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A New Notion of Neighbourhood and Continuity in Neutrosophic Topological Spaces

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Abstract: Owing to a wide range of applications in various fields, the neutrosophic theory initiated by Smarandache has been highly featured in research. This concept led to the evolution of neutrosophic topological spaces which is being explored extensively. The focus of this paper is to introduce and study the concept of neutrosophic $\Upsilon-$neighbourhood and neutrosophic $\Upsilon-$continuity in neutrosophic topological spaces. Further, we define the notion of neutrosophic $\Upsilon-$irresolute functions. We also observe their attributes and relationship with functions existing in literature. Moreover, we present some equivalent conditions for the existence of these functions in which the concept of neighbourhood has been wielded.

Keywords: neutrosophic $\Upsilon-$open, neutrosophic $\Upsilon-$closed, neutrosophic $\Upsilon-$neighbourhood, neutrosophic $\Upsilon-$continuous, neutrosophic $\Upsilon-$irrseolute.

1. Introduction

Several theories were developed as mathematical approaches to rectify the difficulties pertained to uncertainty. Accordingly, the concept of neutrosophy initiated by Florentine Smarandache[1] evolved as a branch of philosophy to study the scope and nature of neutralities. This induced the concept of neutrosophic logic which further led to the conceptualization of neutrosophic sets as a generalization of fuzzy sets and intuitionistic fuzzy sets. A neutrosophic set is characterized by three independent components namely membership, indeterminacy and non-membership functions defined on the non-standard unit interval. Later, Salama and Albowi[3] in 2012 induced the concept of neutrosophic sets in topological spaces which originated as neutrosophic topological spaces. In addition, some basic notions and properties of topological structures such as interior, closure, subspaces and separation axioms have been presented in [4-8]. G. C. Ray and Sudeep[9] proposed the definitions of neutrosophic point and neighbourhood structure. They have also explored the relation of quasi coincidence between neutrosophic sets and characterized the neutrosophic topological spaces by means of quasi-neighbourhood. Meanwhile, Salama et.al[10] in 2014, studied the concept of continuous functions in neutrosophic topological spaces. Further, P. Iswarya and

K. Bageerathi[11], in 2016 introduced the concept of semi-open sets in neutrosophic topological spaces and later the notion of semi-continuous functions[12,13] were also studied. Dhavaseelan and Saeid Jafari[14], in 2017 established the idea of generalized closed sets and continuous functions in neutrosophic topological spaces. C. Maheshwari and S. Chandrasekar[15] defined the notion of gb-closed sets and continuous functions in 2019. Moreover, some novel concepts of continuous functions and other topological structures have been defined and studied by various authors[16-18] in the subsequent years. Recently, the authors[19] of this paper introduced and analyzed a new class of neutrosophic sets namely neutrosophic $\Upsilon-$open sets and neutrosophic $\Upsilon-$closed sets. The main objective of this paper is to introduce and study the concepts of neutrosophic $\Upsilon-$neighbourhood, neutrosophic $\Upsilon-$continuous and irresolute functions in neutrosophic topological spaces. The characterization and composition of these functions have been presented through results and counter examples. Further, various equivalent conditions for the existence of these concepts have also been observed.

The structure of the paper is as follows: section 2 comprises of the prerequisites essential for this work. Section 3 establishes a novel concept of neighbourhood namely neutrosophic $\Upsilon-$neighbourhood and $\Upsilon-$quasi neighbourhood. Section 4 imparts the notion of neutrosophic $\Upsilon-$continuous functions and its attributes. Further, section 5 presents the idea of neutrosophic $\Upsilon-$irresolute functions and the article is ceased with a conclusion in section 6.

2. Preliminaries

In this section, we have presented some basic notions and results required for the progression of this work.

Definition 2.1[3]**:** Let U be a non-empty fixed set. A **neutrosophic set** L is an object having the form $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ where $\mu_L(u), \sigma_L(u)$ and $\gamma_L(u)$ represent the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element $u \in U$. A neutrosophic set $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ can be identified to an ordered triple $< \mu_L, \sigma_L, \gamma_L >$ in $]^-0,1^+[$ on U.

Definition 2.2[3]: Let U be a non-empty set and $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$, $M = \{< u, \mu_M(u), \sigma_M(u), \gamma_M(u) >: u \in U\}$ be neutrosophic sets in U. Then

(i) $L \subseteq M$ if $\mu_L(u) \leq \mu_M(u), \sigma_L(u) \leq \sigma_M(u)$ and $\gamma_L(u) \geq \gamma_M(u)$ for all $u \in U$.

(ii) $L \cup M = \{< u,\ \max\{\mu_L(u), \mu_M(u)\},\ \max\{\sigma_L(u), \sigma_M(u)\},\ \min\{\gamma_L(u), \gamma_M(u)\}>: u \in U\}$

(iii) $L \cap M = \{< u,\ \min\{\mu_L(u), \mu_M(u)\},\ \min\{\sigma_L(u), \sigma_M(u)\},\ \max\{\gamma_L(u), \gamma_M(u)\}>: u \in U\}$

(iv) $L^c = \{< u, \gamma_L(u), 1 - \sigma_L(u), \mu_L(u) >: u \in U\}$

(v) $0_{N_{tr}} = \{< u, 0,0,1 >: u \in U\}$ and $1_{N_{tr}} = \{< u, 1,1,0 >: u \in U\}$

Definition 2.3[3]: A **neutrosophic topology** on a non-empty set U is a family $\tau_{N_{tr}}$ of neutrosophic sets in U satisfying the following axioms:

(i) $0_{N_{tr}}, 1_{N_{tr}} \in \tau_{N_{tr}}$

(ii) $\cup L_i \in \tau_{N_{tr}} \forall \{L_i: i \in I\} \subseteq \tau_{N_{tr}}$

(iii) $L_1 \cap L_2 \in \tau_{N_{tr}}$ for any $L_1, L_2 \in \tau_{N_{tr}}$

The pair $(U, \tau_{N_{tr}})$ is called a neutrosophic topological space. The members of $\tau_{N_{tr}}$ are called neutrosophic open and its complements are called neutrosophic closed.

Definition 2.4[5]: A neutrosophic set $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ is called a **neutrosophic point** if for any element $v \in U, \mu_L(v) = a, \sigma_L(v) = b, \gamma_L(v) = c$ for $u = v$ and $\mu_L(v) = 0, \sigma_L(v) = 0, \gamma_L(v) = 1$ for $u \neq v$, where a, b, c are real standard or non standard subsets of $]^-0,1^+[$. A neutrosophic point is denoted by $u_{a,b,c}$. For the neutrosophic point $u_{a,b,c}, u$ will be called its support.

Definition 2.5[4]: Let $(U, \tau_{N_{tr}})$ be a neutrosophic topological space and S be a non-empty subset of U. Then, a neutrosophic relative topology on S is defined by

$$\tau^S_{N_{tr}} = \{L \cap 1^S_{N_{tr}} : L \in \tau_{N_{tr}}\}$$

where

$$1^S_{N_{tr}} = \begin{cases} < 1,1,0 >, & if s \in S \\ < 0,0,1 >, & otherwise \end{cases}$$

Thus, $(S, \tau^S_{N_{tr}})$ is called a **neutrosophic subspace** of $(U, \tau_{N_{tr}})$.

