Plithogenic functions value

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Abstract: The goal of this paper is to find a formula through which we find all the values of the plithogenic functions. we found this formula and proved it, in addition to providing a definition of the exponential, logarithmic, trigonometric plithogenic function. Also, the absolute value of the plithogenic number was defined.

Keywords: plithogenic functions, plithogenic number, plithogenic exponential.

1. Introduction

The genesis, origination, formation, development, and evolution of new entities through dynamics of contradictory and/or neutral and/or noncontradictory multiple old entities is known as plithogenic. Plithogeny advocates for the integration of theories from several fields.

We use numerous "knowledges" from domains like soft sciences, hard sciences, arts and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on Plithogeny, Plithogenic Set, Logic, Probability, and Statistics [2], in addition to presenting Introduction to the Symbolic Plithogenic Algebraic Structures (revisited), through which he discussed several ideas, including mathematical operations on Plithogenic numbers [1]. Also, An Overview of Plithogenic Set and Symbolic Plithogenic Algebraic Structures was discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-5-6-7].

Paper is divided into four parts. provides an introduction in the first portion, which includes a review of Plithogenic science. A few definitions of a Plithogenic and operations with plithogenic numbers are covered in the second section. the third section defined plithogenic functions. The paper's conclusion is provided in the fourth section.

2. Preliminaries

2.1. Definition of Plithogenic Numbers (PN) [1]

The numbers of the form $PN = a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$ defined as above are called Plithogenic Numbers, where a_nP_n is called the leading (strongest) term.

2.2 Operations with Plithogenic Numbers [1]

Let's consider two plithogenic numbers:

$$PN_1 = a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$$

$$PN_2 = b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n$$

2.2.1. Addition of Plithogenic Numbers

$$PN_1 + PN_2 = (a_0 + b_0) + \sum_{i=1}^{n} (a_i + b_i)\,P_i$$

2.2.2. Subtraction of Plithogenic Numbers

$$PN_1 - PN_2 = (a_0 - b_0) + \sum_{i=1}^{n} (a_i - b_i)\,P_i$$

(SPS, +) is a Symbolic Plithogenic Commutative Group

2.2.3. Scalar Multiplication of Plithogenic Numbers

$$c.PN_1 = c.(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n) = c.a_0 + c.a_1P_1 + c.a_2P_2 + \cdots + c.a_nP_n$$

2.2.4. Multiplication of and Ppower of Plithogenic Numbers

$$PN_1.PN_2 = (a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n).(b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n)$$

and then one multiplies them, term by term $(a_iP_i).(a_jP_j) = a_i.a_j.P_{max\{i,j\}}$,where · is the classical multiplication, as in classical algebra, using the above multiplication of symbolic plithogenic components.

2.2.5. Division of Symbolic Plithogenic Components

$$\frac{P_i}{P_j} = \begin{cases} x_0 + x_1P_1 + x_2P_2 + \cdots + x_jP_j + P_i & x_0 + x_1 + x_2 + \cdots + x_j = 0 \quad i > j \\ x_0 + x_1P_1 + x_2P_2 + \cdots + x_iP_i & x_0 + x_1 + x_2 + \cdots + x_i = 1 \quad i = j \\ \emptyset & i < j \end{cases}$$

where all coefficients $x_0, x_1, x_2, \ldots, x_i, \ldots \in$ SPS.

