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On The Conditions for Symbolic 3-Plithogenic Pythagoras Quadruples

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Abstract: The objective of this paper is to find the necessary and sufficient conditions for a symbolic 3-plithogenic quadruple

$(t_0 + t_1P_1 + t_2P_2 + t_3P_3, s_0 + s_1P_1 + s_2P_2 + s_3P_3, k_0 + k_1P_1 + k_2P_2 + k_3P_3, l_0 + l_1P_1 + l_2P_2 + l_3P_3)$ to be a Pythagoras quadruple, i.e. to be a solution for the non-linear Diophantine equation in four variables $X^2 + Y^2 + Z^2 = T^2$.

Also, many examples will be illustrated and presented to explain how the theorems work.

Keywords: symbolic 3-plithogenic ring, Pythagoras quadruple, Pythagoras Diophantine equation

Introduction and Preliminaries.

Symbolic n-plithogenic sets were defined by Smarandache in [1-3], where these sets were used in generalizing classical algebraic structures such as symbolic 2-plithogenic and symbolic 3-plithogenic structures [4-9], with many applications in other fields [10-12].

It is useful to refer that symbolic n-plithogenic algebraic structures are very similar to neutrosophic and refined neutrosophic structures, see [13-22].

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In this paper, we continue other efforts to study Pythagoras triples in many different rings [23-26].

We present the concept of Pythagoras triple in a symbolic 3-plithogenic commutative ring with many clear examples that clarify the validity of our work.

Definition.

Let R be a ring, the symbolic 3-plithogenic ring is defined as follows:

$3-SP_R=\{a_0+a_1P_1+a_2P_2+a_3P_3;\ a_i\in R, {P_j}^2=P_j, P_i\times P_j=P_{max(i,j)}\}$.

Smarandache has defined algebraic operations on $3-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2+a_3P_3]+[b_0+b_1P_1+b_2P_2+b_3P_3]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2+(a_3+b_3)P_3$.

Multiplication:

$[a_0+a_1P_1+a_2P_2+a_3P_3].[b_0+b_1P_1+b_2P_2+b_3P_3]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_0b_3P_3+a_1b_0{P_1}^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2{P_2}^2+a_1b_3P_3P_1+a_2b_3P_2P_3+a_3b_3(P_3)^2+a_3b_0P_3+a_3b_1P_3P_1+a_3b_2P_2P_3+a_1b_1P_1P_1=a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2+(a_0b_3+a_1b_3+a_2b_3+a_3b_3+a_3b_0+a_3b_1+a_3b_2)P_3$.

Definition.

Let $T=t_0+t_1P_1+t_2P_2+t_3P_3, S=s_0+s_1P_1+s_2P_2+s_3P_3, K=k_0+k_1P_1+k_2P_2+k_3P_3, L=l_0+l_1P_1+l_2P_2+l_3P_3$ be four symbolic 3-plithogenic elements of a symbolic 3-plithogenic commutative ring $3-SP_R$, then (T,S,K,L) is called a symbolic 3-plithogenic Pythagoras quadruple if and only if $T^2+S^2+K^2=L^2$.

Theorem.

Let $T=t_0+t_1P_1+t_2P_2+t_3P_3, S=s_0+s_1P_1+s_2P_2+s_3P_3, K=k_0+k_1P_1+k_2P_2+k_3P_3, L=l_0+l_1P_1+l_2P_2+l_3P_3\in 3-SP_R$, then (T,S,K,L) is a Pythagoras quadruple if and only if:

$(t_0,s_0,k_0,l_0),(t_0+t_1,s_0+s_1,k_0+k_1,l_0+l_1),(t_0+t_1+t_2,s_0+s_1+s_2,k_0+k_1+k_2,l_0+l_1+l_2),(t_0+t_1+t_2+t_3,s_0+s_1+s_2+s_3,k_0+k_1+k_2+k_3,l_0+l_1+l_2+l_3)$ are four Pythagoras quadruples in R.

Proof.

