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e-Open Maps, e-Closed Maps and e-Homeomorphisms in N-Neutrosophic Crisp Topological Spaces

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Abstract. The concept of $N_{nc}eO$ and $N_{nc}eC$ mappings in $N_{nc}ts$ are introduced and studied some of their related properties in this article. In addition, $N_{nc}eHom$, $N_{nc}eCHom$ and $N_{nc}eT_{\frac{1}{2}}$-space in $N_{nc}ts$ are discussed and establishes some of their related characterizations.

Keywords: $N_{nc}e$-open map, $N_{nc}e$-closed map, $N_{nc}eT_{\frac{1}{2}}$-space, $N_{nc}e$-homeomorphism, $N_{nc}e$-C homeomorphism.

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1. Introduction

Smarandache [14] defined the neutrosophic set on three component neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Lellis Thivagar et al. [11] was the first given the geometric existence of N topology and in his paper [10] introduced the notion of N_n-open (closed) sets and N_n continuous in N-neutrosophic topological spaces. The concept of N-neutrosophic crisp topological spaces from neutrosophic crisp topological spaces was first explored and investigated by Al-Hamido [1]. As a generalization of closed sets, e-closed sets were introduced and studied by Ekici [7–9]. In 2020, Vadivel and Sundar introduced the concept of N_{nc} γ-open [15], N_{nc} β-open [16] and N_{nc} δ-open sets [18] and their continuous functions [17, 20, 28] and open mappings [19, 21, 22]. The new N_{nc} open sets called N_{nc} e-open sets and its continuous functions are introduced in $N_{nc}ts$ by Vadivel et al. [23–27]. Recently, Das et al. [2–6] introduced b-open sets in different types of neutrosophic topological spaces. In this paper, $N_{nc}e$ open mapping, $N_{nc}e$ closed mapping, $N_{nc}e$ homeomorphism and $N_{nc}e$-C homeomorphism are introduced and some results in $N_{nc}ts$.

2. Preliminaries

Definition 2.1. [13] Let X be a non-empty set. Then F is called a neutrosophic crisp set (in short, ncs) in X if F has the form $F = (F_{01}, F_{02}, F_{03})$, where F_{01}, F_{02}, and F_{03} are subsets of X, then neutrosophic crisp set of types

(i) $F_{01} \cap F_{02} = F_{02} \cap F_{03} = F_{03} \cap F_{01} = \phi$

(ii) $F_{01} \cap F_{02} = F_{02} \cap F_{03} = F_{03} \cap F_{01} = \phi$ and $F_{01} \cup F_{02} \cup F_{03} = X$

(iii) $F_{01} \cap F_{02} \cap F_{03} = \phi$ and $F_{01} \cup F_{02} \cup F_{03} = X$

Definition 2.2. [13] Let $F = (F_{01}, F_{02}, F_{03}), G = (G_{01}, G_{02}, G_{03}) \in ncs(X)$. Then

(i) $\phi_n = (\phi, \phi, X)$,

(ii) $X_n = (X, X, \phi)$,

(iii) $F \subseteq G$, if $F_{01} \subseteq G_{01}, F_{02} \subseteq G_{02}$ and $F_{03} \supseteq G_{03}$.

(iv) $F = G$, if $F \subseteq G$ and $F \subseteq H$

(v) $F^c = (F_{03}, F_{02}^c, F_{01})$

(vi) $F \cap G = (F_{01} \cap G_{01}, F_{02} \cap G_{02}, F_{03} \cup G_{03})$

(vii) $F \cup G = (F_{01} \cup G_{01}, F_{02} \cup G_{02}, F_{03} \cap G_{03})$.

Definition 2.3. [12] A neutrosophic crisp topology (briefly, nct) on a non-empty set X is a family Γ of nc subsets of X satisfying the following axioms

(i) $\phi_n,\ X_n \in \Gamma$.

(ii) $F_1 \cap F_2 \in \Gamma\ \forall\ F_1\ \&\ F_2 \in \Gamma$.

(iii) $\bigcup\limits_b F_b \in \Gamma$, for any $\{F_b : b \in K\} \subseteq \Gamma$.

Then (X, Γ) is a neutrosophic crisp topological space (briefly, $ncts$) in X. The Γ elements are called neutrosophic crisp open sets (briefly, $ncos$) in X and its complement is called neutrosophic crisp closed set (briefly, $nccs$).

Definition 2.4. [1] Let X be a non-empty set. Then ${}_{nc}\Psi_1$, ${}_{nc}\Psi_2$, $\cdots$, ${}_{nc}\Psi_N$ are N-arbitrary crisp topologies defined on X and the collection $N_{nc}\Psi = \{B \subseteq X : B = (\bigcup\limits_{k=1}^{N} F_k) \cup (\bigcap\limits_{k=1}^{N} L_k)$, $F_k, L_k \in {}_{nc}\Psi_k\}$ is called N_{nc}-topology on X if the axioms are satisfied:

(i) $\phi_n,\ X_n \in N_{nc}\Psi$.

(ii) $\bigcup\limits_{k=1}^{\infty} K_k \in N_{nc}\Psi\ \forall\ \{K_k\}_{k=1}^{\infty} \in N_{nc}\Psi$.

(iii) $\bigcap_{k=1}^{n} K_k \in N_{nc}\Psi \; \forall \; \{K_k\}_{k=1}^{n} \in N_{nc}\Psi$.

Then $(X, N_{nc}\Psi)$ is called a N_{nc}-topological space (briefly, $N_{nc}ts$) on X. The $N_{nc}\Psi$ elements are called N_{nc}-open sets ($N_{nc}os$) on X and its complement is called N_{nc}-closed sets ($N_{nc}cs$) on X. The elements of X are known as N_{nc}-sets ($N_{nc}s$) on X.

