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An Introduction to The Split-Complex Symbolic 2-Plithogenic Numbers

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Abstract:

The objective of this paper is to combine split-complex numbers with symbolic 2-plithogenic numbers in one algebraic structure called split-complex symbolic 2-plithogenic real numbers.

Also, many elementary properties of the suggested system will be handled such as Invertibility and idempotency by many related theorems and examples.

Keywords: Split-complex, symbolic 2-plithogenic number, invertible, idempotent.

Introduction and basic concepts

Split-complex numbers are considered as a generalization of real numbers, where they are defined as follows:

$S = \{a + bJ;\ J^2 = 1, a, b \in R\}$ [1].

Split-complex numbers together make a commutative ring with many interesting properties [2-4, 24].

Addition on S is defined as follows:

$(t_0 + t_1J) + (k_0 + k_1J) = (t_0 + k_0) + J(t_1 + k_1)$.

Multiplication on S is defined as follows:

$(t_0 + t_1 J) \times (k_0 + k_1 J) = (t_0 k_0 + t_1 k_1) + J(t_0 k_1 + t_1 k_0)$.

In [5], Smarandache presented symbolic n-plithogenic sets, then they were used in generalizing many famous algebraic structures such as ring, matrices, and other structures [6-9,15-18].

We refer to many similar numerical systems that generalize real numbers, such as neutrosophic, refined neutrosophic numbers and weak fuzzy numbers [10-14,19-23].

Through this paper, we use symbolic 2-plithogenic real numbers with split-complex numbers to build a new generalization of real numbers, and we present some of its elementary algebraic properties.

Definition.

The symbolic 2-plithogenic ring of real numbers is defined as follows:

$2 - SP_R = \{t_0 + t_1 P_1 + t_2 P_2; t_i \in R, P_1 \times P_2 = P_2 \times P_1 = P_2, {P_1}^2 = {P_2}^2 = P_2\}$.

The addition operation on $2 - SP_R$ is defined as follows:

$(t_0 + t_1 P_1 + t_2 P_2) + (t_0' + t_1' P_1 + t_2' P_2) = (t_0 + t_0') + (t_1 + t_1')P_1 + (t_2 + t_2')P_2$

The multiplication on $2 - SP_R$ is defined as follows:

$(t_0 + t_1 P_1 + t_2 P_2)(t_0' + t_1' P_1 + t_2' P_2) = t_0 t_0' + (t_0 t_1' + t_1 t_0' + t_1 t_1')P_1 + (t_0 t_2' + t_1 t_2' + t_2 t_2' + t_2 t_0' + t_2 t_1')P_2$.

Main concepts.

Definition.

The set of split-complex symbolic 2-plithogenic numbers is defined as follows:

$2 - SP_S = \{(x_0 + x_1 P_1 + x_2 P_2) + J(y_0 + y_1 P_1 + y_2 P_2); x_i, y_i \in R\ J^2 = 1\}$.

Definition.

Addition on $2 - SP_S$ is defined as follows:

$[(x_0 + x_1 P_1 + x_2 P_2) + J(y_0 + y_1 P_1 + y_2 P_2)] + [(z_0 + z_1 P_1 + z_2 P_2) + J(t_0 + t_1 P_1 + t_2 P_2)] = [(x_0 + z_0) + (x_1 + z_1)P_1 + (x_2 + z_2)P_2] + J[(y_0 + t_0) + (y_1 + t_1)P_1 + (y_2 + t_2)P_2]$.

Multiplication on $2 - SP_S$ is defined as follows

For $X=(x_0+x_1P_1+x_2P_2)+J(\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2), Y=(y_0+y_1P_1+y_2P_2)+J(\acute{y}_0+\acute{y}_1P_1+\acute{y}_2P_2)$,

then

$X.Y=(x_0+x_1P_1+x_2P_2)(y_0+y_1P_1+y_2P_2)+(\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2)(\acute{y}_0+\acute{y}_1P_1+\acute{y}_2P_2)+J[(x_0+x_1P_1+x_2P_2)(\acute{y}_0+\acute{y}_1P_1+\acute{y}_2P_2)+(\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2)(y_0+y_1P_1+y_2P_2)]$.

Example.

