Neutrosophic Sets and Systems, Vol. 32, 2020
University of New Mexico

Neutrosophic Geometric Programming (NGP) with (Max, Product) Operator; An Innovative Model

Huda E. Khalid
Telafer University, Head of the Scientific Affairs and Cultural Relations Department, Telafer, Iraq.
E-mail: hodaesmail@yahoo.com

Abstract. In this paper, a neutrosophic optimization model has been first constructed for the neutrosophic geometric programming subject to (max-product) neutrosophic relation constraints. For finding the maximum solution, two new operations (i.e. $\bowtie, \Theta$) between a_{ij} and b_i have been defined, which have a key role in the structure of the maximum solution. Also, two new theorems and some propositions are introduced that discussed the cases of the incompatibility in the relational equations $A o x = b$, with some properties of the operation Θ. Numerical examples have been solved to illustrate new concepts.

Keyword: Neutrosophic Geometric Programming (NGP); (max-product) Operator; Neutrosophic Relation Constraints; Maximum Solution; Incompatible Problem; Pre-Maximum Solution; Relational Neutrosophic Geometric Programming (RNGP).

1. Introduction

The first scientist who put forward the fuzzy relational equations was Elie Sanchez, a famous fuzzy biology mathematician in 1976 [2], while the theoretical concept of the neutrosophic logic has been put by the popular polymath Florentin Smarandache at 1995 [11]. B. Y. Cao constructed the mathematical models of fuzzy relation geometric programming (FRGP) at 2005 [1], his works include the structuring of the maximum and minimum solution of the (FRGP) depending upon the original model for the maximum solution and the minimum solution for the fuzzy relation equations that was put by Elie Sanchez. At 2015, Huda E. Khalid introduced an original structure of the maximum solution for the fuzzy neutrosophic relation geometric programming (FNRGP) [6], Also at 2016, she put a novel algorithm for finding the minimum solution for the same (FNRGP) problems [7]. As of 2016 so far Huda E. Khalid et al [3-10] introducing a big qualitative shift in the concept of neutrosophic geometric programming (NGP) by establishing new concepts for the notion of (over, off, under) in the same (NGP), as well as she introduced and for the first time, a new type of the neutrosophic geometric programming using (over, off, under) neutrosophic less than or equal which contained a new version of the convex condition, furthermore, new decomposition theorems of neutrosophic sets were presented, and new representations for the neutrosophic sets using (α, β, γ)-cuts, with strong (α, β, γ)-cuts had been defined.

In this article, section 2 contains the preliminaries which are necessary for the sake of this paper, while in section 3, a max- product neutrosophic relation geometric programming model has been proposed with an innovative investigation of the maximum solution for this model and two new theorems with some propositions, section 4 presents numerical examples to illustrate the proposed method. The final section was dedicated to the conclusion.

2. Basic Concepts

Without loss of generality, the elements of b must be rearranged in decreasing or increasing order and the elements of the matrix A are correspondingly rearranged.

2.1 Definition [7]

In this definition, the author proposed the following axioms:

a- decreasing partial order

1-The greatest element in $[0,1) \cup \mathrm{I}$ is equal to I, $\max(\mathrm{I}, \mathrm{x}) = \mathrm{I} \quad \forall\ \mathrm{x} \in [0,1)$

2- The fuzzy values in a decreasing order will be rearranged as follows: $1 > \mathrm{x}_1 > \mathrm{x}_2 > \mathrm{x}_3 > \cdots > \mathrm{x}_\mathrm{n} \geq 0$

3- One is the greatest element in $[0,1] \cup \mathrm{I}$, $\max(\mathrm{I}, 1) = 1$

b- Increasing partial order

1- the smallest element in $(0,1] \cup \mathrm{I}$ is I , $\min(\mathrm{I}, \mathrm{x}) = \mathrm{I} \quad \forall\ \mathrm{x} \in (0,1]$

2- The fuzzy values in increasing order will be rearranged as follows: $0 < \mathrm{x}_1 < \mathrm{x}_2 < \mathrm{x}_3 < \cdots < \mathrm{x}_\mathrm{n} \leq 1$

3- Zero is the smallest element in $[0,1] \cup \mathrm{I}$, $\min(\mathrm{I}, 0) = 0$

2.2 Definition [7]

If there exists a solution to $Aox = b$ it's called compatible. Suppose $X(A, b) = \{(x_1, x_2, \dots, x_n)^T \in [0,1]^n \cup I,\ I^n = I, n > 0\ |Aox = b, x_j \in [0,1] \cup I\ \}$ is a solution set of $Aox = b$ we define $x^1 \leq x^2 \Leftrightarrow x_j^1 \leq x_j^2\ (1 \leq j \leq n), \forall\, x^1, x^2 \in \mathrm{X}(\mathrm{A}, \mathrm{b})$. Where " $\leq$ " is a partial order relation on $\mathrm{X}(\mathrm{A}, \mathrm{b})$.