We have:

$$T^2 = {t_0}^2 + [(t_0 + t_1)^2 - {t_0}^2]P_1 + [(t_0 + t_1 + t_2)^2 - (t_0 + t_1)^2]P_2 + [(t_0 + t_1 + t_2 + t_3)^2 - (t_0 + t_1 + t_2)^2]P_3$$

$$S^2 = {s_0}^2 + [(s_0 + s_1)^2 - {s_0}^2]P_1 + [(s_0 + s_1 + s_2)^2 - (s_0 + s_1)^2]P_2 + [(s_0 + s_1 + s_2 + s_3)^2 - (s_0 + s_1 + s_2)^2]P_3$$

$$K^2 = {k_0}^2 + \left[(k_0 + k_1)^2 - {k_0}^2\right]P_1 + [(k_0 + k_1 + k_2)^2 - (k_0 + k_1)^2]P_2 + [(k_0 + k_1 + k_2 + k_3)^2 - (k_0 + k_1 + k_2)^2]P_3$$

$$L^2 = {l_0}^2 + \left[(l_0 + l_1)^2 - {l_0}^2\right]P_1 + [(l_0 + l_1 + l_2)^2 - (l_0 + l_1)^2]P_2 + [(l_0 + l_1 + l_2 + l_3)^2 - (l_0 + l_1 + l_2)^2]P_3$$

The equation $T^2 + S^2 + K^2 = L^2$ I equivalent to:

$${t_0}^2 + {s_0}^2 + {k_0}^2 = {l_0}^2 \quad (1)$$

$$(t_0 + t_1)^2 + (s_0 + s_1)^2 + (k_0 + k_1)^2 = (l_0 + l_1)^2 \quad (2)$$

$$(t_0 + t_1 + t_2)^2 + (s_0 + s_1 + s_2)^2 + (k_0 + k_1 + k_2)^2 = (l_0 + l_1 + l_2)^2 \quad (3)$$

$$(t_0 + t_1 + t_2 + t_3)^2 + (s_0 + s_1 + s_2 + s_3)^2 + (k_0 + k_1 + k_2 + k_3)^2 = (l_0 + l_1 + l_2 + l_3)^2 \quad (4)$$

Thus, the proof holds.

Theorem.

Let $(t_0, s_0, k_0, l_0), (t_1, s_1, k_1, l_1), (t_2, s_2, k_2, l_2), (t_3, s_3, k_3, l_3)$ be four Pythagoras quadruples in R, then the corresponding pythagoras quadruple in $3 - SP_R$ is (T, S, K, L), where:

$$T = t_0 + [t_1 - t_0]P_1 + [t_2 - t_1]P_2 + [t_3 - t_2]P_3$$

$$S = s_0 + [s_1 - s_0]P_1 + [s_2 - s_1]P_2 + [s_3 - s_2]P_3$$

$$K = k_0 + [k_1 - k_0]P_1 + [k_2 - k_1]P_2 + [k_3 - k_2]P_3$$

$$L = l_0 + [l_1 - l_0]P_1 + [l_2 - l_1]P_2 + [l_3 - l_2]P_3$$

Proof.

We must compute $T^2 + S^2 + K^2$,

$$T^2+S^2+K^2 = {t_0}^2+({t_1}^2-{t_0}^2)P_1+({t_2}^2-{t_1}^2)P_2+({t_3}^2-{t_2}^2)P_3+{s_0}^2$$
$$+({s_1}^2-{s_0}^2)P_1+({s_2}^2-{s_1}^2)P_2+({s_3}^2-{s_2}^2)P_3+{k_0}^2+\left({k_1}^2-{k_0}^2\right)P_1$$
$$+\left({k_2}^2-{k_1}^2\right)P_2+\left({k_3}^2-{k_2}^2\right)P_3$$
$$=\left({t_0}^2+{s_0}^2+{k_0}^2\right)+\left({t_1}^2+{s_1}^2+{k_1}^2-{t_0}^2-{s_0}^2-{k_0}^2\right)P_1$$
$$+\left({t_2}^2+{s_2}^2+{k_2}^2-{t_1}^2-{s_1}^2-{k_1}^2\right)P_2$$
$$+\left({t_3}^2+{s_3}^2+{k_3}^2-{t_2}^2-{s_2}^2-{k_2}^2\right)P_3$$
$$={l_0}^2+\left({l_1}^2-{l_0}^2\right)P_1+\left({l_2}^2-{l_1}^2\right)P_2+\left({l_3}^2-{l_2}^2\right)P_3=L^2$$

O that, the proof is complete.

