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An Introduction to The Dual Symbolic 3-Plithogenic And 4-Plithogenic Numbers

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Abstract:

The objective of this paper is to use dual numbers with symbolic 3-plithogenic and 4-plithogenic numbers in one numerical system called dual symbolic 3-plithogenic/4-plithogenic numbers.

Also, the elementary algebraic properties of the suggested systems will be discussed in terms of theorems and related examples that explain the validity of these algebraic number systems.

Keywords: Symbolic 3-plithogenic number, dual number, dual symbolic 3-plithogenic number, Symbolic 4-plithogenic number, dual symbolic 4-plithogenic number.

Introduction and preliminaries.

Dual numbers are considered as a generalization of real numbers, where they are defined as follows:

$D = \{a + bt;\ t^2 = 0, a, b \in R\}$[1]. Dual numbers make together a commutative ring with many interesting properties.

Addition on D is defined as follows:

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$(a_0 + b_0 t) + (a_1 + b_1 t) = (a_0 + a_1) + (b_0 + b_1)t$

Multiplication on D is defined as follows:

$(a_0 + b_0 t).(a_1 + b_1 t) = (a_0 a_1) + (a_0 b_1 + b_0 a_1)t$

In [2-4], smarandache presented symbolic n-plithogenic sets, then they were used in generalizing many famous algebraic structures such as rings, matrices, and other structures [6-11].

We refer to many similar numerical systems that generalize real number, such as neutrosophic numbers, split-complex number, and weak fuzzy numbers [12-18]. These generalized numbers were applicable in cryptography and matrix theory[19-24].

Through this paper, we use symbolic 3-plithogenic real numbers and symbolic 4-plithogenic real numbers to build a new generalization of real numbers, and we present some of its elementary algebraic properties.

Main concepts.

Definition.

The set of symbolic 3-plithogenic dual numbers is defined as follows:

$3 - SP_D = \{(x_0 + x_1 t) + (y_0 + y_1 t)P_1 + (z_0 + z_1 t)P_2 + (s_0 + s_1 t)P_3; x_i, y_i, z_i, s_i \in R, t^2 = 0\}$.

Definition.

Addition of $3 - SP_D$ is defined:

$[(m_0 + m_1 t) + (k_0 + k_1 t)P_1 + (s_0 + s_1 t)P_2 + (r_0 + r_1 t)P_3] + [(n_0 + n_1 t) + (l_0 + l_1 t)P_1 + (q_0 + q_1 t)P_2 + (g_0 + g_1 t)P_3] = (m_0 + n_0) + (m_1 + n_1)t + [(k_0 + l_0) + (k_1 + l_1)t]P_1 + [(s_0 + q_0) + (s_1 + q_1)t]P_2 + [(r_0 + g_0) + (r_1 + g_1)t]P_3$.

$(3 - SP_D, +)$ is an abelian group.

Remark.

A symbolic 3-plithogenic dual number $X = (x_0 + x_1 t) + (y_0 + y_1 t)P_1 + (z_0 + z_1 t)P_2 + (s_0 + s_1 t)P_3$

can be written:

$X = (x_0 + y_0 P_1 + z_0 P_2 + s_0 P_3) + t(x_1 + y_1 P_1 + z_1 P_2 + s_1 P_3)$.

Definition.

Let

$X = (x_0 + x_1P_1 + x_2P_2 + x_3P_3) + t(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2 + \acute{x}_3P_3) = M_1 + M_2t,$

$Y = (y_0 + y_1P_1 + y_2P_2 + y_3P_3) + t(\acute{y}_0 + \acute{y}_1P_1 + \acute{y}_2P_2 + \acute{y}_3P_3) = N_1 + N_2t \in 3 - SP_D,$

then:

Multiplication on $3 - SP_D$ is defined as follows:

$X.Y = M_1N_1 + t(M_1N_2 + N_1M_2)$

Example.

Consider $X = (1 + P_1 + P_2 + P_3) + t(2 - P_3), Y = P_1 + t(1 - P_3)$, we have:

$X + Y = (1 + 2P_1 + P_2 + P_3) + t(3 - 2P_3)$

$X.Y = (1 + P_1 + P_2 + P_3)P_1 + t[(1 + P_1 + P_2 + P_3)(1 - P_3) + (2 - P_3)P_1] =$

$(2P_1 + P_2 + P_3) + t[(1 - P_3 + P_2 - P_3 + P_3 - P_3 + P_1 - P_3) + 2P_1 - P_3] =$

$(2P_1 + P_2 + P_3) + t(1 + P_1 + P_2 - 3P_3).$

Remark.

