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Approximations of interval neutrosophic hyperideals in semi-hyper-rings

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Abstract. This paper deals with the combination of rough sets and interval neutrosophic sets. We introduce the interval neutrosophic hyper-ideals in semi-hyper-rings. Also we study the rough interval neutrosophic hyper-ideals in semi-hyper-rings.

Keywords: Rough sets, neutrosophic sets, interval neutrosophic sets , rough interval neutrosophic sets, semi-hyper-rings.

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1. Introduction

In 1982, Pawlak [11] introduced the concept of rough set, as a formal tool for modeling and processing incomplete information in information systems. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. The concept of a fuzzy set, introduced by Zadeh [24] , provides a natural framework for generalizing some of the notions of classical algebraic structures. As a generalization of fuzzy sets, the intuitionistic fuzzy set was introduced by Atanassov [1] in 1986. One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [12]. The term neutrosophy means knowledge of neutral thought and this neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. It is a logic in which each proposition is estimated to have a degree of truth, a degree of indeterminacy and a degree of falsity. Unlike in intuitionistic fuzzy sets, where the incorporated uncertainty is dependent of the degree of belongingness and degree of non-belongingness, here the uncertainty present, i.e. the indeterminacy factor, is independent of truth and falsity values. Neutrosophic sets are indeed more general than Intuitionistic fuzzy set as there are no constraints between the degree of truth, degree of inde-terminacy and degree of falsity. All these degrees can individually vary within [0, 1]. The theories of neutrosophic set have achieved great success in various areas. Recently many researchers applied the notion of fuzzy neutrosophic sets to several algebraic structures. Subha et al. [17–23] studied the algebraic structures of interval rough fuzzy sets. In this paper we studied the algebraic properties of rough interval neutrosophic sets.

2. Preliminaries

This section we present some basic definitions related to this work.

Definition 2.1. [2] Let W be a nonempty set, and let $P(W)$ be the set of all nonempty subsets of W. A hyperoperation on W is a map $\circ : W \times W \leftarrow P(W)$, and the couple $(W, \circ)$ is called a hypergrupoid. If A and B are nonempty subsets of W, then we denote,

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b \; x \circ A = \{x\} \circ A,$$
$$A \circ x = A \circ \{x\}$$

Definition 2.2. [2] A hypergrupoid $(W, \circ)$ is called a hyper-semi-group if for all $x, y$and z of W we have $(x \circ y) \circ z = x \circ (y \circ z)$.
That is, $\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$

Definition 2.3. [2] A is an algebraic structure $(W, +, .)$ which satisfies the following conditions.
(i) $(W, +)$ is a commutative semi-hyper-group,
(a) $(a + b) + c = a + (y + z)$ (b) $a + b = b + a$, for all $a, b, c \in W$.
(ii) $(H, .)$ is a semi-hyper-group,
(c) $(a.b).c = a.(b.c)$, for all $a, b, c \in W$.
(iii) The multiplication is distributive with respect to hyperroperation $+$,
(d) $a.(b + c) = a.b + a.c$
(e) $(a + b).c = a.c + b.c$, for all $a, b, c \in W$.

Definition 2.4. [2] A nonempty subset A of a hyper-semi-ring $(W, +, .)$ is called sub-hyper-semi-ring if $x + y \subseteq A$ and $x.y \subseteq A$ for all $x, y \in A$.

Definition 2.5. [2] A left(right) hyper-ideal of a hyper-semi-ring W is a nonempty subset I of W satisfying the following:
(i) $x + y \subseteq I$, for all $x, y \in I$.
(ii) $x.a \subseteq I(a.x \subseteq I)$, for all $a \in I$ and $x \in W$.

