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Interpretation of Neutrosophic Soft cubic T-ideal in the Environment of PS-Algebra

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Abstract. This study provides an innovative approach to neutrosophic algebraic structures by introducing a new structure called Neutrosophic Soft Cubic T-ideal (NSCTID), which combines T-ideal (TID) and Neutrosophic Soft Cubic Sets (NSCSs) within the framework of PS-Algebra. Within the already-existing neutrosophic cubic structures, the addition of soft sets with the characteristics of TID makes this structure more desirable. The theoretical development of the proposed structure includes the application of fundamental ideas as union, intersection, the Cartesian product, and homomorphism. We also introduce the notions of NSCTID-translation and NSCTID-multiplication to further enhance the structure of NSCTID. By applying the idea of translation and multiplication, we offer improved algorithm for neutrosophic cubic sets to deal with different parameters that are satisying the TID's distinctive characteristics. Through this thorough research, we offer an elementary understanding of NSCTID and its capabilities, providing the way to new algebraic structures.

Keywords: Neutrosophic soft cubic set; T-ideal; PS-algebra; Cartesian product; Homomorphism; Translation and Multiplication.

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1. Introduction

Zadeh was the first who put up the notion of Fuzzy Sets (FSs) in 1965, which contained a membership degree for each element say "t" [1,2]. The Intuitionistic FSs (IFSs) was established by Atanassov [3], which is a general form of FS on a universe U in which non-membership degree was taken into consideration and presented Interval-Valued IFSs which are undoubtedly both

IVFS and IFS extensions. Rahman et al. [4] employed a fresh approach of a refined intuitionistic fuzzy set to conceptualize its fundamental characteristics through set theoretic procedures such as extended union as well as intersection and same for restricted. The Neutrosophic Sets (NSs) was developed by Smarandache [5] by proposing the concept of the indeterminate degree of an element as an independent element in his 1995 manuscript, which was later published in 1998. The Interval -Valued NSs was first given by Wang et al. [6]. A new methodology for simulating fuzziness and uncertainty was established by Molodtsov in 1999 [7], which is called "soft set theory". Saeed et al. [8] conducted an exhaustive investigation of the idea of soft elements as well as soft members in the respect of soft sets (SSs). Maji et al. [9, 10] expanded SSs to IFSSs and NSSs. Smarandache [11] generalized the soft set to the hypersoft set by transforming the function F into a multi-attribute function and then introduced the hybrids of Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic Hypersoft Set. In [12], Distance and similarity measures using Max-Min operators were proposed in the environment of neutrosophic hypersoft sets with application in MCDM. The concept of cubic sets considers only the intervals of membership and ignores the segments of non-membership information. However, expressing the degree of membership in fuzzy sets with exact values can be difficult in real-world situations. In such cases, it may be simple to represent ambiguity both interval and exact values instead of just one. Therefore, Jun et al. [13] proposed the cubic ISs which combines two sets to represent membership degrees: an interval-valued IFS and an IFS. This hybrid approach can be useful in decision-making when dealing with uncertain judgments. A theoretical development of cubic pythagorean fuzzy soft set was established with its application in MADM. Based on the established TOPSIS method and aggregation operators, the decision-making algorithm is proposed under an intuitionistic fuzzy hypersoft environment to resolve uncertain and confusing information [14, 15]. By combining the NS and IVNS, Ali et al. [16] proposed the definition of NCS. In [17], Mumtaz et al. made an adjustable approach to NCS based decision making by similarity measure and is employed in pattern recognition to show that they can be successfully applied to problems that contain uncertainties. The NCSS was first introduced and some of its characteristics were proved by Chinnadurai et al. [18] in 2016. Gulistan et al. [19] developed a more general approach of neutrosophic cubic soft matrices and used it in MCDM-problems. In 1978, Iseki et al. were the first who established the idea of BCK-algebra [20]. In 1980, BCI-algebras were proposed [21]. The algebra named as BCK is a subclass of BCI-algebras. PS-algebras is a generalization of algebras like BCI, BCK, Q and KU and was first explored in [22] by Priya et al.. Shah et al. investigated images as well as pre images of anti-homomorphism for semiprime, strongly prime, irreducible, and highly irreducible fuzzy ideals in rings [23, 24]. They also established the ideas of strongly primary fuzzy ideals and strongly irreducible fuzzy ideals in ring with unity as well as commutativity and examined their importance in a Leskerian ring. The research conducted by Senapati et al. [25] examined the cubic intuitionistic q-ideals within BCI-algebras. Kandasamy et al. [26] were the first who introduced T- ideals, IFT-ideals, and IF closed T-ideals. Priya et al. [27] investigated the concept of fuzzy translation and fuzzy multiplication in the context of PS-algebra, In [28, 29], Khalid et al. established the MBJ-neutrosophic T-ideal in B-algebra and described several of its properties. The concept of NC translation and multiplication of BF-subalgebra and BF-ideal was also introduced. In [30], they presented the ideas of IF Alpha-Translation as well as multiplication and IF Magnified Beta-Alpha-Translation in the context of PS-algebra. This paper introduces the Neutrosophic Soft Cubic Set (NSCS) and its application in PS-algebra through the concept of T-ideal. Section two provides relevant definitions, while section three presents the concept of NSCTID in PS algebra. Section four examines NSCTID's fundamental properties, followed by an exploration of its translation and multiplication in section five. Finally, section six summarizes the results and suggests future directions for research.

2. Preliminaries

To provide a clear understanding of the proposed work, some essential definitions that will be utilized throughout the explanation are given.

Definition 2.1. [22]:Let P be a set with the constant $'0'$ and the binary operation $'*'$ which is nonempty. If P satisfies the axioms below, it is regarded to be PS-algebra.

i. $\vartheta_1 * \vartheta_1 = 0$

ii. $\vartheta_1 * 0 = 0$

iii. $\vartheta_1 * \vartheta_2 = 0$, and $\vartheta_2 * \vartheta_1 = 0$ implies $\vartheta_1 = \vartheta_2$, for all $\vartheta_1, \vartheta_2 \in$ P.

In any PS-Algebra $(\mathrm{P}, *, 0)$, the following characteristics also satisfy for every $\vartheta_1, \vartheta_2 \in$ P.

iv. $\vartheta_1 * (\vartheta_2 * \vartheta_1) = \vartheta_2 * (\vartheta_1 * \vartheta_1)$

v. $\vartheta_2 * (\vartheta_1 * (\vartheta_2 * \vartheta_1)) = 0$

vi. $\vartheta_1 * (\vartheta_1 * (\vartheta_1 * \vartheta_2)) = \vartheta_1 * \vartheta_2$

vii. $\vartheta_2 * (\vartheta_1 * (\vartheta_1 * \vartheta_2)) = 0$

Definition 2.2. [26]:Let I be a nonempty subset of P. I be an ideal of P if:

i. $0 \in$ I,

ii. $\vartheta_1 * \vartheta_2 \in$ I and $\vartheta_2 \in$ I implies $\vartheta_1 \in$ I,

A nonempty subset I be the T-ideal if it satisfies (i) with

iii. $(\vartheta_1 * \vartheta_2) * \vartheta_3 \in$ I and $\vartheta_2 \in$ I implies $\vartheta_1 * \vartheta_3 \in$ I for all $\vartheta_1, \vartheta_2, \vartheta_3 \in$ P.

Definition 2.3. [26]:An IFS is said to be an IFT-ideal of P if it these three conditions holds:

i. $\alpha_{\mathrm{A}}(0) \geq \alpha_{\mathrm{A}}(\vartheta_1), \beta_{\mathrm{A}}(0) \leq \beta_{\mathrm{A}}(\vartheta_1)$,

ii. $\alpha_{\mathrm{A}}(\vartheta_1 * \vartheta_3) \geq \min\{\alpha_{\mathrm{A}}((\vartheta_1 * \vartheta_2) * \vartheta_3), \alpha_{\mathrm{A}}(\vartheta_2)\}$,

iii. $\beta_{\mathrm{A}}(\vartheta_1 * \vartheta_3) \leq \max\{\beta_{\mathrm{A}}((\vartheta_1 * \vartheta_2) * \vartheta_3), \beta_{A}(\vartheta_2)\}$, for all $\vartheta_1, \vartheta_2 \in \mathrm{P}$.

Definition 2.4. [5]:A set in P represented by

$$\mathrm{A} = \{\langle \vartheta_1, \alpha_{\mathrm{A}}(\vartheta_1), \gamma_{\mathrm{A}}(\vartheta_1), \beta_{\mathrm{A}}(\vartheta_1)\rangle \mid \vartheta_1 \in \mathrm{P}\}$$

is called NS in which the mapping $\alpha_{\mathrm{A}}(\vartheta_1) : \mathrm{P} \to]0^-, 1^+[\gamma_{\mathrm{A}}(\vartheta_1) : \mathrm{P} \to]0^-, 1^+[$ and $\beta_{\mathrm{A}}(\vartheta_1) : \mathrm{P} \to]0^-, 1^+[$ denotes the functions of truth, indeterminate, and falsity membership respectively, and satisfy

$$0^- \leq \alpha_{\mathrm{A}}(\vartheta_1) + \gamma_{\mathrm{A}}(\vartheta_1) + \beta_{\mathrm{A}}(\vartheta_1) \leq 3^+.$$

For an interval-valued NS, $\widehat{\mathrm{A}}$ in P, the mappings $\widehat{\alpha}_{\mathrm{A}}(\vartheta_1), \widehat{\gamma}_{\mathrm{A}}(\vartheta_1), \widehat{\beta}_{\mathrm{A}}(\vartheta_1) \subset [0,1]$ with $\widehat{\alpha}_{\mathrm{A}}(\vartheta_1) = [\widehat{\alpha}_{\mathrm{A_L}}(\vartheta_1), \widehat{\alpha}_{\mathrm{A_U}}(\vartheta_1)], \hat{\beta}_{\mathrm{A}}(\vartheta_1) = [\hat{\beta}_{\mathrm{A_L}}(\vartheta_1), \hat{\beta}_{\mathrm{A_U}}(\vartheta_1)]$ and $\hat{\gamma}_{\mathrm{A}}(\vartheta_1) = [\hat{\gamma}_{\mathrm{A_L}}(\vartheta_1), \hat{\gamma}_{\mathrm{A_U}}(\vartheta_1)]$.

Definition 2.5. [16]:A NCS in P is defined as $\mathrm{C} = \langle \widehat{\mathrm{A}}, \mathrm{A}\rangle$ in which $\widehat{\mathrm{A}}$ is an interval-valued NS and A is the NS. The collection of all NCSs in P is represented by C(P).

Definition 2.6. [7]:Let E be the collection of parameters and V be the universal set. The set $\mathrm{S_K}$ which is defined as $\hat{\mathrm{S}}_{\mathrm{K}} = \{\langle \mathrm{v}, \mathrm{S(e)}\rangle, \mathrm{v} \in \mathrm{V}, \mathrm{e} \in \mathrm{K}, \mathrm{S(e)} \in \mathrm{P(V)}\}$ is said to be a soft set over V. Where $\mathrm{S} : \mathrm{E} \to \mathrm{P(V)}$ in which the P(V) represents the power set of V and $\mathrm{K} \subset \mathrm{E}$.

Definition 2.7. [18]:A SS denoted by $\widehat{\mathrm{S}}_{\mathrm{K}}$ is said to be a NCSS in P if $\widehat{\mathrm{S}}_{\mathrm{K}}$ is the mapping from E to the set C(P). i.e. $\widehat{\mathrm{S}}_{\mathrm{K}} : \mathrm{E} \to \mathrm{C(P)}$ where $\mathrm{K} \subset \mathrm{E}$. The NCSS is denoted by $\widehat{\mathrm{S}}_{\mathrm{K}} = \langle \widehat{\mathrm{B}}, \mathrm{B}\rangle$. Where $\widehat{\mathrm{B}} = \{\langle \vartheta_1, \alpha_{\widehat{\mathrm{B}}_{\mathrm{e_i}}}(\vartheta_1), \gamma_{\widehat{\mathrm{B}}_{\mathrm{e_i}}}(\vartheta_1), \beta_{\widehat{\mathrm{B}}_{\mathrm{e_i}}}(\vartheta_1)\rangle \mid \vartheta_1 \in \mathrm{P}, \mathrm{e_i} \in \mathrm{E}\}$ is the interval-valued NSS and $\mathrm{B} = \{\langle \vartheta_1, \alpha_{\mathrm{B_{e_i}}}(\vartheta_1), \gamma_{\mathrm{B_{e_i}}}(\vartheta_1), \beta_{\mathrm{B_i}}(\vartheta_1)\rangle \mid \vartheta_1 \in \mathrm{P}, \mathrm{e_i} \in \mathrm{E}\}$ is the NSS.

