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Generating Random Variables that follow the Beta Distribution Using the Neutrosophic Acceptance-Rejection Method

*[1]Maissam Jdid, and [2]Nada A. Nabeeh
[1]Faculty of Science, Damascus University, Damascus, Syria.
maissam.jdid66@damascusuniversity.edu.sy
[2]Information Systems Department, Faculty of Computers and Information Sciences, Mansoura University, Mansoura 35516, Egypt
nadaadel@mans.edu.eg
* Correspondence: jdidmaisam@gmail.com

Abstract:

Analysis using simulation is a natural and logical extension of the analytical and mathematical models inherent in operations research. Simulation has become a modern tool that helps in studying many systems that we could not study or predict the results that we could obtain during the operation of these systems over time before the existence of Simulation, since the main interest in statistical analysis is to obtain a series of random variables that follow the probability distribution in which the system under study operates, through a series of random numbers that follow a uniform distribution over the domain [0, 1], using scientific methods provided by the efforts of researchers. In the field of modeling and simulation, such as the reverse transformation method, the rejection and acceptance method, and other methods that we have reformulated using the concepts of neutrosophic science in previous research. In summary of what we have done previously, we present in this research a study whose purpose is to generate random variables that follow the beta distribution, which is used in many applications. In administrative processes, especially in analyzing network diagrams using the neutrosophic rejection and acceptance method.

Keywords: modeling and simulation; neutrosophic science; rejection and acceptance method for generating neutrosophic random variables; neutrosophic uniform distribution; neutrosophic random numbers; Neutrosophic beta distribution.

Introduction:

Operations research has provided many scientific methods that have contributed to the great scientific development witnessed in our contemporary world. The importance of these methods increases when they are reformulated using the concepts of neutrosophic science, the science that relies on neutrosophic data that leaves nothing to chance and takes into account all the circumstances and fluctuations facing decision makers. Therefore, researchers have presented many Among the researches through which some operations research methods were reformulated using the concepts and information presented by the founder of this science, we mention [1-18]. Neutrosophic statistical studies are an extension of traditional statistical studies, as they depend on neutrosophic data, which are groups where any a value such as a in a group denoted by a_N which means (a neutrosophic), (a imprecise), or (a non-specific). a_N may be a neighbor of a or an interval containing a, and in general it can be considered any set close to a. See [19].

In any probability distribution, if there is a quantity that contains some indeterminacy, then this distribution is a neutrosophic probability distribution. Therefore, the random numbers and random variables that we obtain based on this probability distribution are neutrosophic random numbers and random variables. See [20].

Discussion:

Probability distributions are the mainstay of the simulation process. Therefore, the interest of researchers and scholars interested in the simulation method has focused on providing scientific studies that help in obtaining random variables that follow the probability distributions most used in practical applications. In classical studies, we find many algorithms that are concerned with transforming random numbers that follow a uniform distribution into The domain [1, 0] refers to the probability distribution with which the system to be simulated operates, and based on this importance of probability distributions and after the emergence of neuroscientific science, which paid great attention to probability distributions, many probability distributions were reformulated using the concepts of this science, and we presented in previous research how Generating neutrosophic random numbers and methods for converting these numbers into neutrosophic random variables that follow the exponential distribution and others that follow the uniform distribution using the inverse transformation method, which was reformulated using the concepts of neutrosophic science [21-23]. In this research, and given the importance of the beta distribution, we present a study whose goal is to use the method Neutrosophic rejection and acceptance, which was presented in the paper [24], to generate neutrosophic random variables that follow the beta distribution, which is defined using classical values as the following probability density function:

$$\boldsymbol{f}(\boldsymbol{x})=\frac{\boldsymbol{\Gamma}(\boldsymbol{\alpha}+\boldsymbol{\beta})}{\boldsymbol{\Gamma}(\boldsymbol{\alpha})\boldsymbol{\Gamma}(\boldsymbol{\beta})}\boldsymbol{x}^{\boldsymbol{\alpha}-\mathbf{1}}(\mathbf{1}-\boldsymbol{x})^{\boldsymbol{\beta}-\mathbf{1}} \quad ; \quad \mathbf{0}\leq\boldsymbol{x}\leq\mathbf{1} \qquad (\mathbf{1})$$

Where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the medians used to define this distribution and $\boldsymbol{\alpha}>\mathbf{0}, \boldsymbol{\beta}>\mathbf{0}$