2.2.5. Division of Symbolic Plithogenic Numbers

Let consider two symbolic plithogenic numbers as below:

$$PN_r = a_0 + a_1P_1 + a_2P_2 + \cdots + a_rP_r$$

$$PN_s = b_0 + b_1P_1 + b_2P_2 + \cdots + b_sP_s$$

$$\frac{PN_r}{PN_s} = \begin{cases} none, one \;\; many & r \geq s \\ \emptyset & r < s \end{cases}$$

3. Plithogenic functions value

Definition 1

Let $a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$ *and* $b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n$ are numbers of plithogenic, then we say that:

$a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n \leq b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n$ if and only if:

$a_0 \le b_0$, $a_0 + a_1 \le b_0 + b_1$, $a_0 + a_1 + a_2 \le b_0 + b_1 + b_2$, ... , and $a_0 + a_1 + a_2 + \cdots + a_n \le b_0 + b_1 + b_2 + \cdots + b_n$

Definition 2

We say that the plithogenic number $a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$ is positive if the following conditions is met:

$a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n \ge 0 + 0P_1 + 0P_2 + \cdots + 0P_n$, then:

$$a_0 \ge 0 \ \ , a_0 + a_1 \ge 0 \ \ , \ldots., and \ \ a_0 + a_1 + a_2 + \cdots + a_n \ge 0$$

Definition 3

Let $a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$ is number of plithogenic, and f is a plithogenic function, then:

$$f(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n) = f(a_0) + P_1[f(a_0 + a_1) - f(a_0)] + P_2[f(a_0 + a_1 + a_2) - f(a_0 + a_1)] + \cdots + P_n[f(a_0 + a_1 + a_2 + \cdots + a_n) - f(a_0 + a_1 + \cdots + a_{n-1})]$$

Proof:

First: we will proof it for $n = 1$

$$f(a_0 + a_1P_1) = f(a_0) + P_1[f(a_0 + a_1) - f(a_0)]$$

$$(a_0 + a_1P_1)^{\acute{n}} = a_0^{\acute{n}} + P_1\left[(a_0 + a_1)^{\acute{n}} - a_0^{\acute{n}}\right]$$

for $\acute{n} = 1$, we get:

$(a_0 + a_1P_1) = a_0 + P_1[a_0 + a_1 - a_0] = a_0 + a_1P_1$ (The formula is correct for $\acute{n} = 1$)

Assume The formula is correct for $\acute{n} = k$

$$(a_0 + a_1P_1)^k = a_0^k + P_1[(a_0 + a_1)^k - a_0^k]$$

for $\acute{n} = 1$, we get:

Let's check it for $\acute{n} = k + 1$

$$(a_0 + a_1P_1)^{k+1} = a_0^{k+1} + P_1[(a_0 + a_1)^{k+1} - a_0^{k+1}]$$

$$(a_0 + a_1P_1)^{k+1} = (a_0 + a_1P_1)^k(a_0 + a_1P_1) = (a_0^k + P_1[(a_0 + a_1)^k - a_0^k])(a_0 + a_1P_1)$$

$$= a_0^{k+1} + a_0^k a_1P_1 + P_1[a_0(a_0 + a_1)^k - a_0^{k+1}] + P_1[a_1P_1(a_0 + a_1)^k - a_0^k a_1P_1]$$

$$= a_0^{k+1} + a_0^k a_1P_1 + a_0(a_0 + a_1)^kP_1 - a_0^{k+1}P_1 + a_1(a_0 + a_1)^kP_1 - a_0^k a_1P_1$$

$$= a_0^{k+1} + (a_0 + a_1)^k(a_0 + a_1)P_1 - a_0^{k+1}P_1$$

$$= a_0^{k+1} + P_1[(a_0 + a_1)^{k+1} - a_0^{k+1}] \ \text{(The formula is correct for } \acute{n} = k + 1)$$

then: $(a_0 + a_1P_1)^{\acute{n}} = a_0^{\acute{n}} + P_1\left[(a_0 + a_1)^{\acute{n}} - a_0^{\acute{n}}\right]$ is true for any $\acute{n}$

