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Equitable Domination in Neutrosophic Graph Using Strong Arc

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Abstract: In this paper, we have initiated the study of domination and equitable domination of Neutrosophic graphs using strong arcs. Strong arcs represent the optimal (minimum) degree of truth membership value, the optimal (minimum) degree of indeterminacy membership value, and the non-optimal (maximum) degree of falsity membership value. Hence, the studies of domination and equitable domination using strong arcs have been explored. Upper bounds and minimality conditions for the existence of the introduced parameters were discussed. Extend the studies on strong and weak equitable domination of Neutrosophic graphs and obtain the relationship between domination and the equitable domination parameter. Furthermore, we have provided some theorems based on equitable domination of Neutrosophic graphs and discussed the upper and lower bounds of the strong and weak equitable domination in terms of order and size with other existing domination parameters of Neutrosophic graphs.

Keywords: Domination, Equitable Domination, Strong arc, Strong and weak

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1. Introduction

In 1965[1], L.A. Zadeh put forth a mathematical framework to describe the occurrence of uncertainty in real-world circumstances. Rosenfeld[2] was the first to propose the concept of fuzzy graphs and different fuzzy analogues of connectedness in graph theory concepts. Berge and Ore[3] began studying the domination sets of graphs. Studies on paired domination were started by Teresa et al. [4]. Biggs [5] and V.R. Kulli [6] both contributed to the development of efficient domination, and he [7] also developed the theory of domination in graphs. Cockayne[8] employed the independent domination number for the first time in graphs. Swaminathan and Dharmalingam introduced equitable domination [9]. A. Meenakshi developed and explored paired equitable domination [10], and it was continued in an inflated graph and its graph complement [11, 12].

Nagoor Gani and M. Basheer Ahmed developed and investigated the concepts of Strong and Weak Domination of Fuzzy Graph[13]. K.T. Atanassov created intuitionistic fuzzy relations and intuitionistic fuzzy graphs (IFGS)[14]. A.Shannon and Atanassov of [15] and M.G. Karunambigai et al. [16] identified IFGS as a particular instance of IFG. A. Nagoor Gani and Shajitha Begum developed the words "order," "degree," and "magnitude" of IFG[17]. A.Nagoor Gani and S. Anu Priya developed split domination in intuitionistic fuzzy graphs [18] and the author studied Dombi Fuzzy Graphs [19].Mullai et al.[20] studied equitable domination parameter in neutrosophic graphs. In this paper we have developed the equitable domination parameter using strong arcs.

The motivation of this research is to study domination and equitable domination in neutrosophic graphs using strong arcs. In [18], the vertex cardinality and edge cardinality of the intuitionistic graphs in the study of split dominations were focused. This study motivates us to define the order, size, and degree of the vertex of a neutrosophic graph that is optimal while initiating studies of another domination parameter, named equitable domination of a neutrosophic graph using a strong arc. The study of weak and strong domination in fuzzy graphs [13] motivated us to focus our research on the strong and weak equitable domination of neutrosophic graphs using the score function. Section 2 focused on the preliminary work related to our studies. Section 3 explored the studies of domination in the neutrosophic graph using a strong arc, and the upper bounds are given in terms of the order and degree of the neutrosophic graph. Sections 4 and 5 focused on the domination parameter equitable domination using a strong arc; weak and strong equitable domination using a score function are illustrated with an example.

The existence of equitable domination in a neutrosophic graph is guaranteed on the degree of the vertex of the neutrosophic graph

2. Preliminaries

Definition 2.1[11].

An intuitionistic fuzzy graph(IFG) is of the form $G_{IF} = (A_{IF}, B_{IF})$ where A_{IF} is a finite vertex set such that (i) $\mu_1 : A_{IF} \to [0,1]\,;\, \gamma_1 : A_{IF} \to [0,1]$ denote the degree of truth membership value and degree of falsity membership value respectively and $0 \le \mu_1(v_s) + \gamma_1(v_s) \le 1$ for every $v_s \in V$.

(ii) $B_{IF} \subseteq A_{IF} \times A_{IF}$ *where* $\mu_2 : A_{IF} \times A_{IF} \to [0,1]$; $\gamma_2 : A_{IF} \times A_{IF} \to [0,1]$ are such that $\mu_2\{(a_i, a_j)\} \le \min\{\mu_1(a_i), \mu_1(a_j)\}$; $\gamma_2\{(a_i, a_j)\} \ge \max\{\gamma_1(a_i), \gamma_1(a_j)\}$ and where

$0 \le \mu_2\{(a_i, a_j)\}\} + \mu_2\{(a_i, a_j)\} \le 1 \ \forall (a_i, a_j) \in B_{IF}$.

Definition 2.2[11]. An arc (u_d, v_d) is said to be strong arc if

$$\mu_2(a_i, a_j) = \min\{\mu_1(a_1), \mu_1(a_2)\} \text{ and } \gamma_2(a_i, a_j) = \max\{\gamma_1(a_1), \gamma_1(a_2)\}$$

Definition 2.3[11]. The degree of a vertex u_d in an IFG,

$G_{IF} = (A_{IF}, B_{IF})$ is defined as the sum of the weight of the strong arcs incident at u_d and is denoted by deg(u_d). The neiborhood of u_d is denoted by

$$N(u_d) = \{v_d \in A_{IF} / (u_d, v_d) \text{ is an strong arc}\}$$

The minimum degree of G_{IF} is $\delta(G_{IF}) = \min\{d_{G_{IF}}(u_d) / u_d \in A_{IF}\}$

The maximum degree of G_{IF} is $\Delta(G_{IF}) = \max\{d_{G_{IIF}}(u_d)/u_d \in A_{IF}\}$