Definition 2.5. [1,18] Let $(X, N_{nc}\Psi)$ be $N_{nc}ts$ on X and F be a $N_{nc}s$ on X, then the N_{nc} interior of F (briefly, $N_{nc}int(F)$), N_{nc} closure of F (briefly, $N_{nc}cl(F)$), $N_{nc}\delta$ interior of F (briefly, $N_{nc}\delta int(F)$) and $N_{nc}\delta$ closure of F (briefly, $N_{nc}\delta cl(F)$) are defined as

$$N_{nc}int(F) = \cup\{C : C \subseteq F \;\&\; C \text{ is a } N_{nc}os \text{ in } X\}$$

$$N_{nc}cl(F) = \cap\{D : F \subseteq D \;\&\; D \text{ is a } N_{nc}cs \text{ in } X\}$$

$$N_{nc}\delta int(F) = \cup\{C : C \subseteq F \;\&\; C \text{ is a } N_{nc}ros \text{ in } X\}$$

$$N_{nc}\delta cl(F) = \cap\{D : F \subseteq D \;\&\; D \text{ is a } N_{nc}rcs \text{ in } X\}.$$

Definition 2.6. [1,15,18,26,28] Let $(X, N_{nc}\Gamma)$ be any $N_{nc}ts$. Let F be a $N_{nc}s$ in $(X, N_{nc}\Psi)$. Then F is said to be a

(i) N_{nc}-regular (resp. N_{nc}-semi, N_{nc}-pre, N_{nc}-α & N_{nc}-β) open set (briefly, $N_{nc}ros$ (resp. $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{P}os$, $N_{nc}\alpha os$ & $N_{nc}\beta os$)) if $F = N_{nc}int(N_{nc}cl(F))$ (resp. $F \subseteq N_{nc}cl(N_{nc}int(F))$, $F \subseteq N_{nc}int(N_{nc}cl(F))$, $F \subseteq N_{nc}int(N_{nc}cl(N_{nc}int(F)))$ & $F \subseteq N_{nc}cl(N_{nc}int(N_{nc}cl(F)))$).

(ii) $N_{nc}\delta$ (resp. $N_{nc}\delta$-pre, $N_{nc}\delta$-semi & $N_{nc}e$) open set (briefly, $N_{nc}\delta os$ (resp. $N_{nc}\delta\mathcal{P}os$, $N_{nc}\delta\mathcal{S}os$ & $N_{nc}eos$)) if $F = N_{nc}\delta int(F)$ (resp. $F \subseteq N_{nc}int(N_{nc}\delta cl(F))$, $F \subseteq N_{nc}cl(N_{nc}\delta int(F))$ & $F \subseteq N_{nc}cl(N_{nc}\delta int(F)) \cup N_{nc}int(N_{nc}\delta cl(F))$).

Definition 2.7. [10,19,21,22,27] Let $(X_1, N_{nc}\Psi)$ and $(X_2, N_{nc}\tau)$ be any two $N_{nc}ts$'s. A map $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is said to be

(i) N_{nc} (resp. $N_{nc}\alpha$, N_{nc} semi, N_{nc} pre, $N_{nc}\gamma$, $N_{nc}\beta$, $N_{nc}\delta$, $N_{nc}\delta$ semi & $N_{nc}\delta$ pre)-open mapping (briefly, $N_{nc}O$ (resp. $N_{nc}\alpha O$, $N_{nc}\mathcal{S}O$, $N_{nc}\mathcal{P}O$, $N_{nc}\gamma O$, $N_{nc}\beta O$, $N_{nc}\delta O$, $N_{nc}\delta\mathcal{S}O$ & $N_{nc}\delta\mathcal{P}O$) if the inverse image of every $N_{nc}os$ in $(X_1, N_{nc}\Psi)$ is a $N_{nc}\alpha os$ (resp. $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{P}os$, $N_{nc}\gamma os$, $N_{nc}\beta os$, $N_{nc}\delta os$, $N_{nc}\delta\mathcal{S}os$ & $N_{nc}\delta\mathcal{P}os$) in $(X_2, N_{nc}\tau)$.

(ii) N_{nc} (resp. $N_{nc}\alpha$, N_{nc} semi, N_{nc} pre, $N_{nc}\gamma$, $N_{nc}\beta$, $N_{nc}\delta$, $N_{nc}\delta$ semi & $N_{nc}\delta$ pre)-closed mapping (briefly, $N_{nc}C$ (resp. $N_{nc}\alpha C$, $N_{nc}\mathcal{S}C$, $N_{nc}\mathcal{P}C$, $N_{nc}\gamma C$, $N_{nc}\beta C$, $N_{nc}\delta C$, $N_{nc}\delta\mathcal{S}C$ & $N_{nc}\delta\mathcal{P}C$)) if the inverse image of every $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$ is a $N_{nc}\alpha cs$ (resp. $N_{nc}\mathcal{S}cs$, $N_{nc}\mathcal{P}cs$, $N_{nc}\gamma cs$, $N_{nc}\beta cs$, $N_{nc}\delta cs$, $N_{nc}\delta\mathcal{S}cs$ & $N_{nc}\delta\mathcal{P}cs$) in $(X_2, N_{nc}\tau)$.

(iii) N_{nc} (resp. $N_{nc}e$)-continuous (briefly, $N_{nc}Cts$ (resp. $N_{nc}eCts$)) if the inverse image of every $N_{nc}os$ in $(X_2, N_{nc}\tau)$ is a $N_{nc}os$ (resp. $N_{nc}eos$) in $(X_1, N_{nc}\Psi)$.

(iv) N_{nc}-homeomorphism (briefly, $N_{nc}Hom$) if ζ & ζ^{-1} are $N_{nc}Cts$.

Throughout this article, let $(X_1, N_{nc}\Psi)$, $(X_2, N_{nc}\tau)$ and $(X_3, N_{nc}\rho)$ are $N_{nc}ts$'s and $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ and $\eta : (X_2, N_{nc}\tau) \to (X_3, N_{nc}\rho)$ are mappings.

3. N-Neutrosophic crisp e-open mapping

Definition 3.1. A mapping ζ is N-neutrosophic crisp e-open (briefly, $N_{nc}eO$) if image of every $N_{nc}eos$ of $(X_1, N_{nc}\Psi)$ is $N_{nc}eos$ in $(X_2, N_{nc}\tau)$.

Theorem 3.2. Let ζ be a function. Then Every

(i) $N_{nc}O$ is a $N_{nc}\alpha O$.
(ii) $N_{nc}\alpha O$ is a $N_{nc}\mathcal{P}O$.
(iii) $N_{nc}\mathcal{P}O$ is a $N_{nc}\gamma O$.
(iv) $N_{nc}\gamma O$ is a $N_{nc}\beta O$.
(v) $N_{nc}\delta O$ is a $N_{nc}O$.
(vi) $N_{nc}\delta O$ is a $N_{nc}\mathcal{S}O$.
(vii) $N_{nc}\delta\mathcal{S}O$ is a $N_{nc}eO$.
(viii) $N_{nc}\mathcal{P}O$ is a $N_{nc}\delta\mathcal{P}O$.
(ix) $N_{nc}\delta\mathcal{P}O$ is a $N_{nc}eO$.
(x) $N_{nc}eO$ is a $N_{nc}\beta O$.

