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Multi-Attribute Decision Making Based on Several Trigonometric Hamming Similarity Measures under Interval Rough Neutrosophic Environment

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Abstract. In this paper, the sine, cosine and cotangent similarity measures of interval rough neutrosophic sets is proposed. Some properties of the proposed measures are discussed. We have proposed multi attribute decision making approaches based on proposed similarity measures. To demonstrate the applicability, a numerical example is solved.

Keywords: sine hamming similarity measure, cosine hamming similarity measure, cotangent hamming similarity measure, interval rough neutrosophic set.

1 Introduction

The basic concept of neutrosophic set grounded by Smarandache [1, 2, 3, 4, 5] is a generalization of classical set or crisp set [6], fuzzy set [7], intuitionistic fuzzy set [8]. Wang et al.[9] extended the concept of neutrosophic set to single valued neutrosophic sets (SVNSs). Broumi et al. [10, 11] proposed new hybrid intelligent structure namely, rough neutrosophic set combing the concept of rough set theory [12] and the concept of neutrosophic set theory to deal with uncertainty and incomplete information. Rough neutrosophic set is the generalization of rough fuzzy sets [13, 14] and rough intuitionistic fuzzy sets [15]. Several studies of rough neutrosophic sets have been reported in the literature. Mondal and Pramanik [16] applied the concept of rough neutrosophic set in multi-attribute decision making based on grey relational analysis. Pramanik and Mondal [17] presented cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [18] also proposed some rough neutrosophic similarity measures namely Dice and Jaccard similarity measures of rough neutrosophic environment. Mondal and Pramanik [19] proposed rough neutrosophic multi attribute decision making based on rough score accuracy function. Pramanik and Mondal [20] presented cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Pramanik and Mondal [21] presented trigonometric Hamming similarity measure of rough neutrosophic sets. Pramanik et al. [22] proposed rough neutrosophic multi attribute decision making based on correlation coefficient. Pramanik et al. [23] also proposed rough neutrosophic projection and bidirectional projection measures. Mondal et al. [24] presented multi attribute decision making based on rough neutrosophic variational coefficient similarity measures. Mondal at al. [25] also presented rough neutrosophic TOPSIS for multi attribute group decision making. Mondal and Pramanik [26] presented tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. In 2015, Broumi and Smarandache [27] combined the concept of rough set theory [12] and interval neutrosophic set theory [28] and defined interval rough neutrosophic set. Pramanik et al. [29] presented multi attribute decision making based on projection and bidirectional projection measures under interval rough neutrosophic environment.

Multi-attribute decision making using trigonometric Hamming similarity measures under interval rough neutrosophic environment is not addressed in the literature.

Research gap MADM strategy using sine, cosine and cotangent similarity measures under interval rough neutrosophic environment.

Research questions

(i) Is it possible to define sine, cosine and cotangent similarity measures between interval rough neutrosophic sets?

(ii) Is it possible to develop new MADM strategies based on the proposed measures in interval rough neutrosophic environment?

The objectives of the paper are

i. to define sine, cosine and cotangent similarity measures between interval rough neutrosophic sets.

ii. to prove the basic properties of sine, cosine and cotangent similarity measures of interval rough neutrosophic sets.

iii. to develop new MADM strategies based on the proposed measures in interval rough neutrosophic environment.

Contributions

(i) In this paper, we propose sine, cosine and cotangent similarity measures under interval rough neutrosophic environment.

(ii) We develop new MADM strategy based on the proposed measures in interval rough neutrosophic environment.

(iii) We also present numerical example to show the feasibility and applicability of the proposed measures.

Rest of the paper is organized in the following way. Section 2 describes preliminaries of neutrosophic sets and rough neutrosophic sets and interval rough neutrosophic sets. Section 3, Section 4 and Section 5 presents definitions and propositions of the proposed measures. Section 6 presents multi attribute decision-making strategies based on the similarity measures. Section 7 provides a numerical example. Section 8 presents the conclusion and future scopes of research.

2 Preliminaries

In this Section, we provide some basic definitions regarding SVNSs, IRNSs which are useful in the paper.

In 1999, Smarandache presented the following definition of neutrosophic set (NS) [1].