Definition 2.6[14]: Let U and V be two non-empty sets and $f_{N_{tr}}: U \longrightarrow V$ be a function. If $M = \{< v, \mu_M(v), \sigma_M(v), \gamma_M(v) >: v \in V\}$ is a neutrosophic set in V, then the preimage of M under $f_{N_{tr}}$, denoted by $f^{-1}_{N_{tr}}(M)$, is the neutrosophic set in U defined by

$$f^{-1}_{N_{tr}}(M) = \{< u, f^{-1}_{N_{tr}}(\mu_M)(u), f^{-1}_{N_{tr}}(\sigma_M)(u), f^{-1}_{N_{tr}}(\gamma_M)(u) >: u \in U\}$$

If $L = \{< u, \mu_L(u), \sigma_L(u), \gamma_L(u) >: u \in U\}$ is a neutrosophic set in U, then the image of L under $f_{N_{tr}}$, denoted by $f_{N_{tr}}(L)$, is the neutrosophic set in V defined by

$$f_{N_{tr}}(L) = \{< v, f_{N_{tr}}(\mu_L)(v), f_{N_{tr}}(\sigma_L)(v), \left(1 - f_{N_{tr}}(1 - \gamma_L)\right)(v) > \ : v \in V\} \text{ where}$$

$$f_{N_{tr}}(\mu_L)(v) = \begin{cases} sup_{u \in f^{-1}_{N_{tr}}(v)} \mu_L(u), & \text{if } f^{-1}_{N_{tr}}(v) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$f_{N_{tr}}(\sigma_L)(v) = \begin{cases} sup_{u \in f^{-1}_{N_{tr}}(v)} \sigma_L(u), & \text{if } f^{-1}_{N_{tr}}(v) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\left(1 - f_{N_{tr}}(1 - \gamma_L)\right)(v) = \begin{cases} inf_{u \in f^{-1}_{N_{tr}}(v)} \gamma_L(u), & \text{if } f^{-1}_{N_{tr}}(v) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

Definition 2.7: Let $(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be **neutrosophic continuous**[10] (respectively, neutrosophic semi−continuous[12], neutrosophic α −continuous[14], neutrosophic β −continuous, neutrosophic $gs-$ continuous, neutrosophic $gb-$ continuous[15]) if $f^{-1}_{N_{tr}}(M)$ is N_{tr} open(respectively N_{tr} semi − open, $N_{tr}\alpha$ − open, $N_{tr}\beta$ − open, $N_{tr}gs$ − open, $N_{tr}gb$ − open) in $(U, \tau_{N_{tr}})$ for every N_{tr}open set M in $(V, \rho_{N_{tr}})$.

Definition 2.8[7]: Let $u_{a,b,c}$ be a neutrosophic point in a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then a neutrosophic set N in U is said to be **neutrosophic neighbourhood**($N_{tr}nbhd$) of $u_{a,b,c}$ if there exists a N_{tr}open set M such that $u_{a,b,c} \in M \subseteq N$.

Definition 2.9[6]: A neutrosophic point $u_{a,b,c}$ is said to be **neutrosophic quasi – coincident** with a neutrosophic set L, denoted by $u_{a,b,c}qL$ if $u_{a,b,c} \notin L^c$. If $u_{a,b,c}$ is not neutrosophic quasi – coincident with L, we denote it by $u_{a,b,c}\hat{q}L$.

Definition 2.10[6]: A neutrosophic set M is said to be neutrosophic quasi – coincident with a neutrosophic set L, denoted by MqL if $M \nsubseteq L^c$. If M is not neutrosophic quasi – coincident with L, we denote it by $M\hat{q}L$.

Definition 2.11[6]: A neutrosophic set N in U is said to be **neutrosophic quasi-neighbourhood**($N_{tr}Qnbhd$) of $u_{a,b,c}$ if there exists a N_{tr}open set M such that $u_{a,b,c}qM \subseteq N$.

Definition 2.12[19]:A neutrosophic set L of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be **neutrosophic $\Upsilon-$open** if for every non-empty N_{tr}closed set $F \neq 1_{N_{tr}}, L \subseteq N_{tr}cl(N_{tr}int(L \cup F))$. The complement of neutrosophic $\Upsilon-$open set is neutrosophic $\Upsilon-$closed. The class of neutrosophic $\Upsilon-$open sets in $(U, \tau_{N_{tr}})$ is denoted by $N_{tr}\Upsilon O(U, \tau_{N_{tr}})$.

Theorem 2.13[19]: The union of an arbitrary collection of $N_{tr}\Upsilon-$open sets is also $N_{tr}\Upsilon-$open.

Theorem 2.14[19]: In any neutrosophic topological space $(U, \tau_{N_{tr}})$,

(i) Every N_{tr}open set is $N_{tr}\Upsilon-$open.
(ii) Every N_{tr}semi−open set is $N_{tr}\Upsilon-$open.
(iii) Every $N_{tr}\alpha-$ open set is $N_{tr}\Upsilon-$open.
(iv) Every $N_{tr}\Upsilon-$open set is $N_{tr}\beta-$ open.
(v) Every $N_{tr}\Upsilon-$open set is $N_{tr}gs-$ open.
(vi) Every $N_{tr}\Upsilon-$open set is $N_{tr}gb-$ open.

Remark 2.15[19]: The above theorem is also true for $N_{tr}\Upsilon-$closed sets.

Theorem 2.16[19]: A neutrosophic set L in a neutrosophic topological space $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon-$open if and only if for every neutrosophic point $u_{a,b,c} \in L$, there exists a $N_{tr}\Upsilon-$ open set $M_{u_{a,b,c}}$ such that $u_{a,b,c} \in M_{u_{a,b,c}} \subseteq L$.

Definition 2.17[19]: Let be a neutrosophic topological space and L be a neutrosophic set in U.

(i) The **neutrosophic $\Upsilon-$interior** of L is the union of all $N_{tr}\Upsilon-$open sets contained in L. It is denoted by $N_{tr}\Upsilon int(L)$.
(ii) The **neutrosophic $\Upsilon-$ closure** of L is the intersection of all $N_{tr}\Upsilon-$ closed sets containing L. It is denoted by $N_{tr}\Upsilon cl(L)$.

3. Neutrosophic $\Upsilon-$neighbourhood

This section conceptualizes the idea of neutrosophic $\Upsilon-$neighbourhood and neutrosophic $\Upsilon-$quasi neighbourhood. Moreover, their characterizations have been depicted through results and illustrations.

Definition 3.1: Let $u_{a,b,c}$ be a neutrosophic point in a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then a neutrosophic set N in U is said to be a

(i) **neutrosophic $\Upsilon-$neighbourhood**$(N_{tr}\Upsilon-nbhd)$ of $u_{a,b,c}$ if there exists a $N_{tr}\Upsilon-$ open set M such that $u_{a,b,c} \in M \subseteq N$.

(ii) **neutrosophic $\Upsilon-$quasi neighbourhood**$(N_{tr}\Upsilon-Qnbhd)$ of $u_{a,b,c}$ if there exists a $N_{tr}\Upsilon-$ open set M such that $u_{a,b,c}qM \subseteq N$.

Example 3.2: Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where$L = \{< a, 0.7, 0.5, 0.3 >< b, 0.2, 0.7, 0.1 >\}$. Now, let us consider a neutrosophic point $a_{0.1,0.2,0.5}$ in U. Then, there is a $N_{tr}\Upsilon-$ open set $M = \{< a, 0.8, 0.8, 0.1 >< b, 0.5, 0.9, 0.1 >\}$ such that $a_{0.1,0.2,0.5} \in M \subseteq N$ where$N = \{< a, 0.9, 0.8, 0.1 > < b, 0.6, 0.9, 0.1 >\}$. Hence N is a $N_{tr}\Upsilon-nbhd$ of $a_{0.1,0.2,0.5}$.

Example 3.3: Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where$L = \{< a, 0.8, 0.7, 0.1 >< b, 0.4, 0.9, 0.1 >\}$. Now, let us consider a neutrosophic point $a_{0.2,0.9,0.7}$ in U. Then, there is a $N_{tr}\Upsilon-$ open set $M = \{< a, 0.9, 0.8, 0.1 >< b, 0.7, 0.9, 0.1 >\}$ such that $a_{0.2,0.9,0.7}qM \subseteq N$ where $N = \{< a, 0.9, 0.9, 0.1 > < b, 0.8, 0.9, 0.1 >\}$. Hence N is a $N_{tr}\Upsilon-Qnbhd$ of $a_{0.2,0.9,0.7}$.

Theorem 3.4: Every $N_{tr}nbhd$ (resp. $N_{tr}Qnbhd$) of a neutrosophic point $u_{a,b,c}$ in a neutrosophic topological space $(U, \tau_{N_{tr}})$ is a $N_{tr}\Upsilon-nbhd$(resp.$N_{tr}\Upsilon-Qnbhd$) of $u_{a,b,c}$.

Proof: Let N be a $N_{tr}nbhd$ (resp. $N_{tr}Qnbhd$) of a neutrosophic point $u_{a,b,c}$ in U. Then, there exists a N_{tr}open set M in U such that $u_{a,b,c} \in M \subseteq N$ (resp. $u_{a,b,c}qM \subseteq N$). Now, by theorem 2.14, M is $N_{tr}\Upsilon -$ open in U. Hence there exists a $N_{tr}\Upsilon -$ open set M in U such that $u_{a,b,c} \in M \subseteq N$ (resp. $u_{a,b,c}qM \subseteq N$). Therefore N is a $N_{tr}\Upsilon - nbhd$(resp. $N_{tr}\Upsilon - Qnbhd$) of $u_{a,b,c}$.

