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On Treating Input Oriented Data Envelopment Analysis Model under Neutrosophic Environment

Basma E. El- Demerdash [1], S. A. Edalatpanah[2*], and Hamiden Abd El- Wahed Khalifa[1,3*]

[1]Department of Operations and Management Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza 12613, Egypt hamiden@cu.edu.eg/ basma.ezzat@cu.edu.eg

[2]Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Iran

[3]Department of Mathematics, College of Science and Arts, Al- Badaya 51951, Qassim University, Saudi Arabia, Ha.Ahmed@qu.edu.sa

* Correspondence: hamiden@cu.edu.eg

Abstract: Data Envelopment Analysis (DEA) stands out as the most commonly employed approach for assessing the overall performance of a group of similar Decision-Making Units (DMUs) that utilize similar resources to produce comparable outputs. Nonetheless, the observed characteristics of symmetry or asymmetry in various types of data in real-world applications can often be imprecise, unclear, insufficient, or contradictory. Neglecting these conditions can potentially result in erroneous decision-making. Certain models take a more restrictive approach by assuming that inputs and outputs possess the same level of determinism. Regrettably, such constraints don't hold true for the majority of real-world scenarios. In actual situations, however, the observed input and output data may sometimes be neutrosophic numbers. So, the primary purpose of this study is to construct a Neutrosophic Input Oriented DEA (NIODEA) Model that incorporates both neutrosophic and deterministic output and/or input variables, handled in accordance with the scoring function. The model we have developed has broad applicability across diverse organizations, aiding decision-makers in making informed choices and optimizing resource allocation, a particularly valuable asset in today's intensely competitive business environment. To underscore the practical utility of the model, we provide an illustrative example that demonstrates its effectiveness and relevance for decision-makers.

Keywords: Optimization**,** Data Envelopment Analysis; Neutrosophic Variables; Single Valued Neutrosophic; Neutrosophic Score Function; Performance Measure; Efficiency Analysis; Decision Making.

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1. Introduction

The concepts of efficiency are utilized to determine whether restrictions have an impact and if so, how substantial. Efficiency is primarily defined as an organization's capacity to produce the greatest amount of output from a given set of inputs [1]. Data Envelopment Analysis (DEA) firstly developed by Charnes et al. [2], a method to assess the effectiveness of decision-making units (DMUs), is a potent analytical instrument for effective and benchmarking evaluation. Almost all applications, including healthcare, banking, transportation, and education, use DEA for various factors, as Golany and Roll [3] observed that it may be utilized to determine the reasons of inefficiency, DMUs ranking, and measure the effectiveness of programs. About 30 years after the landmark study [2] was published, the application field of DEA It has grown to the point that virtually none of the researchers in DEA field can keep up with its progress, especially in terms of how frequently DEA is employed in practical applications.

The DEA approach has various strengths, one of them is that it doesn't need a preference weight or a particular link between the multiple inputs and outputs. Nevertheless, one of the major significant shortcomings of standard DEA issues is that they do not permit for vague variance in the multiple inputs and outputs, even though that many crucial real-world situations may be of the fuzzy form. Consequently, the DEA model's efficiency ratings may be susceptible to various fluctuations in factors. An efficient DMU that is relative efficient to other comparable DMUs may become inefficient if such ambiguous, confusing, inconsistent, and incomplete variance in variables including inputs, outputs, or perhaps both. In other words, because efficiency scores are highly sensitive to the actual levels of inputs or outputs, they will be inaccurate and misleading if the data gained is not displayed in the proper form.

The DEA models have made great attempts in recent years to address the ambiguity in variables, whether fuzzy input or output. Commonly, the applicability of the fuzzy DEA model is split into four categories α-cut, tolerance, possibility, and fuzzy ranking approaches [4 - 8]. The α-cut approach is regarded as the most common fuzzy DEA issue [9 – 17]. Fuzzy sets, however, only take into account the membership function (MF) and are unable to set further vagueness parameters. As a result, Pythagorean fuzzy sets have also been introduced in [18], along with intuitionistic fuzzy sets. Smarandache [19] proposed neutrosophic set theory; it is an extension of fuzzy set, since each element has a truth, indeterminacy, and falsity membership function. Neutrosophic set has been employed for solving models including indeterminacy, uncertainty, imprecision, ambiguity, inconsistency, and incompleteness, among others. Moreover, there are multiple approaches exist for addressing different issues within a neutrosophic environment such that Haque et al. [20], Pal et al. [21], Haque et al. [22], Chakraborty et al. [23], Singh et al. [24], Jdid and Smarandache [25], Singh et al. [26], Sasikala and Divya [27], Gamal and Mohamed [28].

