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On Certain Operations on Strong Interval Valued Neutrosophic Graph with Application in the Cardiac Functioning of the Human Heart

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Abstract. The neutrosophic theory is known for its prominent application in real life, possessing unclear, indeterminate information. Interval valued neutrosophic theory is even more flexible to handle indeterminacy effectively since the membership functions are depicted as intervals that lie in [0, 1]. In this article, the operations on Strong Interval Valued Neutrosophic graph have been newly defined along with their related theorems. The Strong Interval Valued Neutrosophic Digraph has been newly introduced for evaluating the blood pressure that fluctuates during the blood flow of the human heart. By considering the hemodynamic parameters of a healthy adult of age above 35 years without any cardiac malfunction, we model the cardiac functioning of the human heart during the Systolic and Diastolic phases as Strong Interval Valued Neutrosophic Digraph and its evaluation observed to be analogous to the conventional biological approach.

Keywords: Interval Valued Neutrosophic Graph; Strong Interval Valued Neutrosophic Digraph; Cardiac Cycle of the Human Heart; Wright table.

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1. **Introduction**

To address the ambiguity and imprecision on crisp sets, Zadeh, L., [1] in the year 1965, described the fuzzy set (FS) theory and consequently fuzzy logic. This proposed theory is identified with a membership function assigning all the members of a given Universal set X, a degree of membership m_A in a FS A.

Interval-valued fuzzy sets (IVFS) were initially analyzed by Sambuc [2] who termed as ϕ - floues functions, to identify the features of unpredictability by attributing the membership degrees. Atanassov, K., [3] defined the intuitionistic fuzzy sets (IFS) assigning to each member of the Universal set both a degree of membership and one of the non-membership n_A such that $0 \leq m_A(x) + n_A(x) \leq 1$ which relaxes the duality in the fuzzy set and as a consequence, it allows to address the positive and negative side of an imprecise concept.

The IFS by K. Atanassov [3] corresponds with the definition of vague sets introduced by Gau and Byehrer [4] accordingly by Bustince and Burillo [5]. Further, he introduced the interval-valued intuitionistic fuzzy set (IVIFS) as an extension of both IFS and IVFS. Inspired by the real-time situations, winning/defeating or tie scores from sports games, and yes/no/NA from decision making, Florentin Smarandache [6] proposed the concept of the neutrosophic set (NS) to understand the standard as well as the non-standard analysis.

Thus NS is a systematic paradigm that generalizes the concepts in [1], [3]. Wang et. al [7] defined the single-valued neutrosophic set (SVNS) by defining T, I and F from a nonempty set A to $[0, 1]$. The interval-valued neutrosophic set (IVNS) [8], is more efficient than the SVNS, in which their membership functions are all independent as well as their values are included in [0,1].

Fuzzy analogous of several graph - theoretic concepts are described by Azriel Rosenfield [9]. and thus fuzzy graphs (FG) have diversified applications in the areas of Science, Engineering, Technology, etc and it is essential to model those problems in comparison to the classical graph. With additional remarks on Fuzzy graphs, Bhattacharya [10], established that the concepts in fuzzy graphs do not match with the graph-theoretical concepts all the time.

The Intuitionistic fuzzy graph (IFG) [11] arises by taking the vertex and edge sets as IFS. K. Atanassov [12] in 2019, introduced eight different types of interval-valued intuitionistic fuzzy graphs (IVIFG) and their representations by index matrices.

In many situations, as the relations between the vertices (nodes) are indeterminate, the fuzzy graphs along with their extensions fail, Smarandache [13], defined neutrosophic graph (NG). Thus the single-valued neutrosophic graph (SVNG) is a NG model that generalizes the FG and IFG and Said Broumi [14], [15] further extended to interval-valued neutrosophic graphs (IVNG) and its stron form which are used to model the real-life problems with uncertain, irreconcilable, non-deterministic, unpredictable information effectively and further Mohammed Akram extended to interval-valued neutrosophic digraph (IVNDG) [16] to analyze the applied network models.

Furthermore, Shouzhen Zeng et. al [17] introduced maximal product, rejection, symmetric difference, residue product on SVNG having application in FAO for finding the most reasonable organization for the farmers to develop more food grains and to increase yearning.

Recently, Haque et. al [18], [19], [20], [21] defined various operational laws, logarithmic operational law and exponential operational law for evaluating Multi-criteria group decision-making (MCGDM), Multi -attribute decision-making(MADM) problems under spherical fuzzy, interval neutrosophic environments.

1.1. *Motivation and Novelty*

Based on the literature survey, we found that SIVNG has not been explored in detail. This motivates us to study the operations such as maximal product, rejection, symmetric difference and residue product of any two SIVNGs. Further SIVNG helps to maintain the optimal minimum value between any two nodes. By considering this, we model the cardiac functioning of the human heart under SIVN environment. Also, the blood flow cannot be reversed and falls within certain range. This necessitates us to model the cardiac cycle of the human heart as the SIVNDG, which is a novel concept. By modelling the cardiac functioning of the human heart as SIVNDG helps to explore the blood flow of the human heart in each phase.

1.2. *Organization of the article*

In this article, Section 2 contains preliminaries. In Section 3, the maximal product $(*)$, rejection $(|)$, symmetric difference $(\oplus)$, residue product $(\bullet)$ of any two SIVNG have been introduced and if G_1 and G_2 are any two SIVNGs, we prove that $G_1 * G_2$, $G_1 \mid G_2$, $G_1 \oplus G_2$ and $G_1 \bullet G_2$ is again a SIVNG. Further, the degree and total degree of these operations and their related theorems are discussed in detail. In Section 4, we propose the Strong Interval-valued Neutrosophic Digraph (SIVNDG) based on a Strong Interval-valued Neutrosophic graph (SIVNG) [6] to explore the cardiac cycle of the human heart.By converting the blood pressure values to SIVN values, we study the blood flow of the human heart. Section 5 contains the Sensitivity analysis and Comparative study. Section 6 and Section 7 deals with Results and discussion. Section 8 possess the need and limitation and impact of the research work and Section 9 contains the conclusion.

2. **Preliminaries**

Definition 2.1. An Interval Valued Neutrosophic Graph (IVNG) [14] of a graph $G' = (P', Q')$, we mean a pair $G = (P, Q)$, where $P = ([t_P^l, t_P^u], [i_P^l, i_P^u], [f_P^l, f_P^u])$ is an IVN - set on P' and $Q = ([t_Q^l, t_Q^u], [i_Q^l, i_Q^u], [f_Q^l, f_Q^u])$ is an IVN - relation on Q' that satisfies the following conditions:

(1) $P' = \{p_1, p_2, ..., p_n\}$ such that $t_P^l : P' \to [0,1]$, $t_P^u : P' \to [0,1]$, $i_P^l : P' \to [0,1]$, $i_P^u : P' \to [0,1]$, $f_P^l : P' \to [0,1]$, $f_P^u : P' \to [0,1]$ represent the corresponding degree of membership functions of T, I and F of $p_i \in P'$ with $0 \leq t_P(p_i) + i_P(p_i) + f_P(p_i) \leq 3, \forall p_i \in P'(i = 1, 2, ..., n)$.

(2) The mappings $t^l_Q : P' \times P' \to [0,1]$, $t^u_Q : P' \times P' \to [0,1]$, $i^l_Q : P' \times P' \to [0,1]$, $i^u_Q : P' \times P' \to [0,1]$, $f^l_Q : P' \times P' \to [0,1]$, $f^u_Q : P' \times P' \to [0,1]$ are such that

$$(1)\ t^l_Q(p_i,p_j) \le min(t^l_P(p_i), t^l_P(p_j)),$$
$$(2)\ t^u_Q(p_i,p_j) \le min(t^u_P(p_i), t^u_P(p_j)),$$
$$(3)\ i^l_Q(p_i,p_j) \ge max(i^l_P(p_i), i^l_P(p_j)),$$
$$(4)\ i^u_Q(p_i,p_j) \ge max(i^u_P(p_i), i^u_P(p_j)),$$
$$(5)\ f^l_Q(p_i,p_j) \ge max(f^l_P(p_i), f^l_P(p_j)),$$
$$(6)\ f^u_Q(p_i,p_j) \ge max(f^u_P(p_i), f^u_P(p_j)),$$

where $(p_i,p_j) \in Q'$ and $0 \le t_Q(p_i,p_j) + i_Q(p_i,p_j) + f_Q(p_i,p_j) \le 3, \forall (p_i,p_j) \in Q'(i,j = 1,2,...,n)$.

Definition 2.2. An IVNG $G = (P,Q)$ of $G' = (P',Q')$ is called Strong IVNG (SIVNG) [15] if for any pair $(p_i,p_j) \in Q'$ we have :

$$(1)\ t^l_Q(p_i,p_j) = min(t^l_P(p_i), t^l_P(p_j)),$$
$$(2)\ t^u_Q(p_i,p_j) = min(t^u_P(p_i), t^u_P(p_j)),$$
$$(3)\ i^l_Q(p_i,p_j) = max(i^l_P(p_i), i^l_P(p_j)),$$
$$(4)\ i^u_Q(p_i,p_j) = max(i^u_P(p_i), i^u_P(p_j)),$$
$$(5)\ f^l_Q(p_i,p_j) = max(f^l_P(p_i), f^l_P(p_j)),$$
$$(6)\ f^u_Q(p_i,p_j) = max(f^u_P(p_i), f^u_P(p_j)).$$

Definition 2.3. A Strong Interval Valued Neutrosophic Digraph (SIVNDG) on a non-empty Universal set X is a pair $G = (P, \overrightarrow{Q})$, where $P = ([t^l_P, t^u_P], [i^l_P, i^u_P], [f^l_P, f^u_P])$ is an IVN - set corresponds to X and $Q = ([t^l_Q, t^u_Q], [i^l_Q, i^u_Q], [f^l_Q, f^u_Q])$ is an IVN - relation corresponds to X such that

$$(1)\ t^l_Q(\overrightarrow{p_i,p_j}) = t^l_P(p_i) \wedge t^l_P(p_j),$$
$$(2)\ t^u_Q(\overrightarrow{p_i,p_j}) = t^u_P(p_i) \wedge t^u_P(p_j),$$
$$(3)\ i^l_Q(\overrightarrow{p_i,p_j}) = i^l_P(p_i) \vee i^l_P(p_j),$$
$$(4)\ i^u_Q(\overrightarrow{p_i,p_j}) = i^u_P(p_i) \vee i^u_P(p_j),$$
$$(5)\ f^l_Q(\overrightarrow{p_i,p_j}) = f^l_P(p_i) \vee f^l_P(p_j),$$
$$(6)\ f^u_Q(\overrightarrow{p_i,p_j}) = f^u_P(p_i) \vee f^u_P(p_j),$$

$\forall p_i, p_j \in X$.

Example 2.4. Consider a SIVN-digraph $G = (P, \overrightarrow{Q})$ on $X = \{l_1, l_2, l_3, l_4\}$ in Figure 1. The vertices and the edges of G along with their membership functions are given by

$P = \{l_1 < [0.1,0.2],[0.3,0.4],[0.2,0.5] >, l_2 < [0.4,0.5],[0.2,0.3],[0.1,0.5] >, l_3 < [0.2,0.3],[0.3,0.5],[0.6,0.8] >, l_4 < [0.4,0.6],[0.3,0.5],[0.2,0.4] >\}$,

$Q = \{l_1l_2 < [0.1,0.2],[0.3,0.4],[0.2,0.5] >, l_2l_3 < [0.2,0.3],[0.3,0.5],[0.6,0.8] >, l_3l_4 < [0.2,0.3],[0.3,0.5],[0.6,0.8], l_4l_1 < [0.1,0.2],[0.3,0.5],[0.2,0.5] >\}$.

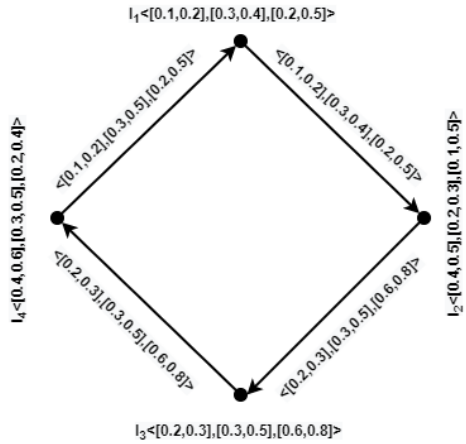

FIGURE 1. Strong Interval Valued Neutrosophic Digraph G

3. Operations on SIVNG

In this section, we introduce two different operations on SIVNG, maximal product and symmetric difference. We prove that for any two SIVNGs, the maximal product and symmetric difference is a SIVNG.

Definition 3.1. The maximal product $G_1 * G_2 = (P_1 * P_2, Q_1 * Q_2)$ of two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ on the crisp graphs $G_1' = (P_1', Q_1')$ and $G_2' = (P_2', Q_2')$ is defined as

(1) $(t^l_{P_1} * t^l_{P_2})(p_1, p_2) = max\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$,
$(t^u_{P_1} * t^u_{P_2})(p_1, p_2) = max\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$,
$(i^l_{P_1} * i^l_{P_2})(p_1, p_2) = min\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$,
$(i^u_{P_1} * i^u_{P_2})(p_1, p_2) = min\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$,
$(f^l_{P_1} * f^l_{P_2})(p_1, p_2) = min\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$,
$(f^u_{P_1} * f^u_{P_2})(p_1, p_2) = min\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$, $\forall (p_1, p_2) \in (P_1' \times P_2')$.

(2) $(t^l_{Q_1} * t^l_{Q_2})((p, p_2)(p, q_2)) = max\{t^l_{P_1}(p), t^l_{Q_2}(p_2q_2)\}$,
$(t^u_{Q_1} * t^u_{Q_2})((p, p_2)(p, q_2)) = max\{t^u_{P_1}(p), t^u_{Q_2}(p_2q_2)\}$,
$(i^l_{Q_1} * i^l_{Q_2})((p, p_2)(p, q_2)) = min\{i^l_{P_1}(p), i^l_{Q_2}(p_2q_2)\}$,
$(i^u_{Q_1} * i^u_{Q_2})((p, p_2)(p, q_2)) = min\{i^u_{P_1}(p), i^u_{Q_2}(p_2q_2)\}$,
$(f^l_{Q_1} * f^l_{Q_2})((p, p_2)(p, q_2)) = min\{f^l_{P_1}(p), f^l_{Q_2}(p_2q_2)\}$,
$(f^u_{Q_1} * f^u_{Q_2})((p, p_2)(p, q_2)) = min\{f^u_{P_1}(p), f^u_{Q_2}(p_2q_2)\}$, $\forall p \in P_1'$ and $p_2q_2 \in Q_2'$.

(3) $(t^l_{Q_1} * t^l_{Q_2})((p_1, r)(q_1, r)) = max\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(r)\}$,
$(t^u_{Q_1} * t^u_{Q_2})((p_1, r)(q_1, r)) = max\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(r)\}$,
$(i^l_{Q_1} * i^l_{Q_2})((p_1, r)(q_1, r)) = min\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(r)\}$,
$(i^u_{Q_1} * i^u_{Q_2})((p_1, r)(q_1, r)) = min\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(r)\}$,

$(f^l_{Q_1} * f^l_{Q_2})((p_1, r)(q_1, r)) = min\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(r)\}$,
$(f^u_{Q_1} * f^u_{Q_2})((p_1, r)(q_1, r)) = min\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(r)\}$, $\forall p_1q_1 \in Q_1'$ and $r \in P_2'$.