Example:

We have $L_1=(1,-1,i,1), L_2=(i,1,-1,-1), L_3=(-i,-1,1,-1), L_4=(1,-i,-1,-1)$ are four Pythagoras quadruples in C.

The corresponding 3-plithogenic Pythagoras quadruple is (T,S,K,L), where:

$T=1+(-1+i)P_1-2iP_2+(1+i)P_3$

$S=-1+2P_1-2P_2+(1-i)P_3$

$K=i+(-1-i)P_1+2P_2-2P_3$

$L=1-2P_1+2P_2-2P_3$

On the other hand, we have:

$$T^2=1-2iP_1-4P_2+2iP_3+2(-1+i)P_1-4iP_2+2(1+i)P_3-4iP_2$$
$$+2(-1+i)(1+i)P_3-4i(1+i)P_3$$
$$=1+(-2i-2+2i)P_1+(-4-4i+4i+4)P_2$$
$$+(2i+2+2i-4-4i+4)P_3=1-2P_1+2P_3$$
$$S^2=1+4P_1+4P_2-2iP_3-4P_1+4P_2+2(-1+i)P_3-8P_2+4(1-i)P_3$$
$$-4(1-i)P_3$$
$$=1+(4-4)P_1+(4+4-8)P_2+(-2i-2+2i+4-4i-4+4i)P_3$$
$$=1-2P_3$$
$$K^2=-1+2P_1$$
$$L^2=1=T^2+S^2+K^2$$

Example.

Consider the following four Pythagoras quadruples in Z_2:

$L_1=(0,0,0,0), L_2=(1,1,1,1), L_3=(1,1,0,0), L_4=(0,0,1,1)$

For every quadruple $(L_i, L_j, L_s, L_k); 1 \leq i, j, , k \leq 4$, we can get a symbolic 3-plithogenic pythagoras quadruple.

We will find some symbolic 3-plithogenic Pythagoras quadruple in $3 - SP_{Z_2}$.

Let us discuss the following cases:

case (1).

$$(L_1, L_1, L_1, L_1): \begin{cases} Y_1 = 0 \\ \acute{Y}_1 = 0 \\ Y_1'' = 0 \\ Y_1''' = 0 \end{cases}$$

case (2).

$$(L_1, L_1, L_1, L_2): \begin{cases} Y_2 = P_3 \\ \acute{Y}_2 = P_3 \\ Y_2'' = P_3 \\ Y_2''' = P_3 \end{cases}$$

case (3).

$$(L_1, L_1, L_1, L_3): \begin{cases} Y_3 = P_3 \\ \acute{Y}_3 = P_3 \\ Y_3'' = 0 \\ Y_3''' = 0 \end{cases}$$

case (4).

$$(L_1, L_1, L_1, L_4): \begin{cases} Y_4 = 0 \\ \acute{Y}_4 = 0 \\ Y_4'' = P_3 \\ Y_4''' = P_3 \end{cases}$$

case (5).

$$(L_1, L_1, L_2, L_1): \begin{cases} Y_5 = P_2 + P_3 \\ \acute{Y}_5 = P_2 + P_3 \\ Y_5'' = P_2 + P_3 \\ Y_5''' = P_2 + P_3 \end{cases}$$

case (6).

$$(L_1, L_1, L_3, L_1): \begin{cases} Y_6 = P_2 + P_3 \\ \acute{Y}_6 = P_2 + P_3 \\ Y_6'' = P_2 + P_3 \\ Y_6''' = P_2 + P_3 \end{cases}$$

case (7).

$$(L_1, L_1, L_4, L_1): \begin{cases} Y_7 = 0 \\ \acute{Y}_7 = 0 \\ Y_7'' = P_2 + P_3 \\ Y_7''' = P_2 + P_3 \end{cases}$$

case (8).

$$(L_1, L_2, L_1, L_1): \begin{cases} Y_8 = P_1 + P_2 \\ \acute{Y}_8 = P_1 + P_2 \\ Y_8'' = P_1 + P_2 \\ Y_8''' = P_1 + P_2 \end{cases}$$

case (9).

