Neutrosophic Sets and Systems, Vol. 60, 2023
University of New Mexico

New Observation on Cesaro Summability in Neutrosophic n-Normed Linear Spaces

Jenifer. P [1], Jeyaraman. M [2]*

1 Research Scholar, P.G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affiliated to Alagappa University, Karaikudi, Tamilnadu, India.; jenifer87.maths@gmail.com, ORCID : https://orcid.org/0009-0002-7569-0125

2 P.G. and Research Department of Mathematics, Raja Doraisingam Govt. Arts College, Sivagangai, Affiliated to Alagappa University, Karaikudi, Tamilnadu, India.; jeya.math@gmail.com, ORCID : https://orcid.org/0000-0002-0364-1845

*Correspondence: jeya.math@gmail.com;

Abstract: We define Cesaro summability in a Neutrosophic n -Normed Linear Space in this article. In a Neutrosophic n-Normed Linear Space, we show the Cesaro summability method to be regular, albeit this does not imply typical convergence in general. We also look for more circumstances in which the opposite is true.

Keywords: Convergence, Cesaro summability, Normed Space, Neutrosophic Normed Linear Space.

1. Introduction

In 1965, Zadeh [20] was the first to present the idea of fuzzy sets, which was an expansion of classical set theoretical concept. This theory has been used in numerous areas of mathematics, including the theory of functions, metric spaces, topological spaces and approximation theory, in addition to numerous branches of engineering, such as population dynamics, nonlinear dynamic systems, and quantum physics. Gunawan and Mashadi [5], Kim and Cho [8] and Malceski [9] and several researchers have studied$n$-normed linear spaces. Vijayabalaji and Narayanan [18] defined fuzzy n-normalized linear space. Following the definition of Intuitionistic Fuzzy n-Normed Space [$\mathcal{IF}n\mathcal{NS}$] given by Vijayabalaji et al. [19], Saadati and Park [11] proposed the idea of Intuitionistic Fuzzy Normed Space [$\mathcal{IFNS}$].

The Neutrosophic Set [$\mathcal{NS}$] is a fresh interpretation of Smarandache's definition of the classical set [14,15]. A neutrosophic set's elements are made up of the triplets true- membership function (T), indeterminacy membership function (I) and falsity membership function (F). When all elements of the universe have a certain degree of T, F, and I, a set is said to be neutrosophic. Some findings on fixed-points were demonstrated in the context of these spaces by Sowndrarajan et. al. [16]. Approximate Fixed Point Theorems for Weak Contractions on Neutrosophic normed Spaces were proved in 2022 by Jeyaraman et. al. [7].

Our goal in this study is to introduce summability theory in a Neutrosophic n-Normed Linear Space [$\mathcal{N}n\mathcal{NLS}$]. We introduce the idea of Cesaro in this context. The definition of convergence for a sequence in $\mathcal{N}n\mathcal{NLS}$ affects our findings. This new definition is the foundation for the development of our current findings. Pertaining to the conventional analogs of the findings reported in this work.

2. Preliminaries

Definition 2.1:

The following axioms define a continuous t-norm as a binary operation $*$: $[0,1] \times [0,1] \to [0,1]$

1. $*$ is continuous, commutative and associative,
2. $\mathfrak{a} * 1 = \mathfrak{a}$ for every $\mathfrak{a} \in [0,1]$,
3. If $\mathfrak{a} \leq \mathfrak{c}$ and $\mathfrak{b} \leq \mathfrak{d}$ then $\mathfrak{a} * \mathfrak{b} \leq \mathfrak{c} * \mathfrak{d}$, for each $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d} \in [0,1]$.

Definition 2.2:

The following axioms define a continuous t-conorm as a binary operation $\diamondsuit$: $[0,1] \times [0,1] \to [0,1]$

1. $\diamondsuit$ is continuous, commutative and associative,
2. $\mathfrak{a} \diamondsuit 0 = \mathfrak{a}$ for every $\mathfrak{a} \in [0,1]$,
3. If $\mathfrak{a} \leq \mathfrak{c}$ and $\mathfrak{b} \leq \mathfrak{d}$ then $\mathfrak{a} \diamondsuit \mathfrak{b} \leq \mathfrak{c} \diamondsuit \mathfrak{d}$, for each $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d} \in [0,1]$.