Theorem 3.2. *The maximal product of two SIVNGs G_1 and G_2 is a SIVNG.*

Proof. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs on $G_1' = (P_1', Q_1')$ and $G_2' = (P_2', Q_2')$ respectively and $((p_1, p_2), (q_1, q_2)) \in Q_1' \times Q_2'$. Then, we have,

Case 1. If $p_1 = q_1 = p$,
$(t^l_{Q_1} * t^l_{Q_2})((p, p_2), (p, q_2)) = max\{t^l_{P_1}(p), t^l_{Q_2}(p_2q_2)\}$
$= max\{t^l_{P_1}(p), min\{t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}\}$
$= min\{max\{t^l_{P_1}(p), t^l_{P_2}(p_2)\}, max\{t^l_{P_1}(p), t^l_{P_2}(q_2)\}\}$
$= min\{(t^l_{P_1} * t^l_{P_2})(p, p_2), (t^l_{P_1} * t^l_{P_2})(p, q_2)\}$
In the same way, the other conditions can also be verified.

Case 2. If $p_2 = q_2 = r$,
$(t^l_{Q_1} * t^l_{Q_2})((p_1, r), (q_1, r)) = max\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(r)\}$
$= max\{min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1)\}, t^l_{P_2}(r)\}$
$= min\{max\{t^l_{P_1}(p_1), t^l_{P_2}(r)\}, max\{t^l_{P_1}(q_1), t^l_{P_2}(r)\}\}$
$= min\{(t^l_{P_1} * t^l_{P_2})(p_1, r), (t^l_{P_1} * t^l_{P_2})(q_1, r)\}$
The other conditions can also be verified using the same approach.
Thus, the maximal product $G_1 * G_2$ is a SIVNG. □

Example 3.3. Consider two SIVNGs G_1 and G_2 as represented in Figure 2. Their maximal product $G_1 * G_2$ is represented in Figure 3.

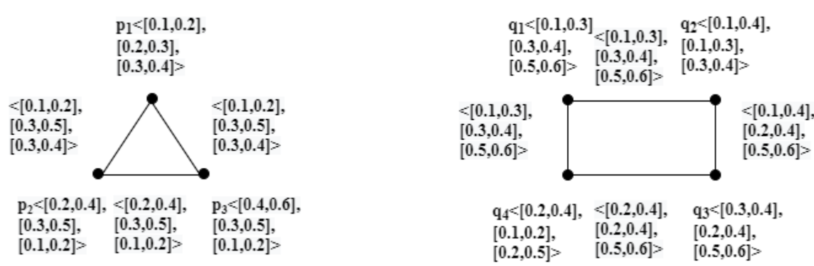

FIGURE 2. Strong Interval Valued Neutrosophic Graphs G_1 and G_2

Definition 3.4. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The degree for any vertex $(p_1, p_2) \in (P_1' \times P_2')$ is,
$(d_{t^l})_{G_1*G_2}(p_1, p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'} (t^l_{Q_1} * t^l_{Q_2})((p_1, p_2), (q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2\in Q_2'} max\{t^l_{P_1}(p_1), t^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1', p_2=q_2} max\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2)\}$,
$(d_{t^u})_{G_1*G_2}(p_1, p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'} (t^u_{Q_1} * t^u_{Q_2})((p_1, p_2), (q_1, q_2))$

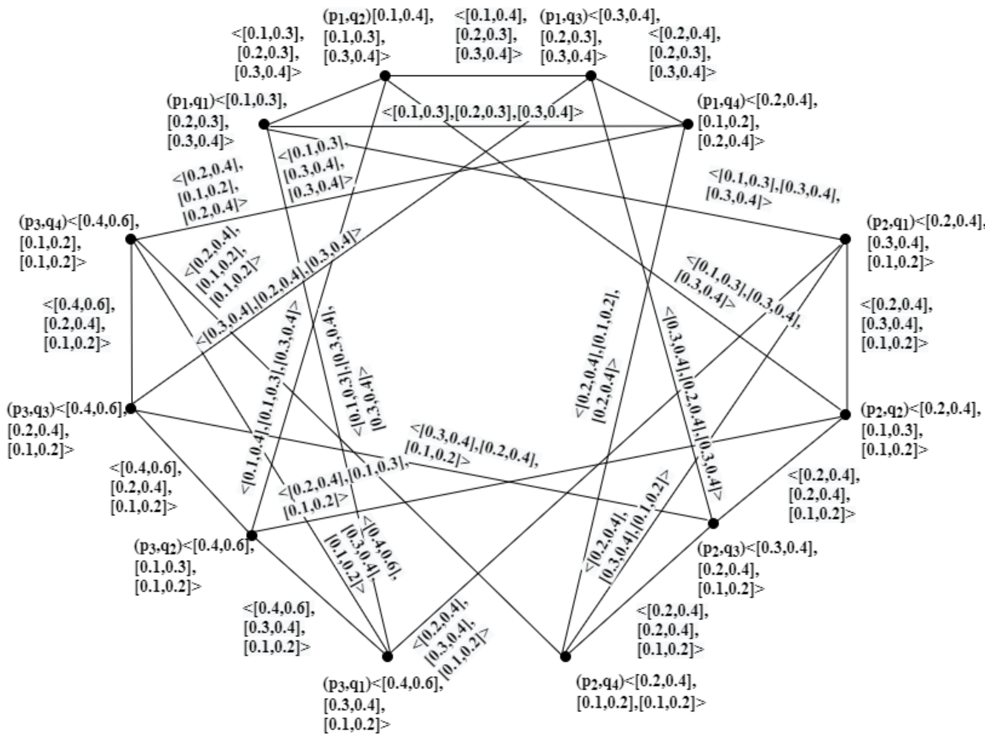

FIGURE 3. Maximal Product $G_1 * G_2$

$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{t^u_{P_1}(p_1), t^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2)\}$,

$(d_{i^l})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'} (i^l_{Q_1} * i^l_{Q_2})((p_1,p_2),(q_1,q_2))$

$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{i^l_{P_1}(p_1), i^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2)\}$,

$(d_{i^u})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'} (i^u_{Q_1} * i^u_{Q_2})((p_1,p_2),(q_1,q_2))$

$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{i^u_{P_1}(p_1), i^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2)\}$,

$(d_{f^l})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'} (f^l_{Q_1} * f^l_{Q_2})((p_1,p_2),(q_1,q_2))$

$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{f^l_{P_1}(p_1), f^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2)\}$,

$(d_{f^u})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'} (f^u_{Q_1} * f^u_{Q_2})((p_1,p_2),(q_1,q_2))$

$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{f^u_{P_1}(p_1), f^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2)\}$.

Theorem 3.5. *Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. If $t^l_{P_1} \geq t^l_{Q_2}, t^u_{P_1} \geq t^u_{Q_2}, i^l_{P_1} \leq i^l_{Q_2}, i^u_{P_1} \leq i^u_{Q_2}, f^l_{P_1} \leq f^l_{Q_2}, f^u_{P_1} \leq f^u_{Q_2}$ and $t^l_{P_2} \geq t^l_{Q_1}, t^u_{P_2} \geq t^u_{Q_1}, i^l_{P_2} \leq i^l_{Q_1}, i^u_{P_2} \leq i^u_{Q_1}, f^l_{P_2} \leq f^l_{Q_1}, f^u_{P_2} \leq f^u_{Q_1}$, then for every $(p_1,p_2) \in (P_1' \times P_2')$, we have,*

$(d_{t^l})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)t^l_{P_1}(p_1) + (d)_{(G_1)}(p_1)t^l_{(P_2)}(p_2)$

$(d_{t^u})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)t^u_{P_1}(p_1) + (d)_{(G_1)}(p_1)t^u_{(P_2)}(p_2)$

$(d_{i^l})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)i^l_{P_1}(p_1) + (d)_{(G_1)}(p_1)i^l_{(P_2)}(p_2)$

$(d_{i^u})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)i^u_{P_1}(p_1) + (d)_{(G_1)}(p_1)i^u_{(P_2)}(p_2)$

$(d_{f^l})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)f^l_{P_1}(p_1) + (d)_{(G_1)}(p_1)f^l_{(P_2)}(p_2)$

$(d_{f^u})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)f^u_{P_1}(p_1) + (d)_{(G_1)}(p_1)f^u_{(P_2)}(p_2)$

Proof. Consider,
$(d_{t^l})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(t^l_{Q_1} * t^l_{Q_2})((p_1,p_2),(q_1,q_2))$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{t^l_{P_1}(p_1), t^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2)\}$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} t^l_{Q_2}(p_2q_2) + \sum_{p_1q_1\in Q_1',p_2=q_2} t^l_{Q_1}(p_1q_1)$
$= (d)_{(G_2)}(p_2)t^l_{P_1}(p_1) + (d)_{(G_1)}(p_1)t^l_{(P_2)}(p_2)$
Similarly, the other conditions can also be proved. □

Definition 3.6. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The total degree for any vertex $(p_1, p_2) \in (P_1' \times P_2')$ is,
$(td_{t^l})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(t^l_{Q_1} * t^l_{Q_2})((p_1,p_2),(q_1,q_2)) + (t^l_{P_1} * t^l_{P_2})(p_1,p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{t^l_{P_1}(p_1), t^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2)\}$
$+ max\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$,
$(td_{t^u})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(t^u_{Q_1} * t^u_{Q_2})((p_1,p_2),(q_1,q_2)) + (t^u_{P_1} * t^u_{P_2})(p_1,p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{t^u_{P_1}(p_1), t^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(p_2)\}$
$+ max\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$,
$(td_{i^l})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(i^l_{Q_1} * i^l_{Q_2})((p_1,p_2),(q_1,q_2)) + (i^l_{P_1} * i^l_{P_2})(p_1,p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{i^l_{P_1}(p_1), i^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2)\}$
$+ min\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$,
$(td_{i^u})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(i^u_{Q_1} * i^u_{Q_2})((p_1,p_2),(q_1,q_2)) + (i^u_{P_1} * i^u_{P_2})(p_1,p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{i^u_{P_1}(p_1), i^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2)\}$
$+ min\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$,
$(td_{f^l})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(f^l_{Q_1} * f^l_{Q_2})((p_1,p_2),(q_1,q_2)) + (f^l_{P_1} * f^l_{P_2})(p_1,p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{f^l_{P_1}(p_1), f^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2)\}$
$+ min\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$,
$(td_{f^u})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(f^u_{Q_1} * f^u_{Q_2})((p_1,p_2),(q_1,q_2)) + (f^u_{P_1} * f^u_{P_2})(p_1,p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{f^u_{P_1}(p_1), f^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2)\}$
$+ min\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$.

Theorem 3.7. *Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. If $t^l_{P_1} \geq t^l_{Q_2}, t^u_{P_1} \geq t^u_{Q_2}, i^l_{P_1} \leq i^l_{Q_2}, i^u_{P_1} \leq i^u_{Q_2}, f^l_{P_1} \leq f^l_{Q_2}, f^u_{P_1} \leq f^u_{Q_2}$ and $t^l_{P_2} \geq t^l_{Q_1}, t^u_{P_2} \geq t^u_{Q_1}, i^l_{P_2} \leq i^l_{Q_1}, i^u_{P_2} \leq i^u_{Q_1}, f^l_{P_2} \leq f^l_{Q_1}, f^u_{P_2} \leq f^u_{Q_1}$, then for every $(p_1, p_2) \in (P_1' \times P_2')$, we have,*
$(td_{t^l})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)t^l_{P_1}(p_1) + (d)_{(G_1)}(p_1)t^l_{(P_2)}(p_2) + max\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$
$(td_{t^u})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)t^u_{P_1}(p_1) + (d)_{(G_1)}(p_1)t^u_{(P_2)}(p_2) + max\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$
$(td_{i^l})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)i^l_{P_1}(p_1) + (d)_{(G_1)}(p_1)i^l_{(P_2)}(p_2) + min\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$
$(td_{i^u})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)i^u_{P_1}(p_1) + (d)_{(G_1)}(p_1)i^u_{(P_2)}(p_2) + min\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$
$(td_{f^l})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)f^l_{P_1}(p_1) + (d)_{(G_1)}(p_1)f^l_{(P_2)}(p_2) + min\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$
$(td_{f^u})_{(G_1*G_2)}(p_1,p_2) = (d)_{(G_2)}(p_2)f^u_{P_1}(p_1) + (d)_{(G_1)}(p_1)f^u_{(P_2)}(p_2) + min\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$

Proof. Consider the case of $(td_{f^u})_{(G_1*G_2)}(p_1,p_2)$, we have,

$(td_{f^u})_{G_1*G_2}(p_1,p_2) = \sum_{((p_1,p_2),(q_1,q_2))\in Q_1'\times Q_2'}(f^u_{Q_1} * f^u_{Q_2})((p_1,p_2),(q_1,q_2)) + (f^u_{P_1} * f^u_{P_2})(p_1,p_2)$

$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{f^u_{P_1}(p_1), f^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2)\}$

$+min\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\},$

$= \sum_{p_1=q_1,p_2q_2\in Q_2'} f^u_{Q_2}(p_2q_2) + \sum_{p_1q_1\in Q_1',p_2=q_2} f^u_{Q_1}(p_1q_1) + min\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\},$

In the same way, the other conditions can also be verified. □

Example 3.8. Consider two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ as represented in Figure 4. Their maximal product $G_1 * G_2$ is represented in Figure 5.

From Figures 4 and 5, d_i, td_f for the vertex (p_3, q_2) are calculated below.

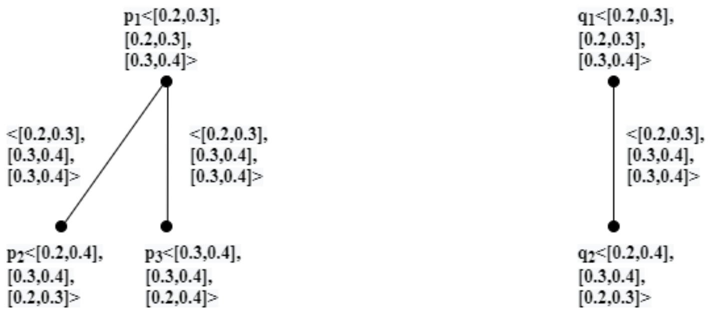

FIGURE 4. Strong Interval Valued Neutrosophic Graphs G_1 and G_2

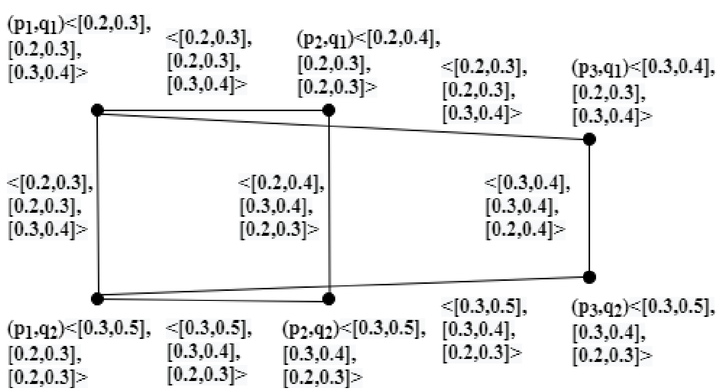

FIGURE 5. Maximal Product $G_1 * G_2$

By direct calculations, $d_{i^l}(p_3, q_2) = 0.3 + 0.3 = 0.6$, $d_{i^u}(p_3, q_2) = 0.4 + 0.4 = 0.8$, $d_i(p_3, q_2) = [0.6, 0.8]$.

