Neutrosophic Sets and Systems, Vol. 60, 2023
University of New Mexico

Interval Valued Pentapartitioned Neutrosophic Set

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Abstract: The notions of interval valued pentapartitioned neutrosophic sets ($IVPNS$s), where the membership values of truth, contradiction, ignorance, unknown, and falsity always fall inside a closed interval [0,1] are introduced in this paper. Also an example of COVID-19 has been discussed using $IVPNS$. Later we have established some basic operations between $IVPNSs$ and useful features of $IVPNS$s have also been presented and discussed.

Keywords: Neutrosophic set, pentapartitioned neutrosophic set, interval neutrosophic set, interval valued pentapartitioned neutrosophic set.

1. Introduction

There are numerous common issues in the disciplines of economics, engineering, environmental research, social science etc in everyday life that can't be solved with classical mathematics. To handle such a circumstance Fuzzy set (FS) [12], rough set (RS) [7] and intuitionistic fuzzy set (IFS) concepts [1] have all been introduced. Traditional FStheory only considers membership values and due to this IFS theory which includes both membership values as well as non-membership values, serves a crucial function in the study of uncertainties. Though the indeterminacy and inconsistent observation that exist in belief systems are not addressed by intuitionistic fuzzy set theory. In order to address this type of indeterminacy, Smarandache developed neutrosophic set (NS) [8] as an addition to IFS theory. Single valued neutrosophic sets were first established by Wang and others [10] in 2010 and this idea is expanded to establish quadripartition single valued neutrosophic sets by Chatterjee et al. [2]. Smarandache [9] classified indeterminacy into three functions in 2013 as the unknown, contradiction and ignorance membership functions and proposed five symbol valued neutrosophic logic using these functions. And he further on extended it to: p types of Truths, $T_1, T_2, \dots, T_p$ and r types of Indeterminacies $I_1, I_2, \dots, I_r$ also s types of falsities $F_1, F_2, \dots, F_s$ where $\mathrm{p}+\mathrm{r}+\mathrm{s} \ = \ \mathrm{n}$ greater than 4, which is the most general form of fuzzy extension of today [9]. Later, adopting this idea, Mallick introduced the pentapartitioned neutrosophic set (PNS) [5], where membership functions of truthiness, disagreement, lack of understanding i.e. ignorance, unknowability, and falsehood were taken into consideration. Das established the concept of single valued pentapartitioned neutrosophic graphs, sub graph, and complete graphs [3] to address graph theoretic challenges including indeterminacy in the form of three distinct elements viz contradictions, ignorances and unknowability. Das et al. has also proposed the Hamming distance in pentapartitioned neutrosophic sets and developed a GRA-based Single valued pentapartitioned neutrosophic sets in MADM method [4]. In a decision-making dilemma, they further validate their findings by choosing a supplier to purchase electronic items for an organization.

In practical scenario to deal with societal uncertainty there are various situations where different membership values belong to some interval. So to overcome from such type of scenario we develop $IVPNS$.

The structure of this article is as follows: Introduction is included in 1st Section, preliminary notions are included in 2nd Section, the concept of $IVPNSs$ is included in 3rd Section, along with certain operations and outcomes and Section 4 concludes and outlines the research's next directions.

2. Preliminaries

Definition 2.1 [8] A NS $\widehat{\pounds}$ on W (the universe), defined as $\widehat{\pounds} = \left\{\left(\hbar, T_{\widehat{\pounds}}(\hbar), I_{\widehat{\pounds}}(\hbar), F_{\widehat{\pounds}}(\hbar)\right): \hbar \in W\right\}$, where $T_{\widehat{\pounds}}, I_{\widehat{\pounds}}, F_{\widehat{\pounds}}: W \to]^-0,1^+[$ satisfying $\forall \hbar \in W, ^-0 \leq T_{\widehat{\pounds}}(\hbar) + I_{\widehat{\pounds}}(\hbar) + F_{\widehat{\pounds}}(\hbar) \leq 3^+$.
Here $T_{\widehat{\pounds}}(\hbar), I_{\widehat{\pounds}}(\hbar)$ and $F_{\widehat{\pounds}}(\hbar)$ represent the truth, indeterminacy and falsity membership values respectively of $\hbar \in W$.

Definition 2.2 [5] A PNS $\widehat{\pounds}$ on the universe W is defined as $\widehat{\pounds} = \left\{\left(\hbar, T_{\widehat{\pounds}}(\hbar), C_{\widehat{\pounds}}(\hbar), G_{\widehat{\pounds}}(\hbar), U_{\widehat{\pounds}}(\hbar), F_{\widehat{\pounds}}(\hbar)\right): \hbar \in W\right\}$, where $T_{\widehat{\pounds}}, C_{\widehat{\pounds}}, G_{\widehat{\pounds}}, U_{\widehat{\pounds}}, F_{\widehat{\pounds}}: W \to [0,1]$ satisfying $\forall \hbar \in W$, $0 \leq T_{\widehat{\pounds}}(\hbar) + C_{\widehat{\pounds}}(\hbar) + G_{\widehat{\pounds}}(\hbar) + U_{\widehat{\pounds}}(\hbar) + F_{\widehat{\pounds}}(\hbar) \leq 5$.
Here $T_{\widehat{\pounds}}(\hbar), C_{\widehat{\pounds}}(\hbar), G_{\widehat{\pounds}}(\hbar), U_{\widehat{\pounds}}(\hbar)$ and $F_{\widehat{\pounds}}(\hbar)$ represent the truthiness, disagreement, lack of understanding i.e. ignorance, unknowability, and falsehood membership values respectively of $\hbar \in W$.

Definition 2.3 [11] An interval neutrosophic set (INS) $\widehat{\pounds}$ on the universe W is defined as $\widehat{\pounds} = \left\{\left(\hbar, T_{\widehat{\pounds}}(\hbar), I_{\widehat{\pounds}}(\hbar), F_{\widehat{\pounds}}(\hbar)\right): \hbar \in W\right\}$, where $T_{\widehat{\pounds}}, I_{\widehat{\pounds}}, F_{\widehat{\pounds}}: W \to]^-0,1^+[$ satisfying $\forall \hbar \in W, ^-0 \leq T_{\widehat{\pounds}}(\hbar) + I_{\widehat{\pounds}}(\hbar) + F_{\widehat{\pounds}}(\hbar) \leq 3^+$.
Here $T_{\widehat{\pounds}}(\hbar), I_{\widehat{\pounds}}(\hbar)$ and $F_{\widehat{\pounds}}(\hbar)$ represent the truthiness, indeterminacy and falsehood membership values respectively of the element $\hbar \in W$.