Proof. Proof of (i) to (iii), (iv) and (v) to (vi) are proved in [19], [21] and [22]. We prove only (vii) to (ix).

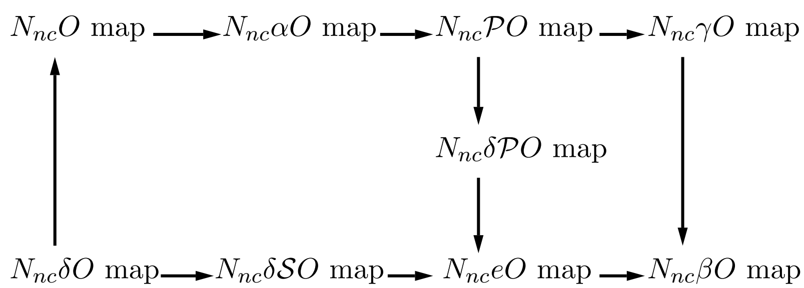
(vii) Let ζ be a $N_{nc}\delta\mathcal{S}O$ mapping and K is a $N_{nc}os$ in X_1. Then $\zeta(K)$ is $N_{nc}\delta\mathcal{S}os$ in X_2. Since every $N_{nc}\delta\mathcal{S}os$ is $N_{nc}eos$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}eos$ in X_2. Therefore ζ is $N_{nc}eO$ mapping.

(viii) Let ζ be a $N_{nc}\mathcal{P}O$ mapping and K is a $N_{nc}os$ in X_1. Then $\zeta(K)$ is $N_{nc}\mathcal{P}os$ in X_2. Since every $N_{nc}\mathcal{P}os$ is $N_{nc}\delta\mathcal{P}os$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}\delta\mathcal{P}os$ in X_2. Therefore ζ is $N_{nc}\delta\mathcal{P}O$ mapping.

(ix) Let ζ be a $N_{nc}\delta\mathcal{P}O$ mapping and K is a $N_{nc}os$ in X_1. Then $\zeta(K)$ is $N_{nc}\delta\mathcal{P}os$ in X_2. Since every $N_{nc}\delta\mathcal{P}os$ is $N_{nc}eos$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}eos$ in X_2. Therefore ζ is $N_{nc}eO$ mapping.

(x) Let ζ be a $N_{nc}eO$ mapping and K is a $N_{nc}os$ in X_1. Then $\zeta(K)$ is $N_{nc}eos$ in X_2. Since every $N_{nc}eos$ is $N_{nc}\beta os$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}\beta os$ in X_2. Therefore ζ is $N_{nc}\beta O$ mapping. □

Remark 3.3. The following diagram shows $N_{nc}eO$ mapping function in $N_{nc}ts$.



None of these implication is reversible as shown in the following examples.

Example 3.4. Let $X = \{a_o, b_o, c_o, d_o, e_o\} = Y$, ${}_{nc}\Psi_1 = \{\phi_n, X_n, A_o\}$, ${}_{nc}\Psi_2 = \{\phi_n, X_n\}$. $A_o = \langle\{a_o\}, \{\phi\}, \{b_o, c_o, d_o, e_o\}\rangle$, then $2_{nc}\Psi = \{\phi_n, X_n, A_o\}$. Let ${}_{nc}\tau_1 = \{\phi_n, Y_n, B_o, C_o, D_o\}$, ${}_{nc}\tau_2 = \{\phi_n, Y_n\}$. $B_o = \langle\{c_o\}, \{\phi\}, \{a_o, b_o, d_o, e_o\}\rangle$, $C_o = \langle\{a_o, b_o\}, \{\phi\}, \{c_o, d_o, e_o\}\rangle$, $D_o = \langle\{a_o, b_o, c_o\}, \{\phi\}, \{d_o, e_o\}\rangle$, then $2_{nc}\tau = \{\phi_n, Y_n, B_o, C_o, D_o\}$. Define $\zeta : (X, 2_{nc}\Psi) \to (Y, 2_{nc}\tau)$ as identity map, then $2_{nc}eO$ map but not $2_{nc}\delta\mathcal{S}O$ map, then $\zeta(\langle\{a_o\}, \{\phi\}, \{b_o, c_o, d_o, e_o\}\rangle) = \langle\{a_o\}, \{\phi\}, \{b_o, c_o, d_o, e_o\}\rangle$ is a $2_{nc}eos$ but not $2_{nc}\delta\mathcal{S}os$ in Y.

Example 3.5. Let $X = \{a_o, b_o, c_o, d_o, e_o\} = Y$, ${}_{nc}\Psi_1 = \{\phi_n, X_n, A_o\}$, ${}_{nc}\Psi_2 = \{\phi_n, X_n\}$. $A_o = \langle\{c_o, d_o\}, \{\phi\}, \{a_o, b_o, e_o\}\rangle$, then $2_{nc}\Psi = \{\phi_n, X_n, A_o\}$. Let ${}_{nc}\tau_1 = \{\phi_n, Y_n, B_o, C_o, D_o\}$, ${}_{nc}\tau_2 = \{\phi_n, Y_n\}$. $B_o = \langle\{c_o\}, \{\phi\}, \{a_o, b_o, d_o, e_o\}\rangle$, $C_o = \langle\{a_o, b_o\}, \{\phi\}, \{c_o, d_o, e_o\}\rangle$, $D_o = \langle\{a_o, b_o, c_o\}, \{\phi\}, \{d_o, e_o\}\rangle$, then $2_{nc}\tau = \{\phi_n, Y_n, B_o, C_o, D_o\}$. Define $\zeta : (X, 2_{nc}\Psi) \to (Y, 2_{nc}\tau)$ as identity map, then $2_{nc}eO$ map but not $2_{nc}\delta\mathcal{P}O$ map, then $\zeta(\langle\{c_o, d_o\}, \{\phi\}, \{a_o, b_o, e_o\}\rangle) = \langle\{c_o, d_o\}, \{\phi\}, \{a_o, b_o, e_o\}\rangle$ is a $2_{nc}eos$ but not $2_{nc}\delta\mathcal{P}os$ in Y.