Definition 2.4[11]. A vertex $u_d \in A_{IF}$ in an IFG, G_{IF} = (A_{IF}, B_{IF}) is said to be an isolate vertex if

$\mu_2(a_i, a_j) = 0$ and $\gamma_2(a_i, a_j) = 0$

Definition 2.5[11]. Let G_{IF} = (A_{IF}, B_{IF}) be a intuitionistic fuzzy graph. Then the cardinality of G_{NS} is defined

$$|G| = \left| \sum_{a_i \in A_{NS}} \frac{1 + T_{A_{NS}} - F_{A_{NS}}}{2} + \sum_{a_i a_j \in B_{NS}} \frac{1 + T_{B_{NS}} - F_{B_{NS}}}{2} \right|$$

Definition 2.6 [11]. Let G_{IF} = (A_{IF}, B_{IF}) be an intuitionistic fuzzy graph and let u_{if} and v_{if} ∈ A_{IF}, we say that u_{if} dominates u_{if} in G_{IF} if there exists a strong arc between them. A subset $D_d \subseteq A_{IF}$ is said to be dominating set in G_{IF} if for every $v_{if} \in A_{IF} - D_d$, there exists $u_{if} \in D_d$ dominates v_{if}.

Definition 2.7[4].

Let X_{sp} be a space of points (objects) with generic elements in X_{sp} is denoted by X_{sp}. A single valued neutrosophic set A_{NS} (SVNS) is characterized by truth membership function $T_{A_{NS}}(X_{sp})$, an indeterminacy membership function $I_{A_{NS}}(X_{sp})$, and a falsity membership function $F_{A_{NS}}(X_{sp})$. For each point x' in X_{sp}, $T_{A_{NS}}(X_{sp})$, $I_{A_{NS}}(X_{sp})$, and $F_{A_{NS}}(X_{sp}) \in [0, 1]$. A SVNS A can be written as A_{NS} ={< x' : $T_{A_{NS}}(x')$, $I_{A_{NS}}(x')$, $F_{A_{NS}}(x')$ >, $x' \in X_{sp}$}.

Definition 2.8[4]

Let A_{NS} = ($T_{A_{NS}}$, $I_{A_{NS}}$, $F_{A_{NS}}$) and B_{NS} = ($T_{B_{NS}}$, $I_{B_{NS}}$, $F_{B_{NS}}$) be single valued neutrosophic sets on a set X_{sp}. If A_{NS} = ($T_{A_{NS}}$, $I_{A_{NS}}$, $F_{A_{NS}}$) is a single valued neutrosophic relation on a set X_{sp}, then A_{NS} = ($T_{A_{NS}}$, $I_{A_{NS}}$, $F_{A_{NS}}$) is called a single valued neutrosophic relation on B_{NS} = ($T_{B_{NS}}$, $I_{B_{NS}}$, $F_{B_{NS}}$), if $T_{B_{NS}}(x', y') \le \min\{T_{A_{NS}}(x'), T_{A_{NS}}(y')\}$

$I_{B_{NS}}(x', y') \le \min\{I_{A_{NS}}(x'), I_{A_{NS}}(y')\}$, $F_{B_{NS}}(x', y') \ge \max\{F_{A_{NS}}(x'), F_{A_{NS}}(y')\}$ for all x', y' in X_{sp}.

Definition 2.9[4]. An arc (a_i, a_j) of a neutrosophic graph, G_{NS} = (A_{NS}, B_{NS}) is said to be strong if

$T_{E_s}\{(a_i, a_j)\} = \min\{T_{V_s}(a_i), T_{V_s}(a_j)\}$; $I_{E_s}\{(a_i, a_j)\} = \min\{I_{V_s}(a_i), I_{V_s}(a_j)\}$;

$F_{E_s}\{(a_i, a_j)\} = \max\{F_{V_s}(a_i), F_{V_s}(a_j)\}$ where $(a_i, a_j) \in E_s$ *and* a_i & $a_j \in V_s$.

3. Domination of a Neutrosophic graph(NSG) Using Strong Arc

Definition 3.1. Let u_d be a vertex in a NSG, $G_{NS} = (A_{NS}, B_{NS})$. The degree of a vertex u_d is defined as the sum of the weight of the strong arcs incident at u_d and is denoted by deg (u_d). The neighbourhood of u_d is denoted by $N(u_d) = \{v_d \in A_{NS} /(u_d, v_d)$ is a strong arc$\}$

The minimum degree of G_{NS} is $\delta(G_{NS}) = \min\{d_{G_{NS}}(u_d)/u_d \in A_{NS}\}$

The maximum degree of G_{NS} is $\Delta(G_{NS}) = \max\{d_{G_{INS}}(u_d)/u_d \in A_{NS}\}$

- ❖ The degree of indeterminancy membership value (I) is not a complement of degree of truth membership value (T) and degree of falsity membership value (F) and the values of T, I, and F are independent of one another, value (I) does not depend on either the Truth (T) or Falsity (F) value.
- ❖ Despite the fact that the value of indeterminacy is unknown, we presume it by using 0.5 for both the possibilities of truth and falsity. This truthness makes our study more significant .Hence to attain feasibility, the order of G_{NS} is defined as follows

Definition 3.2. Let $G_{NS} = (A_{NS}, B_{NS})$ be a neutrosophic graph. Then order of G_{NS} is defined

$$|A_{NS}| = \left| \sum_{a_i \in A_{NS}} \frac{2 + T_{A_{NS}} - (0.5)I_{A_{NS}} - F_{A_{NS}}}{3} \right|$$

Definition 3.3. Let $G_{NS} = (A_{NS}, B_{NS})$ be a neutrosophic graph. Then size of G_{NS} is defined

$$|B_{NS}| = \left| \sum_{a_i \in A_{NS}} \frac{2 + T_{B_{NS}} - (0.5)I_{B_{NS}} - F_{B_{NS}}}{3} \right|$$

Definition 3.4. Let $G_{NS} = (A_{NS}, B_{NS})$ be a neutrosophic graph and let u_d, $v_d \in A_{NS}$, we say that u_d dominates v_d in G_{NS} if there exists a strong arc between them. A subset $D_{NS} \subseteq A_{NS}$ is said to be dominating set if for every $v_d \in A_{IF} - D_{NS}$ there exists at least one $u_d \in D_{NS}$ dominates v_d. The minimum cardinality of a dominating set is called a domination number and is denoted by $\gamma_{NG}(G_{NS})$.