The following example substantiates that the converse of the above-stated theorem need not be true.

Example 3.5: (i) Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.6,0.6,0.2 >< b, 0.2,0.9, 0.1 >\}$. Now, let us consider a neutrosophic point $a_{0.7,0.1,0.5}$ in U. Then there is a $N_{tr}\Upsilon -$ open set $M = \{< a, 0.8,0.7,0.2 >< b, 0.3,0.9,0.1 >\}$ such that $a_{0.7,0.1,0.5} \in M \subseteq N$ where $N = \{< a, 0.8,0.9, 0.1 >< b, 0.4,0.9,0.1 >\}$. This implies N is a $N_{tr}\Upsilon - nbhd$ of $a_{0.7,0.1,0.5}$. However, N is not a $N_{tr}nbhd$ of $a_{0.7,0.1,0.5}$.

(ii) Let $U = \{a, b\}$ and $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ where $L = \{< a, 0.7,0.9,0.1 >< b, 0.5,0.7,0.4 >\}$. Now, let us consider a neutrosophic point $a_{0.1,0.1,0.7}$ in U. Then there is a $N_{tr}\Upsilon -$ open set $M = \{< a, 0.8,0.9,0.1 >< b, 0.7,0.7,0.2 >\}$ such that $a_{0.1,0.1,0.7}qM \subseteq N$ where $N = \{< a, 0.9,0.9,0.1 > < b, 0.9,0.8,0.1 >\}$. This implies N is a $N_{tr}\Upsilon - Qnbhd$ of $a_{0.1,0.1,0.7}$. However, N is not a $N_{tr}nbhd$ of $a_{0.1,0.1,0.7}$.

Theorem 3.6: A neutrosophic set L in a neutrosophic topological space $(U, \tau_{N_{tr}})$ is $N_{tr}\Upsilon -$ open if and only if for every neutrosophic point $u_{a,b,c} \in L, L$ is a $N_{tr}\Upsilon - nbhd$ of $u_{a,b,c}$.

Proof: Let L be $N_{tr}\Upsilon -$ open in U. Also, for each $u_{a,b,c} \in L, L \subseteq L$. Then, by definition 3.1(i), it follows that L is a $N_{tr}\Upsilon - nbhd$ of $u_{a,b,c}$. Conversely, assume that for every $u_{a,b,c} \in L, L$ is a $N_{tr}\Upsilon - nbhd$ of $u_{a,b,c}$. Then, there exists a $N_{tr}\Upsilon -$ open set M in U such that $u_{a,b,c} \in M \subseteq L$. Therefore, by theorem 2.16, L is $N_{tr}\Upsilon -$ open.

Theorem 3.7: Every $N_{tr}\Upsilon -$ open set L in a neutrosophic topological space $(U, \tau_{N_{tr}})$ is a $N_{tr}\Upsilon - Qnbhd$ of every neutrosophic point quasi−coincident with L.

Proof: The proof is obvious since for every neutrosophic point $u_{a,b,c}qL$, we have $u_{a,b,c}qL \subseteq L$.

Theorem 3.8: Let L be a $N_{tr}\Upsilon -$ closed set in a neutrosophic topological space $(U, \tau_{N_{tr}})$ and $u_{a,b,c}qL^c$. Then, there exists a $N_{tr}\Upsilon - Qnbhd$ M of $u_{a,b,c}$ such that $L\hat{q}M$.

Proof: Since L is $N_{tr}\Upsilon -$ closed in U, L^c is $N_{tr}\Upsilon -$ open in U such that $u_{a,b,c}qL^c$. Then, by theorem 3.7, L^c is a $N_{tr}\Upsilon - Qnbhd$ of $u_{a,b,c}$. Hence there exists a $N_{tr}\Upsilon -$ open set M in U such that $u_{a,b,c}qM \subseteq L^c$. Again, by theorem 3.7, M is a $N_{tr}\Upsilon - Qnbhd$ of $u_{a,b,c}$. Also, since $M \subseteq L^c, L\hat{q}M$. Hence there exists a $N_{tr}\Upsilon - Qnbhd$ M of $u_{a,b,c}$ such that $L\hat{q}M$.

Theorem 3.9: Let L be a neutrosophic set in a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then a neutrosophic point $u_{a,b,c} \in N_{tr}\Upsilon cl(L)$ if and only if every $N_{tr}\Upsilon - Qnbhd$ of $u_{a,b,c}$ is quasi-coincident with L.

Proof: Let $u_{a,b,c} \in N_{tr}\Upsilon cl(L)$ and N be a $N_{tr}\Upsilon - Qnbhd$ of $u_{a,b,c}$ such that $N\hat{q}L$. Then, there exists a $N_{tr}\Upsilon -$ open set M such that $u_{a,b,c}qM \subseteq N$. Since $N\hat{q}L, N \subseteq L^c$ and therefore $M \subseteq L^c$ which implies $L \subseteq M^c$. Now, M^c is a $N_{tr}\Upsilon -$ closed set containing L and $N_{tr}\Upsilon cl(L)$ is the smallest $N_{tr}\Upsilon -$ closed set containing L. Hence $N_{tr}\Upsilon cl(L) \subseteq M^c$. Also, since $u_{a,b,c}qM, u_{a,b,c} \notin M^c$. Therefore $u_{a,b,c} \notin N_{tr}\Upsilon cl(L)$ which is a contradiction. Conversely, suppose every $N_{tr}\Upsilon - Qnbhd$ of $u_{a,b,c}$ is quasi-coincident with L. If $u_{a,b,c} \notin N_{tr}\Upsilon cl(L)$, then there exists a $N_{tr}\Upsilon -$ closed set M such that $L \subseteq M$ and $u_{a,b,c} \notin M$. This implies that $u_{a,b,c}qM^c$, where M^c is a $N_{tr}\Upsilon -$ open set in U. Now, by theorem 3.7, M^c is a $N_{tr}\Upsilon - Qnbhd$ of $u_{a,b,c}$ such that $M^c\hat{q}L$ which is a contradiction.

4. Neutrosophic Υ −continuous functions

Topology is constantly intrigued by issues that are either directly or indirectly related to continuity. Accordingly, continuity plays a prominent role in the characterization of topological spaces. This section deals with the origination of neutrosophic $\Upsilon -$ continuous functions in neutrosophic topological spaces. Further, we have observed their properties and discussed the composition of functions.

Definition 4.1: Let $(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be **neutrosophic $\Upsilon -$ continuous** if $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon -$ open in $(U, \tau_{N_{tr}})$ for every N_{tr}open set M in $(V, \rho_{N_{tr}})$.

Example 4.2: Let $U = \{a, b\}, V = \{x, y\}, \tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L, M\}$ and $\rho_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, N\}$ where $L = \{< a, 0.6,0.3,0.5 >< b, 0.5,0.8,0.4 >\}$, $M = \{< a, 0.5,0.2,0.7 >< b, 0.2,0.7,0.9 >\}$ and $N = \{< x, 0.9, 0.9,0.1 >< y, 0.8,0.9,0.2 >\}$. Consider the collections $\mathcal{P} = \{P : L \subset P, M^c \subset P\}$ and $\mathcal{Q} = \{Q : L \subset Q; Q \not\subset M^c;\ M^c \not\subset Q\}$ of neutrosophic sets in U. Then $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, M, \mathcal{P}, \mathcal{Q}, 1_{N_{tr}}\}$. Define $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ as $f_{N_{tr}}(a) = y$ and $f_{N_{tr}}(b) = x$. Then, $f_{N_{tr}}^{-1}(N) = \{< a, 0.8,0.9,0.2 > < b, 0.9,0.9,0.1 >\} \in \mathcal{P}$ which implies $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon -$ open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon$ −continuous.

Theorem 4.3: Every N_{tr}continuous function is $N_{tr}\Upsilon -$ continuous.

Proof: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a N_{tr}continuous function. Let M be a N_{tr}open set in V. Since $f_{N_{tr}}$ is N_{tr}continuous, $f_{N_{tr}}^{-1}(M)$ is N_{tr}open in U. By theorem 2.14, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon -$ open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous.

The following example substantiates that the converse of the above-stated theorem need not be true.

