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A New Framework of Interval-valued Neutrosophic in Ẑ -algebra

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Abstract: This article deals about an interval-valued neutrosophic Ẑ-algebra is a mathematical framework which incorporates the concepts of interval-valued neutrosophic sets, Ẑ-algebra and algebraic operations. This innovative algebraic structure addresses the challenges posed by uncertain, imprecise, and indeterminate information in various fields. In this work, we presented the fundamentals of Ẑ-algebra and int_val neutrosophic sets, as well as several of their attributes such as homomorphism and cartesian product.

Keywords: Fuzzy sets, int_val fuzzy sets, neutrosophic set, Ẑ-subalgebra, int_val Ẑ-subalgebra, neutrosophic Ẑ-subalgebra, int_val neutrosophic Ẑ-subalgebra

1. Introduction

The intuitionistic fuzzy set with interval values is the name given to the new concept (IVIFS) which is presented by Attanassov [1] and outlines the fundamentals of IVIFS theory. Chandramouleeswaran [2] proposed Ẑ-algebra, a novel algebraic structure based upon propositional logic, in 2017. In a neutrosophic set, defined a set-theoretic operators, which is known as an interval neutrosophic set (INS), and then several INS properties related to operations and relations over INS [3]. In [4] introduces the phenomenon of int_val fuzzy β- subalgebras and examines a few of their features. This involves some of the information relevant to the theory of an int_val intuitionistic fuzzy subalgebras of β-algebra. Generalized double statistical convergence sequences on ideals in neutrosophic normed spaces were analysed by Jeyaraman et al. [5]. Henceforth, [6] established that each neutrosophic algebra is a direct product of neutrosophic algebras over the neutrosophic field. The ideology of neutrosophic sets in Ẑ-subalgebras is described, also some characteristics of int_val neutrosophic sets in Ẑ-algebras is also discussed. Maissam Jdid et l. [7] formulated Lagrange multipliers and neutrosophic nonlinear programming problems constrained by equality constraints. Manas Karak et al. [8] have introduced an innovative technique aimed at addressing transportation problems using a neutrosophic framework. This novel approach represents a significant stride in effectively handling uncertainty and indeterminacy within transportation scenarios.

Metawee[9] denotes a novel idea of interval_valued neutrosophic in UP-algebra, UP-subalgebras, as well as proved some results and their generalizations. The basic ideology of fuzzy Ẑ-ideal of a Ẑ-algebra under Ẑ-homomorphisms was evaluated, and some of its Cartesian product properties of fuzzy Ẑ-ideals were explored. Every quotient neutrosophic algebra is shown to be quotient algebra, and the concepts of neutrosophic algebra, the ideal, kernel, & neutrosophic quotient algebra are described [10]. The theme of neutrosophic cubic sets is used in β-subalgebra and then $\wp$-union, $\wp$-intersection, $\mathcal{R}$-union, and $\mathcal{R}$-intersection results based on neutrosophic cubic subalgebra is determined. Moreover, the captivating properties of lower and upper-level sets, as well as the homomorphism of neutrosophic cubic β−subalgebras, were explained [11]. The theory of neutrosophic algebra, including its ideal, kernel, and neutrosophic quotient algebra, as well as characterizing some neutrosophic algebra properties and claiming that every quotient neutrosophic algebra is quotient algebra [12]. The authors [13] started exploring an innovative concept for the Fermatean neutrosophic Dombi fuzzy graph. They also discovered a few outcomes of Fermatean neutrosophic Dombi fuzzy graphs' direct, cartesian composition. Shanmugapriya et al. [14] presented a novel concept of neutrosophic fuzzy Sets in Ẑ-algebra.