$td_{f^l}(p_1, q_1) = 0.2 + 0.2 + 0.2 = 0.6$, $td_{f^u}(p_1, q_1) = 0.4 + 0.3 + 0.3 = 1.0$, $td_f(p_1, q_1) = [0.6, 1]$.

By using theorem, $d_{i^l}(p_3, q_2) = 1(0.3) + 1(0.3) = 0.6$, $d_{i^u}(p_3, q_2) = 1(0.4) + 1(0.4) = 0.8$, $d_i(p_3, q_2) = [0.6, 0.8]$.

$td_{f^l}(p_1, q_1) = 1(0.2) + 1(0.2) + min\{0.2, 0.2\} = 0.6$, $td_{f^u}(p_1, q_1) = 1(0.4) + 1(0.3) +$

$min\{0.4, 0.3\} = 1.0$, $td_f(p_1, q_1) = [0.6, 1]$.

Definition 3.9. The rejection $G_1 \mid G_2 = (P_1 \mid P_2, Q_1 \mid Q_2)$of two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ is defined as

(1) $(t^l_{P_1} \mid t^l_{P_2})(p_1, p_2) = min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$,
$(t^u_{P_1} \mid t^u_{P_2})(p_1, p_2) = min\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$,
$(i^l_{P_1} \mid i^l_{P_2})(p_1, p_2) = max\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$,
$(i^u_{P_1} \mid i^u_{P_2})(p_1, p_2) = max\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$,
$(f^l_{P_1} \mid f^l_{P_2})(p_1, p_2) = max\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$,
$(f^u_{P_1} \mid f^u_{P_2})(p_1, p_2) = max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}, \forall (p_1, p_2) \in (P'_1 \times P'_2)$.

(2) $(t^l_{Q_1} \mid t^l_{Q_2})((p, p_2), (p, q_2)) = min\{t^l_{P_1}(p), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}$,
$(t^u_{Q_1} \mid t^u_{Q_2})((p, p_2), (p, q_2)) = min\{t^u_{P_1}(p), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\}$,
$(i^l_{Q_1} \mid i^l_{Q_2})((p, p_2), (p, q_2)) = max\{i^l_{P_1}(p), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\}$,
$(i^u_{Q_1} \mid i^u_{Q_2})((p, p_2), (p, q_2)) = max\{i^u_{P_1}(p), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\}$,
$(f^l_{Q_1} \mid f^l_{Q_2})((p, p_2), (p, q_2)) = max\{f^l_{P_1}(p), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\}$,
$(f^u_{Q_1} \mid f^u_{Q_2})((p, p_2), (p, q_2)) = max\{f^u_{P_1}(p), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}, \forall p \in P'_1, p_2q_2 \notin Q'_2$.

(3) $(t^l_{Q_1} \mid t^l_{Q_2})((p_1, r), (q_1, r)) = min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{P_2}(r)\}$,
$(t^u_{Q_1} \mid t^u_{Q_2})((p_1, r), (q_1, r)) = min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{P_2}(r)\}$,
$(i^l_{Q_1} \mid i^l_{Q_2})((p_1, r), (q_1, r)) = max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{P_2}(r)\}$,
$(i^u_{Q_1} \mid i^u_{Q_2})((p_1, r), (q_1, r)) = max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{P_2}(r)\}$,
$(f^l_{Q_1} \mid f^l_{Q_2})((p_1, r), (q_1, r)) = max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{P_2}(r)\}$,
$(f^u_{Q_1} \mid f^u_{Q_2})((p_1, r), (q_1, r)) = max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{P_2}(r)\}, \forall p_1q_1 \notin Q'_1, r \in P'_2$.

(4) $(t^l_{Q_1} \mid t^l_{Q_2})((p_1, p_2), (q_1, q_2)) = min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}$,
$(t^u_{Q_1} \mid t^u_{Q_2})((p_1, p_2), (q_1, q_2)) = min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\}$,
$(i^l_{Q_1} \mid i^l_{Q_2})((p_1, p_2), (q_1, q_2)) = max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\}$,
$(i^u_{Q_1} \mid i^u_{Q_2})((p_1, p_2), (q_1, q_2)) = max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\}$,
$(f^l_{Q_1} \mid f^l_{Q_2})((p_1, p_2), (q_1, q_2)) = max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\}$,
$(f^u_{Q_1} \mid f^u_{Q_2})((p_1, p_2), (q_1, q_2)) = max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}$,
$\forall p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2$.

Example 3.10. Consider two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ as represented in Figure 6. Their rejection $G_1 \mid G_2$ is represented in Figure 7.

Theorem 3.11. *The rejection of two SIVNGs G_1 and G_2 is a SIVNG.*

Proof. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs on $G'_1 = (P'_1, Q'_1)$ and $G'_2 = (P'_2, Q'_2)$ respectively and $((p_1, p_2), (q_1, q_2)) \in Q'_1 \times Q'_2$. Then, we have,

Case 1. If $p_1 = q_1, p_2q_2 \notin Q'_2$

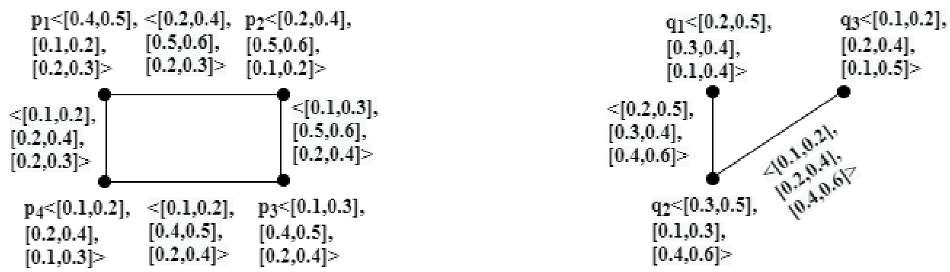

FIGURE 6. Strong Interval Valued Neutrosophic Graphs G_1 and G_2

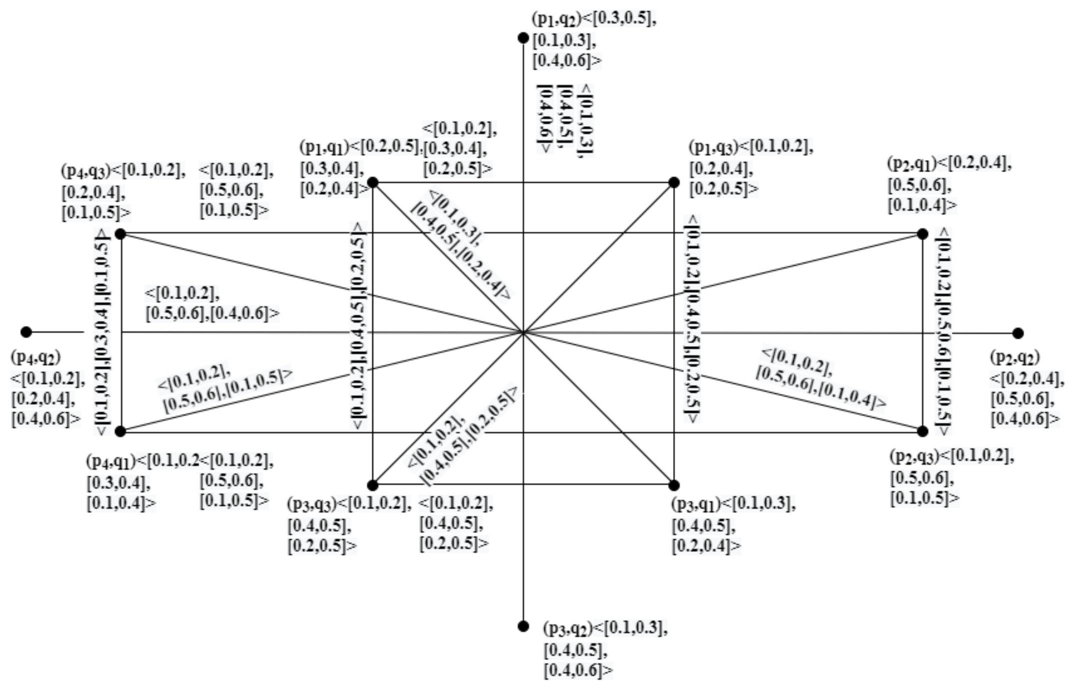

FIGURE 7. Rejection $G_1 \mid G_2$

$(t^l_{Q_1} \mid t^l_{Q_2})((p_1,p_2),(p_1,q_2)) = min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}$
$= min\{min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1)\}, min\{t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}\}$
$= min\{(t^l_{P_1} \mid t^l_{P_2})(p_1,p_2), (t^l_{P_1} \mid t^l_{P_2})(q_1,q_2)\}$

In the same way, the other conditions can also be verified.

Case 2. If $p_2 = q_2, p_1q_1 \notin Q_1^{'}$

$(i^l_{Q_1} \mid i^l_{Q_2})((p_1,p_2),(q_1,q_2)) = max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{P_2}(p_2)\}$
$= max\{max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1)\}, max\{i^l_{P_2}(p_2), i^l_{P_2}(q_2)\}\}$
$= max\{(i^l_{P_1} \mid i^l_{P_2})(p_1,p_2), (i^l_{P_1} \mid i^l_{P_2})(q_1,q_2)\}$

Similarly, the other conditions can also be verified.

Case 3. If $p_1q_1 \notin Q_1^{'}, p_2q_2 \notin Q_2^{'}$

$(f^l_{Q_1} \mid f^l_{Q_2})((p_1,p_2),(q_1,q_2)) = max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\}$

$= max\{max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1)\}, max\{f^l_{P_2}(p_2), f^l_{P_2}(q_2)\}\}$
$= max\{(f^l_{P_1} \mid f^l_{P_2})(p_1, p_2), (f^l_{P_1} \mid f^l_{P_2})(q_1, q_2)\}$
Similarly, the other conditions can also be verified. □

Definition 3.12. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The degree for any vertex $(p_1, p_2) \in (P'_1 \times P'_2)$ is,

$(d_{t^l})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (t^l_{Q_1} \mid t^l_{Q_2})((p_1, p_2), (p_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \notin Q'_2} min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}$
$(d_{t^u})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (t^u_{Q_1} \mid t^u_{Q_2})((p_1, p_2), (p_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \notin Q'_2} min\{t^u_{P_1}(p_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\}$
$(d_{i^l})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (i^l_{Q_1} \mid i^l_{Q_2})((p_1, p_2), (p_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \notin Q'_2} max\{i^l_{P_1}(p_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\}$
$(d_{i^u})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (i^u_{Q_1} \mid i^u_{Q_2})((p_1, p_2), (p_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \notin Q'_2} max\{i^u_{P_1}(p_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\}$
$(d_{f^l})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (f^l_{Q_1} \mid f^l_{Q_2})((p_1, p_2), (p_1, q_2)) =$
$\sum_{p_1=q_1, p_2q_2 \notin Q'_2} max\{f^l_{P_1}(p_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\}$
$(d_{f^u})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (f^u_{Q_1} \mid f^u_{Q_2})((p_1, p_2), (p_1, q_2)) =$
$\sum_{p_1=q_1, p_2q_2 \notin Q'_2} max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}$

Definition 3.13. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The total degree for any vertex $(p_1, p_2) \in (P'_1 \times P'_2)$ is,

$(td_{t^l})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (t^l_{Q_1} \mid t^l_{Q_2})((p_1, p_2), (p_1, q_2)) + (t^l_{P_1} \mid t^l_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1, p_2q_2 \notin Q'_2} min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\} + min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$
$(td_{t^u})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (t^u_{Q_1} \mid t^u_{Q_2})((p_1, p_2), (p_1, q_2)) + (t^u_{P_1} \mid t^u_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1, p_2q_2 \notin Q'_2} min\{t^u_{P_1}(p_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\} + min\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$
$(td_{i^l})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (i^l_{Q_1} \mid i^l_{Q_2})((p_1, p_2), (p_1, q_2)) + (i^l_{P_1} \mid i^l_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1, p_2q_2 \notin Q'_2} max\{i^l_{P_1}(p_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q'_1, p_1=q_1} max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \notin Q'_2} max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\} + max\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$
$(td_{i^u})_{G_1|G_2}(p_1, p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q'_1 \times Q'_2} (i^u_{Q_1} \mid i^u_{Q_2})((p_1, p_2), (p_1, q_2)) + (i^u_{P_1} \mid i^u_{P_2})(p_1, p_2)$

$= \sum_{p_1=q_1, p_2q_2 \notin Q_2'} max\{i^u_{P_1}(p_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q_1', p_1=q_1} max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \notin Q_2'} max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\} + max\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$
$(td_{f^l})_{G_1|G_2}(p_1,p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q_1' \times Q_2'} (f^l_{Q_1} \mid f^l_{Q_2})((p_1,p_2),(p_1,q_2)) + (f^l_{P_1} \mid f^l_{P_2})(p_1,p_2) =$
$\sum_{p_1=q_1, p_2q_2 \notin Q_2'} max\{f^l_{P_1}(p_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q_1', p_1=q_1} max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \notin Q_2'} max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\} + max\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$
$(td_{f^u})_{G_1|G_2}(p_1,p_2) = \sum_{((p_1,p_2),(p_1,q_2)) \in Q_1' \times Q_2'} (f^u_{Q_1} \mid f^u_{Q_2})((p_1,p_2),(p_1,q_2)) + (f^u_{P_1} \mid f^u_{P_2})(p_1,p_2) =$
$\sum_{p_1=q_1, p_2q_2 \notin Q_2'} max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\} + \sum_{p_1q_1 \notin Q_1', p_1=q_1} max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \notin Q_2'} max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\} + max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$

From Figure 7, $d_i(p_1,q_3)$ and $td_i(p_1,q_3)$ for the vertex (p_1,q_3) are calculated below.

$d_{i^l}(p_1,q_3) = 0.3 + 0.4 + 0.4 = 1.1, d_{i^u}(p_1,q_3) = 0.4 + 0.5 + 0.5 = 1.4, d_i(p_1,q_3) = [1.1, 1.4]$.

$td_{i^l}(p_1,q_3) = 0.3 + 0.4 + 0.4 + 0.2 = 1.3, td_{i^u}(p_1,q_3) = 0.4 + 0.5 + 0.5 + 0.4 = 1.8, td_i(p_1,q_3) = [1.3, 1.8]$.

