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Neutrosophic Fuzzy Magic Labeling Graph with its Application in Academic Performance of the Students

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Abstract: Graph labelling is the assignment of labels to the edges, vertices, or both. The issue of limiting the spread of non-interfering frequencies allotted to radio transmitters serves as the motivation for the research on labelling graphs according to different limitations. However, compared to classical models, fuzzy labelling models provide the system with greater accuracy, adaptability, and compatibility. The definition of the neutrosophic fuzzy magic labelling graph and a detailed discussion of its properties using numerical examples for the path, cycle, and star graphs in a neutrosophic environment are presented in this study. The proposed work has also been used in decision-making situations to choose the optimal subject combinations based on student interests for the best academic performance. In order to demonstrate the validity of the suggested work, a comparative analysis with the current methodology has also been conducted.

Keywords: Fuzzy magic labeling, Neutrosophic star graph, Neutrosophic cycle graph, Neutrosophic path graph, Neutrosophic fuzzy magic labeling.

1. Introduction

In order to address the issues of uncertainty and ambiguity in real-world settings, fuzzy relations were first introduced by Zadeh [1]. Fuzzy relations have a wide range of applications in pattern recognition. By substituting Zadeh's fuzzy sets for traditional sets, one can improve theoretical validity and reliability, in addition to application productivity and system connectivity. Numerous mathematical examples exist for the fuzzy graph. Nageswara Rao et al. [2] exhibited several forms of dominance, such as edge, total, strong, regular, linked, split, and, in practical applications, inverse dominance in fuzzy graphs. For visualising data on the connections between items, a graph is a valuable tool. Vertices identify the object, whereas edges highlight relationships. The use of graph theory is essential for illuminating numerous practical problems. Graphs no longer accurately represent every system because of the haziness or uncertainty of the system parameters. In general, when characterising the objects, their relationships, or both are uncertain, a fuzzy graph model needs to be created.

In order to assess the relationships between accounts as good or poor based on how frequently they interact, fuzziness must also be added to the representation. Fuzzy graphs were developed as a result of these and numerous other problems. The concepts of essential blocks and t-components of fuzzy graphs, as well as the creation of t-connected, uniformly connected, and average fuzzy graphs, were developed by John and Sunil Mathew [3]. The principle of the fuzzy equitable association graph was described by Rani and Dharmalingam [4]. The highly irregular and highly total irregular fuzzy graphs, as well as the neighbourly irregular and neighbourly total irregular fuzzy graphs, were all introduced by Huda Mutab [5].

The characteristics of Cartesian multiplication operations in full fuzzy graphs, effective fuzzy graphs, and complement fuzzy graphs were first introduced by Yulianto et al. in [6]. A hesitant fuzzy hypergraph model was suggested by Junhu Wang and Zengtai Gong, based on hesitant fuzzy sets and fuzzy hypergraphs [7]. Using fuzzy graphs in cubic Pythagorean fuzzy sets, Muhiuddin et al. [8] investigated the concept and utilised it to solve a problem involving decision-making. Crisp and fuzzy graphs have equivalent structural characteristics. However, fuzzy graphs emphasise the ambiguity surrounding vertices and edges more. Furthermore, the fuzzy graph is frequently seen in real-life scenarios since there is uncertainty in the world. Building fuzzy graphs draws on a variety of scientific disciplines, including those in mathematics, physics, chemistry, and computer science.

It was suggested that the intuitionistic fuzzy set by Attanassov [9–10] The concept of an intuitionistic fuzzy graph (IFG) was introduced by Atanassov and Shanon [11]. Several variations of the IFG concept were created, such as the very irregular and neighbourly irregular IFG by Nagoor Gani [12]. In [50], Garai developed a ranking technique based on generalised intuitionistic fuzzy numbers. [51] Giri et al. designed the mathematical operations of the generalised non-linear intuitionistic fuzzy number using the alpha-beta cut technique applied in the multi-item inventory model. Mathematics and its applications have seen a sharp increase in research on intuitionistic fuzzy sets. Information sciences and classical mathematics differ from one another. This makes me consider IFGs and how they might be used. Increased issue accuracy, reduced implementation costs, and improved efficacy are all advantages of intuitionistic fuzzy sets and graphs.

The concept has been examined, as have the IFG's properties and structure, according to Karunambigai [13]. The IFS operations were identified by John and Sunil investigators [14], and the suggested strategy was applied to trafficking channels. The concept of effective colouring was developed by Revathy et al. [15] of IFG. The colouring concept for IFG was described by Rifayathali et al. [16]. Akmaland Akram (2017 developed the organisational structure and layout of IFG [17]. In 2022, Amsaveni and Nandhini suggested using IFG in a bipolar complex intuitionistic fuzzy set [18]. The three further IFG activities of product, semi-strong product, and strong product were proposed to be added by Talal and Bayan [19]. The concept and attributes of IFG were first presented by Muhammad et al. in [20].

The neutrosophic graph, which is a fuzzy logic extension with indeterminacy, was proposed by Florentin Smarandache [21]. It has become imperative that the idea of a neutrosophic graph play a significant role in a number of real-world challenges, including computer technology, communication, genetics, economics, sociology, linguistics, legal, medical, finance, engineering IT, networking, and so forth. A fresh aspect of graph theory was introduced by Florentin Smarandache et al. [22]. The graph notion was invented by Euler. The phrase "fuzzy graph" was first used by Rosenfield [23]. Graph theory is very effective in simulating the characteristics of finite-component systems. A graph is a visual representation of information that shows how things are connected, and its vertices and edges show the objects and their connections. Graphical models are employed to describe a variety of networks, including telephone networks, railroad networks, communication networks, traffic networks, and other networks. Data mining, image segmentation, categorization, laser scanners, communication, preparation, and programming all make growing use of fuzzy graphs.

The novel idea of a Pythagorean neutrosophic fuzzy graph (NFG) was introduced by Ajay and Chellamani [24], who also examined its characteristics. Broumi et al. [25] described the many types of single-valued neutrosophic graphs (SVNG) and examined some of their characteristics in relevant scenarios. The effectiveness of the bipartite, regular, and irregular neutrosophic graphs was demonstrated by Huang [26]. In addition to a number of SVNG operations, such as rejection, symmetric difference, maximum product, and residue product, Mohanta [27] provided numerous additional SVNG concepts.

The graph labeling method was introduced by Rosa [28]. A mapping from a collection of edges, vertices, or both to a number of tags is known as graph labelling. Graph labelling has proven useful in many areas. Multiple labels are obtained depending on the demands made of the labelling. Among the most common labels are those that are graceful and attractive. In graphs, there are numerous forms of labelling, including beautiful, friendly, and mean labelling. We want the total number of labels associated with a vertex or edge to be constant across the graph when we apply the "magic" idea to graphs. Magic graph labelling is a logical continuation of the well-known magic squares and magic rectangles. Magic-type labelling is useful when avoiding a look-up table or when a check total is required. A straightforward graph can be used to depict a network that consists of nodes, links, and addresses (labels) assigned to both the links and the nodes. [48] Jafar et al. employed the notion in site selection for solid waste management when they provided length and identity measurements utilizing max-min operators under neutrosophic hypersoft sets. [49] Muhammed elaborates on the principle of neutrosophic hypersoft set to the neutrosophic hypersoft matrices applied in decision-making problems. [52] Garai created a unique ranking method that uses single-valued neutrosophic numbers for multi-attribute decision-making. Garai [53] developed a ranking system using single-valued bipolar neutrosophic numbers to address the challenge of managing water resources in a bipolar neutrosophic environment.

The notion of a magic graph was developed by Sedlack [29]. Magic labelling is a sort of graph labelling that has received a lot of attention and development. The labelling of the whole magic point, the labelling of the super magic point, the labelling of the magic side, and the labelling of the super magic side are also well known in the development of magic labelling. In this work, a neutrosophic number may be computed using various graph types. The neutrosophic fuzzy magic labelling graph has been proposed in the neutrosophic environment using this notion.

The rest of the paper is structured as follows: The literature review for the proposed theory is found in Section 2, and it demonstrates the originality of the methods provided in this work. Section 3 has presented fundamental ideas. In Section 4, the idea of a neutrosophic magic labelling graph was put forth. As an example, Section 5 defines neutrosophic fuzzy magic route graph labelling. In Section 6, an illustration of neutrosophic fuzzy magic labelling of a cycle is given. Neutosophic fuzzy magic labelling of star graphs is described in Section 7 along with an illustration. To choose the best combination of subjects based on the student's interests for the best academic performance, Section 8 applied the indicated strategy in decision-making situations. In Section 9, a comparison analysis using the current methodology is covered, and in Section 10, the current work is concluded with a look towards the future.