Definition 3.6. A dominating set D_{NS} of A_{NS} is said to be a minimal if no proper subset of D_{NS} is a dominating set of G_{NS}.

Example 3.7: Let $G_{NS} = (A_{NS}, B_{NS})$ be the NSG shown in figure (1)

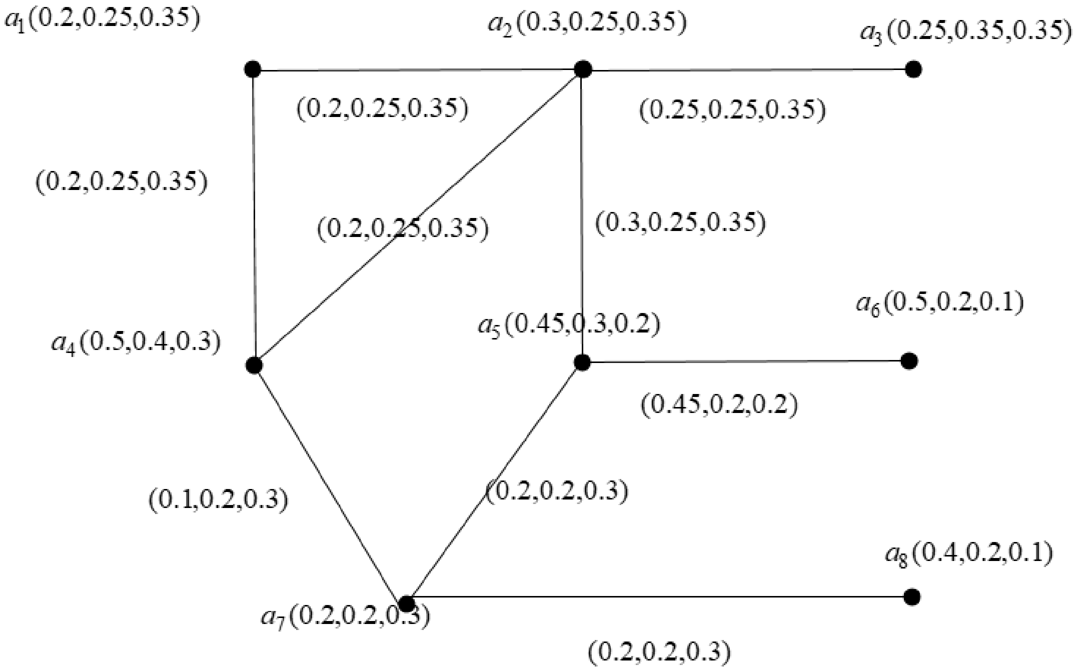

Figure 1. Example of domination of NSG using strong arc

The arcs a_2a_4, a_4a_7 are not strong arcs.

deg(a_1) = (0.4,0.5,0.7), deg(a_2) = (0.75,0.75, 1.05), deg(a_3) = (0.25,0.25,0.35) , deg(a_4) = (0.2,0.25,0.35) , deg(a_5) =(0.95,0.65,0.85) , deg(a_6) =(0.45,0.2,0.2) , deg(a_7) =(0.4,0.4,0.6) , deg(a_8) = (0.2,0.2,0.3).

The minimum degree of truth membership value of G_{NS} is

$$\delta(G_{NS}) = \min\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\} = 0.2$$

The minimum degree of indeterminacy membership value of G_{NS} is

$$\delta(G_{NS}) = \min\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\} = 0.2$$

The minimum degree of falsity membership value of G_{NS} is

$$\delta(G_{NS}) = \min\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\} = 0.2$$

The minimum degree of G_{NS} is $\delta(G_{NS}) = \min\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\} = (0.2,0.2,0.2)$

The maximum degree of truth membership value of G_{NS} is

$$\delta(G_{NS}) = \max\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\} = 0.95$$

The maximum degree of indeterminacy membership value of G_{NS} is

$$\delta(G_{NS}) = \max\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\} = 0.75$$

The maximum degree of falsity membership value of G_{NS} is

$$\delta(G_{NS}) = \max\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\} = 1.05$$

The maximum degree of G_{NS} is $\Delta(G_{NS}) = \max\{d_{G_{INS}}(u_d) / u_d \in A_{NS}\}$ =(0.95,0.75,1.05)

Order of G_{NS} = 5.2248

One of the dominating set is $D_{NS}{}^1$ = { a_2, a_5 ,a_7}, since for every vertex a_i in A_{NS} - $D_{NS}{}^1$ dominated by at least one $a_j \in D_{NS}{}^1$. Hence $\gamma_{NG}(D_{NS}{}^1)$ = 0.6083+0.7+0.6 = 1.9083

Other dominating sets are

(ii) $D_{NS}{}^2$ = {a_1, a_3, a_6, a_8} (iii) $D_{NS}{}^3$ = { a_3, a_4, a_5, a_7} (iv)$D_{NS}{}^4$ = {a_1, a_3, a_5, a_7}

Hence $\gamma_{NG}(D_{NS}{}^2)$ = 2.6499 , $\gamma_{NG}(D_{NS}{}^3)$ = 2.5416 and $\gamma_{NG}(D_{NS}{}^4)$ = 2.45

Domination number of G_{NS} is $\gamma_{NG}(G_{NS})$ =1.9083.