Consider $X=(P_1+P_2)+J(1+3P_2), Y=(1-P_2)+J(2-P_1)$, then:

$X+Y=(1+P_1)+J(3-P_1+3P_2)$

$X.Y=(P_1+P_2)(1-P_2)+(1+3P_2)(2-P_1)+J[(P_1+P_2)(2-P_1)+(1+3P_2)(1-P_2)]=P_1-P_2+P_2-P_2+2-P_1+6P_2-3P_2+J[2P_1-P_1+2P_2-P_2+1-P_2+3P_2-3P_2]=(2+2P_2)+J(1+P_1)$.

Remark.

$(2-SP_S,+,.)$ Is a commutative ring.

Invertibility.

Theorem.

Let $X=(m_0+m_1P_1+m_2P_2)+J(n_0+n_1P_1+n_2P_2)\in 2-SP_S$, then X is invertible if and only if:

$$\begin{cases}(m_0-n_0)+(m_1-n_1)P_1+(m_2-n_2)P_2\\(m_0+n_0)+(m_1+n_1)P_1+(m_2+n_2)P_2\end{cases}$$

Are invertible in $2-SP_R$.

On the other hand:

$$X^{-1}=\frac{1}{X}=\frac{m_0}{{m_0}^2-{n_0}^2}+P_1\left[\frac{m_0+m_1}{(m_0+m_1)^2-(n_0+n_1)^2}-\frac{m_0}{{m_0}^2-{n_0}^2}\right]$$
$$+P_2\left[\frac{m_0+m_1+m_2}{(m_0+m_1+m_2)^2-(n_0+n_1+n_2)^2}-\frac{m_0+m_1}{(m_0+m_1)^2-(n_0+n_1)^2}\right]$$
$$-J\left[\frac{n_0}{{m_0}^2-{n_0}^2}+P_1\left[\frac{n_0+n_1}{(m_0+m_1)^2-(n_0+n_1)^2}-\frac{n_0}{{m_0}^2-{n_0}^2}\right]\right.$$
$$\left.+P_2\left[\frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2-(n_0+n_1+n_2)^2}-\frac{n_0+n_1}{(m_0+m_1)^2-(n_0+n_1)^2}\right]\right]$$

Proof.

Put $X = M + NJ; M = m_0 + m_1P_1 + m_2P_2, N = n_0 + n_1P_1 + n_2P_2$.

According to [2], X is invertible if and only if $M + N, M - N$ are invertible, hence:

$$\begin{cases}(m_0 - n_0) + (m_1 - n_1)P_1 + (m_2 - n_2)P_2 \\ (m_0 + n_0) + (m_1 + n_1)P_1 + (m_2 + n_2)P_2\end{cases}$$

Are invertible in $2 - SP_R$.

Also, $X^{-1} = \frac{1}{X} = \frac{M-NJ}{M^2-N^2} = \frac{M}{M^2-N^2} - J\frac{N}{M^2-N^2}$.

According to [ ], we can write:

$$\frac{M}{M^2 - N^2} = M.\frac{1}{M^2 - N^2}$$

$$= (m_0 + m_1P_1 + m_2P_2)\left[\frac{1}{{m_0}^2 - {n_0}^2}\right.$$

$$+ P_1\left[\frac{1}{(m_0 + m_1)^2 - (n_0 + n_1)^2} - \frac{1}{{m_0}^2 - {n_0}^2}\right]$$

$$\left.+ P_2\left[\frac{1}{(m_0 + m_1 + m_2)^2 - (n_0 + n_1 + n_2)^2} - \frac{1}{(m_0 + m_1)^2 - (n_0 + n_1)^2}\right]\right]$$

$$= \frac{m_0}{{m_0}^2 - {n_0}^2} + P_1\left[\frac{m_0 + m_1}{(m_0 + m_1)^2 - (n_0 + n_1)^2} - \frac{m_0}{{m_0}^2 - {n_0}^2}\right]$$

$$+ P_2\left[\frac{m_0 + m_1 + m_2}{(m_0 + m_1 + m_2)^2 - (n_0 + n_1 + n_2)^2} - \frac{m_0 + m_1}{(m_0 + m_1)^2 - (n_0 + n_1)^2}\right]$$