Example 3.6. Let $X = \{a_o, b_o, c_o, d_o, e_o\} = Y$, ${}_{nc}\Psi_1 = \{\phi_n, X_n, A_o\}$, ${}_{nc}\Psi_2 = \{\phi_n, X_n\}$. $A_o = \langle\{a_o, d_o\}, \{\phi\}, \{b_o, c_o, e_o\}\rangle$, then $2_{nc}\Psi = \{\phi_n, X_n, A_o\}$. Let ${}_{nc}\tau_1 = \{\phi_n, Y_n, B_o, C_o, D_o\}$, ${}_{nc}\tau_2 = \{\phi_n, Y_n\}$. $B_o = \langle\{c_o\}, \{\phi\}, \{a_o, b_o, d_o, e_o\}\rangle$, $C_o = \langle\{a_o, b_o\}, \{\phi\}, \{c_o, d_o, e_o\}\rangle$, $D_o = \langle\{a_o, b_o, c_o\}, \{\phi\}, \{d_o, e_o\}\rangle$, then $2_{nc}\tau = \{\phi_n, Y_n, B_o, C_o, D_o\}$. Define $\zeta : (X, 2_{nc}\Psi) \to (Y, 2_{nc}\tau)$ as identity map, then $2_{nc}\beta O$ map but not $2_{nc}eO$ map, then $\zeta(\langle\{a_o, d_o\}, \{\phi\}, \{b_o, c_o, e_o\}\rangle) = \langle\{a_o, d_o\}, \{\phi\}, \{b_o, c_o, e_o\}\rangle$ is a $2_{nc}\beta os$ but not $2_{nc}eos$ in Y.

Theorem 3.7. A mapping $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is $N_{nc}eO$ iff for every $N_{nc}s$ φ of $(X_1, N_{nc}\Psi)$, $\zeta(N_{nc}int(\varphi)) \subseteq N_{nc}eint(\zeta(\varphi))$.

Proof. Necessity: Let ζ be a $N_{nc}eO$ & φ be a $N_{nc}os$ in $(X_1, N_{nc}\Psi)$. Now, $N_{nc}int(\varphi) \subseteq \varphi$ implies $\zeta(N_{nc}int(\varphi)) \subseteq \zeta(\varphi)$. Since ζ is a $N_{nc}eO$, $\zeta(N_{nc}int(\varphi))$ is $N_{nc}eos$ in $(X_2, N_{nc}\tau)$ such that $\zeta(N_{nc}int(\varphi)) \subseteq \zeta(\varphi)$ therefore $\zeta(N_{nc}int(\varphi)) \subseteq N_{nc}eint(\zeta(\varphi))$.

Sufficiency: Assume φ is a $N_{nc}os$ of $(X_1, N_{nc}\Psi)$. Then $\zeta(\varphi) = \zeta(N_{nc}int(\varphi)) \subseteq N_{nc}eint(\zeta(\varphi))$. But $N_{nc}eint(\zeta(\varphi)) \subseteq \zeta(\varphi)$. So $\zeta(\varphi) = N_{nc}eint(\varphi)$ which implies $\zeta(\varphi)$ is a $N_{nc}eos$ of $(X_2, N_{nc}\tau)$ and hence ζ is a $N_{nc}eO$. □

Theorem 3.8. If $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is a $N_{nc}eO$ mapping then $N_{nc}int(\zeta^{-1}(\lambda)) \subseteq \zeta^{-1}(N_{nc}eint(\lambda))$ for every $N_{nc}s$ λ of $(X_2, N_{nc}\tau)$.

Proof. Let λ be a $N_{nc}s$ of $(X_2, N_{nc}\tau)$. Then $N_{nc}int(\zeta^{-1}(\lambda))$ is a $N_{nc}os$ in $(X_1, N_{nc}\Psi)$. Since ζ is $N_{nc}eO$, $\zeta(N_{nc}int(\zeta^{-1}(\lambda))$ is $N_{nc}eo$ in $(X_2, N_{nc}\tau)$ and hence $\zeta(N_{nc}int(\zeta^{-1}(\lambda))) \subseteq N_{nc}eint(\zeta(\zeta^{-1}(\lambda))) \subseteq N_{nc}eint(\lambda)$. Thus $N_{nc}int(\zeta^{-1}(\lambda)) \subseteq \zeta^{-1}(N_{nc}eint(\lambda))$. □

Theorem 3.9. A mapping $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is $N_{nc}eO$ iff for each $N_{nc}s$ μ of $(X_2, N_{nc}\tau)$ and for each $N_{nc}cs$ ρ of $(X_1, N_{nc}\Psi)$ containing $\zeta^{-1}(\mu)$ there is a $N_{nc}ecs$ μ of $(X_2, N_{nc}\tau) \ni \mu \subseteq \rho$ & $\zeta^{-1}(\mu) \subseteq \rho$.

Proof. Necessity: Assume ζ is a $N_{nc}eO$. Let μ be the $N_{nc}cs$ of $(X_2, N_{nc}\tau)$ & ρ is a $N_{nc}cs$ of $(X_1, N_{nc}\Psi) \ni \zeta^{-1}(\mu) \subseteq \rho$. Then $\mu = (\zeta^{-1}(\rho^c))^c$ is $N_{nc}ecs$ of $(X_2, N_{nc}\tau) \ni \zeta^{-1}(\mu) \subseteq \rho$.

Sufficiency: Assume ν is a $N_{nc}os$ of $(X_1, N_{nc}\Psi)$. Then $\zeta^{-1}(((\zeta(\nu))^c \subseteq \nu^c$ & ν^c is $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$. By hypothesis there is a $N_{nc}ecs$ μ of $(X_2, N_{nc}\tau) \ni (\zeta(\nu))^c \subseteq \mu$ & $\zeta^{-1}(\mu) \subseteq \nu^c$. Therefore $\nu \subseteq (\zeta^{-1}(\mu))^c$. Hence $\mu^c \subseteq \zeta(\nu) \subseteq \zeta((\zeta^{-1}(\mu))^c) \subseteq \mu^c$ which implies $\zeta(\nu) = \mu^c$. Since μ^c is $N_{nc}eos$ of $(X_2, N_{nc}\tau)$. Hence $\zeta(\nu)$ is $N_{nc}eo$ in $(X_2, N_{nc}\tau)$ and thus ζ is $N_{nc}eO$. □

Theorem 3.10. A mapping $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is $N_{nc}eO$ iff $\zeta^{-1}(N_{nc}ecl(\rho)) \subseteq N_{nc}cl(\zeta^{-1}(\rho))$ for every $N_{nc}s$ ρ of $(X_2, N_{nc}\tau)$.