2. **Review of Literature**

The authors [1–8] introduced and developed graphs and fuzzy graphs in different types of fuzzy environments. The authors [10–14] proposed graphs and irregular graphs in the IFS environment. The authors [20–25] developed a neutrosophic graph in different types of neutrosophic environments. The authors Fathalian et al. [30] examined whether simple graphs are fuzzy magic labels, as well as whether every linked network is a fuzzy magic labelling network. New concepts for the labelling and calculation of Pythagorean fuzzy magic constants and Pythagorean fuzzy magic graphs have been presented by Rani and Ashwin [31], and the notion of magic labelling in hesitancy fuzzy graphs was proposed, and outcomes in hesitancy fuzzy graphs such as the route, cycle, and star graphs were obtained. Fathalian et al. [32] are credited with introducing the concept of Hesitancy fuzzy magic labelling for basic graphs. Fuzzy magic and bimagic labelling of neutrosophic route graphs were studied to see whether they included magic value in intuitionistic fuzzy graphs and to further understand bi-magic labelling on intuitionistic fuzzy graphs. Krishnaraj et al. [33] looked at how it differs from traditional labelling methods on the graphs. Fuzzy sequential vertex magic labelling with z-index in trees was studied along with numerous extensions by the authors Nishanthini et al. [34]. It was observed that magic labelling may be used for intuitively fuzzy graphs such as routes, cycles, and stars. The bridge management problem was solved using new neutrosophic labelling graph connection concepts suggested by Seemaand Majeet [35]. A relationship between strongly c-elegant labelling, super-edge magic total labelling, edge antimagic labelling, and super-t-1 magical labelling was proposed by Wang and Bing Ya [36], and it was investigated. Farida et al. [37] investigated the magic covering and edge magic labelling on a simple graph, and Krishnaraj and Vikramaprasad [38] extended the Bi-Magic concepts. There was an introduction to image fuzzy labelling of graphs and the notions of strong arc, partial cut node, and bridge of picture fuzzy labelling graphs, as well as their properties, explained by Ajay and Chellamani [38]. According to a proposal made by Jeyanthi and Jeya [39], Zk-magic graphs also contain the flower, double wheel, shell, cylinder, gear, generalised Jahangir, lotus inside a circle, wheel, and closed helm graphs. The authors, Wasim Hani and Muhamad [40], proposed that the direct product of a directed graph might be labelled using orientable group distance magic labelling. Maheswar et al. [41] introduced anti-Magic labelling, which involves assigning distinct values to various vertices in a network such that the total of the labels has different restrictions.

However. Fuzzy labelling models offer the system higher accuracy, adaptability, and compatibility when compared to classical methods. But in fuzzy, the magic values are discussed only for the membership grades, which should be constant. Whereas in the Intuitionistic Fuzzy Magic Labelling Graph (IFMLG), the magic values are discussed in both membership and non-membership grades. The magic values for membership grades and non-membership grades are both constant in IFMLG. As of the above research and findings, there is less contribution in the neutrosophic fuzzy magic labelling graph (NFMLG), which also shows that the magic labelling graph has not yet been properly proposed and that there has been very little progress in that direction in a neutrosophic environment. This study is inspired by that fact.

The NFMLG discusses magic values in both membership and non-membership grades, as well as indeterminacy grades. This is one of the main advantages that FMLG and IFMLG fail to prove.

3. Preliminaries

In this part, we will go over some fundamental terminology as well as the findings of our study.

Definition 3.1: Fuzzy Graph [42]

A Fuzzy graph denoted by $G(\sigma,\mu)$ is a couple of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where $\forall\, u, v \in V,\ \mu(u,v) = \mu(uv) \le \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 3.2: Fuzzy Labeling Graph [43]

If $\delta(\sigma) : V \to [0,1]$ and $\delta(\mu) : V \times V \to [0,1]$ are bijective such that the membership value of the edges and vertices are distinct and $\mu(\delta(u),\delta(v)) \le \sigma(\delta(u)) \wedge \sigma(\delta(v)), \forall\, \delta(u),\delta(v) \in V$,then the graph $\mathrm{G} = (\delta(\sigma),\delta(\mu))$ is said to be a fuzzy labeling graph.

Definition 3.3: Fuzzy Magic Labeling Graph (FMLG)[44]

A fuzzy labeling graph $G = (\sigma,\mu)$ is called a FMLG if there exist an 'm' such that $\sigma(\delta(u)) + \sigma(\delta(v)) + \mu(\delta(uv)) = constant \quad \forall uv \in E$ and $\delta(u),\delta(v) \in V$.

Definition 3.4: Intuitionistic Fuzzy Graph (IFG) [34]

A IFG of the form $G_g = (V_v, E_e)$ where $V_v = \{\delta(v_1),\delta(v_2),\delta(v_3)..........\delta(v_n)\}$ such that $\delta(\mu_1) : V_v \to [0,1],\ \delta(\vartheta_1) : V_v \to [0,1]$ represent the order of membership function, and non-membership function of the element $\delta(v_i) \in V$ respectively, and $0 \le \mu_1(\delta(v_i)) + \vartheta_1(\delta(v_i)) \le 1$ for every $v_i \in V_v\ (i = 1,2,3,..........n),\ E_e \subseteq V_v \times V_v$ where $\mu_2 : V_v \times V_v \to [0,1], \vartheta_2 : V_v \times V_v \to [0,1]$ are such that

$$\mu_2\left(\delta(\mathrm{v_i}),\delta(\mathrm{v_j})\right) \le \min\mathrm{ima}\left[\mu_1\left(\delta(\mathrm{v_i})\right),\mu_1\left(\delta(\mathrm{v_j})\right)\right],$$
$$\vartheta_2\left(\delta(\mathrm{v_i}),\delta(\mathrm{v_j})\right) \le \max\mathrm{ima}\left[\vartheta_1\left(\delta(\mathrm{v_i})\right),\vartheta_1\left(\delta(\mathrm{v_j})\right)\right]$$

fulfills the condition $0 \le \mu_1\left(\delta(v_i),\delta(v_j)\right) + \vartheta_1\left(\delta(v_i),\delta(v_j)\right) \le 1\ \forall\ v_i, v_j \in E(i, j = 1,2,3.............n)$.

Definition 3.5:Intuitionistic Fuzzy Labeling Graph (IFLG) [35]

A IFLG is of the form $G_g = (V_v, E_e)$ is called an IFLGif $\delta(\mu_1) : V_v \to [0,1], \delta(\vartheta_1) : V_v \to [0,1]\ \ \&\, \delta(\mu_2) : V_v \times V_v \to [0,1], \delta(\vartheta_2) : V_v \times V_v \to [0,1]$ are bijective in order forif $\delta\{\mu_1(m)\}, \delta\{\vartheta_1(m)\}, \delta\{\mu_2(m)\}, \delta\{\vartheta_2(m)\} \in [0,1]$ all are unique $\forall$ vertices and edges, where

$\delta\{\mu_V(m)\},\delta\{\vartheta_V(m)\}$ is order of membership function and $\delta\{\mu_E(m)\},\delta(\vartheta_E(m))$ degree of non-membership function.

Definition 3.6: Intuitionistic Fuzzy Magic Labeling Graph (IFMLG) [35]

A IFLG is an IFMLG if thedegree of membership value $\delta\{\mu_1(m)\}+\delta\{\mu_2(m,n)\}+\delta\{\mu_1(n)\}$ remains equal $\forall m,n\in V$ and degree of non-membership value $\delta(\vartheta_1(m))+\delta(\vartheta_2(m,n))+\delta(\vartheta_1(n))$ remain equal $\forall m,n\in V$. The magic membership value denoted M, therefore $M=\{\delta(\mu_1(m))+\delta(\mu_2(m,n))+\delta(\mu_1(n)),\delta\{\vartheta_1(m)\}+\delta(\vartheta_2(m,n))+\delta(\vartheta_1(n))\}$.

Definition 3.7: Single–valued Neutrosophic Fuzzy Graph (SVNFG)[45]

A SVNFG is of the form $G=(V_v,\sigma,\mu)$ where $\sigma=(Tr_1,Ind_1,Fal_1)\,\&\,(Tr_2,Ind_2,Fal_2)$

$V_v=\{v_1,v_2,v_3..........v_n\}$ such that $Tr_1:V_v\to[0,1], Ind_1:V_v\to[0,1]\,\&\,Fal_1:V_v\to[0,1]$ denote the degree of truth-membership function, indeterminacy and falsity-membership function of the element $v_i\in V_v$ respectively, and $0\le Tr_1(\delta(v_i))+Ind_1(\delta(v_i))+Fal_1(\delta(v_i))\le 3\quad\forall\quad\delta(v_i)\in V(i=1,2,3,..........n)$, where $Tr_2:V_v\times V_v\to[0,1], Ind_2:V_v\times V_v\to[0,1]\,\&\,Fal_2:V_v\times V_v\to[0,1]$ of the edge

$$\mathrm{Tr}_2(\delta(\mathrm{v_i}),\delta(\mathrm{v_j}))\le \min\mathrm{ima}\left[\mathrm{Tr}_1(\delta(\mathrm{v_i})),\mathrm{Tr}_1(\delta(\mathrm{v_j}))\right],$$
$$\mathrm{Ind}_2(\delta(\mathrm{v_i}),\delta(\mathrm{v_j}))\le \min\mathrm{ima}\left[\mathrm{Ind}_1(\delta(\mathrm{v_i})),\mathrm{Ind}_1(\delta(\mathrm{v_j}))\right],$$
$$\mathrm{Fal}_2(\delta(\mathrm{v_i}),\delta(\mathrm{v_j}))\le \max\mathrm{ima}\left[\mathrm{Fal}_1(\delta(\mathrm{v_i})),\mathrm{Fal}_1(\delta(\mathrm{v_j}))\right]$$

satisfies the condition $0\le Tr_1(\delta(v_i),\delta(v_j))+Ind_1(\delta(v_i),\delta(v_j))+Fal_1(\delta(v_i),\delta(v_j))\le 3$ for every $v_i,v_j\in E(i,j=1,2,3.............n)$.

Definition 3.8: Score function SVNS [46]

Let (Tr_N,Ind_N,Fal_N) be a single-valued neutrosophic number. Then the score function is classified by $S(\alpha)=\frac{1+Tr-2Ind-Fal}{2}$ where $S(\alpha)\in[-1,1]$.

4. Proposed definition for Neutrosophic Fuzzy Magic Labeling Graph

The definition of neutrosophic fuzzy magic labeling graph has been proposed in this section.

Definition 4.1: Neutrosophic Fuzzy Labeling Graph (NFLG).