Definition 2.1.1. Let X be a space of points (objects) with generic element in X denoted by x. A NS A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A , I_A and F_A are real standard or non-standard subsets of $(^-0,1^+)$ that is T_A:X $\rightarrow$ $(^-0, 1^+)$, I_A:X $\rightarrow$ $(^-0, 1^+)$ and F_A:X $\rightarrow$ $(^-0, 1^+)$. It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $^-0 \le T_A(X) + I_A(X) + F_A(X) \le 1^+$

Definition 2.1.2: (Single-valued neutrosophic set) [9]. Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function *$T_A(x)$* , a falsity membership function *$F_A(x)$* and indeterminacy function *$I_A(x)$* with

$$T_A(x), I_A(x) \text{ and } F_A(x) \in [0,1] \ \forall \text{ x in X}$$

When X is continuous, a SNVS S can be written as follows

$$A = \int_x < T_A(x), F_A(x), I_A(x) > / \forall x \in X$$

and when X is discrete, a SVNS S can be written as follows

$$A = \sum < T_A(x), F_A(x), I_A(x) > / \forall x \in X$$

For a SVNS S, 0 ≤ $\sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

2.2 Rough neutrosophic set

Rough neutrosophic sets [10, 11] are the generalization of rough fuzzy sets [13, 14] and rough intuitionistic fuzzy sets [15].

Definition 2.2.1: Let Y be a non-null set and R be an equivalence relation on Y. Let P be neutrosophic set in Y with the membership function T_P , indeterminacy function I_P and non-membership function F_P . The lower and the upper approximations of P in the approximation (Y, R) denoted by are respectively defined as:

$$\underline{N}(P) =<< x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / y \in [x]_R,\ x \in Y >$$

and

$$\overline{N}(P) =<< x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / y \in [x]_R,\ x \in Y >$$

where,

$T_{\underline{N(P)}}(x) = \wedge z \in [x]_R T_P(Y), I_{\underline{N(P)}}(x) = \wedge z \in [x]_R I_P(Y),$
$F_{\underline{N(P)}}(x) = \wedge z \in [x]_R F_P(Y)$

and

$T_{\overline{N(P)}}(x) = \vee z \in [x]_R T_P(Y), I_{\overline{N(P)}}(x) = \vee z \in [x]_R I_P(Y),$
$F_{\overline{N(P)}}(x) = \vee z \in [x]_R F_P(Y)$.

So,

$0 \le T_{\underline{N(P)}}(x) + I_{\underline{N(P)}}(x) + F_{\underline{N(P)}}(x) \le 3$

and

$0 \le T_{\overline{N(P)}}(x) + I_{\overline{N(P)}}(x) + F_{\overline{N(P)}}(x) \le 3$

Here $\vee$ and $\wedge$ denote "max" and "min" operators respectively, $T_P(y)$, $I_P(y)$ and $F_P(y)$ are the membership, indeterminacy and non-membership of Y with respect to P.

Thus NS mapping,

$\underline{N}, \overline{N} : N(Y) \to N(Y)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N(P)}, \overline{N(P)})$ is called the rough neutrosophic set in (Y, R).

2.3 Interval rough neutrosophic set

Interval rough neutrosophic set (IRNS) [22] is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache, IRNS is the generalizations of interval valued intuitionistic fuzzy rough set.

Definition 2.3.1

Let R be an equivalence relation on the universal set U.Then the pair (U, R) is called a Pawlak approximationspace. An equivalence class of R containing x will bedenoted by $[x]_R$ for X $\in$ U, the lower and upper approximationof X with respect to (U, R) are denoted by respectively

$\underline{R}X$ and $\overline{R}X$ and are defined by

$\underline{R}X = \{x \in U : [x]_R \subseteq X\}$,

$\overline{R}X = \{x \in U : [x]_R \cap X \ne \emptyset\}$.

Now if $\underline{R}X = \overline{R}X$, then X is called definable; otherwise Xis called a rough set.

Definition 2.3.2

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function μA:U→ [0, 1] and non-membership functionν_A: U→ [0, 1] such that $\mu_A(\underline{R}X)$=1and $\nu_A(\underline{R}X) = 0$

ie, $[\mu_A(x), \nu_A(x)]=[1,0]$ if $x \in (\underline{R}X)$ and $\mu_A(U-\overline{R}X)= 0$, $\nu_A(U-\overline{R}X)=1$

ie,

$0 \le \mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X) \le 1$

Definition 2.3.3

Assume that, (U, R) be a Pawlak approximation space, for an interval neutrosophic set

$A = \{<x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]> : x \in U\}$

The lower approximation $\underline{A}_R$ and the upper approximation $\overline{A}_R$ of A in the Pawlak approximation space (U, R) are expressed as follows:

$\underline{A}_R = \{<x, [\wedge_{y\in[x]_R} \{T_A^L(y)\}, \ _{y\ [x]} \{T_A(y)\}],$
$[\vee_{y\in[x]_R} \{I_A^L(y)\}, \ _{y\ [x]} \{I_A(y)\}],$
$[\vee_{y\in[x]_R} \{F_A^L(y)\}, \vee_{y\ [x]} \{F_A(y)\}]> : x \in U\}$

$\overline{A}_R = \{<x, [\vee_{y\in[x]_R} \{T_A^L(y)\}, \ _{y\ [x]} \{T_A(y)\}],$
$[\wedge_{y\in[x]_R} \{I_A^L(y)\}, \ _{y\ [x]} \{I_A(y)\}],$
$[\wedge_{y\in[x]_R} \{F_A^L(y)\}, \wedge_{y\ [x]} \{F_A(y)\}]> : x \in U\}$

The symbols $\wedge$ and $\vee$ indicate "min" and "max" operators respectively. R denotes an equivalence relation for interval neutrosophic set A. Here $[x]_R$ is the equivalence class of the element x. It is obvious that

$[\wedge_{y\in[x]_R} \{T_A^U(y)\}, \wedge_{y\ [x]} \{T_A(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R} \{I_A(y)\}, \vee_{y\ [x]} \{I_A^L(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R} \{F_A(y)\}, \vee_{y\ [x]} \{F_A^L(y)\}] \subset [0,1].$
and $0 \le \wedge_{y\in[x]_R} \{T_A(y)\} + \vee_{y\ [x]} \{I_A^U(y)\} +$
$\vee_{y\in[x]_R} \{F_A^U(y)\} \quad 3$

Then $\underline{A}_R$ is an interval neutrosophic set (INS)

Similarly, we have

$[\vee_{y\in[x]_R} \{T_A^L(y)\}, \vee_{y\ [x]} \{T_A(y)\}] \subset [0,1],$
$[\wedge_{y\in[x]_R} \{I_A^L(y)\}, \wedge_{y\ [x]} \{I_A(y)\}] \subset [0,1],$
$[\wedge_{y\in[x]_R} \{F_A^L(y)\}, \wedge_{y\ [x]} \{F_A(y)\}] \subset [0,1]$ and
$0 \le \vee_{y\in[x]_R} \{T_A^U(y)\} + \wedge_{y\ [x]} \{I_A(y)\} +$
$\wedge_{y\in[x]_R} \{F_A^U(y)\}] \quad 3$

Then $\underline{A}_R$ is an interval neutrosophic set.

If $\underline{A}_R = \overline{A}_R$ then A is a definable set, otherwise A is an interval valued neutrosophic rough set. Here, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval neutrosophic set with respect to approximation space (U,R) respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$ respectively.

2.4 Hamming distance

Hamming distance between two neutrosophic sets $M=(T_M(x), I_M(x), F_M(x))$ and $N=(T_N(x), I_N(x), F_N(x))$ is defined as

$$H(M,N)=\frac{1}{3}\sum_{i=1}^{n}(|T_M(x_i)-T_N(x_i)|+|I_M(x_i)-I_N(x_i)|+|F_M(x_i)-F_N(x_i)|).$$

3. Cosine Hamming Similarity Measure of IRNS

Assume that

$$M=\{<x_i,([\underline{T}_{iM}^{-},\underline{T}_{iM}^{+}],[\underline{I}_{iM}^{-},\underline{I}_{iM}^{+}],[\underline{F}_{iM}^{-},\underline{F}_{iM}^{+}],[\overline{T}_{iM}^{-},\overline{T}_{iM}^{+}],[\overline{I}_{iM}^{-},\overline{I}_{iM}^{+}],[\overline{F}_{iM}^{-},\overline{F}_{iM}^{+}]>: i=1,2,...,n\}$$

and

$$N=\{<x_i,[\underline{T}_{iN}^{-},\underline{T}_{iN}^{+}],[\underline{I}_{iN}^{-},\underline{I}_{iN}^{+}],[\underline{F}_{iN}^{-},\underline{F}_{iN}^{+}],[\overline{T}_{iN}^{-},\overline{T}_{iN}^{+}],[\overline{I}_{iN}^{-},\overline{I}_{iN}^{+}],[\overline{F}_{iN}^{-},\overline{F}_{iN}^{+}]>: i=1,2,...,n\}$$

in X = {x_1, x_2, …, x_n} be any two IRNSs. A cosine Hamming similarity operator between IRNS M and N is defined as follows:

$$\cos(M,N)=\frac{1}{n}\sum_{i=1}^{n}\cos\left(\frac{\Pi}{6}\left(|\Delta T_M(x_i)-\Delta T_N(x_i)|+|\Delta I_M(x_i)-\Delta I_N(x_i)|+|\Delta F_M(x_i)-\Delta F_N(x_i)|\right)\right).$$

$$\Delta T_M(x_i)=\frac{(\underline{T}_{iM}^{-}+\underline{T}_{iM}^{+}+\overline{T}_{iM}^{-}+\overline{T}_{iM}^{+})}{4},$$

$$\Delta I_M(x_i)=\frac{(\underline{I}_{iM}^{-}+\underline{I}_{iM}^{+}+\overline{I}_{iM}^{-}+\overline{I}_{iM}^{+})}{4},$$

$$\Delta F_M(x_i)=\frac{(\underline{F}_{iM}^{-}+\underline{F}_{iM}^{+}+\overline{F}_{iM}^{-}+\overline{F}_{iM}^{+})}{4},$$

$$\Delta T_N(x_i)=\frac{(\underline{T}_{iN}+\underline{T}_{iN}+\overline{T}_{iN}+\overline{T}_{iN})}{4},$$

$$\Delta I_N(x_i)=\frac{(\underline{I}_{iN}^{-}+\underline{I}_{iN}^{+}+\overline{I}_{iN}^{-}+\overline{I}_{iN}^{+})}{4},$$

$$\Delta F_N(x_i)=\frac{(\underline{F}_{iN}^{-}+\underline{F}_{iN}^{+}+\overline{F}_{iN}^{-}+\overline{F}_{iN}^{+})}{4}.$$

Properties 3.1

The defined rough neutrosophic cosine hamming similarity operator cos(M, *N*) between IRNSs M and N satisfies the following properties:

1. $0\le\cos(M,N)\le 1$.
2. cos(M,N) = 1 if and only if M = N.
3. cos(M,N) = cos(N,M).

Proof:

1. Since the functions $\Delta T_M(x), \Delta I_M(x), \Delta F_M(x), \Delta T_N(x), \Delta I_N(x)$ and $\Delta F_N(x)$ the value of the cosine function are within [0, 1], the similarity measure based on interval rough neutrosophic cosine Hamming similarity function also lies within [0, 1]. Hence $0\le\cos(M,N)\le 1$.

This completes the proof.

2. For any two RNSs M and N, if M = N, then the following relations hold

$$\Delta T_M(x_i)=\Delta T_N(x_i), \Delta I_M(x_i)=\Delta I_N(x_i),$$
$$\Delta F_M(x_i)=\Delta F_N(x_i).$$

Hence,

$$|\Delta T_M(x_i)-\Delta T_N(x_i)|=0, |\Delta I_M(x_i)-\Delta I_N(x_i)|=0,$$
$$|\Delta F_M(x_i)-\Delta F_N(x_i)|=0.$$

Thus cos(M,N) = 1

Conversely,

If cos(M,N) = 1, then

$$|\Delta T_M(x_i)-\Delta T_N(x_i)|=0, |\Delta I_M(x_i)-\Delta I_N(x_i)|=0,$$
$$|\Delta F_M(x_i)-\Delta F_N(x_i)|=0.$$

Since cos(0) = 1. So we can write

$$\Delta T_M(x_i)=\Delta T_N(x_i), \Delta I_M(x_i)=\Delta I_N(x_i),$$
$$\Delta F_M(x_i)=\Delta F_N(x_i).$$

Hence M = N**.**

3. As

$$\cos(M,N)=\frac{1}{n}\sum_{i=1}^{n}\cos\left(\frac{\Pi}{6}\left(|\Delta T_M(x_i)-\Delta T_N(x_i)|+|\Delta I_M(x_i)-\Delta I_N(x_i)|+|\Delta F_M(x_i)-\Delta F_N(x_i)|\right)\right)$$
$$=\frac{1}{n}\sum_{i=1}^{n}\cos\left(\frac{\Pi}{6}\left(|\Delta T_N(x_i)-\Delta T_M(x_i)|+|\Delta I_N(x_i)-\Delta I_M(x_i)|+|\Delta F_N(x_i)-\Delta F_M(x_i)|\right)\right)$$
$$=\cos(N,M)$$

This completes the proof.