Proof. Necessity: Assume ζ is a $N_{nc}eO$. For any $N_{nc}s$ ρ of $(X_2, N_{nc}\tau)$, $\zeta^{-1}(\rho) \subseteq N_{nc}cl(\zeta^{-1}(\rho))$. Therefore by Theorem 3.9 there exists a $N_{nc}ecs$ μ in $(X_2, N_{nc}\tau) \ni \rho \subseteq \mu$ & $\zeta^{-1}(\mu) \subseteq N_{nc}cl(\zeta^{-1}(\rho))$. Therefore we obtain that $\zeta^{-1}(N_{nc}ecl(\rho)) \subseteq \zeta^{-1}(\mu) \subseteq N_{nc}cl(\zeta^{-1}(\rho))$.

Sufficiency: Assume ρ is a $N_{nc}s$ of $(X_2, N_{nc}\tau)$ & μ is a $N_{nc}cs$ of $(X_1, N_{nc}\Psi)$ containing $\zeta^{-1}(\rho)$. Put $\alpha = N_{nc}cl(\rho)$, then $\rho \subseteq \alpha$ and α is $N_{nc}ec$ & $\zeta^{-1}(\alpha) \subsetneq N_{nc}cl(\zeta^{-1}(\rho)) \subseteq \mu$. Then by Theorem 3.9, ζ is $N_{nc}eO$. □

Theorem 3.11. If ζ & η be two neutrosophic crisp mappings and $\eta \circ \zeta : (X_1, N_{nc}\Psi) \to (X_3, N_{nc}\rho)$ is $N_{nc}eO$. If $\eta : (X_2, N_{nc}\tau) \to (X_3, N_{nc}\rho)$ is $N_{nc}eIrr$ then $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is $N_{nc}eO$ mapping.

Proof. Let μ be a $N_{nc}os$ in $(X_1, N_{nc}\Psi)$. Then $\eta \circ \zeta(\mu)$ is $N_{nc}eos$ of $(X_3, N_{nc}\rho)$ because $\eta \circ \zeta$ is $N_{nc}eO$. Since η is $N_{nc}eIrr$ & $\eta \circ \zeta(\mu)$ is $N_{nc}eos$ of $(X_3, N_{nc}\rho)$ therefore $\eta^{-1}(\eta \circ \zeta(\mu)) = \zeta(\mu)$ is $N_{nc}eos$ in $(X_2, N_{nc}\tau)$. Hence ζ is $N_{nc}eO$. □

Theorem 3.12. If ζ is $N_{nc}O$ and η is $N_{nc}eO$ mappings then $\eta \circ \zeta : (X_1, N_{nc}\Psi) \to (X_3, N_{nc}\rho)$ is $N_{nc}eO$.

Proof. Let μ be a $N_{nc}os$ in $(X_1, N_{nc}\Psi)$. Then $\zeta(\mu)$ is a $N_{nc}os$ of $(X_2, N_{nc}\tau)$ because ζ is a $N_{nc}O$. Since η is $N_{nc}eO$, $\eta(\zeta(\mu)) = (\eta \circ \zeta)(\mu)$ is $N_{nc}eos$ of $(X_3, N_{nc}\rho)$. Hence $\eta \circ \zeta$ is $N_{nc}eO$. □

4. N-Neutrosophic crisp e-closed mapping

Definition 4.1. A mapping $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is N-neutrosophic crisp e-closed (briefly, $N_{nc}eC$) if image of every $N_{nc}cs$ of $(X_1, N_{nc}\Psi)$ is $N_{nc}ecs$ in $(X_2, N_{nc}\tau)$.

Theorem 4.2. Let ζ be a function. Then Every

(i) $N_{nc}C$ is a $N_{nc}\alpha C$.
(ii) $N_{nc}\alpha C$ is a $N_{nc}\mathcal{P}C$.
(iii) $N_{nc}\mathcal{P}C$ is a $N_{nc}\gamma C$.
(iv) $N_{nc}\gamma C$ is a $N_{nc}\beta C$.
(v) $N_{nc}\delta C$ is a $N_{nc}C$.
(vi) $N_{nc}\delta C$ is a $N_{nc}\mathcal{S}C$.
(vii) $N_{nc}\delta\mathcal{S}C$ is a $N_{nc}eC$.
(viii) $N_{nc}\mathcal{P}C$ is a $N_{nc}\delta\mathcal{P}C$.
(ix) $N_{nc}\delta\mathcal{P}C$ is a $N_{nc}eC$.
(x) $N_{nc}eC$ is a $N_{nc}\beta C$.

Proof. Proof of (i) to (iii), (iv) and (v) to (vi) are proved in [19], [21] and [22]. We prove only (vii) to (ix).

(vii) Let ζ be a $N_{nc}\delta\mathcal{S}C$ mapping and K is a $N_{nc}cs$ in X_1. Then $\zeta(K)$ is $N_{nc}\delta\mathcal{S}cs$ in X_2. Since every $N_{nc}\delta\mathcal{S}cs$ is $N_{nc}ecs$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}ecs$ in X_2. Therefore ζ is $N_{nc}eC$ mapping.

(viii) Let ζ be a $N_{nc}\mathcal{P}C$ mapping and K is a $N_{nc}cs$ in X_1. Then $\zeta(K)$ is $N_{nc}\mathcal{P}cs$ in X_2. Since every $N_{nc}\mathcal{P}cs$ is $N_{nc}\delta\mathcal{P}cs$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}\delta\mathcal{P}cs$ in X_2. Therefore ζ is $N_{nc}\delta\mathcal{P}C$ mapping.

(ix) Let ζ be a $N_{nc}\delta\mathcal{P}C$ mapping and K is a $N_{nc}cs$ in X_1. Then $\zeta(K)$ is $N_{nc}\delta\mathcal{P}cs$ in X_2. Since every $N_{nc}\delta\mathcal{P}cs$ is $N_{nc}ecs$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}ecs$ in X_2. Therefore ζ is $N_{nc}eC$ mapping.