Definition 2.6. Let R be a commutative semihypergroup and Γ be a commutative group. Then R is called a Γ -semihyperring if there exists a map $R\Gamma R \rightarrow P(R)(a, \alpha, b) \rightarrow a\alpha b)$

$\forall a, b \in R, \alpha \in \Gamma$ and $P(R)$ the set of all non-empty subsets of R, satisfying the following conditions: (i) $(a+b)\alpha c = a\alpha c + b\alpha c$,

(ii)$a\alpha(b+c) = a\alpha b + a\alpha c$,

(iii) $a(\alpha+\beta)b = a\alpha b + a\beta b$,

(iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$

$\forall a, b, c \in R$ and $\forall\ \alpha, \beta \in \Gamma$

We say that R is a Γ-semihyperring with zero, if there exists $0 \in R$ such that $a \in a + 0$ and $0 \in 0\alpha a, 0 \in a\alpha 0$ for all $a \in R$ and $\alpha \in \Gamma$

Definition 2.7. [10] Let W be the universe. The neutrosophic set is an object having the form $A = \{(e, l_t(e), l_i(e), l_f(e)), e \in W\}$

where the functions $l_t, l_i, l_f : W \longrightarrow [0,1]$ define respectively the truth, the degree of indeterminacy and the degree of non-membership of the element $e \in W$ to the set A with the condition $0 \leq l_t + l_i + l_q \leq 3$

3. Interval neutrosophic hyper-ideals(INHI) in semi-hyper-rings

In this section we studied the concept of $INLHI$ in semi-hyper-ring W. Also we proved nonempty intersection of $INLHI$ is also an $INLHI$. More over we discuss the pre image and image of an $INLHI$ of W is also an $INLHI$. At last we proved the cartesian product of two $INLHI$ is also an $INLHI$.

Definition 3.1. A nonempty IN subset l of W is said to be an $INLHI$ of W if the following conditions are holds:

(C1) $\bigwedge_{e \in s+q} l_t(e) \geq l_t(s) \wedge l_t(q)$

(C2) $\bigwedge_{e \in s+q} l_i(e) \geq \frac{l_i(s)+l_i(q)}{2}$

(C3) $\bigvee_{e \in s+q} l_f(e) \leq l_t(s) \vee l_t(q)$

(C4) $\bigwedge_{e \in sq} l_t(e) \geq l_t(q)$

(C5) $\bigwedge_{e \in sq} l_i(e) \geq l_i(q)$

(C6) $\bigvee_{e \in sq} l_f(e) \leq l_f(q)$ for all $e, s, q \in W$

Definition 3.2. A nonempty IN subset l of W is said to be an $INRHI$ of W if the conditions (C1) (C2) and (C3) holds. Moreover

(C7) $\bigwedge_{e \in sq} l_t(e) \geq l_t(s)$

(C8) $\bigwedge_{e \in sq} l_i(e) \geq l_i(s)$

(C9) $\bigvee_{e \in sq} l_f(e) \leq l_f(s)$for all $e, s, q \in W$

Definition 3.3. Let l and m be any two IN subsets of W. Then $l \cap m$ defined by
$l_t \cap m_t(e) = l_t \wedge m_t$, $l_i \cap m_i(e) = l_i \wedge m_i$ and $l_f \cap m_f(e) = l_f \vee m_f$ for all $e \in W$

Proposition 3.4. *A nonempty intersection of an $INLHI$ is an $INLHI$.*

Proof : Assume that $\{l^k : k \in I\}$ be a family of an $INLHI$ of W. Let $r, s \in W$. Then

$$\bigwedge_{e\in r+s} (\bigcap_{k\in I} l_t^k)(e) = \bigwedge_{e\in r+s} \inf_{k\in I} l_t^k(e) \geq \inf_{k\in I} \left(l_t^k(r) \wedge l_t^k(s)\right) = \inf_{k\in I} l_t^k(r) \wedge \inf_{k\in I} l_t^k(s)$$
$$= \bigcap_{k\in I} l_t^k(r) \wedge \bigcap_{k\in I} l_t^k(s)$$

and

$$\bigwedge_{e\in r+s} (\bigcap_{k\in I} l_i^k)(e) = \bigwedge_{e\in r+s} \inf_{k\in I} l_i^k(e) \geq \inf_{k\in I} \left[\frac{l_i^k(r)+l_i^k(s)}{2}\right] = \frac{\inf_{k\in I} l_i^k(r)+\inf_{k\in I} l_i^k(s)}{2} = \frac{\bigcap_{k\in I} l_i^k(r)+\bigcap_{k\in I} l_i^k(s)}{2}$$

also

$$\bigvee_{e\in r+s} (\bigcap_{k\in I} l_f^k)(e) = \bigvee_{e\in r+s} \sup_{k\in I} l_f^k(e) \leq \sup_{k\in I} \left(l_f^k(r) \vee l_f^k(s)\right) = \sup_{k\in I} l_f^k(r) \vee \sup_{k\in I} l_f^k(s)$$
$$= \bigcap_{k\in I} l_f^k(r) \vee \bigcap_{k\in I} l_f^k(s)$$