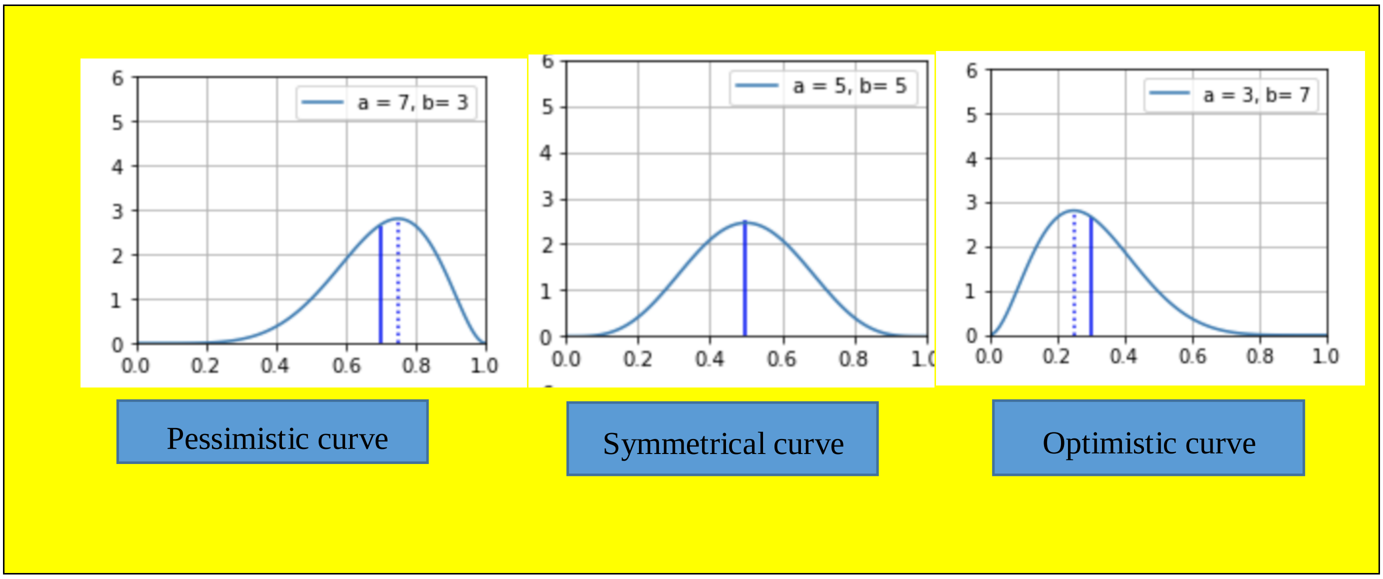
The symbol $\boldsymbol{\Gamma}(\boldsymbol{c})$ is the value of the integral (gamma), defined by the following relationship:

$$\boldsymbol{\Gamma}(\boldsymbol{c})=\int_{\mathbf{0}}^{\infty}\boldsymbol{x}^{\boldsymbol{c}-\mathbf{1}}\,\boldsymbol{e}^{-\boldsymbol{x}}\boldsymbol{dx}$$

The beta distribution curve takes many shapes depending on the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ and we can be classified into three types:

1- Pessimistic curve in this case is $\boldsymbol{\alpha}>\boldsymbol{\beta}$
2- Symmetrical curve $\boldsymbol{\alpha}=\boldsymbol{\beta}$
3- Optimistic curve $\boldsymbol{\alpha}<\boldsymbol{\beta}$

As in the following figures:



Neutrosophic function: [25]
The neutrosophic function $f: A \rightarrow B$ is a function that has some indeterminacy, taking into account the definition of its domain, its corresponding domain, and the relationship between the elements of the domain and the elements of the corresponding domain.
If α , β , or both of them carry some indeterminacy, that is, they take one or the other
Where α_N and β_N are neutrosophic values written as follows:

$$\beta_N = \beta + \delta \text{ و } \alpha_N = \alpha + \varepsilon$$

Where ε is an indeterminacy and takes one form $\varepsilon \in \{\lambda_1, \lambda_2\}$ or $\varepsilon \in [\lambda_1, \lambda_2]$ or other than that.
Where δ is an indeterminacy and takes one form $\delta \in \{\mu_1, \mu_2\}$ or $\delta \in [\mu_1, \mu_2]$ or other than that.
Then we obtain the neutrosophic beta distribution, which has a probability density function given by the following formula:

$$f(x) = \frac{\Gamma(\alpha_N + \beta_N)}{\Gamma(\alpha_N)\Gamma(\beta_N)} x^{\alpha_N - 1}(1-x)^{\beta_N - 1} \quad ; \quad 0 \le x \le 1 \qquad (2)$$