2.3 Corollary [1]

If $\mathrm{X}(\mathrm{A}, \mathrm{b}) \neq \emptyset$. Then $\hat{x} \in X(A, b)$.

Similar to fuzzy relation equations, the above corollary works on neutrosophic relation equations.

2.4 Basic Notes [3, 10]

1. A component I to the zero power is undefined value, (i.e. I^{0} is undefined), since. $I^{0} = I^{1+(-1)} = I^{1} * I^{-1} = \frac{I}{I}$, which is an impossible case (avoid to divide by I).

2. The value of I to the negative power is undefined (i.e. I^{-n}, $n > 0$ is undefined).

3. The Innovative Structure of the Maximum Solution.

We call

$$\left.\begin{array}{l} \min f(x) = \left(c_1 . x_1^{\gamma_1}\right) \vee \left(c_2 . x_2^{\gamma_2}\right) \vee \dots \vee \left(c_n . x_n^{\gamma_n}\right) \\ s.t. \qquad Aox = b \\ x_j \in [0,1] \cup I, \quad 1 \le j \le n \end{array}\right\} \tag{1}$$

A $(\vee, .)$ (max- product) neutrosophic geometric programming, where $A = (a_{ij})$, $1 \le i \le m, 1 \le j \le n$, is $(m \times n)$ dimensional neutrosophic matrix, $x = (x_1, x_2, \dots, x_n)^T$ an n-dimensional variable vector, $b = (b_1, b_2, \dots, b_m)^T$ $(b_i \in [0,1] \cup I)$ an m- dimensional constant vector, $c = (c_1, c_2, \dots, c_n)^T$ $(c_j \ge 0)$ an n- dimensional constant vector, γ_j is an arbitrary real number, and the composition operator $''o''$ is $(\vee, .)$, i.e. $\bigvee_{j=1}^{n} (a_{ij} . x_j) = b_i$.

Note that the program (1) is undefined and has no minimal solution in the case of $\gamma_j < 0$ with some $x_j's$ taking indeterminacy value. Therefore, if $\gamma_j < 0$ with indeterminacy value in some $x_j's$, then the greatest solution $\hat{x}_j$ is an optimal solution for problem (1), the author introduced theorem 3.4 to treat this issue.

3.1 The Shape of the Maximum Solution $\hat{\mathbf{x}}$.

Since 1976, the biological mathematician Elie Sanchez put the formula of the maximum solution in both composite fuzzy relation equations of type $(\vee, \wedge)$ operator and $(\vee, .)$ operator [2], these definitions won't be adequate with neutrosophic relation equations especially neutrosophic geometric programming type, therefore and for the importance of relational neutrosophic geometric programming (RNGP) in real-world problems, the author established a new structure for the maximum solution of (RNGP) with the $(\vee, \wedge)$operator in ref. [6], while this article was dedicated to set up the maximum solution of (RNGP) with the $(\vee, .)$ operator.

Every mathematician who works with neutrosophic theory know that the generality which characterizes the neutrosophic theory are determined in many ways of which,

$\max(I, x) = \min(I, x) = I \quad \forall\, x \in (0,1)$

This property gives some vague and difficulty for determining the maximum solution of the relation equations $Aox = b$, the author still searches about the answer of the following question.

How will be the shape of the greatest solution $\hat{x}$?

Actually, any single solution (the same solution that suggested by Elie Sanchez 1976) would not be accepted and won't be appropriate for the program (1), unless there are two integrated pre-maximum solutions gathered to get the final shape of $\hat{x}$, as follow:

1. The first integrated pre-maximum solution named $\hat{x}_{v1}$ which supports the fuzzy part of the problem, this solution has an adjoint matrix named A_{v1}, this adjoint matrix is derived from the matrix A.
2. The second integrated pre-maximum solution named $\hat{x}_{v2}$ which supports the neutrosophic part of the problem, this solution has an adjoint matrix named A_{v2}, which is derived from the matrix A too.