A NFG is of the form $G=(\delta(V_v),\delta(E_e),\delta(\sigma))$ is called aNFLGif $Tr_1:\delta(V_v)\to[0,1], Ind_1:\delta(V_v)\to[0,1]$ & $Fal_1:\delta(V_v)\to[0,1]$ & $Tr_2:\delta(V_v)\times\delta(V_v)\to[0,1]$, $Ind_2:\delta(V_v)\times\delta(V_v)\to[0,1]$ & $Fal_2:\delta(V_v)\times\delta(V_v)\to[0,1]$is bijective such that vertices and edges each have a separate degree of truth, indeterminacy and falsity-membership function for all$(\delta(v_i),\delta(v_j))$,

$$\mathrm{Tr}_2(\delta(\mathrm{v_i}),\delta(\mathrm{v_j}))\le \min\mathrm{ima}\left[\mathrm{Tr}_1(\delta(\mathrm{v_i})),\mathrm{Tr}_1(\delta(\mathrm{v_j}))\right],$$
$$\mathrm{Ind}_2(\delta(\mathrm{v_i}),\delta(\mathrm{v_j}))\le \min\mathrm{ima}\left[\mathrm{Ind}_1(\delta(\mathrm{v_i})),\mathrm{Ind}_1(\delta(\mathrm{v_j}))\right],$$
$$\mathrm{Fal}_2(\delta(\mathrm{v_i}),\delta(\mathrm{v_j}))\le \max\mathrm{ima}\left[\mathrm{Fal}_1(\delta(\mathrm{v_i})),\mathrm{Fal}_1(\delta(\mathrm{v_j}))\right]$$

and $0\le Tr_1(\delta(v_i),\delta(v_j))+Ind_1(\delta(v_i),\delta(v_j))+Fal_1(\delta(v_i),\delta(v_j))\le 3$.

Definition 4.2:Neutrosophic Fuzzy Magic Labeling Graph (NFMLG)

A NFLG is a NFMLG if there exists amagic graph 'M' such that

degree of truth-membership function equals $Tr_1(\delta(v_i))+Tr_2(\delta(v_i),\delta(v_j))+Tr_1(\delta(v_j))\ \forall\delta(v_i),\delta(v_j)\in E$

degree of indeterminacy function equals $Ind_1(\delta(v_i))+Ind_2(\delta(v_i),\delta(v_j))+Ind_1(\delta(v_j))\ \forall\delta(v_i),\delta(v_j)\in E$

anddegree of falsity function equals $Fal_1(\delta(v_i))+Fal_2(\delta(v_i),\delta(v_j))+Fal_1(\delta(v_j))\quad\forall\delta(v_i),\delta(v_j)\in E$

That is

$$M=\left\{Tr_1(\delta(v_i))+Tr_2(\delta(v_i),\delta(v_j))+Tr_1(\delta(v_j)),Ind_1(\delta(v_i))+Ind_2(\delta(v_i),\delta(v_j))+Ind_1(\delta(v_j)),Fal_1(\delta(v_i))+Fal_2(\delta(v_i),\delta(v_j))+Fal_1(\delta(v_j))\right\}=cons\tan t$$

The magic truth membership value represented by $m_{Tr}(G)$

The magic indeterminacy value represented by $m_{Ind}(G)$

The magic falsity membership value represented by $m_{Fal}(G)$

We represent NFMLG by $M_{m(G)}(G)=(m_{Tr}(G),m_{Ind}(G),m_{Fal}(G))$.

Definition 4.3:Difference between Fuzzy Magic Labeling and NFMLG

Fuzzy Magic Labeling Graph	**NMFG**
Fuzzy magic labeling graph contains only membership function.	NMFG depends on membership, non-membership also indeterminacy.

Membership value alone constant	Membership, non-membership and indeterminacy values are equal to constant.
Fuzzy number is of the form: Example: 0.5	Neutrosphic Number is of the form: Example: (0.8,0.6,0.2)

5. NFMLG of Path Graph

The magic value of a neutrosophic route graph is examined in this section because it satisfies the requirements for a neutrosophic magic labeling graph.

Theorem 5.1: For all $\rho \geq 1(\rho \in \mathbb{N})$ theneutrosophic path P_ρ admits fuzzy magic labeling.

Proof. Let 'P' be any path with distance $n \geq 1(n \in N)$ and $v_1, v_2, v_3 .... v_\rho$ and $v_1, v_2, v_3 .... v_{\rho-1} v_\rho$ are vertices and edges of P. Let $\delta_1, \delta_2, \delta_3 \in [0,1]$ such that we choose $\delta_1 = 0.001, \delta_2 = 0.01 \& \delta_2 = 0.1$ if $\rho \leq 3$ and $\delta_1 = 0.0001, \delta_2 = 0.001 \& \delta_3 = 0.01$ if $\rho \geq 4$ and $\delta_1 = 0.00001, \delta_2 = 0.0001 \& \delta_3 = 0.001$ if $\rho \geq 5$. Where δ_1, δ_2 & δ_3 choose for truth, indeterminacy, and falsity are all degrees of NFMLG membership.

Therefore, NFMLG is given as:

When length is odd:

$$Tr_V(v_{2i-1}) = (2\rho + 2 - i)\delta_1, 1 \leq i \leq \frac{\rho+1}{2}$$
$$Ind_V(v_{2i-1}) = (2\rho + 2 - i)\delta_2, 1 \leq i \leq \frac{\rho+1}{2}$$
$$Fal_V(v_{2i-1}) = (2\rho + 2 - i)\delta_3, 1 \leq i \leq \frac{\rho+1}{2}$$

$$\mathrm{Tr_V}(\mathrm{v_{2i}}) = \mathrm{min\,ima}\left\{\mathrm{Tr}(\mathrm{v_{2i-1}}) / 1 \leq \mathrm{i} \leq \frac{\rho+1}{2}\right\} - \mathrm{i}\delta_1, 1 \leq \mathrm{i} \leq \frac{\rho+1}{2}$$
$$\mathrm{Ind_V}(\mathrm{v_{2i}}) = \mathrm{min\,ima}\left\{\mathrm{Ind}(\mathrm{v_{2i-1}}) / 1 \leq \mathrm{i} \leq \frac{\rho+1}{2}\right\} - \mathrm{i}\delta_2, 1 \leq \mathrm{i} \leq \frac{\rho+1}{2}$$
$$\mathrm{Fal_V}(\mathrm{v_{2i}}) = \mathrm{min\,ima}\left\{\mathrm{Fal}(\mathrm{v_{2i-1}}) / 1 \leq \mathrm{i} \leq \frac{\rho+1}{2}\right\} - \mathrm{i}\delta_3, 1 \leq \mathrm{i} \leq \frac{\rho+1}{2}$$

$$\mathrm{Tr_E}(\mathrm{v}_{\rho-\mathrm{i}+2}, \mathrm{v}_{\rho+1-\mathrm{i}}) = \mathrm{max\,ima}\{\mathrm{Tr_V}(\mathrm{v_i}) / 1 \leq \mathrm{i} \leq \rho+1\} - \mathrm{min\,ima}\{\mathrm{Tr_V}(\mathrm{v_i}) / 1 \leq \mathrm{i} \leq \rho+1\} - (\mathrm{i}-1)\delta_1, 1 \leq \mathrm{i} \leq \rho.$$
$$\mathrm{Ind_E}(\mathrm{v}_{\rho-\mathrm{i}+2}, \mathrm{v}_{\rho+1-\mathrm{i}}) = \mathrm{max\,ima}\{\mathrm{Ind_V}(\mathrm{v_i}) / 1 \leq \mathrm{i} \leq \rho+1\} - \mathrm{min\,ima}\{\mathrm{Ind_V}(\mathrm{v_i}) / 1 \leq \mathrm{i} \leq \rho+1\} - (\mathrm{i}-1)\delta_2, 1 \leq \mathrm{i} \leq \rho.$$
$$\mathrm{Fal_E}(\mathrm{v}_{\rho-\mathrm{i}+2}, \mathrm{v}_{\rho+1-\mathrm{i}}) = \mathrm{max\,ima}\{\mathrm{Fal_V}(\mathrm{v_i}) / 1 \leq \mathrm{i} \leq \rho+1\} - \mathrm{min\,ima}\{\mathrm{Fal_V}(\mathrm{v_i}) / 1 \leq \mathrm{i} \leq \rho+1\} - (\mathrm{i}-1)\delta_3, 1 \leq \mathrm{i} \leq \rho.$$

Case (i) 'i' is even

Then i=2m, where $m \in Z^+$ and for each edge v_i, v_{i+1}

$$\begin{aligned}
m_{Tr}(P_\rho) &= Tr_V(v_i) + Tr_E(v_i, v_{i+1}) + Tr_V(v_{i+1}) \\
&= Tr_V(v_{2m}) + Tr_E(v_{2m}, v_{2m+1}) + Tr_V(v_{2m+1}) \\
&= \min ima\left\{Tr_V(v_{2i-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} - m\delta_1 + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - (\rho-2m)\delta_1 + (2\rho - m + 1)\delta_1 \\
m_{Tr}(P_\rho) &= \min ima\left\{Tr_V(v_{2i-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} - m\delta_1 + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} + (\rho+1)\delta_1
\end{aligned} \tag{1}$$

$$\begin{aligned}
m_{Ind}(P_\rho) &= Ind_V(v_i) + Ind_E(v_i, v_{i+1}) + Ind_V(v_{i+1}) \\
&= Ind_V(v_{2m}) + Ind_E(v_{2m}, v_{2m+1}) + Ind_V(v_{2m+1}) \\
&= \min ima\left\{Ind_V(v_{2m-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} - a\delta_2 + \max ima\{Ind_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Ind_V(v_i)/1 \le i \le \rho+1\} - (\rho-2m)\delta_2 + (2\rho - m + 1)\delta_2 \\
m_{Ind}(P_\rho) &= \min ima\left\{Ind_V(v_{2i-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} - m\delta_2 + \max ima\{Ind_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Ind_V(v_k)/1 \le i \le \rho+1\} + (\rho+1)\delta_2
\end{aligned} \tag{2}$$

$$\begin{aligned}
m_{Fal}(P_\rho) &= Fal_V(v_i) + Fal_E(v_i, v_{i+1}) + Fal_V(v_{i+1}) \\
&= Fal_V(v_{2m}) + Fal_E(v_{2m}, v_{2m+1}) + Fal_V(v_{2m+1}) \\
&= \min ima\left\{Fal_V(v_{2i-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} - m\delta_3 + \max ima\{Fal_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Fal_V(v_i)/1 \le i \le \rho+1\} - (\rho-2m)\delta_3 + (2\rho - m + 1)\delta_3 \\
m_{Fal}(P_\rho) &= \min\left\{Fal_V(v_{2i-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} - m\delta_3 + \max ima\{Fal_V(v_k)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Fal_V(v_k)/1 \le i \le \rho+1\} + (\rho+1)\delta_3
\end{aligned} \tag{3}$$

so that $M_{m(G)}(P_\rho) = (m_{Tr}(G), m_{Ind}(G), m_{Fal}(G))$ = constant.