Example 4.4: Let $U = \{a, b\}, V = \{x, y\}, \tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L, M\}$ and $\rho_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, N\}$ where $L = \{< a, 0.6,0.4,0.9 >< b, 0.5,0.7,1 >\}, M = \{< a, 0.7,0.6,0.8 >< b, 0.6,0.8,0.9 >\}$ and $N = \{< x, 0.6, 0.9,0.3 >< y, 0.7,0.6,0.2 >\}$. Consider the collections $\mathcal{P} = \{P : M \subset P, L^c \subset P\}$ and $\mathcal{Q} = \{Q : M \subset Q; Q \not\subset L^c;\ L^c \not\subset Q\}$ of neutrosophic sets in U.Then $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, M, \mathcal{P}, \mathcal{Q}, 1_{N_{tr}}\}$. Define $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ as $f_{N_{tr}}(a) = y$ and $f_{N_{tr}}(b) = x$. Then, $f_{N_{tr}}^{-1}(N) = \{< a, 0.7,0.6,0.2 > < b, 0.6,0.9,0.3 >\} \in \mathcal{Q}$ which implies $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon -$ open but not N_{tr}open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous but not N_{tr}continuous.

Theorem 4.5: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a function between two neutrosophic topological spaces.

(i) If $f_{N_{tr}}$ is N_{tr}semi − continuous, then $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous.

(ii) If $f_{N_{tr}}$ is $N_{tr}\alpha -$ continuous, then $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous.

(iii) If $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous, then $f_{N_{tr}}$ is $N_{tr}\beta -$ continuous.

(iv) If $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous, then $f_{N_{tr}}$ is $N_{tr}gs -$ continuous.

(v) If $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous, then $f_{N_{tr}}$ is $N_{tr}gb -$ continuous.

Proof: Proof is obvious.

However, the ensuing examples reveal that the converse of these implications is not necessarily true in general.

Example 4.6: Let $U = \{a, b\}, V = \{x, y\}, \tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ and $\rho_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, M\}$ where $L = \{< a, 0.2,0.4,0.7 >< b, 0.1,0.2,0.3 >\}$, $M = \{< x, 0,0.1,0.6 >< y, 0.1,0.2,0.9 >\}$. Consider the collections $\mathcal{P} = \{P :\ 0_{N_{tr}} \subset P \subset L\}$, $\mathcal{Q} = \{Q :\ L \not\subset Q\ ; Q \not\subset L\ ; Q \subset L^c\}$ and $\mathcal{R} = \{R : L \subset R \subset L^c\}$ of neutrosophic sets in U. Then, $N_{tr}\alpha O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, 1_{N_{tr}}, L\}, N_{tr}SO(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{R}, 1_{N_{tr}}\}$

and $N_{tr}\Upsilon O(U,\tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{tr}}\}$. Define $f_{N_{tr}}: (U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ as $f_{N_{tr}}(a) = y$ and $f_{N_{tr}}(b) = x$. Then, $f_{N_{tr}}^{-1}(M) = \{< a, 0.1,0.2,0.9 >< b, 0,0.1,0.6 >\} \in \mathcal{P}$ which implies $f_{N_{tr}}^{-1}(M)$is $N_{tr}\Upsilon-$open. However, it is neither N_{tr}semi − open nor $N_{tr}\alpha -$ open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous but not N_{tr}semi − continuous and $N_{tr}\alpha -$ continuous.

Example 4.7: Let $U = \{a,b\}, V = \{x,y\}, \tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ and $\rho_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, M\}$ where $L = \{< a, 0.7,0.8,0.6 >< b, 0.7,0.7,0.5 >\}$ and $M = \{< x, 0.5,0.7,0.2 >< y, 0.6,0.9,0.1 >\}$. Consider the collections $\mathcal{P} = \{P : L^c \subset P \subset L\}$, $\mathcal{Q} = \{Q : L \subset Q \subset 1_{N_{tr}}\}$, $\mathcal{R} = \{R : L^c \not\subset R\ ; R \not\subset L^c\ ; R \subset L\}$, $\mathcal{S} = \{S : L^c \not\subset S\ ; S \not\subset L^c\ ; S \not\subset L\}$, $\mathcal{T} = \{T : L^c \subset T \not\subset L\}$ and $\mathcal{W} = \{W : 0_{N_{tr}} \subset W \subset L^c\}$ of neutrosophic sets in U. Then, $N_{tr}\beta O(U,\tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}, \mathcal{T}, 1_{N_{tr}}\}$, $N_{tr}gsO(U,\tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{Q}, \mathcal{R}, \mathcal{S}, \mathcal{W}, 1_{N_{tr}}\}$, $N_{tr}gbO(U,\tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}, \mathcal{T}, \mathcal{W}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(U,\tau_{N_{tr}}) = \{0_{N_{tr}}, L, \mathcal{Q}, 1_{N_{tr}}\}$. Define $f_{N_{tr}}: (U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ as $f_{N_{tr}}(a) = x$ and $f_{N_{tr}}(b) = y$. Then, $f_{N_{tr}}^{-1}(M) = \{< a, 0.5,0.7,0.2 >< b, 0.6,0.9,0.1 >\} \in \mathcal{S}$ which implies $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\beta -$ open, $N_{tr}gs -$ open and $N_{tr}gb -$ open but not $N_{tr}\Upsilon -$ open. Hence $f_{N_{tr}}$ is $N_{tr}\beta -$ continuous, $N_{tr}gs -$ continuous and $N_{tr}gb -$ continuous but not $N_{tr}\Upsilon -$ continuous.

Theorem 4.8: Let $f_{N_{tr}}: (U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ be a function between two neutrosophic topological spaces. Then the following statements are equivalent:

(i) $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous.

(ii) The inverse image of every N_{tr}closed set in $(V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon$ −closed in $(U,\tau_{N_{tr}})$.

(iii) $f_{N_{tr}}(N_{tr}\Upsilon cl(L)) \subseteq N_{tr}cl\left(f_{N_{tr}}(L)\right)$ for every neutrosophic set L in U.

(iv) $N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(N_{tr}cl(M))$ for every neutrosophic set M in V.

Proof:

(i)⟹(ii) Let $f_{N_{tr}}$ be a $N_{tr}\Upsilon -$ continuous function and N be a N_{tr}closed set in V. Then N^c is N_{tr}open in V. Since $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous, $f_{N_{tr}}^{-1}(N^c)$ is $N_{tr}\Upsilon -$ open in U. That is, $(f_{N_{tr}}^{-1}(N))^c$ is $N_{tr}\Upsilon -$ open in U. Hence $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon -$ closed in U.

(ii)⟹(i) Let M be N_{tr}open in V. Then M^c is N_{tr}closed in V. By assumption, $f_{N_{tr}}^{-1}(M^c)$ is $N_{tr}\Upsilon -$ closed in U. That is, $(f_{N_{tr}}^{-1}(M))^c$is $N_{tr}\Upsilon -$ closed in U. Hence $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon -$ open in U. Therefore, $f_{N_{tr}}$ is $N_{tr}\Upsilon -$ continuous.

(ii)⟹(iii) Let L be a neutrosophic set in U. Now, $L \subseteq f_{N_{tr}}^{-1}\left(f_{N_{tr}}(L)\right)$ implies $L \subseteq f_{N_{tr}}^{-1}(N_{tr}cl\left(f_{N_{tr}}(L)\right)$

Since $N_{tr}cl\left(f_{N_{tr}}(L)\right)$ is N_{tr}closed in V, by assumption $f_{N_{tr}}^{-1}\left(N_{tr}cl\left(f_{N_{tr}}(L)\right)\right)$ is a $N_{tr}\Upsilon -$ closed set containing L. Also, $N_{tr}\Upsilon cl(L)$ is the smallest $N_{tr}\Upsilon -$ closed set containing L. Hence, $N_{tr}\Upsilon cl(L) \subseteq f_{N_{tr}}^{-1}(N_{tr}cl\left(f_{N_{tr}}(L)\right)$. Therefore, $f_{N_{tr}}(N_{tr}\Upsilon cl(L)) \subseteq N_{tr}cl\left(f_{N_{tr}}(L)\right)$.

(iii)⟹(ii) Let N be a N_{tr}closed set in V. Then, by assumption

$f_{N_{tr}}\left(N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(N)\right)\right) \subseteq N_{tr}cl\left(f_{N_{tr}}\left(f_{N_{tr}}^{-1}(N)\right)\right) \subseteq N_{tr}cl(N) = N$ implies $N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(N)\right) \subseteq f_{N_{tr}}^{-1}(N)$.