Moreover

$$\bigwedge_{e\in rs} (\bigcap_{k\in I} l_t^k)(e) = \bigwedge_{e\in r+s} \inf_{k\in I} l_t^k(e) \geq \inf_{k\in I} l_t^k(s) = \bigcap_{k\in I} l_t^k(s)$$

Similarly we can prove for

$\bigwedge_{e\in rs} (\bigcap_{k\in I} l_i^k)(e) \geq \bigcap_{k\in I} l_i^k(s)$ **and** $\bigvee_{e\in rs} (\bigcap_{k\in I} l_f^k)(e) \leq \bigcap_{k\in I} l_f^k(s)$

Hence the theorem.

Definition 3.5. Let $\sigma : F \longrightarrow E$ be a mapping from SHR W to E. Then σ is said to be homomorphism if

(1) $\sigma(e + s) \subseteq \sigma(e) + \sigma(s)$

(2) $\sigma(es) \subseteq \sigma(e)\sigma(s)$

(3) $\sigma(0_F) = 0_E$ for all $e, s \in W$

where 0_F and 0_E are zeros of F and E respectively.

Proposition 3.6. *Let $\sigma : F \longrightarrow E$ be a homomorphism of semi-hyper-ring. If l is an $INLHI$ of W. Then pre-image of l is an $INLHI$ of W.*

Proof : Since $\sigma : F \longrightarrow E$ be a homomorphism of W. Also since l is an $INLHI$ of W and $u, e, k \in W$.

$$\bigwedge_{u\in e+k} \sigma^{-1}(l_t)(u) = \bigwedge_{t\in e+k} l_t(\sigma(u))$$
$$= \bigwedge_{\sigma(u)\subseteq\sigma(e)+\sigma(k)} l_t(\sigma(u))$$

$$\geq l_t(\sigma(e)) \wedge l_t(\sigma(k))$$
$$= \sigma^{-1}(l_t)(e) \wedge \sigma^{-1}(l_t)(k)$$

Also

$$\begin{aligned}\bigwedge_{u\in e+k} \sigma^{-1}(l_i)(u) &= \bigwedge_{t\in e+k} l_i(\sigma(u)) \\ &= \bigwedge_{\sigma(u)\subseteq\sigma(e)+\sigma(k)} l_i(\sigma(u)) \\ &\geq l_i(\sigma(e)) \wedge l_i(\sigma(k)) \\ &= \sigma^{-1}(l_i)(e) \wedge \sigma^{-1}(l_i)(k)\end{aligned}$$

Moreover

$$\begin{aligned}\bigvee_{u\in e+k} \sigma^{-1}(l_f)(u) &= \bigvee_{t\in e+k} l_f(\sigma(u)) \\ &= \bigvee_{\sigma(u)\subseteq\sigma(e)+\sigma(k)} l_f(\sigma(u)) \\ &\leq l_f(\sigma(e)) \vee l_i(\sigma(k)) \\ &= \sigma^{-1}(l_f)(e) \vee \sigma^{-1}(l_f)(k)\end{aligned}$$

Again

$$\begin{aligned}\bigwedge_{u\in ek} \sigma^{-1}(l_t)(u) &= \bigwedge_{t\in ek} l_t(\sigma(u)) \\ &= \bigwedge_{\sigma(u)\subseteq\sigma(e)\sigma(k)} l_t(\sigma(u)) \\ &\geq l_t(\sigma(k)) = \sigma^{-1}(l_t)(k)\end{aligned}$$

Also

$$\begin{aligned}\bigwedge_{u\in ek} \sigma^{-1}(l_i)(u) &= \bigwedge_{t\in ek} l_i(\sigma(u)) \\ &= \bigwedge_{\sigma(u)\subseteq\sigma(e)\sigma(k)} l_i(\sigma(u)) \\ &\geq l_i(\sigma(k)) = \sigma^{-1}(l_i)(k)\end{aligned}$$

and

$$\begin{aligned}\bigvee_{u\in ek} \sigma^{-1}(l_f)(u) &= \bigvee_{t\in ek} l_f(\sigma(u)) \\ &= \bigvee_{\sigma(u)\subseteq\sigma(e)\sigma(k)} l_f(\sigma(u)) \\ &\leq l_f(\sigma(k)) = \sigma^{-1}(l_f)(k)\end{aligned}$$

Hence pre-image of l is an $INLHI$ of W.