Recent attempts have been made to include neutrosophic data into the DEA model, either as neutrosophic input or neutrosophic output. Edalatpanah [29] devolved a new form of

DEA involved neutrosophic input and output. Abdelfattah [30] provided a DEA model using triangular neutrosophic for both inputs and outputs variables that takes the truth, indeterminacy, and falsity degrees of each data value into consideration. Kahraman et al. [31] introduced a novel Neutrosophic Analytic Hierarchy Process that was subsequently combined with neutrosophic DEA to be employed in performance evaluation. All inputs and outputs in the DEA model suggested by Yang et al. [32] are single-valued neutrosophic triangular numbers. Mao, et al. [33] proposed a neutrosophic DEA model with undesirable outputs, it is constructed simply and is based on the aggregation operator.

Motivation and contribution

In real-world scenarios, it's not uncommon for observed values of inputs and/or outputs to exhibit neutrosophic characteristics. However, one of the significant limitations of the traditional DEA model is its inability to account for uncertainty or variations in input and output data. It assumes that all data are precisely known or represented as crisp values. Consequently, DEA efficiency measurements can be highly sensitive to such variations. A DMU that appears efficient relative to others may become inefficient when these uncertainties are considered, or vice versa. In other words, if the collected data for a variable are not accurately represented in their true neutrosophic nature, the resulting efficiency scores can be inaccurate and misleading due to their sensitivity to the actual levels of inputs or outputs. Additionally, like any empirical technique, DEA relies on simplifying assumptions that researchers must acknowledge when interpreting the results. Recent research in DEA has aimed to address these limitations, but certain challenges persist. Firstly, the developed DEA models are not universally applicable for handling both deterministic variables and variables with neutrosophic variations. Secondly, the DEA models designed to accommodate neutrosophic variables often assume that all variables (whether inputs or outputs) share the same neutrosophic nature.

Our primary focus is on assessing the performance of comparable DMUs with the goal of ensuring quality, identifying areas of deficiency, and ultimately enhancing their efficiency. Given this problem context, the principal objective of this research is to develop a Neutrosophic Input-oriented Data Envelopment Analysis (NIODEA) model. This model will account for a blend of neutrosophic and deterministic input and/or output variables, effectively addressing the complexities of real-world scenarios.

The remaining sections are categorized as follows. Some definitions pertaining to the triangular neutrosophic fuzzy number are introduced in the next section. The third section talks over the conventional DEA models. The suggested NIODEA model is presented in the fourth section. The next section includes an illustrative example. The study concludes with the customary findings and the future implications.

2. Preliminaries

This section gives some brief overview for essential definitions of triangular neutrosophic concept to help in understanding the proposed model.

Neutrosophic theory, a groundbreaking branch of mathematics and philosophy, ventures into the heart of uncertainty, ambiguity, and imprecision. It grapples with the fundamental notion that in the real world, many phenomena are not entirely true or false, but rather possess shades of truth, falsity, and indeterminacy. Traditional mathematics, rooted in classical logic, often struggles to capture the complexity of such situations. At the core of Neutrosophic theory are three fundamental components: neutrosophic sets, neutrosophic logic, and neutrosophic probability. These concepts provide a powerful framework for dealing with indeterminate and contradictory information, opening doors to a deeper understanding of complex systems and uncertain data.