$(3 - SP_D, +, .)$ Is a commutative ring.

Invertibility:

Theorem.

Let $X = (m_0 + m_1P_1 + m_2P_2 + m_3P_3) + t(n_0 + n_1P_1 + n_2P_2 + n_3P_3) \in 3 - SP_D$, then X is invertible if and only if $m_0 \neq 0, m_0 + m_1 \neq 0, m_0 + m_1 + m_2 \neq 0, m_0 + m_1 + m_2 + m_3 \neq 0$ and:

$$X^{-1} = \frac{1}{X} = \left[\frac{1}{m_0} + \left(\frac{1}{m_0 + m_1} - \frac{1}{m_0}\right)P_1 + \left(\frac{1}{m_0 + m_1 + m_2} - \frac{1}{m_0 + m_1}\right)P_2 + \left(\frac{1}{m_0 + m_1 + m_2 + m_3} - \frac{1}{m_0 + m_1 + m_2}\right)P_3\right]$$
$$- t\left[\frac{n_0}{(m_0)^2} + \left(\frac{n_0 + n_1}{(m_0 + m_1)^2} - \frac{n_0}{(m_0)^2}\right)P_1 + \left(\frac{n_0 + n_1 + n_2}{(m_0 + m_1 + m_2)^2} - \frac{n_0 + n_1}{(m_0 + m_1)^2}\right)P_2 + \left(\frac{n_0 + n_1 + n_2 + n_3}{(m_0 + m_1 + m_2 + m_3)^2} - \frac{n_0 + n_1 + n_2}{(m_0 + m_1 + m_2)^2}\right)P_3\right]$$

Proof.

X is invertible if and only if $\frac{1}{X}$ is defined as follows:

$$\frac{1}{X}=\frac{1}{(m_0+m_1P_1+m_2P_2+m_3P_3)+t(n_0+n_1P_1+n_2P_2+n_3P_3)}$$
$$=\frac{(m_0+m_1P_1+m_2P_2+m_3P_3)-t(n_0+n_1P_1+n_2P_2+n_3P_3)}{[(m_0+m_1P_1+m_2P_2+m_3P_3)+t(n_0+n_1P_1+n_2P_2+n_3P_3)][(m_0+m_1P_1+m_2P_2+m_3P_3)-t(n_0+n_1P_1+n_2P_2+n_3P_3)]}$$
$$=\frac{(m_0+m_1P_1+m_2P_2+m_3P_3)-t(n_0+n_1P_1+n_2P_2+n_3P_3)}{(m_0+m_1P_1+m_2P_2+m_3P_3)^2}$$

So that $m_0+m_1P_1+m_2P_2+m_3P_3$ is invertible in $3-SP_R$.

This is equivalent to $m_0\neq 0, m_0+m_1\neq 0, m_0+m_1+m_2\neq 0, m_0+m_1+m_2+m_3\neq 0$.

On the other hand, $\frac{1}{X}=\frac{1}{m_0+m_1P_1+m_2P_2+m_3P_3}-t\frac{(n_0+n_1P_1+n_2P_2+n_3P_3)^2}{(m_0+m_1P_1+m_2P_2+m_3P_3)^2}$

Put $Y=\left[\frac{1}{m_0}+\left(\frac{1}{m_0+m_1}-\frac{1}{m_0}\right)P_1+\left(\frac{1}{m_0+m_1+m_2}-\frac{1}{m_0+m_1}\right)P_2+\left(\frac{1}{m_0+m_1+m_2+m_3}-\frac{1}{m_0+m_1+m_2}\right)P_3\right]-t\left[\frac{n_0}{(m_0)^2}+\left(\frac{n_0+n_1}{(m_0+m_1)^2}-\frac{n_0}{(m_0)^2}\right)P_1+\left(\frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2}-\frac{n_0+n_1}{(m_0+m_1)^2}\right)P_2+\left(\frac{n_0+n_1+n_2+n_3}{(m_0+m_1+m_2+m_3)^2}-\frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2}\right)P_3\right]$.

Compute the result of XY to get:

$XY=1$

So that, $X^{-1}=\frac{1}{X}=Y$

Example.

Take $X=(1+P_3)+t(2+P_3)\in 3-SP_D$:

$X^{-1}=\frac{1}{1}+\left(\frac{1}{2}-\frac{1}{1}\right)P_2-t\left[\frac{2}{1}+\left(\frac{2}{1}-\frac{2}{1}\right)P_1+\left(\frac{2}{1}-\frac{2}{1}\right)P_2+\left(\frac{3}{4}-\frac{2}{1}\right)P_2\right]=1-\frac{1}{2}P_3-t\left(2-\frac{5}{4}P_3\right)$.