Definition 2.4 [11] Let $\widehat{\pounds}$, $\widehat{¥}$ are two INSs on W defined by $\widehat{\pounds} = \left\{\left(\hbar, T_{\widehat{\pounds}}(\hbar), I_{\widehat{\pounds}}(\hbar), F_{\widehat{\pounds}}(\hbar)\right): \hbar \in W\right\}$ and $\widehat{¥} = \left\{\left(\hbar, T_{\widehat{¥}}(\hbar), I_{\widehat{¥}}(\hbar), F_{\widehat{¥}}(\hbar)\right): \hbar \in W\right\}$. Then for all $\hbar \in W$

i. $\widehat{\pounds}$ is contained in $\widehat{¥}$ if and only if
$$glbT_{\widehat{\pounds}}(\hbar) \leq glbT_{\widehat{¥}}(\hbar),\ lubT_{\widehat{\pounds}}(\hbar) \leq lubT_{\widehat{¥}}(\hbar),$$
$$glbI_{\widehat{\pounds}}(\hbar) \geq glbI_{\widehat{¥}}(\hbar),\ lubI_{\widehat{\pounds}}(\hbar) \geq lubI_{\widehat{¥}}(\hbar),$$
$$glbF_{\widehat{\pounds}}(\hbar) \geq glbF_{\widehat{¥}}(\hbar),\ lubF_{\widehat{\pounds}}(\hbar) \geq lubF_{\widehat{¥}}(\hbar).$$

ii. The union of $\widehat{\pounds}$ and $\widehat{¥}$ is an INS $\widehat{\varpi}$, defined by
$$\widehat{\varpi} = \widehat{\pounds} \cup \widehat{¥} = \{(\hbar, T_{\widehat{\varpi}}(\hbar), I_{\widehat{\varpi}}(\hbar), F_{\widehat{\varpi}}(\hbar)): \hbar \in W\}$$
where, $glbT_{\widehat{\varpi}}(\hbar) = \vee\left(glbT_{\widehat{\pounds}}(\hbar), glbT_{\widehat{¥}}(\hbar)\right),\ lubT_{\widehat{\varpi}}(\hbar) = \vee\left(lubT_{\widehat{\pounds}}(\hbar), lubT_{\widehat{¥}}(\hbar)\right)$

$$glbI_{\widetilde{\omega}}(\hbar) = \wedge\left(glbI_{\widetilde{£}}(\hbar), glbI_{\widetilde{¥}}(\hbar)\right),\ lubI_{\widetilde{\omega}}(\hbar) = \wedge\left(lubI_{\widetilde{£}}(\hbar), lubI_{\widetilde{¥}}(\hbar)\right)$$

$$glbF_{\widetilde{\omega}}(\hbar) = \wedge\left(glbF_{\widetilde{£}}(\hbar), glbF_{\widetilde{¥}}(\hbar)\right),\ lubF_{\widetilde{\omega}}(\hbar) = \wedge\left(lubF_{\widetilde{£}}(\hbar), lubF_{\widetilde{¥}}(\hbar)\right).$$

iii. The intersection of $\widetilde{£}$ and $\widetilde{¥}$ is an INS $\widetilde{\omega}$, defined by

$$\widetilde{\omega} = \widetilde{£} \cap \widetilde{¥} = \{(\hbar, T_{\widetilde{\omega}}(\hbar), I_{\widetilde{\omega}}(\hbar), F_{\widetilde{\omega}}(\hbar)) : \hbar \in W\}$$

where, $$glbT_{\widetilde{\omega}}(\hbar) = \wedge\left(glbT_{\widetilde{£}}(\hbar), glbT_{\widetilde{¥}}(\hbar)\right),\ lubT_{\widetilde{\omega}}(\hbar) = \wedge\left(lubT_{\widetilde{£}}(\hbar), lubT_{\widetilde{¥}}(\hbar)\right)$$

$$glbI_{\widetilde{\omega}}(\hbar) = \vee\left(glbI_{\widetilde{£}}(\hbar), glbI_{\widetilde{¥}}(\hbar)\right),\ lubI_{\widetilde{\omega}}(\hbar) = \vee\left(lubI_{\widetilde{£}}(\hbar), lubI_{\widetilde{¥}}(\hbar)\right)$$

$$glbF_{\widetilde{\omega}}(\hbar) = \vee\left(glbF_{\widetilde{£}}(\hbar), glbF_{\widetilde{¥}}(\hbar)\right),\ lubF_{\widetilde{\omega}}(\hbar) = \vee\left(lubF_{\widetilde{£}}(\hbar), lubF_{\widetilde{¥}}(\hbar)\right).$$

iv. The complement of $\widetilde{£}$ is $\widetilde{£}^{c}$, defined by $\widetilde{£}^{c} = \left\{\left(\hbar, T_{\widetilde{£}^c}(\hbar), I_{\widetilde{£}^c}(\hbar), F_{\widetilde{£}^c}(\hbar)\right) : \hbar \in W\right\}$ where $T_{\widetilde{£}^c}(\hbar) = F_{\widetilde{£}}(\hbar)$, $I_{\widetilde{£}^c}(\hbar) = [1 - lubI_{\widetilde{£}}(\hbar), 1 - glbI_{\widetilde{£}}(\hbar)]$ and $F_{\widetilde{£}^c}(\hbar) = T_{\widetilde{£}}(\hbar)$.

3. Interval Valued Pentapartitioned Neutrosophic Sets

Here, we provide a novel idea of interval valued pentapartitioned neutrosophic sets and examine some fundamental characteristics.

In Neutrosophic sets there are three characteristic aspects including membership, non membership and indeterminacy whereas in Pentapartitioned Neutrosophic sets, the indeterminacy membership function has been subdivided into three parts: contradictory membership, ignorance membership and unknown membership. However, it has been observed that in issues involving group decision-making, the expert's opinion values differ from individual to individual and as a consequence, it is essential to present the idea of interval valued neutrosophic sets, where each characteristic aspect values are subsets of [0, 1] as opposed to single valued pentapartitioned neutrosophic sets.

Definition 3.1 An interval valued pentapartitioned neutrosophic set ($IVPNS$) $\widetilde{£}$ on the universe W is defined as $\widetilde{£} = \left\{\left(\hbar, T_{\widetilde{£}}(\hbar), C_{\widetilde{£}}(\hbar), G_{\widetilde{£}}(\hbar), U_{\widetilde{£}}(\hbar), F_{\widetilde{£}}(\hbar)\right) : \hbar \in W\right\}$, where $T_{\widetilde{£}}, C_{\widetilde{£}}, G_{\widetilde{£}}, U_{\widetilde{£}}, F_{\widetilde{£}} : W \to Int([0,1])$ satisfying $\forall \hbar \in W$, $0 \leq lubT_{\widetilde{£}}(\hbar) + lubC_{\widetilde{£}}(\hbar) + lubG_{\widetilde{£}}(\hbar) + lubU_{\widetilde{£}}(\hbar) + lubF_{\widetilde{£}}(\hbar) \leq 5$.

Here $T_{\widetilde{£}}(\hbar), C_{\widetilde{£}}(\hbar), G_{\widetilde{£}}(\hbar), U_{\widetilde{£}}$ and $F_{\widetilde{£}}(\hbar)$ represent the truthiness, disagreement, lack of understanding i.e. ignorance, unknowability and falsehood membership values respectively of $\hbar \in W$.

Example 3.2 Consider the statement, "Does humans are immune to COVID-19 infection after vaccination?"

Suppose the statement is given to two groups of peoples for their personal views where each group consists of five peoples, say, $M = \{m_{11}, m_{12}, m_{13}, m_{14}, m_{15}, m_{21}, m_{22}, m_{23}, m_{24}, m_{25}\}$. Now it is obvious that different perspective will be observed regarding the statement with distinct membership value. The possible perspective may be expressed as degrees of "agreement (T)", "both agreement and disagreement (C)", "ignorance (G)", "neither agreement not disagreement (U)", "disagreement (F)".

Now from the group of peoples, suppose m_{i1}, $(i = 1,2)$ make agreement (T) with distinct membership value which may lie in an interval$\in Int([0,1])$. Similarly $m_{i2}, m_{i3}, m_{i4}, m_{i5}$, $(i = 1,2)$ make their perspectives C, G, U, F respectively which may also lie in an interval$\in Int([0,1])$.

Here some fundamental operators are defined in interval valued pentapartitioned neutrosophic sets ($IVPNS$s) which are further utilized to examine various $IVPNS$ features.

