_0 I}\int_{h_1(y,I)}^{h_2(y,I)} f(x,y,I)\ dxdy$$

Example 4

Let $R \cup I = \{(x,y,I):\ 1+I \le y \le 3+3I \ \ and \ \ (1+I)y \le x \le y^2\}$, then let find:

$$\iint_{R\cup I} 5\ dA$$

Solution:

$$\int_{1+I}^{3+3I}\int_{(1+I)y}^{y^2} 5\ dxdy = \int_{1+I}^{3+3I} 5x\Big|_{(1+I)y}^{y^2}\ dy$$

$$= 5\int_{1+I}^{3+3I} (y^2-(1+I)y)\ dy$$

$$= 5\left(\frac{y^3}{3}-(1+I)\frac{y^2}{2}\right)\Bigg|_{1+I}^{3+3I}$$

$$= 5\left(\left[\frac{(3+3I)^3}{3}-(1+I)\frac{(3+3I)^2}{2}\right]-\left[\frac{(1+I)^3}{3}-(1+I)\frac{(1+I)^2}{2}\right]\right)$$

$$= 5\left(\left[\frac{9(1+I)^3}{2}\right]-\left[\frac{(1+I)^3}{6}\right]\right)=\frac{65(1+I)^3}{3}$$

$$= \frac{65(1+7I)}{3} = \frac{65}{3} + \frac{455}{3}$$

Theorem 5

Let $f(x,y,I)$ and $g(x,y,I)$ be integrable over region $R \cup I \subset \mathbb{R}^2$, and let $\dot{c} + \dot{c}_0 I$ any constant, where $\dot{c}, \dot{c}_0$ are real numbers, while I = indeterminacy. Then:

(i) $$\iint_{R\cup I} (\dot{c} + \dot{c}_0 I) f(x,y,I) \ dA = (\dot{c} + \dot{c}_0 I) \iint_{R\cup I} f(x,y,I) \ dA$$

(ii) $$\iint_{R\cup I} [f(x,y,I) + g(x,y,I)] \ dA = \iint_{R\cup I} f(x,y,I) \ dA + \iint_{R\cup I} f(x,y,I) \ dA$$

(iii) If $R \cup I = R_1 \cup R_2 \cup I$, where R_1, R_2 are nonoverlapping region, then:

$$\iint_{R\cup I} f(x,y,I) \ dA = \iint_{R\cup I} f(x,y,I) \ dA + \iint_{R\cup I} f(x,y,I) \ dA$$

3.2 Applications of double neutrosophic integrals

The area of region t can be calculated using a double neutrosophic integrals:

$$A = \iint_{R\cup I} dxdy = \iint_{R\cup I} dydx$$

Example 5

Using a double neutrosophic integrals to find the area of the plane region bounded by the curve of $y = x^2$ and $y = (1+I)x$.

Solution:

Let find the intersection points of the two equations:

$$x^2 = (1+I)x$$

$$x^2 - (1+I)x = 0$$

$$x(x-1-I) = 0 \qquad \Rightarrow \begin{cases} x = 0 \\ x = 1+I \end{cases}$$

$$\Rightarrow \begin{cases} y = 0 \\ y = 1+3I \end{cases}$$

so, the two equations intersect at the points: $(0 \ ,0)$ and $(1+I \ ,1+3I)$

- Integration according to the horizontal slice:

$$\int_0^{1+I} \int_{x^2}^{(1+I)x} dydx = \int_0^{1+I} y|_{x^2}^{(1+I)x} dx$$

$$= \int_0^{1+I} \left((1+I)x - x^2\right) dx$$

$$= \left((1+I)\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^{1+I}$$

$$= (1+I)\frac{(1+I)^2}{2} - \frac{(1+I)^3}{3}$$

$$= \frac{(1+I)^3}{6} = \boxed{\frac{1}{6} + \frac{7}{6}I}$$

➢ Integration according to the vertical slice:

$$\int_0^{1+3I} \int_{(1-\frac{1}{2}I)y}^{\sqrt{y}} dxdy = \int_0^{1+3I} x|_{(1-\frac{1}{2}I)y}^{\sqrt{y}} dx$$