Neutrosophic sets allow us to represent elements with imprecise or conflicting attributes, offering a flexible alternative to the crisp sets of classical mathematics. Neutrosophic logic extends this flexibility by embracing degrees of truth, falsity, and indeterminacy in reasoning, enabling more realistic and nuanced decision-making. Neutrosophic probability, in turn, quantifies the likelihood of neutrosophic events, offering a richer perspective on uncertainty compared to traditional probability theory. Let W to be a set of positive real numbers coupled with a variable, and w to be a general element of W. A fuzzy set A in W is defined mathematically as the collection of ordered pairs: "$A = \{(w, \mu_A(w)) \mid w \epsilon W\}$, where μ_A: is the MF and usually assumed to vary in the interval [0,1]".

A MF is a mapping that allocates $\forall\ w \in W$ a number, $\mu_A(w) \in [0,1]$ and represents the membership degree of w in A. The closer value of $\mu_A(w)$ is to one, the largest membership of w in A. Hence, a fuzzy set A may be accurately described by associating a number ranging from 0 to 1 with each element w, which indicates its membership degree in A. The MF of a fuzzy set A may also be denoted by $A(w)$ [34].

Definition 1: [13] A fuzzy number $\widetilde{w}_i = (w^1, w^2, w^3)$, where $w^1 \le w^2 \le w^3$ on $\mathbb{R}$ is a triangular fuzzy number if its MF define as follows:

$$\mu_{\widetilde{w}_i} = \begin{cases} 0 & , y \le w^1 \\ \frac{y-w^1}{w^2-w^1} & , w^1 < y \le w^2 \\ 1 & , x = w^2 \\ \frac{w^3-y}{w^3-w^2} & , w^2 < y \le w^3 \\ 0 & , y \ge w^3. \end{cases}$$

(1)

Definition 2: [35] Let us denote the space of objects by Y and its generic element as y, $y \in Y$. The neutrosophic set of $\tilde{Q}^M$ has the form $\tilde{Q}^M = \{\langle y: T_{\tilde{Q}^M}(y), I_{\tilde{Q}^M}(y), F_{\tilde{Q}^M}(y)\rangle, y \in Y, T_{\tilde{Q}^M}(y), I_{\tilde{Q}^M}(y), F_{\tilde{Q}^M}(y) \in]0^-, 1^+[\}$, where $T_{\tilde{Q}^M}(y), I_{\tilde{Q}^M}(y), F_{\tilde{Q}^M}(y)$ are truth, indeterminacy, falsity MFs with the no restriction condition on their sum, $0^- \le T_{\tilde{Q}^M}(y) + I_{\tilde{Q}^M}(y) + F_{\tilde{Q}^M}(y) \le 3^+$, and $]0^-, 1^+[$ is an irregular unit interval.

Definition 3: [35] A single valued neutrosophic set $\tilde{Q}^{SVN}$ of a nonempty set y is constructed as $\tilde{Q}^{SVN} = \{\langle y, T_{\tilde{Q}^M}(y), I_{\tilde{Q}^M}(y), F_{\tilde{Q}^M}(y)\rangle, y \in Y\}$, where $T_{\tilde{Q}^M}(y), I_{\tilde{Q}^M}(y)$, and $F_{\tilde{Q}^M}(y) \in [0,1]$ and $0 \leq T_{\tilde{Q}^M}(y) + I_{\tilde{Q}^M}(y) + F_{\tilde{Q}^M}(y) \leq 3$.

Definition 4: [36] Let $\mathcal{L}_{\tilde{s}}, \delta_{\tilde{s}}, \mathcal{F}_{\tilde{s}} \in [0,1]$ $w^1, w^2, w^3 \in \mathbb{R}$ such that $w^1 \leq w^2 \leq w^3$. Then a single valued triangular fuzzy neutrosophic set (SVTFN), $\tilde{s}^{TN} = \langle (w^1, w^2, w^3); \mathcal{L}_{\tilde{s}}, \delta_{\tilde{s}}, \mathcal{F}_{\tilde{s}}\rangle$ is a special neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy, falsity MFs are:

$$T_{\tilde{Q}^M}(y) = \begin{cases} 0 & , y < w^1 \\ \frac{(y-w^1)\mathcal{L}_{\tilde{s}^{TN}}}{w^2-w^1} & , w^1 \leq y \leq w^2 \\ \frac{(c-y)\mathcal{L}_{\tilde{s}^{TN}}}{w^3-w^2} & , w^2 \leq y \leq w^3 \\ 0 & , y > w^3 \end{cases} \quad (2)$$