Definition 3.3 Let $\widetilde{\widehat{£}}$ and $\widetilde{\widehat{¥}}$ are two $IVPNS$s on W defined by $\widetilde{\widehat{£}} = \left\{\left(\hbar, T_{\widehat{£}}(\hbar), C_{\widehat{£}}(\hbar), G_{\widehat{£}}(\hbar), U_{\widehat{£}}(\hbar), F_{\widehat{£}}(\hbar)\right) : \hbar \in W\right\}$ and $\widetilde{\widehat{¥}} = \left\{\left(\hbar, T_{\widehat{¥}}(\hbar), C_{\widehat{¥}}(\hbar), G_{\widehat{¥}}(\hbar), U_{\widehat{¥}}(\hbar), F_{\widehat{¥}}(\hbar)\right) : \hbar \in W\right\}$. Then for every $\hbar \in W$

i. $\widetilde{\widehat{£}}$ is contained in $\widetilde{\widehat{¥}}$ iff

$$\begin{gathered}
glbT_{\widehat{£}}(\hbar) \leq glbT_{\widehat{¥}}(\hbar), \quad lubT_{\widehat{£}}(\hbar) \leq lubT_{\widehat{¥}}(\hbar),\\
glbC_{\widehat{£}}(\hbar) \leq glbC_{\widehat{¥}}(\hbar), \quad lubC_{\widehat{£}}(\hbar) \leq lubC_{\widehat{¥}}(\hbar),\\
glbG_{\widehat{£}}(\hbar) \geq glbG_{\widehat{¥}}(\hbar), \quad lubG_{\widehat{£}}(\hbar) \geq lubG_{\widehat{¥}}(\hbar),\\
glbU_{\widehat{£}}(\hbar) \geq glbU_{\widehat{¥}}(\hbar), \quad lubU_{\widehat{£}}(\hbar) \geq lubU_{\widehat{¥}}(\hbar),\\
glbF_{\widehat{£}}(\hbar) \geq glbF_{\widehat{¥}}(\hbar), \quad lubF_{\widehat{£}}(\hbar) \geq lubF_{\widehat{¥}}(\hbar).
\end{gathered}$$

ii. The union of $\widetilde{\widehat{£}}$ and $\widetilde{\widehat{¥}}$ is an $IVPNS$ $\widetilde{\widehat{\omega}}$, defined by

$$\widetilde{\widehat{\omega}} = \widetilde{\widehat{£}} \cup \widetilde{\widehat{¥}} = \{(\hbar, T_{\widehat{\omega}}(\hbar), C_{\widehat{\omega}}(\hbar), G_{\widehat{\omega}}(\hbar), U_{\widehat{\omega}}(\hbar), F_{\widehat{\omega}}(\hbar)) : \hbar \in W\}$$

where, $glbT_{\widehat{\omega}}(\hbar) = \vee\left(glbT_{\widehat{£}}(\hbar), glbT_{\widehat{¥}}(\hbar)\right)$, $lubT_{\widehat{\omega}}(\hbar) = \vee\left(lubT_{\widehat{£}}(\hbar), lubT_{\widehat{¥}}(\hbar)\right)$

$glbC_{\widehat{\omega}}(\hbar) = \vee\left(glbC_{\widehat{£}}(\hbar), glbC_{\widehat{¥}}(\hbar)\right)$, $lubC_{\widehat{\omega}}(\hbar) = \vee\left(lubC_{\widehat{£}}(\hbar), lubC_{\widehat{¥}}(\hbar)\right)$

$glbG_{\widehat{\omega}}(\hbar) = \wedge\left(glbG_{\widehat{£}}(\hbar), glbG_{\widehat{¥}}(\hbar)\right)$, $lubG_{\widehat{\omega}}(\hbar) = \wedge\left(lubG_{\widehat{£}}(\hbar), lubG_{\widehat{¥}}(\hbar)\right)$

$glbU_{\widehat{\omega}}(\hbar) = \wedge\left(glbU_{\widehat{£}}(\hbar), glbU_{\widehat{¥}}(\hbar)\right)$, $lubU_{\widehat{\omega}}(\hbar) = \wedge\left(lubU_{\widehat{£}}(\hbar), lubU_{\widehat{¥}}(\hbar)\right)$

$glbF_{\widehat{\omega}}(\hbar) = \wedge\left(glbF_{\widehat{£}}(\hbar), glbF_{\widehat{¥}}(\hbar)\right)$, $lubF_{\widehat{\omega}}(\hbar) = \wedge\left(lubF_{\widehat{£}}(\hbar), lubF_{\widehat{¥}}(\hbar)\right)$

or simply we can write

$$\begin{aligned}
&\widetilde{\widehat{£}} \cup \widetilde{\widehat{¥}}\\
&= \Big\{\hbar, \Big[\vee\left(glbT_{\widehat{£}}(\hbar), glbT_{\widehat{¥}}(\hbar)\right), \vee\left(lubT_{\widehat{£}}(\hbar), lubT_{\widehat{¥}}(\hbar)\right)\Big], \Big[\vee\left(glbC_{\widehat{£}}(\hbar), glbC_{\widehat{¥}}(\hbar)\right),\\
&\vee\left(lubC_{\widehat{£}}(\hbar), lubC_{\widehat{¥}}(\hbar)\right)\Big], \Big[\wedge\left(glbG_{\widehat{£}}(\hbar), glbG_{\widehat{¥}}(\hbar)\right),\\
&\wedge\left(lubG_{\widehat{£}}(\hbar), lubG_{\widehat{¥}}(\hbar)\right)\Big], \Big[\wedge\left(glbU_{\widehat{£}}(\hbar), glbU_{\widehat{¥}}(\hbar)\right),\\
&\wedge\left(lubU_{\widehat{£}}(\hbar), lubU_{\widehat{¥}}(\hbar)\right)\Big], \Big[\wedge\left(glbF_{\widehat{£}}(\hbar), glbF_{\widehat{¥}}(\hbar)\right), \wedge\left(lubF_{\widehat{£}}(\hbar), lubF_{\widehat{¥}}(\hbar)\right)\Big]\Big\}
\end{aligned}$$

iii. The intersection is an $IVPNS$ $\widetilde{\widehat{\omega}}$, defined by

$$\widetilde{\widehat{\omega}} = \widetilde{\widehat{£}} \cap \widetilde{\widehat{¥}} = \{(\hbar, T_{\widehat{\omega}}(\hbar), C_{\widehat{\omega}}(\hbar), G_{\widehat{\omega}}(\hbar), U_{\widehat{\omega}}(\hbar), F_{\widehat{\omega}}(\hbar)) : \hbar \in W\}$$

where, $glbT_{\widehat{\omega}}(\hbar) = \wedge\left(glbT_{\widehat{£}}(\hbar), glbT_{\widehat{¥}}(\hbar)\right)$, $lubT_{\widehat{\omega}}(\hbar) = \wedge\left(lubT_{\widehat{£}}(\hbar), lubT_{\widehat{¥}}(\hbar)\right)$

$$glbC_{\widehat{\varpi}}(\hbar) = \wedge\left(glbC_{\widehat{\text{£}}}(\hbar), glbC_{\widehat{\text{¥}}}(\hbar)\right),\ lubC_{\widehat{\varpi}}(\hbar) = \wedge\left(lubC_{\widehat{\text{£}}}(\hbar), lubC_{\widehat{\text{¥}}}(\hbar)\right)$$

$$glbG_{\widehat{\varpi}}(\hbar) = \vee\left(glbG_{\widehat{\text{£}}}(\hbar), glbG_{\widehat{\text{¥}}}(\hbar)\right),\ lubG_{\widehat{\varpi}}(\hbar) = \vee\left(lubG_{\widehat{\text{£}}}(\hbar), lubG_{\widehat{\text{¥}}}(\hbar)\right)$$

$$glbU_{\widehat{\varpi}}(\hbar) = \vee\left(glbU_{\widehat{\text{£}}}(\hbar), glbU_{\widehat{\text{¥}}}(\hbar)\right),\ lubU_{\widehat{\varpi}}(\hbar) = \vee\left(lubU_{\widehat{\text{£}}}(\hbar), lubU_{\widehat{\text{¥}}}(\hbar)\right)$$