Based on what has been reported about this beta distribution in classical studies, we can present these neutrosophic types of beta distribution: [24]
The graphic curve of the beta neutrosophic distribution takes many shapes depending on the values of α_N and β_N and can be classified into three types:

1- Pessimistic curve in this case is $\alpha_N > \beta_N$
2- Symmetrical curve $\alpha_N = \beta_N$
3- Optimistic curve $\alpha_N < \beta_N$

Generating random variables that follow a beta distribution:

We know that the process of neutrosophic simulation depends on generating neutrosophic random numbers that follow the regular neutrosophic distribution and then converting these random numbers into random variables that follow the probability distribution with which the system to be simulated operates. There are several methods that can be used for the conversion process, including we mentioned in previous research the inverse transformation method and the rejection method. And acceptance [21-23], where the appropriate method is used for the probability density function because in the inverse transformation method we need the inverse function of the cumulative distribution function. As we know, in many functions, the inverse function does not exist or obtaining it requires complex operations. Here we resort to the method Rejection and acceptance. As is clear from relationship (1), it is not easy to obtain the inverse function of the probability density function for the beta distribution. Therefore, we will use the inverse transformation method and apply it to the beta distribution according to the following algorithms:

Rejection and acceptance algorithm: [23,24]

We calculate $\boldsymbol{M_N}$, the maximum value taken by the probability density function over its defined domain, and we obtain it by calculating the derivative of this function and setting it equal to zero, i.e.:

$$M_N = \left.\frac{df_N(x)}{dx}\right|_{x=0}$$

And in simplified form, we define the derivative of the neutrosophic function as follows [24]:

$$f_N(x) = \lim_{h\to 0} \frac{[inf\ f(x+h) - inf\ f(x) sup f(x+h) - sup\ f(x)]}{h}$$

This is a generalization of the traditional derivative definition, where the function and variables are conventional. It can be written as:

$$[inf\ H\ , sup\ H] = h$$
$$inf\ (x+H) = sup(x+H) = f(x+h)$$
$$inf\ f(x) = sup\ f(x) = f(x)$$

Here, $\boldsymbol{H}$ is a closed, open, half-closed, or half-open interval.

By applying the above definition to the function defined by the relationship (2), we obtain:

$$f'(x) = c.(\alpha_N - 1)x^{\alpha_N - 2}(1-x)^{\beta_N - 1} - c(\beta_N - 1)x^{\alpha_N - 1}(1-x)^{\beta_N - 2}$$

To find the solution, we set the derivative equal to zero:

$$f'(x) = c.(\alpha_N - 1)x^{\alpha_N - 2}(1-x)^{\beta_N - 1} - c(\beta_N - 1)x^{\alpha_N - 1}(1-x)^{\beta_N - 2} = 0$$

In short, by finding the common limits, we get:

$$Nx = \frac{\alpha_N - 1}{\alpha_N + \beta_N - 2} = mod \qquad (3)$$

This value of x corresponds to the maximum value of the function, and therefore:

$$M_N = \frac{\alpha_N - 1}{\alpha_N + \beta_N - 2}$$

After obtaining the values of $\boldsymbol{\alpha_N}$ and $\boldsymbol{\beta_N}$ and substituting them into equations (2) and (3), we apply the following algorithm:

1. Generate two random numbers, $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ following a uniform distribution in the range [0, 1]. We use the method of squaring the average to generate the random numbers $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ as follows [24,25]:
$$R_{i+1} = Mid[R_i^2] \quad ; i = 0,1,2,---- \qquad (4)$$

where, Mid refers to the middle four digits of $\boldsymbol{R_1^2}$ and $\boldsymbol{R_0}$ is an initial random number consisting of four digits (called the seed) that doesn't contain zero in any of its four digits.