Theorem 3.7: A dominating set D_{NS} of a neutrosophic graph G_{NS} = (A_{NS}, B_{NS}) is minimal if and only if for each vertex $v_d \in D_{NS}$ one of the following conditions holds

(i)There exists a vertex $u_d \in A_{NS}$ - D_{NS} such that $N(u_d) \cap D_{NS} = \{v_d\}$

(ii)v_d is an isolate in $\langle D_{NS} \rangle$

Proof: Suppose D_{NS} is a minimal dominating set of G_{NS} there exists a vertex v_d of D_{NS} which does not satisfy any of the above conditions. Hence there exists a vertex $u_d \in A_{NS}$ - D_{NS} such that $N(u_d) \cap D_{NS} \neq \{v_d\}$. Furthermore by condition(ii) v_d is not an isolate in $\langle D_{NS} \rangle$, then D_{NS} - v_d will be a minimal dominating set of G_{NS} which is a contradiction to the assumption.

Theorem 3.8: A subset D_{NS} of A_{NS} of a NSG, G_{NS} = (A_{NS}, B_{NS}) is a dominating then there exists two vertices u_d , $v_d \in A_{NS}$ - D_{NS} such that every u_d - v_d path contains at least one vertex of D_{NS}.

Proof: Suppose D_{NS} is a dominating set of G_{NS}. Since every vertex in A_{NS} - D_{NS} is dominated by at least one vertex of D_{NS}, there exists a u_d - v_d path contains at least one vertex of D_{NS}.

Theorem 3.9: For any NSG, G_{NS} = (A_{NS}, B_{NS}) with order p

(i) $\gamma_{NG}(G_{NS}) \leq p - \Delta_T(G_{NS})$

(ii) $\gamma_{NG}(G_{NS}) \leq p - \Delta_I(G_{NS})$

(iii) $\gamma_{NG}(G_{NS}) \leq p - \Delta_F(G_{NS})$

Proof: Let G_{NS} = (A_{NS}, B_{NS}) be a NSG with order p.

Since $\sum_{i=1}^{n} d(v_i) \leq p$, where v_i represents the vertices present in the strong arc, the no of vertices present in a dominating set is less than n. Furthermore, by the definition of $\Delta_T(G_{NS})$, the maximum truth membership values of v_i among all the vertices of G_{NS} and by the definition of minimal dominating set we have

$$\gamma_{NG}(G_{NS}) \le \sum_{i=1}^{n} d(v_i) \le p - \Delta_T(G_{NS})$$

Similarly, we prove $\gamma_{NG}(G_{NS}) \le p - \Delta_I(G_{NS})$ and $\gamma_{NG}(G_{NS}) \le p - \Delta_F(G_{NS})$

4. Equitable domination of a Neutrosophic graph(NSG)

Definition 4.1. A dominating set D_{NS} of A_{NS} of a neutrosophic graph $G_{NS} = (A_{NS}, B_{NS})$ is a equitable dominating set if for every $u_d \in A_{NS} - D_{NS}$ there exists $u_d \; v_d \in B_{NS}$ such that $|\deg(u_d) - \deg(v_d)| \le 1$. The minimum cardinality of an equitable dominating set is called an equitable domination number and is denoted by $\gamma_{ef-NG}(G_{NS})$.

Definition 4.2. An equitable dominating set D_{NS} of A_{NS} is said to be a minimal if no proper subset of D_{NS} is a equitable dominating set of G_{NS}.

Example 4.3. Let $G_{NS} = (A_{NS}, B_{NS})$ be the NSG shown in figure (2)