$$glbF_{\widehat{\varpi}}(\hbar) = \vee\left(glbF_{\widehat{\text{£}}}(\hbar), glbF_{\widehat{\text{¥}}}(\hbar)\right),\ lubF_{\widehat{\varpi}}(\hbar) = \vee\left(lubF_{\widehat{\text{£}}}(\hbar), lubF_{\widehat{\text{¥}}}(\hbar)\right)$$

or simply we can write $\widehat{\text{£}} \cap \widehat{\text{¥}} = \left\{\hbar, \left[\wedge\left(glbT_{\widehat{\text{£}}}(\hbar), glbT_{\widehat{\text{¥}}}(\hbar)\right), \wedge\left(lubT_{\widehat{\text{£}}}(\hbar), lubT_{\widehat{\text{¥}}}(\hbar)\right)\right], \left[\wedge\left(glbC_{\widehat{\text{£}}}(\hbar), glbC_{\widehat{\text{¥}}}(\hbar)\right), \wedge\left(lubC_{\widehat{\text{£}}}(\hbar), lubC_{\widehat{\text{¥}}}(\hbar)\right)\right], \left[\vee\left(glbG_{\widehat{\text{£}}}(\hbar), glbG_{\widehat{\text{¥}}}(\hbar)\right), \vee\left(lubG_{\widehat{\text{£}}}(\hbar), lubG_{\widehat{\text{¥}}}(\hbar)\right)\right], \left[\vee\left(glbU_{\widehat{\text{£}}}(\hbar), glbU_{\widehat{\text{¥}}}(\hbar)\right), \vee\left(lubU_{\widehat{\text{£}}}(\hbar), lubU_{\widehat{\text{¥}}}(\hbar)\right)\right], \left[\vee\left(glbF_{\widehat{\text{£}}}(\hbar), glbF_{\widehat{\text{¥}}}(\hbar)\right), \vee\left(lubF_{\widehat{\text{£}}}(\hbar), lubF_{\widehat{\text{¥}}}(\hbar)\right)\right]\right\}$

iv. The complement of $\widehat{\text{£}}$ is $\widehat{\text{£}}^{c}$, defined by $\widehat{\text{£}}^{c} = \left\{\left(\hbar, T_{\widehat{\text{£}}^c}(\hbar), C_{\widehat{\text{£}}^c}(\hbar), G_{\widehat{\text{£}}^c}(\hbar), U_{\widehat{\text{£}}^c}(\hbar), F_{\widehat{\text{£}}^c}(\hbar)\right) : \hbar \in W\right\}$ where $T_{\widehat{\text{£}}^c}(\hbar) = F_{\widehat{\text{£}}}(\hbar)$, $C_{\widehat{\text{£}}^c}(\hbar) = U_{\widehat{\text{£}}}(\hbar)$ $G_{\widehat{\text{£}}^c}(\hbar) = [1 - lubG_{\widehat{\text{£}}}(\hbar), 1 - glbG_{\widehat{\text{£}}}(\hbar)]$, $U_{\widehat{\text{£}}^c}(\hbar) = C_{\widehat{\text{£}}}(\hbar)$ and $F_{\widehat{\text{£}}^c}(\hbar) = T_{\widehat{\text{£}}}(\hbar)$.

or simply we can write $\widehat{\text{£}}^{c} = \left\{\left(\hbar, F_{\widehat{\text{£}}}(\hbar), U_{\widehat{\text{£}}}(\hbar), [1 - lubG_{\widehat{\text{£}}}(\hbar), 1 - glbG_{\widehat{\text{£}}}(\hbar)], C_{\widehat{\text{£}}}(\hbar), T_{\widehat{\text{£}}}(\hbar)\right) : \hbar \in W\right\}$.

Example 3.4 Consider two $IVPNS$s $\widehat{\text{£}}$ and $\widehat{\text{¥}}$ defined over W as

$\widehat{\text{£}} = \{(\hbar_1, [0.32,0.54], [0.23,0.65], [0.56,0.79], [0.32,0.43], [0.85,0.96]), (\hbar_2, [0.67,0.78], [0.55,0.78], [0.11,0.32], [0.23,0.84], [0.15,0.38]), [(\hbar_3, [0.24,0.56], [0.17,0.52], [0.25,0.75], [0.21,0.63], [0.31,0.56])\}$

$\widehat{\text{¥}} = \{(\hbar_1, [0.57,0.91], [0.52,0.83], [0.57,0.78], [0.23,0.39], [0.61,0.84]), (\hbar_2, [0.52,0.71], [0.24,0.56], [0.20,0.52], [0.75,0.80], [0.41,0.62]), [(\hbar_3, [0.12,0.31], [0.38,0.56], [0.55,0.74], [0.19,0.86], [0.16,0.83])\}$

Then

$\widehat{\text{£}} \cup \widehat{\text{¥}} = \{(\hbar_1, [0.57,0.91], [0.52,0.83], [0.56,0.78], [0.23,0.39], [0.61,0.84]), (\hbar_2, [0.67,0.78], [0.55,0.78], [0.11,0.32], [0.23,0.80], [0.15,0.38]), [(\hbar_3, [0.24,0.56], [0.38,0.56], [0.25,0.74], [0.19,0.63], [0.16,0.56])\}$

$\widehat{\text{£}} \cap \widehat{\text{¥}} = \{(\hbar_1, [0.32,0.54], [0.23,0.65], [0.57,0.79], [0.32,0.43], [0.85,0.96]), (\hbar_2, [0.52,0.71], [0.24,0.56], [0.20,0.52], [0.75,0.84], [0.41,0.62]), [(\hbar_3, [0.24,0.56], [0.38,0.56], [0.25,0.74], [0.19,0.63], [0.16,0.56])\}$

$\widehat{\text{£}}^{c} = \{(\hbar_1, [0.85,0.96], [0.32,0.43], [0.21,0.34], [0.23,0.65], [0.32,0.54]), (\hbar_2, [0.15,0.38], [0.23,0.84], [0.68,0.91], [0.55,0.78], [0.67,0.78]), [(\hbar_3, [0.31,0.56], [0.21,0.63], [0.25,0.75], [0.17,0.52], [0.24,0.56])\}$

Theorem 3.5 For any three $IVPNS$s $\widehat{\text{£}}$, $\widehat{\text{¥}}$ and $\widehat{\varpi}$

i. $\widehat{\text{£}} \cup \widehat{\text{£}} = \widehat{\text{£}}$, $\widehat{\text{£}} \cap \widehat{\text{£}} = \widehat{\text{£}}$ (Idempotent Law)

ii. $\widehat{\text{£}} \cup \widehat{\text{¥}} = \widehat{\text{¥}} \cup \widehat{\text{£}}$, $\widehat{\text{£}} \cap \widehat{\text{¥}} = \widehat{\text{¥}} \cap \widehat{\text{£}}$ (Commutative Law)

iii. $\left(\widehat{\text{£}} \cup \widehat{\text{¥}}\right) \cup \widehat{\varpi} = \widehat{\text{£}} \cup \left(\widehat{\text{¥}} \cup \widehat{\varpi}\right)$, $\left(\widehat{\text{£}} \cap \widehat{\text{¥}}\right) \cap \widehat{\varpi} = \widehat{\text{£}} \cap \left(\widehat{\text{¥}} \cap \widehat{\varpi}\right)$ (Associative Law)

iv. $\widehat{\text{£}} \cup \left(\widehat{\text{¥}} \cap \widehat{\varpi}\right) = \left(\widehat{\text{£}} \cup \widehat{\text{¥}}\right) \cap \left(\widehat{\text{£}} \cup \widehat{\varpi}\right)$, $\widehat{\text{£}} \cap \left(\widehat{\text{¥}} \cup \widehat{\varpi}\right) = \left(\widehat{\text{£}} \cap \widehat{\text{¥}}\right) \cup \left(\widehat{\text{£}} \cap \widehat{\varpi}\right)$ (Distributive Law)

v. $\left(\widehat{\pounds} \cup \widehat{\yen}\right)^c = \widehat{\pounds}^c \cap \widehat{\yen}^c$, $\left(\widehat{\pounds} \cap \widehat{\yen}\right)^c = \widehat{\pounds}^c \cup \widehat{\yen}^c$ (De Morgan's Law)

vi. $\widehat{\pounds} \cup \left(\widehat{\pounds} \cap \widehat{\yen}\right) = \widehat{\pounds}$, $\widehat{\pounds} \cap \left(\widehat{\pounds} \cup \widehat{\yen}\right) = \widehat{\pounds}$ (Absorption Law)

vii. $\left(\widehat{\pounds}^c\right)^c = \widehat{\pounds}$ (Involution Law)