Definition 2.8. [29]:Let $\mathrm{C} = \langle \widehat{\mathrm{A}}, \mathrm{A}\rangle$ be an NCS of P and for the set $\widehat{\mathrm{A}}$, the $\mu, v \in [[0,0], \Theta]$ and $\lambda \in [[0,0], \mathrm{I}]$, where for the set $\mathrm{A}, \mu, v \in [0, \Gamma]$ and $\lambda \in [0, \epsilon]$. An object of the form $\mathrm{C}^{\mathrm{T}}_{\mu,v,\lambda} = \langle\ (\widehat{\mathrm{A}})^{\mathrm{T}}_{\mu,v,\lambda}, (\mathrm{A})^{\mathrm{T}}_{\mu,v,\lambda}\rangle$ is called an NC-Translation of C, when

$$(\widehat{\mathrm{A}}_{\mathrm{T}})^{\mathrm{Tr}}_{\mu}(\mathrm{t}_1) = \widehat{\mathrm{A}}_{\mathrm{T}}(\mathrm{t}_1) + \mu, (\widehat{\mathrm{A}}_{\mathrm{I}})^{\mathrm{Tr}}_{v}(\mathrm{t}_1) = \widehat{\mathrm{A}}_{\mathrm{I}}(\mathrm{t}_1) + v, (\widehat{\mathrm{A}}_{\mathrm{F}})^{\mathrm{Tr}}_{\lambda}(\mathrm{t}_1) = \widehat{\mathrm{A}}_{\mathrm{F}}(\mathrm{t}_1) - \lambda$$

$$(\mathrm{A}_{\mathrm{T}})^{\mathrm{Tr}}_{\mu}(\mathrm{t}_1) = \mathrm{A}_{\mathrm{T}}(\mathrm{t}_1) + \mu, (\mathrm{A}_{\mathrm{I}})^{\mathrm{Tr}}_{v}(\mathrm{t}_1) = \mathrm{A}_{\mathrm{I}}(\mathrm{t}_1) + v, (\mathrm{A}_{\mathrm{F}})^{\mathrm{Tr}}_{\lambda}(\mathrm{t}_1) = \mathrm{A}_{\mathrm{F}}(\mathrm{t}_1) - \lambda$$

for all $\mathrm{t}_1 \in \mathrm{P}$.

Definition 2.9. [29]:Let $\mathrm{C} = \langle \widehat{\mathrm{A}}, \mathrm{A}\rangle$ be an NCS of P and $\sigma \in [0,1]$. A set having the representation as $\mathrm{C}^{\mathrm{Mp}}_{\sigma} = \langle((\kappa_{\mathrm{T}})^{\mathrm{Mp}}_{\sigma}, (\kappa_{\mathrm{I}})^{\mathrm{Mp}}_{\sigma}, (\kappa_{\mathrm{F}})^{\mathrm{Mp}}_{\sigma}), ((\upsilon_{\mathrm{T}})^{\mathrm{Mp}}_{\sigma}, (\upsilon_{\mathrm{I}})^{\mathrm{Mp}}_{\sigma}, (\upsilon_{\mathrm{F}})^{\mathrm{Mp}}_{\sigma})\rangle$ is called an NC-Multiplication of C, when

$$(\kappa_{\mathrm{T}})^{\mathrm{Mp}}_{\delta}(\mathrm{t}_1) = \delta \cdot \kappa_{\mathrm{T}}(\mathrm{t}_1), (\kappa_{\mathrm{I}})^{\mathrm{Mp}}_{\delta}(\mathrm{t}_1) = \delta \cdot \kappa_{\mathrm{I}}(\mathrm{t}_1), (\kappa_{\mathrm{F}})^{\mathrm{Mp}}_{\delta}(\mathrm{t}_1) = \delta \cdot \kappa_{\mathrm{F}}(\mathrm{t}_1)$$

$$(\upsilon_{\mathrm{T}})^{\mathrm{Mp}}_{\delta}(\mathrm{t}_1) = \delta \cdot \upsilon_{\mathrm{T}}(\mathrm{t}_1), (\upsilon_{\mathrm{I}})^{\mathrm{Mp}}_{\delta}(\mathrm{t}_1) = \delta \cdot \upsilon_{\mathrm{I}}(\mathrm{t}_1), (\upsilon_{\mathrm{F}})^{\mathrm{Mp}}_{\delta}(\mathrm{t}_1) = \delta \cdot \upsilon_{\mathrm{F}}(\mathrm{t}_1)$$

for all $\mathrm{t}_1 \in \mathrm{P}$.

3. Neutrosophic soft cubic T-ideal

This section aims to present the notion of a NSCTID, accompanied by an illustrative example. Additionally, we explore several properties that are pertinent to this concept.

Definition 3.1. Let an NSC-set which is denoted as $K = \langle (\widehat{N}_K)^{T,I,F}_{e_i}, (A_K)^{T,F}_{e_i} \rangle$, where $\widehat{N}_{K_{e_i}}$ and $A_{K_{e_i}}$ represents an interval-valued NSS and NSS respectively in PS-algebra P. The set K is termed as Neutrosophic soft cubic T-ideal (NSCTID) in P if it achieves the following assertions:

i. $(\widehat{N}_K)^T_{e_i}(0) \geq (\widehat{N}_K)^T_{e_i}(t_1), (\widehat{N}_K)^I_{e_i}(0) \geq (\widehat{N}_K)^I_{e_i}(t_1), (\widehat{N}_K)^F_{e_i}(0) \geq (\widehat{N}_K)^F_{e_i}(t_1),$
$(A_K)^T_{e_i}(0) \leq (A_K)^T_{e_i}(t_1), (A_K)^I_{e_i}(0) \leq (A_K)^I_{e_i}(t_1), (A_K)^F_{e_i}(0) \leq (A_K)^F_{e_i}(t_1).$

ii. $(\widehat{N}_K)^T_{e_i}(t_1 * t_3) \geq \text{rmin}\{(\widehat{N}_K)^T_{e_i}((t_1 * t_2) * t_3), (\widehat{N}_K)^T_{e_i}(t_2)\},$
$(\widehat{N}_K)^I_{e_i}(t_1 * t_3) \geq \text{rmin}\{(\widehat{N}_K)^I_{e_i}((t_1 * t_2) * t_3), (\widehat{N}_K)^I_{e_i}(t_2)\},$
$(\widehat{N}_K)^F_{e_i}(t_1 * t_3) \geq \text{rmin}\{(\widehat{N}_K)^F_{e_i}((t_1 * t_2) * t_3), (\widehat{N}_K)^F_{e_i}(t_2)\},$

iii. $(A_K)^T_{e_i}(t_1 * t_3) \leq \max\{(A_K)^T_{e_i}((t_1 * t_2) * t_3), (A_K)^T_{e_i}(t_2)\}$
$(A_K)^I_{e_i}(t_1 * t_3) \leq \max\{(A_K)^I_{e_i}((t_1 * t_2) * t_3), (A_K)^I_{e_i}(t_2)\},$
$(A_K)^F_{e_i}(t_1 * t_3) \leq \max\{(A_K)^F_{e_i}((t_1 * t_2) * t_3), (A_K)^F_{e_i}(t_2)\},$

For all $t_1, t_2, t_3 \in P$. Where $(\widehat{N}_K)^{T,I,F}_{e_i}$ and $(A_K)^{T,I,F}_{e_i} \in [0, 1]$.

Example 3.2. Let $P = \{0, t_1, t_2, t_3\}$ be a PS-algebra with the following Cayley table. We

TABLE 1. Cayley table of $(P, *, 0)$.

$*$	0	t_1	t_2	t_3
0	0	t_2	t_1	t_3
t_1	0	0	0	t_2
t_2	0	0	0	t_2
t_3	0	t_2	t_2	0

define a NSCS represented as $K = \langle \widehat{N}_{e_i}, A_{e_i} \rangle$ in P as in Table **2** and Table **3**. The set K with the aforementioned values satisfies all of the requirements of the definition **3.1** above.

The calculations below show a few outcomes.

$[0.5, 0.7] = (\widehat{N}_K)^T_{e_i}(t_1 * t_3) = (\widehat{N}_K)^T_{e_i}(t_2) \geq \text{rmin}\{(\widehat{N}_K)^T_{e_i}(t_3), (\widehat{N}_K)^T_{e_i}(t_2)\} = \text{rmin}\{[0.4, 0.6], [0.5, 0.7]\},$

$[0.4, 0.6] = (\widehat{N}_K)^I_{e_i}(t_1 * t_3) = (\widehat{N}_K)^I_{e_i}(t_2) \geq \text{rmin}\{(\widehat{N}_K)^I_{e_i}(t_3), (\widehat{N}_K)^I_{e_i}(t_2)\} = \text{rmin}\{[0.3, 0.5], [0.4, 0.6]\},$

$[0.4, 0.5] = (\widehat{N}_K)^F_{e_i}(t_1 * t_3) = (\widehat{N}_K)^F_{e_i}(t_2) \geq \text{rmin}\{(\widehat{N}_K)^F_{e_i}(t_3), (\widehat{N}_K)^F_{e_i}(t_2)\} = \text{rmin}\{[0.2, 0.4], [0.4, 0.5]\},$

Table 2. Interval-valued NSS ($\widehat{\mathrm{N}}_{\mathrm{e_i}}$).

$*$	0	t_1	t_2	t_3
$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}$	$[0.6, 0.8]$	$[0.3, 0.6]$	$[0.5, 0.7]$	$[0.4, 0.6]$
$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}$	$[0.5, 0.7]$	$[0.3, 0.4]$	$[0.4, 0.6]$	$[0.3, 0.5]$
$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}$	$[0.4, 0.9]$	$[0.3, 0.6]$	$[0.4, 0.5]$	$[0.2, 0.4]$

Table 3. NSS ($\mathrm{A}_{\mathrm{e_i}}$).

$*$	0	t_1	t_2	t_3
$(\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}$	0.2	0.3	0.5	0.6
$(\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}$	0.5	0.6	0.8	1
$(\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}$	0.3	0.7	0.4	0.5

$$0.5 = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_1 * t_3) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_2) \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_2)\} = 0.6,$$
$$0.8 = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_1 * t_3) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_2) \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_2)\} = 1,$$
$$0.4 = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_1 * t_3) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_2) \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(t_2)\} = 0.5$$

Hence $\mathrm{K} = \langle(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{e_i}, \mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{e_i}\rangle$ is NSCTID in P.

Theorem 3.3. *Every NSCTID* $K = \langle(\widehat{N}_K)^{\mathrm{T,I,F}}_{e_i}, (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}\rangle$ *of* P *fulfills the following inequalities for all* $x, t_1, t_2, t_3 \in \mathrm{P}$*.*

i. *If* $t_1 * t_2 \leq t_3$*, then*

$$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_1 * x) \geq \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,}}_{\mathrm{e_i}}(t_3), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_2)\}$$
$$(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_1 * x) \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_2)\}.$$

Proof. Let $x, t_1, t_2, t_3 \in \mathrm{P}$ such that $t_1 * t_2 = t_3$. Now, $(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_1 * x) \geq \{\mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}((t_1 * t_2) * x), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_2)\} \geq \mathrm{rmin}\{\mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(((t_1 * t_3) * t_2) * x), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_3)\}, (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,F}}_{\mathrm{e_i}}(t_2)\}$ $= \mathrm{rmin}\{\mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(0), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_3)\}, (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_2)\} = \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I}}_{\mathrm{e_i}}(t_3), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_2)\}$ $(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_1 * x) \leq \{\max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I}}_{\mathrm{e_i}}((t_1 * t_2) * x), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,IF}}_{\mathrm{e_i}}(t_2)\} \leq \max\{\max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(((t_1 * t_3) * t_2) * x)), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_3)\}, (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,F}}_{\mathrm{e_i}}(t_2)\} = \max\{\max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(0), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_3)\}, (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,F}}_{\mathrm{e_i}}(t_2)\} =$ $\max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,,F}}_{\mathrm{e_i}}(t_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,F}}_{\mathrm{e_i}}(t_2)\}$.

Hence

$$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_1 * x) \geq \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,IF}}_{\mathrm{e_i}}(t_3), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(t_2)\}$$
$$(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,IF}}_{\mathrm{e_i}}(t_1 * x) \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T,IF}}_{\mathrm{e_i}}(t_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{e_i}(t_2)\}$$

forall $t_1, t_2, t_3 \in \mathrm{P}$. □

ii. *If* $t_1 \leq t_2$*, then*

$$(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * x) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)$$
$$(A_K)_{e_i}^{T,I,F}(t_1 * x) \leq (A_K)_{e_i}^{T,I,F}(t_2)$$

for all $x, t_1, t_2, t_3 \in P$.