Definition 3.14. The symmetric difference $G_1 \oplus G_2 = (P_1 \oplus P_2, Q_1 \oplus Q_2)$ of two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ is defined as

(1) $(t^l_{P_1} \oplus t^l_{P_2})(p_1,p_2) = min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$,
$(t^u_{P_1} \oplus t^u_{P_2})(p_1,p_2) = min\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$,
$(i^l_{P_1} \oplus i^l_{P_2})(p_1,p_2) = max\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$,
$(i^u_{P_1} \oplus i^u_{P_2})(p_1,p_2) = max\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$,
$(f^l_{P_1} \oplus f^l_{P_2})(p_1,p_2) = max\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$,
$(f^u_{P_1} \oplus f^u_{P_2})(p_1,p_2) = max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$, $\forall (p_1,p_2) \in (P_1' \times P_2')$.

(2) $(t^l_{Q_1} \oplus t^l_{Q_2})((p,p_2)(p,q_2)) = min\{t^l_{P_1}(p), t^l_{Q_2}(p_2q_2)\}$,
$(t^u_{Q_1} \oplus t^u_{Q_2})((p,p_2)(p,q_2)) = min\{t^u_{P_1}(p), t^u_{Q_2}(p_2q_2)\}$,
$(i^l_{Q_1} \oplus i^l_{Q_2})((p,p_2)(p,q_2)) = max\{i^l_{P_1}(p), i^l_{Q_2}(p_2q_2)\}$,
$(i^u_{Q_1} \oplus i^u_{Q_2})((p,p_2)(p,q_2)) = max\{i^u_{P_1}(p), i^u_{Q_2}(p_2q_2)\}$,
$(f^l_{Q_1} \oplus f^l_{Q_2})((p,p_2)(p,q_2)) = max\{f^l_{P_1}(p), f^l_{Q_2}(p_2q_2)\}$,
$(f^u_{Q_1} \oplus f^u_{Q_2})((p,p_2)(p,q_2)) = max\{f^u_{P_1}(p), f^u_{Q_2}(p_2q_2)\}$, $\forall p \in P_1'$ and $p_2q_2 \in Q_2'$.

(3) $(t^l_{Q_1} \oplus t^l_{Q_2})((p_1,r)(q_1,r)) = min\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(r)\}$
$(t^u_{Q_1} \oplus t^u_{Q_2})((p_1,r)(q_1,r)) = min\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(r)\}$,
$(i^l_{Q_1} \oplus i^l_{Q_2})((p_1,r)(q_1,r)) = max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(r)\}$,
$(i^u_{Q_1} \oplus i^u_{Q_2})((p_1,r)(q_1,r)) = max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(r)\}$,
$(f^l_{Q_1} \oplus f^l_{Q_2})((p_1,r)(q_1,r)) = max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(r)\}$,
$(f^u_{Q_1} \oplus f^u_{Q_2})((p_1,r)(q_1,r)) = max\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(r)\}$, $\forall p_1q_1 \in Q_1'$ and $r \in P_2'$.

(4) $(t^l_{Q_1} \oplus t^l_{Q_2})(p_1,p_2)(q_1,q_2) = min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{Q_2}(p_2q_2)\}$,
$(t^u_{Q_1} \oplus t^u_{Q_2})(p_1,p_2)(q_1,q_2) = min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{Q_2}(p_2q_2)\}$,
$(i^l_{Q_1} \oplus i^l_{Q_2})(p_1,p_2)(q_1,q_2) = max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{Q_2}(p_2q_2)\}$,
$(i^u_{Q_1} \oplus i^u_{Q_2})(p_1,p_2)(q_1,q_2) = max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{Q_2}(p_2q_2)\}$,

$(f^l_{Q_1} \oplus f^l_{Q_2})(p_1,p_2)(q_1,q_2) = max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{Q_2}(p_2q_2)\}$,
$(f^u_{Q_1} \oplus f^u_{Q_2})(p_1,p_2)(q_1,q_2) = max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{Q_2}(p_2q_2)\}$,
$\forall p_1q_1 \notin Q'_1, p_2q_2 \in Q'_2$.

(5) $(t^l_{Q_1} \oplus t^l_{Q_2})(p_1,p_2)(q_1,q_2) = min\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}$,
$(t^u_{Q_1} \oplus t^u_{Q_2})(p_1,p_2)(q_1,q_2) = min\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\}$,
$(i^l_{Q_1} \oplus i^l_{Q_2})(p_1,p_2)(q_1,q_2) = max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\}$,
$(i^u_{Q_1} \oplus i^u_{Q_2})(p_1,p_2)(q_1,q_2) = max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\}$,
$(f^l_{Q_1} \oplus f^l_{Q_2})(p_1,p_2)(q_1,q_2) = max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\}$,
$(f^u_{Q_1} \oplus f^u_{Q_2})(p_1,p_2)(q_1,q_2) = max\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}$,
$\forall p_1q_1 \in Q'_1, p_2q_2 \notin Q'_2$.

Example 3.15. Consider two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ as represented in Figure 8. Their symmetric difference $G_1 \oplus G_2$ is represented in Figure 9. For instance, consider the vertex p_1q_1 in Figure 9. Then from the above definition, $(t^l_{P_1} \oplus t^l_{P_2})(p_1, q_1) = min\{t^l_{P_1}(p_1), t^l_{P_2}(q_1)\} = min\{0.2, 0.1\} = 0.1$ and $(t^u_{P_1} \oplus t^u_{P_2})(p_1, q_1) = min\{t^u_{P_1}(p_1), t^u_{P_2}(q_1)\} = min\{0.4, 0.3\} = 0.3$.The other membership values can be found accordingly. Further, $(t^l_{Q_1} \oplus t^l_{Q_2})(p_1, q_1)(p_1, q_2) = min\{t^l_{P_1}(p_1), t^l_{Q_2}(q_1, q_2)\} = min\{0.2, 0.1\} = 0.1$ and $(t^u_{Q_1} \oplus t^u_{Q_2})(p_1, q_1)(p_1, q_2) = min\{t^u_{P_1}(p_1), t^u_{Q_2}(q_1, q_2)\} = min\{0.4, 0.3\} = 0.3$. Similarly, all the other membership values can be calculated.

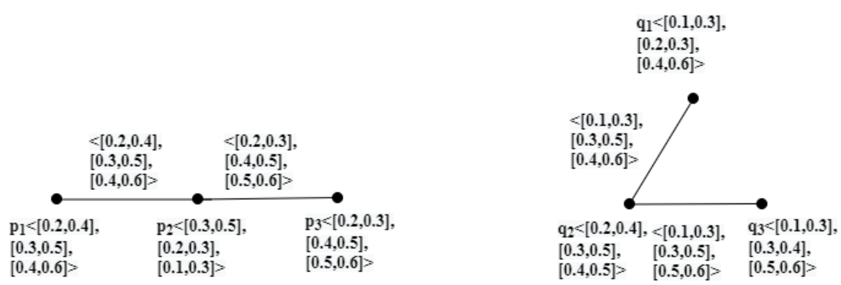

FIGURE 8. Strong Interval Valued Neutrosophic Graphs G_1 and G_2

Theorem 3.16. *The symmetric difference of two SIVNGs G_1 and G_2 is a SIVNG.*

Proof. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs on $G'_1 = (P'_1, Q'_1)$ and $G'_2 = (P'_2, Q'_2)$ respectively and $((p_1, p_2), (q_1, q_2)) \in Q'_1 \times Q'_2$. Then, we have,

Case 1. If $p_1 = q_1 = p, p_2q_2 \in Q'_2$,

$(t^u_{Q_1} \oplus t^u_{Q_2})((p, p_2)(p, q_2)) = min\{t^u_{P_1}(p), t^u_{Q_2}(p_2q_2)\}$
$= min\{t^u_{P_1}(p), min\{t^u_{P_2}(p_2), t^u_{P_2}(q_2)\}\}$
$= min\{min\{t^u_{P_1}(p), t^u_{P_2}(p_2)\}, min\{t^u_{P_1}(p), t^u_{P_2}(q_2)\}\}$
$= min\{(t^u_{P_1} \oplus t^u_{P_2})(p, p_2), (t^u_{P_1} \oplus t^u_{P_2})(p, q_2)\}$

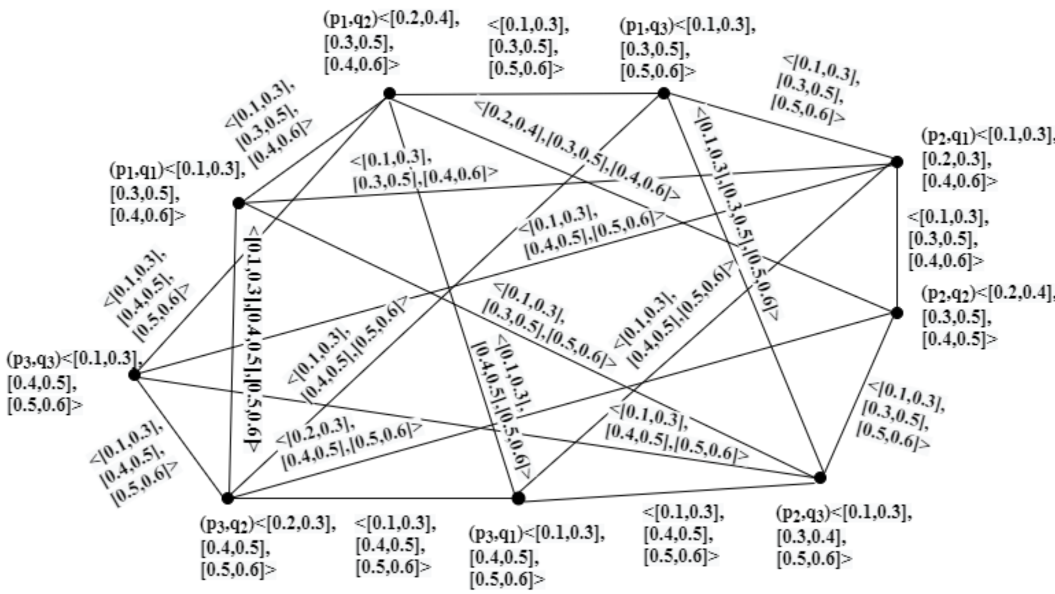

FIGURE 9. Symmetric difference $G_1 \oplus G_2$

Using the same approach, the other conditions can also be evaluated.

Case 2. If $p_2 = q_2 = r, p_1q_1 \in Q_1'$,

$(i^u_{Q_1} \oplus i^u_{Q_2})((p_1, r)(q_1, r)) = max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(r)\}$

$= max\{max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1)\}, i^u_{P_2}(r)\}$

$= max\{max\{i^u_{P_1}(p_1), i^u_{P_2}(r)\}, max\{i^u_{P_1}(q_1), i^u_{P_2}(r)\}\}$

$= max\{(i^u_{P_1} \oplus i^u_{P_2})(p_1, r), (i^u_{P_1} \oplus i^u_{P_2})(q_1, r)\}$

In the same way, the other conditions can also be verified.

Case 3. If $p_1q_1 \notin Q_1', p_2q_2 \in Q_2'$,

$(f^u_{Q_1} \oplus f^u_{Q_2})((p_1, p_2)(q_1, q_2)) = max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{Q_2}(p_2q_2)\}$

$= max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), max\{f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}\}$

$= max\{max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}, max\{, f^u_{P_1}(q_1), f^u_{P_2}(q_2)\}\},$

$= max\{(f^u_{P_1} \oplus f^u_{P_2})(p_1, p_2), (f^u_{P_1} \oplus f^u_{P_2})(q_1, q_2)\}$

In the same way, the other conditions can also be verified.

Case 4. If $p_1q_1 \in Q_1', p_2q_2 \notin Q_2'$,

$(f^u_{Q_1} \oplus f^u_{Q_2})((p_1, p_2)(q_1, q_2)) = max\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}$

$= max\{max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1)\}, f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}$

$= max\{max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}, max\{f^u_{P_1}(q_1), f^u_{P_2}(q_2)\}\}$

$= max\{(f^u_{P_1} \oplus f^u_{P_2})(p_1, p_2), (f^u_{P_1} \oplus f^u_{P_2})(q_1, q_2)\}$

Similarly, the other conditions can also be verified. □

Definition 3.17. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The degree for any vertex $(p_1, p_2) \in (P_1' \times P_2')$ is,

$(d_{t^l})_{G_1 \oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (t^l_{Q_1} \oplus t^l_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \in Q_2'} min\{t^l_{P_1}(p_1), t^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q_1', p_2=q_2} min\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \in Q_2'} min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{Q_2}(p_2q_2)\}$
$+ \sum_{p_1q_1 \in Q_1', p_2q_2 \notin Q_2'} min\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\}$
$(d_{t^u})_{G_1 \oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (t^u_{Q_1} \oplus t^u_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \in Q_2'} min\{t^u_{P_1}(p_1), t^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q_1', p_2=q_2} min\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \in Q_2'} min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{Q_2}(p_2q_2)\}$
$+ \sum_{p_1q_1 \in Q_1', p_2q_2 \notin Q_2'} min\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\}$
$(d_{i^l})_{G_1 \oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (i^l_{Q_1} \oplus i^l_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \in Q_2'} max\{i^l_{P_1}(p_1), i^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q_1', p_2=q_2} max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \in Q_2'} max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{Q_2}(p_2q_2)\}$
$+ \sum_{p_1q_1 \in Q_1', p_2q_2 \notin Q_2'} max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\}$
$(d_{i^u})_{G_1 \oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (i^u_{Q_1} \oplus i^u_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \in Q_2'} max\{i^u_{P_1}(p_1), i^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q_1', p_2=q_2} max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \in Q_2'} max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{Q_2}(p_2q_2)\}$
$+ \sum_{p_1q_1 \in Q_1', p_2q_2 \notin Q_2'} max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\}$
$(d_{f^l})_{G_1 \oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (f^l_{Q_1} \oplus f^l_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \in Q_2'} max\{f^l_{P_1}(p_1), f^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q_1', p_2=q_2} max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \in Q_2'} max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{Q_2}(p_2q_2)\}$
$+ \sum_{p_1q_1 \in Q_1', p_2q_2 \notin Q_2'} max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\}$
$(d_{f^u})_{G_1 \oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (f^u_{Q_1} \oplus f^u_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \in Q_2'} max\{f^u_{P_1}(p_1), f^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q_1', p_2=q_2} max\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \in Q_2'} max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{Q_2}(p_2q_2)\}$
$+ \sum_{p_1q_1 \in Q_1', p_2q_2 \notin Q_2'} max\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\}$

Theorem 3.18. *Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. If $t^l_{P_1} \geq t^l_{Q_2}, t^u_{P_1} \geq t^u_{Q_2}, i^l_{P_1} \leq i^l_{Q_2}, i^u_{P_1} \leq i^u_{Q_2}, f^l_{P_1} \leq f^l_{Q_2}, f^u_{P_1} \leq f^u_{Q_2}$ and $t^l_{P_2} \geq t^l_{Q_1}, t^u_{P_2} \geq t^u_{Q_1}, i^l_{P_2} \leq i^l_{Q_1}, i^u_{P_2} \leq i^u_{Q_1}, f^l_{P_2} \leq f^l_{Q_1}, f^u_{P_2} \leq f^u_{Q_1}$, then for every $(p_1, p_2) \in (P_1' \times P_2')$,*
$(d)_{G_1 \oplus G_2}(p_1, p_2) = q'(d)_{G_1}(p_1) + s'(d)_{G_2}(p_2)$, where $s' = \mid P_1' \mid - (d)_{G_1}(p_1)$ and $q' = \mid P_2' \mid - (d)_{G_2}(p_2)$.