$$(L_1, L_3, L_1, L_1): \begin{cases} Y_9 = P_1 + P_2 \\ \acute{Y}_9 = P_1 + P_2 \\ Y_9'' = 0 \\ Y_9''' = 0 \end{cases}$$

case (10).

$$(L_1, L_4, L_1, L_1): \begin{cases} Y_{10} = 0 \\ \acute{Y}_{10} = 0 \\ Y_{10}'' = P_1 + P_2 \\ Y_{10}''' = P_1 + P_2 \end{cases}$$

case (11).

$$(L_2, L_1, L_1, L_1): \begin{cases} Y_{11} = 1 + P_1 \\ \acute{Y}_{11} = 1 + P_1 \\ Y_{11}'' = 1 + P_1 \\ Y_{11}''' = 1 + P_1 \end{cases}$$

case (12).

$$(L_3, L_1, L_1, L_1): \begin{cases} Y_{12} = 1 + P_1 \\ \acute{Y}_{12} = 1 + P_1 \\ Y_{12}'' = 0 \\ Y_{12}''' = 0 \end{cases}$$

case (13).

$$(L_4, L_1, L_1, L_1): \begin{cases} Y_{13} = 0 \\ \acute{Y}_{13} = 0 \\ Y_{13}'' = 1 + P_1 \\ Y_{13}''' = 1 + P_1 \end{cases}$$

case (14).

$$(L_2, L_2, L_2, L_2): \begin{cases} Y_{14} = 1 \\ Y_{14}' = 1 \\ Y_{14}'' = 1 \\ Y_{14}''' = 1 \end{cases}$$

case (15).

$$(L_2, L_2, L_2, L_1): \begin{cases} Y_{15} = 1 + P_3 \\ Y_{15}' = 1 + P_3 \\ Y_{15}'' = 1 + P_3 \\ Y_{15}''' = 1 + P_3 \end{cases}$$

case (16).

$$(L_2, L_2, L_2, L_3): \begin{cases} Y_{16} = 1 \\ Y_{16}' = 1 \\ Y_{16}'' = 1 + P_3 \\ Y_{16}''' = 1 + P_3 \end{cases}$$

case (17).

$$(L_2, L_2, L_2, L_4): \begin{cases} Y_{17} = 1 + P_3 \\ Y_{17}' = 1 + P_3 \\ Y_{17}'' = 1 \\ Y_{17}''' = 1 \end{cases}$$

case (18).

$$(L_2, L_2, L_1, L_2): \begin{cases} Y_{18} = 1 + P_2 + P_3 \\ Y_{18}' = 1 + P_2 + P_3 \\ Y_{18}'' = 1 + P_2 + P_3 \\ Y_{18}''' = 1 + P_2 + P_3 \end{cases}$$

case (19).

$$(L_2, L_2, L_3, L_2): \begin{cases} Y_{19} = 1 \\ Y_{19}' = 1 \\ Y_{19}'' = 1 + P_3 \\ Y_{19}''' = 1 + P_3 \end{cases}$$

case (20).

$$(L_2, L_2, L_4, L_2): \begin{cases} Y_{20} = 1 + P_2 + P_3 \\ Y_{20}' = 1 + P_2 + P_3 \\ Y_{20}'' = 1 \\ Y_{20}''' = 1 \end{cases}$$

case (21).

$$(L_2, L_1, L_2, L_2): \begin{cases} Y_{21} = 1 + P_1 + P_2 \\ \acute{Y}_{21} = 1 + P_1 + P_2 \\ Y_{21}{}'' = 1 + P_1 + P_2 \\ Y_{21}{}''' = 1 + P_1 + P_2 \end{cases}$$

case (22).

$$(L_2, L_3, L_2, L_2): \begin{cases} Y_{22} = 1 \\ \acute{Y}_{22} = 1 \\ Y_{22}{}'' = 1 + P_1 + P_2 \\ Y_{22}{}''' = 1 + P_1 + P_2 \end{cases}$$

case (23).

$$(L_2, L_4, L_2, L_2): \begin{cases} Y_{23} = 1 + P_1 + P_2 \\ \acute{Y}_{23} = 1 + P_1 + P_2 \\ Y_{23}{}'' = 1 \\ Y_{23}{}''' = 1 \end{cases}$$