When 'i' is even, Equations (1), (2), and (3) satisfy the requirement for NFMLG.

Case (i) 'i' is odd

Then i=2m+1, where $m \in Z^+$ and for each edge v_i, v_{i+1}

$$\begin{aligned}
m_{Tr}(P_\rho) &= Tr_V(v_i) + Tr_E(v_i, v_{i+1}) + Tr_V(v_{i+1}) \\
&= Tr_V(v_{2m+1}) + Tr_E(v_{2m+1}, v_{2m+2}) + Tr_V(v_{2m+2}) \\
&= (2\rho - m + 1)\delta_1 + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - (\rho - 2m - 1)\delta_1 + \min ima\left\{Tr_V(v_{2i-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} - (m+1)\delta_1 \\
m_{Tr}(P_\rho) &= \min ima\left\{Tr_V(v_{2i-1}) / 1 \le i \le \frac{\rho+1}{2}\right\} + \max ima\{Tr_V(v_i)/1 \le i \le n+1\} \\
&\quad - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} + (\rho+1)\delta_1
\end{aligned} \tag{4}$$

$$\begin{aligned} m_{Ind}(P_\rho) &= Ind_V(v_i) + Ind_E(v_i, v_{i+1}) + Ind_V(v_{i+1}) \\ &= Ind_V(v_{2m+1}) + Ind_E(v_{2m+1}, v_{2m+2}) + Ind_V(v_{2m+2}) \\ &= (2\rho - m + 1)\delta_2 + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\ &\quad -(\rho - 2m - 1)\delta_2 + \min ima\left\{Tr_V(v_{2i-1})/1 \le i \le \frac{\rho+1}{2}\right\} - (m+1)\delta_2 \end{aligned}$$

$$\begin{aligned} m_{Ind}(P_\rho) &= \min ima\left\{Ind_V(v_{2i-1})/1 \le i \le \frac{\rho+1}{2}\right\} + \max ima\{Ind_V(v_i)/1 \le i \le \rho+1\} \\ &\quad - \min ima\{Ind_V(v_k)/1 \le i \le \rho+1\} + (\rho+1)\delta_2 \end{aligned} \quad (5)$$

$$\begin{aligned} m_{Fal}(P_\rho) &= Fal_V(v_i) + Fal_E(v_i, v_{i+1}) + Fal_V(v_{i+1}) \\ &= Fal_V(v_{2m+1}) + Fal_E(v_{2m+1}, v_{2m+2}) + Fal_V(v_{2m+2}) \\ &= (2\rho - m + 1)\delta_3 + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\ &\quad -(\rho - 2m - 1)\delta_3 + \min ima\left\{Fal_V(v_{2i-1})/1 \le i \le \frac{\rho+1}{2}\right\} - (m+1)\delta_3 \end{aligned}$$

$$\begin{aligned} m_{Fal}(P_\rho) &= \min ima\left\{Fal_V(v_{2i-1})/1 \le i \le \frac{\rho+1}{2}\right\} + \max ima\{Fal_V(v_i)/1 \le i \le n+1\} \\ &\quad - \min ima\{Fal_V(v_i)/1 \le i \le \rho+1\} + (\rho+1)\delta_3 \end{aligned} \quad (6)$$

so that $M_{m(G)}(P_\rho) = (m_{Tr}(G), m_{Ind}(G), m_{Fal}(G))$ = constant.

When k is odd, Equations (4), (5), and (6) satisfy the requirement for NFMLG.

When the length is even:

$$Tr_V(v_{2i}) = (2\rho + 2 - i)\delta_1, 1 \le i \le \frac{\rho}{2}$$

$$Ind_V(v_{2i}) = (2\rho + 2 - i)\delta_2, 1 \le i \le \frac{\rho}{2}$$

$$Fal_V(v_{2i}) = (2\rho + 2 - i)\delta_3, 1 \le i \le \frac{\rho}{2}$$

$$Tr_V(v_{2i-1}) = \min ima\left\{Tr(v_{2i})/1 \le i \le \frac{\rho}{2}\right\} - i\delta_1, 1 \le i \le \frac{\rho+2}{2}$$

$$Ind_V(v_{2i-1}) = \min ima\left\{Tr(v_{2i})/1 \le i \le \frac{\rho}{2}\right\} - i\delta_2, 1 \le i \le \frac{\rho+2}{2}$$

$$Fal_V(v_{2i-1}) = \min ima\left\{Tr(v_{2i})/1 \le i \le \frac{\rho}{2}\right\} - i\delta_3, 1 \le i \le \frac{\rho+2}{2}$$

$$Tr_E(v_{\rho-i+2}, v_{n+1-i}) = \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - (i-1)\delta_1, 1 \le i \le \rho.$$

$$Ind_E(v_{\rho-i+2}, v_{n+1-i}) = \max\{Ind_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Ind_V(v_i)/1 \le i \le \rho+1\} - (i-1)\delta_2, 1 \le i \le \rho.$$ Case

$$Fal_E(v_{\rho-i+2}, v_{n+1-i}) = \max ima\{Fal_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Fal_V(v_i)/1 \le i \le \rho+1\} - (i-1)\delta_3, 1 \le i \le \rho.$$

(i) 'i' is even then i=2m, where $m \in Z^+$ and for each edge v_i, v_{i+1}

$$\begin{aligned}
m_{Tr}(P_\rho) &= Tr_V(v_i) + Tr_E(v_i, v_{i+1}) + Tr_V(v_{i+1}) \\
&= Tr_V(v_{2m}) + Tr_E(v_{2m}, v_{2m+1}) + Tr_V(v_{2m+1}) \\
&= (2\rho - m + 2)\delta_1 + \max ima\{Tr_V(v_i)/1 \le i \le n+1\} - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - (\rho - 2m)\delta_1 \\
&\quad + \min ima\left\{Tr_V(v_{2i})/1 \le i \le \frac{\rho}{2}\right\} - (m+1)\delta_1
\end{aligned}$$

$$\begin{aligned}
m_{Tr}(P_\rho) &= \min ima\left\{Tr_V(v_{2i-1})/1 \le i \le \frac{\rho}{2}\right\} + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} + (\rho+1)\delta_1
\end{aligned} \tag{7}$$

$$\begin{aligned}
m_{Ind}(P_\rho) &= Ind_V(v_i) + Ind_E(v_i, v_{i+1}) + Ind_V(v_{i+1}) \\
&= Ind_V(v_{2m}) + Ind_E(v_{2m}, v_{2m+1}) + Ind_V(v_{2m+1}) \\
&= (2\rho - m + 2)\delta_2 + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - (\rho - 2i)\delta_2 + \min ima\left\{Tr_V(v_{2i})/1 \le i \le \frac{\rho}{2}\right\} - (m+1)\delta_2
\end{aligned}$$

$$\begin{aligned}
m_{Ind}(P_\rho) &= \min ima\left\{Tr_V(v_{2i-1})/1 \le i \le \frac{\rho}{2}\right\} + \max ima\{Tr_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Tr_V(v_i)/1 \le i \le \rho+1\} + (\rho+1)\delta_2
\end{aligned} \tag{8}$$

$$\begin{aligned}
m_{Fal}(P_\rho) &= Fal_V(v_i) + Fal_E(v_i, v_{i+1}) + Fal_V(v_{i+1}) \\
&= Fal_V(v_{2m}) + Fal_E(v_{2m}, v_{2m+1}) + Fal_V(v_{2m+1}) \\
&= (2\rho - m + 2)\delta_3 + \max ima\{Fal_V(v_i)/1 \le i \le \rho+1\} - \min ima\{Fal_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - (\rho - 2m)\delta_3 + \min ima\left\{Fal_V(v_{2i})/1 \le i \le \frac{\rho}{2}\right\} - (m+1)\delta_3
\end{aligned}$$

$$\begin{aligned}
m_{Fal}(P_\rho) &= \min ima\left\{Fal_V(v_{2i-1})/1 \le i \le \frac{\rho}{2}\right\} + \max ima\{Fal_V(v_i)/1 \le i \le \rho+1\} \\
&\quad - \min ima\{Fal_V(v_i)/1 \le i \le \rho+1\} + (\rho+1)\delta_3
\end{aligned} \tag{9}$$

so that $M_{m(G)}(P_\rho) = (m_{Tr}(G), m_{Ind}(G), m_{Fal}(G))$ = constant.

When i=2m, equations (7), (8), and (9) satisfy the requirement for NFMLG.