Definition 2.3:

A $\mathcal{NnNLS}$ is the 7-tuple $(\mathfrak{H}, \zeta, \vartheta, \varpi, *, \diamondsuit, \odot)$ where $\mathfrak{H}$ is a linear space over a field F, $*$ is a continuous t-norm, $\diamondsuit$ and $\odot$ continuous t-conorm, μ, ϑ *and* ω are fuzzy sets on $\mathfrak{H}^n \times (0,\infty)$, μ denotes the degree of membership, ϑ denotes the indeterminacy and ω denotes the non –membership of $(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) \in \mathfrak{H}^n \times (0,\infty)$ satisfying the following conditions for every $(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n) \in \mathfrak{H}^n$ and $s, t > 0$.

a) $0 \leq \zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) \leq 1$; $0 \leq \vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) \leq 1$; $0 \leq \varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) \leq 1$;
b) $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) + \vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) + \varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) \leq 3$,
c) $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) > 0$,
d) $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 1$ if and only if $\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n$ are linearly dependent,
e) $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t)$ is constant for any combination of$\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n$,
f) $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \propto \mathfrak{h}_n, t) = \zeta\left(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, \frac{t}{|\propto|}\right)$ for each $\propto \neq 0, \propto \in$ F,
g) $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, s) * \zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}'_n, t) \leq \zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n + \mathfrak{h}'_n, s + t)$,
h) $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) : (0,\infty) \to [0,1]$ is continuous,
i) $\lim_{t\to\infty} \zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 1$ and $\lim_{t\to 0} \zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 0$,
j) $\vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) < 1$,
k) $\vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 0$ if and only if $\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n$ are linearly dependent,
l) $\vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t)$ is constant for any combination of$\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n$,
m) $\vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \propto \mathfrak{h}_n, t) = \vartheta\left(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, \frac{t}{|\propto|}\right)$ for each $\propto \neq 0, \propto \in$ F,
n) $\vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, s) \diamondsuit \vartheta(\mathfrak{x}_1, \mathfrak{x}_2, \dots, \mathfrak{x}'_n, t) \geq \vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n + \mathfrak{x}'_n, s + t)$,
o) $\vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) : (0,\infty) \to [0,1]$ is continuous,
p) $\lim_{t\to\infty} \vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 0$ and $\lim_{t\to 0} \vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 1$,
q) $\varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) < 1$,
r) $\varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 0$ if and only if $\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n$ are linearly dependent,
s) $\varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t)$ is constant for any combination of$\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n$,
t) $\varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \propto \mathfrak{h}_n, t) = \varpi\left(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, \frac{t}{|\propto|}\right)$ for each $\propto \neq 0, \propto \in$ F,
u) $\varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, s) \odot \varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}'_n, t) \geq \varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n + \mathfrak{h}'_n, s + t)$,
v) $\varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) : (0,\infty) \to [0,1]$ is continuous,
w) $\lim_{t\to\infty} \varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 0$ and $\lim_{t\to 0} \varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = 1$.

Example 2.4:

Let $(\mathfrak{H}, \|\cdot, \dots, \cdot\|)$ be a linear space with n norms. Also, let $\mathfrak{a} * \mathfrak{b} = \mathfrak{a}b$, $\mathfrak{a} \diamondsuit \mathfrak{b} = min\{\mathfrak{a} + \mathfrak{b}, 1\}$ and $\mathfrak{a} \odot \mathfrak{b} = min\{\mathfrak{a} + \mathfrak{b}, 1\}$, for every $\mathfrak{a}, \mathfrak{b} \in [0,1]$, $\zeta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = \frac{t}{t + \|\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n\|}$, $\vartheta(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = \frac{\|\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n\|}{t + \|\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n\|}$ and $\varpi(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n, t) = \frac{\|\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n\|}{t}$. Then $(\mathfrak{H}, \zeta, \xi, \varpi, *, \diamondsuit, \odot)$ is a $\mathcal{NnNLS}$.

Lemma 2.5.