   The following definition describes the mathematical formula of $\hat{x}_{v1}$ and $\hat{x}_{v2}$.

3.2 Definition

$$a_{ij} \bowtie b_i = \begin{cases} \frac{b_i}{a_{ij}}, & if \;\; a_{ij} > b_i\,,\;\; a_{ij} \in [0,1],\;\; b_i \in [0,1] \\ 1, & if \;\; a_{ij} \le b_i\,,\;\; a_{ij} \in [0,1],\;\; b_i \in [0,1] \\ 1\,, & if \;\; a_{ij} \in [0,1],\;\; b_i = nI, n \in (0,1] \end{cases} \tag{2}$$

$$a_{ij} \Theta b_i = \begin{cases} \frac{nI}{a_{ij}}, & if \;\; a_{ij} > n\,,\;\; a_{ij} \in [0,1], b_i = nI, n \in (0,1] \\ 1, & if \;\; a_{ij} \le n\,,\;\; a_{ij} \in [0,1], b_i = nI, n \in (0,1] \\ not\ comp. & if \;\; a_{ij} = mI\,, m \in (0,1]\,, b_i \in [0,1] \cup I \\ 1 & if \;\; a_{ij}, b_{ij} \in [0,1] \end{cases} \tag{3}$$

Where $\bowtie$ is an operator defined at [0,1], while the operator Θ is defined at $[0,1] \cup I$.

Let $\hat{x}_j = \wedge_{i=1}^{m}(a_{ij} \bowtie b_i), \quad (1 \le j \le n)\,, \qquad (4)$

be the components of the pre-maximum solution $\hat{x}_{v1}$, (i.e. $\hat{x}_{v1} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$).

Let $\hat{x}_j = \wedge_{i=1}^{m}(a_{ij} \Theta b_i), \quad (1 \le j \le n)\,, \qquad (5)$

be the components of the pre maximum solution $\hat{x}_{v2}$, (i.e. $\hat{x}_{v2} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$).

Now the following question will be raised,

Which one $\hat{x}_{v1}$ or $\hat{x}_{v2}$ should be the exact maximum solution?

Neither $\hat{x}_{v1}$ nor $\hat{x}_{v2}$ will be the exact solution! the exact solution is the integration between them. Before solving $Ao\hat{x} = b$, we first define the matrices A_{v1}, A_{v2}.

Let A_{v1} be a matrix has the same dimension and the same rows elements of A except for those rows of the indexes $i = i_o$ corresponding to those indexes of $b_{i_o} = nI$, those special rows of A_{v1} will be zeros.

Let A_{v2} be a matrix has the same dimension and the same rows elements of A except for those rows of the indexes $i = i_o$ corresponding to those indexes of $b_{i_o} \in [0,1]$, those special rows of A_{v2} will be zeros.

Consequently,

$$Ao\hat{x} = b = (A_{v1}o\hat{x}_{v1}) + (A_{v2}o\hat{x}_{v2}) \tag{6}$$

The formula (6) is the greatest solution in $X(A, b)$.

The maximum value of the objective function $f(\hat{x}) = f(\hat{x}_{v1}) \vee f(\hat{x}_{v2})$.

3.3 Theorem

If $a_{ij} = mI$, $m \in (0,1], b_i \in [0,1] \cup I$ then $Aox = b$, is not compatible.

Proof

Let $a_{ij} = mI$, $b_i \in [0,1] \cup I$, the essential question in this case is

What is the value of $x_j \in [0,1] \cup I$ satisfying

$$\vee_{1 \le j \le n}(a_{ij}.x_j) = b_i \ ? \tag{7}$$

It is well known that the equation (7) can be written as an upper-bound constraint and a lower-bound constraint, that is,

$$\vee_{1 \le j \le n}(a_{ij}.x_j) \le b_i \tag{8}$$

$$\vee_{1 \le j \le n}(a_{ij}.x_j) \ge b_i \tag{9}$$

First,

The inequality (8) can be written in n constraints:

$a_{ij}.x_j \leq b_i$, $i.e.$ $x_j \leq \frac{b_i}{a_{ij}}$,$1 \leq j \leq n$.