Proof. Let $x, t_1, t_2, t_3 \in P$ such that $t_1 \leq t_2 \to t_1 * t_2 = 0$, Now, $(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * x) \geq \{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * t_2) * x), (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)\} = \{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,F}(0), (\widehat{N}_K)_{e_i}^{T,F}(t_2)\} = (\widehat{N}_K)_{e_i}^{T,F}(t_2)$.
And
$(A_K)_{e_i}^{T,I,F}(t_1 * x) \leq \{\max\{(A_K)_{e_i}^{T,I,F}((t_1 * t_2) * x), (A_K)_{e_i}^{T,I,F}(t_2)\}. = \{\max\{(A_K)_{e_i}^{T,I,F}(0), (A_K)_{e_i}^{T,I,F}(t_2)\}\} = (A_K)_{e_i}^{T,IF}(t_2)$.
Hence $(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * x) \geq (\widehat{N}_K)_{e_i}^{T_i,IF}(t_2)$ and $(A_K)_{e_i}^{T,I,F}(t_1 * x) \leq (A_K)_{e_i}^{T,I,F}(t_2)$ for all $t_1, t_2, t_3 \in P$.

□

iii.

$$((\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * (t_2 * t_1)) * x) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)$$
$$((A_K)_{e_i}^{T,I,F}(t_1 * (t_2 * t_1)) * x) \leq (A_K)_{e_i}^{T,I,F}(t_2).$$

Proof. Let K is an NSCTID. Then, $(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * (t_2 * t_1)) * x) \geq \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I}(((t_1 * (t_2 * t_1)) * t_2) * x), (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)\} = \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I}(0), (\widehat{N}_K)_{e_i}^{T,IF}(t_2)\} = (\widehat{N}_K)_{e_i}^{T,I}(t_2)$, $(A_K)_{e_i}^{T,I,F}((t_1 * (t_2 * t_1)) * x) \leq \max\{(A_K)_{e_i}^{T,I,F}(((t_1 * (t_2 * t_1)) * t_2) * x), (A_K)_{e_i}^{T,I,F}(t_2)\} = \max\{(A_K)_{e_i}^{T,F}(0), (A_K)_{e_i}^{T,F}(t_2)\} = (A_K)_{e_i}^{T,I,F}(t_2)$.
Hence

$$(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * (t_2 * t_1)) * x) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)$$
$$(A_K)_{e_i}^{T,I,F}((t_1 * (t_2 * t_1)) * x) \leq (A_K)_{e_i}^{T,F}(t_2),$$

for all $x, t_1, t_2, t_3 \in P$. □

Theorem 3.4. *Suppose* $K = \langle (\widehat{N}_K)_{e_i}^{T,I,F}, (A_K)_{e_i}^{T,,F} \rangle$ *and* $L = \langle (\widehat{N}_L)_{e_i}^{T,I,F}, (A_L)_{e_i}^{T,I,F} \rangle$ *are NSCTIDs of* P*. Then their union* $K \cup L$ *is also an NSCTID of* P*.*

Proof. If $t_1, t_2 \in K \cup L$, then $t_1, t_2 \in K$ and $t_1, t_2 \in L$. Then, $(\widehat{N}_{e_i})_{K \cup L}^{T,I,F}(0) = (\widehat{N}_{e_i})_{K \cup L}^{T,I,F}(t_1 * t_1) = \text{rmax}\{(\widehat{N}_K)_{e_i}^{T,IF}(t_1 * t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1 * t_1)\} \geq \text{rmax}\{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)\}, \text{rmin}\{(\widehat{N}_L)_{e_i}^{T,I,F}(t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1)\}\} = \text{rmax}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1)\} = (\widehat{N}_{e_i})_{K \cup L}^{T,I,F}(t_1)$, And $(A_{e_i})_{K \cup L}^{T,I,F}(0) = (A_{e_i})_{K \cup L}^{T,I,F}(t_1 * t_1) = \min\{(A_K)_{e_i}^{T,I,F}(t_1 * t_1), (A_L)_{e_i}^{T,I,F}(t_1 * t_1)\} \leq \min\{\max\{(A_K)_{e_i}^{T,I,F}(t_1), (A_K)_{e_i}^{T,I,F}(t_1)\}, \max\{(A_L)_{e_i}^{T,I,F}(t_1), (A_L)_{e_i}^{T,I,F}(t_1)\}\} = \min\{(A_K)_{e_i}^{T,I,F}(t_1), (A_L)_{e_i}^{T,I,F}(t_1)\} = (A_{e_i})_{K \cup L}^{T,I,F}(t_1)$. Thus $(\widehat{N}_{e_i})_{K \cup L}^{T,I,F}(0) \geq (\widehat{N}_{e_i})_{K \cup L}^{T,I,F}(t_1)$ and $(A_{e_i})_{K \cup L}^{T,I,F}(0) \leq (A_{e_i})_{K \cup L}^{TI,F}(t_1)$.
Now, $(\widehat{N}_{e_i})_{K \cup L}^{T,I,F}(t_1 * t_3) = \text{rmax}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * t_3), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1 * t_3)\} \geq \text{rmax}\{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * t_2) * t_3), (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)\}, \text{rmin}\{(\widehat{N}_L)_{e_i}((t_1 * t_2) * t_3), (\widehat{N}_L)_{e_i}^{T,I,F}(t_2)\}\} = \text{rmin}\{\text{rmax}\{(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * t_2) * t_3), (\widehat{N}_L)_{e_i}^{T,I,F}((t_1 * t_2) * t_3)\} \text{rmax}\{(\widehat{N}_K)_{e_i}^{T,IF}(t_2), (\widehat{N}_L)_{e_i}^{T,I,F}(t_2)\}\} = \text{rmin}\{(\widehat{N}_{e_i})_{K \cup L}^{T,I,F}((t_1 * t_2) * t_3), (\widehat{N}_{e_i})_{K \cup L}^{T,I,F}(t_2)\}$, And $(A_{e_i})_{K \cup L}^{T,I,F}(t_1 * t_3) = \min\{(A_K)_{e_i}^{T,I,F}(t_1 * t_3), (A_L)_{e_i}^{T,I,F}(t_1 * t_3)\}$

$\leq \min\{\max\{(A_K)_{e_i}^{T,F}((t_1 * t_2) * t_3), (A_K)_{e_i}^{T,I,F}(t_2)\}, \max\{(A_L)_{e_i}^{T,F}((t_1 * t_2) * t_3), (A_L)_{e_i}^{T,I,}(t_2)\}\}$
$= \max\{\min\{(A_K)_{e_i}^{T,I,F}((t_1 * t_2) * t_3), (A_L)_{e_i}^{T,I,F}((t_1 * t_2) * t_3)\}, \min\{(A_K)_{e_i}^{T,I,F}(t_2), (A_L)_{e_i}^{T,I,F}(t_2)\}\}$
$= \max\{(A_{e_i})_{KUL}^{T,I,F}((t_1 * t_2) * t_3), (A_{e_i})_{KUL}^{T,I,}(t_2)\}$
Hence the union " $K \cup L$ " is also an NSCTID of P. □

Theorem 3.5. *Suppose* $K = \langle(\widehat{N}_K)_{e_i}^{T,I,F}, (A_K)_{e_i}^{T,I,F}\rangle$ *and* $L = \langle(\widehat{N}_L)_{e_i}^{T,I,F}, (A_L)_{e_i}^{T,I,F}\rangle$ *are two NSCTIDs of a PS-algebra* P*. Then the intersection* $K \cap L$ *is also NSCTID of* P.

Proof. Let $t_1, t_2 \in K \cap L$, then $t_1, t_2 \in K$ and $t_1, t_2 \in L$. As $(\widehat{N}_{e_i})_{K\cap L}^{T,I,F}(0) = (\widehat{N}_{e_i})_{K\cap L}^{T,I,F}(t_1 * t_1)$ $= \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1 * t_1)\} \geq \text{rmin}\{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)\}, \text{rmin}\{(\widehat{N}_L)_{e_i}^{T,I,F}(t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1)\}\} = \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1)\} = (\widehat{N}_{e_i})_{K\cap L}^{T,I,F}(t_1)$. And $(A_{e_i})_{K\cap L}^{T,I,F}(0) = (A_{e_i})_{K\cap L}^{T,I,F}(t_1 * t_1) = \max\{(A_K)_{e_i}^{T,I,F}(t_1 * t_1), (A_L)_{e_i}^{T,I,F}(t_1 * t_1)\} \leq \max\{\max\{(A_K)_{e_i}^{T,I,F}(t_1), (A_K)_{e_i}^{T,I,F}(t_1)\}, \max\{(A_L)_{e_i}^{T,I,F}(t_1), (A_L)_{e_i}^{T,I,F}(t_1)\}\} = \max\{(A_K)_{e_i}^{T,I,F}(t_1), (A_L)_{e_i}^{T,I,F}(t_1)\} = (A_{e_i})_{K\cap L}^{T,I,F}(t_1)$. Thus$(\widehat{N}_{e_i})_{K\cap L}^{T,I,F}(0) \geq (\widehat{N}_{e_i})_{K\cap L}^{T,I,F}(t_1)$ and $(A_{e_i})_{K\cap L}^{T,I,F}(0) \leq (A_{e_i})_{K\cap L}^{T,IF}(t_1)$.
Now, $(\widehat{N}_{e_i})_{K\cap L}^{T,I,F}(t_1 * t_3) = \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * t_3), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1 * t_3)\} \geq \text{rmin}\{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * t_2) * t_3), (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)\}, \text{rmin}\{(\widehat{N}_L)_{e_i}^{T,F}((t_1 * t_2) * t_3), (\widehat{N}_L)_{e_i}^{T,I,F}(t_2)\}\} = \text{rmin}\{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,IF}((t_1 * t_2) * t_3), (\widehat{N}_L)_{e_i}^{T,I,F}((t_1 * t_2) * t_3)\}\ \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,IF}(t_2), (\widehat{N}_L)_{e_i}^{T,I,F}(t_2)\}\} = \text{rmin}\{(\widehat{N}_{e_i})_{K\cap L}^{T,I,F}((t_1 * t_2) * t_3), (\widehat{N}_{e_i})_{K\cap L}^{T,I,F}(t_2)\}$, And $A_{e_{i_{K\cap L}}}(t_1 * t_3) = \max\{(A_K)_{e_i}^{T,I,F}(t_1 * t_3), (A_L)_{e_i}^{T,I,F}(t_1 * t_3)\} \leq$ $\max\{\max\{(A_K)_{e_i}^{T,I,F}((t_1 * t_2) * t_3), (A_K)_{e_i}^{T,I,F}(t_2)\}, \max\{(A_L)_{e_i}^{T,I,F}((t_1 * t_2) * t_3), (A_L)_{e_i}^{T,I,F}(t_2)\}\}$
$= \max\{\max\{(A_K)_{e_i}^{T,I,F}((t_1 * t_2) * t_3), (A_L)_{e_i}^{T,I,F}((t_1 * t_2) * t_3)\}, \max\{(A_K)_{e_i}^{T,I,F}(t_2), (A_L)_{e_i}^{T,I,F}(t_2)\}\}$
$= \max\{(A_{e_i})_{K\cap L}^{T,I,F}((t_1 * t_2) * t_3), (A_{e_i})_{K\cap L}^{T,I,F}(t_2)\}$
Hence, $K \cap L$ is an NSCTID of P. □

4. **Cartesian product and Homomorphism of NSCTID**

In this section, the interpretation of the cartesian product and homomorphism of NSCTID is given by proving some theorems.

Definition 4.1. Let $K = ((\widehat{N}_K)_{e_i}^{T,I,F}, (A_K)_{e_i}^{T,I,F})$ and $L = ((\widehat{N}_L)_{e_i}^{T,I,F}, (A_L)_{e_i}^{T,I,F})$ are two NSCTIDs of R and P respectively. The cartesian product $K \times L = (R \times P, (\widehat{N}_K)_{e_i}^{T,I,F} \times \widehat{N}_L)_{e_i}^{T,I,F}, (A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})$ is defined as:

$$(\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F}(t_1, t_2) \geq \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,IF}(t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_2)\}$$
$$(A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F}(t_1, t_2) \leq \max\{(A_K)_{e_i}^{T,I,F}(t_1), (A_L)_{e_i}^{T,I,F}(t_2)\}.$$

Where $(\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F} : R \times P \to [0, 1]$ and $(A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F} : R \times P \to [0, 1]$ for all $t_1 \in R$ and $t_2 \in P$.

Theorem 4.2. *Let* K *and* L *be two NSCTIDs of* P*, then* $K \times L$ *is an NSCTID of* $R \times P$.