Proposition 3.7. *Let $\sigma : F \longrightarrow E$ be a surjective homomorphism of semi-hyper-ring. If l is an $INLHI$ of W. Then image of l is an $INLHI$ of W.*

Proof : Since l is an $INLHI$ of W and $u_0, e_0, k_0 \in W$. Then

$$\begin{aligned}\bigwedge_{u_0\in e_0+k_0} \sigma(l_t)(u_0) &= \bigwedge_{u_0\in e_0+k_0} \sup_{u\in\sigma^{-1}(u_0)} l_t(u) \\ &= \bigwedge_{u_0\in e_0+k_0} \sup_{e\in\sigma^{-1}(e_0),k\in\sigma^{-1}(k_0)} l_t(u) \\ &\geq \sup_{e\in\sigma^{-1}(e_0),k\in\sigma^{-1}(k_0)} \{l_t(u) \vee l_t(k)\} \\ &= \sup_{e\in\sigma^{-1}(e_0)} l_t(u) \wedge \sup_{k\in\sigma^{-1}(k_0)} l_t(k)\end{aligned}$$

$$= \sigma(l_t)(e_0) \wedge \sigma(l_t)(k_0)$$

Also

$$\begin{aligned}
\bigwedge_{u_0 \in e_0 + k_0} \sigma(l_i)(u_0) &= \bigwedge_{u_0 \in e_0 + k_0} \sup_{u \in \sigma^{-1}(u_0)} l_i(\sigma(u)) \\
&= \bigwedge_{u_0 \in e_0 + k_0} \sup_{e \in \sigma^{-1}(e_0), k \in \sigma^{-1}(k_0)} l_i(\sigma(u)) \\
&\geq \sup_{e \in \sigma^{-1}(e_0), k \in \sigma^{-1}(k_0)} \tfrac{l_i(e) + l_i(k)}{2} \\
&= 1/2 \left[\sup_{e \in \sigma^{-1}(e_0)} l_i(u) + \sup_{k \in \sigma^{-1}(k_0)} l_i(k) \right] \\
&= 1/2 \left[\sigma(l_i)(e_0) + \sigma(l_i)(k_0) \right]
\end{aligned}$$

$$\begin{aligned}
\bigvee_{u_0 \in e_0 + k_0} \sigma(l_f)(u_0) &= \bigvee_{u_0 \in e_0 + k_0} \inf_{u \in \sigma^{-1}(u_0)} l_f(u) \\
&= \bigvee_{u_0 \in e_0 + k_0} \inf_{e \in \sigma^{-1}(e_0), k \in \sigma^{-1}(k_0)} l_f(u) \\
&\leq \inf_{e \in \sigma^{-1}(e_0), k \in \sigma^{-1}(k_0)} \{ l_t(e) \vee l_t(k) \} \\
&= \inf_{e \in \sigma^{-1}(e_0)} l_t(e) \vee \inf_{k \in \sigma^{-1}(k_0)} l_t(k) \\
&= \sigma(l_t)(e_0) \vee \sigma(l_t)(k_0)
\end{aligned}$$

Moreover

$$\begin{aligned}
\bigwedge_{u_0 \in e_0 k_0} \sigma(l_t)(u_0) &= \bigwedge_{u_0 \in e_0 k_0} \sup_{u \in \sigma^{-1}(u_0)} l_t(u) \\
&= \bigwedge_{u_0 \in e_0 + k_0} \sup_{e \in \sigma^{-1}(e_0), k \in \sigma^{-1}(k_0)} l_t(u) \\
&\geq \sup_{k \in \sigma^{-1}(k_0)} l_t(k) \\
&= \sigma(l_t)(k_0)
\end{aligned}$$

$$\begin{aligned}
\bigwedge_{u_0 \in e_0 k_0} \sigma(l_i)(u_0) &= \bigwedge_{u_0 \in e_0 k_0} \sup_{u \in \sigma^{-1}(u_0)} l_i(u) \\
&= \bigwedge_{u_0 \in e_0 + k_0} \sup_{e \in \sigma^{-1}(e_0), k \in \sigma^{-1}(k_0)} l_i(u) \\
&\geq \sup_{k \in \sigma^{-1}(k_0)} l_i(k) \\
&= \sigma(l_i)(k_0)
\end{aligned}$$