Proof: Let $\widehat{\pounds}$, $\widehat{\yen}$ and $\widehat{\omega}$ are two $IVPNS$s on W defined by

$\widehat{\pounds} = \left\{\left(\hbar, T_{\widehat{\pounds}}(\hbar), C_{\widehat{\pounds}}(\hbar), G_{\widehat{\pounds}}(\hbar), U_{\widehat{\pounds}}(\hbar), F_{\widehat{\pounds}}(\hbar)\right) : \hbar \in W\right\}$, $\widehat{\yen} = \left\{\left(\hbar, T_{\widehat{\yen}}(\hbar), C_{\widehat{\yen}}(\hbar), G_{\widehat{\yen}}(\hbar), U_{\widehat{\yen}}(\hbar), F_{\widehat{\yen}}(\hbar)\right) : \hbar \in W\right\}$ and $\widehat{\omega} = \{(\hbar, T_{\widehat{\omega}}(\hbar), C_{\widehat{\omega}}(\hbar), G_{\widehat{\omega}}(\hbar), U_{\widehat{\omega}}(\hbar), F_{\widehat{\omega}}(\hbar)) : \hbar \in W\}$ respectively. Then for every $\hbar \in W$

(i) Straight forward.

(ii) We know that,

$$\begin{aligned}
\widehat{\pounds} \cup \widehat{\yen} = \Big\{\hbar, &\left[\vee\left(glbT_{\widehat{\pounds}}(\hbar), glbT_{\widehat{\yen}}(\hbar)\right), \vee\left(lubT_{\widehat{\pounds}}(\hbar), lubT_{\widehat{\yen}}(\hbar)\right)\right],\\
&\left[\vee\left(glbC_{\widehat{\pounds}}(\hbar), glbC_{\widehat{\yen}}(\hbar)\right), \vee\left(lubC_{\widehat{\pounds}}(\hbar), lubC_{\widehat{\yen}}(\hbar)\right)\right],\\
&\left[\wedge\left(glbG_{\widehat{\pounds}}(\hbar), glbG_{\widehat{\yen}}(\hbar)\right), \wedge\left(lubG_{\widehat{\pounds}}(\hbar), lubG_{\widehat{\yen}}(\hbar)\right)\right],\\
&\left[\wedge\left(glbU_{\widehat{\pounds}}(\hbar), glbU_{\widehat{\yen}}(\hbar)\right), \wedge\left(lubU_{\widehat{\pounds}}(\hbar), lubU_{\widehat{\yen}}(\hbar)\right)\right],\\
&\left[\wedge\left(glbF_{\widehat{\pounds}}(\hbar), glbF_{\widehat{\yen}}(\hbar)\right), \wedge\left(lubF_{\widehat{\pounds}}(\hbar), lubF_{\widehat{\yen}}(\hbar)\right)\right] : \hbar \in W\Big\}\\
= \Big\{\hbar, &\left[\vee\left(glbT_{\widehat{\yen}}(\hbar), glbT_{\widehat{\pounds}}(\hbar)\right), \vee\left(lubT_{\widehat{\yen}}(\hbar), lubT_{\widehat{\pounds}}(\hbar)\right)\right],\\
&\left[\vee\left(glbC_{\widehat{\yen}}(\hbar), glbC_{\widehat{\pounds}}(\hbar)\right), \vee\left(lubC_{\widehat{\yen}}(\hbar), lubC_{\widehat{\pounds}}(\hbar)\right)\right],\\
&\left[\wedge\left(glbG_{\widehat{\yen}}(\hbar), glbG_{\widehat{\pounds}}(\hbar)\right), \wedge\left(lubG_{\widehat{\yen}}(\hbar), lubG_{\widehat{\pounds}}(\hbar)\right)\right],\\
&\left[\wedge\left(glbU_{\widehat{\yen}}(\hbar), glbU_{\widehat{\pounds}}(\hbar)\right), \wedge\left(lubU_{\widehat{\yen}}(\hbar), lubU_{\widehat{\pounds}}(\hbar)\right)\right],\\
&\left[\wedge\left(glbF_{\widehat{\yen}}(\hbar), glbF_{\widehat{\pounds}}(\hbar)\right), \wedge\left(lubF_{\widehat{\yen}}(\hbar), lubF_{\widehat{\pounds}}(\hbar)\right)\right] : \hbar \in W\Big\}\\
= \widehat{\yen} \cup \widehat{\pounds}&
\end{aligned}$$

$$\therefore \widehat{\pounds} \cup \widehat{\yen} = \widehat{\yen} \cup \widehat{\pounds}$$

Similarly, $\widehat{\pounds} \cap \widehat{\yen} = \widehat{\yen} \cap \widehat{\pounds}$.

(iii) We know that,

$$\left(\widetilde{\widehat{\text{£}}} \cup \widetilde{\widehat{\text{¥}}}\right) \cup \widetilde{\widehat{\omega}} = \Big\{\hbar, \Big[\vee\left(glbT_{\widehat{\text{£}}}(\hbar), glbT_{\widehat{\text{¥}}}(\hbar)\right), \vee\left(lubT_{\widehat{\text{£}}}(\hbar), lubT_{\widehat{\text{¥}}}(\hbar)\right)\Big],$$

$$\Big[\vee\left(glbC_{\widehat{\text{£}}}(\hbar), glbC_{\widehat{\text{¥}}}(\hbar)\right),$$

$$\vee\left(lubC_{\widehat{\text{£}}}(\hbar), lubC_{\widehat{\text{¥}}}(\hbar)\right)\Big],$$

$$\Big[\wedge\left(glbG_{\widehat{\text{£}}}(\hbar), glbG_{\widehat{\text{¥}}}(\hbar)\right),$$

$$\wedge\left(lubG_{\widehat{\text{£}}}(\hbar), lubG_{\widehat{\text{¥}}}(\hbar)\right)\Big],$$

$$\Big[\wedge\left(glbU_{\widehat{\text{£}}}(\hbar), glbU_{\widehat{\text{¥}}}(\hbar)\right),$$

$$\wedge\left(lubU_{\widehat{\text{£}}}(\hbar), lubU_{\widehat{\text{¥}}}(\hbar)\right)\Big],$$

$$\Big[\wedge\left(glbF_{\widehat{\text{£}}}(\hbar), glbF_{\widehat{\text{¥}}}(\hbar)\right),$$

$$\wedge\left(lubF_{\widehat{\text{£}}}(\hbar), lubF_{\widehat{\text{¥}}}(\hbar)\right)\Big] : \hbar \in W\Big\} \cup \widetilde{\widehat{\omega}}$$