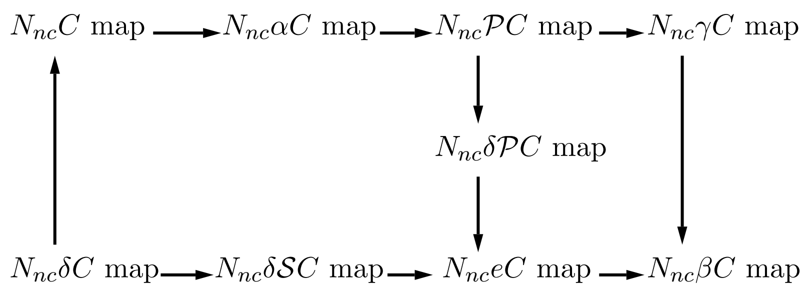
(x) Let ζ be a $N_{nc}eC$ mapping and K is a $N_{nc}cs$ in X_1. Then $\zeta(K)$ is $N_{nc}ecs$ in X_2. Since every $N_{nc}ecs$ is $N_{nc}\beta cs$ by Proposition 3.1 in [26], $\zeta(K)$ is $N_{nc}\beta cs$ in X_2. Therefore ζ is $N_{nc}\beta C$ mapping. □

Example 4.3. In Example 3.4, then $2_{nc}eC$ map but not $2_{nc}\delta\mathcal{S}C$ map, then $\zeta(\langle\{b_o, c_o, d_o, e_o\}, \{\phi\}, \{a_o\}\rangle) = \langle\{b_o, c_o, d_o, e_o\}, \{\phi\}, \{a_o\}\rangle$ is a $2_{nc}ecs$ but not $2_{nc}\delta\mathcal{S}cs$.

Example 4.4. In Example 3.5, then $2_{nc}eC$ map but not $2_{nc}\delta\mathcal{P}C$ map, then $\zeta(\langle\{a_o, b_o, e_o\}, \{\phi\}, \{c_o, d_o\}\rangle) = \langle\{a_o, b_o, e_o\}, \{\phi\}, \{c_o, d_o\}\rangle$ is a $2_{nc}ecs$ but not $2_{nc}\delta\mathcal{P}cs$.

Example 4.5. In Example 3.6, then $2_{nc}\beta C$ map but not $2_{nc}eC$ map, then $\zeta(\langle\{b_o, c_o, e_o\}, \{\phi\}, \{a_o, d_o\}\rangle) = \langle\{b_o, c_o, e_o\}, \{\phi\}, \{a_o, d_o\}\rangle$ is a $2_{nc}\beta cs$ but not $2_{nc}ecs$.

Remark 4.6. The following diagram shows $N_{nc}eC$ mapping function in $N_{nc}ts$.



None of these implication is reversible as shown in the following examples.

Theorem 4.7. A mapping $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is $N_{nc}eC$ iff for each $N_{nc}s$ μ of $(X_2, N_{nc}\tau)$ and for each $N_{nc}os$ λ of $(X_1, N_{nc}\Psi)$ containing $\zeta^{-1}(\mu)$ there is a $N_{nc}eos$ ρ of $(X_2, N_{nc}\tau) \ni \mu \subseteq \rho$ & $\zeta^{-1}(\rho) \subseteq \lambda$.

Proof. Necessity: Assume ζ is a $N_{nc}eC$. Let μ be the $N_{nc}cs$ of $(X_2, N_{nc}\tau)$ & λ is a $N_{nc}os$ of $(X_1, N_{nc}\Psi) \ni \zeta^{-1}(\mu) \subseteq \lambda$. Then $\rho = X_2 - \zeta^{-1}(\lambda^c)$ is $N_{nc}eos$ of $(X_2, N_{nc}\tau) \ni \zeta^{-1}(\rho) \subseteq \lambda$.

Sufficiency: Assume ν is a $N_{nc}cs$ of $(X_1, N_{nc}\Psi)$. Then $(\zeta(\nu))^c$ is a $N_{nc}s$ of $(X_2, N_{nc}\tau)$ & ν^c is $N_{nc}os$ in $(X_1, N_{nc}\Psi) \ni \zeta^{-1}((\zeta(\nu))^c) \subseteq \nu^c$. By hypothesis there is a $N_{nc}eos$ ρ of $(X_2, N_{nc}\tau)$ $\ni (\zeta(\nu))^c \subseteq \rho$ & $\zeta^{-1}(\rho) \subseteq \nu^c$. Therefore $\nu \subseteq (\zeta^{-1}(\rho))^c$. Hence $\rho^c \subseteq \zeta(\rho) \subseteq \zeta((\zeta^{-1}(\rho))^c) \subseteq \rho^c$ which implies $\zeta(\nu) = \rho^c$. Since ρ^c is $N_{nc}ecs$ of $(X_2, N_{nc}\tau)$. Hence $\zeta(\nu)$ is $N_{nc}ec$ in $(X_2, N_{nc}\tau)$ and thus ζ is $N_{nc}eC$. □

Theorem 4.8. If ζ is $N_{nc}C$ & η is $N_{nc}eC$. Then $\eta \circ \zeta : (X_1, N_{nc}\Psi) \to (X_3, N_{nc}\rho)$ is $N_{nc}eC$.

Proof. Let μ be a $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$. Then $\zeta(\mu)$ is $N_{nc}cs$ of $(X_2, N_{nc}\tau)$ because ζ is $N_{nc}C$. Now $(\eta \circ \zeta)(\mu) = \eta(\zeta(\mu))$ is $N_{nc}ecs$ in $(X_3, N_{nc}\rho)$ because η is $N_{nc}eC$. Thus $\eta \circ \zeta$ is $N_{nc}eC$. □

Theorem 4.9. If $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is $N_{nc}eC$ map, then $N_{nc}ecl(\zeta(\rho)) \subsetneq \zeta(N_{nc}cl(\rho))$.

Theorem 4.10. Let ζ & η are $N_{nc}eC$ mappings. If every $N_{nc}ecs$ of $(X_2, N_{nc}\tau)$ is $N_{nc}c$ then, $\eta \circ \zeta : (X_1, N_{nc}\Psi) \to (X_3, N_{nc}\rho)$ is $N_{nc}eC$.

Proof. Let μ be a $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$. Then $\zeta(\mu)$ is $N_{nc}ecs$ of $(X_2, N_{nc}\tau)$ because ζ is $N_{nc}eC$ mapping. By hypothesis $\zeta(\mu)$ is $N_{nc}cs$ of $(X_2, N_{nc}\tau)$. Now $\eta(\zeta(\mu)) = (\eta \circ \zeta)(\mu)$ is $N_{nc}ecs$ in $(X_3, N_{nc}\rho)$ because η is $N_{nc}eC$. Thus $\eta \circ \zeta$ is $N_{nc}eC$. □

Theorem 4.11. The following statements are equivalent for a mapping ζ:

(i) ζ is a $N_{nc}eO$.

(ii) ζ is a $N_{nc}eC$.