2. Convert the numbers $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ into neutrosophic function random numbers. To convert random numbers following a uniform distribution into neutrosophic random numbers, we follow the method introduced in [22], where three forms of neutrosophic random numbers are distinguished based on the uncertainty associated with the range [0, 1]:

a. **Form 1**: Uncertainty in the lower limit, i.e., $[\mathbf{0}+\boldsymbol{\varepsilon}, \mathbf{1}]$. where, each classical random number is mapped to a neutrosophic function random number using the following relationship:
$$NR_0 = \frac{R_0 - \varepsilon}{1-\varepsilon} = \frac{[R_0\,,\ R_0 - n]}{[1\,, 1-n]} \in [\,R_0, \frac{R_0 - n}{1-n}]$$

b. **Form 2**: Uncertainty in the upper limit, i.e., $[\mathbf{0}\,, \mathbf{1}+\boldsymbol{\varepsilon}]$. where, each classical random number is mapped to a neutrosophic function random number using the following relationship:
$$NR_0 = \frac{R_0}{1+\varepsilon} = \frac{R_0}{[1\,,\ n+1]} \in [R_0\,, \frac{R_0}{n+1}]$$

c. **Form 3**: Uncertainty in both upper and lower limits, i.e. $[\mathbf{0}\,, \mathbf{1}+\boldsymbol{\varepsilon}]$. where, each classical random number is mapped to a neutrosophic function random number using the following relationship:
$$NR_0 = R_0 - \varepsilon \in [R_0\,, R_0 - n]$$

In the three previous forms, we have $\boldsymbol{\varepsilon} \in [\mathbf{0}\,, \boldsymbol{n}]$ $\quad \boldsymbol{and} \quad \mathbf{0} < \boldsymbol{n} < \mathbf{1}$.

In this study, we will use the third form: non-deterministic in the upper and lower bounds, i.e., $[\mathbf{0}+\boldsymbol{\varepsilon}\,, \mathbf{1}+\boldsymbol{\varepsilon}]$. Here, we associate each classical random number with a non-deterministic random number using the following relationship:
$$NR_0 = R_0 - \varepsilon \in [R_0\,, R_0 - n]$$

Where $\boldsymbol{\varepsilon} \in [\mathbf{0}\,, \boldsymbol{n}]$ $\quad \boldsymbol{and} \quad \mathbf{0} < \boldsymbol{n} < \mathbf{1}$

3. We take one of the two numbers, let it be $\boldsymbol{NR_1}$, and transform it appropriately for the uniform distribution. We know that when the boundaries of the domain are neutrosophic values, we apply the following [22] for the upper and lower limits of the domain:

$[\boldsymbol{b}+\boldsymbol{\varepsilon}\,, \boldsymbol{a}+\boldsymbol{\varepsilon}]$, where $\boldsymbol{a} = \boldsymbol{a_N} = \boldsymbol{a} + \varepsilon$ and $\boldsymbol{b_N} = \mathbf{b} + \varepsilon$, and $\boldsymbol{\varepsilon} \in [\mathbf{0}\,, \boldsymbol{n}]$ $\boldsymbol{and}$ $\boldsymbol{a} < \boldsymbol{n} < \boldsymbol{b}$.

We use the following relationship:
$$Nx_1 = (b-a)NR_1 + a$$

4. We test whether $\boldsymbol{NR_2}$ satisfies the inequality:
$$NR_2 \le \frac{f(NR_1)}{M} \qquad (5)$$

That is,

$$NR_2 \leq \frac{f_N(NR_1)}{M_N} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}(NR_1)^{\alpha_N-1}\left[1-(NR_1)^{\beta_N-1}\right]$$

5. If the inequality (5) is satisfied, then we accept that $\boldsymbol{Nx_1} = \boldsymbol{NR_1}$follows the Beta distribution defined by equation (1).

6. If $\boldsymbol{NR_2}$ does not satisfy the inequality (5), then we reject the two numbers $\boldsymbol{NR_1}$, $\boldsymbol{NR_2}$and return to the first step to generate new random numbers.