Permutation (24).

$$(L_1, L_2, L_2, L_2): \begin{cases} Y_{24} = P_1 \\ \acute{Y}_{24} = P_1 \\ Y_{24}{}'' = P_1 \\ Y_{24}{}''' = P_1 \end{cases}$$

case (25).

$$(L_3, L_2, L_2, L_2): \begin{cases} Y_{25} = 1 \\ \acute{Y}_{25} = 1 \\ Y_{25}{}'' = P_1 \\ Y_{25}{}''' = P_1 \end{cases}$$

case (26).

$$(L_4, L_2, L_2, L_2): \begin{cases} Y_{26} = P_1 \\ \acute{Y}_{26} = P_1 \\ Y_{26}{}'' = 1 \\ Y_{26}{}''' = 1 \end{cases}$$

case (27).

$$(L_3, L_3, L_3, L_3): \begin{cases} Y_{27} = 1 \\ \acute{Y}_{27} = 1 \\ Y_{27}{}'' = 0 \\ Y_{27}{}''' = 0 \end{cases}$$

case (28).

$$(L_3, L_3, L_3, L_1): \begin{cases} Y_{28} = 1 + P_3 \\ Y'_{28} = 1 + P_3 \\ Y_{28}'' = 0 \\ Y_{28}''' = 0 \end{cases}$$

case (29).

$$(L_3, L_3, L_3, L_2): \begin{cases} Y_{29} = 1 \\ Y'_{29} = 1 \\ Y_{29}'' = P_3 \\ Y_{29}''' = P_3 \end{cases}$$

case (30).

$$(L_3, L_3, L_3, L_4): \begin{cases} Y_{30} = 1 + P_3 \\ Y'_{30} = 1 + P_3 \\ Y_{30}'' = P_3 \\ Y_{30}''' = P_3 \end{cases}$$

case (31).

$$(L_3, L_3, L_1, L_3): \begin{cases} Y_{31} = 1 + P_3 \\ Y'_{31} = 1 + P_3 \\ Y_{31}'' = P_3 \\ Y_{31}''' = P_3 \end{cases}$$

case (32).

$$(L_3, L_3, L_2, L_3): \begin{cases} Y_{32} = 1 \\ Y'_{32} = 1 \\ Y_{32}'' = P_2 + P_3 \\ Y_{32}''' = P_2 + P_3 \end{cases}$$

case (33).

$$(L_3, L_3, L_4, L_3): \begin{cases} Y_{33} = 1 + P_2 + P_3 \\ Y'_{33} = 1 + P_2 + P_3 \\ Y_{33}'' = P_2 + P_3 \\ Y_{33}''' = P_2 + P_3 \end{cases}$$

case (34).

$$(L_3, L_1, L_3, L_3): \begin{cases} Y_{34} = 1 + P_2 + P_3 \\ Y'_{34} = 1 + P_2 + P_3 \\ Y_{34}'' = 0 \\ Y_{34}''' = 0 \end{cases}$$

case (35).

$$(L_3, L_2, L_3, L_3): \begin{cases} Y_{35} = 1 \\ Y_{35}' = 1 \\ Y_{35}'' = P_1 + P_2 \\ Y_{35}''' = P_1 + P_2 \end{cases}$$

case (36).

$$(L_3, L_4, L_3, L_3): \begin{cases} Y_{36} = 1 + P_1 + P_2 \\ Y_{36}' = 1 + P_1 + P_2 \\ Y_{36}'' = P_1 + P_2 \\ Y_{36}''' = P_1 + P_2 \end{cases}$$

case (37).

$$(L_1, L_3, L_3, L_3): \begin{cases} Y_{37} = P_1 \\ Y_{37}' = P_1 \\ Y_{37}'' = 0 \\ Y_{37}''' = 0 \end{cases}$$

case (38).

$$(L_2, L_3, L_3, L_3): \begin{cases} Y_{38} = 1 \\ Y_{38}' = 1 \\ Y_{38}'' = 1 + P_1 \\ Y_{38}''' = 1 + P_1 \end{cases}$$

case (39).