Hence $x_j \leq \wedge\left(\frac{b_i}{a_{ij}}\right)$, where the notation '' $\wedge$'' denotes the minimum operator.

So, we have $x_j \in [0, \wedge\left(\frac{b_i}{a_{ij}}\right)] \cup I$, but $a_{ij} = mI$, this is a contradict for the fact that the variables of the system $Aox = b$ are being in the interval $[0,1] \cup I$.

Second,

The inequality (9) can be written in n constraints:

$(a_{ij}.x_j) \geq b_i$, $i.e.$ $x_j \geq \frac{b_i}{a_{ij}}$,$1 \leq j \leq n$.

Hence, $x_j \geq \vee\left(\frac{b_i}{a_{ij}}\right)$, where the notation '' $\vee$'' denotes the maximum operator.

Thus, we have $x_j \in [\vee\left(\frac{b_i}{a_{ij}}\right), 1] \cup I$, but $a_{ij} = mI$, in this proof we faced the division on the indeterminate component (I) which is prohibited behavior. Consequently the variable x_j will either belong to the interval $[0, \wedge (b_i/I)] \cup I$ or belong to the interval$[\vee (b_i/I), 1] \cup I$, this implies that the system of the relation equation $Aox = b$ will be not compatible.
Therefore, the system of the relative equations $Aox = b$ is incompatible at $a_{ij} = mI, m \in (0,1]$. So, the restriction of $Aox = b$ for being compatible is that all elements of the matrix A $(i.e. a_{ij})$ are belonging to the interval [0,1]. □

3.4 Theorem

If $\gamma_j < 0$ $(1 \leq j \leq n)$, then the greatest solution to the problem (1) is an optimal solution.

Proof

Since $\gamma_j < 0$ $(1 \leq j \leq n)$, with $x_j \in [0,1] \cup I$, then $\frac{d(x_j^{\gamma_j})}{dx_j} = \gamma_j x_j^{\gamma_j - 1} < 0$ for each $x_j \in [0,1] \cup I$, this means that ${x_j}^{\gamma_j}$ is monotone decreasing function of x_j. It is clear that $c_j x_j^{\gamma_j}$ is also a monotone decreasing function about x_j. Therefore, $\forall\, x \in X(A, b)$, when $x \leq \hat{x}$, then $c_j. x_j^{\gamma_j} \geq c_j. \hat{x}_j^{\gamma_j}$ $(1 \leq j \leq n)$, such that $f(x) \geq f(\hat{x})$, so $\hat{x}$ is an optimal solution to the problem (1).

It remains to study the case that if $\gamma_j < 0$ with the component $\hat{x}_j$ in $\hat{x}_{v2}$ equal to I, we know that I^n is undefined for $n \leq 0$, in this case, the component $x_j = I$ that has a power $\gamma_j < 0$ will be replaced by that corresponding x_j in the $\hat{x}_{v1}$. □

3.5 Proposition

Let $a \in (0,1), b = mI$ & $c = nI, n, m \in (0,1]$, $if\ m \geq n$, then $a\, \Theta b \geq a\, \Theta\, c$.

Proof

1) Let $a > m \Rightarrow a > n,$

But we have $m \geq n \Rightarrow b \geq c \Rightarrow \frac{b}{a} \geq \frac{c}{a} \Rightarrow a\, \Theta b \geq a\, \Theta\, c$.

2) Let $a \leq m \Rightarrow a\, \Theta\, b = 1$, since $m \geq n \Rightarrow a\, \Theta\, c \leq 1$

Hence, $a\, \Theta\, c \leq a\, \Theta\, b$.

3.6 Corollary

Let $a \in (0,1), b = mI,\ \ c = nI, m, n \in (0,1]$, if $m \geq n$ then $a\, \Theta\, (b \vee c) \geq a\, \Theta\, c$

Proof

Since $m \geq n \Rightarrow b \geq c \Rightarrow b \vee c = b$, from proposition 2.5, we have

$a\, \Theta\, b \geq a\, \Theta\, c \qquad (replacing\ b \vee c\ instead\ of\ b) \Rightarrow a\, \Theta\, (b \vee c) \geq a\, \Theta\, c$.

3.7 Proposition

Let $a \in (0,1), b = mI, m \in (0,1]$, then $a.(a\, \Theta\, b) = a \wedge b$.