We explain the above through the following example:

Example 1:

We have a system that operates according to the beta function defined by the following probability density function:

$$\boldsymbol{f(x) = \frac{\Gamma(10)}{\Gamma(3)\Gamma(7)} x^2(1-x)^6 \quad ; \quad 0 \leq x \leq 1}$$

That is, $\boldsymbol{\alpha = 3}$ and $\boldsymbol{\beta = 7}$

What is required is to generate random variables that follow this distribution. In this example, we will take the beta function in the classical form and neutrosophic random numbers:

We apply the rejection and acceptance algorithm according to the previously mentioned steps:

1. We calculate the largest value of the density function over its defined field from the following relationship:

$$\boldsymbol{M = \frac{\alpha - 1}{\alpha + \beta - 2}}$$

We find:

$$\boldsymbol{M = \frac{3-1}{3+7-2} = \frac{2}{8} = \frac{1}{4}}$$

2. We use the method of squaring to generate two random numbers that follow the uniform distribution over the domain $[\boldsymbol{0}\,,\boldsymbol{1}]$**.** We take the seed $\boldsymbol{R_0 = 0.1273}$ and obtain the following random numbers:

$$\boldsymbol{R_1 = 0.6205 \quad , R_2 = 0.5020}$$

3. We convert the classical random numbers to random numbers that follow the non-deterministic uniform distribution over the domain $[0+\varepsilon\,, 1+\varepsilon]$. We take the non-deterministic bounds $\boldsymbol{\varepsilon} \in [\boldsymbol{0}\ ,\boldsymbol{0.02}]$and obtain the following neutrosophic non-deterministic random numbers:

$$NR_1 = R_1 - \varepsilon \in [R_1\,, R_1 - 0.02]$$
$$NR_1 = R_1 - \varepsilon = 0.6205 - [0\ \ ,0.02] = [0.6205\,,0.6005] \Rightarrow NR_1 \in [0.6205\,,0.6005]$$
$$NR_2 = R_2 - \varepsilon \in [R_2\,, R_2 - 0.02]$$
$$NR_2 = R_2 - \varepsilon = 0.5020 - [0\ \ ,0.02] = [0.5020\,,0.482] \Rightarrow NR_2 \in [0.5020\,,0.482]$$

4. We take one of the numbers and form an appropriate transformation for the uniform

distribution over the domain $[\mathbf{0}\,,\mathbf{1}]$ **.** Here, since the distribution is defined over the domain $[\mathbf{0}\,,\mathbf{1}]$**,** we take one of the non-deterministic random numbers, let's say $\boldsymbol{NR_1} \in [\mathbf{0.6205}\,,\mathbf{0.6005}]$, and calculate the value of the probability density function at that point:

$$\boldsymbol{f(NR_1)} = \frac{\boldsymbol{\Gamma(10)}}{\boldsymbol{\Gamma(3)\Gamma(7)}}([\mathbf{0.6205}\,,\mathbf{0.6005}])^{\mathbf{2}}(\mathbf{1}-([\mathbf{0.6205}\,,\mathbf{0.6005}]))^{\mathbf{6}}$$

We know that if $\boldsymbol{n}$ is a positive integer, then $(\boldsymbol{n}+\mathbf{1}) = \boldsymbol{n}!$ **,** therefore:

$$\frac{\mathbf{\Gamma(10)}}{\mathbf{\Gamma(3)\Gamma(7)}} = \frac{\mathbf{9!}}{\mathbf{2!\,6!}} = \mathbf{252}$$

$$\boldsymbol{f(NR_1)} \in (\mathbf{252}\times[\mathbf{0.385},\mathbf{0.3606}]\times[\mathbf{0.003},\mathbf{0.0041}]) = [\mathbf{0.2911},\mathbf{0.3726}]$$

1. We test the inequality (5) and for this, we calculate:

$$\frac{f(NR_1)}{M} \in \frac{[0.2911,0.3726]}{0.25} = [1.1644,1.4904]$$

We have $NR_2 \in [0.5020\,,0.482]$. We note that:

$$[0.5020\,,0.482] \le [1.1644,1.4904]$$

Therefore, inequality (5) is satisfied, i.e.:

$$NR_2 \le \frac{f(NR_1)}{M}$$

Here we accept that $N(R_1) \in [1.1644,1.4904]$follows the beta distribution given in the example.

Generating random numbers following the neutrosophic beta distribution from classical random numbers:

Example 2: We have a system that operates according to the beta function defined by the neutrosophic probability density function as in the following relationship:

$$f(x) = \frac{\Gamma(\alpha_N+\beta_N)}{\Gamma(\alpha_N)\Gamma(\beta_N)}x^{\alpha_N-1}(1-x)^{\beta_N-1} \quad ; \quad 0 \le x \le 1$$

$\alpha_N$and β_N are neutrosophic values of the form $\alpha_N = \alpha+\varepsilon$, $\beta_N = \beta+\delta$, which are adjacent to the real values $\alpha = 3$ and $\beta = 7$.