We define $<\varrho>= \varrho-[\varrho]$, for every $\varrho>0$, where $[\cdot]$ stands for the largest integer function. What follows is accurate:

(i) If $\varrho > 1$, then $\varrho_n > n$ for every $n \in \mathbb{N}\backslash\{0\}$ with $n \geq \frac{1}{\langle\varrho\rangle}$,

(ii) If $0 < \varrho < 1$, then $\varrho_n < n$ for every $n \in \mathbb{N}\backslash\{0\}$, where $\varrho_n = [n\varrho]$.

Lemma 2.6.

The following claims are accurate:

(i) If $\varrho > 1$, then for every $n \in \mathbb{N}\backslash\{0\}$ with $n \geq \frac{3\varrho-1}{\varrho(\varrho-1)}$, we have $\frac{\varrho}{\varrho-1} < \frac{\varrho_n+1}{\varrho_n-n} < \frac{2\varrho}{\varrho-1}$,

(ii) If $0 < \varrho < 1$, then for every $n \in \mathbb{N}\backslash\{0\}$ with $n > \frac{1}{\varrho}$, we have $0 < \frac{\varrho_n+1}{n-\varrho_n} < \frac{2\varrho}{\varrho-1}$.

3. In $\mathcal{NnNLS}$ Cesaro Summability

Definition 3.1.

In $\mathcal{NnNLS}$ $(\mathfrak{H},\zeta,\vartheta,\varpi,*,\diamond,\odot)$, choose the sequence to be $\{\mathfrak{a}_n\}$. The Arithmetic means of $\{\mathfrak{a}_n\}$, is defined and denoted by

$$\chi_n = \frac{1}{n+1}\sum_{k=0}^{n} \mathfrak{a}_k.$$

If $\lim_{n\to\infty} \chi_n = \mathfrak{a}$, then $\{\mathfrak{a}_n\}$ is said to be Cesaro summable to $\mathfrak{a} \in \mathfrak{H}$.

Theorem 3.2.

In $\mathcal{NnNLS}$ $(\mathfrak{H},\zeta,\vartheta,\varpi,*,\diamond,\odot)$, if the sequence to be $\{\mathfrak{a}_n\}$converges to $\mathfrak{a} \in \mathfrak{H}$, then $\{\mathfrak{a}_n\}$ is Cesaro summable to $\mathfrak{a}$.

Proof.

Choose $\mathfrak{a} \in \mathfrak{H}$ be the converging point of the sequence $\{\mathfrak{a}_n\}$.
Fix $r > 0$ and $\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1} \in \mathfrak{H}$.
Then for given $\varepsilon > 0$, there exists $n_0 \in \mathfrak{N}$ such that
$\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n - \mathfrak{a},\frac{r}{2}\right) > 1-\varepsilon, \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n - \mathfrak{a},\frac{r}{2}\right) < \varepsilon$ and
$\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n - \mathfrak{a},\frac{r}{2}\right) < \varepsilon$, for all $n > n_0$.
Also, from Definition (2.3), we have that
$\lim_{n\to\infty}\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k - \mathfrak{a}),\frac{(n+1)r}{2}\right) = 1, \lim_{n\to\infty}\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k - \mathfrak{a}),\frac{(n+1)r}{2}\right) = 0$ and
$\lim_{n\to\infty}\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k - \mathfrak{a}),\frac{(n+1)r}{2}\right) = 0.$
Consequently, there are $n_1 \in \mathfrak{N}$such that

$$\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k - \mathfrak{a}),\frac{(n+1)r}{2}\right) > 1-\varepsilon,$$

$\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k - \mathfrak{a}),\frac{(n+1)r}{2}\right) < \varepsilon$ and

$$\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k - \mathfrak{a}),\frac{(n+1)r}{2}\right) < \varepsilon, \text{for all } n > n_1.$$