Also, $f_{N_{tr}}^{-1}(N) \subseteq N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(N)\right)$. Hence $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon -$ closed in U.

(iii)⟹(iv) Let M be a neutrosophic set in V and let $L = f_{N_{tr}}^{-1}(M)$. By assumption, $f_{N_{tr}}(N_{tr}\Upsilon cl(L)) \subseteq N_{tr}cl\left(f_{N_{tr}}(L)\right) = N_{tr}cl(M)$.This implies $N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(N_{tr}cl(M))$.

(iv)⟹(iii) Let $M = f_{N_{tr}}(L)$. Then, by assumption, $N_{tr}\Upsilon cl(L) = N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(N_{tr}cl(M)) \subseteq f_{N_{tr}}^{-1}\left(N_{tr}cl\left(f_{N_{tr}}(L)\right)\right)$. This implies $f_{N_{tr}}(N_{tr}\Upsilon cl(L)) \subseteq N_{tr}cl\left(f_{N_{tr}}(L)\right)$.

(iv) ⟹ (i) Let M be N_{tr}open in V. Then M^c is N_{tr}closed in V. By assumption, $f_{N_{tr}}^{-1}(M^c) = f_{N_{tr}}^{-1}(N_{tr}cl(M^c)) \supseteq N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M^c)\right)$. Also, we know that $f_{N_{tr}}^{-1}(M^c) \subseteq N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M^c)\right)$. Hence $f_{N_{tr}}^{-1}(M^c) = N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M^c)\right)$. Therefore, $f_{N_{tr}}^{-1}(M^c)$ is $N_{tr}\Upsilon -$closed in U. That is, $\left(f_{N_{tr}}^{-1}(M)\right)^c$ is $N_{tr}\Upsilon -$closed in U. Hence $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon -$open in U. Therefore $f_{N_{tr}}$ is $N_{tr}\Upsilon -$continuous.

Example 4.9: (i) Consider the topological spaces and the functions defined in example 4.2. Here $f_{N_{tr}}$ is $N_{tr}\Upsilon -$continuous and $N_{tr}\Upsilon C(U,\tau_{N_{tr}}) = \{0_{N_{tr}}, L^c, M^c, \mathcal{P}', \mathcal{Q}', 1_{N_{tr}}\}$ where $\mathcal{P}' = \{P^c : P \in \mathcal{P}\}$ and $\mathcal{Q}' = \{Q^c : Q \in \mathcal{Q}\}$. Now, $f_{N_{tr}}^{-1}(N^c) = \{< a, 0.2\ \ 0.1, 0.8 >< b, 0.1,0.1,0.9 >\} \in \mathcal{P}'$. Hence the inverse image of every N_{tr}closed set in $(V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon -$closed in $(U,\tau_{N_{tr}})$ if $f_{N_{tr}}$ is $N_{tr}\Upsilon -$continuous.
(ii) Let $U = \{a,b\}, V = \{x,y\}, \tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}$ and $\rho_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, M\}$ where $L = \{< a, 0.2,0.4, 0.9 >< b, 0.3,0.8,0.7 >\}$ and $M = \{< x, 0.9,0.7,0.1 >< y, 0.8,0.9,0.2 >\}$. Consider the collections $\mathcal{P} = \{P : P \subset L, P \subset L^c\}$ and $\mathcal{Q} = \{Q : Q \subset L^c; Q \not\subset L;\ \ L \not\subset Q\}$ of neutrosophic sets in U. Then $N_{tr}\Upsilon C(U,\tau_{N_{tr}}) = \{0_{N_{tr}}, L^c, \mathcal{P}, \mathcal{Q}, 1_{N_{tr}}\}$. Now, define $f_{N_{tr}}: (U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ as $f_{N_{tr}}(a) = x$ and $f_{N_{tr}}(b) = y$. Then, $f_{N_{tr}}^{-1}(M^c) = \{< a, 0.1,0.3,0.9 >< b, 0.2,0.1,0.8 >\} \in \mathcal{P}$. Now, $f_{N_{tr}}^{-1}(M^c) = (f_{N_{tr}}^{-1}(M))^c$ is $N_{tr}\Upsilon -$closed implies $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon -$open. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon -$continuous if the inverse image of every N_{tr}closed set in $(V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon -$closed in $(U,\tau_{N_{tr}})$.

Theorem 4.10: A function $f_{N_{tr}}: (U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon -$continuous if and only if $f_{N_{tr}}^{-1}(N_{tr}int(M)) \subseteq N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right)$ for every neutrosophic set M in V.

Proof: Let $f_{N_{tr}}$ be a $N_{tr}\Upsilon -$continuous function and M be a neutrosophic set in V. Then $N_{tr}int(M)$ is N_{tr}open in V. By assumption, $f_{N_{tr}}^{-1}(N_{tr}int(M))$ is $N_{tr}\Upsilon -$open in U. Now, $f_{N_{tr}}^{-1}(N_{tr}int(M)) \subseteq f_{N_{tr}}^{-1}(M)$ and $N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right)$ is the largest $N_{tr}\Upsilon -$open set contained in $f_{N_{tr}}^{-1}(M)$. Hence $f_{N_{tr}}^{-1}(N_{tr}int(M)) \subseteq N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right)$. Conversely, let M be a N_{tr}open set in V. Then $f_{N_{tr}}^{-1}(M) = f_{N_{tr}}^{-1}(N_{tr}int(M)) \subseteq N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right)$. Also, $N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(M)$. This implies $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon -$open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon -$continuous.

Theorem 4.11: Let $f_{N_{tr}}: (U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ be a function between two neutrosophic topological spaces. Then the following statements are equivalent:
(i) $f_{N_{tr}}$ is $N_{tr}\Upsilon -$continuous.
(ii) For each neutrosophic point $u_{a,b,c}$, the inverse image of every $N_{tr}nbhd$ of $f_{N_{tr}}(u_{a,b,c})$ is $N_{tr}\Upsilon -nbhd$ of $u_{a,b,c}$.
(iii) For each neutrosophic point $u_{a,b,c}$in U and every $N_{tr}nbhd\ N$ of $f_{N_{tr}}(u_{a,b,c})$, there exists a $N_{tr}\Upsilon -$open set L in U such that $u_{a,b,c} \in L$ and $f_{N_{tr}}(L) \subseteq N$.

Proof:

(i)⟹(ii) Let $u_{a,b,c}$ be a neutrosophic point in U and let N be a $N_{tr}nbhd$ of $f_{N_{tr}}(u_{a,b,c})$. Then there exists a N_{tr} open set M in V such that $f_{N_{tr}}(u_{a,b,c}) \in M \subseteq N$. Since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Also, $u_{a,b,c} \in f_{N_{tr}}^{-1}\left(f_{N_{tr}}(u_{a,b,c})\right) \in f_{N_{tr}}^{-1}(M) \subseteq f_{N_{tr}}^{-1}(N)$. Hence there exists a $N_{tr}\Upsilon-$open set $f_{N_{tr}}^{-1}(M)$ such that $u_{a,b,c} \in f_{N_{tr}}^{-1}(M) \subseteq f_{N_{tr}}^{-1}(N)$. This implies $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-nbhd$ of $u_{a,b,c}$.

(ii)⟹(iii) Let $u_{a,b,c}$ be a neutrosophic point in U and let N be a $N_{tr}nbhd$ of $f_{N_{tr}}(u_{a,b,c})$. Then by assumption, $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-nbhd$ of $u_{a,b,c}$. Then there exists a $N_{tr}\Upsilon-$open set L in U such that $u_{a,b,c} \in L \subseteq f_{N_{tr}}^{-1}(N)$. Thus $u_{a,b,c} \in L$ and $f_{N_{tr}}(L) \subseteq f_{N_{tr}}\left(f_{N_{tr}}^{-1}(N)\right) \subseteq N$.

(iii) ⟹ (i) Let M be a N_{tr} open set in V and let $u_{a,b,c} \in f_{N_{tr}}^{-1}(M)$. Since M is N_{tr} open and $f_{N_{tr}}(u_{a,b,c}) \in M, M$ is a $N_{tr}nbhd$ of $f_{N_{tr}}(u_{a,b,c})$. Hence it follows (iii) that there exists a $N_{tr}\Upsilon-$open set L in U such that $u_{a,b,c} \in L$ and $f_{N_{tr}}(L) \subseteq M$. This implies $u_{a,b,c} \in L \subseteq f_{N_{tr}}^{-1}\left(f_{N_{tr}}(L)\right) \subseteq f_{N_{tr}}^{-1}(M)$. By theorem 2.16, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Therefore $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous.