$$(L_4, L_3, L_3, L_3): \begin{cases} Y_{39} = P_1 \\ Y_{39}' = P_1 \\ Y_{39}'' = 1 + P_1 \\ Y_{39}''' = 1 + P_1 \end{cases}$$

case (40).

$$(L_4, L_4, L_4, L_4): \begin{cases} Y_{40} = 0 \\ Y_{40}' = 0 \\ Y_{40}'' = 1 \\ Y_{40}''' = 1 \end{cases}$$

case (41).

$$(L_4, L_4, L_4, L_1): \begin{cases} Y_{41} = 0 \\ Y_{41}' = 0 \\ Y_{41}'' = 1 + P_3 \\ Y_{41}''' = 1 + P_3 \end{cases}$$

case (42).

$$(L_4, L_4, L_4, L_2): \begin{cases} Y_{42} = P_3 \\ Y_{42}' = P_3 \\ Y_{42}'' = 1 \\ Y_{42}''' = 1 \end{cases}$$

case (43).

$$(L_4, L_4, L_4, L_3): \begin{cases} Y_{43} = P_3 \\ Y_{43}' = P_3 \\ Y_{43}'' = 1 + P_3 \\ Y_{43}''' = 1 + P_3 \end{cases}$$

case (44).

$$(L_4, L_4, L_1, L_4): \begin{cases} Y_{44} = 0 \\ Y_{44}' = 0 \\ Y_{44}'' = 1 + P_2 + P_3 \\ Y_{44}''' = 1 + P_2 + P_3 \end{cases}$$

case (45).

$$(L_4, L_4, L_2, L_4): \begin{cases} Y_{45} = P_2 + P_3 \\ Y_{45}' = P_2 + P_3 \\ Y_{45}'' = 1 \\ Y_{45}''' = 1 \end{cases}$$

case (46).

$$(L_4, L_4, L_3, L_4): \begin{cases} Y_{46} = P_2 + P_3 \\ Y_{46}' = P_2 + P_3 \\ Y_{46}'' = 1 + P_2 + P_3 \\ Y_{46}''' = 1 + P_2 + P_3 \end{cases}$$

case (47).

$$(L_4, L_1, L_4, L_4): \begin{cases} Y_{47} = 0 \\ Y_{47}' = 0 \\ Y_{47}'' = 1 + P_1 + P_2 \\ Y_{47}''' = 1 + P_1 + P_2 \end{cases}$$

case (48).

$$(L_4, L_2, L_4, L_4): \begin{cases} Y_{48} = P_1 + P_2 \\ Y_{48}' = P_1 + P_2 \\ Y_{48}'' = 1 \\ Y_{48}''' = 1 \end{cases}$$

case (49).

$$(L_4, L_3, L_4, L_4): \begin{cases} Y_{49} = P_1 + P_2 \\ \acute{Y_{49}} = P_1 + P_2 \\ Y_{49}'' = 1 + P_1 + P_2 \\ Y_{49}''' = 1 + P_1 + P_2 \end{cases}$$

case (50).

$$(L_1, L_4, L_4, L_4): \begin{cases} Y_{50} = 0 \\ \acute{Y_{50}} = 0 \\ Y_{50}'' = P_1 \\ Y_{50}''' = P_1 \end{cases}$$

case (51).

$$(L_2, L_4, L_4, L_4): \begin{cases} Y_{51} = 1 + P_1 \\ \acute{Y_{51}} = 1 + P_1 \\ Y_{51}'' = 1 \\ Y_{51}''' = 1 \end{cases}$$

case (52).

$$(L_3, L_4, L_4, L_4): \begin{cases} Y_{52} = 1 + P_1 \\ \acute{Y_{52}} = 1 + P_1 \\ Y_{52}'' = P_1 \\ Y_{52}''' = P_1 \end{cases}$$

case (53).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{53} = P_1 + P_3 \\ \acute{Y_{53}} = P_1 + P_3 \\ Y_{53}'' = P_1 + P_2 + P_3 \\ Y_{53}''' = P_1 + P_2 + P_3 \end{cases}$$

case (54).

$$(L_1, L_2, L_4, L_3): \begin{cases} Y_{54} = P_1 + P_2 + P_3 \\ \acute{Y_{54}} = P_1 + P_2 + P_3 \\ Y_{54}'' = P_1 + P_2 \\ Y_{54}''' = P_1 + P_2 \end{cases}$$

case (55).