(iii) ζ^{-1} is $N_{nc}eCts$.

5. N-Neutrosophic crisp e-homeomorphism

Definition 5.1. A bijection ζ is called a $N_{nc}e$-homeomorphism (briefly $N_{nc}eHom$) if ζ & ζ^{-1} are $N_{nc}eCts$.

Theorem 5.2. Each $N_{nc}Hom$ is a $N_{nc}eHom$.

Proof. Let ζ be $N_{nc}Hom$, then ζ and ζ^{-1} are $N_{nc}Cts$. But every $N_{nc}Cts$ is $N_{nc}eCts$. Hence, ζ and ζ^{-1} is $N_{nc}eCts$. Therefore, ζ is a $N_{nc}eHom$. □

Theorem 5.3. Let ζ be a bijective mapping. The following statements are equivalent, if ζ is $N_{nc}eCts$:

(i) ζ is a $N_{nc}eC$.

(ii) ζ is a $N_{nc}eO$.

(iii) ζ^{-1} is a $N_{nc}eHom$.

Definition 5.4. A $N_{nc}ts$ $(X_1, N_{nc}\Psi)$ is said to be a neutrosophic crisp $eT_{\frac{1}{2}}$ (briefly, $N_{nc}eT_{\frac{1}{2}}$)-space if every $N_{nc}ecs$ is $N_{nc}c$ in $(X_1, N_{nc}\Psi)$.

Theorem 5.5. Let ζ be a $N_{nc}eHom$, then ζ is a $N_{nc}Hom$ if $(X_1, N_{nc}\Psi)$ and $(X_2, N_{nc}\tau)$ are $N_{nc}eT_{\frac{1}{2}}$-space.

Proof. Assume that μ is a $N_{nc}cs$ in $(X_2, N_{nc}\tau)$, then $\zeta^{-1}(\mu)$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Since $(X_1, N_{nc}\Psi)$ is a $N_{nc}eT_{\frac{1}{2}}$-space, $\zeta^{-1}(\mu)$ is a $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$. Therefore, ζ is $N_{nc}Cts$. By hypothesis, ζ^{-1} is $N_{nc}eCts$. Let ν be a $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$. Then, $(\zeta^{-1})^{-1}(\nu) = \zeta(\nu)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\tau)$, by presumption. Since $(X_2, N_{nc}\tau)$ is a $N_{nc}eT_{\frac{1}{2}}$-space, $\zeta(\nu)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\tau)$. Hence, ζ^{-1} is $N_{nc}Cts$. Hence, ζ is a $N_{nc}Hom$. □

Theorem 5.6. The following statements are equivalent for ζ, if $(X_2, N_{nc}\tau)$ is a $N_{nc}eT_{\frac{1}{2}}$-space:

(i) ζ is $N_{nc}eC$.

(ii) If μ is a $N_{nc}os$ in $(X_1, N_{nc}\Psi)$, then $\zeta(\mu)$ is $N_{nc}eos$ in $(X_2, N_{nc}\tau)$.

(iii) $\zeta(N_{nc}int(\mu)) \subseteq N_{nc}cl(N_{nc}int(\zeta(\mu)))$ for every $N_{nc}s$ μ in $(X_1, N_{nc}\Psi)$.

Proof. (i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (iii): Let μ be a $N_{nc}s$ in $(X_1, N_{nc}\Psi)$. Then, $N_{nc}int(\mu)$ is a $N_{nc}os$ in $(X_1, N_{nc}\Psi)$. Then, $\zeta(N_{nc}int(\mu))$ is a $N_{nc}eos$ in $(X_2, N_{nc}\tau)$. Since $(X_2, N_{nc}\tau)$ is a $N_{nc}eT_{\frac{1}{2}}$-space, so $\zeta(N_{nc}int(\mu))$ is a $N_{nc}os$ in $(X_2, N_{nc}\tau)$. Therefore, $\zeta(N_{nc}int(\mu)) = N_{nc}int(\zeta(N_{nc}int(\mu))) \subseteq N_{nc}cl(N_{nc}int(\zeta(\mu)))$.

(iii) $\Rightarrow$ (i): Let μ be a $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$. Then, μ^c is a $N_{nc}os$ in $(X_1, N_{nc}\Psi)$. From, $\zeta(N_{nc}int(\mu^c)) \subseteq N_{nc}cl(N_{nc}int(\zeta(\mu^c)))$. Hence, $\zeta(\mu^c) \subseteq N_{nc}cl(N_{nc}int(\zeta(\mu^c)))$. Therefore, $\zeta(\mu^c)$ is $N_{nc}eos$ in $(X_2, N_{nc}\tau)$. Therefore, $\zeta(\mu)$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Hence, ζ is a $N_{nc}C$. □

Theorem 5.7. Let ζ & η be $N_{nc}eC$, where $(X_1, N_{nc}\Psi)$ and $(X_3, N_{nc}\rho)$ are two $N_{nc}ts$'s and $(X_2, N_{nc}\tau)$ a $N_{nc}eT_{\frac{1}{2}}$-space, then the composition $\eta \circ \zeta$ is $N_{nc}eC$.

Proof. Let μ be a $N_{nc}cs$ in $(X_1, N_{nc}\Psi)$. Since ζ is $N_{nc}ec$ & $\zeta(\mu)$ is a $N_{nc}ecs$ in $(X_2, N_{nc}\tau)$, by assumption, $\zeta(\mu)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\tau)$. Since η is $N_{nc}ec$, then $\eta(\zeta(\mu))$ is $N_{nc}ec$ in $(X_3, N_{nc}\rho)$ & $\eta(\zeta(\mu)) = (\eta \circ \zeta)(\mu)$. Therefore, $\eta \circ \zeta$ is $N_{nc}eC$. □

Theorem 5.8. The following statements are hold for ζ & η:

(i) If $\eta \circ \zeta$ is $N_{nc}eO$ & ζ is $N_{nc}Cts$, then η is $N_{nc}eO$.

(ii) If $\eta \circ \zeta$ is $N_{nc}O$ & η is $N_{nc}eCts$, then ζ is $N_{nc}eO$.

Proof. Obvious. □

6. N-Neutrosophic crisp e-C Homeomorphism

Definition 6.1. A bijection ζ is called a $N_{nc}e$-C homeomorphism (briefly, $N_{nc}eCHom$) if ζ & ζ^{-1} are $N_{nc}eIrr$ mappings.