Proof. Let R and P be two PS-algebras.For any $t_1, t_2 \in R \times P$, We have $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(0,0) = \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(0), (\widehat{N}_L)_{e_i}^{T,I,F}(0)\} \geq \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,F}(t_1), (\widehat{N}_L)_{e_i}^{T,F}(t_2)\} = ((\widehat{N}_K)_{e_i}^{T,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(t_1,t_2)((A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})(0,0) = \max\{(A_K)_{e_i}^{T,F}(0), (A_L)_{e_i}^{T,I,F}(0)\} \leq \max\{(A_K)_{e_i}^{T,I,F}(t_1), (A_L)_{e_i}^{T,I,F}(t_2)\} = ((A_K)_{e_i}^{T,F} \times (A_L)_{e_i}^{T,I,F})(t_1,t_2)$ That is $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,F})(0,0) \geq ((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I})(t_1,t_2)$ $((A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,F})(0,0) \leq ((A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,F})(t_1,t_2)$.

Now, Let $(x_1,x_2),(t_1,t_2)$ and $(y_1,y_2) \in R \times P$.Then, $((\widehat{N}_K)_{e_i}^{T,I} \times (\widehat{N}_L)_{e_i}^{T,I})(t_1 * x_1, t_2 * x_2) = \text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * x_1), (\widehat{N}_L)_{e_i}^{T,IF}(t_2 * x_2)\} \geq \text{rmin}\{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I}((t_1 * y_1) * x_1), (\widehat{N}_K)_{e_i}^{T,IF}(y_1)\}, \text{rmin}\{(\widehat{N}_L)_{e_i}^{T,I,F}((t_2 * y_2) * x_2), (\widehat{N}_L)_{e_i}^{T,I,F}(y_2)\}\} = \text{rmin}\{\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * y_1) * x_1), (\widehat{N}_L)_{e_i}^{T,I,F}((t_2 * y_2) * x_2)\}$ $\text{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(y_1), (\widehat{N}_L)_{e_i}^{T,I,F}(y_2)\}\} = \text{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(((t_1 * y_1)x_1)\ ((t_2 * y_2) * x_2)), ((\widehat{N}_K)_{e_i}^{T,IF} \times (\widehat{N}_L)_{e_i}^{T,IF})(y_1,y_2)\} = \text{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})((t_1 * x_1, t_2 * x) * (y_1,y_2)), ((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(y_1,y_2)$.

And $(A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})(t_1 * x_1, t_2 * x_2) = \max\{(A_K)_{e_i}^{T,I,F}(t_1 * x_1), (A_L)_{e_i}^{T,I,F}(t_2 * x_2)\}$ $\leq \max\{\max\{(A_K)_{e_i}^{T,I,F}((t_1 * y_1) * x_1), (A_K)_{e_i}^{T,F}(y_1)\}, \max\{(A_L)_{e_i}^{T,I}((t_2 * y_2) * x), (A_L)_{e_i}^{T,I,F}(y_2)\}\}$ $= \max\{\max\{(A_K)_{e_i}^{T,I,F}((t_1 * y_1) * x_1), (A_L)_{e_i}^{T,I,F}((t_2 * y_2) * x_2)\}, \max\{(A_K)_{e_i}^{T,I,F}(y_1), (A_L)_{e_i}^{T,I,F}(y_2)\}$ $= \max\{((A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})(((t_1 * y_1) * x_1), ((t_2 * y_2) * x_2)), (A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})(y_1,y_2)\} = \max\{(A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})((t_1 * x_1, t_2 * x_2) * (y_1,y_2)), (A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})(y_1,y_2)$.

Thus $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(t_1 * x_1, t_2 * x_2) \geq \text{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})((t_1 * x_1, t_2 * x_2) * (y_1,y_2)), ((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(y_1,y_2)$.$(A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})(t_1 * x_1, t_2 * x_2) \leq \max\{(A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})((t_1 * x_1, t_2 * x_2) * (y_1,y_2)), (A_K)_{e_i}^{T,I,F} \times (A_L)_{e_i}^{T,I,F})(y_1,y_2)$. □

Theorem 4.3. *Let* K *and* L *are two NSCSs of* R *and* P *such that* $K \times L$ *is an NSCTID of* $R \times P$*, Then*

i. *For all* $t_1 \in R$ *and* $t_2 \in P$,

$$(\widehat{N}_K)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (A_K)_{e_i}^{T,I,F}(0) \leq (A_K)_{e_i}^{T,I,F}(t_2),$$
$$(\widehat{N}_L)_{e_i}^{T,I}, F(0) \geq (\widehat{N}_L)_{e_i}^{T,I,F}(t_1), (A_L)_{e_i}^{T,I,F}(0) \leq (A_L)_{e_i}^{T,I,F}(t_2).$$

ii. *For all* $t_1 \in P$*, If* $(\widehat{N}_K)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)$ *then,*

$$(\widehat{N}_L)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_1) \text{ and } (\widehat{N}_L)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_L)_{e_i}^{T,I,F}(t_1),$$

Also if $(A_K)e_{e_i}^{T,F}(0) \leq (A_K)_{e_i}^{T,I,F}(t_1)$ *then*

$$(A_L)_{e_i}^{T,I,F}(0) \leq (A_K)_{e_i}^{T,I,F}(t_1) \text{ and } (A_L)_{e_i}^{T,I,F}(0) \leq (A_L)_{e_i}^{T,I,F}(t_1).$$

iii. *For all* $t_1 \in P$*, If* $(\widehat{N}_L)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_L)_{e_i}^{T,I,F}(t_1)$ *then,*

$$(\widehat{N}_K)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_1) \text{and } (\widehat{N}_K)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_L)_{e_i}^{T,I,F}(t_1),$$

Also if $(\mathrm{A_L})_{e_i}^{T,I,F}(0) \leq (\mathrm{A_L})_{e_i}^{T,I,F}(t_1)$ *then*

$$(\mathrm{A_K})_{e_i}^{T,I,F}(0) \leq (\mathrm{A_K})_{e_i}^{T,I,F}(t_1) and (\mathrm{A_K})_{e_i}^{T,I,F}(0) \leq (\mathrm{A_L})_{e_i}^{T,I,F}(t_1).$$

Proof. ***i.*** Suppose $(\widehat{N}_K)_{e_i}^{T,I,F}(t_1) > (\widehat{N}_K)_{e_i}^{T,I,F}(0)$ or $(\widehat{N}_L)_{e_i}^{T,I,F}(t_2) > (\widehat{N}_L)_{e_i}^{T,I,F}(0)$ for all $t_1 \in R$ and $t_2 \in P$. Then, $(\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(t_1, t_2) = \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_2)\} > \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(0), (\widehat{N}_L)_{e_i}^{T,I,F}(0)\} = ((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(0,0)$. Thus $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(t_1, t_2) > ((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(0,0)$ for all $t_1 \in R$ and $t_2 \in P$, which is the contradiction to $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})$ is a NSCTID of $R \times P$.
Also, if $(\mathrm{A_K})_{e_i}^{T,I,F}(t_1) < (\mathrm{A_K})_{e_i}^{T,I,F}(0)$ or $(\mathrm{A_L})_{e_i}^{T,I,F}(t_2) < (\mathrm{A_L})_{e_i}^{T,I,F}(0)$ for all $t_1 \in R$ and $t_2 \in P$. Then, $(\mathrm{A_K})_{e_i}^{T,I,F} \times (\mathrm{A_L})_{e_i}^{T,I,F})(t_1, t_2) = \max\{(\mathrm{A_K}))_{e_i}^{T,I,F}(t_1), (\mathrm{A_L})_{e_i}^{T,I,F}(t_2)\} > \max\{(\mathrm{A_K})_{e_i}^{T,I,F}(0), (\mathrm{A_L})_{e_i}^{T,I,F}(0)\} = ((\mathrm{A_K})_{e_i}^{T,I,F} \times (\mathrm{A_L})_{e_i}^{T,I,F})(0,0)$.
Thus $(\mathrm{A_K})_{e_i}^{T,I,F} \times (\mathrm{A_L})_{e_i}^{T,I,F})(t_1, t_2) < (\mathrm{A_K})_{e_i}^{T,I,F} \times (\mathrm{A_L})_{e_i}^{T,I,F})(0,0)$ for all $t_1 \in R$ and $t_2 \in P$, which is the contradiction to $(\mathrm{A_K})_{e_i}^{T,I,F} \times (\mathrm{A_L})_{e_i}^{T,I,F})$ is an NSCTID of $R \times P$. □

Proof. ***ii.*** Suppose $(\widehat{N}_L)_{e_i}^{T_iI,F}(0) < (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)$ or $(\widehat{N}_K)_{e_i}^{T,I,F}(0) < (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)$ for all $t_1 \in R, P$. Then, $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(0,0) = \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,I}(0), (\widehat{N}_L)_{e_i}^{T,IF}(0)\} = (\widehat{N}_L)_{e_i}^{T,I,F}(0)$, And $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,I})(t_1, t_1) = \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (\widehat{N}_L)_{e_i}^{T,I,F}(t_1)\} > (\widehat{N}_L)_{e_i}^{T,F}(0) = ((\widehat{N}_K)_{e_i}^{T,F} \times (\widehat{N}_L)_{e_i}^{T,I,F})(0,0)$. This implies $((\widehat{N}_K)_{e_i}^{T,I,F} \times (\widehat{N}_L)_{e_i}^{T,IF})(t_1, t_1) = ((\widehat{N}_K)_{e_i}^{T,IF} \times (\widehat{N}_L)_{e_i}^{T,I,F})(0,0)$. Which is the contradiction to $((\widehat{N}_K)_{e_i}^{TI,F} \times (\widehat{N}_L)_{e_i}^{T,F,F})$ is an NSCTID of $R \times P$. Hence if $(\widehat{N}_K)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)$ then $(\widehat{N}_L)_{e_i}^{T,I,F}(0) \geq (\widehat{N}_L)_{e_i}^{T,I,F}(t_1)$ and $(\widehat{N}_L)_{e_i}(0) \geq (\widehat{N}_L)_{e_i}(t_1)$ for all $t_1 \in R, P$. Now suppose $(\mathrm{A_L})_{e_i}^{T,I,F}(0) > (\mathrm{A_K})_{e_i}^{T,I,F}(t_1)$ or $(\mathrm{A_K})_{e_i}^{T,I,F}(0) > (\mathrm{A_K})_{e_i}^{T,I,F}(t_1)$ for all $t_1 \in R, P$. Then, $(\mathrm{A_K})_{e_i}^{T,I,F} \times (\mathrm{A_L})_{e_i}^{T,I,F})(0,0) = \max\{(\mathrm{A_K})_{e_i}^{T,I,F}(0), (\mathrm{A_L})_{e_i}^{T,I}(0)\} = (\mathrm{A_L})_{e_i}(0)$.
And $(\mathrm{A_K})_{e_i}^{T,IF} \times (\mathrm{A_L})_{e_i}^{T,I,F})(t_1, t_1) = \max\{(\mathrm{A_K})_{e_i}^{T,I,F}(t_1), (\mathrm{A_L})_{e_i}^{T,I,F}(t_1)\} > (\mathrm{A_L})_{e_i}^{T,I,F}(0) = (\mathrm{A_K})_{e_i}^{T,IF} \times (\mathrm{A_L})_{e_i}^{T,I,F})(0,0)$.
This implies $(\mathrm{A_K})_{e_i}^{T,IF} \times (\mathrm{A_L})_{e_i}^{T,I,F})(t_1, t_1) = (\mathrm{A_K})_{e_i}^{T,IF} \times (\mathrm{A_L})_{e_i}^{T,I,F})(0,0)$. Which is a contradiction to $(\mathrm{A_K})_{e_i}^{T,I,F} \times (\mathrm{A_L})_{e_i^{T,I,F}})$ is an NSCTID of $R \times P$. □

Proof. ***iii.*** The proof is quite the same as ***ii***. □

Theorem 4.4. *Let* $\Sigma : P \to R$ *is a homomorphism of PS -algebra. If* $K = \langle(\widehat{N}_K)_{e_i}^{T,I,F}, (\mathrm{A_K})_{e_i}^{T,I,F}\rangle$ *be an NSCTID of* R *then the pre-image* $\Sigma^{-1}(K) = (\Sigma^{-1}(\widehat{N}_K)_{e_i}^{T,I,F}), \Sigma^{-1}((\mathrm{A_K})_{e_i}^{T,I,F}))$ *of* P *under* Σ *is an NSCTID in* P.