Remark 4.12: The statements of theorem 4.8, 4.10 and 4.11 are all equivalent.

Definition 4.13: A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be $\boldsymbol{N_{tr}T_{\Upsilon}}$ **−space** if every $N_{tr}\Upsilon-$open set in $(U, \tau_{N_{tr}})$ is N_{tr}open.

Remark 4.14: The composition of two $N_{tr}\Upsilon-$continuous functions need not be $N_{tr}\Upsilon-$continuous.

Example 4.15: Let $U = \{a, b\}, V = \{x, y\}$and $W = \{p, q\}$. Consider the neutrosophic topologies $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, L\}, \rho_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, M\}$ and $\xi_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, N\}$ where $L = \{< a, 0.3,0.4,0.9 > \quad < b, 0.4,0.5,0.8 >\}$, $M = \{< x, 0.9,0.6,0.3 >< y, 0.8,0.5,0.4 >\}$ and $N = \{< p, 0.9,0.6,0.1 >< q, 0.9,0.7\ 0.2 >\}$. Consider the collections $\mathcal{P} = \{P :\ 0_{N_{tr}} \subset P \subset L\}, \mathcal{Q} = \{Q : L \subset Q \subset L^c\}, \mathcal{R} = \{R : R \not\subset L; L \not\subset R; R \subset L^c\}$ of neutrosophic sets in U and $\mathcal{S} = \{S : M \subset S \subset 1_{N_{tr}}\}$, the collection of neutrosophic sets in V. Then, $N_{tr}\Upsilon O(U, \tau_{N_{tr}}) = \{0_{N_{tr}}, L, L^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(V, \rho_{N_{tr}}) = \{0_{N_{tr}}, M, \mathcal{S}, 1_{N_{tr}}\}$. Define $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ as $f_{N_{tr}}(a) = x$ and $f_{N_{tr}}(b) = y$. Then $f_{N_{tr}}^{-1}(M) = \{< a, 0.9,0.6,0.3 >< b, 0.8,0.9,0.4 >\}$ is $N_{tr}\Upsilon-$open in $(U, \tau_{N_{tr}})$. Also, define $g_{N_{tr}}: (V, \rho_{N_{tr}}) \longrightarrow (W, \xi_{N_{tr}})$ as $g_{N_{tr}}(x) = q$ and $g_{N_{tr}}(y) = p$. Then $g_{N_{tr}}^{-1}(N) = \{< x, 0.9,0.7,0.2 >< y, 0.9,0.6,0.1 >\} \in \mathcal{S}$ which implies $g_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-$open in $(V, \rho_{N_{tr}})$. This implies that both $f_{N_{tr}}$ and $g_{N_{tr}}$ are $N_{tr}\Upsilon-$continuous. Now, let $g_{N_{tr}} \circ f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (W, \xi_{N_{tr}})$ be the composition of two $N_{tr}\Upsilon-$continuous functions. Then, $g_{N_{tr}} \circ f_{N_{tr}}$ is not $N_{tr}\Upsilon-$continuous since $\left(g_{N_{tr}} \circ f_{N_{tr}}\right)^{-1}(N) = f_{N_{tr}}^{-1}\left(g_{N_{tr}}^{-1}(N)\right) = \{< a, 0.9,0.7,0.2 >< b, 0.9,0.6,0.1 >\}$ is not $N_{tr}\Upsilon-$open in $(U, \tau_{N_{tr}})$.

Theorem 4.16: Let $(U, \tau_{N_{tr}}), (V, \rho_{N_{tr}})$ and $(W, \xi_{N_{tr}})$ be neutrosophic topological space and let $(V, \rho_{N_{tr}})$ be $N_{tr}T_{\Upsilon}-$space. Then the composition $g_{N_{tr}} \circ f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (W, \xi_{N_{tr}})$ of two $N_{tr}\Upsilon-$continuous functions $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ and $g_{N_{tr}}: (V, \rho_{N_{tr}}) \longrightarrow (W, \xi_{N_{tr}})$ is $N_{tr}\Upsilon-$continuous.

Proof: Let N be any N_{tr}open set in W. Since $g_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous, $g_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-$open in V. Then, by assumption $g_{N_{tr}}^{-1}(N)$ is N_{tr} open in V. Also, since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous, $f_{N_{tr}}^{-1}\left(g_{N_{tr}}^{-1}(N)\right) = \left(g_{N_{tr}} \circ f_{N_{tr}}\right)^{-1}(N)$ is $N_{tr}\Upsilon-$open in U. Hence $g_{N_{tr}} \circ f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous.

Theorem 4.17: Let $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ be a $N_{tr}\Upsilon-$continuous function and $g_{N_{tr}}:(V,\rho_{N_{tr}}) \longrightarrow (W,\xi_{N_{tr}})$ be a N_{tr}continuous function. Then their composition $g_{N_{tr}} \circ f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (W,\xi_{N_{tr}})$is $N_{tr}\Upsilon-$continuous.

Proof: Let N be any N_{tr}open set in W. Since $g_{N_{tr}}$ is N_{tr}continuous, $g_{N_{tr}}^{-1}(N)$ is N_{tr}open in V.Also, since $g_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous, $f_{N_{tr}}^{-1}\left(g_{N_{tr}}^{-1}(N)\right) = \left(g_{N_{tr}} \circ f_{N_{tr}}\right)^{-1}(N)$ is $N_{tr}\Upsilon-$open in U. Hence $g_{N_{tr}} \circ f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous.

Theorem 4.18: Let $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ be a $N_{tr}\Upsilon-$continuous function where $(U,\tau_{N_{tr}})$ is a $N_{tr}T_{\Upsilon}-$space. If S is a subset of U, then the restriction $f_{N_{tr}}|_S : (S,\tau_{N_{tr}}^S) \longrightarrow (V,\rho_{N_{tr}})$ is also $N_{tr}\Upsilon-$continuous.

Proof: Let M be a N_{tr}open set in V. Since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Now, since U is a $N_{tr}T_{\Upsilon}-$space, $f_{N_{tr}}^{-1}(M)$ is N_{tr}open in U. Hence $f_{N_{tr}}|_S{}^{-1}(M) = f_{N_{tr}}^{-1}(M) \cap 1_{N_{tr}}^S$ is N_{tr} open in S. By theorem 2.14, $f_{N_{tr}}|_S{}^{-1}(M)$ is $N_{tr}\Upsilon-$open in S. Hence $f_{N_{tr}}|_S$ is $N_{tr}\Upsilon-$continuous.

5. Neutrosophic $\Upsilon-$irresolute functions

Analogous to the previous section, this segment deals with the concept of neutrosophic $\Upsilon-$irresolute functions and its behavior.

Definition 5.1: Let $(U,\tau_{N_{tr}})$ and $(V,\rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ is said to be neutrosophic $\Upsilon-$irresolute if $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in $(U,\tau_{N_{tr}})$ for every $N_{tr}\Upsilon-$open set M in $(V,\rho_{N_{tr}})$.

Example 5.2: Let $U=\{a,b\}, V=\{x,y\}, \tau_{N_{tr}}=\{0_{N_{tr}},1_{N_{tr}},L\}$ and $\rho_{N_{tr}}=\{0_{N_{tr}},1_{N_{tr}},M\}$ where $L=\{<a,0.5,0.6,0.3><b,0.6,0.7,0.2>\}$ and $M=\{<x,0.5,0.7,0.3><y,0.8,0.7,0.2>\}$. Also, consider the collections $\mathcal{P}=\{P : L \subset P \subset 1_{N_{tr}}\}$ and $\mathcal{Q}=\{Q : M \subset Q \subset 1_{N_{tr}}\}$ of neutrosophic sets in U and V respectively. Then, $N_{tr}\Upsilon O(U,\tau_{N_{tr}})=\{0_{N_{tr}},L,\mathcal{P},1_{N_{tr}}\}$ and $N_{tr}\Upsilon O(V,\rho_{N_{tr}})=\{<0_{N_{tr}},M,\mathcal{Q},1_{N_{tr}}\}$. Now, let us define $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ as $f_{N_{tr}}(a)=x$ and $f_{N_{tr}}(b)=y$. Then,$f_{N_{tr}}^{-1}(M)=\{<a,0.5,0.7,0.3><b,0.8,0.7,0.2>\} \in \mathcal{P}$ and for each $Q \in \mathcal{Q}$, there exists some $P \in \mathcal{P}$ such that $f_{N_{tr}}^{-1}(Q)=P$. Hence the inverse image of every $N_{tr}\Upsilon-$open set in V is $N_{tr}\Upsilon-$open in U. Therefore $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute.