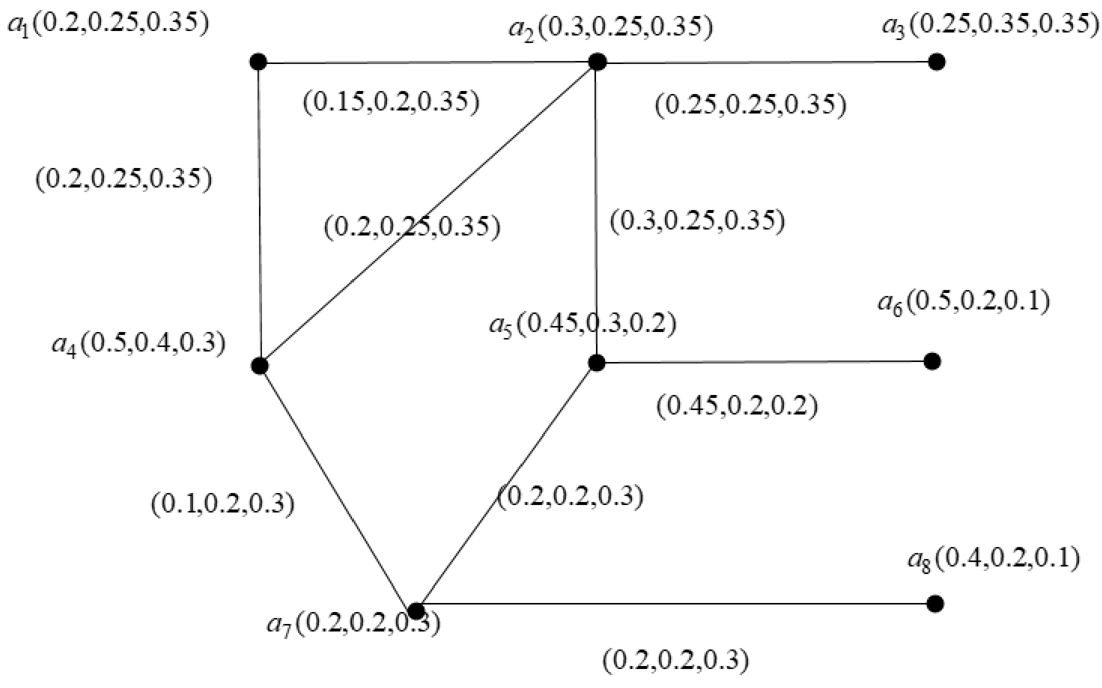

Figure 2. Example of Equitable domination of NSG Using Strong arc

The arcs a_1a_2, a_2a_4, a_4a_7 are not strong arcs.

$\deg(a_1) = (0.2,0.25,0.35)$, $\deg(a_2) = (0.55,0.5,0.7)$, $\deg(a_3) = (0.25,0.25,0.35)$, $\deg(a_4) = (0.2,0.25,0.35)$, $\deg(a_5) = (0.95,0.65,0.85)$, $\deg(a_6) = (0.45,0.2,0.2)$, $\deg(a_7) = (0.4,0.4,0.6)$, $\deg(a_8) = (0.2,0.2,0.3)$.

One of the equitable dominating set is $D_{ed\text{-}NS}{}^1$ = { a_2, a_4 ,a_5 ,a_7}, since $|\deg(a_i)-\deg(a_j)|\le 1$ for every vertex a_i in A_{NS} - $D_{ed\text{-}NS}{}^1$ there exists $a_j \in D_{ed\text{-}NS}{}^1$ such that $a_ia_j \in A_{NS}$. Hence $\gamma_{ef-NG}(D_{ed-NS}{}^1)$ = 0.6083+0.6666+0.7+0.6 = 2.5749

Other equitable dominating sets are

(ii) $D_{NS}{}^2$ = {a_1, a_3, a_6, a_8} (iii) $D_{NS}{}^3$ = { a_3, a_4, a_5, a_7}

Hence $\gamma_{ef-NG}(D_{ed-NS}{}^2)$ = 2.6499 and $\gamma_{ef-NG}(D_{ed-NS}{}^3)$ = 2.5416.

Equitable domination number of G_{NS} is $\gamma_{ef-NG}(G_{ed-NS})=\gamma_{ef-NG}(D_{ed-NS}{}^3)$ = 2.5416.

Domination number of G_{NS} is $\gamma_{NG}(D_{ed-NS}{}^3)$ = 2.5416.

Theorem 4.4. An equitable dominating set D_{NS} of a neutrosophic graph G_{NS} = (A_{NS}, B_{NS}) is minimal if and only if for each vertex $v_d \in D_{NS}$ one of the following conditions holds

(i) There exists a vertex $u_d \in A_{NS}$ - D_{NS} such that $N(u_d)\cap D_{NS}=\{v_d\}$

(ii) v_d is an isolate in $\langle D_{NS}\rangle$

Proof: Suppose D_{NS} is a minimal equitable dominating set of G_{NS} there exists a vertex v_d of D_{NS} which does not satisfy any of the above conditions. Hence there exists a vertex $u_d \in A_{NS}$ - D_{NS} such that $N(u_d)\cap D_{NS}\ne\{v_d\}$. Furthermore by condition(ii) v_d is not an isolate in $\langle D_{NS}\rangle$, then D_{NS} - v_d will be a minimal equitable dominating set of G_{NS} which is a contradiction to the assumption.

Theorem 4.5. For any NSG, G_{NS} = (A_{NS}, B_{NS})

$\gamma_{NG}(G_{NS})\le\gamma_{ed-NG}(G_{NS})$

Proof: Let D_d and D_{NS} be the minimal dominating set and minimal equitable dominating set of G_{NS} respectively. Let $v_d \in D_{NS}$ be a vertex which is adjacent to r- number of vertices such that deg(v_d) = t, where r > t and rest of the vertices in A_{NS}, A_{NS} - v_d is adjacent to exactly one vertex say v_d . By the definition of equitable domination, A_{NS} - v_d will be the members of D_{NS}. But $v_d \in D_d$ is the only member of dominating set of G_{NS}. Hence the inequality holds. In the case of proving equality, Let H_{NS} be a neutrosophic path P_4. Clearly $\gamma_{NG}(G_{NS})=\gamma_{ed-NG}(G_{NS})=2$.