$$= \Big\{\hbar, \Big[\vee\left(glbT_{\widehat{\text{£}}}(\hbar), glbT_{\widehat{\text{¥}}}(\hbar), glbT_{\widehat{\omega}}(\hbar)\right), \vee\left(lubT_{\widehat{\text{£}}}(\hbar), lubT_{\widehat{\text{¥}}}(\hbar), lubT_{\widehat{\omega}}(\hbar)\right)\Big],$$

$$\Big[\vee\left(glbC_{\widehat{\text{£}}}(\hbar), glbC_{\widehat{\text{¥}}}(\hbar), glbC_{\widehat{\omega}}(\hbar)\right),$$

$$\vee\left(lubC_{\widehat{\text{£}}}(\hbar), lubC_{\widehat{\text{¥}}}(\hbar), lubC_{\widehat{\omega}}(\hbar)\right)\Big],$$

$$\Big[\wedge\left(glbG_{\widehat{\text{£}}}(\hbar), glbG_{\widehat{\text{¥}}}(\hbar), glbG_{\widehat{\omega}}(\hbar)\right),$$

$$\wedge\left(lubG_{\widehat{\text{£}}}(\hbar), lubG_{\widehat{\text{¥}}}(\hbar), lubG_{\widehat{\omega}}(\hbar)\right)\Big],$$

$$\Big[\wedge\left(glbU_{\widehat{\text{£}}}(\hbar), glbU_{\widehat{\text{¥}}}(\hbar), glbU_{\widehat{\omega}}(\hbar)\right),$$

$$\wedge\left(lubU_{\widehat{\text{£}}}(\hbar), lubU_{\widehat{\text{¥}}}(\hbar), lubU_{\widehat{\omega}}(\hbar)\right)\Big],$$

$$\Big[\wedge\left(glbF_{\widehat{\text{£}}}(\hbar), glbF_{\widehat{\text{¥}}}(\hbar), glbF_{\widehat{\omega}}(\hbar)\right),$$

$$\wedge\left(lubF_{\widehat{\text{£}}}(\hbar), lubF_{\widehat{\text{¥}}}(\hbar), lubF_{\widehat{\omega}}(\hbar)\right)\Big] : \hbar \in W\Big\}$$

$$= \widetilde{\widehat{\text{£}}} \cup \Big\{\hbar, \Big[\vee\left(glbT_{\widehat{\text{¥}}}(\hbar), glbT_{\widehat{\omega}}(\hbar)\right), \vee\left(lubT_{\widehat{\text{¥}}}(\hbar), lubT_{\widehat{\omega}}(\hbar)\right)\Big],$$

$$\Big[\vee\left(glbC_{\widehat{\text{¥}}}(\hbar), glbC_{\widehat{\omega}}(\hbar)\right), \vee\left(lubC_{\widehat{\text{¥}}}(\hbar), lubC_{\widehat{\omega}}(\hbar)\right)\Big],$$

$$\Big[\wedge\left(glbG_{\widehat{\text{¥}}}(\hbar), glbG_{\widehat{\omega}}(\hbar)\right), \wedge\left(lubG_{\widehat{\text{¥}}}(\hbar), lubG_{\widehat{\omega}}(\hbar)\right)\Big],$$

$$\left[\wedge\left(glbU_{\widetilde{\yen}}(\hbar), glbU_{\widetilde{\omega}}(\hbar)\right), \wedge\left(lubU_{\widetilde{\yen}}(\hbar), lubU_{\widetilde{\omega}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbF_{\widetilde{\yen}}(\hbar), glbF_{\widetilde{\omega}}(\hbar)\right), \wedge\left(lubF_{\widetilde{\yen}}(\hbar), lubF_{\widetilde{\omega}}(\hbar)\right)\right] : \hbar \in W\Big\}$$

$$= \widetilde{\pounds} \cup \left(\widetilde{\yen} \cup \widetilde{\omega}\right)$$

$$\therefore \left(\widetilde{\pounds} \cup \widetilde{\yen}\right) \cup \widetilde{\omega} = \widetilde{\pounds} \cup \left(\widetilde{\yen} \cup \widetilde{\omega}\right)$$

Similarly, $\left(\widetilde{\pounds} \cap \widetilde{\yen}\right) \cap \widetilde{\omega} = \widetilde{\pounds} \cap \left(\widetilde{\yen} \cap \widetilde{\omega}\right)$.

(iv) We know that,

$$\widetilde{\pounds} \cup \left(\widetilde{\yen} \cap \widetilde{\omega}\right)$$

$$= \widetilde{\pounds} \cup \Big\{\hbar, \left[\wedge\left(glbT_{\widetilde{\yen}}(\hbar), glbT_{\widetilde{\omega}}(\hbar)\right), \wedge\left(lubT_{\widetilde{\yen}}(\hbar), lubT_{\widetilde{\omega}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbC_{\widetilde{\yen}}(\hbar), glbC_{\widetilde{\omega}}(\hbar)\right), \wedge\left(lubC_{\widetilde{\yen}}(\hbar), lubC_{\widetilde{\omega}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbG_{\widetilde{\yen}}(\hbar), glbG_{\widetilde{\omega}}(\hbar)\right), \vee\left(lubG_{\widetilde{\yen}}(\hbar), lubG_{\widetilde{\omega}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbU_{\widetilde{\yen}}(\hbar), glbU_{\widetilde{\omega}}(\hbar)\right), \vee\left(lubU_{\widetilde{\yen}}(\hbar), lubU_{\widetilde{\omega}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbF_{\widetilde{\yen}}(\hbar), glbF_{\widetilde{\omega}}(\hbar)\right), \vee\left(lubF_{\widetilde{\yen}}(\hbar), lubF_{\widetilde{\omega}}(\hbar)\right)\right] : \hbar \in W\Big\}$$

$$= \Big\{\hbar, \left[\vee\left(glbT_{\widetilde{\pounds}}(\hbar), \wedge\left(glbT_{\widetilde{\yen}}(\hbar), glbT_{\widetilde{\omega}}(\hbar)\right)\right), \vee\left(lubT_{\widetilde{\pounds}}(\hbar), \wedge\left(lubT_{\widetilde{\yen}}(\hbar), lubT_{\widetilde{\omega}}(\hbar)\right)\right)\right],$$

$$\left[\vee\left(glbC_{\widetilde{\pounds}}(\hbar), \wedge\left(glbC_{\widetilde{\yen}}(\hbar), glbC_{\widetilde{\omega}}(\hbar)\right)\right), \vee\left(lubC_{\widetilde{\pounds}}(\hbar), \wedge\left(lubC_{\widetilde{\yen}}(\hbar), lubC_{\widetilde{\omega}}(\hbar)\right)\right)\right],$$

$$\left[\wedge\left(glbG_{\widetilde{\pounds}}(\hbar), \vee\left(glbG_{\widetilde{\yen}}(\hbar), glbG_{\widetilde{\omega}}(\hbar)\right)\right), \wedge\left(lubG_{\widetilde{\pounds}}(\hbar), \vee\left(lubG_{\widetilde{\yen}}(\hbar), lubG_{\widetilde{\omega}}(\hbar)\right)\right)\right],$$

$$\left[\wedge\left(glbU_{\widetilde{\pounds}}(\hbar), \vee\left(glbU_{\widetilde{\yen}}(\hbar), glbU_{\widetilde{\omega}}(\hbar)\right)\right), \wedge\left(lubU_{\widetilde{\pounds}}(\hbar), \vee\left(lubU_{\widetilde{\yen}}(\hbar), lubU_{\widetilde{\omega}}(\hbar)\right)\right)\right],$$

$$\left[\wedge\left(glbF_{\widetilde{£}}(\hbar),\vee\left(glbF_{\widetilde{¥}}(\hbar),glbF_{\widetilde{\varpi}}(\hbar)\right)\right),\wedge\left(lubF_{\widetilde{£}}(\hbar),\vee\left(lubF_{\widetilde{¥}}(\hbar),lubF_{\widetilde{\varpi}}(\hbar)\right)\right)\right]:\hbar \in W\Big\}$$