Proof. For any $t_1 \in P$, we have
$\Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F})(t_1) = (\widehat{N}_K)_{e_i}^{T,I,F}(\Sigma(t_1)) \leq (\widehat{N}_K)_{e_i}^{T,I,F}(0) = (\widehat{N}_K)_{e_i}^{T,I,F}(\Sigma(0)) = \Sigma^{-1}(\widehat{N}_K)_{e_i}^{T,I,F}(0)$,
$\Sigma^{-1}((\mathrm{A_K})_{e_i}^{T,I,F})(t_1) = (\mathrm{A_K})_{e_i}^{T,I,F}(\Sigma(t_1)) \geq (\mathrm{A_K})_{e_i}^{T,I,F}(0) = (\mathrm{A_K})_{e_i}^{T,I,F}(\Sigma(0)) = \Sigma^{-1}(\mathrm{A_K})_{e_i}^{T,I,F}(0)$.
Also,

$\Sigma^{-1}(\widehat{N}_K)_{e_i}^{T,I,F})(t_1 * x) = (\widehat{N}_K)_{e_i}^{T,I,F}(\Sigma(t_1 * x)) \geq \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,F}((\Sigma(t_1) * \Sigma(t_2)) * \Sigma(x)), (\widehat{N}_K)_{e_i}^{T,I,F}$ $(\Sigma(t_2))\} = \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}(\Sigma((t_1 * t_2) * x), (\widehat{N}_K)_{e_i}^{T,F}(\Sigma(t_2))\} = \mathrm{rmin}\{\Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F})((t_1 * t_2)$ $x), \Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F})(t_2)\}$,$\Sigma^{-1}((A_K)_{e_i}^{T,I,F})(t_1 * x) = (A_K)_{e_i}^{T,I,F}(\Sigma(t_1 * x)) \leq \max\{(A_K)_{e_i}^{T,I,F}(\Sigma(((t_1)$ $\Sigma(t_2)) * \Sigma(x)), (A_K)_{e_i}^{T,F}(\Sigma(t_2))\} = \max\{(A_K)_{e_i}^{T,I,F}(\Sigma((t_1 * t_2) * x), (A_K)_{e_i}^{T,F}(\Sigma(t_2))\} = \max\{\Sigma^{-1}$ $((A_K)_{e_i}^{T,F})((t_1 * t_2) * x), \Sigma^{-1}((A_K)_{e_i}^{T,I,F})(t_2)\}$. □

Theorem 4.5. *Let $\Sigma : P \to R$ be an epimorphism of PS-algebra. Then* $K = \langle(\widehat{N}_K)_{e_i}^{T,I,F}, (A_K)_{e_i}^{T,I,F}\rangle$ *is an NSCTID of* P, *if* $\Sigma^{-1}(K) = ((\Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F}), \Sigma^{-1}((A_K)_{e_i}^{T,I,F}$)) *of* P *under* Σ *is an NSCTID in* P.

Proof. For any $t_1 \in R$, there exist $a \in P$ such that $\Sigma(a) = t_1$. Then, $(\widehat{N}_K)_{e_i}^{T,I,F}(t_1) = (\widehat{N}_K)_{e_i}^{T,I,F}(\Sigma(a)) = \Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F})(a) \leq \Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F})(0) = (\widehat{N}_K)_{e_i}^{T,I,F}(\Sigma(0)) = (\widehat{N}_K)_{e_i}^{T,I,F}(0)$, And $(A_K)_{e_i}^{T,I,F}(t_1) = (A_K)_{e_i}^{T,I,F}(\Sigma(a)) = \Sigma^{-1}((A_K)_{e_i}^{T,I,F})(a) \geq \Sigma^{-1}((A_K)_{e_i}^{T,F})(0) = (A_K)_{e_i}^{T,IF}(\Sigma(0)) = (A_K)_{e_i}^{T,I,F}(0)$.
Let $y, t_1, t_2 \in R$ then $\Sigma(x) = y, \Sigma(a) = t_1$ and $\Sigma(b) = t_2$ for some $a, b, x \in P$.
Now, $(\widehat{N}_K)_{e_i}^{T,I,F}(t_1 * y) = (\widehat{N}_K)_{e_i}^{T,I,}(\Sigma(a * x)) = \Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F})(a * x) \geq \mathrm{rmin}\{\Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,F}((a * b) * x)), \Sigma^{-1}((\widehat{N}_K)_{e_i}^{T,I,F}(b))\} = \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,F}(\Sigma((a * b) * x)), (\widehat{N}_K)_{e_i}^{T,I}(\Sigma(b))\} = \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}$ $(\Sigma(a) * \Sigma(b)) * \Sigma(x)), (\widehat{N}_K)_{e_i}(\Sigma(b))\} = \mathrm{rmin}\{(\widehat{N}_K)_{e_i}^{T,I,F}((t_1 * t_2) * y), (\widehat{N}_K)_{e_i}^{T,I,F}(t_2)\}$.
And $(A_K)_{e_i}^{T,I,F}(t_1 * y) = (A_K)_{e_i}^{T,I,F}(\Sigma(a * x)) = \Sigma^{-1}((A_K)_{e_i}^{T,F}(a * x) \leq \max\{\Sigma^{-1}((A_K)_{e_i}^{T,I,F}((a * b) * x)), \Sigma^{-1}((A_K)_{e_i}^{T,I}(b))\} = \max\{(A_K)_{e_i}^{TI,F}(\Sigma((a * b) * x)), (A_K)_{e_i}^{T,I,F}(\Sigma(b))\} = \max\{(A_K)_{e_i}^{T,I,F}$ $(\Sigma(a) * \Sigma(b)) * \Sigma(x))\}, (A_K)_{e_i}^{T,I,F}(\Sigma(b))\} = \max\{(A_K)_{e_i}^{T,I,F}((t_1 * t_2) * y), (A_K)_{e_i}^{T,I,F}(t_2)\}$ □

5. Translation and Multiplication of Neutrosophic Soft Cubic T-Ideal

In this section, the interpretation of NSCTID-translation and NSCTID-multiplication is given. For the simplicity, the notation $K =< t_1, (\widehat{N}_K)_{e_i}^{T,I,F}(t_1), (A_K)_{e_i}^{T,I}(t_1) \mid t_1 \in P >$ for the NSCS is used.
In this paper, we use $\xi = [1,1] - \mathrm{rsup}\{(\widehat{N}_K)_{e_i}^{T,I,F}(t_1) \mid t_1 \in P\}$, $\Psi = \mathrm{r}\inf\{(\widehat{N}_K)_{e_i}^{F}(t_1) \mid t_1 \in P\}$, $\zeta = 1- \sup\{(A_K)_{e_i}^{T,I}(t_1) \mid t_1 \in P\}, \Phi = \inf\{(A_K)_{e_i}^{F}(t_1)\} \mid t_1 \in P$ for any NSCS $K = \langle(\widehat{N}_K)_{e_i}^{T,I,F}, (A_K)_{e_i}^{T,I,F}\rangle$ of P.

Definition 5.1. Let $K = \langle(\widehat{N}_K)_{e_i}^{T,I,,F}, (A_K)_{e_i}^{T,IF}\rangle$ be an NSCS of P. For $(\widehat{N}_K)_{e_i}^{T,I,F}, \alpha, \beta \in [[0,0], \xi]$ and $\gamma \in [[0,0], \Psi]$, where for $(A_K)_{e_i}^{T,I,F}, \alpha, \beta \in [0, \zeta]$ and $\gamma \in [0, \Phi]$. A set of the form $\widetilde{K}^{Tr}_{\alpha,\beta,\gamma} = \langle((\widehat{N}_K)_{e_i}^{T,I,F})^{Tr}_{\alpha,\beta,\gamma}((A_K)_{e_i}^{T,I,F})^{Tr}_{\alpha,\beta,\gamma}\rangle$ is called an Neutrosophic Soft Cubic Translation **NSCTr** of K, As for all $t_1 \in P$.

$((\widehat{N}_K)_{e_i}^{T})^{Tr}_{\alpha}(t_1) = (\widehat{N}_K)_{e_i}^{T}(t_1)+\alpha, ((\widehat{N}_K)_{e_i}^{I})^{Tr}_{\beta}(t_1) = (\widehat{N}_K)_{e_i}^{I}(t_1)+\beta, ((\widehat{N}_K)_{e_i}^{F})^{Tr}_{\gamma}(t_1) = (\widehat{N}_K)_{e_i}^{F}(t_1)-\gamma,$

$((A_K)_{e_i}^{T})^{Tr}_{\alpha}(t_1) = (A_K)_{e_i}^{T}(t_1)+\alpha, ((A_K)_{e_i}^{I})^{Tr}_{\beta}(t_1) = (A_K)_{e_i}^{I}(t_1)+\beta, ((A_K)_{e_i}^{F})^{Tr}_{\gamma}(t_1) = (A_K)_{e_i}^{F}(t_1)-\gamma.$

Definition 5.2. Let K be an NSCS of P and $\eta \in [0,1]$. An object having the form $\widetilde{\mathrm{K}}^{\mathrm{Mp}}_{\eta} = \langle(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}, ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}, ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}), (((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}, ((\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}, ((\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}))\rangle$ is called a Neutrosophic Soft Cubic Translation **NSCMp** of K, Where

$((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_1) = \eta\cdot(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_1), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_1) = \eta\cdot(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_1), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_1) = \eta\cdot(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_1),$

$((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_1) = \eta\cdot\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_1), (\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_1) = \eta\cdot(\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_1), ((\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_1) = \eta\cdot(\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_1)$

for all $\mathrm{t}_1 \epsilon P$.

5.1. *Neutrosophic Soft Cubic T-Ideal Translation*

This section defines neutrosophic soft cubic T-Ideal translation with a theorem and example.

Theorem 5.3. *If* K *is an NSCTID of* P*, then NSCTr* $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ *of K is an NSCTID of* P.

Proof. Let $\mathrm{K} = \langle(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}\rangle$ of P be an NSCTID of P. Then we have $((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(0) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(0) + \alpha \geq (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_1) + \alpha = ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t}_1)$, $((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(0) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(0) + \beta \geq (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_1) + \beta = ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t}_1), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(0) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(0) + \gamma \geq (\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_1) + \gamma = ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t}_1)$,
And $((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(0) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(0) + \alpha \leq ((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_1) + \alpha = ((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t}_1)((\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(0) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(0) + \beta \leq (\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_1) + \beta = ((\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t}_1)\ ((\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(0) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(0) + \gamma \leq (\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_1) + \gamma = ((\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t}_1)$ Now $((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t}_1 * \mathrm{t}_3) = \widehat{\mathrm{N}}_{\mathrm{e_i}}(\mathrm{t}_1 * \mathrm{t}_3) + \alpha \geq \mathrm{rmin}\{\widehat{\mathrm{N}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), \widehat{\mathrm{N}}_{\mathrm{e_i}\mathrm{T}}(\mathrm{t}_2)\} + \alpha = \mathrm{rmin}\{\widehat{\mathrm{N}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3) + \alpha, \widehat{\mathrm{N}}_{\mathrm{e_i}}(\mathrm{t}_2) + \alpha\} = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t}_2)\}$, $((\widehat{N}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t}_1 * \mathrm{t}_3) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_1 * \mathrm{t}_3) + \beta \geq \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_2)\} + \beta = \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3) + \beta, (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_2) + \beta\} = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t}_2)\}((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t}_1 * \mathrm{t}_3) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_1 * \mathrm{t}_3) + \gamma \geq \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_2)\} + \gamma = \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3) + \gamma, (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_2) + \gamma\} = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t}_2)\}$ And $((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t}_1 * \mathrm{t}_3) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_1 * \mathrm{t}_3) + \alpha \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_2)\} + \alpha = \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3) + \alpha, (\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_2) + \alpha\} = \max\{((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), ((\mathrm{A}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t}_2)\}, ((\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{Tr}_{\beta}(\mathrm{t}_1 * \mathrm{t}_3) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_1 * \mathrm{t}_3) + \beta \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_2)\} + \beta = \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3) + \beta, (\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_2) + \beta\} = \max\{((\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), ((\mathrm{A}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t}_2)\}, ((\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t}_1 * \mathrm{t}_3) = (\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_1 * \mathrm{t}_3) + \gamma \leq \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), (\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_2)\} + \gamma = \max\{(\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3) + \gamma, (\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_2) + \gamma\} = \max\{((\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), ((\mathrm{A}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t}_2)\}$, Hence $\widetilde{K}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ of K is an NSCTID of P. □

Example 5.4. Let $P = \{0, \mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3\}$ be a PS-algebra with the cayley's table as shown in **Table 1**. The NSCS $\mathrm{K} = \langle(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{e_i}, (\mathrm{A}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}\rangle$ of P is defined as

$$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{[0.7, 0.9] \text{ if } \mathrm{t_i} = 0 \text{ and } [0.3, 0.6] \text{ if otherwise}$$

$$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{[0.6, 0.8] \text{ if } \mathrm{t_i} = 0 \text{ and } [0.4, 0.5] \text{ if otherwise}$$

$$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{[0.5, 1] \text{ if } \mathrm{t_i} = 0 \text{ and } [0.2, 0.7] \text{ if otherwise}$$