$$I_{\tilde{Q}^M}(y) = \begin{cases} 0 & , y < w^1 \\ \frac{(w^2-y)+(y-w^1)\delta_{\tilde{s}^{TN}}}{w^2-w^1} & , w^1 < y \leq w^2 \\ \frac{(y-w^2)+(w^3-y)\delta_{\tilde{s}^{TN}}}{w^3-w^2} & , w^2 < y \leq w^3 \\ 0 & , w^3 > c \end{cases} \quad (3)$$

$$F_{\tilde{Q}^M}(y) = \begin{cases} 0 & , y < w^1 \\ \frac{(w^2-y)+(y-w^1)\mathcal{F}_{\tilde{s}^{TN}}}{w^2-w^1} & , w^1 < y \leq w^2 \\ \frac{(y-w^2)+(w^3-y)\mathcal{F}_{\tilde{s}^{TN}}}{w^3-w^2} & , w^2 < y \leq w^3 \\ 0 & , y > w^3 \end{cases} \quad (4)$$

***Definition 5*:** [36] let $\tilde{v}^{TN} = \langle (a,b,c); \mathcal{L}_{\tilde{v}^{TN}}, \delta_{\tilde{v}^{TN}}, \mathcal{F}_{\tilde{v}^{TN}}\rangle$ be SVTFN, then

1. Score function $SF(\tilde{v}^{TN}) = \left(\frac{1}{4}(a+2b+c)\right)\left(\frac{1}{3}(2+\mathcal{L}_{\tilde{v}^{TN}} - \delta_{\tilde{v}^{TN}} - \mathcal{F}_{\tilde{v}^{TN}})\right)$ (5)

2. Accuracy function $AF(\tilde{v}^{TN}) = \left(\frac{1}{4}(a+2b+c)\right)\left(\frac{1}{3}(2+\mathcal{L}_{\tilde{v}^{TN}} - \delta_{\tilde{v}^{TN}} + \mathcal{F}_{\tilde{v}^{TN}})\right)$ (6)

***Definition 6*:** [35] let $\tilde{v}^{TN} = \langle (a,b,c); \mathcal{L}_{\tilde{v}^{TN}}, \delta_{\tilde{v}^{TN}}, \mathcal{F}_{\tilde{v}^{TN}}\rangle$ and $\tilde{u}^{TN} = \langle (a_1,b_1,c_1); \mathcal{L}_{\tilde{u}^{TN}}, \delta_{\tilde{u}^{TN}}, \mathcal{F}_{\tilde{u}^{TN}}\rangle$ be two SVTFN, the arithmetic operations on $\tilde{v}^{TN}$ and $\tilde{u}^{TN}$ as follows:

1. $\tilde{v}^{TN} \oplus \tilde{u}^{TN} = \langle (a+a_1, b+b_1, c+c_1,); \mathcal{L}_{\tilde{v}^{TN}} \wedge \mathcal{L}_{\tilde{u}^{TN}}, \delta_{\tilde{v}^{TN}} \vee \delta_{\tilde{u}^{TN}}, \mathcal{F}_{\tilde{v}^{TN}} \vee \mathcal{F}_{\tilde{u}^{TN}}\rangle$ (7)
2. $\tilde{v}^{TN} \ominus \tilde{u}^{TN} = \langle (a-a_1, b-b_1, c-c_1,); \mathcal{L}_{\tilde{v}^{TN}} \wedge \mathcal{L}_{\tilde{u}^{TN}}, \delta_{\tilde{v}^{TN}} \vee \delta_{\tilde{u}^{TN}}, \mathcal{F}_{\tilde{v}^{TN}} \vee \mathcal{F}_{\tilde{u}^{TN}}\rangle$ (8)
3. $n\tilde{v}^{TN} = \begin{cases} \langle (na, nb, nc); \mathcal{L}_{\tilde{v}^{TN}}, \delta_{\tilde{v}^{TN}}, \mathcal{F}_{\tilde{v}^{TN}}\rangle, n > 0 \\ \langle (nc, nb, na); \mathcal{L}_{\tilde{v}^{TN}}, \delta_{\tilde{v}^{TN}}, \mathcal{F}_{\tilde{v}^{TN}}\rangle, n < 0 \end{cases}$ (9)
4. ${\tilde{v}^{TN}}^{-1} = \langle (a^{-1}, b^{-1}, c^{-1}); \mathcal{L}_{\tilde{v}^{TN}}, \delta_{\tilde{v}^{TN}}, \mathcal{F}_{\tilde{v}^{TN}}\rangle, \tilde{v}^{TN} \neq 0$ (10)