By a similar argument, we get:

$$\frac{N}{M^2 - N^2} = \frac{n_0}{{m_0}^2 - {n_0}^2} + P_1\left[\frac{n_0 + n_1}{(m_0 + m_1)^2 - (n_0 + n_1)^2} - \frac{n_0}{{m_0}^2 - {n_0}^2}\right]$$

$$+ P_2\left[\frac{n_0 + n_1 + n_2}{(m_0 + m_1 + m_2)^2 - (n_0 + n_1 + n_2)^2} - \frac{n_0 + n_1}{(m_0 + m_1)^2 - (n_0 + n_1)^2}\right]$$

Thus, the proof holds.

Example.

Take $X = (1 + P_1) + J(2 - P_1 + 2P_2) \in 2 - SP_S$, then:

$M = 1 + P_1, N = 2 - P_1 + 2P_2, M + N = 3 + P_2, M - N = -1 + 2P_1 - 2P_2$

$M + N, M - N$ are invertible in $2 - SP_R$.

$$X^{-1} = \frac{1}{-3} + P_1\left(\frac{2}{3} - \frac{1}{-3}\right) + P_2\left(\frac{2}{-5} - \frac{2}{3}\right) - J\left[\frac{2}{-3} + P_1\left(\frac{1}{3} - \frac{1}{-3}\right) + P_2\left(\frac{3}{-5} - \frac{1}{3}\right)\right]$$

$$= \frac{-1}{3} + P_1 + P_2\left(-\frac{16}{15}\right) + J\left(\frac{2}{3} - \frac{2}{3}P_1 + \frac{4}{15}P_2\right)$$

Idempotenty.

Let $X = M + NJ; M = m_0 + m_1P_1 + m_2P_2, N = n_0 + n_1P_1 + n_2P_2 \in 2 - SP_R$.

then X is called idempotent if and only if $X^2 = X$.

First, we compute X^2:

$X^2 = M^2 + N^2 + 2MNJ == {m_0}^2 + {n_0}^2 + P_1[(m_0 + m_1)^2 + (n_0 + n_1)^2 - {m_0}^2 - {n_0}^2] +$
$P_2[(m_0 + m_1 + m_2)^2 + (n_0 + n_1 + n_2)^2 - (m_0 + m_1)^2 - (n_0 + n_1)^2]$.

The equation $X^2 = X$ is equivalent to:

$$\begin{cases} {m_0}^2 + {n_0}^2 = m_0 \quad (1) \\ (m_0 + m_1)^2 + (n_0 + n_1)^2 = m_0 + m_1 \quad (2) \\ (m_0 + m_1 + m_2)^2 + (n_0 + n_1 + n_2)^2 = m_0 + m_1 + m_2 \quad (3) \\ N(2M - 1) = 0 \quad (4) \end{cases}$$

Equation (4) is equivalent to:

$$\begin{cases} n_0(2m_0 - 1) = 0 \quad (5) \\ (n_0 + n_1)(2m_0 + 2m_1 - 1) = 0 \quad (6) \\ (n_0 + n_1 + n_2)(2m_0 + 2m_1 + 2m_2 - 1) = 0 \quad (7) \end{cases}$$

Equation (5) implies that $n_0 = 0$ or $m_0 = \frac{1}{2}$

If $n_0 \neq 0, m_0 \neq \frac{1}{2}$, then form (1), we get $m_1 = 0$ or $m_0 = 1$.

If $n_0 \neq 0, m_0 = \frac{1}{2}$, then form (1), we get $n_0 = \frac{1}{2}$ or $n_0 = -\frac{1}{2}$.

If $n_0 = 0, m_0 = \frac{1}{2}$, we get a contradiction:

Equation (6) implies that $n_0 + n_1 = 0$ or $m_0 + m_1 = \frac{1}{2}$

If $n_0 + n_1 \neq 0, m_0 + m_1 \neq \frac{1}{2}$, then $m_0 + m_1 = 0$ or $m_0 + m_1 = 1$.

If $n_0 + n_1 \neq 0, m_0 + m_1 = \frac{1}{2}$, then $n_0 + n_1 = \frac{1}{2}$ or $n_0 + n_1 = -\frac{1}{2}$.