Proof

1) Let $a > m \Rightarrow \frac{mI}{a} = \frac{b}{a} = a\, \Theta\, b$ [multiply both sides by a] $\Rightarrow$

$$b = a.(a\, \Theta\, b) \tag{10}$$

2) Let $a \leq m \Rightarrow a\, \Theta\, b = 1$ [multiply both sides by a] $\Rightarrow$

$$a = a.(a\, \Theta\, b) \tag{11}$$

From (10) & (11) we have $a.(a\, \Theta\, b) = a \wedge b$.

3.8 Proposition

Let $a \in (0,1), b = mI, m \in (0,1]$, then $a.(a \,\Theta\, b) = \begin{cases} b & a > am \\ 1 & a \le am \end{cases}$.

Proof

1) Let $a > am$, from definition (3.2) we have $a \,\Theta\, (a.m) = \frac{a.mI}{a} = mI = b$.
2) Let $a \le am$, again from definition (3.2) we have $a \,\Theta\, (a.b) = 1$.

Hence, $a \,\Theta\, (a.b) = \begin{cases} b & a > am \\ 1 & a \le am \end{cases}$

4 Numerical examples

In the upcoming examples, the (max- product) neutrosophic geometric problem is considered.

4.1 Example

Let $\min f(x) = (0.3.x_1^2) \vee (1.8I\ .x_2^{\frac{1}{3}}) \vee (I\ .x_3^{\frac{1}{4}})$

s. t. $Aox = b$

$x_j \in [0,1] \cup I \quad (1 \le j \le n)$

Where $b = (1, \frac{1}{3}I, \frac{1}{5}I)^T$, $A = \begin{pmatrix} .6 & 1 & .2 \\ .5 & .2 & .1 \\ .3 & .5 & .1 \end{pmatrix}_{3\times 3}$

Using the formula (2), we can find the components of $\hat{x}_{v1}$ as follows

$$\hat{x}_1 = \bigwedge_{i=1}^{3} (a_{i1} \bowtie b_i) = (a_{11} \bowtie b_1) \wedge (a_{21} \bowtie b_2) \wedge (a_{31} \bowtie b_3)$$

$$= (0.6 \bowtie 1) \wedge \left(0.5 \bowtie \frac{1}{3}I\right) \wedge (0.3 \bowtie 0.2I) = 1 \wedge 1 \wedge 1 = 1$$

$$\hat{x}_2 = \bigwedge_{i=1}^{3} (a_{i2} \bowtie b_i) = (a_{12} \bowtie b_1) \wedge (a_{22} \bowtie b_2) \wedge (a_{32} \bowtie b_3)$$

$$= (1 \bowtie 1) \wedge \left(0.2 \bowtie \frac{1}{3}I\right) \wedge (0.5 \bowtie 0.2I) = 1 \wedge 1 \wedge 1 = 1$$

$$\hat{x}_3 = \bigwedge_{i=1}^{3} (a_{i3} \bowtie b_i) = (a_{13} \bowtie b_1) \wedge (a_{23} \bowtie b_2) \wedge (a_{33} \bowtie b_3)$$

$$= (0.2 \bowtie 1) \wedge \left(0.1 \bowtie \frac{1}{3}I\right) \wedge (0.1 \bowtie 0.2I) = 1 \wedge 1 \wedge 1 = 1$$

$\therefore \hat{x}_{v1} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)^T = (1,1,1)^T$

Using the formula (3), we can find the components of $\hat{x}_{v2}$ as follows

$$\hat{x}_1 = \bigwedge_{i=1}^{3} (a_{i1}\Theta\, b_i) = (a_{11}\Theta\, b_1) \wedge (a_{21}\Theta\, b_2) \wedge (a_{31}\Theta\, b_3)$$

$$= (0.6\;\Theta\;1) \wedge \left(0.5\;\Theta\;\frac{1}{3}I\right) \wedge (0.3\;\Theta\;0.2I) = 1 \wedge \frac{1/3}{0.5}I \wedge \frac{0.2}{0.3}I = \frac{2}{3}I$$

$$\hat{x}_2 = \bigwedge_{i=1}^{3} (a_{i2}\Theta\, b_i) = (a_{12}\Theta\, b_1) \wedge (a_{22}\Theta\, b_2) \wedge (a_{32}\Theta\, b_3)$$