4. Sine Hamming Similarity Measure of IRNS

Assume that

$$M=\{<x_i,([\underline{T}_{iM}^{-},\underline{T}_{iM}^{+}],[\underline{I}_{iM}^{-},\underline{I}_{iM}^{+}],[\underline{F}_{iM}^{-},\underline{F}_{iM}^{+}],[\overline{T}_{iM}^{-},\overline{T}_{iM}^{+}],[\overline{I}_{iM}^{-},\overline{I}_{iM}^{+}],[\overline{F}_{iM}^{-},\overline{F}_{iM}^{+}]>: i=1,2,...,n\}$$

and

$$N=\{<x_i,[\underline{T}_{iN}^{-},\underline{T}_{iN}^{+}],[\underline{I}_{iN}^{-},\underline{I}_{iN}^{+}],[\underline{F}_{iN}^{-},\underline{F}_{iN}^{+}],[\overline{T}_{iN}^{-},\overline{T}_{iN}^{+}],[\overline{I}_{iN}^{-},\overline{I}_{iN}^{+}],[\overline{F}_{iN}^{-},\overline{F}_{iN}^{+}]>: i=1,2,...,n\}$$

in $X = \{x_1, x_2, \ldots, x_n\}$be any two IRNSs. A sine Hamming similarity operator between IRNS M and N is defined as follows:

$$\sin(M,N) = 1 - [\frac{1}{n}\sum_{i=1}^{n}\sin(\frac{\Pi}{}(|\Delta T_M(x_i) - \Delta T_N(x_i)| + |\Delta I_M(x_i) - \Delta I_N(x_i)| + |\Delta F_M(x_i) - \Delta F_N(x_i)|))].$$

Here,

$$\Delta T_M(x_i) = \frac{(\underline{T_{iM}^-} + \underline{T_{iM}^+} + \overline{T_{iM}^-} + \overline{T_{iM}^+})}{4},$$

$$\Delta I_M(x_i) = \frac{(\underline{I_{iM}^-} + \underline{I_{iM}^+} + \overline{I_{iM}^-} + \overline{I_{iM}^+})}{4},$$

$$\Delta F_M(x_i) = \frac{(\underline{F_{iM}^-} + \underline{F_{iM}^+} + \overline{F_{iM}^-} + \overline{F_{iM}^+})}{4},$$

$$\Delta T_N(x_i) = \frac{(\underline{T_{iN}^-} + \underline{T_{iN}^+} + \overline{T_{iN}^-} + \overline{T_{iN}^+})}{4},$$

$$\Delta I_N(x_i) = \frac{(\underline{I_{iN}^-} + \underline{I_{iN}^+} + \overline{I_{iN}^-} + \overline{I_{iN}^+})}{4},$$

$$\Delta F_N(x_i) = \frac{(\underline{F_{iN}^-} + \underline{F_{iN}^+} + \overline{F_{iN}^-} + \overline{F_{iN}^+})}{4}.$$

Properties 4.1

The defined rough neutrosophic sine hamming similarity operator sin (M, *N*) between IRNSs M and N satisfies the following properties:

1. $0 \le \sin(M,N) \le 1$.
2. sin (M,N) = 1 if and only if M = N.
3. sin (M,N) = sin (N,M).

Proof:

1.Since the functions $\Delta T_M(x), \Delta I_M(x), \Delta F_M(x), \Delta T_N(x), \Delta I_N(x)$ and $\Delta F_N(x)$ the value of the sine function are within [0,1], the similarity measure based on interval rough neutrosophic cosine Hamming similarity function also lies within [0,1]. Hence $0 \le \sin(M,N) \le 1$.

This completes the proved.

2.For any two RNSs M and N, if M = N, then the following relations hold

$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i),$

$\Delta F_M(x_i) = \Delta F_N(x_i).$

Hence,

$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0,$

$|\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$

Thus sin(M,N) = 1

Conversely,

If sin(M,N) = 1, then

$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0,$

$|\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$

Since sin(0) = 1. So we can write

$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i),$

$\Delta F_M(x_i) = \Delta F_N(x_i).$

Hence M = N**.**

3. As

$$\sin(M,N) = 1 - [\frac{1}{n}\sum_{i=1}^{n}\sin(\frac{\Pi}{}(|\Delta T_M(x_i) - \Delta T_N(x_i)| + |\Delta I_M(x_i) - \Delta I_N(x_i)| + |\Delta F_M(x_i) - \Delta F_N(x_i)|))]$$
$$= 1 - [\frac{1}{n}\sum_{i=1}^{n}\sin(\frac{\Pi}{}(|\Delta T_N(x_i) - \Delta T_M(x_i)| + |\Delta I_N(x_i) - \Delta I_M(x_i)| + |\Delta F_N(x_i) - \Delta F_M(x_i)|))]$$
$$= \sin(N,M).$$

This completes the proof.