And

$$(\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t}_i) = \{0.2 \text{ if } \mathrm{t}_i = 0 \text{ and } 0.7 \text{ if otherwise}$$

$$(\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_i) = \{0.5 \text{ if } \mathrm{t}_i = 0 \text{ and } 0.9 \text{ if otherwise}$$

$$(\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t}_i) = \{0.4 \text{ if } \mathrm{t}_i = 0 \text{ and } 1 \text{ if otherwise.}$$

The set K is an NSCTID of P as it satisfies (i) but fulfills (ii) and (iii) with the condition: If $\mathrm{i} = \mathrm{j}$ then $\mathrm{t_i} * \mathrm{t_j} = 0$ with

$$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.7, 0.9], (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.6, 0.8], (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.5, 1],$$

Otherwise

$$(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.3, 0.6], (\mathrm{t_i} * \mathrm{t_j}) = [0.4, 0.5], (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.2, 0.7]$$

And

$$(\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = 0.2, (\ \mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i}) = 0.5, (\ \mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i}) = 0.4,$$

Otherwise $(\mathrm{A_K})^{T}_{\mathrm{e_i}}(\mathrm{t}_i) = 0.7, (\mathrm{A_K})^{I}_{\mathrm{e_i}}(\mathrm{t}_i) = 0.9, (\mathrm{A_K})^{F}_{\mathrm{e_i}}(\mathrm{t}_i) = 1$ given in definition 3.1. Now, for $(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}$ we choose $\alpha = [0.04, 0.08], \beta = [0.05, 0.09], \gamma = [0.03, 0.07]$ and for $(\mathrm{A})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, \alpha = 0.03, \beta = 0.04, \gamma = 0.05$. Then the mapping $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha,\beta,\gamma} \mid \mathrm{P} \to [0,1]$ is given by

$$((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{[0.04,0.08]}(0) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(0) + [0.04, 0.08] = [0.74, 0.98]$$

$$((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{[0.05,0.09]}(0) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(0) + [0.05, 0.09] = [0.65, 0.89]$$

$$((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{[0.03,0.07]}(0) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(0) - [0.03, 0.07] = [0.47, 0.93]$$

$$((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{[0.04,0.08]}(\mathrm{t_i}) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) + [0.04, 0.08] = [0.34, 0.68]$$

$$((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{[0.05,0.09]}(\mathrm{t_i}) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) + [0.05, 0.09] = [0.45, 0.59]$$

$$((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{[0.03,0.07]}(\mathrm{t_i}) = (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i}) - [0.03, 0.07] = [0.17, 0.63]$$

$$((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{0.03}(0) = (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(0) + 0.03 = [0.23]$$

$$((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{0.04}(0) = (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(0) + 0.04 = [0.54]$$

$$((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{0.05}(0) = (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(0) - 0.05 = [0.35]$$

$$((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{0.03}(\mathrm{t_i}) = (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) + 0.03 = [0.73]$$

$$((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{0.04}(\mathrm{t_i}) = (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) + 0.04 = [0.94]$$

$$((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{0.05}(\mathrm{t_i}) = (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) - 0.05 = [0.95]$$

Hence $\widetilde{K}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ is an NSCTID of P.

Theorem 5.5. *The union of any two NSC-translations of an NSCTID is an NSCTID of P.*

Proof. Suppose $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ and $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}$ are two NSC-translations of NSCTID of P respectively. In $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ for $(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I}}_{\mathrm{e_i}}, \alpha, \beta \in [[0,0], \xi]$ and $\gamma \in [[0,0], \Psi]$, where for $(\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, \alpha, \beta \in [0, \zeta]$ and $\gamma \in [0, \Phi]$, and in $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}$, for $(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, \alpha', \beta' \in [[0,0], \xi]$ and $\gamma' \in [[0,0], \Psi]$, where for

$(A_K)^{T,I,F}_{e_i}, \alpha', \beta' \in [0,\zeta]$ and $\gamma' \in [0,\Phi]$. and $\alpha \geq \alpha', \beta \geq \beta', \gamma \geq \gamma'$ as we know that, $\widetilde{K}^{Tr}_{\alpha,\beta,\gamma}$ and $\widetilde{K}^{Tr}_{\alpha',\beta',\gamma'}$ are NSCTID of P. Then

$(((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha} \cup ((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha'})(0) = (((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha} \cup ((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha'})(t_1 * t_1) = rmax\{((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha}(t_1 * t_1), ((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha'}(t_1 * t_1)\} \geq rmax\{rmin\{((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha}(t_1), ((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha}(t_1)\}, rmin\{((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha'}(t_1), ((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha'}(t_1)\}\} = rmax\{((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha}(t_1), ((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha'}(t_1)\} = rmax\{(\widehat{N}_K)^T_{e_i}(t_1) + \alpha, (\widehat{N}_K)^T_{e_i}(t_1) + \alpha'\} = ((\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha} \cup (\widehat{N}_K)^T_{e_i})^{Tr}_{\alpha'})(t_1)$, $(((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta} \cup ((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta'})(0) = (((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta} \cup ((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta'})(t_1 * t_1) = rmax\{((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta}(t_1 * t_1), ((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta'}(t_1 * t_1)\} \geq rmax\{rmin\{((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta}(t_1), ((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta}(t_1)\}, rmin\{((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta'}(t_1), ((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta'}(t_1)\}\} = rmax\{((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta}(t_1), ((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta'}(t_1)\} = rmax\{(\widehat{N}_K)^I_{e_i}(t_1) + \beta, (\widehat{N}_K)^I_{e_i}(t_1) + \beta'\} = ((\widehat{N}_K)^I_{e_i})^{Tr}_{\beta} \cup (\widehat{N}_K)^I_{e_i})^{Tr}_{\beta'})(t_1)$, $(((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma} \cup ((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma'})(0) = (((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma} \cup ((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma'})(t_1 * t_1) = rmax\{((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma}(t_1 * t_1), ((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma'}(t_1 * t_1)\} \geq rmax\{rmin\{((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma}(t_1), ((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma}(t_1)\}, rmin\{((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma'}(t_1), ((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma'}(t_1)\}\} = rmax\{((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma}(t_1), ((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma'}(t_1)\} = rmax\{(\widehat{N}_K)^F_{e_i}(t_1) + \gamma, (\widehat{N}_K)^F_{e_i}(t_1) + \gamma'\} = ((\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma} \cup (\widehat{N}_K)^F_{e_i})^{Tr}_{\gamma'})(t_1)$, $(((A_K)^T_{e_i})^{Tr}_{\alpha} \cup ((A_K)^T_{e_i})^{Tr}_{\alpha'})(0) = (((A_K)^T_{e_i})^{Tr}_{\alpha} \cup ((A_K)^T_{e_i})^{Tr}_{\alpha'})(t_1 * t_1) = min\{((A_K)^T_{e_i})^{Tr}_{\alpha}(t_1 * t_1), ((A_K)^T_{e_i})^{Tr}_{\alpha'}(t_1 * t_1)\} \leq min\{max\{((A_K)^T_{e_i})^{Tr}_{\alpha}(t_1), ((A_K)^T_{e_i})^{Tr}_{\alpha}(t_1)\} max\{((A_K)^T_{e_i})^{Tr}_{\alpha'}(t_1), ((A_K)^T_{e_i})^{Tr}_{\alpha'}(t_1)\}\} = min\{((A_K)^T_{e_i})^{Tr}_{\alpha}(t_1), ((A_K)^T_{e_i})^{Tr}_{\alpha'}(t_1)\} = min\{(A_K)^T_{e_i}(t_1) + \alpha, (A_K)^T_{e_i}(t_1) + \alpha'\} = ((A_K)^T_{e_i})^{Tr}_{\alpha} \cup (A_K)^T_{e_i})^{Tr}_{\alpha'})(t_1)$. $(((A_K)^I_{e_i})^{Tr}_{\beta} \cup ((A_K)^I_{e_i})^{Tr}_{\beta'})(0) = (((A_K)^I_{e_i})^{Tr}_{\beta} \cup ((A_K)^I_{e_i})^{Tr}_{\beta'})(t_1 * t_1) = min\{((A_K)^I_{e_i})^{Tr}_{\beta}(t_1 * t_1), ((A_K)^I_{e_i})^{Tr}_{\beta'}(t_1 * t_1)\} \leq min\{max\{((A_K)^I_{e_i})^{Tr}_{\beta}(t_1), ((A_K)^I_{e_i})^{Tr}_{\beta}(t_1)\}, max\{((A_K)^I_{e_i})^{Tr}_{\beta'}(t_1), ((A_K)^I_{e_i})^{Tr}_{\beta'}(t_1)\}\} = min\{((A_K)^I_{e_i})^{Tr}_{\beta}(t_1), ((A_K)^I_{e_i})^{Tr}_{\beta'}(t_1)\} = min\{(A_K)^I_{e_i}(t_1) + \beta, (A_K)^I_{e_i}(t_1) + \beta'\} = ((A_K)^{I}_{e_i})^{Tr}_{\beta} \cup (A_K)^{I}_{e_i})^{Tr}_{\beta'})(t_1)$ $(((A_K)^F_{e_i})^{Tr}_{\gamma} \cup ((A_K)^F_{e_i})^{Tr}_{\gamma'})(0) = (((A_K)^F_{e_i})^{Tr}_{\gamma} \cup ((A_K)^F_{e_i})^{Tr}_{\gamma'})(t_1 * t_1) = min\{((A_K)^F_{e_i})^{Tr}_{\gamma}(t_1 * t_1), ((A_K)^F_{e_i})^{Tr}_{\gamma'}(t_1 * t_1)\} \leq min\{max\{((A_K)^F_{e_i})^{Tr}_{\gamma}(t_1), ((A_K)^F_{e_i})^{Tr}_{\gamma}(t_1)\} max\{((A_K)^F_{e_i})^{Tr}_{\gamma'}(t_1), ((A_K)^F_{e_i})^{Tr}_{\gamma'}(t_1)\}\} = min\{((A_K)^F_{e_i})^{Tr}_{\gamma}(t_1), ((A_K)^F_{e_i})^{Tr}_{\gamma'}(t_1)\} = min\{(A_K)^F_{e_i}(t_1) + \gamma, (A_K)^F_{e_i}(t_1) + \gamma'\} = ((A_K)^F_{e_i})^{Tr}_{\gamma} \cup (A_K)^F_{e_i})^{Tr}_{\gamma'})(t_1)$.

Now

$(((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma} \cup ((\widehat{N}_K)^{T,IF}_{e_i})^{Tr}_{\alpha',\beta',\gamma'})(t_1 * t_3) = rmax\{((\widehat{N}_K)^{TI,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}(t_1 * t_3), ((\widehat{N}_K)^{T,I}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}(t_1 * t_3)\} \geq rmax\{rmin\{((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}((t_1 * t_2) * t_3), ((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}(t_2), rmin\{((\widehat{N}_K)^{TI,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}((t_1 * t_2) * t_3), ((\widehat{N}_K)^{T,IF}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}(t_2)\}, = rmin\{rmax\{((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}((t_1 * t_2) * t_3), ((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}((t_1 * t_2) * t_3)\}, rmax\{((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}(t_2), ((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}(t_2)\}\} = rmin\{(((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma} \cup ((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'})((t_1 * t_2) * t_3), (((\widehat{N}_K)^{T,IF}_{e_i})^{Tr}_{\alpha,\beta,\gamma} \cup ((\widehat{N}_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'})(t_2)\}$.

And

$((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma} \cup ((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'})(t_1 * t_3) = min\{((A_K)^{T,I}_{e_i})^{Tr}_{\alpha,\beta,\gamma}(t_1 * t_3), ((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}(t_1 * t_3)\} \leq min\{max\{((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}((t_1 * t_2) * t_3), ((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}(t_2)\}, max\{((A_K)^{T,IF}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}((t_1 * t_2) * t_3), ((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}(t_2)\}\} = max\{min\{((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}((t_1 * t_2) * t_3), ((A_K)^{T,IF}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}((t_1 * t_2) * t_3)\}, min\{((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma}(t_2), ((A_K)^{TI,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'}(t_2)\}\} = max\{(((A_K)^{T,IF}_{e_i})^{Tr}_{\alpha,\beta,\gamma} \cup ((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'})((t_1 * t_2) * t_3), (((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha,\beta,\gamma} \cup ((A_K)^{T,I,F}_{e_i})^{Tr}_{\alpha',\beta',\gamma'})(t_2)\}$.