Case (ii) 'i' is odd

Then i=2m+1, where $m \in Z^+$ and for each edge v_i, v_{i+1}

$$\begin{aligned}
m_{Tr}(P_\rho) &= Tr_V(v_i)+Tr_E(v_i,v_{i+1})+Tr_V(v_{i+1}) \\
&= Tr_V(v_{2m+1})+Tr_E(v_{2m+1},v_{2m+2})+Tr_V(v_{2m+2}) \\
&= \min ima\left\{Tr_V(v_{2i})/1\le i\le \frac{\rho}{2}\right\}-(m+1)\delta_1+\max ima\left\{Tr_V(v_i)/1\le i\le \rho+1\right\} \\
&\quad -\min ima\left\{Tr_V(v_i)/1\le i\le \rho+1\right\}-(\rho-2m-1)\delta_1-(2\rho-m+1)\delta_1
\end{aligned}$$

$$\begin{aligned}
m_{Tr}(P_\rho) &= \min ima\left\{Tr_V(v_{2i})/1\le i\le \frac{\rho}{2}\right\}+\max ima\left\{Tr_V(v_i)/1\le i\le \rho+1\right\} \\
&\quad -\min ima\left\{Tr_V(v_i)/1\le i\le \rho+1\right\}+(\rho+1)\delta_1
\end{aligned} \tag{10}$$

$$\begin{aligned}
m_{Ind}(P_\rho) &= Ind_V(v_i)+Ind_E(v_i,v_{i+1})+Ind_V(v_{i+1}) \\
&= Ind_V(v_{2m+1})+Ind_E(v_{2m+1},v_{2m+2})+Ind_V(v_{2m+2}) \\
&= \min ima\left\{Ind_V(v_{2i})/1\le i\le \frac{\rho}{2}\right\}-(m+1)\delta_2+\max ima\left\{Ind_V(v_i)/1\le i\le \rho+1\right\} \\
&\quad -\min ima\left\{Ind_V(v_i)/1\le i\le \rho+1\right\}-(\rho-2m-1)\delta_1-(2\rho-m+1)\delta_2
\end{aligned}$$

$$\begin{aligned}
m_{Ind}(P_\rho) &= \min ima\left\{Ind_V(v_{2i})/1\le i\le \frac{\rho}{2}\right\}+\max ima\left\{Ind_V(v_k)/1\le i\le \rho+1\right\} \\
&\quad -\min ima\left\{Ind_V(v_i)/1\le i\le \rho+1\right\}+(\rho+1)\delta_2
\end{aligned} \tag{11}$$

$$\begin{aligned}
m_{Fal}(P_n) &= Fal_V(v_i)+Fal_E(v_i,v_{i+1})+Fal_V(v_{i+1}) \\
&= Fal_V(v_{2m+1})+Fal_E(v_{2m+1},v_{2m+2})+Fal_V(v_{2m+2}) \\
&= \min ima\left\{Fal_V(v_{2i})/1\le i\le \frac{\rho}{2}\right\}-(m+1)\delta_2+\max ima\left\{Fal_V(v_i)/1\le i\le \rho+1\right\} \\
&\quad -\min ima\left\{Fal_V(v_i)/1\le i\le \rho+1\right\}-(\rho-2m-1)\delta_1-(2\rho-m+1)\delta_3
\end{aligned}$$

$$\begin{aligned}
m_{Fal}(P_n) &= \min ima\left\{Fal_V(v_{2i})/1\le i\le \frac{\rho}{2}\right\}+\max ima\left\{Fal_V(v_i)/1\le i\le \rho+1\right\} \\
&\quad -\min ima\left\{Fal_V(v_i)/1\le i\le \rho+1\right\}+(\rho+1)\delta_3
\end{aligned} \tag{12}$$

so that $M_{m(G)}(P_\rho)=(m_{Tr}(G),m_{Ind}(G),m_{Fal}(G))$ = constant.

Equations (10), (11), (12) satisfies the condition for NFMLG when i=2m+1.

Example. 5.2:

Figure 1 represents NFMLG path graph with eight nodes and seven edges.

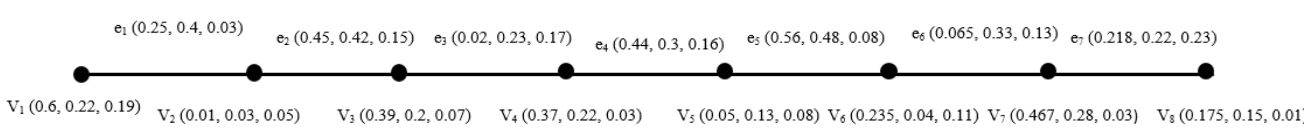

Figure1: NFMLG of path graph

The aforementionedNeutrosophic path graph P_8's magic value is $(0.86, 0.65, 0.27)$.

The value 0.86 indicates truth membership

The value 0.65 indicates indeterminacy

The value 0.27 indicates falsity

Using definition 3.8, the scorevalue of the magic value of the NFMLG of path graph is given by $S(\alpha)=0.145$ .Here in NFMLG-path graph the score value indicates that our result fits the requirement for a neutrosophic set because it falls with the [-1, -1] score limit.

6. NFMGL of Cycle Graph

In this part, we examine the magic value of a neutrosophic cycle graph, which satisfies the requirements for a neutrosophic magic labeling graph.

Theorem 6.1: If ' ρ **' is odd, then the cycle** C_ρ **is an NFMLG.**

Proof: Let C_ρ be any cycle with odd integers $v_1, v_2, v_3 \ldots\ldots\ldots\ldots\ldots v_n$ and $v_1, v_2, v_3 \ldots\ldots\ldots\ldots\ldots v_{n-1} v_1$ are vertices and edges of C_ρ. Let $\delta_1, \delta_2, \delta_3 \in [0,1]$ such that we choose $\delta_1 = 0.001, \delta_2 = 0.01 \& \delta_2 = 0.1$ if $\rho \le 3$ and $\delta_1 = 0.0001, \delta_2 = 0.001 \& \delta_3 = 0.01$ if $\rho \ge 4$ and $\delta_1 = 0.00001, \delta_2 = 0.0001 \& \delta_3 = 0.001$ if $\rho \ge 5$. Where $\delta_1, \delta_2 \ \& \ \delta_3$ choose for collection of truth, indeterminacy and falsity membership degree in NFMLG.

Therefore, NFMLG is given as:

When length is odd:

$$Tr_V(v_{2i}) = (2\rho + 1 - i)\delta_1, 1 \le i \le \frac{\rho - 1}{2}$$

$$Ind_V(v_{2i}) = (2\rho + 1 - i)\delta_2, 1 \le i \le \frac{\rho - 1}{2}$$

$$Fal_V(v_{2i}) = (2\rho + 1 - i)\delta_3, 1 \le i \le \frac{\rho - 1}{2}$$

$$\mathrm{Tr}_V(\mathrm{v}_{2i}) = \min\mathrm{ima}\left\{\mathrm{Tr}(\mathrm{v}_{2i}) / 1 \le \mathrm{i} \le \frac{\rho - 1}{2}\right\} - \mathrm{i}\delta_1, 1 \le \mathrm{i} \le \frac{\rho + 1}{2}$$

$$\mathrm{Ind}_V(\mathrm{v}_{2i}) = \min\mathrm{ima}\left\{\mathrm{Tr}(\mathrm{v}_{2i}) / 1 \le \mathrm{i} \le \frac{\rho - 1}{2}\right\} - \mathrm{i}\delta_2, 1 \le \mathrm{i} \le \frac{\rho + 1}{2}$$

$$\mathrm{Fal}_V(\mathrm{v}_{2i}) = \min\mathrm{ima}\left\{\mathrm{Tr}(\mathrm{v}_{2i}) / 1 \le \mathrm{i} \le \frac{\rho - 1}{2}\right\} - \mathrm{i}\delta_3, 1 \le \mathrm{i} \le \frac{\rho + 1}{2}$$

$$\mathrm{Tr_E}\left(\mathrm{v}_1,\mathrm{v}_\rho\right)=\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Tr_V}\left(\mathrm{v_i}\right)/1\le \mathrm{i}\le\rho\right\}$$

$$\mathrm{Fal_E}\left(\mathrm{v}_1,\mathrm{v}_\rho\right)=\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Fal_V}\left(\mathrm{v_i}\right)/1\le \mathrm{i}\le\rho\right\}$$

$$\mathrm{Ind_E}\left(\mathrm{v}_1,\mathrm{v}_\rho\right)=\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Ind_V}\left(\mathrm{v_i}\right)/1\le \mathrm{i}\le\rho\right\}$$

$$Tr_E\left(v_{\rho-i+1},v_{\rho-i}\right)=Tr_E\left(v_1,v_\rho\right)-i\delta_1,\quad 1\le i\le\rho-1$$
$$Ind_E\left(v_{\rho-i+1},v_{\rho-i}\right)=Fal_E\left(v_1,v_\rho\right)-i\delta_2,\quad 1\le i\le\rho-1$$
$$Tr_E\left(v_{\rho-i+1},v_{\rho-i}\right)=Ind_E\left(v_1,v_\rho\right)-i\delta_3,\quad 1\le i\le\rho-1.$$