Proof. Consider,
$(d_{i^l})_{G_1 \oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (i^l_{Q_1} \oplus i^l_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1=q_1, p_2q_2 \in Q_2'} max\{i^l_{P_1}(p_1), i^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q_1', p_2=q_2} max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2)\}$
$+ \sum_{p_1q_1 \notin Q_1', p_2q_2 \in Q_2'} max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{Q_2}(p_2q_2)\}$

$+\sum_{p_1q_1\in Q_1',p_2q_2\notin Q_2'} max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\}$
$= q^{'}(d^l_i)_{G_1}(p_1) + s^{'}(d^l_i)_{G_2}(p_2)$

In the same way, the other conditions can also be verified. □

Definition 3.19. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The total degree for any vertex $(p_1, p_2) \in (P_1' \times P_2')$ is,

$(td_{t^l})_{G_1\oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2))\in Q_1'\times Q_2'}(t^l_{Q_1} \oplus t^l_{Q_2})((p_1, p_2)(q_1, q_2)) + (t^l_{P_1} \oplus t^l_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{t^l_{P_1}(p_1), t^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2)\}$
$+\sum_{p_1q_1\notin Q_1',p_2q_2\in Q_2'} min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1), t^l_{Q_2}(p_2q_2)\}$
$+\sum_{p_1q_1\in Q_1',p_2q_2\notin Q_2'} min\{t^l_{Q_1}(p_1q_1), t^l_{P_2}(p_2), t^l_{P_2}(q_2)\} + min\{t^l_{P_1}(p_1), t^l_{P_1}(p_2)\}$

$(td_{t^u})_{G_1\oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2))\in Q_1'\times Q_2'}(t^u_{Q_1} \oplus t^u_{Q_2})((p_1, p_2)(q_1, q_2)) + (t^u_{P_1} \oplus t^u_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} min\{t^u_{P_1}(p_1), t^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} min\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(p_2)\}$
$+\sum_{p_1q_1\notin Q_1',p_2q_2\in Q_2'} min\{t^u_{P_1}(p_1), t^u_{P_1}(q_1), t^u_{Q_2}(p_2q_2)\}$
$+\sum_{p_1q_1\in Q_1',p_2q_2\notin Q_2'} min\{t^u_{Q_1}(p_1q_1), t^u_{P_2}(p_2), t^u_{P_2}(q_2)\} + min\{t^u_{P_1}(p_1), t^u_{P_1}(p_2)\}$

$(td_{i^l})_{G_1\oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2))\in Q_1'\times Q_2'}(i^l_{Q_1} \oplus i^l_{Q_2})((p_1, p_2)(q_1, q_2)) + (i^l_{P_1} \oplus i^l_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{i^l_{P_1}(p_1), i^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2)\}$
$+\sum_{p_1q_1\notin Q_1',p_2q_2\in Q_2'} max\{i^l_{P_1}(p_1), i^l_{P_1}(q_1), i^l_{Q_2}(p_2q_2)\}$
$+\sum_{p_1q_1\in Q_1',p_2q_2\notin Q_2'} max\{i^l_{Q_1}(p_1q_1), i^l_{P_2}(p_2), i^l_{P_2}(q_2)\} + max\{i^l_{P_1}(p_1), i^l_{P_1}(p_2)\}$

$(td_{i^u})_{G_1\oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2))\in Q_1'\times Q_2'}(i^u_{Q_1} \oplus i^u_{Q_2})((p_1, p_2)(q_1, q_2)) + (i^u_{P_1} \oplus i^u_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{i^u_{P_1}(p_1), i^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2)\}$
$+\sum_{p_1q_1\notin Q_1',p_2q_2\in Q_2'} max\{i^u_{P_1}(p_1), i^u_{P_1}(q_1), i^u_{Q_2}(p_2q_2)\}$
$+\sum_{p_1q_1\in Q_1',p_2q_2\notin Q_2'} max\{i^u_{Q_1}(p_1q_1), i^u_{P_2}(p_2), i^u_{P_2}(q_2)\} + max\{i^u_{P_1}(p_1), i^u_{P_1}(p_2)\}$

$(td_{f^l})_{G_1\oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2))\in Q_1'\times Q_2'}(f^l_{Q_1} \oplus f^l_{Q_2})((p_1, p_2)(q_1, q_2)) + (f^l_{P_1} \oplus f^l_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{f^l_{P_1}(p_1), f^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2)\}$
$+\sum_{p_1q_1\notin Q_1',p_2q_2\in Q_2'} max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{Q_2}(p_2q_2)\}$
$+\sum_{p_1q_1\in Q_1',p_2q_2\notin Q_2'} max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\} + max\{f^l_{P_1}(p_1), f^l_{P_1}(p_2)\}$

$(td_{f^u})_{G_1\oplus G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2))\in Q_1'\times Q_2'}(f^u_{Q_1} \oplus f^u_{Q_2})((p_1, p_2)(q_1, q_2)) + (f^u_{P_1} \oplus f^u_{P_2})(p_1, p_2)$
$= \sum_{p_1=q_1,p_2q_2\in Q_2'} max\{f^u_{P_1}(p_1), f^u_{Q_2}(p_2q_2)\} + \sum_{p_1q_1\in Q_1',p_2=q_2} max\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2)\}$
$+\sum_{p_1q_1\notin Q_1',p_2q_2\in Q_2'} max\{f^u_{P_1}(p_1), f^u_{P_1}(q_1), f^u_{Q_2}(p_2q_2)\}$
$+\sum_{p_1q_1\in Q_1',p_2q_2\notin Q_2'} max\{f^u_{Q_1}(p_1q_1), f^u_{P_2}(p_2), f^u_{P_2}(q_2)\} + max\{f^u_{P_1}(p_1), f^u_{P_1}(p_2)\}$

Theorem 3.20. *Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. If $t^l_{P_1} \geq t^l_{Q_2}, t^u_{P_1} \geq t^u_{Q_2}, i^l_{P_1} \leq i^l_{Q_2}, i^u_{P_1} \leq i^u_{Q_2}, f^l_{P_1} \leq f^l_{Q_2}, f^u_{P_1} \leq f^u_{Q_2}$ and $t^l_{P_2} \geq t^l_{Q_1}, t^u_{P_2} \geq t^u_{Q_1}, i^l_{P_2} \leq i^l_{Q_1}, i^u_{P_2} \leq i^u_{Q_1}, f^l_{P_2} \leq f^l_{Q_1}, f^u_{P_2} \leq f^u_{Q_1}$, then for every $(p_1, p_2) \in (P_1' \times P_2')$,*

$(td_{t^l})_{G_1\oplus G_2}(p_1, p_2) = q'(td_{t^l})_{G_1}(p_1) + s'(td_{t^l})_{G_2}(p_2) - (q' - 1)t^l_{G_1}(p_1) - (s' - 1)t^l_{G_2}(p_2) - max\{t^l_{G_1}(p_1), t^l_{G_2}(p_2)\}$

$(td_{t^u})_{G_1\oplus G_2}(p_1, p_2) = q'(td_{t^u})_{G_1}(p_1) + s'(td_{t^u})_{G_2}(p_2) - (q' - 1)t^u_{G_1}(p_1) - (s' - 1)t^u_{G_2}(p_2) -$

$max\{t^u_{G_1}(p_1), t^u_{G_2}(p_2)\}$

$(td_{i^l})_{G_1 \oplus G_2}(p_1,p_2) = q'(td_{i^l})_{G_1}(p_1) + s'(td_{i^l})_{G_2}(p_2) - (q'-1)i^l_{G_1}(p_1) - (s'-1)i^l_{G_2}(p_2) - min\{i^l_{G_1}(p_1), i^l_{G_2}(p_2)\}$

$(td_{i^u})_{G_1 \oplus G_2}(p_1,p_2) = q'(td_{i^u})_{G_1}(p_1) + s'(td_{i^u})_{G_2}(p_2) - (q'-1)i^u_{G_1}(p_1) - (s'-1)i^u_{G_2}(p_2) - min\{i^u_{G_1}(p_1), i^u_{G_2}(p_2)\}$

$(td_{f^l})_{G_1 \oplus G_2}(p_1,p_2) = q'(td_{f^l})_{G_1}(p_1) + s'(td_{f^l})_{G_2}(p_2) - (q'-1)f^l_{G_1}(p_1) - (s'-1)f^l_{G_2}(p_2) - min\{f^l_{G_1}(p_1), f^l_{G_2}(p_2)\}$

$(td_{f^u})_{G_1 \oplus G_2}(p_1,p_2) = q'(td_{f^u})_{G_1}(p_1) + s'(td_{f^u})_{G_2}(p_2) - (q'-1)f^u_{G_1}(p_1) - (s'-1)f^u_{G_2}(p_2) - min\{f^u_{G_1}(p_1), f^u_{G_2}(p_2)\}$

Proof. Consider,

$(td_{f^l})_{G_1 \oplus G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q'_1 \times Q'_2} (f^l_{Q_1} \oplus f^l_{Q_2})((p_1,p_2)(q_1,q_2)) + (f^l_{P_1} \oplus f^l_{P_2})(p_1,p_2)$

$= \sum_{p_1=q_1, p_2q_2 \in Q'_2} max\{f^l_{P_1}(p_1), f^l_{Q_2}(p_2q_2)\} + \sum_{p_1q_1 \in Q'_1, p_2=q_2} max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2)\}$

$+ \sum_{p_1q_1 \notin Q'_1, p_2q_2 \in Q'_2} max\{f^l_{P_1}(p_1), f^l_{P_1}(q_1), f^l_{Q_2}(p_2q_2)\}$

$+ \sum_{p_1q_1 \in Q'_1, p_2q_2 \notin Q'_2} max\{f^l_{Q_1}(p_1q_1), f^l_{P_2}(p_2), f^l_{P_2}(q_2)\} + max\{f^l_{P_1}(p_1), f^l_{P_1}(p_2)\}$

$= \sum_{p_1=q_1, p_2q_2 \in Q'_2}, f^l_{Q_2}(p_2q_2) + \sum_{p_1q_1 \in Q'_1, p_2=q_2} f^l_{Q_1}(p_1q_1) + \sum_{p_1q_1 \notin Q'_1, p_2q_2 \in Q'_2} f^l_{Q_2}(p_2q_2)$

$+ \sum_{p_1q_1 \in Q'_1, p_2q_2 \notin Q'_2} f^l_{Q_1}(p_1q_1) + max\{f^l_{P_1}(p_1), f^l_{P_1}(p_2)\}$

$= \sum_{p_1=q_1, p_2q_2 \in Q'_2}, f^l_{Q_2}(p_2q_2) + \sum_{p_1q_1 \in Q'_1, p_2=q_2} f^l_{Q_1}(p_1q_1) + \sum_{p_1q_1 \notin Q'_1, p_2q_2 \in Q'_2} f^l_{Q_2}(p_2q_2)$

$+ \sum_{p_1q_1 \in Q'_1, p_2q_2 \notin Q'_2} f^l_{Q_1}(p_1q_1) - min\{f^l_{P_1}(p_1), f^l_{P_1}(p_2)\}$

$= q'(td_{f^l})_{G_1}(p_1) + s'(td_{f^l})_{G_2}(p_2) - (q'-1)f^l_{G_1}(p_1) - (s'-1)f^l_{G_2}(p_2) - min\{f^l_{G_1}(p_1), f^l_{G_2}(p_2)\}.$

Similarly, the other conditions can also be proved. □

Example 3.21. The symmetric difference $G_1 \oplus G_2$ of two SIVNGs G_1 and $G_2 = (P_2, Q_2)$ is represented in Figure 10 and 11.

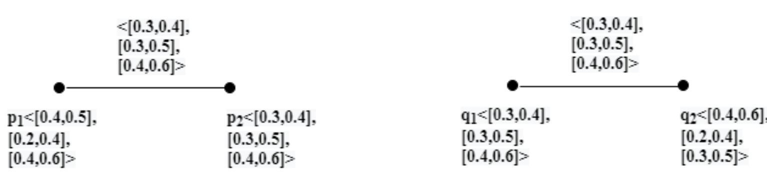

FIGURE 10. Strong Interval Valued Neutrosophic Graphs G_1 and G_2

By direct calculations, $d_{t^l}(p_1,q_1) = 0.3 + 0.3 = 0.6$; $d_{t^u}(p_1,q_1) = 0.4 + 0.4 = 0.8$; $d_t(p_1,q_1) = [0.6, 0.8]$; $td_{t^l}(p_1,q_1) = 0.3 + 0.3 + 0.3 = 0.9$; $td_{t^u}(p_1,q_1) = 0.4 + 0.4 + 0.4 = 1.2$; $td_t(p_1,q_1) = [0.9, 1.2]$.

By using theorem, $s' =| P'_1 | -(d)_{G_1}(p_1) = 2 - 1 = 1$; $q' =| P'_2 | -(d)_{G_2}(p_2) = 2 - 1 = 1$; $d_{t^l}(p_1,q_1) = 1(0.3) + 1(0.3) = 0.6$; $d_{t^u}(p_1,q_1) = 1(0.4) + 1(0.4) = 0.8$; $d_t(p_1,q_1) = [0.6, 0.8]$; $td_{t^l}(p_1,q_1) = 1(0.7) + 1(0.6) - 0(0.4) - 0(0.3) - max\{0.4, 0.3\} = 0.9$; $td_{t^u}(p_1,q_1) = 1(0.9) +$

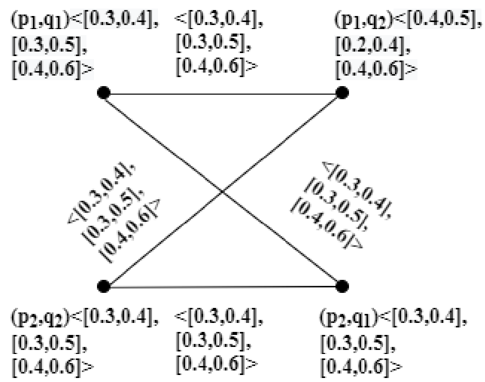

Figure 11. Symmetric difference $G_1 \oplus G_2$

$1(0.8) - 0(0.5) - 0(0.4) - max\{0.5, 0.4\} = 1.2$; $td_t(p_1, q_1) = [0.9, 1.2]$.

Definition 3.22. The residue product $G_1\ G_2 = (P_1 \bullet P_2, Q_1 \bullet Q_2)$ of two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ is defined as

(1) $(t^l_{P_1} \bullet t^l_{P_2})(p_1, p_2) = max\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$,
$(t^u_{P_1} \bullet t^u_{P_2})(p_1, p_2) = max\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$,
$(i^l_{P_1} \bullet i^l_{P_2})(p_1, p_2) = min\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$,
$(i^u_{P_1} \bullet i^u_{P_2})(p_1, p_2) = min\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$,
$(f^l_{P_1} \bullet f^l_{P_2})(p_1, p_2) = min\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$,
$(f^u_{P_1} \bullet f^u_{P_2})(p_1, p_2) = min\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$.