$$= \int_0^{1+3I} \left(\sqrt{y} - (1 - \frac{1}{2}I)y\right) dx$$

$$= \left(\frac{2}{3}\sqrt{y^3} - (1 - \frac{1}{2}I)\frac{y^2}{2}\right)\Big|_0^{1+3I}$$

$$= \left[\frac{2}{3}\sqrt{(1+3I)^3} - (1 - \frac{1}{2}I)\frac{(1+3I)^2}{2}\right] - \left[\frac{2}{3}\sqrt{0^3} - (1 - \frac{1}{2}I)\frac{0^2}{2}\right]$$

$$= \frac{2}{3}\sqrt{1+63I} - (1 - \frac{1}{2}I)\frac{(1+15I)}{2}$$

$$= \frac{2}{3}(1+7I) - \left(\frac{1}{2} + \frac{15}{2}I - \frac{1}{4}I - \frac{15}{4}I\right)$$

$$= \frac{2}{3}(1+7I) - \left(\frac{1}{2} + \frac{7}{2}I\right)$$

$$= \frac{2}{3} + \frac{14I}{3} - \frac{1}{2} - \frac{7}{2}I = \boxed{\frac{1}{6} + \frac{7}{6}I}$$

note that we got the same result.

Example 6

Using a double neutrosophic integrals to find the area of the plane region bounded by the curve of $y = 9 + 7I - x^2$ and $x - axis$.

Solution:

$$9 + 7I - x^2 = 0$$

$$x^2 = 9 + 7I$$

by root the both sides, we get on:

$$\Longrightarrow \begin{cases} x = 3 + I \\ x = -3 - I \end{cases} \quad (1) \qquad or \qquad \begin{cases} x = 3 - 7I \\ x = -3 + 7I \end{cases} \quad (2)$$

where:

$$\sqrt{9 + 7I} = \gamma + \delta I$$

$$9 + 7I = \gamma^2 + 2\gamma\delta I + \delta^2 I$$

$$9 + 7I = \gamma^2 + (2\gamma\delta + \delta^2)I$$

then:

$$\begin{cases} \gamma^2 = 9 \\ 2\gamma\delta + \delta^2 = 7 \end{cases}$$

$$\begin{cases} \gamma = \pm 3 \\ \delta^2 + 2\gamma\delta - 7 = 0 \end{cases}$$

find the values of β:

- When $\gamma = 3 \quad \Longrightarrow \quad \delta^2 + 6\delta - 7 = 0$

$$(\delta + 7)(\delta - 1) = 0 \qquad \Longrightarrow \delta = -7 \ , \delta = 1$$

- When $\gamma = -3 \quad \Longrightarrow \quad \delta^2 - 6\delta - 7 = 0$

$$(\delta - 7)(\delta + 1) = 0 \qquad \Longrightarrow \delta = 7 \ , \delta = -1$$

$$\begin{aligned} \sqrt{9 + 7I} &= 3 + I \\ or \quad &= -3 - I \\ or \quad &= 3 - 7I \\ or \quad &= -3 + 7I \end{aligned}$$

case (1):

$$A = \int_{-3-I}^{3+I} \int_{0}^{9+7I-x^2} dydx = \int_{-3-I}^{3+I} y|_0^{9+7I-x^2} dx$$

$$= \int_{-3-I}^{3+I} (9 + 7I - x^2)dx$$

$$= \left((9 + 7I)x - \frac{x^3}{3} \right) \Bigg|_{-3-I}^{3+I}$$

$$= \left[(9 + 7I)(3 + I) - \frac{(3 + I)^3}{3} \right] - \left[(9 + 7I)(-3 - I) - \frac{(-3 - I)^3}{3} \right]$$

$$= 2\left[(9 + 7I)(3 + I) - \frac{(3 + I)^3}{3} \right]$$

$$= 2\left[27 + 37I - \left(\frac{27 + 27I + 9I + I}{3}\right)\right]$$

$$= 2\left[27 + 37I - 9 - \frac{37I}{3}\right] = 36 + \frac{148}{3}I$$

case (2):

$$A = \int_{-3+7I}^{3-7I} \int_{0}^{9+7I-x^2} dydx = \int_{-3+7I}^{3-7I} y|_0^{9+7I-x^2} dx$$