If $n_0 + n_1 = 0, m_0 + m_1 = \frac{1}{2}$, we get a contradiction:

Equation (7) implies that $n_0 + n_1 + n_2 = 0$ or $m_0 + m_1 + m_2 = \frac{1}{2}$

If $n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 \neq \frac{1}{2}$, then $m_0 + m_1 + m_2 = 0$ or $m_0 + m_1 +$
$m_2 = 1$.

If $n_0 + n_1 + n_2 \neq 0, m_0 + m_1 + m_2 = \frac{1}{2}$, then $n_0 + n_1 + n_2 = \frac{1}{2}$ or $n_0 + n_1 + n_2 =$
$-\frac{1}{2}$.

If $n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = \frac{1}{2}$, we get a contradiction:

The possible cases are:

Case 1.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = 0$.

Case 2.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = P_1 - P_2$.

Case 3.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = P_1$.

Case 4.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = P_2$.

Case 5.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = 1 - P_1$.

Case 6.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = 1 - P_1 + P_2$.

Case 7.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = 1 - P_2$.

Case 8.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = 1$

Case 9.

$n_0 = 0, m_0 = 0, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = \left(\frac{1}{2}P_1 - \frac{1}{2}P_2\right) + J\left(\frac{1}{2}P_1 - \frac{1}{2}P_2\right)$.

Case 10.

$n_0 = 0, m_0 = 0, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0,$

then $X = \left(\frac{1}{2}P_1 - \frac{1}{2}P_2\right) + J\left(-\frac{1}{2}P_1 + \frac{1}{2}P_2\right).$

Case 11.

$n_0 = 0, m_0 = 0, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1,$ then

$X = \left(\frac{1}{2}P_1 + \frac{1}{2}P_2\right) + J\left(\frac{1}{2}P_1 - \frac{1}{2}P_2\right).$

Case 12.

$n_0 = 0, m_0 = 0, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1,$

then $X = \left(\frac{1}{2}P_1 + \frac{1}{2}P_2\right) + J\left(-\frac{1}{2}P_1 + \frac{1}{2}P_2\right).$

Case 13.

$n_0 = 0, m_0 = 1, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0,$ then

$X = \left(1 - \frac{1}{2}P_1 - \frac{1}{2}P_2\right) + J\left(\frac{1}{2}P_1 - \frac{1}{2}P_2\right).$

Case 14.

$n_0 = 0, m_0 = 1, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1,$

then $X = \left(1 - \frac{1}{2}P_1 + \frac{1}{2}P_2\right) + J\left(-\frac{1}{2}P_1 + \frac{1}{2}P_2\right).$

Case 15.

$n_0 = 0, m_0 = 1, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1,$ then

$X = \left(1 - \frac{1}{2}P_1 + \frac{1}{2}P_2\right) + J\left(\frac{1}{2}P_1 - \frac{1}{2}P_2\right).$

Case 16.

$n_0 = 0, m_0 = 1, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0,$

then $X = \left(1 - \frac{1}{2}P_1 - \frac{1}{2}P_2\right) + J\left(-\frac{1}{2}P_1 + \frac{1}{2}P_2\right).$

Case 17.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then

$X = \frac{1}{2}P_2 + J\left(\frac{1}{2}P_2\right).$

Case 18.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \frac{1}{2}P_2 - \frac{1}{2}P_2 J.$

Case 19.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then

$X = P_1 - \frac{1}{2}P_2 + \frac{1}{2}P_2 J.$

Case 20.

$n_0 = 0, m_0 = 0, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = P_1 - \frac{1}{2}P_2 - \frac{1}{2}P_2 J.$

Case 21.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then

$X = \left(1 - P_1 + \frac{1}{2}P_2\right) + \frac{1}{2}P_2 J.$

Case 22.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \left(1 - P_1 + \frac{1}{2}P_2\right) - \frac{1}{2}P_2 J.$

Case 23.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then

$X = \left(1 - \frac{1}{2}P_2\right) + \frac{1}{2}P_2 J.$

Case 24.

$n_0 = 0, m_0 = 1, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \left(1 - \frac{1}{2}P_2\right) - \frac{1}{2}P_2 J.$

Case 25.