(2) $(t^l_{Q_1} \bullet t^l_{Q_2})((p_1, p_2)(q_1, q_2)) = t^l_{Q_1}(p_1 q_1)$,
$(t^u_{Q_1} \bullet t^u_{Q_2})((p_1, p_2)(q_1, q_2)) = t^u_{Q_1}(p_1 q_1)$,
$(i^l_{Q_1} \bullet i^l_{Q_2})((p_1, p_2)(q_1, q_2)) = i^l_{Q_1}(p_1 q_1)$,
$(i^u_{Q_1} \bullet i^u_{Q_2})((p_1, p_2)(q_1, q_2)) = i^u_{Q_1}(p_1 q_1)$,
$(f^l_{Q_1} \bullet f^l_{Q_2})((p_1, p_2)(q_1, q_2)) = f^l_{Q_1}(p_1 q_1)$,
$(f^u_{Q_1} \bullet f^u_{Q_2})((p_1, p_2)(q_1, q_2)) = f^u_{Q_1}(p_1 q_1), \forall p_1 q_1 \in Q'_1, p_2 \neq q_2$.

Theorem 3.23. *The residue product of two SIVNGs G_1 and G_2, need not be a SIVNG.*

From example 3.24, Figure 13, it is clear that t, i and f values of the vertices and the edges in $G_1 \bullet G_2$ do not satisfy the strong condition and hence it is an IVNG.

Example 3.24. Consider two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ as represented in Figure 12. Their residue product $G_1 \bullet G_2$ is represented in Figure 13.

Theorem 3.25. *Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. If $t^l_{P_1} \geq t^l_{Q_2}, t^u_{P_1} \geq t^u_{Q_2}, i^l_{P_1} \leq i^l_{Q_2}, i^u_{P_1} \leq i^u_{Q_2}, f^l_{P_1} \leq f^l_{Q_2}, f^u_{P_1} \leq f^u_{Q_2}$ and $t^l_{P_2} \geq t^l_{Q_1}, t^u_{P_2} \geq t^u_{Q_1}, i^l_{P_2} \leq i^l_{Q_1}, i^u_{P_2} \leq i^u_{Q_1}, f^l_{P_2} \leq f^l_{Q_1}, f^u_{P_2} \leq f^u_{Q_1}$, then the residue product of two SIVNGs G_1 and G_2 is a SIVNG.*

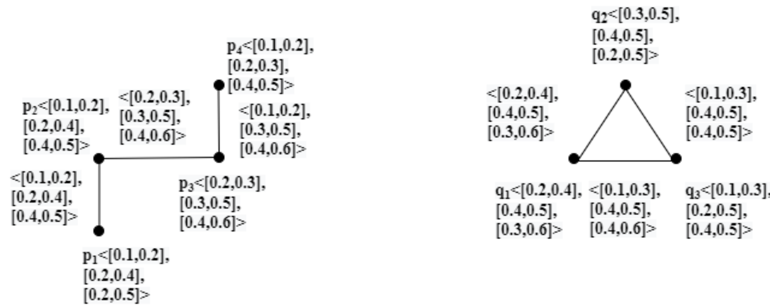

FIGURE 12. Strong Interval Valued Neutrosophic Graphs G_1 and G_2

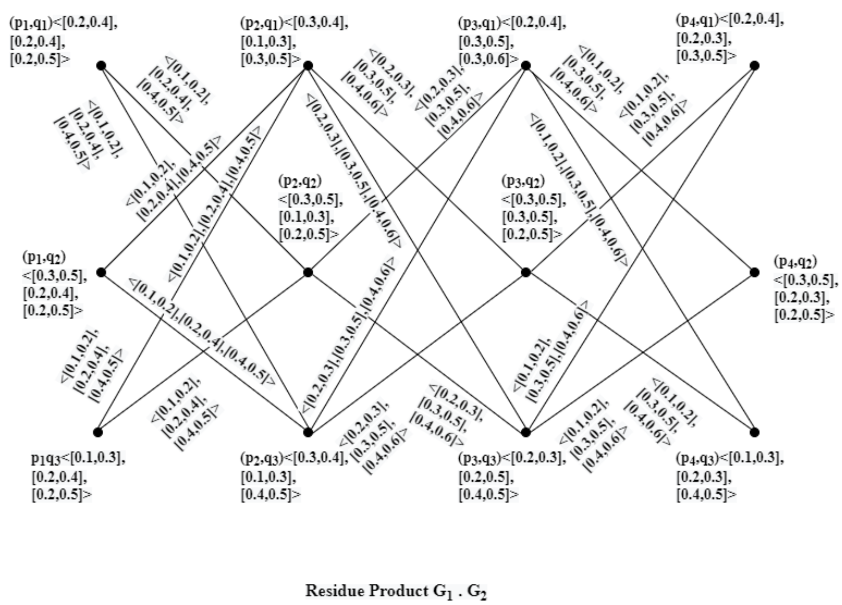

FIGURE 13. Residue product $G_1 \bullet G_2$

Proof. For $p_1q_1 \in Q_1^{'}, p_2 \neq q_2$,
$(t^l_{Q_1} \bullet t^l_{Q_2})((p_1,p_2)(q_1,q_2)) = t^l_{Q_1}(p_1q_1), = min\{t^l_{P_1}(p_1), t^l_{P_1}(q_1)\}$,
$= min\{max\{t^l_{P_1}(p_1), t^l_{P_1}(p_2)\}, max\{t^l_{P_1}(q_1), t^l_{P_1}(q_2)\}\}$,
$= min\{(t^l_{P_1} \bullet t^l_{P_2})(p_1,p_2), (t^l_{P_1} \bullet t^l_{P_2})(q_1,q_2)\}$. □

Example 3.26. Consider two SIVNGs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ as represented in Figure 14. Their residue product $G_1 \bullet G_2$ is represented in Figure 15.

Definition 3.27. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The degree for any vertex $(p_1,p_2) \in (P_1^{'} \times P_2^{'})$ is,
$(d_{t^l})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1^{'} \times Q_2^{'}} (t^l_{Q_1} \bullet t^l_{Q_2})((p_1,p_2)(q_1,q_2))$
$= \sum_{p_1q_1 \in Q_1^{'}, p_2 \neq q_2} t^l_{Q_1}(p_1q_1) = (d^l_t)_{G_1}(p_1)$
$(d_{t^u})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1^{'} \times Q_2^{'}} (t^u_{Q_1} \bullet t^u_{Q_2})((p_1,p_2)(q_1,q_2))$

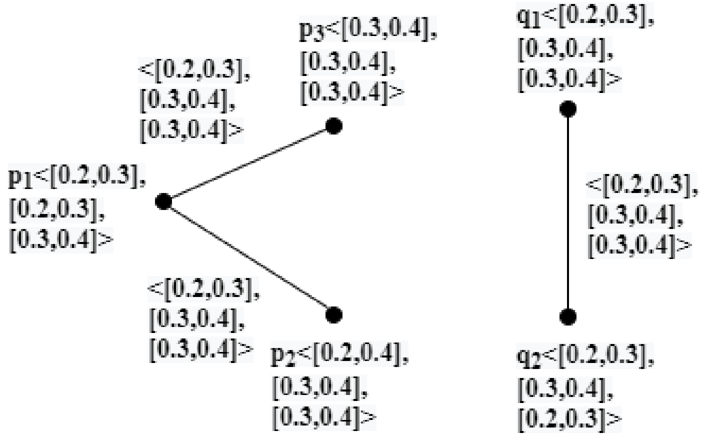

FIGURE 14. Strong Interval Valued Neutrosophic Graphs G_1 and G_2

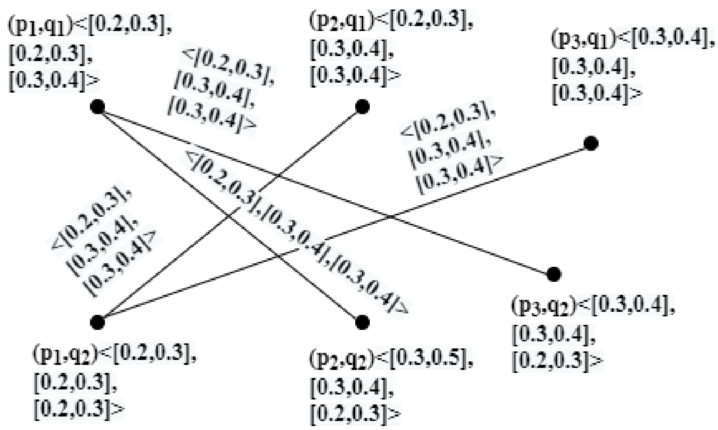

FIGURE 15. Residue product $G_1 \bullet G_2$

$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} t^u_{Q_1}(p_1 q_1) = (d^u_t)_{G_1}(p_1)$
$(d_{i^l})_{G_1 \bullet G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (i^l_{Q_1} \bullet i^l_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} i^l_{Q_1}(p_1 q_1) = (d^l_i)_{G_1}(p_1)$
$(d_{i^u})_{G_1 \bullet G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (i^u_{Q_1} \bullet i^u_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} i^u_{Q_1}(p_1 q_1) = (d^u_i)_{G_1}(p_1)$
$(d_{f^l})_{G_1 \bullet G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (f^l_{Q_1} \bullet f^l_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} f^l_{Q_1}(p_1 q_1) = (d^l_f)_{G_1}(p_1)$
$(d_{f^u})_{G_1 \bullet G_2}(p_1, p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (f^u_{Q_1} \bullet f^u_{Q_2})((p_1, p_2)(q_1, q_2))$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} f^u_{Q_1}(p_1 q_1) = (d^u_f)_{G_1}(p_1)$

Definition 3.28. Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two SIVNGs. The total degree for any vertex $(p_1, p_2) \in (P_1' \times P_2')$ is,

$(td_{t^l})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (t^l_{Q_1} \bullet t^l_{Q_2})((p_1,p_2)(q_1,q_2)) + (t^l_{P_1} \bullet t^l_{P_2})(p_1,p_2)$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} t^l_{Q_1}(p_1 q_1) + max\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} t^l_{Q_1}(p_1 q_1) + t^l_{P_1}(p_1) + t^l_{P_2}(p_2) - min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)\}$
$= (td_{t^l})_{G_1}(p_1) + t^l_{P_2}(p_2) - min\{t^l_{P_1}(p_1), t^l_{P_2}(p_2)$
$(td_{t^u})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (t^u_{Q_1} \bullet t^u_{Q_2})((p_1,p_2)(q_1,q_2)) + (t^u_{P_1} \bullet t^u_{P_2})(p_1,p_2)$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} t^u_{Q_1}(p_1 q_1) + max\{t^u_{P_1}(p_1), t^l_{P_2}(p_2)\}$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} t^u_{Q_1}(p_1 q_1) + t^u_{P_1}(p_1) + t^u_{P_2}(p_2) - min\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)\}$
$= (td_{t^u})_{G_1}(p_1) + t^u_{P_2}(p_2) - min\{t^u_{P_1}(p_1), t^u_{P_2}(p_2)$
$(td_{i^l})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (i^l_{Q_1} \bullet i^l_{Q_2})((p_1,p_2)(q_1,q_2)) + (i^l_{P_1} \bullet i^l_{P_2})(p_1,p_2)$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} i^l_{Q_1}(p_1 q_1) + min\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} i^l_{Q_1}(p_1 q_1) + i^l_{P_1}(p_1) + i^l_{P_2}(p_2) - max\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)\}$
$= (td_{i^l})_{G_1}(p_1) + i^l_{P_2}(p_2) - max\{i^l_{P_1}(p_1), i^l_{P_2}(p_2)$
$(td_{i^u})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (i^u_{Q_1} \bullet i^u_{Q_2})((p_1,p_2)(q_1,q_2)) + (i^u_{P_1} \bullet i^u_{P_2})(p_1,p_2)$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} i^u_{Q_1}(p_1 q_1) + min\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} i^u_{Q_1}(p_1 q_1) + i^u_{P_1}(p_1) + i^u_{P_2}(p_2) - max\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)\}$
$= (td_{i^u})_{G_1}(p_1) + i^u_{P_2}(p_2) - max\{i^u_{P_1}(p_1), i^u_{P_2}(p_2)$
$(td_{f^l})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (f^l_{Q_1} \bullet f^l_{Q_2})((p_1,p_2)(q_1,q_2)) + (f^l_{P_1} \bullet f^l_{P_2})(p_1,p_2)$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} f^l_{Q_1}(p_1 q_1) + min\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} f^l_{Q_1}(p_1 q_1) + f^l_{P_1}(p_1) + f^l_{P_2}(p_2) - max\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)\}$
$= (td_{f^l})_{G_1}(p_1) + f^l_{P_2}(p_2) - max\{f^l_{P_1}(p_1), f^l_{P_2}(p_2)$
$(td_{f^u})_{G_1 \bullet G_2}(p_1,p_2) = \sum_{((p_1,p_2)(q_1,q_2)) \in Q_1' \times Q_2'} (f^u_{Q_1} \bullet f^u_{Q_2})((p_1,p_2)(q_1,q_2)) + (f^u_{P_1} \bullet f^u_{P_2})(p_1,p_2)$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} f^u_{Q_1}(p_1 q_1) + min\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$
$= \sum_{p_1 q_1 \in Q_1', p_2 \neq q_2} f^u_{Q_1}(p_1 q_1) + f^u_{P_1}(p_1) + f^u_{P_2}(p_2) - max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)\}$
$= (td_{f^u})_{G_1}(p_1) + f^u_{P_2}(p_2) - max\{f^u_{P_1}(p_1), f^u_{P_2}(p_2)$

From Figure 15, $d_f(p_1,q_2)$ and $td_f(p_1,q_2)$ for the vertex (p_1,q_2) are calculated below.

$d_{f^l}(p_1,q_2) = 0.3 + 0.3 = 0.6$, $d_{f^u}(p_1,q_2) = 0.4 + 0.4 = 0.8$, $d_f(p_1,q_2) = [0.6, 0.8]$

$td_{f^l}(p_1,q_2) = 0.9 + 0.2 - 0.3 = 0.8$, $td_{f^u}(p_1,q_2) = 1.2 + 0.3 - 0.4 = 1.1$, $td_f(p_1,q_2) = [0.8, 1.1]$.