Let's proof it for:

$$e^{a_0 + a_1P_1} = e^{a_0} + P_1[e^{a_0 + a_1} - e^{a_0}]$$

$$e^{a_0+a_1P_1} = \sum_{ń=0}^{\infty} \frac{1}{ń!}(a_0 + a_1P_1)^{ń}$$

$$= \sum_{ń=0}^{\infty} \frac{1}{ń!}\left(a_0{}^{ń} + P_1\left[(a_0 + a_1)^{ń} - a_0{}^{ń}\right]\right)$$

$$= \sum_{ń=0}^{\infty} \frac{a_0{}^{ń}}{ń!} + P_1\left[\sum_{ń=0}^{\infty} \frac{(a_0 + a_1)^{ń}}{ń!} + \sum_{ń=0}^{\infty} \frac{a_0{}^{ń}}{ń!}\right]$$

$$= e^{a_0} + P_1[e^{a_0+a_1} - e^{a_0}]$$

Second: we will proof it for $n = 2$

$$f(a_0 + a_1P_1 + a_2P_2) = f(a_0) + P_1[f(a_0 + a_1) - f(a_0)] + P_2[f(a_0 + a_1 + a_2) - f(a_0 + a_1)]$$

$$(a_0 + a_1P_1 + a_2P_2)^{ń} = a_0{}^{ń} + P_1\left[(a_0 + a_1)^{ń} - a_0{}^{ń}\right] + P_2\left[(a_0 + a_1 + a_2)^{ń} - (a_0 + a_1)^{ń}\right]$$

for $ń = 1$, we get:

$$(a_0 + a_1P_1 + a_2P_2) = a_0 + P_1[a_0 + a_1 - a_0] + P_2[a_0 + a_1 + a_2 - (a_0 + a_1)] = a_0 + a_1P_1 + a_2P_2$$

(The formula is correct for $ń = 1$)

Assume The formula is correct for $ń = k$

$$(a_0 + a_1P_1 + a_2P_2)^k = a_0{}^k + P_1[(a_0 + a_1)^k - a_0{}^k] + P_2[(a_0 + a_1 + a_2)^k - (a_0 + a_1)^k]$$

for $n = 1$, we get:

Let's check it for $ń = k + 1$

$$(a_0 + a_1P_1 + a_2P_2)^{k+1} = a_0{}^{k+1} + P_1[(a_0 + a_1)^{k+1} - a_0{}^{k+1}] + P_2[(a_0 + a_1 + a_2)^{k+1} - (a_0 + a_1)^{k+1}]$$

$$(a_0 + a_1P_1 + a_2P_2)^{k+1} = (a_0 + a_1P_1 + a_2P_2)^k(a_0 + a_1P_1 + a_2P_2)$$

$$= (a_0{}^k + P_1[(a_0 + a_1)^k - a_0{}^k] + P_2[(a_0 + a_1 + a_2)^k - (a_0 + a_1)^k])(a_0 + a_1P_1 + a_2P_2)$$

$$= a_0a_0{}^k + P_1[a_0(a_0 + a_1)^k - a_0a_0{}^k] + P_2[a_0(a_0 + a_1 + a_2)^k - a_0(a_0 + a_1)^k] + a_0{}^k a_1P_1 + a_1P_1P_1[(a_0 + a_1)^k - a_0{}^k] + a_1P_1P_2[(a_0 + a_1 + a_2)^k - (a_0 + a_1)^k] + a_0{}^k a_2P_2 + a_2P_2P_1[(a_0 + a_1)^k - a_0{}^k] + a_2P_2P_2[(a_0 + a_1 + a_2)^k - (a_0 + a_1)^k]$$

$$= a_0a_0{}^k + P_1[a_0(a_0 + a_1)^k - a_0a_0{}^k] + P_2[a_0(a_0 + a_1 + a_2)^k - a_0(a_0 + a_1)^k] + a_0{}^k a_1P_1 + P_1[a_1(a_0 + a_1)^k - a_1a_0{}^k] + P_2[a_1(a_0 + a_1 + a_2)^k - a_1(a_0 + a_1)^k] + a_0{}^k a_2P_2 + P_2[a_2(a_0 + a_1)^k - a_2a_0{}^k] + P_2[a_2(a_0 + a_1 + a_2)^k - a_2(a_0 + a_1)^k]$$