Example 4.6

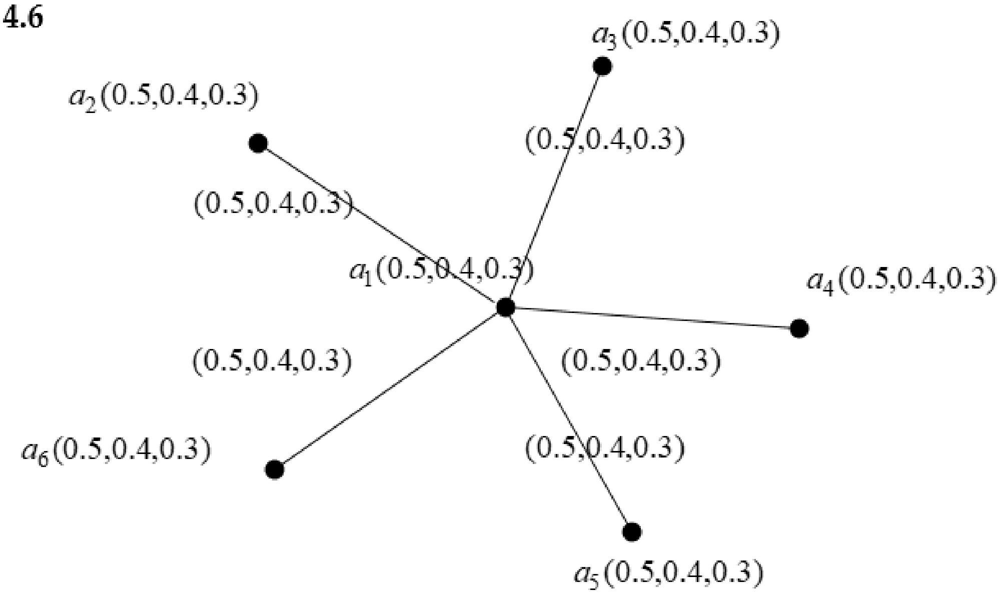

Figure 3. Example of Equitable domination

Let $G_{NS} = (A_{NS}, B_{NS})$ be the NSG shown in figure (3)
All arcs are strong.

Only possible equitable dominating set is $D_{ed\text{-}NS} = \{ a_1, a_2, a_3, a_4, a_5, a_6\}$, since $|\deg(a_1) - \deg(a_j)| \geq 1$ for every j = 2,3,4&5. Hence $\gamma_{ef-NG}(D_{ed-NS}) = 4$

But the dominating set is $D_{NS} = \{ a_1 \}$, hence $\gamma_{NG}(D_{NS}) = 0.6666$

By example 4.3, $\gamma_{ef-NG}(G_{ed-NS}) = \gamma_{NG}(D_{NS}) = 2.5416$.

By example 4.6, $\gamma_{ef-NG}(G_{ed-NS}) > \gamma_{NG}(D_{NS})$.

Theorem 4.7. If a dominating set $D_{ed\text{-}NS}$ of a NSG, $G_{NS} = (A_{NS}, B_{NS})$ is a equitable dominating then there exists two vertices u_d, $v_d \in A_{NS} - D_{ed\text{-}NS}$ such that every u_d - v_d path contains at least one vertex of $D_{ed\text{-}NS}$.

Proof: Suppose $D_{ed\text{-}NS}$ is equitable dominating set of G_{NS}. Since every vertex in $A_{NS} - D_{NS}$ is equitably dominated by at least one vertex of D_{NS}, there exists a u_d - v_d path contains at least one vertex of $D_{ed\text{-}NS}$.

Theorem 4.8. For any NSG, $G_{NS} = (A_{NS}, B_{NS})$ with order p

(i) $\gamma_{ef-NG}(G_{NS}) \leq p - \Delta_T(G_{NS})$

(ii) $\gamma_{ef-NG}(G_{NS}) \leq p - \Delta_I(G_{NS})$

(iii) $\gamma_{ef-NG}(G_{NS}) \leq p - \Delta_F(G_{NS})$

Proof: Let $G_{NS} = (A_{NS}, B_{NS})$ be a NSG with order p.

Since $\sum_{i=1}^{n} d(v_i) \le p$, where v_i represents the vertices present in the strong arc, the no of vertices present in an equitable dominating set is less than n. Furthermore, by the definition of $\Delta_T(G_{NS})$, the maximum truth membership values of v_i among all the vertices of G_{NS} and by the definition of minimal equitable dominating set we have

$$\gamma_{ef-NG}(G_{NS}) \le \sum_{i=1}^{n} d(v_i) \le p - \Delta_T(G_{NS})$$

Similarly, we prove $\gamma_{ef-NG}(G_{NS}) \le p - \Delta_I(G_{NS})$ and $\gamma_{ef-NG}(G_{NS}) \le p - \Delta_F(G_{NS})$

5. Strong and Weak Equitable Domination in NSG

The concept strong and weak domination in neutrosophic graph is more difficult to handle the values on degree of truth membership, indeterminacy membership and falsity membership, as the degree of edge membership values follows from the degree of incident vertex membership values as in the order of minimum of degree of truth membership values, minimum of degree of indeterminacy values and maximum of degree of falsity membership values respectively. To overcome this difficulty as in the concept of strong and weak equitable domination, we use the score function of vertex cardinality for each vertex and edge cardinality for each edge. The existence of strong and weak equitable domination in a neutrosophic graph is guaranteed on the degree of the neutrosophic graph

Definition 5.1. Let $G_{NS} = (A_{NS}, B_{NS})$ be a neutrosophic graph. Then the vertex score function v_{sf} of G_{NS} is defined

$$v_{sf} = \frac{2 + T_{A_{NS}} - (0.5)I_{A_{NS}} - F_{A_{NS}}}{3}$$

Definition 5.2. Let $G_{NS} = (A_{NS}, B_{NS})$ be a neutrosophic graph. Then vertex score function e_{sf} of G_{NS} is defined

$$e_{sf} = \frac{2 + T_{B_{NS}} - (0.5)I_{B_{NS}} - F_{B_{NS}}}{3}$$

Definition 5.3. Let u_d be a vertex in a NSG, $G_{NS} = (A_{NS}, B_{NS})$. The degree of a vertex u_d is defined as the sum of the weight of the score function of strong arcs incident at u_d and is denoted by deg(u_d). The neighbourhood of u_d is denoted by
$N(u_d) = \{v_d \in A_{NS} / (u_d, v_d)$ is a strong arc$\}$

The minimum degree of G_{NS} is $\delta(G_{NS}) = \min\{d_{G_{NS}}(u_d) / u_d \in A_{NS}\}$

The maximum degree of G_{NS} is $\Delta(G_{NS}) = \max\{d_{G_{INS}}(u_d)/u_d \in A_{NS}\}$

Definition 5.4. Order of $G_{NS} = (A_{NS}, B_{NS})$ is the sum of the score function of vertex cardinality of each vertex and is denoted by $O(G_{NS})$ and size of $G_{NS} = (A_{NS}, B_{NS})$ is the sum of the score function of edge cardinality of each edge.