4. **Application**

4.1. *The Cardiac Cycle of a Human Heart*

The right atrium (RA) of the heart receives deoxygenated blood from both Superior Vena Cava (SVC) and Inferior Vena Cava (IVC). Then, the tricuspid valve (TVL) opens due to the contraction of the right atrium and the deoxygenated blood has directed to the right ventricle (RV). After the ventricular filling, the tricuspid valve (TVL) shuts. Now, the right ventricle (RV) gets contracted, which causes the opening of the pulmonary valve (PVL) and the blood is transferred to the pulmonary artery (PA) and then to the lungs for oxygenation. After the blood gets oxygenated, it enters the left atrium (LA) via pulmonary veins (PV). Now, the left atrium gets contracted and the mitral valve (MVL) opens for transferring the oxygenated blood to the left ventricle. After passing out the blood to the left ventricle (LV), the mitral valve (MVL) closes. Now, the left ventricle contracts for ejecting the blood to the aorta (A) through the aortic valve (AVL). From there, the oxygenated blood passes to all the parts of the human body. The blood flow through the human heart has presented in Figure 16. Biologically during the period of cardiac cycle, it is observed that

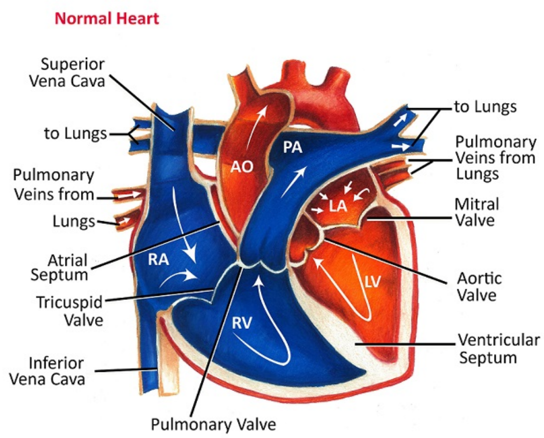

Figure 16. The Human Heart

(1) Left ventricular systole and diastole is the most effective phase on the whole.
(2) The Left side of the human heart has comparatively higher pressure than on the right side. i.e., Left Atrial (Ventricular) Systole has higher pressure than Right Atrial (Ventricular) Systole and vice versa.
(3) Systolic (ventricular) pressure is higher than diastolic (ventricular) pressure.

The flowchart given in Figure 17 illustrates the method for evaluating the cardiac functioning of the human heart.

4.2. *The Wright Table - Study of blood flow along with their blood pressure values*

Wright's table [22], a teaching tool to learn and understand the cardiac cycle, has elaborated the path of blood flow with the blood pressure changes. The Wright table explains how the pressures and flows of each compartment fluctuate over time, as well as how the heart functions as a pump, first filling and then emptying the ventricles and thereby transferring blood from low-pressure venous to high-pressure arterial compartments. The Wright's table provided in Table 1 and Table 2 elaborates the path of blood flow along with the blood pressure changes during AS/VD and AD/VS phase of the human heart observed for a healthy adult of age

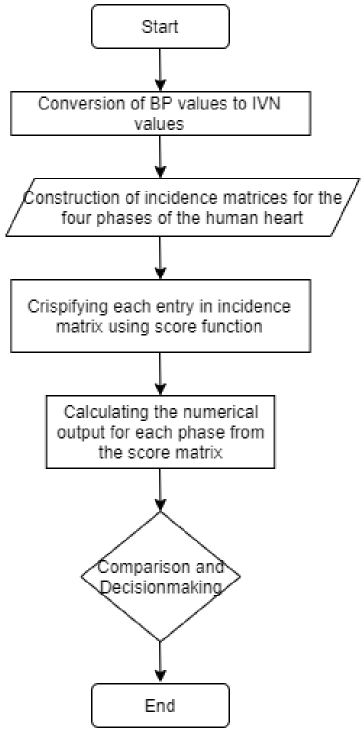

FIGURE 17. Flowchart for evaluating Cardiac Functioning of the Human Heart

above 35 years without any cardiac malfunction along with their corresponding hemodynamic parameters.

TABLE 1. The Wright's table representation of AS/VD and AD/VS phase (Right Side)

	SVC and IVC		RA		TVL		RV		PVL		PA	
AS/VD	2-5	→	4-6	→	12.6-29.3	→	0-8	(closed valve)	0	(closed valve)	8-15	→
AD/VS	2-5	→	0	(closed valve)	0	(closed valve)	15-25	→	15-25	→	15-25	→

4.3. *Conversion of Blood pressure values into Interval Valued Neutrosophic values (IVN-values)*

As the rate of blood pressure changes from time to time under a certain interval and it is highly impracticable for getting the same blood pressure value in each prediction, a minute level of indeterminacy and falsity have been observed.

TABLE 2. The Wright's table representation of AS/VD and AD/VS phase (Left Side)

	PV		LA		MVL		LV		AVL		A	
AS/VD	2-5	$\longrightarrow$	7-8	$\longrightarrow$	0	$\longrightarrow$	0-12	(closed valve)	0	(closed valve)	60-90	$\longrightarrow$
AD/VS	2-5	$\longrightarrow$	0	(closed valve)	0	(closed valve)	100-140	$\longrightarrow$	8-12	$\longrightarrow$	100-140	$\longrightarrow$

The blood pressure values given in Table 1 and Table 2 are converted to fit under an IVN-environment. The truth-membership values are exactly the blood pressure values taken for consideration and the indeterminacy-membership values and the falsity-membership values are estimated accordingly.

Since the IVN-values lie in the range of [0, 1], the blood pressure values (mm/Hg) given in Table 1 and Table 2 are re-scaled using bar conversion. For instance, the blood pressure value in Superior Vena Cava is 2-5 mm/Hg and its bar conversion becomes $[0.00266645, 0.00666612] \approx [0.003, 0.007]$.

Thus with reference to the bar conversion, Table 3 shows the re-scaled Interval Valued Neutrosophic blood pressure values observed in Table 1 and Table 2.

TABLE 3. Rescaled IVN Blood Pressure Values

	AS/VD	AD/VS
SVC & IVC	$< [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >$	$< [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >$
RA	$< [0.005, 0.008], [0.001, 0.003], [0.001, 0.002] >$	$< [0, 0], [0.001, 0.001], [0.001, 0.001] >$
TVL	$< [0.017, 0.04], [0.001, 0.002], [0.001, 0.002] >$	$< [0, 0], [0, 0], [0, 0] >$
RV	$< [0, 0.01], [0.001, 0.002], [0.002, 0.004] >$	$< [0.02, 0.03], [0.001, 0.002], [0.001, 0.0015] >$
PVL	$< [0, 0], [0, 0], [0, 0] >$	$< [0.02, 0.03], [0.001, 0.002], [0.001, 0.002] >$
PA	$< [0.01, 0.02], [0.002, 0.004], [0.001, 0.003] >$	$< [0.02, 0.03], [0.002, 0.003], [0.001, 0.003] >$
PV	$< [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >$	$< [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >$
LA	$< [0.009, 0.01], [0.001, 0.002], [0.001, 0.002] >$	$< [0, 0], [0.001, 0.001], [0.001, 0.001] >$
MVL	$< [0, 0], [0.001, 0.001], [0.001, 0.001] >$	$< [0, 0], [0, 0], [0, 0] >$
LV	$< [0, 0.016], [0.001, 0.0012], [0.001, 0.0015] >$	$< [0.13, 0.19], [0.002, 0.003], [0.001, 0.002] >$
AVL	$< [0, 0], [0, 0], [0, 0] >$	$< [0.01, 0.016], [0.0012, 0.0016], [0.001, 0.0015] >$
A	$< [0.08, 0.12], [0.005, 0.007], [0.003, 0.005] >$	$< [0.13, 0.19], [0.003, 0.005], [0.002, 0.004] >$

4.4. *Modeling of Human Heart as SIVN - Digraph*

The blood flow through right and left heart as given in Figure 16 is represented as a digraph $G = (P, \overrightarrow{Q})$ with the vertex set $X = \{p_1, p_2, p_3, ..., p_{16}\}$ along with the directed edges in Figure 18.

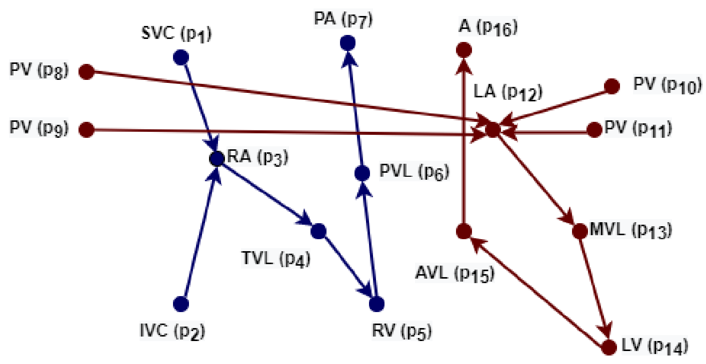

FIGURE 18. The Human Heart digraph

4.5. *SIVNDG representation of the cardiac cycle functioning during AS/VD and AD/VS Phases*

During the cardiac cycle functioning, both the right and the left atria narrow down at first, pumping blood to the right ventricle and the left ventricle, respectively. During this period, both the right and the left atria are in systolic phase and the corresponding right and the left ventricles are in diastolic phase. In response to electrical impulses the right and left ventricles contract instantly, allowing blood to flow to the lungs and to the rest of the body.At this time, the atria remain in diastolic phase and the ventricles are in systolic phase and their corresponding strong interval-valued neutrosophic values during this AS/VD and AD/VS phases are represented in Figure 19 and Figure 20 with reference to Table 3.

During AS/VD phase, the vertices and the edges along with their membership functions for the directed subgraphs $H_1 = (P_{H_1}, \overrightarrow{Q_{H_1}})$ and $H_2 = (P_{H_2}, \overrightarrow{Q_{H_2}})$ for $X = \{p_1, p_2, p_3, ..., p_{16}\}$ are defined by

$P_{H_1} = \{p_1 < [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >, p_2 < [0.003, 0.007],$
$[0.001, 0.0015], [0.001, 0.002] >, p_3 < [0.005, 0.008], [0.001, 0.003], [0.001,$
$0.002] >, p_4 < [0.017, 0.04], [0.001, 0.002], [0.001, 0.002] >, p_5 < [0, 0.01],$
$[0.001, 0.002], [0.002, 0.004] >, p_6 < [0, 0], [0, 0], [0, 0] >, p_7 < [0.01, 0.02],$
$[0.002, 0.004], [0.001, 0.003] >\}.$
$\overrightarrow{Q_{H_1}} = \{\overrightarrow{p_1p_3} < [0.003, 0.007], [0.001, 0.003], [0.001, 0.002] >,$
$\overrightarrow{p_2p_3} < [0.003, 0.007], [0.001, 0.003], [0.001, 0.002] >, \overrightarrow{p_3p_4} < [0.005, 0.008],$

$[0.001, 0.003], [0.001, 0.002] >, \overrightarrow{p_4p_5} < [0, 0.01], [0.001, 0.002], [0.002, 0.004] >,$
$\overrightarrow{p_5p_6} < [0, 0], [0.001, 0.002], [0.002, 0.004] >, \overrightarrow{p_6p_7} < [0, 0], [0.002, 0.004],$
$[0.001, 0.003] >\}.$

$P_{H_2} = \{p_8 < [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >,$
$p_9 < [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >, p_{10} < [0.003, 0.007],$
$[0.001, 0.0015], [0.001, 0.002] >, p_{11} < [0.003, 0.007], [0.001, 0.0015],$
$[0.001, 0.002] >, p_{12} < [0.009, 0.01], [0.001, 0.002], [0.001, 0.002] >, p_{13} < [0, 0],$
$[0.001, 0.001], [0.001, 0.001] >, p_{14} < [0, 0.016], [0.001, 0.0012], [0.001, 0.0015] >, p_{15} <$
$[0, 0], [0, 0], [0, 0] >, p_{16} < [0.08, 0.12], [0.005, 0.007], [0.003, 0.005] >\}.$

$\overrightarrow{Q_{H_2}} = \{\overrightarrow{p_8p_{12}} < [0.003, 0.007], [0.001, 0.002], [0.001, 0.002] >,$
$\overrightarrow{p_9p_{12}} < [0.003, 0.007], [0.001, 0.002], [0.001, 0.002] >,$
$\overrightarrow{p_{10}p_{12}} < [0.003, 0.007], [0.001, 0.002], [0.001, 0.002] >,$
$\overrightarrow{p_{11}p_{12}} < [0.003, 0.007], [0.001, 0.002], [0.001, 0.002] >,$
$\overrightarrow{p_{12}p_{13}} < [0, 0], [0.001, 0.002], [0.001, 0.002] >,$
$\overrightarrow{p_{13}p_{14}} < [0, 0], [0.001, 0.0012], [0.001, 0.0015] >,$
$\overrightarrow{p_{14}p_{15}} < [0, 0], [0.001, 0.0012], [0.001, 0.0015] >,$
$\overrightarrow{p_{15}p_{16}} < [0, 0], [0.005, 0.007], [0.003, 0.005] >\}.$

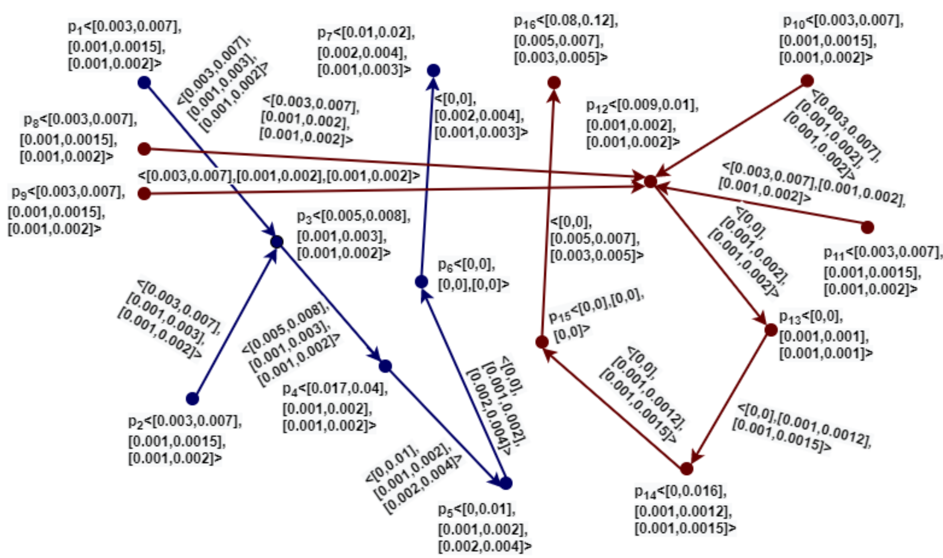

FIGURE 19. SIVN Digraph $G = H_1 \cup H_2$ during AS/ VD Phase

During AD / VS phase, the vertices and the edges along with their membership functions for the directed subgraphs $H_3 = (P_{H_3}, \overrightarrow{Q_{H_3}})$ and $H_4 = (P_{H_4}, \overrightarrow{Q_{H_4}})$ on X are defined by,

$P_{H_3} = \{p_1 < [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >, p_2 < [0.003, 0.007],$
$[0.001, 0.0015], [0.001, 0.002] >, p_3 < [0, 0], [0.001, 0.001], [0.001, 0.001] >,$
$p_4 < [0, 0], [0, 0], [0, 0] >, p_5 < [0.02, 0.03], [0.001, 0.002], [0.001, 0.0015] >,$