$$= a_0{}^{k+1} + P_1[a_0(a_0 + a_1)^k + a_1(a_0 + a_1)^k - a_0{}^{k+1}] + P_2[a_0(a_0 + a_1 + a_2)^k + a_1(a_0 + a_1 + a_2)^k + a_2(a_0 + a_1 + a_2)^k - a_0(a_0 + a_1)^k - a_1(a_0 + a_1)^k]$$

$$= a_0{}^{k+1} + P_1[(a_0 + a_1)(a_0 + a_1)^k - a_0{}^{k+1}] + P_2[(a_0 + a_1 + a_2)(a_0 + a_1 + a_2)^k - (a_0 + a_1)(a_0 + a_1)^k]$$

$$= a_0{}^{k+1} + P_1[(a_0 + a_1)^{k+1} - a_0{}^{k+1}] + P_2[(a_0 + a_1 + a_2)^{k+1} - (a_0 + a_1)^{k+1}]$$

(The formula is correct for $\acute{n} = k + 1$)

then: $(a_0 + a_1P_1 + a_2P_2)^{\acute{n}} = a_0{}^{\acute{n}} + P_1\left[(a_0 + a_1)^{\acute{n}} - a_0{}^{\acute{n}}\right] + P_2\left[(a_0 + a_1 + a_2)^{\acute{n}} - (a_0 + a_1)^{\acute{n}}\right]$ is true for any $\acute{n}$

Let's proof it for:

$$e^{a_0+a_1P_1+a_2P_2} = e^{a_0} + P_1[e^{a_0+a_1} - e^{a_0}] + P_2[e^{a_0+a_1+a_2} - e^{a_0+a_1}]$$

$$e^{a_0+a_1P_1+a_2P_2} = \sum_{\acute{n}=0}^{\infty} \frac{1}{\acute{n}!}(a_0 + a_1P_1 + a_2P_2)^{\acute{n}}$$

$$= \sum_{\acute{n}=0}^{\infty} \frac{1}{\acute{n}!}\left(a_0{}^{\acute{n}} + P_1\left[(a_0 + a_1)^{\acute{n}} - a_0{}^{\acute{n}}\right] + P_2\left[(a_0 + a_1 + a_2)^{\acute{n}} - (a_0 + a_1)^{\acute{n}}\right]\right)$$

$$= \sum_{\acute{n}=0}^{\infty} \frac{a_0{}^{\acute{n}}}{\acute{n}!} + P_1\left[\sum_{\acute{n}=0}^{\infty} \frac{(a_0 + a_1)^{\acute{n}}}{\acute{n}!} + \sum_{\acute{n}=0}^{\infty} \frac{a_0{}^{\acute{n}}}{\acute{n}!}\right] + P_2\left[\sum_{\acute{n}=0}^{\infty} \frac{(a_0 + a_1 + a_2)^{\acute{n}}}{\acute{n}!} + \sum_{\acute{n}=0}^{\infty} \frac{(a_0 + a_1)^{\acute{n}}}{\acute{n}!}\right]$$

$$= e^{a_0} + P_1[e^{a_0+a_1} - e^{a_0}] + P_2[e^{a_0+a_1+a_2} - e^{a_0+a_1}]$$

Hence, we can apply:

$$f(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)$$
$$= f(a_0) + P_1[f(a_0 + a_1) - f(a_0)] + P_2[f(a_0 + a_1 + a_2) - f(a_0 + a_1)] + \cdots$$
$$+ P_n[f(a_0 + a_1 + a_2 + \cdots + a_n) - f(a_0 + a_1 + \cdots + a_{n-1})]$$

for any n.