Case (i) 'i' is even

Then i=2m, where $m\in Z^+$ and for each edge v_i,v_{i+1}

$$\begin{aligned}\mathrm{m_{Tr}}(\mathrm{C}_\rho)&=\mathrm{Tr_V}\left(\mathrm{v_i}\right)+\mathrm{Tr_E}\left(\mathrm{v_i},\mathrm{v_{i+1}}\right)+\mathrm{Tr_V}\left(\mathrm{v_{i+1}}\right)\\&=\mathrm{Tr_V}\left(\mathrm{v_{2m}}\right)+\mathrm{Tr_E}\left(\mathrm{v_{2m}},\mathrm{v_{2m+1}}\right)+\mathrm{Tr_V}\left(\mathrm{v_{2m+1}}\right)\\&=\left(2\rho-\mathrm{m}+1\right)\delta_1+\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Tr_V}\left(\mathrm{v_i}\right)/1\le\mathrm{i}\le\rho\right\}-\left(\rho-2\mathrm{m}\right)\delta_1\\&\quad+\min\mathrm{ima}\left\{\mathrm{Tr_V}\left(\mathrm{v_{2i}}\right)/1\le\mathrm{i}\le\frac{\rho-1}{2}\right\}-\left(\mathrm{m}+1\right)\delta_1\\\mathrm{m_{Tr}}(\mathrm{C}_\rho)&=\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Tr_V}\left(\mathrm{v_i}\right)/1\le\mathrm{i}\le\rho\right\}+\min\mathrm{ima}\left\{\mathrm{Tr_V}\left(\mathrm{v_{2i}}\right)/1\le\mathrm{i}\le\frac{\rho-1}{2}\right\}+\rho\delta_1\end{aligned}\tag{13}$$

$$\begin{aligned}\mathrm{m_{Ind}}(\mathrm{C}_\rho)&=\mathrm{Ind_V}\left(\mathrm{v_i}\right)+\mathrm{Ind_E}\left(\mathrm{v_i},\mathrm{v_{i+1}}\right)+\mathrm{Ind_V}\left(\mathrm{v_{i+1}}\right)\\&=\mathrm{Ind_V}\left(\mathrm{v_{2m}}\right)+\mathrm{Ind_E}\left(\mathrm{v_{2m}},\mathrm{v_{2m+1}}\right)+\mathrm{Ind_V}\left(\mathrm{v_{2m+1}}\right)\\&=\left(2\rho-\mathrm{m}+1\right)\delta_2+\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Ind_V}\left(\mathrm{v_i}\right)/1\le\mathrm{i}\le\rho\right\}-\left(\rho-2\mathrm{m}\right)\delta_2\\&\quad+\min\mathrm{ima}\left\{\mathrm{Ind_V}\left(\mathrm{v_{2i}}\right)/1\le\mathrm{i}\le\frac{\rho-1}{2}\right\}-\left(\mathrm{m}+1\right)\delta_2\\\mathrm{m_{Ind}}(\mathrm{C}_\rho)&=\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Ind_V}\left(\mathrm{v_i}\right)/1\le\mathrm{i}\le\rho\right\}+\min\mathrm{ima}\left\{\mathrm{Ind_V}\left(\mathrm{v_{2i}}\right)/1\le\mathrm{i}\le\frac{\rho-1}{2}\right\}+\rho\delta_2\end{aligned}\tag{14}$$

$$\begin{aligned}\mathrm{m_{Fal}}(\mathrm{C}_\rho)&=\mathrm{Fal_V}\left(\mathrm{v_i}\right)+\mathrm{Fal_E}\left(\mathrm{v_i},\mathrm{v_{i+1}}\right)+\mathrm{Fal_V}\left(\mathrm{v_{i+1}}\right)\\&=\mathrm{Fal_V}\left(\mathrm{v_{2m}}\right)+\mathrm{Fal_E}\left(\mathrm{v_{2m}},\mathrm{v_{2m+1}}\right)+\mathrm{Fal_V}\left(\mathrm{v_{2m+1}}\right)\\&=\left(2\rho-\mathrm{m}+1\right)\delta_3+\frac{1}{2}\max\left\{\mathrm{Fal_V}\left(\mathrm{v_i}\right)/1\le\mathrm{i}\le\rho\right\}-\left(\rho-2\mathrm{m}\right)\delta_3\\&\quad+\min\mathrm{ima}\left\{\mathrm{Ind_V}\left(\mathrm{v_{2i}}\right)/1\le\mathrm{i}\le\frac{\rho-1}{2}\right\}-\left(\mathrm{m}+1\right)\delta_3\\\mathrm{m_{Fal}}(\mathrm{C}_\rho)&=\frac{1}{2}\max\mathrm{ima}\left\{\mathrm{Fal_V}\left(\mathrm{v_i}\right)/1\le\mathrm{i}\le\rho\right\}+\min\mathrm{ima}\left\{\mathrm{Fal_V}\left(\mathrm{v_{2i}}\right)/1\le\mathrm{i}\le\frac{\rho-1}{2}\right\}+\rho\delta_3\end{aligned}\tag{15}$$

so that $M_{m(G)}\left(P_\rho\right)=\left(m_{Tr}(G),m_{Ind}(G),m_{Fal}(G)\right)$ = constant.

Equations (13), (14), (15) satisfy the condition for NFMLG when k is even.

Case (ii) 'i' is odd

Then i=2m+1, where $m \in Z^+$ and for each edge v_i, v_{i+1}

$$m_{Tr}(C_\rho) = Tr_V(v_i) + Tr_E(v_i, v_{i+1}) + Tr_V(v_{i+1})$$
$$= Tr_V(v_{2m+1}) + Tr_E(v_{2m+1}, v_{2m+2}) + Tr_V(v_{2m+2})$$
$$= \min ima\left\{Tr_V(v_{2m}) / 1 \le i \le \frac{\rho-1}{2}\right\} - (m+1)\delta_1 + \frac{1}{2}\max ima\left\{Tr_V(v_i) / 1 \le i \le \rho\right\}$$
$$-(\rho - 2m - 1)\delta_1 - (2\rho - m)\delta_1$$
$$m_{Tr}(C_\rho) = \frac{1}{2}\max ima\left\{Tr_V(v_i) / 1 \le i \le \rho\right\} + \min ima\left\{Tr_V(v_{2i}) / 1 \le i \le \frac{\rho-1}{2}\right\} + \rho\delta_1 \quad (16)$$

$$m_{Ind}(C_\rho) = Ind_V(v_i) + Ind_E(v_k, v_{i+1}) + Ind_V(v_{i+1})$$
$$= Ind_V(v_{2m+1}) + Ind_E(v_{2m+1}, v_{2m+2}) + Ind_V(v_{2m+2})$$
$$= \min ima\left\{Ind_V(v_{2i}) / 1 \le i \le \frac{\rho-1}{2}\right\} - (m+1)\delta_2 + \frac{1}{2}\max ima\left\{Ind_V(v_i) / 1 \le i \le \rho\right\}$$
$$-(\rho - 2m - 1)\delta_2 - (2\rho - m)\delta_2$$
$$m_{Ind}(C_\rho) = \frac{1}{2}\max ima\left\{Ind_V(v_i) / 1 \le i \le \rho\right\} + \min ima\left\{Ind_V(v_{2i}) / 1 \le i \le \frac{\rho-1}{2}\right\} + v\delta_2 \quad (17)$$

$$m_{Fal}(C_\rho) = Fal_V(v_i) + Fal_E(v_i, v_{i+1}) + Fal_V(v_{i+1})$$
$$= Fal_V(v_{2m+1}) + Fal_E(v_{2m+1}, v_{2m+2}) + Fal_V(v_{2m+2})$$
$$= \min ima\left\{Fal_V(v_{2i}) / 1 \le i \le \frac{\rho-1}{2}\right\} - (m+1)\delta_3 + \frac{1}{2}\max ima\left\{Fal_V(v_i) / 1 \le i \le \rho\right\}$$
$$-(\rho - 2m - 1)\delta_3 - (2\rho - m)\delta_3$$
$$m_{Fal}(C_\rho) = \frac{1}{2}\max ima\left\{Fal_V(v_i) / 1 \le i \le \rho\right\} + \min ima\left\{Fal_V(v_{2i}) / 1 \le i \le \frac{\rho-1}{2}\right\} + \rho\delta_3 \quad (18)$$

Hence $M_{m(G)}(C_\rho) = (m_{Tr}(G), m_{Ind}(G), m_{Fal}(G)) =$ constant.

When 'i' is odd, equations (16), (17), and (18) satisfy the requirement for NFMLG.

The magic value $M_{m(G)}(C_\rho)$ is same and unique in above cases. Thus C_ρ is an NFMLG.

Example 6.2:

Figure 2 represents NFMLG cycle Graph with five nodes and five edges.

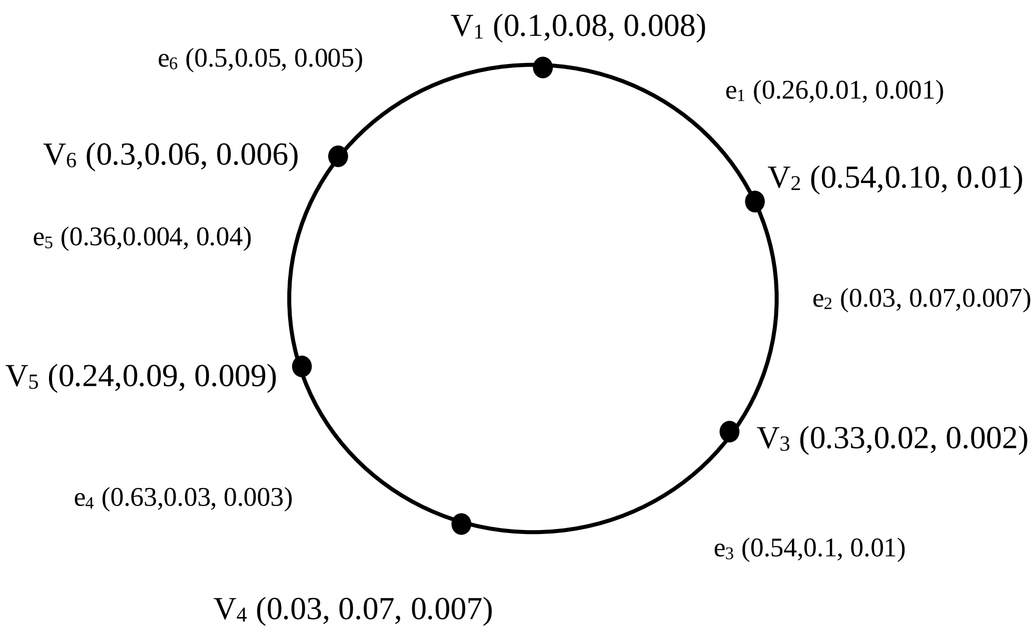

Figure2: NFMLG of cycle graph

The magic value of the aforementionedneutrosophic cycle graph $C_6 \quad is\left(0.9, 0.19, 0.019\right)$

The values are 0.9,0.19,0.019 indicates the truthmembership,indeterminacy and falsity function for NFMLG-cycle graph.