$p_6 < [0.02, 0.03], [0.001, 0.002], [0.001, 0.002] >, p_7 < [0.02, 0.03], [0.002, 0.003],$
$[0.001, 0.003] >\}.$
$\overrightarrow{Q_{H_3}} = \{\overrightarrow{p_1p_3} < [0, 0], [0.001, 0.0015], [0.001, 0.002] >, \overrightarrow{p_2p_3} < [0, 0],$
$[0.001, 0.0015], [0.001, 0.002] >, \overrightarrow{p_3p_4} < [0, 0], [0.001, 0.001], [0.001, 0.001] >,$
$\overrightarrow{p_4p_5} < [0, 0], [0.001, 0.002], [0.001, 0.0015] >, \overrightarrow{p_5p_6} < [0.02, 0.03], [0.001,$
$0.002], [0.001, 0.002] >, \overrightarrow{p_6p_7} < [0.02, 0.03], [0.002, 0.003], [0.001, 0.003] >\}.$
$P_{H_4} = \{p_8 < [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >, p_9 < [0.003,$
$0.007], [0.001, 0.0015], [0.001, 0.002] >, p_{10} < [0.003, 0.007], [0.001, 0.0015],$
$[0.001, 0.002] >, p_{11} < [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >,$
$p_{12} < [0, 0], [0.001, 0.001], [0.001, 0.001] >, p_{13} < [0, 0], [0, 0], [0, 0] >,$
$p_{14} < [0.13, 0.19], [0.002, 0.003], [0.001, 0.002] >, p_{15} < [0.01, 0.016], [0.0012,$
$0.0016], [0.001, 0.0015] >, p_{16} < [0.13, 0.19], [0.003, 0.005], [0.002, 0.004] >\}.$
$\overrightarrow{Q_{H_4}} = \{\overrightarrow{p_8p_{12}} < [0, 0], [0.001, 0.0015], [0.001, 0.002] >, \overrightarrow{p_9p_{12}} < [0, 0],$
$[0.001, 0.0015], [0.001, 0.002] >, \overrightarrow{p_{10}p_{12}} < [0, 0], [0.001, 0.0015], [0.001, 0.002] >, \overrightarrow{p_{11}p_{12}} <$
$[0, 0], [0.001, 0.0015], [0.001, 0.002] >, \overrightarrow{p_{12}p_{13}} < [0, 0], [0.001, 0.001],$
$[0.001, 0.001] >, \overrightarrow{p_{13}p_{14}} < [0, 0], [0.002, 0.003], [0.001, 0.002] >, \overrightarrow{p_{14}p_{15}} < [0.01,$
$0.016], [0.002, 0.003], [0.001, 0.002] >, \overrightarrow{p_{15}p_{16}} < [0.01, 0.016], [0.003, 0.005],$
$[0.002, 0.004] >\}.$

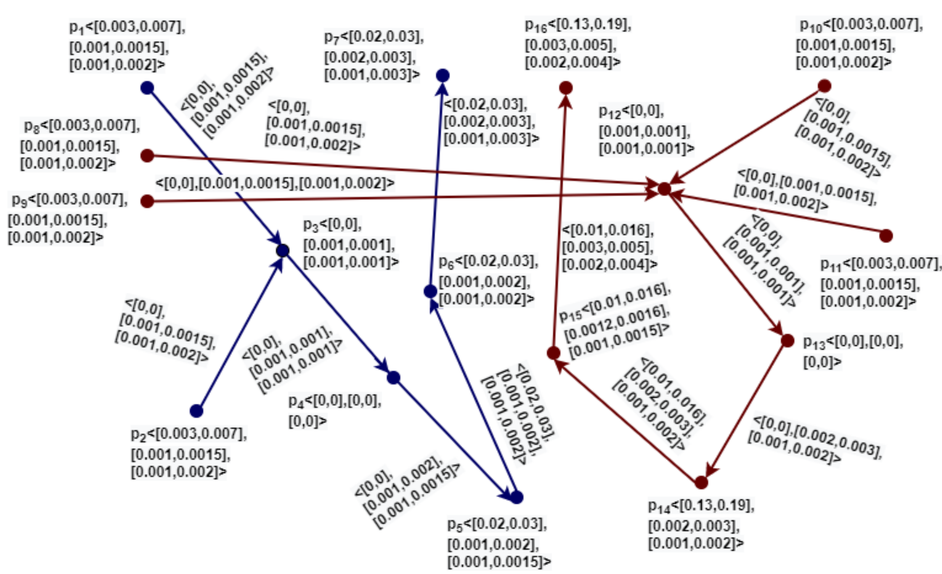

FIGURE 20. SIVN Digraph $G = H_3 \cup H_4$ during AD/ VS Phase

4.6. *Matrix form of SIVNDG during AS/VD and AD/VS phase*

The strong Interval Valued Neutrosophic directed subgraphs $\overrightarrow{H_l}(l = 1, 2, 3, 4)$ during AS/ VD and AD/ VS Phase are represented by the following incident matrices $m_{\overrightarrow{H_l}} = (\overrightarrow{p_ip_j})$ where

$i, j = 1, 2, .., 16$ and $l = 1, 2, 3, 4$.

$$m_{\overrightarrow{H_1}} = \begin{cases} < [0.003, 0.007], [0.001, 0.0015], [0.001, 0.002] >, & i = 1, 2, j = 3 \\ < [0.005, 0.008], [0.001, 0.003], [0.001, 0.002] >, & i = 3, j = 4 \\ < [0, 0.01], [0.001, 0.002], [0.002, 0.004] >, & i = 4, j = 5 \\ < [0, 0], [0.001, 0.002], [0.002, 0.004] >, & i = 5, j = 6 \\ < [0, 0], [0.002, 0.004], [0.001, 0.003] >, & i = 6, j = 7 \\ < [0, 0], [0, 0], [1, 1] > & \text{otherwise} \end{cases}$$

$$m_{\overrightarrow{H_2}} = \begin{cases} < [0.003, 0.007], [0.001, 0.002], [0.001, 0.002] >, & i = 8, 9, 10, 11, j = 12 \\ < [0, 0], [0.001, 0.002], [0.001, 0.002] >, & i = 12, j = 13 \\ < [0, 0], [0.001, 0.0012], [0.001, 0.0015] >, & i = 13, j = 14 \\ < [0, 0], [0.001, 0.0012], [0.001, 0.0015] >, & i = 14, j = 15 \\ < [0, 0], [0.005, 0.007], [0.003, 0.005] >, & i = 15, j = 16 \\ < [0, 0], [0, 0], [1, 1] > & \text{otherwise} \end{cases}$$

$$m_{\overrightarrow{H_3}} = \begin{cases} < [0, 0], [0.001, 0.0015], [0.001, 0.002] >, & i = 1, 2, j = 3 \\ < [0, 0], [0.001, 0.001], [0.001, 0.001] >, & i = 3, j = 4 \\ < [0, 0], [0.001, 0.002], [0.001, 0.0015] >, & i = 4, j = 5 \\ < [0.02, 0.03], [0.001, 0.002], [0.001, 0.002] >, & i = 5, j = 6 \\ < [0.02, 0.03], [0.002, 0.003], [0.001, 0.003] >, & i = 6, j = 7 \\ < [0, 0], [0, 0], [1, 1] > & \text{otherwise} \end{cases}$$

$$m_{\overrightarrow{H_4}} = \begin{cases} < [0, 0], [0.001, 0.0015], [0.001, 0.002] >, & i = 8, 9, 10, 11, j = 12 \\ < [0, 0], [0.001, 0.001], [0.001, 0.001] >, & i = 12, j = 13 \\ < [0, 0], [0.002, 0.003], [0.001, 0.002] >, & i = 13, j = 14 \\ < [0.01, 0.016], [0.002, 0.003], [0.001, 0.002] >, & i = 14, j = 15 \\ < [0.01, 0.016], [0.003, 0.005], [0.002, 0.004] >, & i = 15, j = 16 \\ < [0, 0], [0, 0], [1, 1] > & \text{otherwise} \end{cases}$$

For any given Strong Interval Valued Neutrosophic Number $a_P = ([t_P^l, t_P^u], [i_P^l, i_P^u], [f_P^l, f_P^u])$ with the score function [23]

$$S(a_P) = \left(\frac{2 + (t_P^l + t_P^u) - 2(i_P^l + i_P^u) - (f_P^l + f_P^u)}{4} \right) \tag{1}$$

In order to obtain the crisp values from the corresponding SIVN values from the above incidence matrices $\overrightarrow{H_l}(l = 1, 2, 3, 4)$, the score function is used. The score values of each entry of the corresponding incidence matrices $\overrightarrow{H_l}(l = 1, 2, 3, 4)$ are consolidated in Table 4, 5, 6, 7.

Table 4. Score matrix for AS/VD on the Right side of the heart

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	$RowTotal$
p_1	0.0	0.0	0.49975	0.0	0.0	0.0	0.0	0.49975
p_2	0.0	0.0	0.49975	0.0	0.0	0.0	0.0	0.49975
p_3	0.0	0.0	0.0	0.5005	0.0	0.0	0.0	0.5005
p_4	0.0	0.0	0.0	0.0	0.4995	0.0	0.0	0.4995
p_5	0.0	0.0	0.0	0.0	0.0	0.497	0.0	0.497
p_6	0.0	0.0	0.0	0.0	0.0	0.0	0.496	0.496
p_7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
				Total				2.9925

Table 5. Score matrix for AD/VS on the Right side of the heart

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	$RowTotal$
p_1	0.0	0.0	0.498	0.0	0.0	0.0	0.0	0.498
p_2	0.0	0.0	0.498	0.0	0.0	0.0	0.0	0.498
p_3	0.0	0.0	0.0	0.4985	0.0	0.0	0.0	0.4985
p_4	0.0	0.0	0.0	0.0	0.497875	0.0	0.0	0.497875
p_5	0.0	0.0	0.0	0.0	0.0	0.51025	0.0	0.51025
p_6	0.0	0.0	0.0	0.0	0.0	0.0	0.509	0.509
p_7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
				Total				3.011625

Table 6. Score matrix for AS/VD on the Left side of the heart

	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	$RowTotal$
p_8	0.0	0.0	0.0	0.0	0.50025	0.0	0.0	0.0	0.0	0.50025
p_9	0.0	0.0	0.0	0.0	0.50025	0.0	0.0	0.0	0.0	0.50025
p_{10}	0.0	0.0	0.0	0.0	0.50025	0.0	0.0	0.0	0.0	0.50025
p_{11}	0.0	0.0	0.0	0.0	0.50025	0.0	0.0	0.0	0.0	0.50025
p_{12}	0.0	0.0	0.0	0.0	0.0	0.49775	0.0	0.0	0.0	0.49775
p_{13}	0.0	0.0	0.0	0.0	0.0	0.0	0.498275	0.0	0.0	0.498275
p_{14}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.498275	0.0	0.498275
p_{15}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.492	0.492
p_{16}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
						Total				3.9873

5. **Sensitivity Analysis and Comparative Study**

The Sensitivity Analysis focuses on the uncertainty analysis of a mathematical model or a system. In decision making problems, it helps to determine the significance of each criterion

Table 7. Score matrix for AD/VS on the Left side of the heart

	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	$RowTotal$
p_8	0.0	0.0	0.0	0.0	0.498	0.0	0.0	0.0	0.0	0.498
p_9	0.0	0.0	0.0	0.0	0.498	0.0	0.0	0.0	0.0	0.498
p_{10}	0.0	0.0	0.0	0.0	0.498	0.0	0.0	0.0	0.0	0.498
p_{11}	0.0	0.0	0.0	0.0	0.498	0.0	0.0	0.0	0.0	0.498
p_{12}	0.0	0.0	0.0	0.0	0.0	0.4985	0.0	0.0	0.0	0.4985
p_{13}	0.0	0.0	0.0	0.0	0.0	0.0	0.49675	0.0	0.0	0.49675
p_{14}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.50325	0.0	0.50325
p_{15}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.501	0.501
p_{16}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
						Total				3.9915

used. Since both SVC and IVC push the deoxygenated blood to the RA with high pressure, the RA remains in Systolic phase at this time. Then the deoxygenated blood passes to the RV which remains in diastolic phase. Table 4 represents the crisp value that depicts the flow of deoxygenated blood during AS/VD phase on the Right side of the human heart. After the ventricular filling, the deoxygenated blood is transferred to PA. During this time, the RV stays in Systolic phase and RA remains in Diastolic phase. Table 5 gives the numerical values of the blood flow of the human heart during AD/VS phase on the Right side. Then, the oxygenated blood passes to LA and then to the LV. At this time, the LA is in Systolic phase and the corresponding LV is in Diastolic phase. Table 6 gives the values of the blood flow during AS/VD phase on the Left side of the human heart. Finally, the LV pushes out the blood to the Aorta and simultaneously the LV is in Systolic phase whereas the LA is in Diastolic phase. Table 7 illustrates the values of the blood flow during AD/VS phase on the Left side of the human heart. Now, by comparing the score values in Table 4, Table 5, Table 6 and Table 7, the most crucial phase during the cardiac cycle is evaluated. From the cumulative numerical values of the score matrices for the AS/VD and AD/VS phases on the Right and the Left side of the human heart, the sensitivity analysis is tabulated in Table 8.

Comparatively, from Table 4, Table 5, Table 6 and Table 7, the AS/VD phase on the Left side of the human heart is highly significant phase.

6. Results

It is evident that AS/VD phase on the Left side of the human heart is the most crucial phase. Also, it is observed that,

Table 8. Sensitivity Analysis

Phases in the Human Heart	Row Total	Ordering
AD/VS Phase(Left Side)	3.9915	1
AS/VD Phase (Left Side)	3.9873	2
AD/VS Phase (Right Side)	3.011625	3
AS/VD Phase (Right Side)	2.9925	4

(1) Atrial Systole / Ventricular Diastole on the left-hand side of the human heart (3.9873) has comparatively higher pressure than Atrial Systole / Ventricular Diastole on the right-hand side of the human heart (2.9925).
(2) Atrial Diastole / Ventricular Systole on the left-hand side of the human heart (3.9915) has comparatively higher pressure than Atrial Diastole / Ventricular Systole on the right-hand side of the human heart (3.011625).

7. Discussion

From Table 8, it is clear that

(1) Ventricular Systole and Diastole on the Left side of the human heart is the most significant process as compared to the Right side.
(2) Systolic ventricular phase is comparatively greater than diastolic ventricular phase.

The above analysis are analogous to the cardiac functioning of a normal and healthy individual.

8. Need, Limitation and Impact

(1) Since the blood flow is uni-directional and the blood pressure values fluctuates within certain range, it is necessary to depict the blood flow under a directed interval valued neutrosophic environment. Also, the blood usually flows from high to low pressure, in order to maintain the optimal level between any two compartments of the human heart, we model the cardiac functioning of the human heart as SIVNDG.
(2) The score function helps to make the deneutrosophication of SIVN values to a crisp value.
(3) Modelling the cardiac cycle of the human heart as SIVNDG helps to evaluate the blood flow in each phase effectively.
(4) The blood pressure is dynamic in nature as it changes while sleeping or doing exercise or a rest etc. The study of blood flow under these circumstances can be studied by our proposed model only if the necessary blood pressure values available.

9. Conclusions

For any two SIVNGs, it is shown that $G_1 * G_2, G_1 \mid G_2, G_1 \oplus G_2$ and $G_1 \bullet G_2$ is again a SIVNG. By modeling the cardiac functioning of the human heart, it is observed that the cardiac cycle is fit under the SIVNDG since the blood flow is unidirectional and the hemodynamic parameters show a varying pattern. Furthermore, the indeterminacy observed in the interval of blood pressure values is limited within and not more or less that range.With the observation of score function, we found that our result is identical to the conventional biological approach. Hence, evaluating the cardiac functioning of the heart by modeling as SIVNDG is the most reasonable choice.

Conflicts of Interest

The authors declare no conflicts of interest.

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