Natural power.

Theorem.

Let $X=(m_0+m_1P_1+m_2P_2+m_3P_3)+t(n_0+n_1P_1+n_2P_2+n_3P_3)\in 3-SP_D$, then:

$X^n=(m_0)^n+((m_0+m_1)^n-(m_0)^n)P_1+((m_0+m_1+m_2)^n-(m_0+m_1)^n)P_2+((m_0+m_1+m_2+m_3)^n-(m_0+m_1+m_2)^n)P_3+n(n_0+n_1P_1+n_2P_2+n_3P_3)[(m_0)^{n-1}+((m_0+m_1)^{n-1}-(m_0)^{n-1})P_1+((m_0+m_1+m_2)^{n-1}-(m_0+m_1)^{n-1})P_2((m_0+m_1+m_2+m_3)^{n-1}-(m_0+m_1+m_2)^{n-1})P_3]$ for $n\in N$.

Proof.

Let $X=A+Bt; A,B\in 3-SP_D$, then:

$A^n = A^n + nA^{n-1}Bt$, we get:

$A^n = (m_0)^n + ((m_0 + m_1)^n - (m_0)^n)P_1 + ((m_0 + m_1 + m_2)^n - (m_0 + m_1)^n)P_2 + ((m_0 + m_1 + m_2 + m_3)^n - (m_0 + m_1 + m_2)^n)P_3$, then the proof holds.

Example.

Take $X = (1 + P_3) + t(2 - P_3) \in 3 - SP_D$

$X^3 = 1 + (1 - 1)P_1 + (1 - 1)P_2 + (8 - 1)P_3 + 3t(2 - P_3)[1 + (1 - 1)P_1 + (1 - 1)P_2 + (4 - 1)P_3] = 1 + 7P_3 + 3t[(2 - P_3)(1 + 3P_3)] = 1 + 7P_3 + t(6 + 6P_3)$.

Idempotency.

Definition.

Let $X \in 3 - SP_D$, then X is called idempotent if and only if $X^2 = X$.

Theorem.

Let $X = (m_0 + m_1P_1 + m_2P_2 + m_3P_3) + t(n_0 + n_1P_1 + n_2P_2 + n_3P_3) \in 3 - SP_D$, then X is called idempotent if and only if:

1. $m_0 + m_1P_1 + m_2P_2 + m_3P_3$ is idempotent.
2. $(n_0 + n_1P_1 + n_2P_2 + n_3P_3)[2m_0 - 1 + 2m_1P_1 + 2m_2P_2 + 2m_3P_3] = 0$

Proof.

$X = M + Nt$ is idempotent if and only if:

$$X^2 = X \Longrightarrow \begin{cases} M^2 = M \\ 2MN = N \Longrightarrow N(2M - 1) = 0 \end{cases}$$

For $M = m_0 + m_1P_1 + m_2P_2 + m_3P_3, N = n_0 + n_1P_1 + n_2P_2 + n_3P_3 \in 3 - SP_R$.

This implies the proof.

Definition.

The set of symbolic 4-plithogenic dual numbers is defined as follows:

$4 - SP_D = \{(x_0 + x_1t) + (y_0 + y_1t)P_1 + (z_0 + z_1t)P_2 + (s_0 + s_1t)P_3 + (l_0 + l_1t)P_4; x_i, y_i, z_i, s_i, l_i \in R, t^2 = 0\}$.

Definition.

Addition of $4 - SP_D$ is defined:

$[(m_0 + m_1t) + (k_0 + k_1t)P_1 + (s_0 + s_1t)P_2 + (r_0 + r_1t)P_3 + (d_0 + d_1t)P_4] + [(n_0 + n_1t) + (l_0 + l_1t)P_1 + (q_0 + q_1t)P_2 + (g_0 + g_1t)P_3 + (c_0 + c_1t)P_4] =$

$(m_0+n_0)+(m_1+n_1)t+[(k_0+l_0)+(k_1+l_1)t]P_1+[(s_0+q_0)+(s_1+q_1)t]P_2+[(r_0+g_0)+(r_1+g_1)t]P_3+[(d_0+c_0)+(d_1+c_1)t]P_4.$

$(4-SP_D,+)$ is an abelian group.

Remark.

A symbolic 4-plithogenic dual number $X=(x_0+x_1t)+(y_0+y_1t)P_1+(z_0+z_1t)P_2+(s_0+s_1t)P_3+(d_0+d_1t)P_4$

can be written:

$X=(x_0+y_0P_1+z_0P_2+s_0P_3+d_0P_4)+t(x_1+y_1P_1+z_1P_2+s_1P_3+d_1P_4).$

Definition.