Also

$$\begin{aligned}
\bigvee_{u_0 \in e_0 k_0} \sigma(l_f)(u_0) &= \bigvee_{u_0 \in e_0 k_0} \inf_{u \in \sigma^{-1}(u_0)} l_f(u) \\
&= \bigvee_{u_0 \in e_0 + k_0} \inf_{e \in \sigma^{-1}(e_0), k \in \sigma^{-1}(k_0)} l_f(u) \\
&\leq \inf_{k \in \sigma^{-1}(k_0)} l_f(k) \\
&= \sigma(l_f)(k_0)
\end{aligned}$$

Definition 3.8. Cartesian product of two IN subsets l and m of W is defined by,

$(l_t \times m_t)(e, k) = l_t \wedge m_t$

$(l_i \times m_i)(e,k) = \frac{l_i+m_i}{2}$

$(l_f \times m_f)(e,k) = l_f \vee m_f$ for all $e,k \in W$

Theorem 3.9. *Cartesian product of two $INLHI$ is also an $INLHI$.*

Proof: Let l and m be two $INLHI$ of W. Let $(e_1,e_2),(k_1,k_2),(u_1,u_2) \in W \times W$. Then

$$\begin{aligned}\bigwedge_{(e_1,e_2)\in(k_1,k_2)+(u_1,u_2)} (l_t \times m_t)(e_1,e_2) &= \bigwedge_{e_1\in(k_1+u_1),e_2\in(k_2+u_2)} (l_t \times m_t)(e_1,e_2)\\ &= \bigwedge_{e_1\in(k_1+u_1),e_2\in(k_2+u_2)} (l_t(e_1) \wedge m_t(e_2))\\ &\geq min\left\{(l_t(k_1) \wedge l_t(u_1)), (m_t(k_1) \wedge m_t(u_1))\right\}\\ &= min\left\{(l_t(k_1) \wedge l_t(k_2)), (m_t(u_1) \wedge m_t(u_2))\right\}\\ &= min\left\{(l_t \times m_t)(k_1,k_2), (l_t \times m_t)(u_1,u_2)\right\}\end{aligned}$$

Also

$$\begin{aligned}\bigwedge_{(e_1,e_2)\in(k_1,k_2)+(u_1,u_2)} (l_i \times m_i)(e_1,e_2) &= \bigwedge_{e_1\in(k_1+u_1),e_2\in(k_2+u_2)} (l_i \times m_i)(e_1,e_2)\\ &= \bigwedge_{e_1\in(k_1+u_1),e_2\in(k_2+u_2)} \frac{l_i(e_1)+m_i(e_2)}{2}\\ &\geq 1/2\left[\frac{l_i(k_1)+m_i(u_1)}{2} + \frac{l_i(k_2)+m_i(u_2)}{2}\right]\\ &= 1/2\left[\frac{l_i(k_1)+m_i(k_2)}{2} + \frac{l_i(u_1)+m_i(u_2)}{2}\right]\\ &= 1/2\left[(l_i \times m_i)(k_1,k_2) + (l_i \times m_i)(u_1,u_2)\right]\end{aligned}$$

and

$$\begin{aligned}\bigvee_{(e_1,e_2)\in(k_1,k_2)+(u_1,u_2)} (l_f \times m_f)(e_1,e_2) &= \bigvee_{e_1\in(k_1+u_1),e_2\in(k_2+u_2)} (l_f \times m_f)(e_1,e_2)\\ &= \bigvee_{e_1\in(k_1+u_1),e_2\in(k_2+u_2)} (l_t(e_1) \vee m_t(e_2))\\ &\leq max\left\{(l_t(k_1) \vee l_t(u_1)), (m_t(k_1) \vee m_t(u_1))\right\}\\ &= max\left\{(l_t(k_1) \vee l_t(k_2)), (m_t(u_1) \vee m_t(u_2))\right\}\\ &= max\left\{(l_t \times m_t)(k_1,k_2), (l_t \times m_t)(u_1,u_2)\right\}\end{aligned}$$