5. Cotangent Hamming Similarity Measure of IRNS

Assume that

$$M = \{<x_i, ([\underline{T_{iM}^-}, \underline{T_{iM}^+}], [\underline{I_{iM}^-}, \underline{I_{iM}^+}], [\underline{F_{iM}^-}, \underline{F_{iM}^+}], [\overline{T_{iM}^-}, \overline{T_{iM}^+}], [\overline{I_{iM}^-}, \overline{I_{iM}^+}], [\overline{F_{iM}^-}, \overline{F_{iM}^+}] >: i = 1, 2, \ldots, n\}$$

and

$$N = \{<x_i, [\underline{T_{iN}^-}, \underline{T_{iN}^+}], [\underline{I_{iN}^-}, \underline{I_{iN}^+}], [\underline{F_{iN}^-}, \underline{F_{iN}^+}], [\overline{T_{iN}^-}, \overline{T_{iN}^+}], [\overline{I_{iN}^-}, \overline{I_{iN}^+}], [\overline{F_{iN}^-}, \overline{F_{iN}^+}] >: i = 1, 2, \ldots, n\}$$

in $X = \{x_1, x_2, \ldots, x_n\}$be any two IRNSs. A cosine Hamming similarity operator between IRNS M and N is defined as follows:

$$\cot(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cot(\frac{\Pi}{4} + \frac{\Pi}{12}(|\Delta T_M(x_i) - \Delta T_N(x_i)| + |\Delta I_M(x_i) - \Delta I_N(x_i)| + |\Delta F_M(x_i) - \Delta F_N(x_i)|)).$$

Here,

$$\Delta T_M(x_i) \quad \frac{(\underline{T_{iM}^-} + \underline{T_{iM}} \quad \overline{T_{iM}} \quad \overline{T_{iM}})}{4},$$

$$\Delta I_M(x_i) \quad \frac{(\underline{I_{iM}^-} + \underline{I_{iM}} \quad \overline{I_{iM}} \quad \overline{I_{iM}})}{4},$$

$$\Delta F_M(x_i) = \frac{(\underline{F^-_{iM}} + \underline{F^+_{iM}} + \overline{F^-_{iM}} + \overline{F^+_{iM}})}{4},$$

$$\Delta T_N(x_i) = \frac{(\underline{T^-_{iN}} + \underline{T^+_{iN}} + \overline{T^-_{iN}} + \overline{T^+_{iN}})}{4},$$

$$\Delta I_N(x_i) = \frac{(\underline{I^-_{iN}} + \underline{I^+_{iN}} + \overline{I^-_{iN}} + \overline{I^+_{iN}})}{4},$$

$$\Delta F_N(x_i) = \frac{(\underline{F^-_{iN}} + \underline{F^+_{iN}} + \overline{F^-_{iN}} + \overline{F^+_{iN}})}{4}.$$

Properties 5.1

The defined rough neutrosophic cosine hamming similarity operator cot(M, *N*) between IRNSs M and N satisfies the following properties:

1. cot(M, N) = 1 if and only if M = N.
2. cot(M, N) = cot(N, M).

Proof:

1.For any two RNSs M and N, if M = N, then the following relations hold

$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i), \Delta F_M(x_i) = \Delta F_N(x_i).$

Hence,

$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0,$

$|\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$

Thus cot(M,N) = 1

Conversely,

If cot(M,N) = 1, then

$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0,$

$|\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$

Since cot($\frac{\Pi}{4}$) = 1. So we can write

$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i),$

$\Delta F_M(x_i) = \Delta F_N(x_i).$

Hence M = N**.**

2. As,

$$\cot(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cot\Big(\frac{\Pi}{4} + \frac{\Pi}{12}\big(|\Delta T_M(x_i) - \Delta T_N(x_i)| + |\Delta I_M(x_i) - \Delta I_N(x_i)| + |\Delta F_M(x_i) - \Delta F_N(x_i)|\big)\Big)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\cot\Big(\frac{\Pi}{4} + \frac{\Pi}{12}\big(|\Delta T_N(x_i) - \Delta T_M(x_i)| + |\Delta I_N(x_i) - \Delta I_M(x_i)| + |\Delta F_N(x_i) - \Delta F_M(x_i)|\big)\Big)$$

$$= \cot(N, M).$$

This completes the proof.