Hence $\widetilde{K}^{Tr}_{\alpha,\beta,\gamma} \cup \widetilde{K}^{Tr}_{\alpha',\beta',\gamma'}$ is an NSCTID of P. □

Theorem 5.6. *The intersection of any two NSC-translations of an NSCTID is an NSCTID of P.*

Proof. Suppose $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ and $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}$ are two NSC-translations of NSCTID of P respectively. Where in $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ for $(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, \alpha, \beta \in [[0,0],\xi]$ and $\gamma \in [[0,0],\Psi]$, where for $(\mathrm{A_K})^{\mathrm{T,I,F,F}}_{\mathrm{e_i}}, \alpha, \beta \in [0,\zeta]$ and $\gamma \in [0,\Phi]$, and in $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}$, for $(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,F}}_{\mathrm{e_i}}, \alpha', \beta' \in [[0,0],\xi]$ and $\gamma' \in [[0,0],\Psi]$, where for $(\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, \alpha', \beta' \in [0,\zeta]$ and $\gamma' \in [0,\Phi]$. and $\alpha \leq \alpha', \beta \leq \beta', \gamma \leq \gamma'$ as we know that, $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha,\beta,\gamma}$ and $\widetilde{\mathrm{K}}^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}$ are NSCTID of P.Then
$(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'})(0) = (((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'})(\mathrm{t_1} * \mathrm{t_1}) = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1} * \mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1} * \mathrm{t_1})\} \geq \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1})\}, \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1})\}\} = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1})\} = \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_1}) + \alpha, (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_1}) + \alpha'\} = ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha} \cap (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'})(\mathrm{t_1})$, $(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'})(0) = (((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'})(\mathrm{t_1} * \mathrm{t_1}) = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1} * \mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1} * \mathrm{t_1})\} \geq \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1})\}, \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1})\}\} = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1})\} = \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_1}) + \beta, (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_1}) + \beta'\} = ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta} \cap (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'})(\mathrm{t_1})$, $(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'})(0) = (((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'})(\mathrm{t_1} * \mathrm{t_1}) = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1} * \mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1} * \mathrm{t_1})\} \geq \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1})\}, \mathrm{rmin}\{((\widehat{N}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1})\}\} = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1}) = \mathrm{rmin}\{(\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_1}) + \gamma, (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_1}) + \gamma'\} = ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma} \cap (\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'})(\mathrm{t_1})$,
$(((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha} \cap ((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'})(0) = (((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha} \cap ((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'})(\mathrm{t_1} * \mathrm{t_1}) = \max\{((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1} * \mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1} * \mathrm{t_1})\} \leq \max\{\max\{((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1})\}, \max\{((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1})\}\} = \max\{((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'}(\mathrm{t_1})\} = \max\{(\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_1}) + \alpha, (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_1}) + \alpha'\} = ((\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha} \cap (\mathrm{A_K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha'})(\mathrm{t_1})$. $(((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta} \cap ((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'})(0) = (((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta} \cap ((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'})(\mathrm{t_1} * \mathrm{t_1}) = \max\{((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1} * \mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1} * \mathrm{t_1})\} \leq \max\{\max\{((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1})\}, \max\{((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1})\}\} = \max\{((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'}(\mathrm{t_1})\} = \max\{(\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_1}) + \beta, (\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_1}) + \beta'\} = ((\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta} \cap (\mathrm{A_K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\beta'})(\mathrm{t_1})$ $(((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma} \cap ((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'})(0) = (((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma} \cap ((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'})(\mathrm{t_1} * \mathrm{t_1}) = \max\{((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1} * \mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1} * \mathrm{t_1})\} \leq \max\{\max\{((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1})\}, \max\{((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1})\}\} = \max\{((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'}(\mathrm{t_1})\} = \max\{(\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_1}) + \gamma, (\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_1}) + \gamma'\} = ((\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma} \cap (\mathrm{A_K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\gamma'})(\mathrm{t_1})$.
Now
$(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'})(\mathrm{t_1} * \mathrm{t_3}) = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}(\mathrm{t_1} * \mathrm{t_3}) \geq ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}(\mathrm{t_1} * \mathrm{t_3})\} \geq \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}(\mathrm{t_2})\}, \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}(\mathrm{t_2})\}\} = \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3})\}, \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}(\mathrm{t_2}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}(\mathrm{t_2})\}\} = \{rmin\{(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'})((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), (((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{Tr,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'})(\mathrm{t_2})\}$.
And
$(((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma} \cap ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'})(\mathrm{t_1} * \mathrm{t_3}) = \max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}(\mathrm{t_1} * \mathrm{t_3}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}(\mathrm{t_1} * \mathrm{t_3})\} \leq \max\{\max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha,\beta,\gamma}(\mathrm{t_2})\} \max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Tr}}_{\alpha',\beta',\gamma'}$

$((t_1*t_2)*t_3),((A_K)_{e_i}^{T,I,F})_{\alpha',\beta',\gamma'}^{Tr}(t_2)\}\}=\max\{\max\{((A_K)_{e_i}^{T,I,F})_{\alpha,\beta,\gamma}^{Tr}((t_1*t_2)*t_3),((A_K)_{e_i}^{T,I,F})$ $_{\alpha',\beta',\gamma'}^{Tr}((t_1*t_2)\ t_3)\},\max\{((A_K)_{e_i}^{T,I,F})_{\alpha,\beta,\gamma}^{Tr}(t_2),((A_K)_{e_i}^{T,I,F})_{\alpha',\beta',\gamma'}^{Tr}(t_2)\ \{\max\{(((A_K)_{e_i}^{T,I,F})_{\alpha,\beta,\gamma}^{Tr}\cap$ $((A_K)_{e_i}^{T,F})_{\alpha',\beta',\gamma'}^{Tr})((t_1*t_2)*t_3),\ (((A_K)_{e_i}^{T,I,F})_{\alpha,\beta,\gamma}^{Tr}\cap((A_K)_{e_i}^{T,I,F})_{\alpha',\beta',\gamma'}^{Tr})(t_2)\}.$
Hence $\widetilde{K}_{\alpha,\beta,\gamma}^{Tr}\cap\widetilde{K}_{\alpha',\beta',\gamma'}^{Tr}$ is an NSCTID of P. □

5.2. *Neutrosophic Soft Cubic T-Ideal Multiplication*

This section defines neutrosophic soft cubic T-Ideal multiplication with a theorem and example.

Theorem 5.7. *If* K *is an NSCTID of* P*, then NSCMp* $\widetilde{K}_\eta^{Mp}$ *of* K *is an NSCTID of* P*, for all* $\eta\epsilon\ [0,1]$.

Proof. Let K be an NSCTID of P and $\eta\in[0,1]$. Then we have $((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(0)=\eta\cdot(\widehat{N}_K)_{e_i}^T(0)\geq\eta\cdot(\widehat{N}_K)_{e_i}^T(t_1)\rightarrow((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(0)\geq((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(t_1)$, $((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(0)=\eta\cdot(\widehat{N}_K)_{e_i}^I(0)\geq\eta\cdot(\widehat{N}_K)_{e_i}^I(t_1)\rightarrow((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(0)\geq((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(t_1)$, $((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(0)=\eta\cdot(\widehat{N}_K)_{e_i}^F(0)\geq\eta\cdot(\widehat{N}_K)_{e_i}^F(t_1)\rightarrow((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(0)\geq((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(t_1)$,
And $((A_K)_{e_i}^T)_\eta^{Mp}(0)=\eta\cdot(A_K)_{e_i}^T(0)\leq\eta\cdot(A_K)_{e_i}^T(t_1)\rightarrow((A_K)_{e_i}^T)_\eta^{Mp}(0)\leq((A_K)_{e_i}^T)_\eta^{Mp}(t_1)$, $((A_K)_{e_i}^I)_\eta^{Mp}(0)=\eta\cdot(A_K)_{e_i}^I(0)\leq\eta\cdot(A_K)_{e_i}^I(t_1)\rightarrow((A_K)_{e_i}^I)_\eta^{Mp}(0)\leq((A_K)_{e_i}^I)_\eta^{Mp}(t_1)$, $((A_K)_{e_i}^F)_\eta^{Mp}(0)=\eta\cdot(A_K)_{e_i}^F(0)\leq\eta\cdot(A_K)_{e_i}^F(t_1)\rightarrow((A_K)_{e_i}^F)_\eta^{Mp}(0)\leq((A_K)_{e_i}^F)_\eta^{Mp}(t_1)$, Now $((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(t_1*t_3)=\eta\cdot(\widehat{N}_K)_{e_i}^T(t_1*t_3)\geq\eta\cdot\text{rmin}\{(\widehat{N}_K)_{e_i}^T((t_1*t_2)*t_3),(\widehat{N}_K)_{e_i}^T(t_2)\}=\text{rmin}\{\eta\cdot(\widehat{N}_K)_{e_i}^T((t_1*t_2)*t_3),\eta\cdot(\widehat{N}_K)_{e_i}^T(t_2)\}\ ((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(t_1*t_3)=\text{rmin}\{((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}((t_1*t_2)*t_3),((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(t_2)\}\ ((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(t_1*t_3)\geq\text{rmin}\{((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}((t_1*t_2)*t_3),((\widehat{N}_K)_{e_i}^T)_\eta^{Mp}(t_2)\}$, $((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(t_1*t_3)=\eta\cdot(\widehat{N}_K)_{e_i}^I(t_1*t_3)\geq\eta\cdot\text{rmin}\{(\widehat{N}_K)_{e_i}^I((t_1*t_2)*t_3),(\widehat{N}_K)_{e_i}^I(t_2)\}=\text{rmin}\{\eta\cdot(\widehat{N}_K)_{e_i}^I((t_1*t_2)*t_3),\eta\cdot(\widehat{N}_K)_{e_i}^I(t_2)\}\ ((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(t_1*t_3)=\text{rmin}\{((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}((t_1*t_2)*t_3),((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(t_2)\}\ ((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(t_1*t_3)\geq\text{rmin}\{((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}((t_1*t_2)*t_3),((\widehat{N}_K)_{e_i}^I)_\eta^{Mp}(t_2)\}$, $((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(t_1*t_3)=\eta\cdot(\widehat{N}_K)_{e_i}^F(t_1*t_3)\geq\eta\cdot\text{rmin}\{(\widehat{N}_K)_{e_i}^F((t_1*t_2)*t_3),(\widehat{N}_K)_{e_i}^F(t_2)\}=\text{rmin}\{\eta\cdot(\widehat{N}_K)_{e_i}^F((t_1*t_2)*t_3),\eta\cdot(\widehat{N}_K)_{e_i}^F(t_2)\}\ ((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(t_1*t_3)=\text{rmin}\{((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}((t_1*t_2)*t_3),((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(t_2)\}\ ((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(t_1*t_3)\geq\text{rmin}\{((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}((t_1*t_2)*t_3),((\widehat{N}_K)_{e_i}^F)_\eta^{Mp}(t_2)\}$
And $((A_K)_{e_i}^T)_\eta^{Mp}(t_1*t_3)=\eta\cdot(A_K)_{e_i}^T(t_1*t_3)\leq\eta\cdot\max\{(A_K)_{e_i}^T((t_1*t_2)*t_3),(A_K)_{e_i}^T(t_2)\}=\max\{\eta\cdot(A_K)_{e_i}^T((t_1*t_2)*t_3),\eta\cdot(A_K)_{e_i}^T(t_2)\}\ ((A_K)_{e_i}^T)_\eta^{Mp}(t_1*t_3)=\max\{((A_K)_{e_i}^T)_\eta^{Mp}((t_1*t_2)*t_3),((A_K)_{e_i}^T)_\eta^{Mp}(t_2)\}\ ((A_K)_{e_i}^T)_\eta^{Mp}(t_1*t_3)\leq\max\{((A_K)_{e_i}^T)_\eta^{Mp}((t_1*t_2)*t_3),((A_K)_{e_i}^T)_\eta^{Mp}(t_2)\}$, $((A_K)_{e_i}^I)_\eta^{Mp}(t_1*t_3)=\eta\cdot(A_K)_{e_i}^I(t_1*t_3)\leq\eta\cdot\max\{(A_K)_{e_i}^I((t_1*t_2)*t_3),(A_K)_{e_i}^I(t_2)\}=\max\{\eta\cdot(A_K)_{e_i}^I((t_1*t_2)*t_3),\eta\cdot(A_K)_{e_i}^I(t_2)\}\ ((A_K)_{e_i}^I)_\eta^{Mp}(t_1*t_3)=\max\{((A_K)_{e_i}^I)_\eta^{Mp}((t_1*t_2)*t_3),((A_K)_{e_i}^I)_\eta^{Mp}(t_2)\}\ ((A_K)_{e_i}^I)_\eta^{Mp}(t_1*t_3)\leq\max\{((A_K)_{e_i}^I)_\eta^{Mp}((t_1*t_2)*t_3),((A_K)_{e_i}^I)_\eta^{Mp}(t_2)\}c$, $((A_K)_{e_i}^F)_\eta^{Mp}(t_1*t_3)=\eta\cdot(A_K)_{e_i}^F(t_1*t_3)\leq\eta\cdot\max\{(A_K)_{e_i}^F((t_1*t_2)*t_3),(A_K)_{e_i}^F(t_2)\}=\max\{\eta\cdot(A_K)_{e_i}^F((t_1*t_2)*t_3),\eta\cdot(A_K)_{e_i}^F(t_2)\}\ ((A_K)_{e_i}^F)_\eta^{Mp}(t_1*t_3)=\max\{((A_K)_{e_i}^F)_\eta^{Mp}((t_1*t_2)*$