$n_0 = 0, m_0 = 1, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then

$X = \left(\frac{1}{2}P_1\right) + \frac{1}{2}P_1 J.$

Case 26.

$n_0 = 0, m_0 = 1, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \left(\frac{1}{2}P_1\right) + \left(\frac{1}{2}P_1 - P_2\right)J$.

Case 27.

$n_0 = 0, m_0 = 1, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \left(\frac{1}{2}P_1\right) + \left(-\frac{1}{2}P_1 + P_2\right)J$.

Case 28.

$n_0 = 0, m_0 = 1, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \left(\frac{1}{2}P_1\right) + \left(-\frac{1}{2}P_1\right)J$.

Case 29.

$n_0 = 0, m_0 = 1, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \left(1 - \frac{1}{2}P_1\right) + \left(\frac{1}{2}P_1\right)J$.

Case 30.

$n_0 = 0, m_0 = 1, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \left(1 - \frac{1}{2}P_1\right) + \left(\frac{1}{2}P_1 - P_2\right)J$.

Case 31.

$n_0 = 0, m_0 = 1, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \left(1 - \frac{1}{2}P_1\right) + \left(-\frac{1}{2}P_1 + P_2\right)J$.

Case 32.

$n_0 = 0, m_0 = 1, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \left(1 - \frac{1}{2}P_1\right) + \left(-\frac{1}{2}P_1\right)J$.

Case 33.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = \left(\frac{1}{2} - \frac{1}{2}P_1\right) + \left(\frac{1}{2} - \frac{1}{2}P_1\right)J$.

Case 34.

$n_0=\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=0, n_0+n_1+n_2=0, m_0+m_1+m_2=1$, then $X=\left(\frac{1}{2}-\frac{1}{2}P_1+P_2\right)+\left(\frac{1}{2}-\frac{1}{2}P_1\right)J$.

Case 35.

$n_0=\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=1, n_0+n_1+n_2=0, m_0+m_1+m_2=0$, then $X=\left(\frac{1}{2}+\frac{1}{2}P_1-P_2\right)+\left(\frac{1}{2}-\frac{1}{2}P_1\right)J$.

Case 36.

$n_0=\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=1, n_0+n_1+n_2=0, m_0+m_1+m_2=0$, then $X=\left(\frac{1}{2}+\frac{1}{2}P_1-P_2\right)+\left(\frac{1}{2}-\frac{1}{2}P_1\right)J$.

Case 37.

$n_0=-\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=0, n_0+n_1+n_2=0, m_0+m_1+m_2=0$, then $X=\left(\frac{1}{2}-\frac{1}{2}P_1\right)+\left(-\frac{1}{2}+\frac{1}{2}P_1\right)J$.

Case 38.

$n_0=-\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=0, n_0+n_1+n_2=0, m_0+m_1+m_2=1$, then $X=\left(\frac{1}{2}-\frac{1}{2}P_1+P_2\right)+\left(-\frac{1}{2}+\frac{1}{2}P_1\right)J$.

Case 39.

$n_0=-\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=1, n_0+n_1+n_2=0, m_0+m_1+m_2=0$, then $X=\left(\frac{1}{2}+\frac{1}{2}P_1-P_2\right)+\left(-\frac{1}{2}+\frac{1}{2}P_1\right)J$.

Case 40.

$n_0=-\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=1, n_0+n_1+n_2=0, m_0+m_1+m_2=1$, then $X=\left(\frac{1}{2}-\frac{1}{2}P_1\right)+\left(-\frac{1}{2}+\frac{1}{2}P_1\right)J$.

Case 41.

$n_0=\frac{1}{2}, m_0=\frac{1}{2}, n_0+n_1=0, m_0+m_1=0, n_0+n_1+n_2=\frac{1}{2}, m_0+m_1+m_2=\frac{1}{2}$, then $X=\left(\frac{1}{2}-\frac{1}{2}P_1+\frac{1}{2}P_2\right)+\left(\frac{1}{2}-\frac{1}{2}P_1+\frac{1}{2}P_2\right)J$.