Now, we have that

$$\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{n+1}\sum_{k=0}^{n}\mathfrak{a}_k - \mathfrak{a},r\right) = \zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{n+1}\sum_{k=0}^{n}\mathfrak{a}_k - \mathfrak{a},r\right)$$

$$\geq \min\left\{\begin{array}{c}\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=n_0+1}^{n}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right)\end{array}\right\}$$

$$\geq \min\left\{\begin{array}{c}\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=n_0+1}^{n}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n-n_0)r}{2}\right)\end{array}\right\}$$

$$\geq \min\left\{\begin{array}{c}\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_{n_0+1}-\mathfrak{a}\frac{r}{2}\right),\\ \zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_{n_0+2}-\mathfrak{a},\frac{r}{2}\right),\\ \dots,\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\mathfrak{a},\frac{r}{2}\right)\end{array}\right\} > 1-\varepsilon,$$

$$\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{n+1}\sum_{k=0}^{n}\mathfrak{a}_k-\mathfrak{a},r\right)=\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{n+1}\sum_{k=0}^{n}\mathfrak{a}_k-\mathfrak{a},r\right)$$

$$\leq \max\left\{\begin{array}{c}\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=n_0+1}^{n}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right)\end{array}\right\}$$

$$\leq \max\left\{\begin{array}{c}\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=n_0+1}^{n}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n-n_0)r}{2}\right)\end{array}\right\}$$

$$\leq \max\left\{\begin{array}{c}\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_{n_0+1}-\mathfrak{a},\frac{r}{2}\right),\\ \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_{n_0+2}-\mathfrak{a},\frac{r}{2}\right),\dots,\\ \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\mathfrak{a},\frac{r}{2}\right)\end{array}\right\} < \varepsilon,$$

and in a similar manner, we also have that

$$\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{n+1}\sum_{k=0}^{n}\mathfrak{a}_k-\mathfrak{a},r\right)=\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{n+1}\sum_{k=0}^{n}\mathfrak{a}_k-\mathfrak{a},r\right)$$

$$\leq \max\left\{\begin{array}{l}\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=n_0+1}^{n}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right)\end{array}\right\}$$

$$\leq \max\left\{\begin{array}{l}\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=n_0+1}^{n}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n-n_0)r}{2}\right)\end{array}\right.$$

$$\leq \max\left\{\begin{array}{c}\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\sum_{k=0}^{n_0}(\mathfrak{a}_k-\mathfrak{a}),\frac{(n+1)r}{2}\right),\\ \varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_{n_0+1}-\mathfrak{a},\frac{r}{2}\right),\\ \varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_{n_0+2}-\mathfrak{a},\frac{r}{2}\right),\dots,\\ \varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_{n}-\mathfrak{a},\frac{r}{2}\right)\end{array}\right\}<\varepsilon,$$

for all $n > \max\{n_0, n_1\}$. Thus, the proof is completed.

Example 3.3.

Let $\mathfrak{H} = \mathfrak{R}^n$ with

$$\|\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1}\| = abs\left(\begin{vmatrix}\mathfrak{h}_{11} & \cdots & \mathfrak{h}_{1n}\\ \vdots & \ddots & \vdots\\ \mathfrak{h}_{n1} & \cdots & \mathfrak{h}_{nn}\end{vmatrix}\right)$$

where $\mathfrak{h}_i = (\mathfrak{h}_{i1},\mathfrak{h}_{i2},\dots,\mathfrak{h}_{in}) \in \mathfrak{R}^n$ for every$i = 1,2,\dots,n$ and $\mathfrak{a} * \mathfrak{b} = \mathfrak{a}\,\mathfrak{b}, \mathfrak{a} \diamondsuit \mathfrak{b} = \min\{\mathfrak{a}+\mathfrak{b},1\}$ and $a \odot b = \min\{a+b,1\}$ for all $\mathfrak{a},\mathfrak{b} \in [0,1]$.
Now for all $v_1, v_2, \dots, v_n \in \mathfrak{R}^n$ and $r > 0$, define
$\zeta(y_1,y_2,\dots,y_n,r) = \frac{r}{r+\|y_1,y_2,\dots,y_n\|}$, $\vartheta(y_1,y_2,\dots,y_n,r) = \frac{\|y_1,y_2,\dots,y_n\|}{r+\|y_1,y_2,\dots,y_n\|}$ and

$$\varpi(y_1,y_2,\dots,\quad y_n,t) = \frac{\|y_1,y_2,\dots,y_n\|}{r}.$$

Then $(\mathfrak{R}^n,\zeta,\vartheta,\varpi,*,\diamondsuit,\odot)$ is a $\mathcal{N}n\mathcal{NLS}$.
Choose the sequence $\{\mathfrak{a}_k\} = \left((-1)^{k+1},0,0,\dots,0\right) \in \mathfrak{R}^n$.
Then $\lim_{n\to\infty}\zeta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\chi_{2n},r) = \lim_{n\to\infty}\frac{r}{r+\left\|\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},-\frac{1}{2n+1}\right\|} = \lim_{n\to\infty}\frac{r}{r+\left|-\frac{1}{2n+1}\right|\mathfrak{A}} = 1,$
where the value of $\mathfrak{A}$, which is always a finite number, relies on the selection of $\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1}$.