$$= (1\;\Theta\;1) \wedge \left(0.2\;\Theta\;\frac{1}{3}I\right) \wedge (0.5\;\Theta\;0.2I) = 1 \wedge 1 \wedge \frac{2}{5}I = \frac{2}{5}I$$

$$\hat{x}_3 = \bigwedge_{i=1}^{3} (a_{i3}\Theta b_i) = (a_{13}\Theta\, b_1) \wedge (a_{23}\Theta\, b_2) \wedge (a_{33}\Theta\, b_3)$$

$$= (0.2\;\Theta\;1) \wedge \left(0.1\;\Theta\;\frac{1}{3}I\right) \wedge (0.1\;\Theta\;0.2I) = 1 \wedge 1 \wedge 1 = 1$$

$$\therefore \hat{x}_{v2} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)^T = \left(\frac{2}{3}I, \frac{2}{5}I, 1\right)^T$$

In this example, $A_{v1} = \begin{pmatrix} .6 & 1 & .2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $A_{v2} = \begin{pmatrix} 0 & 0 & 0 \\ .5 & .2 & .1 \\ .3 & .5 & .1 \end{pmatrix}$,

$$Ao\hat{x} = (A_{v1}o\hat{x}_{v1}) + (A_{v2}o\hat{x}_{v2}) = \begin{pmatrix} .6 & 1 & .2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} o \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ .5 & .2 & .1 \\ .3 & .5 & .1 \end{pmatrix} o \begin{bmatrix} \frac{2}{3}I \\ \frac{2}{5}I \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \frac{1}{3}I \\ \frac{1}{5}I \end{bmatrix} = b$$

Since $Ao\hat{x} = b$, then there is a solution in $X(A, b)$ and $\hat{x}$ is the greatest solution to $Aox = b$. The value of $f(\hat{x})$ is calculated as follow,

$f(\hat{x}) = f(\hat{x}_{v1}) \vee f(\hat{x}_{v2})$

$f(\hat{x}) = \langle (0.3\,.\,(1)^2) \vee \left(1.8I\,.\,(1)^{\frac{1}{3}}\right) \vee \left(I\,.\,(1)^{\frac{1}{4}}\right) \rangle \vee \langle \left(0.3.\,(\frac{2}{3}I)^2\right) \vee \left(1.8I\,.\left(\frac{2}{5}I\right)^{\frac{1}{3}}\right) \vee$

$\left(I\,.\,(1)^{\frac{1}{4}}\right) \rangle = \langle (0.3\,) \vee (1.8I) \vee (I\,) \rangle \vee \langle (0.133I) \vee (1.33I) \vee (I) \rangle = 1.8I$

Do not forget that the indeterminate component I to the power n where $n > 0$ is equal to I (i.e. $I^n = I \;\; for\, n > 0$).

4.2 Example

Let $A = \begin{pmatrix} 0.1 & 1 & 0.4 \\ I & 0.9 & 0 \\ 0.5 & 0.2I & 0.7 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 0.3I \\ 0.6 \end{pmatrix},$

It easy to see that some components of the matrix A are of the form $a_{ij} = mI, m \in (0,1]$, while $b_i \in [0,1] \cup I$, in this case, and by theorem (3.2), the system of the relation equation $Aox = b$ is incompatible.

4.3 Example

Let $\min f(x) = (0.2I.x_1^{-\frac{2}{3}}) \vee (1.3.x_2^{\frac{1}{3}}) \vee (I\,.x_3^{\frac{1}{2}}) \vee (0.35.x_4^{-2})$

s. t. $\;\; Aox = b$

$x_j \in [0,1] \cup I \quad (1 \le j \le n)$

Where $\;\; b = (0.3, 0.7I, 0.5, 0.2I)^T \;,\; A = \begin{pmatrix} .2 & .3 & .4 & .6 \\ .3 & .2 & .9 & .8 \\ 1 & 0 & .1 & 1 \\ 0 & .5 & 1 & 0 \end{pmatrix}_{4\times 4}$