In similar manner we prove

$$\begin{aligned}\bigwedge_{(e_1,e_2)\in(k_1,k_2)(u_1,u_2)} (l_t \times m_t)(e_1,e_2) &= \bigwedge_{e_1\in(k_1u_1),e_2\in(k_2u_2)} (l_t \times m_t)(e_1,e_2)\\ &= \bigwedge_{e_1\in(k_1u_1),e_2\in(k_2u_2)} (l_t(e_1) \wedge m_t(e_2))\\ &\geq min\left\{l_t(u_1) \wedge m_t(u_2)\right\}\\ &= min\left\{(l_t \times m_t)(u_1,u_2)\right\}\end{aligned}$$

also

$$\begin{aligned}\bigwedge_{(e_1,e_2)\in(k_1,k_2)(u_1,u_2)} (l_i \times m_i)(e_1,e_2) &= \bigwedge_{e_1\in(k_1u_1),e_2\in(k_2u_2)} (l_i \times m_i)(e_1,e_2)\\ &= \bigwedge_{e_1\in(k_1u_1),e_2\in(k_2u_2)} \frac{l_i(e_1)+m_i(e_2)}{2}\\ &\geq \frac{l_i(u_1)+m_i(u_2)}{2} = (l_i \times m_i)(u_1,u_2)\end{aligned}$$

Moreover

$$\begin{aligned}\bigvee_{(e_1,e_2)\in(k_1,k_2)(u_1,u_2)}(l_f\times m_f)(e_1,e_2) &= \bigvee_{e_1\in(k_1u_1),e_2\in(k_2u_2)}(l_f\times m_f)(e_1,e_2)\\ &= \bigvee_{e_1\in(k_1+u_1),e_2\in(k_2+u_2)}(l_t(e_1)\vee m_t(e_2))\\ &\leq l_f(u_1)\vee m_f(u_2) = (l_f\times m_f)(u_1,u_2)\end{aligned}$$

4. Rough interval neutrosophic hyper-ideal ($RINHI$) in semihyperrings

This section deals with the new concept $RINHI$ of semihyperrings. Let ϕ be a congruence relation on W.
ϕ is an equivalence relation on W such that $(e,s)\in\phi \implies (ew,sw)\in\phi$ and $(we,ws)\in\phi$ for every $w\in W$.

Definition 4.1. An $INHI$ is called an ϕ-lower(upper)$INHI$ of W if its lower(upper) approximation is also an $INHI$.

Definition 4.2. An $INHI$ is said to be an $RINHI$ if it is both ϕ-lower and ϕ-upper $INHI$ of W.

Theorem 4.3. *Let l be an $INHI$ of W. Then l is an $RINHI$.*

Proof: Since l is an $INHI$ of W. Let $e,s,q\in W$ then

$$\begin{aligned}\bigwedge_{e\in s+q}\overline{\phi}(l_t)(e) &= \bigwedge_{e\in s+q}\bigvee_{r\in[s+q]_\phi} l_t(r)\\ &\geq \bigwedge_{e\in s+q}\bigvee_{r\in[s]_\phi+[q]_\phi} l_t(r)\\ &\geq \bigwedge_{r\in i+j}\bigvee_{i+j\subseteq[s]_\phi+[q]_\phi} l_t(r)\\ &= \bigvee_{i\in[s]_\phi,j\in[q]_\phi}\bigwedge_{r\in i+j} l_t(r)\\ &\geq \bigvee_{i\in[s]_\phi,j\in[q]_\phi}\{l_t(i)\wedge l_t(j)\}\\ &= \bigvee_{i\in[s]_\phi} l_t(s)\wedge\bigvee_{j\in[q]_\phi} l_t(q)\\ &= \overline{\phi}(l_t)(s)\wedge\overline{\phi}(l_t)(q)\end{aligned}$$