Definition 5.5. Let $G_{NS} = (A_{NS}, B_{NS})$ be a neutrosophic graph. For any u_d , $v_d \in A_{NS}$, we say u_d strongly equitable dominates v_d if $\deg(u_d) \geq \deg(v_d)$ and u_d is a member of equitable dominating set

.**Definition 5.6.** Let $G_{NS} = (A_{NS}, B_{NS})$ be a neutrosophic graph. For any u_d , $v_d \in A_{NS}$, we say u_d weakly equitable dominates v_d if $\deg(u_d) \leq \deg(v_d)$ and u_d is a member of equitable dominating set.

Definition 5.7. A dominating set $D_{ed\text{-}NS}^S$ of A_{NS} of a neutrosophic graph $G_{NS} = (A_{NS}, B_{NS})$ is a strong equitable dominating set if for every $v_d \in A_{NS} - D_{ed\text{-}NS}^S$ there exists at least one $u_d \in D_{ed\text{-}NS}$ such that u_d strongly equitable dominates v_d .The minimum cardinality of a strong equitable dominating set is called a strong equitable domination number and is denoted by $\gamma_{ed-NG}{}^S(G_{NS})$.

Definition 5.8. A dominating set $D_{ed\text{-}NS}{}^W$ of A_{NS} of a neutrosophic graph $G_{NS} = (A_{NS}, B_{NS})$ is a weak equitable dominating set if for every $v_d \in A_{NS} - D_{ed\text{-}NS}{}^W$ there exists at least one $u_d \in D_{ed\text{-}NS}{}^W$ such that u_d weakly equitable dominates v_d .The minimum cardinality of a weak equitable dominating set is called a weak equitable domination number and is denoted by $\gamma_{ef-NG}{}^W(G_{NS})$.

Example 5.9. Let $G_{NS} = (A_{NS}, B_{NS})$ be the NSG represented in figure1. The following figure 4 is the Neutrosophic graph with score function of vertex cardinality and edge cardinality.

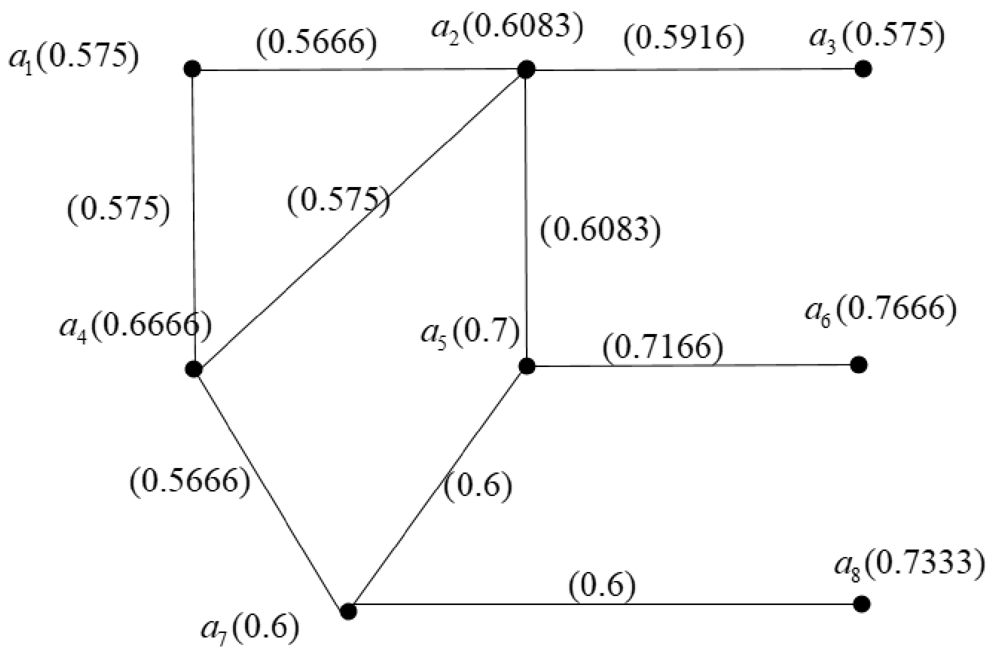

Figure 4. Neutrosophic graph with score function

The arcs a_1a_2, a_2a_5 , a_4a_7 are not strong arcs.

deg(a_1) = 0.575, deg(a_2) = 1.1999, deg(a_3) = 0.5916, deg(a_4) = 0.575, deg(a_5) = 1.9249, deg(a_6) = 0.7166, deg(a_7) = 1.2 , deg(a_8) = 0.6.