Case 42.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \left(\frac{1}{2} - \frac{1}{2}P_1 + \frac{1}{2}P_2\right) + \left(\frac{1}{2} - \frac{1}{2}P_1 - \frac{1}{2}P_2\right)J.$

Case 43.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \left(\frac{1}{2} - \frac{1}{2}P_1 + \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + \frac{1}{2}P_1 + \frac{1}{2}P_2\right)J.$

Case 44.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \left(\frac{1}{2} - \frac{1}{2}P_1 + \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + \frac{1}{2}P_1 - \frac{1}{2}P_2\right)J.$

Case 45.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 0, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \left(\frac{1}{2} + \frac{1}{2}P_1 - \frac{1}{2}P_2\right) + \left(\frac{1}{2} - \frac{1}{2}P_1 + \frac{1}{2}P_2\right)J.$

Case 46.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \left(\frac{1}{2} + \frac{1}{2}P_1 - \frac{1}{2}P_2\right) + \left(\frac{1}{2} - \frac{1}{2}P_1 - \frac{1}{2}P_2\right)J.$

Case 47.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \left(\frac{1}{2} + \frac{1}{2}P_1 - \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + \frac{1}{2}P_1 + \frac{1}{2}P_2\right)J.$

Case 48.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = 0, m_0 + m_1 = 1, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \left(\frac{1}{2} + \frac{1}{2}P_1 - \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + \frac{1}{2}P_1 - \frac{1}{2}P_2\right)J.$

Case 49.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0,$ then $X = \left(\frac{1}{2} - \frac{1}{2}P_2\right) + \left(\frac{1}{2} - \frac{1}{2}P_2\right)J.$

Case 50.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = \left(\frac{1}{2} + \frac{1}{2}P_2\right) + \left(\frac{1}{2} - \frac{1}{2}P_2\right)J$.

Case 51.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = \left(\frac{1}{2} - \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + P_1 - \frac{1}{2}P_2\right)J$.

Case 52.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = \left(\frac{1}{2} + \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + P_1 - \frac{1}{2}P_2\right)J$.

Case 53.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = \left(\frac{1}{2} - \frac{1}{2}P_2\right) + \left(\frac{1}{2} - P_1 + \frac{1}{2}P_2\right)J$.

Case 54.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = \left(\frac{1}{2} + \frac{1}{2}P_2\right) + \left(\frac{1}{2} - P_1 + \frac{1}{2}P_2\right)J$.

Case 55.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 0$, then $X = \left(\frac{1}{2} - \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + \frac{1}{2}P_2\right)J$.

Case 56.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = 0, m_0 + m_1 + m_2 = 1$, then $X = \left(\frac{1}{2} + \frac{1}{2}P_2\right) + \left(-\frac{1}{2} + \frac{1}{2}P_2\right)J$.

Case 57.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2}$, then $X = \frac{1}{2} + \frac{1}{2}J$.

Case 58.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \frac{1}{2} + \left(\frac{1}{2} - P_2\right)J.$

Case 59.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \frac{1}{2} + \left(\frac{1}{2} - P_1 + P_2\right)J.$

Case 60.

$n_0 = \frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \frac{1}{2} + \left(\frac{1}{2} - P_1\right)J.$

Case 61.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \frac{1}{2} + \left(-\frac{1}{2} + P_1\right)J.$

Case 62.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = \frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \frac{1}{2} + \left(-\frac{1}{2} + P_1 - P_2\right)J.$

Case 63.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = \frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$

then $X = \frac{1}{2} + \left(-\frac{1}{2} + P_2\right)J.$

Case 64.

$n_0 = -\frac{1}{2}, m_0 = \frac{1}{2}, n_0 + n_1 = -\frac{1}{2}, m_0 + m_1 = \frac{1}{2}, n_0 + n_1 + n_2 = -\frac{1}{2}, m_0 + m_1 + m_2 = \frac{1}{2},$ then $X = \frac{1}{2} - \frac{1}{2}J.$

Conclusion

In this paper, we have defined for the first time the ring of split-complex symbolic 2-plithogenic numbers by combining split-complex numbers with symbolic 2-plithogenic numbers.

We have studied many elementary properties of the novel generalized system, where necessary and sufficient conditions for Invertibility and idempotency of symbolic 2-plithogenic split-complex numbers were handled by many related theorems and valid examples.

In the future, we aim to study matrix systems and functional systems generated by the novel algebraic structure.

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