Using the formula (2), the components of $\hat{x}_{v1}$ are

$$\hat{x}_1 = \bigwedge_{i=1}^{4} (a_{i1} \bowtie b_i) = 0.5$$

$$\hat{x}_2 = \bigwedge_{i=1}^{4} (a_{i2} \bowtie b_i) = 1$$

$$\hat{x}_3 = \bigwedge_{i=1}^{4} (a_{i3} \bowtie b_i) = \frac{3}{4}$$

$$\hat{x}_4 = \bigwedge_{i=1}^{4} (a_{i4} \bowtie b_i) = \frac{1}{2}$$

$$\therefore\ \hat{x}_{v1} = (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4)^T = \left(0.5, 1, \frac{3}{4}, 0.5\right)^T$$

Using the formula (3), the components of $\hat{x}_{v2}$ are

$$\hat{x}_1 = \bigwedge_{i=1}^{4} (a_{i1} \Theta\, b_i) = 1$$

$$\hat{x}_2 = \bigwedge_{i=1}^{4} (a_{i2} \Theta\, b_i) = \frac{2}{5} I$$

$$\hat{x}_3 = \bigwedge_{i=1}^{4} (a_{i3} \Theta b_i) = 0.2I$$

$$\hat{x}_4 = \bigwedge_{i=1}^{4} (a_{i4}\, \Theta b_i) = 0.875I$$

$$\therefore\ \hat{x}_{v2} = (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4)^T = \left(\frac{2}{5} I, 1, 0.2I, 0.875I\right)^T$$

In this example, $A_{v1} = \begin{pmatrix} .2 & .3 & .4 & .6 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & .1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, $A_{v2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ .3 & .2 & .9 & .8 \\ 0 & 0 & 0 & 0 \\ 0 & .5 & 1 & 0 \end{pmatrix}$,

$$A o \hat{x} = (A_{v1} o \hat{x}_{v1}) + (A_{v2} o \hat{x}_{v2})$$

$$= \begin{pmatrix} .2 & .3 & .4 & .6 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & .1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} o \begin{bmatrix} 0.5 \\ 1 \\ \frac{3}{4} \\ 0.5 \end{bmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ .3 & .2 & .9 & .8 \\ 0 & 0 & 0 & 0 \\ 0 & .5 & 1 & 0 \end{pmatrix} o \begin{bmatrix} \frac{2}{5} I \\ 1 \\ 0.2I \\ 0.875I \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 \\ 0.7I \\ 0.5 \\ 0.2I \end{bmatrix} = b$$

Since $Ao\hat{x} = b$, then there is a solution in $X(A,b)$ and $\hat{x}$ is the greatest solution to $Aox = b$. The value of $f(\hat{x})$ is calculated as follow,

$$f(\hat{x}) = f(\hat{x}_{v1}) \vee f(\hat{x}_{v2})$$

$$f(\hat{x}) = \langle\left(0.2I\,.\,(\tfrac{1}{2})^{-\frac{3}{2}}\right) \vee \left(1.3.\,(1)^{\frac{1}{3}}\right) \vee \left(I\,.\left(\tfrac{3}{4}\right)^{\frac{1}{2}}\right) \vee (0.35.\,(0.5)^{-2})\rangle \vee$$

$$\langle\left(0.2I\,.\,(1)^{-\frac{3}{2}}\right) \vee \left(1.3.\,(0.4I)^{\frac{1}{3}}\right) \vee \left(I\,.\,(0.2I)^{\frac{1}{2}}\right) \vee (0.35.\,(0.5)^{-2})\rangle = \langle(0.57I\,) \vee (1.3) \vee (0.87I\,) \vee (0.5I\,)\rangle \vee \langle(0.2I) \vee (0.96I) \vee (0.45I) \vee (0.5I)\rangle = 1.3$$

5 Conclusion

It is important to know that the fuzzy geometric programming problems (FGPP) have wide applications in the business management, communication system, civil engineering, mechanical engineering, structural design and optimization, chemical engineering, optimal control, decision making, and electrical engineering, unfortunately, the fuzzy logic lacks to cover the indeterminate solution of any real-world problems, this pushed the author to construct a new branch of the neutrosophic geometric programming (NGP) problems subject to neutrosophic relation equations (NRE) and made a series of articles in an attempt to cover the theoretical sides of (NGP) problems. This paper contains a new (NGP) model subject to (NRE) with setting up a definition for the maximum solution of this program as well as some new theorems dealt with the consistency of the problem and some propositions of the new operation Θ. The future prospects are to make a deep study for the above-mentioned applications from the point of view of relational neutrosophic geometric programming (RNGP) problems.

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Received: 20 Feb, 2020. Accepted: 21 Mar, 2020