$$\lim_{n\to\infty}\vartheta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\chi_{2n},r) = \lim_{n\to\infty}\frac{\left\|\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},-\frac{1}{2n+1}\right\|}{r+\left\|\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},-\frac{1}{2n+1}\right\|} = \lim_{n\to\infty}\frac{\left|-\frac{1}{2n+1}\right|\mathfrak{B}}{r+\left|-\frac{1}{2n+1}\right|\mathfrak{B}} = 0,$$

where the value of $\mathfrak{B}$, which is always a finite number, relies on the selection of $\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1}$.
and $\lim_{n\to\infty}\varpi(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\chi_{2n},r) = \lim_{n\to\infty}\frac{\left\|\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},-\frac{1}{2n+1}\right\|}{r} = \lim_{n\to\infty}\frac{\left|-\frac{1}{2n+1}\right|\mathfrak{C}}{r} = 0,$
where the value of $\mathfrak{C}$, which is always a finite number, relies on the selection of $\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1}$.
Therefore, we have that $\chi_{2n} \to \bar{0} = (0,0,\dots,0) \in \mathfrak{R}^n$.
Also, $\lim_{n\to\infty}\zeta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_{2n+1},r) = \lim_{n\to\infty}\zeta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\bar{0},r) = 1,$
$\lim_{n\to\infty}\vartheta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_{2n+1},r) = \lim_{n\to\infty}\xi(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\bar{0},r) = 0$ and
$\lim_{n\to\infty}\varpi(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_{2n+1},r) = \lim_{n\to\infty}\varpi(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\bar{0},r) = 0.$
Thus, we have $\chi_{2n+1} \to \bar{0}$. From the reasoning listed above, we conclude that $\chi_n \to \bar{0}$, i.e., the sequence $\{\mathfrak{a}_k\}$ is Cesaro summable to $\bar{0}$. However, it is clear that $\{\mathfrak{a}_k\}$ is not convergent because $\{\mathfrak{a}_{2k}\} \to (-1,0,0,\dots,0)$ and $\{\mathfrak{a}_{2k+1}\} \to (1,0,0,\dots,0)$.

Theorem 3.4.

Let $\{\mathfrak{a}_n\}$ be a sequence in a $\mathcal{LS}(\mathfrak{H},\zeta,\vartheta,\varpi,*,\diamond,\odot)$. If $\{\mathfrak{a}_n\}$ is Cesaro summable to $\mathfrak{a}$, then it is convergent to $\mathfrak{a}$ if and only if for any $\mathfrak{x}_1,\mathfrak{x}_2,\dots,\mathfrak{x}_{n-1}\in\mathfrak{H}$ and $r>0$ the following conditions are met:

(3.1) $\sup\limits_{\lambda>1}\lim\limits_{n\to\infty}\inf\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)=1,$

(3.2) $\inf\limits_{\lambda>1}\lim\limits_{n\to\infty}\sup\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)=0$ and

(3.3) $\inf\limits_{\lambda>1}\lim\limits_{n\to\infty}\sup\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)=0.$

Proof.

Assume that $\{\mathfrak{a}_n\}$ is summable to Cesaro. Assume that $\{\mathfrak{a}_n\}$ converges to $\mathfrak{a}$.
Fix $\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1}\in\mathfrak{H}$ and $r>0$. For any $\lambda>1$,utilising Lemma (2.5), for each $n\in\mathbb{N}\setminus\{0\}$ with $n\geq\langle\lambda\rangle^{-1}$, we have

(3.4) $\mathfrak{a}_n-\chi_n=\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n)-\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n).$