$$= \int_{-3+7I}^{3-7I} (9 + 7I - x^2)dx$$

$$= \left((9 + 7I)x - \frac{x^3}{3}\right)\Bigg|_{-3+7I}^{3-7I}$$

$$= \left[(9 + 7I)(3 - 7I) - \frac{(3 - 7I)^3}{3}\right] - \left[(9 + 7I)(-3 + 7I) - \frac{(-3 + 7I)^3}{3}\right]$$

$$= \left[(9 + 7I)(3 - 7I) - \frac{(3 - 7I)^3}{3}\right] + \left[(9 + 7I)(3 - 7I) - \frac{(3 - 7I)^3}{3}\right]$$

$$= 2\left[(9 + 7I)(3 - 7I) - \frac{(3 - 7I)^3}{3}\right]$$

$$= 2\left[27 - 91I - \left(\frac{27 - 189I + 441I - 343I}{3}\right)\right]$$

$$= 2\left[27 + 91I - 9 + \frac{91I}{3}\right] = 36 + \frac{364}{3}I$$

4. Conclusions

The significance of this paper stems from the fact that it explained the concept of double neutrosophic integrals. where double neutrosophic integrals over a rectangle and over a general region were presented. In addition, integrations were calculated according to the horizontal and vertical slice, and we got the same results in both cases. Also, we introduced the applications of double neutrosophic integrals.

Acknowledgments " This study is supported via funding from Prince sattam bin Abdulaziz University project number (PSAU/2023/R/1445)".

References

[1] Smarandache, F., "Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability", Sitech-Education Publisher, Craiova – Columbus, 2013.

[2] Alhasan,Y. "Concepts of Neutrosophic Complex Numbers", International Journal of Neutrosophic Science, Volume 8 , Issue 1, pp. 9-18, 2020.

[3] Vasantha Kandasamy, W.B., Smarandache, F., "Finite Neutrosophic Complex Numbers", Zip Publisher, Columbus, Ohio, USA, pp.1-16, 2011.

[4] Smarandache, F., "Neutrosophy. / Neutrosophic Probability, Set, and Logic, American Research Press", Rehoboth, USA, 1998.

[5] Smarandache, F., "Introduction to Neutrosophic statistics", Sitech-Education Publisher, pp.34-44, 2014.

[6] Smarandache, F., "A Unifying Field in Logics: Neutrosophic Logic", Preface by Charles Le, American Research Press, Rehoboth, 1999, 2000. Second edition of the Proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Statistics, University of New Mexico, Gallup, 2001.

[7] Smarandache, F., "Proceedings of the First International Conference on Neutrosophy", Neutrosophic Set, Neutrosophic Probability and Statistics, University of New Mexico, 2001.

[8] Smarandache, F., "Neutrosophic Precalculus and Neutrosophic Calculus", book, 2015.

[9] Alhasan,Y.; "The definite neutrosophic integrals and its applications", Neutrosophic Sets and Systems, Volume 49, pp. 278-293, 2022.

[10] Alhasan,Y., "The General Exponential form of a Neutrosophic Complex Number", International Journal of Neutrosophic Science, Volume 11, Issue 2, pp. 100-107, 2020.

[11] Merkepci, H., Abobala, M., " The Application of AH-Isometry in the Study of Neutrosophic Conic Sections", Galoitica Journal Of Mathematical Structures And Applications, Volume 2, Issue 2, pp. 18-22, 2022.

[12] Alhasan,Y.; "The neutrosophic integrals by parts", Neutrosophic Sets and Systems, Volume 45, pp. 309-319, 2021.

[13] Madeleine Al- Tahan, "Some Results on Single Valued Neutrosophic (Weak) Polygroups", International Journal of Neutrosophic Science, Volume 2, Issue 1, PP: 38-46, 2020.

[14] Alfahal, A., Alhasan, Y., Abdulfatah, R., Ali, R., " On the Split-Complex Neutrosophic Numbers and Their Algebraic Properties", International Journal of Neutrosophic Science, Volume 11, Issue 2, pp. 100-107, 2020.

[15] Alhasan,Y.; "The neutrosophic integrals and integration methods", Neutrosophic Sets and Systems, Volume 43, pp. 291-301, 2021.

[16] Abobala, M., A, Hatip, " An Algebraic Approach to Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Volume 43, pp. 114-123, 2021.

Received: June 6, 2023. Accepted: Sep 27, 2023