Strong equitable dominating set is $D_{ed\text{-}NS}{}^{S}$ = { a_2, a_4 ,a_6 ,a_8}, since $|\deg(u_d)-\deg(v_d)|\le 1$, for every vertex v_d in A_{NS} - $D_{ed\text{-}NS}{}^{S}$ there exists at least one $u_d \in D_{NS}$ such that u_d strongly equitable dominates v_d . Hence $\gamma_{ef-NG}(D_{ed-NS}{}^{S})$ = 0.6083+0.7666+0.7333+0.6666= 2.7748

Weak equitable dominating set is $D_{ed\text{-}NS}{}^{W}$ = { a_1, a_3 ,a_5 ,a_7}, since $|\deg(u_d)-\deg(v_d)|\le 1$, for every vertex v_d in A_{NS} - $D_{ed\text{-}NS}{}^{W}$ there exists at least one $u_d \in D_{NS}$ such that u_d weakly equitable dominates v_d . Hence $\gamma_{ef-NG}(D_{ed-NS}{}^{W})$ = 0.575+0.575+0.7+0.6= 2.45

Theorem 5.10. For any NSG, $G_{NS} = (A_{NS}, B_{NS})$

$\gamma_{ed-NG}{}^{W}(G_{NS}) \le \gamma_{ed-NG}{}^{S}(G_{NS})$ or $\gamma_{ed-NG}{}^{W}(G_{NS}) \ge \gamma_{ed-NG}{}^{S}(G_{NS})$

Proof. Let $D_{NS}{}^{w}$ and $D_{NS}{}^{S}$ be the weak and strong equitable dominating set of G_{NS}.
Case(i) Let the number of vertices present in the strong and weak domination is same then by the definition of strong and weak equitable domination, we have $\gamma_{ef-NG}{}^{W}(G_{NS}) < \gamma_{ef-NG}{}^{S}(G_{NS})$.

Example 5.8 shows that $\gamma_{ef-NG}{}^{W}(G_{NS}) < \gamma_{ef-NG}{}^{S}(G_{NS})$

Case(ii) Let the number of vertices present in the weak domination is more than by strong (with nearly equal membership values) then we have $\gamma_{ef-NG}{}^{W}(G_{NS}) > \gamma_{ef-NG}{}^{S}(G_{NS})$.

Case(iii) Let the arcs present in the given neutrosophic graph is strong with equal number of vertices present in strong and weak equitable dominating set then we have $\gamma_{ef-NG}{}^{W}(G_{NS}) = \gamma_{ef-NG}{}^{S}(G_{NS})$

Theorem 5.9. For any NSG, $G_{NS} = (A_{NS}, B_{NS})$

(i) $\gamma_{ed-NG}{}^{S}(G_{NS}) \le O(G_{NS}) - \Delta(G_{NS})$

(ii) $\gamma_{ed-NG}{}^{S}(G_{NS}) \le O(G_{NS}) - \delta(G_{NS})$

Proof: Let $G_{NS} = (A_{NS}, B_{NS})$ be a NSG.

Since $\sum_{i=1}^{n} d(v_i) \le p$, where v_i represents the vertices present in the strong arc, the number of vertices present in an equitable dominating set is less than n. Furthermore, by the definition of $\Delta_T(G_{NS})$, the maximum truth membership values of v_i among all the vertices of G_{NS} and by the definition of minimal strong equitable dominating set we have

$$\gamma_{ed-NG}(G_{NS}{}^{S}) \leq \sum_{i=1}^{n} d(v_i) \leq p - \Delta(G_{NS})$$

Similarly, we prove $\gamma_{ef-NG}(G_{NS}{}^{S}) \leq p - \delta(G_{NS})$.

Theorem 5.10. For any NSG, $G_{NS} = (A_{NS}, B_{NS})$

(i) $\gamma_{ed-NG}{}^{W}(G_{NS}) \leq O(G_{NS}) - \Delta(G_{NS})$

(ii) $\gamma_{ed-NG}{}^{W}(G_{NS}) \leq O(G_{NS}) - \delta(G_{NS})$

Proof: Theorem 5.10. follows from Theorem 5.9.

Theorem 5.11. For any NSG, $G_{NS} = (A_{NS}, B_{NS})$

(i) $\gamma_{ed-NG}{}^{S}(G_{NS}) \geq O(G_{NS})/(\Delta(G_{NS})+1)$

(ii) $\gamma_{ed-NG}{}^{W}(G_{NS}) \geq O(G_{NS})/(\Delta(G_{NS})+1)$

Proof: Let $G_{NS} = (A_{NS}, B_{NS})$ be a NSG and its strong equitable domination number be $\gamma_{ef-NG}{}^{S}(G_{NS})$

$$\left|O(G_{NS}) - \gamma_{ef-NS}{}^{S}(G_{NS})\right| \leq \sum_{i=1}^{n} d(v_i) \leq \gamma_{ed-NS}{}^{S}(G_{NS})\Delta(G_{NS})$$

$$\leq \gamma_{ed-NS}{}^{S}(G_{NS})\Delta(G_{NS})$$

$$O(G_{NS}) \leq \gamma_{ed-NS}{}^{S}(G_{NS})\Delta(G_{NS}) + \gamma_{ed-NS}{}^{S}(G_{NS})$$

$$\leq \gamma_{ed-NS}{}^{S}(G_{NS})(\Delta(G_{NS})+1)$$

Hence $\gamma_{ed-NG}{}^{S}(G_{NS}) \geq O(G_{NS})/(\Delta(G_{NS})+1)$

Similarly prove $\gamma_{ed-NG}{}^{W}(G_{NS}) \geq O(G_{NS})/(\Delta(G_{NS})+1)$

6.Conclusions Strong and weak equitable domination in a neutrosophic graph is difficult to initiate, as the NSG has degrees of truth membership value, degrees of indeterminacy membership value, and degrees of truth membership value. Comparing these three types of degrees of membership values of one vertex to another will help us identify strong and weak equitable dominating vertices. But in implementing this, the research focus is very narrowly focused on strong and weak equitable domination. Hence, we conclude that, using the vertex cardinality score function, we can convert all these three degree of membership values into a single value and then proceed with the concept of strong and weak equitable domination. In future, we have planned to continue the work on paired equitable domination of NSG using strong arcs and furthermore to find relationships between domination, equitable domination and paired equitable domination of neutrosophic graphs.

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