$$=\Big\{\hbar,\left[\vee\left(glbT_{\widetilde{£}}(\hbar),glbT_{\widetilde{¥}}(\hbar)\right),\vee\left(lubT_{\widetilde{£}}(\hbar),lubT_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbC_{\widetilde{£}}(\hbar),glbC_{\widetilde{¥}}(\hbar)\right),\vee\left(lubC_{\widetilde{£}}(\hbar),lubC_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbG_{\widetilde{£}}(\hbar),glbG_{\widetilde{¥}}(\hbar)\right),\wedge\left(lubG_{\widetilde{£}}(\hbar),\vee\, lubG_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbU_{\widetilde{£}}(\hbar),glbU_{\widetilde{¥}}(\hbar)\right),\wedge\left(lubU_{\widetilde{£}}(\hbar),lubU_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbF_{\widetilde{£}}(\hbar),glbF_{\widetilde{¥}}(\hbar)\right),\wedge\left(lubF_{\widetilde{£}}(\hbar),lubF_{\widetilde{¥}}(\hbar)\right)\right]:\hbar \in W\Big\}$$

$$\cap\Big\{\hbar,\left[\vee\left(glbT_{\widetilde{£}}(\hbar),glbT_{\widetilde{\varpi}}(\hbar)\right),\vee\left(lubT_{\widetilde{£}}(\hbar),lubT_{\widetilde{\varpi}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbC_{\widetilde{£}}(\hbar),glbC_{\widetilde{\varpi}}(\hbar)\right),\vee\left(lubC_{\widetilde{£}}(\hbar),lubC_{\widetilde{\varpi}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbG_{\widetilde{£}}(\hbar),glbG_{\widetilde{\varpi}}(\hbar)\right),\wedge\left(lubG_{\widetilde{£}}(\hbar),\vee\, lubG_{\widetilde{\varpi}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbU_{\widetilde{£}}(\hbar),glbU_{\widetilde{\varpi}}(\hbar)\right),\wedge\left(lubU_{\widetilde{£}}(\hbar),lubU_{\widetilde{\varpi}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbF_{\widetilde{£}}(\hbar),glbF_{\widetilde{\varpi}}(\hbar)\right),\wedge\left(lubF_{\widetilde{£}}(\hbar),lubF_{\widetilde{\varpi}}(\hbar)\right)\right]:\hbar \in W\Big\}$$

$$=\left(\widetilde{£}\cup\widetilde{¥}\right)\cap\left(\widetilde{£}\cup\widetilde{\varpi}\right)$$

$$\therefore \widetilde{£}\cup\left(\widetilde{¥}\cap\widetilde{\varpi}\right)=\left(\widetilde{£}\cup\widetilde{¥}\right)\cap\left(\widetilde{£}\cup\widetilde{\varpi}\right)$$

Similarly, $\widetilde{£}\cap\left(\widetilde{¥}\cup\widetilde{\varpi}\right)=\left(\widetilde{£}\cap\widetilde{¥}\right)\cup\left(\widetilde{£}\cap\widetilde{\varpi}\right)$.

(v) We know that,

$$\left(\widetilde{£}\cup\widetilde{¥}\right)^{c}=\Big\{\hbar,\left[\vee\left(glbT_{\widetilde{£}}(\hbar),glbT_{\widetilde{¥}}(\hbar)\right),\vee\left(lubT_{\widetilde{£}}(\hbar),lubT_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbC_{\widetilde{£}}(\hbar),glbC_{\widetilde{¥}}(\hbar)\right),\vee\left(lubC_{\widetilde{£}}(\hbar),lubC_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbG_{\widetilde{£}}(\hbar),glbG_{\widetilde{¥}}(\hbar)\right),\wedge\left(lubG_{\widetilde{£}}(\hbar),lubG_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbU_{\widetilde{£}}(\hbar),glbU_{\widetilde{¥}}(\hbar)\right),\wedge\left(lubU_{\widetilde{£}}(\hbar),lubU_{\widetilde{¥}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbF_{\widetilde{£}}(\hbar),glbF_{\widetilde{¥}}(\hbar)\right),\wedge\left(lubF_{\widetilde{£}}(\hbar),lubF_{\widetilde{¥}}(\hbar)\right)\right]:\hbar \in W\Big\}^{c}$$

$$= \Big\{\hbar, \Big[\wedge\Big(glbF_{\widetilde{\pounds}}(\hbar), glbF_{\widetilde{\yen}}(\hbar)\Big), \wedge\Big(lubF_{\widetilde{\pounds}}(\hbar), lubF_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[\wedge\Big(glbU_{\widetilde{\pounds}}(\hbar), glbU_{\widetilde{\yen}}(\hbar)\Big), \wedge\Big(lubU_{\widetilde{\pounds}}(\hbar), lubU_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[1 - \wedge\Big(lubG_{\widetilde{\pounds}}(\hbar), lubG_{\widetilde{\yen}}(\hbar)\Big), 1 - \wedge\Big(glbG_{\widetilde{\pounds}}(\hbar), glbG_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[\vee\Big(glbC_{\widetilde{\pounds}}(\hbar), glbC_{\widetilde{\yen}}(\hbar)\Big), \vee\Big(lubC_{\widetilde{\pounds}}(\hbar), lubC_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[\vee\Big(glbT_{\widetilde{\pounds}}(\hbar), glbT_{\widetilde{\yen}}(\hbar)\Big), \vee\Big(lubT_{\widetilde{\pounds}}(\hbar), lubT_{\widetilde{\yen}}(\hbar)\Big)\Big] : \hbar \in W\Big\}$$

$$= \Big\{\hbar, \Big[\wedge\Big(glbF_{\widetilde{\pounds}}(\hbar), glbF_{\widetilde{\yen}}(\hbar)\Big), \wedge\Big(lubF_{\widetilde{\pounds}}(\hbar), lubF_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[\wedge\Big(glbU_{\widetilde{\pounds}}(\hbar), glbU_{\widetilde{\yen}}(\hbar)\Big), \wedge\Big(lubU_{\widetilde{\pounds}}(\hbar), lubU_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[\vee\Big(1 - lubG_{\widetilde{\pounds}}(\hbar), 1 - lubG_{\widetilde{\yen}}(\hbar)\Big), \vee\Big(1 - glbG_{\widetilde{\pounds}}(\hbar), 1 - glbG_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[\vee\Big(glbC_{\widetilde{\pounds}}(\hbar), glbC_{\widetilde{\yen}}(\hbar)\Big), \vee\Big(lubC_{\widetilde{\pounds}}(\hbar), lubC_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\Big[\vee\Big(glbT_{\widetilde{\pounds}}(\hbar), glbT_{\widetilde{\yen}}(\hbar)\Big), \vee\Big(lubT_{\widetilde{\pounds}}(\hbar), lubT_{\widetilde{\yen}}(\hbar)\Big)\Big] : \hbar \in W\Big\}$$

$$= \Big\{\Big(\hbar, F_{\widetilde{\pounds}}(\hbar), U_{\widetilde{\pounds}}(\hbar), [1 - lubG_{\widetilde{\pounds}}(\hbar), 1 - glbG_{\widetilde{\pounds}}(\hbar)], C_{\widetilde{\pounds}}(\hbar), T_{\widetilde{\pounds}}(\hbar)\Big) : \hbar \in W\Big\}$$

$$\cap \Big\{\Big(\hbar, F_{\widetilde{\yen}}(\hbar), U_{\widetilde{\yen}}(\hbar), [1 - lubG_{\widetilde{\yen}}(\hbar), 1 - glbG_{\widetilde{\yen}}(\hbar)], C_{\widetilde{\yen}}(\hbar), T_{\widetilde{\yen}}(\hbar)\Big) : \hbar \in W\Big\}$$

$$= \widetilde{\pounds}^{c} \cap \widetilde{\yen}^{c}$$

$$\therefore \Big(\widetilde{\pounds} \cup \widetilde{\yen}\Big)^{c} = \widetilde{\pounds}^{c} \cap \widetilde{\yen}^{c}$$

Similarly, $\Big(\widetilde{\pounds} \cap \widetilde{\yen}\Big)^{c} = \widetilde{\pounds}^{c} \cup \widetilde{\yen}^{c}$.