Theorem 5.3: Every $N_{tr}\Upsilon-$irresolute function is $N_{tr}\Upsilon-$continuous.

Proof: Let $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ be a $N_{tr}\Upsilon-$irresolute function and M be a N_{tr}open set in V. Then, by theorem 2.14, M is $N_{tr}\Upsilon-$open in V. Since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irreolsute, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous.

The following example substantiates that the converse of the above-stated theorem need not be true.

Example 5.4: Let $U=\{a,b\}$, $V=\{x,y\}$, $\tau_{N_{tr}}=\{0_{N_{tr}},1_{N_{tr}},L\}$ and $\rho_{N_{tr}}=\{0_{N_{tr}},1_{N_{tr}},M\}$ where $L=\{<a,0.1,0.3,0.7><b,0.3,0.2,0.8>\}$ and $M=\{<x,0.7,0.7,0.1><y,0.8,0.8,0.3>\}$. Consider the collections $\mathcal{P}=\{P: 0_{N_{tr}} \subset P \subset L\}$, $\mathcal{Q}=\{Q: L \not\subset Q ; Q \not\subset L ; Q \subset L^c\}$ and $\mathcal{R}=\{R : L \subset R \subset L^c\}$ of neutrosophic sets in U. Then, $N_{tr}\Upsilon O(U,\tau_{N_{tr}})=\{0_{N_{tr}},L,L^c,\mathcal{P},\mathcal{Q},\mathcal{R},1_{N_{tr}}\}$. Now, let us define $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ as $f_{N_{tr}}(a)=x$ and $f_{N_{tr}}(b)=y$. Then,$f_{N_{tr}}^{-1}(M)=\{<a,0.7,0.7,0.1><b,0.8,0.8,0.3>\}=L^c$ which implies $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous. However, the inverse image of a $N_{tr}\Upsilon-$open set $S=\{<x,0.8,0.7,0.1><y,0.9,0.8,0.2>\}$ in V is not $N_{tr}\Upsilon-$open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon-$continuous but not $N_{tr}\Upsilon-$irresolute.

Theorem 5.5: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a $N_{tr}\Upsilon-$continuous function where $(V, \rho_{N_{tr}})$ is a $N_{tr}T_{\Upsilon}$ $-$space. Then $f_{N_{tr}}$ is $N_{tr}\Upsilon$ $-$irresolute.

Proof: Let M be $N_{tr}\Upsilon-$open in V. Then, by assumption M is N_{tr}open in V. Since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$ continuous, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon$ $-$irresolute.

Theorem 5.6: Let $f_{N_{tr}}: (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a function between two neutrosophic topological spaces. Then the following statements are equivalent:

(i) $f_{N_{tr}}$ is $N_{tr}\Upsilon$ $-$irresolute.

(ii) The inverse image of every $N_{tr}\Upsilon-$closed set in $(V, \rho_{N_{tr}})$ is $N_{tr}\Upsilon-$closed in $(U, \tau_{N_{tr}})$.

(iii) $f_{N_{tr}}(N_{tr}\Upsilon cl(L)) \subseteq N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)$ for every neutrosophic set L in U.

(iv) $N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(N_{tr}\Upsilon cl(M))$ for every neutrosophic set M in V.

(v) $f_{N_{tr}}^{-1}(N_{tr}\Upsilon int(M)) \subseteq N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right)$ for every neutrosophic set M in V.

(vi) For each neutrosophic point $u_{a,b,c}$, the inverse image of every $N_{tr}\Upsilon-nbhd$ of $f_{N_{tr}}(u_{a,b,c})$ is $N_{tr}\Upsilon-nbhd$ of $u_{a,b,c}$.

(vii) For each neutrosophic point $u_{a,b,c}$in U and every $N_{tr}\Upsilon-nbhdN$ of $f_{N_{tr}}(u_{a,b,c})$, there exists a $N_{tr}\Upsilon$ $-$open set L in U such that $u_{a,b,c} \in L$ and $f_{N_{tr}}(L) \subseteq N$.

Proof:

(i)⟹(ii) Let $f_{N_{tr}}$ be a $N_{tr}\Upsilon$ $-$irresolute function and N be a $N_{tr}\Upsilon-$closed set in V. Then N^c is $N_{tr}\Upsilon-$open in V. Since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute, $f_{N_{tr}}^{-1}(N^c)$ is $N_{tr}\Upsilon-$open in U. That is, $(f_{N_{tr}}^{-1}(N))^c$ is $N_{tr}\Upsilon-$open in U. Hence $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-$closed in U.

(ii)⟹(i) Let M be $N_{tr}\Upsilon-$open in V. Then M^c is $N_{tr}\Upsilon-$closed in V. By assumption, $f_{N_{tr}}^{-1}(M^c)$ is $N_{tr}\Upsilon-$closed in U. That is, $(f_{N_{tr}}^{-1}(M))^c$is $N_{tr}\Upsilon-$closed in U. Hence $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Therefore, $f_{N_{tr}}$ is $N_{tr}\Upsilon$ $-$irresolute.

(ii)⟹(iii) Let L be a neutrosophic set in U. Now, $L \subseteq f_{N_{tr}}^{-1}\left(f_{N_{tr}}(L)\right) \Longrightarrow L \subseteq f_{N_{tr}}^{-1}\left(N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)\right)$.

Since $N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)$ is $N_{tr}\Upsilon-$closed in V, by assumption $f_{N_{tr}}^{-1}\left(N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)\right)$ is a $N_{tr}\Upsilon-$ closed set containing L. Also, $N_{tr}\Upsilon cl(L)$ is the smallest $N_{tr}\Upsilon-$closed set containing L. Hence, $N_{tr}\Upsilon cl(L) \subseteq f_{N_{tr}}^{-1}\left(N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)\right)$. Therefore, $f_{N_{tr}}(N_{tr}\Upsilon cl(L)) \subseteq N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)$.

(iii)⟹(ii) Let N be a $N_{tr}\Upsilon-$closed set in V. Then, by assumption $f_{N_{tr}}\left(N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(N)\right)\right) \subseteq$ $N_{tr}\Upsilon cl\left(f_{N_{tr}}\left(f_{N_{tr}}^{-1}(N)\right)\right) \subseteq N_{tr}\Upsilon cl(N) = N$ implies $N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(N)\right) \subseteq f_{N_{tr}}^{-1}(N)$. Also, $f_{N_{tr}}^{-1}(N) \subseteq$ $N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(N)\right)$. Hence $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-$closed in U.

(iii)⟹(iv) Let M be a neutrosophic set in V and let $L = f_{N_{tr}}^{-1}(M)$. By assumption, $f_{N_{tr}}(N_{tr}\Upsilon cl(L)) \subseteq$ $N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right) = N_{tr}\Upsilon cl(M)$.This implies $N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(N_{tr}\Upsilon cl(M))$.

(iv)⟹(iii) Let $M = f_{N_{tr}}(L)$. Then, by assumption, $N_{tr}\Upsilon cl(L) = N_{tr}\Upsilon cl\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}\left(N_{tr}\Upsilon cl(M)\right)$

$= f_{N_{tr}}^{-1}\left(N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)\right)$. This implies $f_{N_{tr}}\left(N_{tr}\Upsilon cl(L)\right) \subseteq N_{tr}\Upsilon cl\left(f_{N_{tr}}(L)\right)$.

(iv)⟺(v) This can be proved by taking complements.

(v)⟹(i) Let M be a $N_{tr}\Upsilon-$open set in V. Then $f_{N_{tr}}^{-1}(M) = f_{N_{tr}}^{-1}\left(N_{tr}\Upsilon int(M)\right) \subseteq N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right)$.

Also, $N_{tr}\Upsilon int\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(M)$. This implies $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute.