Definition 7: [35] the order relation between $\tilde{v}^{TN}$ and $\tilde{u}^{TN}$ based on score and accuracy functions are:

1. If $SF(\tilde{v}^{TN}) > SF(\tilde{u}^{TN})$, then $\tilde{v} > \tilde{u}$
2. If $SF(\tilde{v}^{TN}) < SF(\tilde{u}^{TN})$, then $\tilde{v} < \tilde{u}$
3. If $SF(\tilde{v}^{TN}) = SF(\tilde{u}^{TN})$, then
   a) If $AF(\tilde{v}^{TN}) > AF(\tilde{u}^{TN})$, then $\tilde{v} > \tilde{u}$
   b) If $AF(\tilde{v}^{TN}) < AF(\tilde{u}^{TN})$, then $\tilde{v} < \tilde{u}$
   c) If $AF(\tilde{v}^{TN}) = AF(\tilde{u}^{TN})$, then $\tilde{v} = \tilde{u}$

3. DEA Mathematical Model

DEA's essential model with 'n' DMUs, 'J' inputs and 'S' outputs was first introduced in [2]. The model provides the relative efficiency scores for all DMUs and it hinges on optimizing a DEA-estimated production function, it is a deterministic frontier function. The DEA estimate value for all inputs provides the maximum output that can be achieved from inputs under all conditions. Conversely, for any outputs, the DEA value estimate the minimum input achieving a certain output under all scenarios. In this regard, it resembles the parametric frontier with one-sided deviations determined utilizing mathematical programming techniques.

The DEA model may be categorized as either having constant returns to scale (CRS) or variable returns to scale (VRS) based on the assumptions connecting the change in outputs to the change in inputs (VRS). In CRS models, the outputs are not impacted by the size of the DMU; rather, they vary directly proportional to the change in inputs, assuming that the scale of operation has no effect on efficiency; hence, output and input oriented measures of efficiency are equivalent. In VRS models, changes in outputs are not always proportionate to changes in inputs; hence, output and input oriented measures of efficiency scores for inefficient units are not equivalent [37]. This work focuses on the input-oriented VRS model, which may be described as follows:

$$
\begin{aligned}
\text{Min} \quad & Z_p = \theta \\
\text{s.t.} \quad & \\
& \sum_{i=1}^{n} \lambda_i x_{ij} \leq \theta x_{pj} \qquad ,\forall \; j = 1, \dots J \\
& \sum_{i=1}^{n} \lambda_i y_{is} \geq y_{ps} \qquad ,\forall \; s = 1, \dots S \\
& \sum_{i=1}^{n} \lambda_i = 1 \\
& \lambda_i \geq 0 \qquad ,\forall \; i = 1, \dots n
\end{aligned}
\tag{11}
$$

Where θ is the efficiency score of DMU p; s is the no. of outputs, $1 \leq s \leq S$; j is the no. of inputs, $1 \leq j \leq J$; i is the no. of DMUs, $1 \leq i \leq n$; y_{is} is the amount of outputs produced by the

i^{th} DMU; x_{ij} is the amount of the j^{th}input utilized by the i^{th}DMU; and λ_i is the weight of the i^{th}DMU.