$$(L_1, L_3, L_2, L_4): \begin{cases} Y_{55} = P_1 + P_3 \\ \acute{Y_{55}} = P_1 + P_3 \\ Y_{55}'' = P_2 \\ Y_{55}''' = P_2 \end{cases}$$

case (56).

$$(L_1, L_3, L_4, L_2): \begin{cases} Y_{56} = P_1 + P_2 + P_3 \\ \acute{Y}_{56} = P_1 + P_2 + P_3 \\ Y_{56}'' = P_2 \\ Y_{56}''' = P_2 \end{cases}$$

case (57).

$$(L_1, L_4, L_2, L_3): \begin{cases} Y_{57} = P_2 \\ \acute{Y}_{57} = P_2 \\ Y_{57}'' = P_1 + P_3 \\ Y_{57}''' = P_1 + P_3 \end{cases}$$

case (58).

$$(L_1, L_4, L_3, L_2): \begin{cases} Y_{58} = P_2 \\ \acute{Y}_{58} = P_2 \\ Y_{58}'' = P_1 + P_2 + P_3 \\ Y_{58}''' = P_1 + P_2 + P_3 \end{cases}$$

case (59).

$$(L_2, L_1, L_3, L_4): \begin{cases} Y_{59} = 1 + P_1 + P_2 + P_3 \\ \acute{Y}_{59} = 1 + P_1 + P_2 + P_3 \\ Y_{59}'' = 1 + P_1 + P_3 \\ Y_{59}''' = 1 + P_1 + P_3 \end{cases}$$

case (60).

$$(L_2, L_1, L_4, L_3): \begin{cases} Y_{60} = 1 + P_1 + P_3 \\ \acute{Y}_{60} = 1 + P_1 + P_3 \\ Y_{60}'' = P_2 + P_3 \\ Y_{60}''' = P_2 + P_3 \end{cases}$$

case (61).

$$(L_3, L_1, L_2, L_4): \begin{cases} Y_{61} = 1 + P_1 + P_2 + P_3 \\ \acute{Y}_{61} = 1 + P_1 + P_2 + P_3 \\ Y_{61}'' = P_2 \\ Y_{61}''' = P_2 \end{cases}$$

case (62).

$$(L_3, L_1, L_4, L_2): \begin{cases} Y_{62} = 1 + P_1 + P_3 \\ \acute{Y}_{62} = 1 + P_1 + P_3 \\ Y_{62}'' = P_2 \\ Y_{62}''' = P_2 \end{cases}$$

case (63).

$$(L_4, L_1, L_2, L_3): \begin{cases} Y_{63} = P_2 \\ Y_{63}' = P_2 \\ Y_{63}'' = 1 + P_2 + P_3 \\ Y_{63}''' = 1 + P_2 + P_3 \end{cases}$$

case (64).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{64} = P_2 \\ Y_{64}' = P_2 \\ Y_{64}'' = 1 + P_3 \\ Y_{64}''' = 1 + P_3 \end{cases}$$

case (65).

$$(L_2, L_3, L_1, L_4): \begin{cases} Y_{65} = 1 + P_2 \\ Y_{65}' = 1 + P_2 \\ Y_{65}'' = 1 + P_3 \\ Y_{65}''' = 1 + P_3 \end{cases}$$

case (66).

$$(L_2, L_3, L_4, L_1): \begin{cases} Y_{66} = 1 + P_2 \\ Y_{66}' = 1 + P_2 \\ Y_{66}'' = 1 + P_1 + P_2 + P_3 \\ Y_{66}''' = 1 + P_1 + P_2 + P_3 \end{cases}$$

case (67).

$$(L_3, L_2, L_1, L_4): \begin{cases} Y_{67} = 1 + P_2 \\ Y_{67}' = 1 + P_2 \\ Y_{67}'' = P_1 + P_2 + P_3 \\ Y_{67}''' = P_1 + P_2 + P_3 \end{cases}$$

case (68).