Using definition 3.8 we find the score value of the magic value of NFMLG of cycle graph is given by $S(\alpha) = 0.7505$. Because our result falls under the [-1,1] score limit, the NFMLG-cycle graph's score value here indicates that out result satisfies the criteria for a neutrosophic set.

7.NFMLG of Star Graph

The magic value of a neutrosophic star graph is examined in this section because it satisfies the requirements for a neutrosophic magic labeling graph.

Theorem 7.1: For any $\rho \geq 2$, star graph $S_{1,\rho}$ **is an NFMLG.**

Proof: Let $S_{1,\rho}$ be any star graph having $v, u_1, u_2, u_3 .... u_\rho$ as vertices and $vu_1, vu_2, vu_3 .... vuv_\rho$ as edges. Let $\delta_1, \delta_2, \delta_3 \in [0,1]$ such that we choose $\delta_1 = 0.001$, $\delta_2 = 0.01 \& \delta_2 = 0.1$ if $\rho \leq 3$ and $\delta_1 = 0.0001$, $\delta_2 = 0.001 \& \delta_3 = 0.01$ if $\rho \geq 4$ and $\delta_1 = 0.00001, \delta_2 = 0.0001 \& \delta_3 = 0.001$ if $\rho \geq 5$. Where $\delta_1, \delta_2 \& \delta_3$ choose for collection of truth membership degree, indeterminacy and falsity membership degree in NFMLG.

Therefore, NFMLG is given as:

$$Tr_V(u_i) = (2(\rho+1)-i)\delta_1, 1 \leq i \leq \rho$$
$$Ind_V(u_i) = (2(\rho+1)-i)\delta_2, 1 \leq i \leq \rho$$
$$Fal_V(u_i) = (2(\rho+1)-i)\delta_3, 1 \leq i \leq \rho$$

$$Tr_V(v_i) = \min\{Tr(u_i)/1 \leq i \leq \rho\} - \delta_1$$
$$Ind_V(v_i) = \min\{Tr(u_i)/1 \leq k \leq \rho\} - \delta_2$$
$$Fal_V(v_i) = \min\{Tr(u_i)/1 \leq i \leq \rho\} - \delta_3$$

$$Tr_E(v, u_{\rho-i}) = \max\{Tr_V(v_i), Tr_V(v)/1 \leq i \leq \rho\} - \min\{Tr_V(v_i), Tr_V(v)/1 \leq i \leq \rho\} - i\delta_1, 0 \leq i \leq \rho-1$$
$$Fal_E(v, u_{\rho-i}) = \max\{Ind_V(v_i), Ind_V(v)/1 \leq i \leq \rho\} - \min\{Ind_V(v_i), Ind_V(v)/1 \leq i \leq \rho\} - i\delta_2, 0 \leq i \leq \rho-1$$
$$Ind_E(v, u_{\rho-i}) = \max\{Fal_V(v_i), Fal_V(v)/1 \leq i \leq \rho\} - \min\{Fal_V(v_i), Fal_V(v)/1 \leq i \leq \rho\} - k\delta_3, 0 \leq i \leq \rho-1$$

Case (i) 'i' is even.

Then i=2m, where $m \in Z^+$ and for each edge v, u_i.

$$\begin{aligned} m_{Tr}(S_{1,\rho}) &= Tr_V(v) + Tr_E(v, u_i) + Tr_V(u_i) \\ &= Tr_V(v) + Tr_E(v, u_{2m}) + Tr_V(u_{2m}) \\ &= \min ima\{Tr_V(u_i)/1 \leq i \leq \rho\} - \delta_1 + \max ima\{Tr_V(v_i), Tr_V(v)/1 \leq i \leq \rho\} \\ &\quad - \min ima\{Tr_V(v_i), Tr_V(v)/1 \leq i \leq \rho\} - (v - 2m)\delta_1 + [2(\rho+1) - 2m]\delta_1 \end{aligned}$$

$$\begin{aligned} m_{Tr}(S_{1,\rho}) &= \min ima\{Tr_V(u_i)/1 \leq i \leq \rho\} + \max ima\{Tr_V(v_i), Tr_V(v)/1 \leq i \leq \rho\} \\ &\quad - \min ima\{Tr_V(v_i), Tr_V(v)/1 \leq i \leq \rho\} + (\rho+1)\delta_1 \end{aligned} \tag{19}$$

$$\begin{aligned} m_{Ind}(S_{1,\rho}) &= Ind_V(v) + Ind_E(v,u_i) + Fal_V(u_i) \\ &= Ind_V(v) + Ind_E(v,u_{2m}) + Ind_V(u_{2m}) \\ &= \min ima\{Ind_V(u_i)/1\le i\le\rho\} - \delta_2 + \max ima\{Ind_V(v_i), Ind_V(v)/1\le i\le\rho\} \\ &\quad - \min ima\{Ind_V(v_i), Ind_V(v)/1\le i\le\rho\} - (\rho-2m)\delta_2 + [2(\rho+1)-2m]\delta_2 \\ m_{Ind}(S_{1,\rho}) &= \min ima\{Ind_V(u_i)/1\le i\le\rho\} + \max ima\{Ind_V(v_i), Ind_V(v)/1\le i\le\rho\} \\ &\quad - \min ima\{Ind_V(v_i), Ind_V(v)/1\le i\le\rho\}(\rho+1)\delta_2 \end{aligned} \tag{20}$$

$$\begin{aligned} m_{Fal}(S_{1,\rho}) &= Fal_V(v) + Fal_E(v,u_i) + Fal_V(u_i) \\ &= Fal_V(v) + Fal_E(v,u_{2m}) + Fal_V(u_{2m}) \\ &= \min\{Fal_V(u_i)/1\le i\le\rho\} - \delta_3 + \max\{Fal_V(v_i), Fal_V(v)/1\le i\le\rho\} \\ &\quad - \min\{Fal_V(v_i), Fal_V(v)/1\le i\le\rho\} - (\rho-2m)\delta_3 + [2(\rho+1)-2m]\delta_3 \\ m_{Fal}(S_{1,n}) &= \min\{Fal_V(u_i)/1\le i\le\rho\} + \max\{Fal_V(v_i), Fal_V(v)/1\le i\le\rho\} \\ &\quad - \min\{Fal_V(v_i), Fal_V(v)/1\le i\le\rho\} + (\rho+1)\delta_3 \end{aligned} \tag{21}$$

so that $M_{m(G)}(S_{1,\rho}) = (m_{Tr}(G), m_{Ind}(G), m_{Fal}(G))$ = constant.

Equations (19), (20), (21) satisfy the condition for NFMLG when 'i' is even.

Case (ii) 'i' is odd

Then i=2m+1, where $m \in Z^+$ and for each edge v, u_i

$$\begin{aligned} m_{Tr}(S_{1,\rho}) &= Tr_V(v) + Tr_E(v,u_i) + Tr_V(u_i) \\ &= Tr_V(v) + Tr_E(v,u_{2m+1}) + Tr_V(u_{2m+1}) \\ &= \min ima\{Tr_V(u_i)/1\le i\le\rho\} - \delta_1 + \max ima\{Tr_V(v_i), Tr_V(v)/1\le i\le\rho\} \\ &\quad - \min\{Tr_V(v_i), Tr_V(v)/1\le i\le\rho\} - (\rho-2m-1)\delta_1 + (2(\rho+1)-2m+1)\delta_1 \\ m_{Tr}(S_{1,\rho}) &= \min ima\{Tr_V(u_i)/1\le i\le\rho\} + \max ima\{Tr_V(v_i), Tr_V(v)/1\le i\le\rho\} \\ &\quad - \min ima\{Tr_V(v_i), Tr_V(v)/1\le i\le\rho\} + (\rho+1)\delta_1 \end{aligned} \tag{22}$$

$$\begin{aligned} m_{Ind}(S_{1,\rho}) &= Ind_V(v) + Ind_E(v,u_i) + Ind_V(u_i) \\ &= Ind_V(v) + Ind_E(v,u_{2m+1}) + Ind_V(u_{2m+1}) \\ &= \min ima\{Ind_V(u_i)/1\le i\le\rho\} - \delta_1 + \max ima\{Ind_V(v_i), Ind_V(v)/1\le i\le\rho\} \\ &\quad - \min\{Ind_V(v_i), Ind_V(v)/1\le i\le\rho\} - (\rho-2m-1)\delta_2 + (2(\rho+1)-2m+1)\delta_2 \\ m_{Ind}(S_{1,\rho}) &= \min ima\{Ind_V(u_i)/1\le i\le\rho\} + \max ima\{Ind_V(v_i), Ind_V(v)/1\le i\le n\} \\ &\quad - \min ima\{Ind_V(v_i), Ind_V(v)/1\le i\le\rho\} + (\rho+1)\delta_2 \end{aligned} \tag{23}$$

$$\begin{aligned} m_{Fal}(S_{1,\rho}) &= Fal_V(v) + Fal_E(v,u_i) + Fal_V(u_i) \\ &= Fal_V(v) + Fal_E(v,u_{2m+1}) + Fal_V(u_{2m+1}) \\ &= \min ima\{Fal_V(u_i)/1\le i\le\rho\} - \delta_3 + \max ima\{Fal_V(v_i), Fal_V(v)/1\le i\le\rho\} \\ &\quad - \min ima\{Fal_V(v_i), Fal_V(v)/1\le i\le\rho\} - (\rho-2m-1)\delta_3 + (2(\rho+1)-2m+1)\delta_3 \\ m_{Fal}(S_{1,\rho}) &= \min ima\{Fal_V(u_i)/1\le i\le\rho\} + \max ima\{Fal_V(v_k), Fal_V(v)/1\le i\le\rho\} \\ &\quad - \min ima\{Fal_V(v_i), Fal_V(v)/1\le i\le\rho\} + (\rho+1)\delta_3 \end{aligned} \tag{24}$$

So that, $M_{m(G)}\left(S_{1,\rho}\right)=\left(m_{Tr}(G), m_{Ind}(G), m_{Fal}(G)\right)$ = constant.