Let

$X=(x_0+x_1P_1+x_2P_2+x_3P_3+x_4P_4)+t(\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2+\acute{x}_3P_3+\acute{x}_4P_4)=M_1+M_2t,$

$Y=(y_0+y_1P_1+y_2P_2+y_3P_3+y_4P_4)+t(\acute{y}_0+\acute{y}_1P_1+\acute{y}_2P_2+\acute{y}_3P_3+\acute{y}_4P_4)=N_1+N_2t\in 4-SP_D,$

then:

Multiplication on $4-SP_D$ is defined as follows:

$X.Y=M_1N_1+t(M_1N_2+N_1M_2)$

Example.

Consider $X=(1+P_4)+t(2-P_3), Y=P_1+t(1-P_4)$, we have:

$X+Y=(1+P_1+P_4)+t(3-P_3-P_4)$

$X.Y=(1+P_4)P_1+t[(1+P_4)(1-P_4)+(2-P_3)P_1]=(P_1+P_4)+t[(1-P_4)+2P_1-P_3]=(P_1+P_4)+t(1+2P_1-P_3-P_4).$

Remark.

$(4-SP_D,+,.)$ Is a commutative ring.

Invertibility:

Theorem.

Let

$X=(m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4)+t(n_0+n_1P_1+n_2P_2+n_3P_3+n_4P_4)\in 4-SP_D,$

then X is invertible if and only if $m_0 \neq 0, m_0 + m_1 \neq 0, m_0 + m_1 + m_2 \neq 0, m_0 + m_1 + m_2 + m_3 \neq 0, m_0 + m_1 + m_2 + m_3 + m_4 \neq 0$ and:

$$X^{-1} = \frac{1}{X} = \left[\frac{1}{m_0} + \left(\frac{1}{m_0+m_1} - \frac{1}{m_0}\right)P_1 + \left(\frac{1}{m_0+m_1+m_2} - \frac{1}{m_0+m_1}\right)P_2 + \left(\frac{1}{m_0+m_1+m_2+m_3} - \frac{1}{m_0+m_1+m_2}\right)P_3 + \left(\frac{1}{m_0+m_1+m_2+m_3+m_4} - \frac{1}{m_0+m_1+m_2+m_3}\right)P_4\right] - t\left[\frac{n_0}{(m_0)^2} + \left(\frac{n_0+n_1}{(m_0+m_1)^2} - \frac{n_0}{(m_0)^2}\right)P_1 + \left(\frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2} - \frac{n_0+n_1}{(m_0+m_1)^2}\right)P_2 + \left(\frac{n_0+n_1+n_2+n_3}{(m_0+m_1+m_2+m_3)^2} - \frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2}\right)P_3 + \left(\frac{n_0+n_1+n_2+n_3+n_4}{(m_0+m_1+m_2+m_3+m_4)^2} - \frac{n_0+n_1+n_2+n_3}{(m_0+m_1+m_2+m_3)^2}\right)P_4\right].$$

Proof.

X is invertible if and only if $\frac{1}{X}$ is defined as follows:

$$\frac{1}{X} = \frac{1}{(m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4)+t(n_0+n_1P_1+n_2P_2+n_3P_3+n_4P_4)} = \frac{(m_0 + m_1P_1 + m_2P_2 + m_3P_3 + m_4P_4) - t(n_0 + n_1P_1 + n_2P_2 + n_3P_3 + n_4P_4)}{[(m_0 + m_1P_1 + m_2P_2 + m_3P_3 + m_4P_4) + t(n_0 + n_1P_1 + n_2P_2 + n_3P_3 + n_4P_4)][(m_0 + m_1P_1 + m_2P_2 + m_3P_3 + m_4P_4) - t(n_0 + n_1P_1 + n_2P_2 + n_3P_3 + n_4P_4)]} = \frac{(m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4)-t(n_0+n_1P_1+n_2P_2+n_3P_3+n_4P_4)}{(m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4)^2},$$

So that $m_0 + m_1P_1 + m_2P_2 + m_3P_3 + m_4P_4$ is invertible in $4 - SP_R$.

This is equivalent to $m_0 \neq 0, m_0 + m_1 \neq 0, m_0 + m_1 + m_2 \neq 0, m_0 + m_1 + m_2 + m_3 \neq 0, m_0 + m_1 + m_2 + m_3 + m_4 \neq 0$.