6. Decision making under trigonometric interval rough neutrosophic Hamming similarity measures

In this section, we apply interval rough cosine, sine and cotangent Hamming similarity measures between IRNSs to the multi-attribute decision making problem. Consider C={C_1,C_2, ... ,C_m} be the set of attributes and *A*={A_1,A_2, ... , A_n} be a set of alternatives. Now we provide an algorithm for MADM problems involving interval rough neutrosophic information.

Algorithm 1. (see Fig 1)

Step 1: Construction of the decision matrix with interval rough neutrosophic number

Decision maker considers the decision matrix with respect to m alternatives and n attributes in
terms of interval rough neutrosophic numbers as follows:

Table1: Interval Rough neutrosophic decision matrix

$$D = \langle Z_{ij} \rangle_{n \times m} = \begin{pmatrix} Z_{11} & Z_{12} & \dots\dots\dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots\dots\dots & Z_{2m} \\ \dots & \dots & \dots\dots\dots\dots & \\ \dots & \dots & \dots\dots\dots\dots & \\ \dots & \dots & \dots\dots\dots\dots & \\ Z_{n1} & Z_{n2} & \dots\dots\dots & Z_{nm} \end{pmatrix}$$

Where

$Zij = \langle([\underline{T^-_{iM}}, \underline{T^+_{iM}}], [\underline{I^-_{iM}}, \underline{I^+_{iM}}], [\underline{F^-_{iM}}, \underline{F^+_{iM}}],$ $[\overline{T^-_{iM}}, \overline{T^+_{iM}}], [\overline{I^-_{iM}}, \overline{I^+_{iM}}], [\overline{F^-_{iM}}, \overline{F^+_{iM}}])\rangle$ with

$0 \le \vee_{y\in[x]_R}\{T^U_A(y)\} + \wedge_{y\in[x]_R}\{I^U_A(y)\} + \wedge_{y\in[x]_R}\{F^U_A(y)\}] \le 3$

Step 2: Determination of the ideal alternative

Generally, the evaluation attribute can be categorized into two types: benefit type attribute and cost type attribute. We define an ideal alternative S^* .

For benefit type attribute,

S^*=

$\{(\min_i \underline{T_{ij}}, \max_i \underline{I_{ij}}, \max_i \underline{F_{ij}}), (\max_i \overline{T_{ij}}, \min_i \overline{I_{ij}}, \min_i \overline{F_{ij}})\}$.

For cost type attribute,

S^*=

$\{(\max_i \underline{T_{ij}}, \min_i \underline{I_{ij}}, \min_i \underline{F_{ij}}), (\min_i \overline{T_{ij}}, \max_i \overline{I_{ij}}, \max_i \overline{F_{ij}})\}$.

Step 3: Determination of the interval rough trigonometric neutrosophic Hamming similarity function of the alternatives

We compute interval rough trigonometric neutrosophic similarity measure between the ideal alternative S^* and each alternative $Z_i = \langle Z_{ij} \rangle_{n \times m}$ for all i = 1, ….., n and j = 1, ….., m.

Step 4: Ranking the alternatives

Using the interval rough trigonometric neutrosophic similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative is selected with the highest similarity value.

Step 5: End.

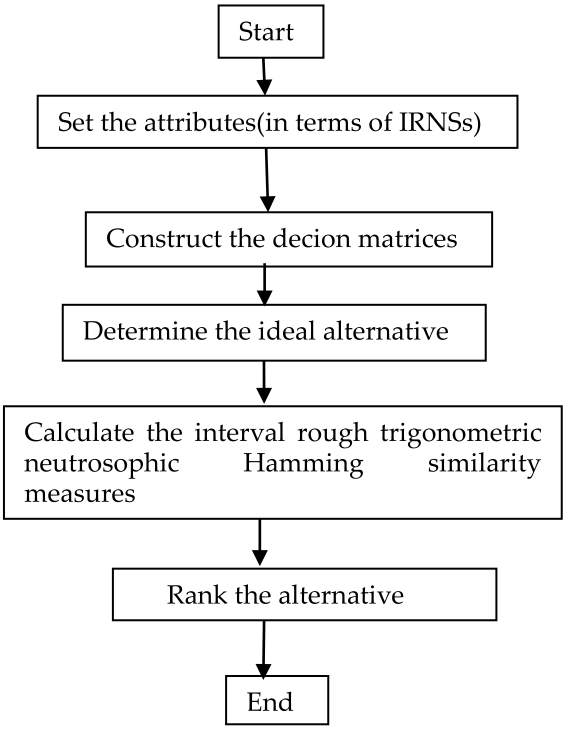

Fig 1. A flowchart of the proposed decision making method

7. Numerical example

Assume that a decision maker intends to select the most suitable laptop for random use from the three initially chosen laptops (S_1, S_2, S_3) by considering four attributes namely: features C_1, reasonable price C_2, customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 5, the considered problem is solved by the following steps:

Step1: Construct the decision matrix with interval rough neutrosophic number

The decision maker construct the decision matrix with respect to the three alternatives and four attributes in terms of interval rough neutrosophic number.