4. Developed neutrosophic input-oriented Data Envelopment Analysis Model

Now we are going to formulate a NIODEA model in order to evaluate quality by comparing the performance of similar organizations, assuming that some of the input and/or output variables may be in neutrosophic settings. Here, we introduce our modification to the conventional DEA model in order to evaluate relative efficiency in the case of neutrosophic variation in a portion of the outputs and/or inputs. The constructed NIODEA model relies on the score function. The restriction affecting some of the input and/or output values in the DEA model will be a neutrosophic inequality that may sometimes be violated. Since an inequality incorporating several neutrosophic variables may never be established with crisp. The suggested model consists of three stages. First, the MF for neutrosophic input and output variables is specified. Finding the score and accuracy function for neutrosophic variables based on the MF is the second stage. In the third step, each DMU's relative efficiency score is determined. The NIODEA model for evaluating the efficiency level of p^{th} DMU is as follows:

$$\text{Min} \quad \tilde{Z}_p^{\,TN} = \theta$$

$s.t.$

$$\sum_{i=1}^{n} \lambda_i x_{ij} \;\leq\; \theta x_{pj} \qquad ,\forall j \in J_D$$

$$\sum_{i=1}^{n} \lambda_i \tilde{x}_{ij}^{\,TN} \;\leq\; \theta \tilde{x}_{pj}^{\,TN} \qquad ,\forall j \in J_N$$

$$\sum_{i=1}^{n} \lambda_i y_{is} \geq y_{ps} \qquad ,\forall s \in S_D$$

$$\sum_{i=1}^{n} \lambda_i \tilde{y}_{is}^{\,TN} \geq \tilde{y}_{ps}^{\,TN} \qquad ,\forall s \in S_N$$

$$\sum_{i=1}^{n} \lambda_i = 1$$

$$\lambda_i \;\geq 0, \qquad (i = 1{:}n)$$

where J_D is the deterministic inputs set, J_N is the neutrosophic inputs set, J is the total inputs set, i.e., $J_D \cup J_N = J$. S_D is the deterministic outputs set, S_N is the neutrosophic outputs set, and S is total outputs set, $S_D \cup S_N = S$.

Comparing model (11) to model (12), it is clear that each of the two constraints controlling the inputs and outputs is split into two constraints in order to manage the deterministic variables separately from the neutrosophic variables.

In the suggested model, it is assumed that the neutrosophic variables have triangle MFs. Depending on the score function described in Section 2, the triangular NIODEA model was transformed to a standard DEA model that can be solved easily.

Min $Z_p = \theta$

$s.t.$

$$\sum_{i=1}^{n} \lambda_i x_{ij} \leq \theta x_{pj} \quad ,\forall j \in J_D$$

$$\sum_{i=1}^{n} \lambda_i SF(\tilde{x}_{ij}{}^{TN}) \leq \theta SF(\tilde{x}_{pj}{}^{TN}) \quad ,\forall j \in J_N$$

$$\sum_{i=1}^{n} \lambda_i y_{is} \geq y_{ps} \quad ,\forall s \in S_D$$

$$\sum_{i=1}^{n} \lambda_i SF(\tilde{y}_{is}{}^{TN}) \geq SF(\tilde{y}_{ps}{}^{TN}) \quad ,\forall s \in S_N$$

$$\sum_{i=1}^{n} \lambda_i = 1$$

$$\lambda_i \geq 0, \quad (i = 1{:}n)$$

5. Illustrative Example

In this section, a numerical model is employed to demonstrate the application of the improved model. Seven DMUs (D1, D2, …., D7) with three inputs (N1, N2 and N3) two are deterministic (N1 and N2) and (N3) is neutrosophic. The outputs are O1 and O2, where (O1) is deterministic and (O2) is neutrosophic. The values are considered in the following hypothetical example. The data for deterministic variables are presented in Table 1, while those for neutrosophic variables are given in Table 2. The objective of this problem is to evaluate the relative efficiency of the DMUs using the NIODEA model that we have developed.

Before formulating and solving the problem, we must compute the score function for each neutrosophic variable (input or output). Table 3 presents the computed values.