(vi) We know that

$$\widetilde{\pounds} \cup \Big(\widetilde{\pounds} \cap \widetilde{\yen}\Big)$$

$$= \widetilde{\pounds} \cup \Big\{\hbar, \Big[\wedge\Big(glbT_{\widetilde{\pounds}}(\hbar), glbT_{\widetilde{\yen}}(\hbar)\Big), \wedge\Big(lubT_{\widetilde{\pounds}}(\hbar), lubT_{\widetilde{\yen}}(\hbar)\Big)\Big],$$

$$\left[\wedge\left(glbC_{\widehat{£}}(\hbar), glbC_{\widehat{¥}}(\hbar)\right),\right.$$

$$\left.\wedge\left(lubC_{\widehat{£}}(\hbar), lubC_{\widehat{¥}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbG_{\widehat{£}}(\hbar), glbG_{\widehat{¥}}(\hbar)\right),\right.$$

$$\left.\vee\left(lubG_{\widehat{£}}(\hbar), \vee\, lubG_{\widehat{¥}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbU_{\widehat{£}}(\hbar), glbU_{\widehat{¥}}(\hbar)\right),\right.$$

$$\left.\vee\left(lubU_{\widehat{£}}(\hbar), lubU_{\widehat{¥}}(\hbar)\right)\right],$$

$$\left[\vee\left(glbF_{\widehat{£}}(\hbar), glbF_{\widehat{¥}}(\hbar\right.\right.$$

$$\left.\left.\vee\left(lubF_{\widehat{£}}(\hbar), lubF_{\widehat{¥}}(\hbar)\right)\right] : \hbar \in W\right\}$$

$$= \left\{\hbar, \left[\vee\left(glbT_{\widehat{£}}(\hbar), \wedge\left(glbT_{\widehat{£}}(\hbar), glbT_{\widehat{¥}}(\hbar)\right)\right), \vee\left(lubT_{\widehat{£}}(\hbar), \wedge\left(lubT_{\widehat{£}}(\hbar), lubT_{\widehat{¥}}(\hbar)\right)\right)\right],\right.$$

$$\left[\vee\left(glbC_{\widehat{£}}(\hbar), \wedge\left(glbC_{\widehat{£}}(\hbar), glbC_{\widehat{¥}}(\hbar)\right)\right), \vee\left(lubC_{\widehat{£}}(\hbar), \wedge\left(lubC_{\widehat{£}}(\hbar), lubC_{\widehat{¥}}(\hbar)\right)\right)\right],$$

$$\left[\wedge\left(glbG_{\widehat{£}}(\hbar), \vee\left(glbG_{\widehat{£}}(\hbar), glbG_{\widehat{¥}}(\hbar)\right)\right), \wedge\left(lubG_{\widehat{£}}(\hbar), \vee\left(lubG_{\widehat{£}}(\hbar), \vee\, lubG_{\widehat{¥}}(\hbar)\right)\right)\right],$$

$$\left[\wedge\left(glbU_{\widehat{£}}(\hbar), \vee\left(glbU_{\widehat{£}}(\hbar), glbU_{\widehat{¥}}(\hbar)\right)\right), \wedge\ lubU_{\widehat{£}}(\hbar), \vee\left(lubU_{\widehat{£}}(\hbar), lubU_{\widehat{¥}}(\hbar)\right)\right],$$

$$\left[\wedge\left(glbF_{\widehat{£}}(\hbar), \vee\left(glbF_{\widehat{£}}(\hbar), glbF_{\widehat{¥}}(\hbar)\right)\right),\right.$$

$$\left.\left.\wedge\left(lubF_{\widehat{£}}(\hbar), \vee\left(lubF_{\widehat{£}}(\hbar), lubF_{\widehat{¥}}(\hbar)\right)\right)\right] : \hbar \in W\right\}$$

$$= \left\{\hbar, \left[glbT_{\widehat{£}}(\hbar), lubT_{\widehat{£}}(\hbar)\right], \left[glbC_{\widehat{£}}(\hbar), lubC_{\widehat{£}}(\hbar)\right], \left[glbG_{\widehat{£}}(\hbar), lubG_{\widehat{£}}(\hbar)\right],\right.$$

$$\left.\left[glbU_{\widehat{£}}(\hbar), lubU_{\widehat{£}}(\hbar)\right], \left[glbF_{\widehat{£}}(\hbar), lubF_{\widehat{£}}(\hbar)\right] : \hbar \in W\right\}$$

$$= \left\{\left(\hbar, T_{\widehat{£}}(\hbar), C_{\widehat{£}}(\hbar), G_{\widehat{£}}(\hbar), U_{\widehat{£}}(\hbar), F_{\widehat{£}}(\hbar)\right) : \hbar \in W\right\}$$

$$= \widehat{£}$$

$$\therefore \widehat{£} \cup \left(\widehat{£} \cap \widehat{¥}\right) = \widehat{£}$$

Similarly, $\widehat{£} \cap \left(\widehat{£} \cup \widehat{¥}\right) = \widehat{£}$.

(vii) $\left(\widehat{£}^{c}\right)^{c}$

$$= \left\{\left(\hbar, F_{\widehat{£}}(\hbar), U_{\widehat{£}}(\hbar), \left[1 - lubG_{\widehat{£}}(\hbar), 1 - glbG_{\widehat{£}}(\hbar)\right], C_{\widehat{£}}(\hbar), T_{\widehat{£}}(\hbar)\right) : \hbar \in W\right\}^{c}$$

$$= \left\{ \left(\hbar, T_{\widehat{£}}\ (\hbar), C_{\widehat{£}}\ (\hbar), \left[1 - \left(1 - glbG_{\widehat{£}}(\hbar) \right), 1 - \left(1 - lubG_{\widehat{£}}(\hbar) \right) \right], U_{\widehat{£}}\ (\hbar), F_{\widehat{£}}\ (\hbar) \right) : \hbar \in W \right\}$$

$$= \left\{ \left(\hbar, T_{\widehat{£}}\ (\hbar), C_{\widehat{£}}\ (\hbar), \left[glbG_{\widehat{£}}(\hbar), lubG_{\widehat{£}}(\hbar) \right], U(\hbar), F_{\widehat{£}}\ (\hbar) \right) : \hbar \in W \right\}$$

$$= \left\{ \left(\hbar, T_{\widehat{£}}\ (\hbar), C(\hbar), G_{\widehat{£}}(\hbar), U_{\widehat{£}}\ (\hbar), F_{\widehat{£}}\ (\hbar) \right) : \hbar \in W \right\}$$

$$= \widehat{£}^{c}$$

$$\therefore \left(\widehat{£}^{c} \right)^{c} = \widehat{£}$$

4. Conclusion

This research includes the idea of $IVPNS$s. Also some important properties of $IVPNS$s have been studied along with examples. A real life example of COVID-19 has been discussed in the paper using $IVPNS$. Some more operations along with aggregation operators on $IVPNS$s can be studied in future with the help of important results obtained here. Further while making decision like MCDM [6], $IVPNS$s also applicable to deal with uncertain observation.

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Received: July 7, 2023. Accepted: Nov 21, 2023