Again, by Lemma (2.6), for $n\geq\frac{3\lambda-1}{\lambda(\lambda-1)}$, we have

$$\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),r\right)=\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},(\chi\lambda_n-\chi_n),\frac{r}{\frac{\lambda_n+1}{\lambda_n-n}}\right)$$
$$\geq\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},(\chi\lambda_n-\chi_n),\frac{r}{\frac{2\lambda}{\lambda-1}}\right),$$

$$\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),r\right)=\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},(\chi\lambda_n-\chi_n),\frac{r}{\frac{\lambda_n+1}{\lambda_n-n}}\right)$$
$$\leq\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},(\chi\lambda_n-\chi_n),\frac{r}{\frac{2\lambda}{\lambda-1}}\right)\text{ and}$$

$$\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),r\right)=\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},(\chi\lambda_n-\chi_n),\frac{r}{\frac{\lambda_n+1}{\lambda_n-n}}\right)$$
$$\leq\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},(\chi\lambda_n-\chi_n),\frac{r}{\frac{2\lambda}{\lambda-1}}\right)$$

Since $\{\chi_n\}$ is a Cauchy sequence, we have

$\lim\limits_{n\to\infty}\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),r\right)=1$,

$\lim\limits_{n\to\infty}\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),r\right)=0$ and

$\lim\limits_{n\to\infty}\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),r\right)=0$, and therefore $\lim\limits_{n\to\infty}\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n)=0.$

Hence using (3.4), we have

$\lim\limits_{n\to\infty}\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)=1,$

$\lim\limits_{n\to\infty}\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)=0$ and

$$\lim_{n\to\infty}\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)=0.$$

As a result, (3.1), (3.2), and (3.3). We presume that (3.1), (3.2), and (3.3) are true in order to demonstrate the converse. Fix $\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1}\in\mathfrak{H}$ and $r>0$. Then for given $\varepsilon>0$, we have the following:

(i) a thing exists $\lambda>1$ and $n_0\in\mathbb{N}$ such that
$\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),\frac{r}{3}\right)>1-\varepsilon$,

$\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)<\varepsilon$ and

$\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),r\right)<\varepsilon$, for every $n>n_0$.

(ii) a thing exists $n_0\in\mathbb{N}$ such that $\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_n-\mathfrak{a},\frac{r}{3}\right)>1-\varepsilon$,

$\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_n-\mathfrak{a},\frac{r}{3}\right)<\varepsilon$ and $\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_n-\mathfrak{a},\frac{r}{3}\right)<\varepsilon$, for all $n>n_1$.

(iii) Also, since $\lim_{n\to\infty}\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n)=0$, there exists $n_2\in\mathbb{N}$ such that

$\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),\frac{r}{3}\right)>1-\varepsilon$,

$\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),\frac{r}{3}\right)<\varepsilon$ and

$\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),\frac{r}{3}\right)<\varepsilon$, for all $n>n_2$.

Therefore, we have

$$\zeta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\mathfrak{a},r)=\zeta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\chi_n+\chi_n-\mathfrak{a},r)$$
$$=\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n)-\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n)+\chi_n-\mathfrak{a},r\right)$$
$$\geq\min\left\{\begin{array}{c}\zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),\frac{r}{3}\right),\\ \zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),\frac{r}{3}\right),\\ \zeta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_n-\mathfrak{a},\frac{r}{3}\right)\end{array}\right\}>1-\varepsilon,$$

$$\vartheta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\mathfrak{a},r)=\vartheta(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\chi_n+\chi_n-\mathfrak{a},r)$$
$$=\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n)-\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n)+\chi_n-\mathfrak{a},r\right)$$
$$\leq\max\left\{\begin{array}{c}\vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),\frac{r}{3}\right),\\ \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),\frac{r}{3}\right),\\ \vartheta\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_n-\mathfrak{a},\frac{r}{3}\right)\end{array}\right\}<\varepsilon \text{ and}$$

$$\varpi(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\mathfrak{a},r)=\varpi(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\mathfrak{a}_n-\chi_n+\chi_n-\mathfrak{a},r)$$
$$=\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n)-\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n)+\chi_n-\mathfrak{a},r\right)$$
$$\leq\max\left\{\begin{array}{c}\varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{\lambda_n+1}{\lambda_n-n}(\chi\lambda_n-\chi_n),\frac{r}{3}\right),\\ \varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}(\mathfrak{a}_k-\mathfrak{a}_n),\frac{r}{3}\right),\\ \varpi\left(\mathfrak{h}_1,\mathfrak{h}_2,\dots,\mathfrak{h}_{n-1},\chi_n-\mathfrak{a},\frac{r}{3}\right)\end{array}\right\}<\varepsilon,$$

for all $n>\max\{n_0,n_1,n_2\}$. This completes the proof.