$\mathrm{t}_3), ((\mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_2)\}$ $((\mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_1 * \mathrm{t}_3) \leq \max\{((\mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}((\mathrm{t}_1 * \mathrm{t}_2) * \mathrm{t}_3), ((\mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t}_2)\}$.
Hence $\widetilde{\mathrm{K}}^{\mathrm{Mp}}_{\eta}$ of K is an NSCTID of P, for all $\eta \in [0,1]$. □

Example 5.8. Let $\mathrm{P} = \{0, \mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3\}$ be a PS-algebra with the cayley's table as shown in Table 1 . The NSC-set $\mathrm{K} = \langle (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, (\mathrm{A}_\mathrm{K})^{\mathrm{T,I,F}}_{\mathrm{e_i}} \rangle$ of P is defined as

$$(\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{[0.7, 0.9] \text{ if } \mathrm{t_i} = 0 \text{ and } [0.3, 0.6]. \text{ if otherwise}$$

$$(\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{[0.6, 0.8] \text{ if } \mathrm{t_i} = 0 \text{ and } [0.4, 0.5]. \text{ if otherwise}$$

$$(\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{[0.5, 1] \text{ if } \mathrm{t_i} = 0 \text{ and } [0.2, 0.7]. \text{ if otherwise.}$$

And

$$(\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{0.2 \text{ if } \mathrm{t_i} = 0 \text{ and } 0.7. \text{ if otherwise}$$

$$(\mathrm{A}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{0.5 \text{ if } \mathrm{t_i} = 0 \text{ and } 0.9. \text{ if otherwise}$$

$$(\mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i}) = \{0.4 \text{ if } \mathrm{t_i} = 0 \text{ and } 1. \text{ if otherwise.}$$

The set K is an NSCTID of P as it satisfies (i) but fulfills (ii) and (iii) with the condition: If $\mathrm{i} = \mathrm{j}$ then $\mathrm{t_i} * \mathrm{t_j} = 0$ with $(\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.7, 0.9], (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.6, 0.8], (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.5, 1]$
Otherwise

$$(\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.3, 0.6], (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.4, 0.5], (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i} * \mathrm{t_j}) = [0.2, 0.7],$$

And

$$(\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = 0.2, (\ \mathrm{A}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i}) = 0.5, (\ \mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i}) = 0.4$$

Otherwise $(\mathrm{A}_\mathrm{K})^{T}_{\mathrm{e_i}}(\mathrm{t}_i) = 0.7, (\mathrm{A}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t}_i) = 0.9, (\mathrm{A}_\mathrm{K})^{F}_{\mathrm{e_i}}(\mathrm{t}_i) = 1$ given in definition 3.1. Now, for $(\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T,I,F}}_{\mathrm{e_i}} \eta \in [0.2, 0.5]$ and for $(\mathrm{A})^{\mathrm{T,I,F}}_{\mathrm{e_i}}, \eta = 0.2$. Then the mapping $\widetilde{\mathrm{K}}^{\mathrm{Mp}}_{\eta} \mid \mathrm{P}arrow[0,1]$ is given by

$$((\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{[0.2,0.5]}(0) = [0.2, 0.5] \cdot (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(0) = [0.14, 0.45]$$

$$((\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{[0.2,0.5]}(0) = [0.2, 0.5] \cdot (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(0) = [0.12, 0.4]$$

$$((\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{[0.2,0.5]}(0) = [0.2, 0.5] \cdot (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}}(0) = [0.1, 0.5]$$

$$((\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{[0.2,0.5]}(\mathrm{t_i}) = [0.2, 0.5] \cdot (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = [0.06, 0.3]$$

$$((\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{[0.2,0.5]}(\mathrm{t_i}) = [0.2, 0.5] \cdot (\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{t_i}) = [0.08, 0.25]$$

$$((\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{[0.2,0.5]}(\mathrm{t_i}) = [0.2, 0.5].(\widehat{\mathrm{N}}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{t_i}) = [0.04, 0.35],$$

And

$$((\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{0.03}(0) = 0.2.(\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(0) = [0.04]$$

$$((\mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{0.2}(0) = 0.2.(\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(0) = [0.08]$$

$$((\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{0.2}(\mathrm{t_i}) = 0.2.(\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = [0.14]$$

$$((\mathrm{A}_\mathrm{K})^{\mathrm{I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{0.2}(\mathrm{t_i}) = 0.2.(\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = [0.18]$$

$$((\mathrm{A}_\mathrm{K})^{\mathrm{F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{0.2}(\mathrm{t_i}) = 0.2.(\mathrm{A}_\mathrm{K})^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{t_i}) = [0.2]$$

Hence $\widetilde{K}_{\eta}^{Mp}$ is an NSCTID of P.

Theorem 5.9. *The union of any two NSC-multiplications of an NSCTID is an NSCTID of* P*.*

Proof. Suppose $\widetilde{K}_{\eta}^{Mp}$ and $\widetilde{K}_{\eta'}^{Mp}$ are two NSC-multiplications of an NSCTID is an NSCTID of P, where $\eta, \eta' \in (0,1]$ and $\eta \le \eta'$. As we know that $\widetilde{K}_{\eta}^{Mp}$ and $\widetilde{K}_{\eta'}^{Mp}$ are NSCTIDs of P. Then $(((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(0) = (((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1 * t_1) = \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1 * t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1 * t_1)\} \ge \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1)\}, \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1)\}\} = \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1)\} = \mathrm{rmin}\{\eta \cdot (\widehat{N}_K)_{e_i}^{T,I,F}(t_1), \eta' \cdot (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)\} = ((\widehat{N}_K)_{e_i}^{TI,F})_{\eta}^{Mp} \cup (\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1)$.

And

$(((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(0) = (((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1 * t_1) = \max\{((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1 * t_1), ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1 * t_1)\} \le \max\{\max\{((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1), ((A_K)_{e_i}^{TI,F})_{\eta}^{Mp}(t_1)\}, \max\{((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1), ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1)\}\} = \max\{((A_K)_{e_i}^{T,IF})_{\eta}^{Mp}(t_1), ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1)\} = \max\{\eta \cdot (A_K)_{e_i}^{T,I,F}(t_1), \eta' \cdot (A_K)_{e_i}^{T,I,F}(t_1)\} = ((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup (A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1)$. $(((\widehat{N}_K)_{e_i}^{T,IF})_{\eta}^{Mp} \cup ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1 * t_3) = \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1 * t_3), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1 * t_3)\} \ge \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}((t_1 * t_2) * t_3), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_2)\}, \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}((t_1 * t_2) * t_3), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_2)\}\} = \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}((t_1 * t_2) * t_3), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}((t_1 * t_2) * t_3)\}, \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_2), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_2)\}\} = \{\mathrm{rmin}\{(((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})((t_1 * t_2) * t_3), (((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_2)\}$.

And

$(((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1 * t_3) = \max\{((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1 * t_3), ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1 * t_3)\} \le \max\{\max\{((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp}((t_1 * t_2) * t_3), ((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_2)\}. \max\{((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}((t_1 * t_2) * t_3), ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_2)\}\} = \max\{\max\{((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp}((t_1 * t_2) * t_3), ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}((t_1 * t_2) * t_3)\}, \max\{((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_2), ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_2)\}\} = \{\max\{(((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp} U ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})((t_1 t_2) * t_3), (((A_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cup ((A_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_2)\}$

Hence $\widetilde{K}_{\eta}^{Mp} \cup \widetilde{K}_{\eta'}^{Mp}$ is an NSCTID of P. □

Theorem 5.10. *The intersection of any two NSC-multiplications of an NSCTID is an NSCTID of* P*.*

Proof. Suppose $\widetilde{K}_{\eta}^{Mp}$ and $\widetilde{K}_{\eta'}^{Mp}$ are two NSC-multiplications of an NSCTID is an NSCTID of P, where $\eta, \eta' \in (0,1]$ and $\eta \le \eta'$. As we know that $\widetilde{K}_{\eta}^{Mp}$ and $\widetilde{K}_{\eta'}^{Mp}$ are NSCTIDs of P. Then $(((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cap ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(0) = (((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cap ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1 * t_1) = \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1 * t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1 * t_1)\} \ge \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1)\}, \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1)\}\} = \mathrm{rmin}\{((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp}(t_1), ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp}(t_1)\} = \mathrm{rmin}\{\eta \cdot (\widehat{N}_K)_{e_i}^{T,I,F}(t_1), \eta' \cdot (\widehat{N}_K)_{e_i}^{T,I,F}(t_1)\} = ((\widehat{N}_K)_{e_i}^{T,I,F})_{\eta}^{Mp} \cap (\widehat{N}_K)_{e_i}^{T,I,F})_{\eta'}^{Mp})(t_1)$.

And

$(((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e}_i})^{\mathrm{Mp}}_{\eta} \cap ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})(0) = (((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})(\mathrm{t_1} * \mathrm{t_1}) = \max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_1} * \mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_1} * \mathrm{t_1})\} \le \max\{\max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_1})\}, \max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_1})\}\} = \max\{((\mathrm{A_K})^{\mathrm{T,I}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_1}), ((\mathrm{A_K})^{\mathrm{T,IF}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_1})\} = \max\{\eta \cdot (\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(\mathrm{t_1}), \eta' \cdot (\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}}(\mathrm{t_1})\} = ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap (\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})(\mathrm{t_1})$. $(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})(\mathrm{t_1} * \mathrm{t_3}) = \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_1} * \mathrm{t_3}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_1} * \mathrm{t_3})\} \ge \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_2})\}, \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_2})\}\} = \mathrm{rmin}\{\mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3})\}, \mathrm{rmin}\{((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_2}), ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_2})\}\} = \{\mathrm{rmin}\{(((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3})\ (((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap ((\widehat{\mathrm{N}}_{\mathrm{K}})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})(\mathrm{t_2})\}$.

And

$(((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})(\mathrm{t_1} * \mathrm{t_3}) = \max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_1} * \mathrm{t_3}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_1} * \mathrm{t_3})\} \le \max\{\max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_2})\}, \max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_2})\}\} = \max\{\max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3})\}, \max\{((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta}(\mathrm{t_2}), ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'}(\mathrm{t_2})\}\} = \{\max\{(((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})((\mathrm{t_1} * \mathrm{t_2}) * \mathrm{t_3}), (((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta} \cap ((\mathrm{A_K})^{\mathrm{T,I,F}}_{\mathrm{e_i}})^{\mathrm{Mp}}_{\eta'})(\mathrm{t_2})\}$.

Hence $\widetilde{\mathrm{K}}^{\mathrm{Mp}}_{\eta} \cap \widetilde{\mathrm{K}}^{\mathrm{Mp}}_{\eta'}$ is an NSCTID of P. □

6. **Conclusion**

This paper extensively explores the application of the neutrosophic soft cubic set to investigate specific properties of the t-ideal in a PS-algebra. The derived definitions and fundamental outcomes hold potential for broader use in other algebraic structures like Lie algebras and lattices in the future. Moreover, there are several areas where this proposed structure can be advantageous, such as DNA identification, formalizing procedures in genetic algorithms, genomics, fuzzy logic-based networks, and particularly complex networks. Within the already-existing neutrosophic cubic structures, the addition of soft sets with T-ideal properties makes this structure a better choice for future implementations. This can also be extended to the hypersoft sets and can be advantegeous for MCDM algorithms in various real life applications such as transportation, healthcare etc.

One potential limitation of our proposed structure NSCTID is the ground algebra because the results and properties in this study may or may not be satisfy for other algebras in future. Also the practical implementation and computational efficiency of the NSCTID framework in real-world applications may pose challenges. The translation of theoretical concepts into efficient algorithms and the handling of computational complexities could require further investigation and optimization to fully realize the benefits of NSCTID in practical scenarios.

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