Example1

$$e^{6+4P_1-P_2+2P_3+7P_4} = e^6 + P_1[e^{10} - e^6] + P_2[e^9 - e^{10}] + P_3[e^{11} - e^9] + P_2[e^{18} - e^{11}]$$

Remark 1

Absolute value of the plithogenic number defined as follows:

$$|a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n|$$
$$= |a_0| + P_1[|a_0 + a_1| - |a_0|] + P_2[|a_0 + a_1 + a_2| - |a_0 + a_1|] + \cdots$$
$$+ P_n[|a_0 + a_1 + a_2 + \cdots + a_n| - |a_0 + a_1 + \cdots + a_{n-1}|] \geq 0$$

Example 2

- $|9 - 4P_1 + 2P_2 - P_3| = 9 + P_1[|9 - 4| - |9|] + P_2[|9 - 4 + 2| - |9 - 4|] + P_3[|9 - 4 + 2 - 1| - |9 - 4 + 2|]$ $= 9 - 4P_1 + 2P_2 - P_3 > 0$

- $|5 - 7P_1 - 8P_2| = 5 + P_1[|5 - 7| - |5|] + P_2[|5 - 7 - 8| - |5 - 7|]$

$$= 5 - 3P_1 + 8P_2 > 0$$

clear the Absolute value of the plithogenic number according to the definition 2.

Definition 4

1) Plithogenic exponential functions is defined as formula:

$$e^{(a_0+a_1P_1+a_2P_2+\cdots+a_nP_n)x} = e^{a_0x} + P_1\left[e^{(a_0+a_1)x} - e^{a_0x}\right] + P_2\left[e^{(a_0+a_1+a_2)x} - e^{(a_0+a_1)x}\right] + \cdots + P_n\left[e^{(a_0+a_1+a_2+\cdots+a_n)x} - e^{(a_0+a_1+a_2+\cdots+a_{n-1})}\right]$$

2) Plithogenic logarithmic functions is defined as formula:

$$ln\big((a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)x\big) = ln(a_0x) + P_1\big[ln\big((a_0 + a_1)x\big) - ln(a_0x)\big] + P_2\big[ln\big((a_0 + a_1 + a_2)x\big) - ln\big((a_0 + a_1)x\big)\big] + \cdots + P_n\big[ln\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big) - ln\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big)\big]$$

where: $(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)x > 0$

Definition 5

Plithogenic trigonometric functions is defined as formulas:

1) $sin\big((a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)x\big) = sin(a_0x) + P_1\big[sin\big((a_0 + a_1)x\big) - sin(a_0x)\big] + P_2\big[sin\big((a_0 + a_1 + a_2)x\big) - sin\big((a_0 + a_1)x\big)\big] + \cdots + P_n\big[sin\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big) - sin\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big)\big]$
2) $cos\big((a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)x\big) = cos(a_0x) + P_1\big[cos\big((a_0 + a_1)x\big) - cos(a_0x)\big] + P_2\big[cos\big((a_0 + a_1 + a_2)x\big) - cos\big((a_0 + a_1)x\big)\big] + \cdots + P_n\big[cos\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big) - cos\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big)\big]$
3) $tan\big((a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)x\big) = tan(a_0x) + P_1\big[tan\big((a_0 + a_1)x\big) - tan(a_0x)\big] + P_2\big[tan\big((a_0 + a_1 + a_2)x\big) - tan\big((a_0 + a_1)x\big)\big] + \cdots + P_n\big[tan\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big) - tan\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big)\big]$
4) $cot\big((a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)x\big) = cot(a_0x) + P_1\big[cot\big((a_0 + a_1)x\big) - cot(a_0x)\big] + P_2\big[cot\big((a_0 + a_1 + a_2)x\big) - cot\big((a_0 + a_1)x\big)\big] + \cdots + P_n\big[cot\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big) - cot\big((a_0 + a_1 + a_2 + \cdots + a_{n-1})x\big)\big]$

4. Conclusions

This paper is considered important in the field of plithogenic, as it presented the concept of the plithogenic function, and how to calculate its values by finding a formula through which we find the values of the plithogenic functions.

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