4. Conclusion

The idea of Cesaro summability in a $\mathcal{N}n\mathcal{NLS}$, one of the most general mathematical structures with both algebraic and analytic features, is discussed in this study. As a result, many current theorems are extended and generalized by the current results in Cesaro summability. Future work on this topic might result in the expansion of neutrosophic normed spaces and finite-dimensional $\mathcal{N}n\mathcal{NLS}$.

References

1. Atanassov .K, intuitionistic fuzzy sets, Fuzzy Sets Syst, 20, 87-96, (1986).
2. Basarir M, On the statistical convergence of sequences. Firat Univ. J. Sci. 2, 1-6 (1995).

3. Bilgin T,Lacunary strongly Δ-convergent sequences of fuzzy numbers.Inf. Sci. 160, 201-206, (2004).
4. Fridy JA, Orhan C,Lacunary statistical convergence. Pac. J. Math.160, 43-51, (1993).
5. Gunawan H, Mashadi M, On n-normed spaces. Int. J. Math. Sci. 27, 631-639, (2001).
6. Jeyaraman M, Mangayarkkarasi AN, Jeyanthi V, Pandiselvi R. Hyers Ulam-Rassias stability for functional equation in Neutrosophic Normed Spaces, International Journal of Neutrosophic Science, Vol.18, No.1,127-143, (2022).
7. Jeyaraman M, Ramachandran A and Shakila V.B, Approximate fixed point Theorems for weak contractions on Neutrosophic Normed space, Journal of computational Mathematics, 6(1), 134-158, (2022).
8. Kim SS, Cho YJ, Strict convexity in linear n-normed spaces. Demonstr. Math.29, 739-744, (1996).
9. Malceski R, Strong n-convex n-normed spaces, Mat. Bilt. 21, 81-102, (1997).
10. Mursaleen M, Mohiuddine SA, On Lacunary statistical convergence with respect to the intuitionistic fuzzy normed space, J. Comput. Appl. Math. 233(2), 142-149, (2009).
11. Saadati R, Park JH, Intuitionistic fuzzy Euclidean normed spaces, Commun. Math. Anal. 12, 85-90, (2006).
12. Sen M, Debnath P, Lacunary statistical convergence in intuitionistic fuzzy n-normed spaces, Math. Comput.Model.54, 2978-2985, (2011).
13. Simsek N, Kirisci M, Fixed point theorems in Neutrosophic Metric Spaces, Sigma J. Eng. Nat. Sci., 10(2), 221-230, (2019).
14. Smarandache F,Neutrosophic set, A generalisation of the intuitionistic fuzzy sets, Int. J. Pure Appl. Math., 24, 287-297, (2005).
15. Smarandache. F, Neutrosophic . Neutrosophic Probability, Set, and Logic, Pri-Quest Information and Learning, Ann Arbor, Michigan , USA (1998).
16. Sowndrarajan S, Jeyaraman M and Smarandache, F. Fixed Point theorems in neutrosophic metric spaces Neutrosophic Sets and Systems, Volume 36, 251-268, (2020).
17. Thillaigovindan N, Anita Shanth S, Jun, YB, On Lacunary statistical convergence in intuitionistic fuzzy n-normed spaces, Ann. Fuzzy Math. Inform. 1, 119-131, (2011).
18. Vijayabalaji S, Narayanan A, Fuzzy n-normed linear space, J. Math. Sci. 24, 3963-3977 (2005).DOI: 10.1155/IJMMS.2005.3963.
19. Vijayabalaji S, Thillaigovindan N, Jun YB, Intuitionistic fuzzy n-normed linear space, Bull. Korean Math. Soc. 44, 291-308, (2007).
20. Zadeh LA, Fuzzy sets, Inf. Control 8, 338-353, (1965).

Received: July 8, 2023. Accepted: Nov 19, 2023