Theorem 6.2. Each $N_{nc}eCHom$ is a $N_{nc}eHom$.

Proof. Let us assume that μ is a $N_{nc}cs$ in $(X_2, N_{nc}\tau)$. This shows that μ is a $N_{nc}ecs$ in $(X_2, N_{nc}\tau)$. By assumption, $\zeta^{-1}(\mu)$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Hence, ζ is a $N_{nc}eCts$. Hence, ζ & ζ^{-1} are $N_{nc}eCts$. Hence ζ is a $N_{nc}eHom$. □

Theorem 6.3. If $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ is a $N_{nc}eCHom$, then $N_{nc}ecl(\zeta^{-1}(\mu)) \subseteq \zeta^{-1}(N_{nc}cl(\mu))$ for each $N_{nc}ts$ μ in $(X_2, N_{nc}\tau)$.

Proof. Let μ be a $N_{nc}ts$ in $(X_2, N_{nc}\tau)$. Then, $N_{nc}cl(\mu)$ is a $N_{nc}cs$ in $(X_2, N_{nc}\tau)$, and every $N_{nc}cs$ is a $N_{nc}ecs$ in $(X_2, N_{nc}\tau)$. Assume ζ is $N_{nc}eIrr$, $\zeta^{-1}(N_{nc}cl(\lambda))$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$, then $N_{nc}cl(\zeta^{-1}(N_{nc}cl(\mu))) = \zeta^{-1}(N_{nc}cl(\mu))$. Here, $N_{nc}ecl\ (\zeta^{-1}(\mu)) \subseteq N_{nc}ecl(\zeta^{-1}(N_{nc}cl(\mu))) = \zeta^{-1}(N_{nc}cl(\mu))$. Therefore, $N_{nc}ecl(\zeta^{-1}(\mu)) \subseteq \zeta^{-1}(N_{nc}cl(\mu))$ for every $N_{nc}s$ μ in $(X_2, N_{nc}\tau)$. □

Theorem 6.4. Let $\zeta : (X_1, N_{nc}\Psi) \to (X_2, N_{nc}\tau)$ be a $N_{nc}eCHom$, then $N_{nc}ecl(\zeta^{-1}(\mu)) = \zeta^{-1}(N_{nc}ecl(\mu))$ for each $N_{nc}s$ μ in $(X_2, N_{nc}\tau)$.

Proof. Since ζ is a $N_{nc}eCHom$, then ζ is a $N_{nc}eIrr$. Let μ be a $N_{nc}s$ in $(X_2, N_{nc}\tau)$. Clearly, $N_{nc}ecl(\mu)$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Then $N_{nc}ecl(\mu)$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Since $\zeta^{-1}(\mu) \subseteq \zeta^{-1}(N_{nc}ecl(\mu))$, then $N_{nc}ecl(\zeta^{-1}(\mu)) \subseteq N_{nc}ecl(\zeta^{-1}(N_{nc}ecl(\mu))) = \zeta^{-1}(N_{nc}ecl(\mu))$. Therefore, $N_{nc}ecl(\zeta^{-1}(\mu)) \subseteq \zeta^{-1}(N_{nc}ecl(\mu))$. Let ζ be a $N_{nc}eCHom$. ζ^{-1} is a $N_{nc}eIrr$. Let us consider $N_{nc}s$ $\zeta^{-1}(\mu)$ in $(X_1, N_{nc}\Psi)$, which implies $N_{nc}ecl(\zeta^{-1}(\mu))$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Hence, $N_{nc}ecl(\zeta^{-1}(\mu))$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. This implies that $(\zeta^{-1})^{-1}(N_{nc}ecl(\zeta^{-1}(\mu))) = \zeta(N_{nc}ecl(\zeta^{-1}(\mu)))$ is a $N_{nc}ecs$ in $(X_2, N_{nc}\tau)$. This proves $\mu = (\zeta^{-1})^{-1}(\zeta^{-1}(\mu)) \subseteq (\zeta^{-1})^{-1}(N_{nc}ecl(\zeta^{-1}(\mu))) = \zeta(N_{nc}ecl(\zeta^{-1}(\mu)))$. Therefore, $N_{nc}ecl(\mu) \subseteq N_{nc}ecl(\zeta(N_{nc}ecl(\zeta^{-1}(\mu)))) = \zeta(N_{nc}ecl(\zeta^{-1}(\mu)))$, since ζ^{-1} is a $N_{nc}eIrr$. Hence, $\zeta^{-1}(N_{nc}ecl(\mu)) \subseteq \zeta^{-1}(\zeta(N_{nc}ecl(\zeta^{-1}(\mu)))) = N_{nc}ecl(\zeta^{-1}(\mu))$. That is, $\zeta^{-1}(N_{nc}ecl(\mu)) \subseteq N_{nc}ecl(\zeta^{-1}(\mu))$. Hence, $N_{nc}ecl(\zeta^{-1}(\mu)) = \zeta^{-1}(N_{nc}\ ecl(\mu))$. □

Theorem 6.5. If ζ & η are $N_{nc}eCHom$'s, then $\eta \circ \zeta$ is a $N_{nc}eCHom$.

Proof. Let ζ and η to be two $N_{nc}eCHom$'s. Assume μ is a $N_{nc}ecs$ in $(X_3, N_{nc}\rho)$. Then, $\eta^{-1}(\mu)$ is a $N_{nc}ecs$ in $(X_2, N_{nc}\tau)$. Then, by hypothesis, $\zeta^{-1}(\eta^{-1}(\mu))$ is a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Hence, $\eta \circ \zeta$ is a $N_{nc}eIrr$ mapping. Now, let ν be a $N_{nc}ecs$ in $(X_1, N_{nc}\Psi)$. Then, by presumption, $\zeta(\eta)$ is a $N_{nc}ecs$ in $(X_2, N_{nc}\tau)$. Then, by hypothesis, $\eta(\zeta(\nu))$ is a $N_{nc}ecs$ in $(X_3, N_{nc}\rho)$. This implies that $\eta \circ \zeta$ is a $N_{nc}eIrr$. Hence, $\eta \circ \zeta$ is a $N_{nc}eCHom$. □

7. Conclusions

In this paper, the new concept of a $N_{nc}eO$ and $N_{nc}eC$ mappings, $N_{nc}Hom$ and a $N_{nc}eHom$ in $N_{nc}ts$ are studied and discussed their properties. Also, we extended to $N_{nc}eCHom$'s and $N_{nc}eT_{\frac{1}{2}}$-space with some of their properties.

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