On the other hand,

$$\frac{1}{X} = \frac{1}{m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4} - t\,\frac{(n_0+n_1P_1+n_2P_2+n_3P_3+n_4P_4)^2}{(m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4)^2}$$

Put

$$Y = \left[\frac{1}{m_0} + \left(\frac{1}{m_0+m_1} - \frac{1}{m_0}\right)P_1 + \left(\frac{1}{m_0+m_1+m_2} - \frac{1}{m_0+m_1}\right)P_2 + \left(\frac{1}{m_0+m_1+m_2+m_3} - \frac{1}{m_0+m_1+m_2}\right)P_3 + \left(\frac{1}{m_0+m_1+m_2+m_3+m_4} - \frac{1}{m_0+m_1+m_2+m_3}\right)P_4\right] - t\left[\frac{n_0}{(m_0)^2} + \left(\frac{n_0+n_1}{(m_0+m_1)^2} - \frac{n_0}{(m_0)^2}\right)P_1 + \left(\frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2} - \frac{n_0+n_1}{(m_0+m_1)^2}\right)P_2 + \left(\frac{n_0+n_1+n_2+n_3}{(m_0+m_1+m_2+m_3)^2} - \frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2}\right)P_3 + \left(\frac{n_0+n_1+n_2+n_3+n_4}{(m_0+m_1+m_2+m_3+m_4)^2} - \frac{n_0+n_1+n_2+n_3}{(m_0+m_1+m_2+m_3)^2}\right)P_4\right].$$

Compute the result of XY to get:

$$XY = 1$$

So that, $X^{-1} = \frac{1}{X} = Y$

Natural power.

Theorem.

Let

$X=(m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4)+t(n_0+n_1P_1+n_2P_2+n_3P_3+n_4P_4)\in$ $4-SP_D$,

then:

$X^n=(m_0)^n+((m_0+m_1)^n-(m_0)^n)P_1+((m_0+m_1+m_2)^n-(m_0+m_1)^n)P_2+((m_0+m_1+m_2+m_3)^n-(m_0+m_1+m_2)^n)P_3+((m_0+m_1+m_2+m_3+m_4)^n-(m_0+m_1+m_2+m_3)^n)P_4+n(n_0+n_1P_1+n_2P_2+n_3P_3+n_4P_4)[(m_0)^{n-1}+((m_0+m_1)^{n-1}-(m_0)^{n-1})P_1+((m_0+m_1+m_2)^{n-1}-(m_0+m_1)^{n-1})P_2((m_0+m_1+m_2+m_3)^{n-1}-(m_0+m_1+m_2)^{n-1})P_3+((m_0+m_1+m_2+m_3+m_4)^{n-1}-(m_0+m_1+m_2+m_3)^{n-1})P_4]$ for $n\in N$.

Proof.

Let $X=A+Bt;A,B\in 4-SP_D$, then:

$A^n=A^n+nA^{n-1}Bt$, we get:

$A^n=(m_0)^n+((m_0+m_1)^n-(m_0)^n)P_1+((m_0+m_1+m_2)^n-(m_0+m_1)^n)P_2+((m_0+m_1+m_2+m_3)^n-(m_0+m_1+m_2)^n)P_3+((m_0+m_1+m_2+m_3+m_4)^n-(m_0+m_1+m_2+m_3)^n)P_4$, then the proof holds.

Idempotency.

Definition.

Let $X\in 4-SP_D$, then X is called idempotent if and only if $X^2=X$.

Theorem.

Let $X=(m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4)+t(n_0+n_1P_1+n_2P_2+n_3P_3+m_4P_4)\in$ $4-SP_D$, then X is called idempotent if and only if:

1. $m_0+m_1P_1+m_2P_2+m_3P_3+m_4P_4$ is idempotent.
2. $(n_0+n_1P_1+n_2P_2+n_3P_3+n_4P_4)[2m_0-1+2m_1P_1+2m_2P_2+2m_3P_3+2m_4P_4]=0$

Proof.

$X=M+Nt$ is idempotent if and only if:

$$X^2=X\Longrightarrow\begin{cases} M^2=M \\ 2MN=N\Longrightarrow N(2M-1)=0\end{cases}$$

For $M = m_0 + m_1P_1 + m_2P_2 + m_3P_3 + m_4P_4, N = n_0 + n_1P_1 + n_2P_2 + n_3P_3 + n_4P_4 \in 4 - SP_R$.

This implies the proof.

Conclusion

In this paper, we have studied for the first time the combination of symbolic 3-plithogenic numbers and 4-plithogenic numbers with dual numbers. The novel algebraic structures generated by them are called dual symbolic 3-plithogenic numbers and dual symbolic 4-plithogenic numbers.

We have determined the invertibility condition and the formula of the inverse for dual symbolic 3-plithogenic and 4-plithogenic numbers.

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