(i)⟹(vi) Let $u_{a,b,c}$ be a neutrosophic point in U and let N be a $N_{tr}\Upsilon-nbhd$ of $f_{N_{tr}}(u_{a,b,c})$. Then there exists a $N_{tr}\Upsilon-$open set M in V such that $f_{N_{tr}}\left(u_{a,b,c}\right) \in M \subseteq N$. Since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Also, $u_{a,b,c} \in f_{N_{tr}}^{-1}\left(f_{N_{tr}}\left(u_{a,b,c}\right)\right) \in f_{N_{tr}}^{-1}(M) \subseteq f_{N_{tr}}^{-1}(N)$. Hence there exists a $N_{tr}\Upsilon-$open set $f_{N_{tr}}^{-1}(M)$ such that $u_{a,b,c} \in f_{N_{tr}}^{-1}(M) \subseteq f_{N_{tr}}^{-1}(N)$. This implies $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-nbhd$ of $u_{a,b,c}$.

(vi)⟹(vii) Let $u_{a,b,c}$ be a neutrosophic point in U and let N be a $N_{tr}\Upsilon-nbhd$ of $f_{N_{tr}}(u_{a,b,c})$. Then by assumption, $f_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-nbhd$ of $u_{a,b,c}$. Then there exists a $N_{tr}\Upsilon-$open set L in U such that $u_{a,b,c} \in L \subseteq f_{N_{tr}}^{-1}(N)$. Thus $u_{a,b,c} \in L$ and $f_{N_{tr}}(L) \subseteq f_{N_{tr}}\left(f_{N_{tr}}^{-1}(N)\right) \subseteq N$.

(vii)⟹(i) Let M be a $N_{tr}\Upsilon-$open set in V and let $u_{a,b,c} \in f_{N_{tr}}^{-1}(M)$. Since M is $N_{tr}\Upsilon-$open and $f_{N_{tr}}\left(u_{a,b,c}\right) \in M, M$ is a $N_{tr}\Upsilon-nbhd$ of $f_{N_{tr}}\left(u_{a,b,c}\right)$. Hence it follows from (vii) that there exists a $N_{tr}\Upsilon-$open set L in U such that $u_{a,b,c} \in L$ and $f_{N_{tr}}(L) \subseteq M$. This implies $u_{a,b,c} \in L \subseteq f_{N_{tr}}^{-1}\left(f_{N_{tr}}(L)\right) \subseteq f_{N_{tr}}^{-1}(M)$. By theorem 3.6, $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open in U. Therefore $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute.

Example 5.7: (i) Consider the topological spaces and the function $f_{N_{tr}}$ defined in example 5.2. Here $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute and $N_{tr}\Upsilon C\left(U,\tau_{N_{tr}}\right) = \left\{0_{N_{tr}}, L^c, \mathcal{P}', 1_{N_{tr}}\right\}$, $N_{tr}\Upsilon C\left(V,\rho_{N_{tr}}\right) = \left\{0_{N_{tr}}, M^c, \mathcal{Q}', 1_{N_{tr}}\right\}$ where $\mathcal{P}' = \{P^c : P \in \mathcal{P}\}$ and $\mathcal{Q}' = \{Q^c : Q \in \mathcal{Q}\}$. Now, $f_{N_{tr}}^{-1}(M^c) = \{< a, 0.3, 0.3, 0.5 >< b, 0.2, 0.3, 0.8 >\} \in \mathcal{P}'$ and for each $Q \in \mathcal{Q}'$, there exists some $P \in \mathcal{P}'$ such that $f_{N_{tr}}^{-1}(Q) = P$. Hence the inverse image of every $N_{tr}\Upsilon-$closed set in $(V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon-$closed in $(U,\tau_{N_{tr}})$ if $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute.

(ii) Let $U = \{a,b\}, V = \{x,y\}, \tau_{N_{tr}} = \left\{0_{N_{tr}}, 1_{N_{tr}}, L\right\}$ and $\rho_{N_{tr}} = \left\{0_{N_{tr}}, 1_{N_{tr}}, M\right\}$ where $L = \{< a, 0.7, 0.5, 0.5 >< b, 0.8, 0.6, 0.4 >\}$ and $M = \{< x, 0.8, 0.6, 0.4 >< y, 0.9, 0.7, 0.1 >\}$. Consider the collections $\mathcal{P} = \{P : 0_{N_{tr}} \subset P \subset L^c\}$ and $\mathcal{Q} = \{Q : 0_{N_{tr}} \subset Q \subset M^c\}$ of neutrosophic sets in U and V respectively. Then, $N_{tr}\Upsilon C\left(U,\tau_{N_{tr}}\right) = \left\{0_{N_{tr}}, L^c, \mathcal{P}, 1_{N_{tr}}\right\}$ and $N_{tr}\Upsilon C\left(V,\rho_{N_{tr}}\right) = \left\{0_{N_{tr}}, M^c, \mathcal{Q}, 1_{N_{tr}}\right\}$. Now, define $f_{N_{tr}}: (U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ as $f_{N_{tr}}(a) = x$ and $f_{N_{tr}}(b) = y$. Then, $f_{N_{tr}}^{-1}(M^c) = \{< a, 0.1, 0.3, 0.9 >< b, 0.2, 0.1, 0.8 >\} \in \mathcal{P}$ and for each $Q \in \mathcal{Q}$, there exists some $P \in \mathcal{P}$ such that $f_{N_{tr}}^{-1}(Q) = P$. Now, $f_{N_{tr}}^{-1}(M^c) = (f_{N_{tr}}^{-1}(M))^c$ is $N_{tr}\Upsilon-$closed implies $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Upsilon-$open . Similarly, we can prove that the inverse image of every $N_{tr}\Upsilon-$open set in $(V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon-$open in $\left(U,\tau_{N_{tr}}\right)$. Hence $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute if the inverse image of every $N_{tr}\Upsilon-$closed set in $(V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon-$closed in $\left(U,\tau_{N_{tr}}\right)$.

Theorem 5.8: If $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ and $g_{N_{tr}}:(V,\rho_{N_{tr}}) \longrightarrow (W,\xi_{N_{tr}})$ are $N_{tr}\Upsilon-$ irresolute functions, then their composition $g_{N_{tr}} \circ f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (W,\xi_{N_{tr}})$ is also $N_{tr}\Upsilon-$irresolute.

Proof: Let N be $N_{tr}\Upsilon-$ open in W. Since $g_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute, $g_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-$ open in V. Again, since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$ irresolute, $f_{N_{tr}}^{-1}\left(g_{N_{tr}}^{-1}(N)\right) = \left(g_{N_{tr}} \circ f_{N_{tr}}\right)^{-1}(N)$ is $N_{tr}\Upsilon-$ open in U. Hence $g_{N_{tr}} \circ f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute.

Theorem 5.9: If $f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (V,\rho_{N_{tr}})$ is $N_{tr}\Upsilon-$ irresolute and $g_{N_{tr}}:(V,\rho_{N_{tr}}) \longrightarrow (W,\xi_{N_{tr}})$ is $N_{tr}\Upsilon-$ continuous, then $g_{N_{tr}} \circ f_{N_{tr}}:(U,\tau_{N_{tr}}) \longrightarrow (W,\xi_{N_{tr}})$ is $N_{tr}\Upsilon-$ continuous.

Proof: Let N be N_{tr}open in W. Since $g_{N_{tr}}$ is $N_{tr}\Upsilon-$ continuous, $g_{N_{tr}}^{-1}(N)$ is $N_{tr}\Upsilon-$ open in V. Also, since $f_{N_{tr}}$ is $N_{tr}\Upsilon-$irresolute, $f_{N_{tr}}^{-1}\left(f_{N_{tr}}^{-1}(N)\right) = \left(g_{N_{tr}} \circ f_{N_{tr}}\right)^{-1}(N)$ is $N_{tr}\Upsilon-$ open in U. Hence $g_{N_{tr}} \circ f_{N_{tr}}$ is $N_{tr}\Upsilon-$ continuous.

6. Conclusions

The theory of neutrosophic sets is essential in many application areas since indeterminacy is ubiquitous and these membership functions are crucial. In this paper, we have introduced and analyzed the concepts of neutrosophic $\Upsilon-$neighbourhood and neutrosophic $\Upsilon-$continuity. In addition, we have also defined neutrosophic $\Upsilon-$irreolute functions in neutrosophic topological spaces. As mentioned earlier, continuity features a prominent position in the characterization of topological spaces. Accordingly, this concept can be wielded in the description of various topological structures in future. Moreover, several other topological concepts such as homeomorphisms, connectedness and separation axioms could be explored by means of neutrsphic $\Upsilon-$open sets and neutrosophic $\Upsilon-$continuity.

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