$$(L_3, L_2, L_4, L_1): \begin{cases} Y_{68} = 1 + P_2 \\ Y_{68}' = 1 + P_2 \\ Y_{68}'' = 1 + P_3 \\ Y_{68}''' = 1 + P_3 \end{cases}$$

case (64).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{64} = P_2 \\ Y_{64}' = P_2 \\ Y_{64}'' = P_1 + P_3 \\ Y_{64}''' = P_1 + P_3 \end{cases}$$

case (69).

$$(L_3, L_4, L_1, L_2): \begin{cases} Y_{69} = 1 + P_1 + P_3 \\ \acute{Y_{69}} = 1 + P_1 + P_3 \\ Y_{69}'' = P_1 + P_3 \\ Y_{69}''' = P_1 + P_3 \end{cases}$$

case (70).

$$(L_3, L_4, L_2, L_1): \begin{cases} Y_{70} = 1 + P_1 + P_2 + P_3 \\ \acute{Y_{70}} = 1 + P_1 + P_2 + P_3 \\ Y_{70}'' = P_1 + P_3 \\ Y_{70}''' = P_1 + P_3 \end{cases}$$

case (71).

$$(L_4, L_3, L_1, L_2): \begin{cases} Y_{71} = P_1 + P_2 + P_3 \\ \acute{Y_{71}} = P_1 + P_2 + P_3 \\ Y_{71}'' = 1 + P_3 \\ Y_{71}''' = 1 + P_3 \end{cases}$$

case (72).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{72} = P_1 + P_3 \\ \acute{Y_{72}} = P_1 + P_3 \\ Y_{72}'' = 1 + P_1 + P_2 + P_3 \\ Y_{72}''' = 1 + P_1 + P_2 + P_3 \end{cases}$$

case (73).

$$(L_4, L_2, L_1, L_3): \begin{cases} Y_{73} = P_1 + P_2 + P_3 \\ \acute{Y_{73}} = P_1 + P_2 + P_3 \\ Y_{73}'' = 1 + P_2 \\ Y_{73}''' = 1 + P_2 \end{cases}$$

case (74).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{74} = P_1 + P_3 \\ \acute{Y_{74}} = P_1 + P_3 \\ Y_{74}'' = 1 + P_2 \\ Y_{74}''' = 1 + P_2 \end{cases}$$

case (75).

$$(L_2, L_4, L_1, L_3): \begin{cases} Y_{75} = 1 + P_1 + P_3 \\ \acute{Y_{75}} = 1 + P_1 + P_3 \\ Y_{75}'' = 1 + P_2 \\ Y_{75}''' = 1 + P_2 \end{cases}$$

case (76).

$$(L_2, L_4, L_3, L_1): \begin{cases} Y_{76} = 1 + P_1 + P_2 + P_3 \\ Y'_{76} = 1 + P_1 + P_2 + P_3 \\ Y_{76}'' = 1 + P_2 \\ Y_{76}''' = 1 + P_2 \end{cases}$$

case (77).

$$(L_2, L_2, L_3, L_3): \begin{cases} Y_{77} = 1 \\ Y'_{77} = 1 \\ Y_{77}'' = 1 + P_2 \\ Y_{77}''' = 1 + P_2 \end{cases}$$

case (78).

$$(L_2, L_2, L_1, L_1): \begin{cases} Y_{78} = 1 + P_2 \\ Y'_{78} = 1 + P_2 \\ Y_{78}'' = 1 + P_2 \\ Y_{78}''' = 1 + P_2 \end{cases}$$

case (79).

$$(L_2, L_2, L_4, L_4): \begin{cases} Y_{79} = 1 + P_2 \\ Y'_{79} = 1 + P_2 \\ Y_{79}'' = 1 \\ Y_{79}''' = 1 \end{cases}$$

case (80).

$$(L_1, L_1, L_2, L_2): \begin{cases} Y_{80} = P_2 \\ Y'_{80} = P_2 \\ Y_{80}'' = P_2 \\ Y_{80}''' = P_2 \end{cases}$$

Conclusion.

In this paper, we have studied Pythagoras quadruples in symbolic 3-plithogenic commutative rings, where necessary and sufficient conditions for a symbolic 3-plithogenic quadruple (x, y, z, t) to be a Pythagoras quadruple.

Also, we have presented some related examples that explain how to find 3-plithogenic quadruples from classical triples.

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