	C_1	C_2	C_3	C_4
S_1	<([.6, .7], [.3, .5], [.3, .4]), ([.8, .9], [.1, .3], [.1, .2])>	<([.5, .7], [.3, .4], [.1, .2]), ([.7, .9], [.3, .5], [.3, .4])>	<([.5, .6], [.4, .5], [.4, .6]), ([.7, .8], [.2, .4], [.3, .4])>	<([.8, .9], [.3, .4], [.5, .6]), ([.7, .8], [.3, .5], [.3, .5])>
S_2	<([.7, .8], [.2, .3], [.0, .2]), ([.7, .9], [.1, .2], [.1, .2])>	<([.6, .7], [.1, .2], [.0, .2]), ([.6, .7], [.1, .3], [.1, .3])>	<([.5, .7], [.2, .3], [.1, .2]), ([.6, .9], [.3, .5], [.2 .4])>	<([.7, .8], [.3, .5], [.1,.3]), ([.5, .7], [.5, .6], [.2, .3])>
S_3	<([.6, .7], [.3, .4], [.0, .3]), ([.6, .9], [.1, .2], [.1, .2])>	<([.5, .7], [.2, .4], [.2, .4]), ([.6, .8], [.1, .3], [.1, .2])>	<([.6, .8], [.2, .4], [.3, .4]), ([.6, .8], [.2, .5], [.3, .5])>	<([.4, .7], [.2, .4], [.4, .5]), ([.5, .8], [.2, .5], [.0, .2])>

Step 2: Determine the benefit type attribute and cost type attribute

Here three benefit type attributes C_1, C_2, C_3 and one cost type attribute C_4. We calculate the ideal alternative as follows:

$$S^* =$$
$$\{<([.6,.7],[.3,.5],[.3,.4]),([.8,.9],[.1,.2],[.1,.2])>,$$
$$<([.5,.7],[.3,.4],[.2,.4]),([.7,.9],[.1,.3],[.1,.2])>,$$
$$<([.5,.6],[.4,.5],[.4,.6]),([.7,.9],[.2,.4],[.2,.4])>,$$
$$<([.8,.9],[.2,.4],[.1,.3]),([.5,.7],[.5,.6],[.3,.5])>)>\}$$

Step3: Calculate the interval rough trigonometric neutrosophic Hamming similarity measure of the alternatives

$$\cos(S_1, S^*) = 0.999998923,$$
$$\cos(S_1, S^*) = 0.999997135,$$
$$\cos(S_1, S^*) = 0.999998505,$$

$$\sin(S_1, S^*) = 0.999531651,$$
$$\sin(S_1, S^*) = 0.997658256,$$

$\sin(S_1, S^*) = 0.998343644,$
$\cot(S_1, S^*) = 70.25049621,$
$\cot(S_1, S^*) = 67.22363275,$
$\cot(S_1, S^*) = 68.81008448.$

Step 4: Rank the alternatives

Ranking of alternatives is prepared based on the descending order of similarity measures. The highest value reflects the best alternatives.
Here,
$\cos(S_1, S^*) > \cos(S_3, S^*) > \cos(S_2, S^*).$
$\sin(S_1, S^*) > \sin(S_3, S^*) > \sin(S_2, S^*).$
$\cot(S_1, S^*) > \cot(S_3, S^*) > \cot(S_2, S^*).$

Hence, the laptop S_1 is the best alternative for random use.

8. Conclusions

In this paper, we have proposed interval rough trigonometric Hamming similarity measures and proved their properties. We have developed three MADM strategies base on sine, cosine and cotangent similarity measures under interval rough neutrosophic environment. Then we solved an illustrative numerical example to demonstrate the feasibility, applicability of the developed strategies. The concept presented in this paper can be applied other multiple attribute decision making problems such as teacher selection [30, 31, 32], school selection [33], weaver selection [34, 35, 36], brick field selection [37, 38], logistics center location selection [39, 40], data mining [41] etc. under interval rough neutrosophic environment.

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