Equations (16), (17), and (18) satisfy the condition for NFMLG when 'i' is odd.

The magic value is the same and distinct in all of the cases before it. The star graph is therefore NFMLG.

Example 7.2:

The NFMLG of star graph shown in Figure 3 has seven nodes and seven edges.

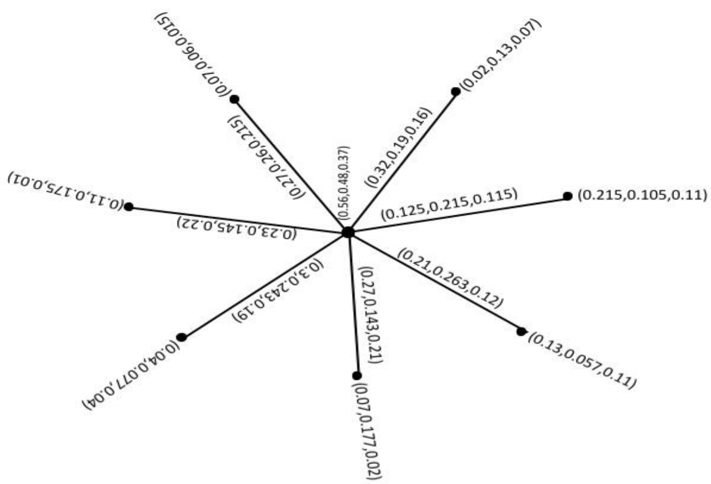

Figure 3:NFMLG of star graph

The magic value of the aforementionedNeutrosophic star graph $S_{1,7}$ is $(0.9, 0.8, 0.6)$

The truth membership,indeterminacy and falsity value of NFMLG-star graph as follows:

Truthmembership value-0.9

Indeterminacy-0.8

Falsity-0.6

Using definition 3.8,the score value of the magic value of NFMLG ofa star graph is calculated andis given by $S(\alpha)=-0.15$. The NFMLG-star graph's score value here indicates that our solution meets the requirements for a neutrosophic set because it is between [-1,1] score limit.

8. Application of Neutrosophicfuzzy magic labelingpathgraphs

In this section, the advantage of NFMLG and why we recommended NFMLG concept to a problem involving decision-making has been elaborated.

Advantage, Uses and Limitation of Neutrosophic fuzzy magic labeling:

The neutrosophic becomes appreared and found their place in research since the world is full of uncertainty (indeterminacy) so we used neutrosophic in our research. These ideas, while applicable to a variety of real-world problems, cannot deal with all forms of uncertainty, such as ambiguous and inconsistent information.

Uses of Neutrosphic magic labeling:

In many field like medical image processing applications, neutrosophic sets (NS) play a vital role in denoising, clustering, segmentation, and classification. In order to reduce uncertainty for effective diagnosis, clustering techniques have been integrated with NS for the efficient creation of computer-aided diagnosis systems.

When it comes to fuzzy magic labelling graphs, we can only talk about the criteria in one category, but when it comes to intuitionistic fuzzy magic labelling graphs, we can discuss the criteria in two different scenarios. However, in the case of NFMLG, we are talking about the criteria in three different scenarios. This is one of the key benefits of NFMLG because it offers more options, greater flexibility, and greater compatibility. Therefore, we are using the NFMLG-path graph in this student's subject selection decision-making problem in the following way:

The subject of education is divisive and has generated numerous arguments. One of them deals with topic choice. Some people think that students should be free to select the subjects they want to learn about, while others think that all subjects should be obligatory. In our opinion, kids need to have the option to select topics based on their interests. Students today are very well educated, and they pick their classes based on what they want to do for a living in the future.

Despite a few very popular courses, students who have a strong interest in a certain subject or career will regard all options available as the best option. Gaining curiosity and confidence is enough to become a master in any field. It is crucial for students whose secondary education is about to end to choose their career path as soon as they graduate. At this time, it is crucial to stress how important it is to give children enough information about career alternatives that are related to their interests.

This section displays the interest, confusion, or lack thereof among students in a subject or a group of topics based on their replies, which were provided by 100 students in class "X" [47]. The data show that NFMLG can be used as a tool because it considers three different membership functions, including membership with indeterminacy (the conundrum that a certain proportion of students in a particular subject or pair face) and non-membership (the extent to which students do not belong to a particular subject or pair) (the disinterest of a percentage of students in a subject or pair of subjects). Using NFMLG, we can determine which courses will likely benefit the most students and result in the highest levels of learning when taken together.

Let $Subject\,(S) =$ {English (ENG), Language (LANG), Mathematics (MAT),Science (SC),Social Science (SSC)} be the collection of vertices. The table below depicts the proportion of students that are interested, undecided, or disinterested in picking a subject or pair of topics.

Table 1: Indicated the subject/subject combination

Subject/Subject Combination	Interest	Dilemma	Disinterest
	*Here 0.72 represents 72%		
ENG	0.16	0.30	0.49
LANG	0.12	0.23	0.13
MAT	0.72	0.15	0.46
SC	0.15	0.63	0.41
SSC	0.63	0.19	0.21
ENG-MAT	0.12	0.55	0.05
ENG-LANG	0.72	0.47	0.38
ENG-SC	0.69	0.07	0.10
ENG-SSC	0.12	0.51	0.30
MAT-SC	0.13	0.22	0.13
LANG-MAT	0.16	0.62	0.41
LANG-SC	0.73	0.14	0.43
SC-SSC	0.22	0.18	0.38

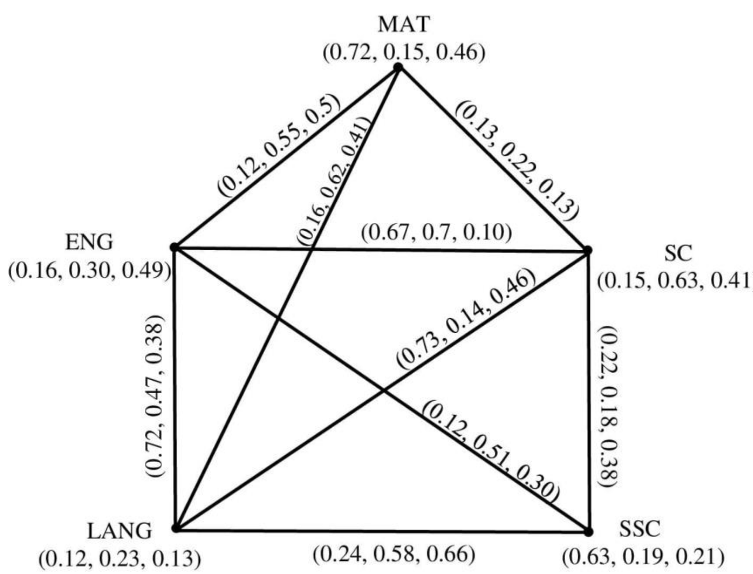

Figure 4.Graph representation the subject/subject combination

Figure 4 displays the percentage of students who are passionate about a certain subject, the percentage of students who are undecided, and the percentage of students who have no interest in a subject.The students' interest in, perplexity over, and lack of interest in integrating any two upper-secondary courses may be seen in the graph edges' membership, indeterminacy, and non-membership.

Figure 4 shows that the majority of students who study both language and science are interested in pursuing a career in medicine, which is shown by an edge (LANG, SC) with the highest degree of membership function. According to research, students who are afraid of math lessons might choose this option. Most students are pulled between studying Language and Math, according to the edge (LANG, MATHS). Language and social science are not subjecting that students who have strong non-membership functions on the edge (LANG, SSC) desire to study together.

Research limitations:

It has limitations for Bipolar neutrosophic set, complex intuitionistic fuzzy hypersoft set, complex neutrosophic hypersoft set and other complex neutrosophic hypersoft set-like models.

10. Comparative analysis

Comparative analysis and the current methodology have been addressed throughout this section.

Table 2: Comparative analysis

Method	Results
Intuitionistic Fuzzy Graph	Membership Function-Interest: (MAT,SC)
	Non-Membership Function-Disinterest: (LANG,SC)
Neutrosophic Fuzzy Magic Labeling of Cycle graph	Membership Function-Interest: (LANG, SC)
	Indeterminacy Membership Function- Dilemma: (LANG, MATHS)
	Non-Membership Function-Disinterest: (LANG,SSC)

The results for the present approach of neutrosophic fuzzy magic labelling of simple graphs are proven in Table 2. The membership function's output reveals that the majority of students are drawn to the idea of merging mathematics and science. The majority of students do not want to study a mix of language and social science courses, according to the results of the non-membership function. This NFMLG-recommended approach reveals that students are interested in choosing Language and Science topics based on the membership function and that they are interested in Math and Language based on the indeterminacy result. Additionally, the non-membership function demonstrates that the majority of students detest the combination of the social science and language fields. In the NFMLG context, we are debating how to divide the subjects into three groups to choose the best selection for the students. Students will instinctively select the better option if there are more options available. This multiple option and different subject combination facility is possible while using only NFMLG; this is the main advantage of the neutrosophic fuzzy magic labelling graph.

11. Conclusion

A Neutrosophic network is an extension of an intuitionistic fuzzy network that offers greater precision, compatibility, and flexibility when organising the modelling in many real-world applications than an intuitionistic fuzzy graph. In neutrosophic graph problems, connectivity principles are the main solution strategy. Especially the magic labelling model offers the system higher accuracy, adaptability, and compatibility when compared to classical methods. Hence, in this paper, we propose the definition of the neutrosophic fuzzy magic labelling graph and a detailed discussion of its properties using numerical examples for the path, cycle, and star graphs in a neutrosophic environment. The proposed work has also been used in decision-making situations to choose the optimal subject combinations based on student interests for the best academic success. In order to demonstrate the validity of the proposed work, a comparative analysis with the current methodology has also been conducted. The current research may be in future expanded into a neutrosophic superhypergraph environment.

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