Table 1 Hypothetical data for the DMUs' deterministic variables

DMUs	Inputs		Output
	N 1	N 2	O 1
D1	6.11	4.36	0.21
D2	3.66	2.54	0.12

D3	1.44	0.48	0.14
D4	1.21	0.23	0.10
D5	2.75	1.40	0.10
D6	4.18	2.74	0.06
D7	6.39	3.36	0.18

Table 2 Hypothetical neutrosophic variables data for DMUs

DMUs	Input	Output
	N 3	O 2
D1	$\langle(1.76, 7.27, 12.27); 0.9, 0.4, 0.1\rangle$	$\langle(0.12, 0.19, 0.27); 1.0, 0.0, 0.0\rangle$
D2	$\langle(3.85, 4.65, 5.53); 0.9, 0.7, 0.1\rangle$	$\langle(0.00, 0.10, 0.24); 1.0, 0.0, 0.0\rangle$
D3	$\langle(1.33, 1.88, 3.38); 0.9, 0.4, 0.1\rangle$	$\langle(0.05, 0.10, 0.16); 1.0, 0.0, 0.0\rangle$
D4	$\langle(0.78, 1.48, 2.06); 0.8, 0.5, 0.1\rangle$	$\langle(0.00, 0.06, 0.16); 1.0, 0.0, 0.0\rangle$
D5	$\langle(3.22, 3.63, 4.61); 0.8, 0.5, 0.2\rangle$	$\langle(0.02, 0.07, 0.17); 1.0, 0.0, 0.0\rangle$
D6	$\langle(4.30, 6.13, 8.03); 0.9, 0.5, 0.2\rangle$	$\langle(0.00, 0.06, 0.15); 1.0, 0.0, 0.0\rangle$
D7	$\langle(4.40, 8.00, 10.68); 0.9, 0.4, 0.1\rangle$	$\langle(0.06, 0.17, 0.30); 1.0, 0.0, 0.0\rangle$

Table 3 Score functions of N3, O2

DMUs	Input	Output
	N 3	O 2
D1	5.71	0.19
D2	3.27	0.11
D3	1.69	0.10
D4	1.06	0.07
D5	2.64	0.08
D6	4.51	0.07
D7	6.22	0.18

Each DMU requires a linear programming formulation to evaluate its relative efficiency. Below is D1's linear programming model.

$$
\begin{aligned}
&\text{Min } Z_A = \theta \\
&s.t. \\
&6.11\lambda_{D1} + 3.66\lambda_{D2} + 1.44\lambda_{D3} + 1.21\lambda_{D4} + 2.75\lambda_{D5} + 4.18\lambda_{D6} + 6.39\lambda_{D7} \leq 6.11\theta \\
&4.36\lambda_{D1} + 2.54\lambda_{D2} + 0.48\lambda_{D3} + 0.23\lambda_{D4} + 1.04\lambda_{D5} + 2.74\lambda_{D6} + 3.36\lambda_{D7} \leq 4.36\theta \\
&5.716\lambda_{D1} + 3.27\lambda_{D2} + 1.69\lambda_{D3} + 1.06\lambda_{D4} + 2.64\lambda_{D5} + 4.51\lambda_{D6} + 6.22\lambda_{D7} \leq 5.71\theta \\
&0.21\lambda_{D1} + 0.12\lambda_{D2} + 0.14\lambda_{D3} + 0.10\lambda_{D4} + 0.10\lambda_{D5} + 0.06\lambda_{D6} + 0.18\lambda_{D7} \geq 0.21 \\
&0.19\lambda_{D1} + 0.11\lambda_{D2} + 0.10\lambda_{D3} + 0.07\lambda_{D4} + 0.08\lambda_{D5} + 0.07\lambda_{D6} + 0.18\lambda_{D7} \geq 0.19 \\
&\lambda_{D1} + \lambda_{D2} + \lambda_{D3} + \lambda_{D4} + \lambda_{D5} + \lambda_{D6} + \lambda_{D7} = 1 \\
&\lambda_{Di} \geq 0, \ (i = 1{:}7).
\end{aligned}
\tag{14}
$$

Furthermore, relative efficiency models are formulated for DMUs D2 to D7. The models are then solved using GAMS software. The relative efficiency of each DMU is listed in Table 4.