and

$$\begin{aligned}\bigwedge_{e\in s+q}\overline{\phi}(l_i)(e) &= \bigwedge_{e\in s+q}\bigwedge_{r\in[s+q]_\phi} l_i(r)\\ &\geq \bigwedge_{e\in s+q}\bigwedge_{r\in[s]_\phi+[q]_\phi} l_i(r)\\ &\geq \bigwedge_{r\in i+j}\bigwedge_{i+j\subseteq[s]_\phi+[q]_\phi} l_i(r)\\ &= \bigwedge_{i\in[s]_\phi,j\in[q]_\phi}\bigwedge_{r\in i+j} l_i(r)\\ &\geq \bigwedge_{i\in[s]_\phi,j\in[q]_\phi}\left[\frac{l_i(i)+l_i(j)}{2}\right]\\ &= \frac{1}{2}\left[\bigwedge_{i\in[s]_\phi} l_i(i)+\bigwedge_{j\in[q]_\phi} l_i(j)\right]\end{aligned}$$

$$= \frac{1}{2}\left[\overline{\phi}(l_i)(s) + \overline{\phi}(l_i)(q)\right]$$

also

$$\begin{aligned}
\bigvee_{e\in s+q} \overline{\phi}(l_f)(e) &= \bigvee_{e\in s+q} \bigvee_{r\in[s+q]_\phi} l_f(r)\\
&\leq \bigvee_{e\in s+q} \bigvee_{r\in[s]_\phi+[q]_\phi} l_f(r)\\
&\leq \bigvee_{r\in i+j} \bigvee_{i+j\subseteq[s]_\phi+[q]_\phi} l_f(r)\\
&= \bigvee_{i\in[s]_\phi, j\in[q]_\phi} \bigvee_{r\in i+j} l_f(r)\\
&\leq \bigvee_{i\in[s]_\phi, j\in[q]_\phi} \{l_f(i) \vee l_f(j)\}\\
&= \bigvee_{i\in[s]_\phi} l_f(s) \vee \bigvee_{j\in[q]_\phi} l_f(q)\\
&= \overline{\phi}(l_f)(s) \vee \overline{\phi}(l_f)(q)
\end{aligned}$$

Moreover

$$\begin{aligned}
\bigwedge_{e\in sq} \overline{\phi}(l_t)(e) &= \bigwedge_{e\in sq} \bigvee_{r\in[sq]_\phi} l_t(r)\\
&= \bigwedge_{e\in sq} \bigvee_{r\in[s]_\phi[q]_\phi} l_t(r)\\
&= \bigwedge_{r\in ij} \bigvee_{ij\subseteq[s]_\phi[q]_\phi} l_t(r)\\
&= \bigvee_{i\in[s]_\phi j\in[q]_\phi} \bigwedge_{r\in ij} l_t(r)\\
&\geq \bigvee_{i\in[s]_\phi j\in[q]_\phi} l_t(j)\\
&\geq \bigvee_{j\in[q]_\phi} l_t(j)\\
&= \overline{\phi}(l_t)(q)
\end{aligned}$$

Similarly we can prove for

$\bigwedge_{e\in sq} \overline{\phi}(l_f)(e) \geq \overline{\phi}(l_f)(q)$ **and** $\bigwedge_{e\in sq} \overline{\phi}(l_i)(e) \leq \overline{\phi}(l_i)(q)$

Consequently we can prove for lower approximation

ie.,

$$\bigwedge_{e\in s+q} \underline{\phi}(l_t)(e) \geq \underline{\phi}(l_t)(s) \wedge \underline{\phi}(l_t)(q)$$

$$\bigwedge_{e\in s+q} \underline{\phi}(l_i)(e) \geq \frac{1}{2}\left[\underline{\phi}(l_i)(s) + \underline{\phi}(l_i)(q)\right]$$

$$\bigwedge_{e\in s+q} \underline{\phi}(l_f)(e) \leq \underline{\phi}(l_f)(s) \wedge \underline{\phi}(l_f)(q)$$

and

$$\bigwedge_{e\in sq} \underline{\phi}(l_t)(e) \geq \underline{\phi}(l_t)(q)$$

$$\bigwedge_{e\in sq} \underline{\phi}(l_f)(e) \geq \underline{\phi}(l_f)(q)$$

$$\bigwedge_{e\in sq} \underline{\phi}(l_i)(e) \leq \underline{\phi}(l_i)(q)$$

Hence l is a $RINLHI$ of W.

5. Conclusions

In this paper we introduce the notion of rough interval neutrosophic hyperideals in semihyperrings. Some basic properties of this ideals are studied. We apply rough interval neutrosophic set to some more algebraic structures. Moreover in future we apply rough interval neutrosophic sets to some applications like multi criteria decision making, medical analysis, decision making, gray analysis etc.,

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