Where ε and δ are the indeterminacy in these values. We take them in this example as follows: $\varepsilon \in [\lambda_1,\lambda_2\,] = [0\,,0.2\,]$and $\delta \in [\mu_1,\mu_2] = [0\,,0.1\,]$ we get $\alpha_N \in [3\,,3.2\,]$ and $\beta_N \in [7\,,7.1\,]$ then the probability density function is written as follows:

$$f_N(x) = \frac{\Gamma([10\,,10\,.3\,])}{\Gamma([3\,,3.2\,])\Gamma([7\,,7.1\,])}x^{[2\,,2.2\,]}(1-x)^{[6\,,6.2\,]} \quad ; \quad 0 \le x \le 1$$

What is required is to generate random numbers that follow the previous distribution, based on classical random numbers. We apply the rejection and acceptance algorithm according to the steps mentioned previously, and as we did in Example 1, we obtain what is required.

Example 3: We have a system that operates according to the beta function defined by the neutrosophic probability density function as in the following relationship:

$$f(x) = \frac{\Gamma(\alpha_N+\beta_N)}{\Gamma(\alpha_N)\Gamma(\beta_N)}x^{\alpha_N-1}(1-x)^{\beta_N-1} \quad ; \quad 0 \le x \le 1$$

$\alpha_N$and $\beta_N$are neutrosophic values of the form $\alpha_N = \alpha+\varepsilon$, $\beta_N = \beta+\delta$, which are adjacent to the real values $\alpha = 3$, $\beta = 7$.

And ε and δ are the indeterminacy in these values. We take them in this example as follows: $\varepsilon \in [\lambda_1,\lambda_2\,] = [0\,,0.2\,]$and $\delta \in [\mu_1,\mu_2] = [0\,,0.1\,]$ we get $\alpha_N \in [3\,,3.2\,]$ and $\beta_N \in [7\,,7.1\,]$ then the probability density function is written as follows:

$$f_N(x) = \frac{\Gamma([10\,,10\,.3\,])}{\Gamma([3\,,3.2\,])\Gamma([7\,,7.1\,])}x^{[2\,,2.2\,]}(1-x)^{[6\,,6.2\,]} \quad ; \quad 0 \le x \le 1$$

What is required is to generate random numbers that follow the previous distribution, based on neutrosophic random numbers. We apply the rejection and acceptance algorithm according to the steps mentioned previously, and as we did in Example 1, we obtain what is required.

The difference between the second and third examples:

In the second example, we generate random numbers that follow a uniform distribution over the domain $[0\,,1]$using the mean square method. For example, we take the seed $R_0 = 0.1273$ from the relationship (3), which is:

$$R_1 = 0.6205 \quad ,R_2 = 0.5020$$

Then we use the two numbers to implement the rejection and acceptance algorithm, as we did in the first example.

In the third example, we generate two random numbers that follow a uniform distribution over the domain $[0\,,1]$using the mean square method. For example, we take the seed $R_0 = 0.1273$ from the relationship (3), which is:

$$R_1 = 0.6205 \quad ,R_2 = 0.5020$$

Then we transform them into two random numbers that follow the uniform neutrosophic distribution over the domain$[0+\varepsilon, 1+\varepsilon]$. We take the indeterminacy $\varepsilon \in [0\ \ ,0.02]$. We get the following two-neutrosophic random numbers:

$$NR_1 \in [0.6205\,,0.6005]\ \ ,\ \ NR_2 \in [0.5020\,,0.482]$$

Then we use two-neutrosophic random numbers to implement the rejection and acceptance algorithm, as we did in the first example.

Conclusion and results:

In order to obtain more accurate results and enjoy a margin of freedom when simulating systems that operate according to the beta distribution, which is one of the important distributions that has many uses in many fields, we presented in this research a study through which we are able to obtain neutrosophic random variables that follow this distribution, using neutrosophic rejection and acceptance method. Thanks to the indeterminacy of neutrosophic values, we are able to provide simulation results suitable for all circumstances and achieve the desired goal for decision makers.

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