Table 4 Relative efficiency

DMUs	NIODEA	Fuzzy IODEA	Stochastic IODEA	Deterministic IODEA
D1	1	1	0.59	1

D2	0.65	0.36	1	0.40
D3	1	1	1	1
D4	1	1	1	1
D5	0.48	0.44	1	0.46
D6	0.29	0.29	0.99	0.29
D7	1	0.80	0.84	1

The provided table displays the efficiency scores for each DMUs obtained from the NIODEA model. A careful look at the efficiency scores for the seven DMUs reveals that four are efficient (DMUs D1, D3, D4, and D7), while the other three are inefficient (DMUs D2, D5, and D6). Two of the three ineffective DMUs are quite unproductive (D5 and D6). We provided suggestions for improving the inefficient DMUs to enhance its performance by conduct a comprehensive analysis with efficient DMUs to identify the factors causing inefficiency and then explore ways to optimize resource utilization or improve the quality of outputs.

Comparatively, we also designed relative efficiency models for three distinct cases: the first with the neutrosophic variables considered fuzzy, the second with the neutrosophic variables considered stochastic, and the third with all variables considered deterministic. The models established by Tharwat et al. [17] and El-Demerdash et al. [38] were applied to the first and second cases, respectively. In the first scenario, we assume the three values for the triangular neutrosophic variable to represent the values for the triangle fuzzy variable so that the fuzzy IODEA model may be executed. To run the stochastic IODEA model for the stochastic variables in the second scenario, we averaged the three values for the neutrosophic function to represent the mean and assumed the variance and covariance between DMUs. In the last scenario, the neutrosophic variables' average values were used as the deterministic values. Table 4 also displays the relative efficacy of these three cases.

Table 4 demonstrates that the nature of the variables may have a significant impact on the relative efficiency of the DMUs. As seen by the data in Table 4, several DMUs have altered their status from efficient to inefficient and conversely. DMUs (D_1, D_3, D_4, and D_7) consistently exhibit high efficiency scores (close to or equal to 1) across all models. This suggests that they are consistently efficient regardless of the modeling approach used. DMU D_2 displays a variable level of efficiency across different models. It is efficient in the Stochastic IODEA model but less so in the Fuzzy IODEA and Deterministic IODEA models. This highlights the sensitivity of its efficiency to the modeling methodology. DMUs (D_5 and D_6) demonstrate consistently lower efficiency scores across all models, indicating a need for improvement in their performance. Therefore, to get accurate findings about the efficacy and inefficacy of the investigated DMUs, it is essential to identify the precise nature of the variables. In addition, the results indicate that the NIODEA model, due to its integration of various uncertainty dimensions, may offer a more comprehensive yet complex view of efficiency. The Fuzzy IODEA model tends to provide lower efficiency scores and may be less suitable for these DMUs. The Stochastic IODEA model appears to be sensitive to variations, assigning high efficiency scores even for DMUs that are less efficient than other models. Finally, the NIODEA model yields more accurate and reliable results than the classic DEA model and its variations, such as the fuzzy and stochastic DEA models.

6. Conclusion and Future Work

In this research, we introduce a novel approach, the NIODEA model, designed to handle both deterministic and neutrosophic variables. This innovative model, utilizing a specified scoring function and triangular Membership Functions (MFs) for neutrosophic variables, enables us to effectively assess the relative efficiency of Decision-Making Units (DMUs). The illustrative example highlights the profound impact of the inherent characteristics of variables on the determination of relative efficiency. It demonstrates how variables can shift the status of DMUs from efficient to inefficient, and vice versa. This underscores the critical importance of precisely defining variable structures and selecting the appropriate DEA model to ensure the generation of dependable results.

Our research emphasizes the sensitivity of DEA efficiency measurements to changes in variable nature. An initially efficient DMU, relative to others, can become inefficient when uncertainty variations are considered, and conversely, due to the high sensitivity of efficiency scores to variable levels of inputs or outputs. Hence, it is imperative to discern the nature of variables from the outset and apply the most suitable DEA model to attain accurate and reliable outcomes. By implementing the four different models in our illustrative example, we observed similarities in efficient DMUs and disparities in inefficient DMUs regarding their efficiency levels. As part of our future research agenda, we intend to apply the developed NIODEA model to real-world scenarios, thereby enhancing its practicality and relevance. Additionally, we aim to augment the model's versatility by exploring alternative MFs for neutrosophic variables. Our ongoing work will concentrate on the development of an integrated IODEA model capable of handling deterministic, neutrosophic, and stochastic variables, further contributing to the field of decision analysis.

Data Availability

In this article, no data were used.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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