Samarandache generalises intuitionistic fuzzy sets to neutrosophic set, and many examples are given to distinguish between neutrosophic set as well as intuitionistic fuzzy set [15]. Neutrosophic set is the general framework that was recently proposed. However, from the point of technical view, the neutrosophic set must be specified. An int_val fuzzy set has been used to discuss these various algebraic structures as well as related topics. In [16], the authors have undertaken an insightful exploration into the concept of a fuzzy Ẑ-ideal within the context of a Ẑ-algebra. Furthermore, it was shown that the Cartesian product of fuzzy Ẑ-ideals is a fuzzy Ẑ-ideal. In [17] the authors provided the evidence of Cartesian product of fuzzy Ẑ-subalgebras is always a fuzzy Ẑ-subalgebra. The fundamental principle of a fuzzy Ẑ-subalgebra of Ẑ-algebra and its properties were investigated, and it also discusses how to resolve the inverse image of fuzzy Ẑ-subalgebras and the Ẑ-homomorphism of its image. The author of [18] has made a noteworthy contribution to the field by introducing a novel concept known as MBJ - neutrosophic set within the context of β-algebras. The MBJ - neutrosophic β- subalgebra's homomorphic and inverse images are presented. In MBJ - neutrosophic β- subalgebra, Cartesian product is often examined. A In 1965, Zadeh discovered the fuzzy set, which is very helpful for finding the uncertainties [19]. And again, extended the concept of an int_val fuzzy set as generalization of traditional fuzzy sets, then invented an int_val fuzzy set by using an int_val membership function to represent an interval on the membership scale [20].

This article's main objective is to explain the int_val neutrosophic in Ẑ-algebra. The following are the sections of the paper. The introduction appears in Section 1. Section 2 explained about the necessary definitions and properties of Ẑ-algebra and so on. Section 3 provides a more accurate explanation of neutrosophic in Ẑ-algebra and int_val neutrosophic in Ẑ-algebra. In Section 4, the int_val neutrosophic Ẑ- subalgebra homomorphism is discussed. The cartesian product of two neutrosophic Ẑ-algebras with int_val is defined in Section 5. Section 6 introduces the conclusion of this work.

2. Preliminaries

This section describes the fundamental definitions of fuzzy sets and Ẑ-algebra, as well as their major properties and examples. In the below discussion, the following notations are used such as X denoted by $\mathfrak{W}$, x denoted by ϵ, y denoted as ϱ, and Y is denoted by $\mathfrak{Y}$.

Definition 2.1.[15] Let the fuzzy set from the universal set $\mathfrak{W}$ and it is defining to be ς (ϵ): $\mathfrak{W} \rightarrow [0,1]$ for every element $\epsilon \in \mathfrak{W}$, and ς (ϵ) is known as the membership value of ϵ.

Definition 2.2.[4] The int_val fuzzy set $\mathfrak{W}$ is to be defined on $\bar{\xi}$ ={ ϵ, $\bar{\varsigma}_{\xi}$ (ϵ)/ $\epsilon \in \mathfrak{W}$ }, briefly denoted by, $\bar{\varsigma}_{\xi}(\epsilon)$= $[\varsigma_{\xi}^{L}(\epsilon), \varsigma_{\xi}^{U}(\epsilon)]$, where $\varsigma_{\xi}^{L}(\epsilon)$ & $\varsigma_{\xi}^{U}(\epsilon)$ are the two fuzzy sets in $\mathfrak{W}$ such that $\varsigma_{\xi}^{L}(\epsilon) \leq \varsigma_{\xi}^{U}(\epsilon)$ for all $\epsilon \in \mathfrak{W}$. Let $\bar{\varsigma}_{\xi}(\epsilon) = [\varsigma_{\xi}^{L}(\epsilon), \varsigma_{\xi}^{U}(\epsilon)]$ $\forall$ $\epsilon \in \mathfrak{W}$.

Let $\mathfrak{D}$[0,1] denote the collection of all closed sub-intervals of [0,1]. If $\varsigma_{\xi}^{L}(\epsilon) = \varsigma_{\xi}^{U}(\epsilon)$ =c, where 0≤c≤1, then there exist $\bar{\varsigma}_{\xi}(\epsilon)$ = [c, c] = $\bar{c}$.

For the convenience, ϵ belongs to $\mathfrak{D}$ [0,1] $\forall$ $\epsilon \in \mathfrak{W}$,

Thus, the int_val fuzzy set $\bar{\xi}$ is represented as $\bar{\xi}$ ={ ϵ, $\bar{\varsigma}_{\xi}$ (ϵ)/ $\epsilon \in \mathfrak{W}$}, where $\bar{\varsigma}_{\xi}$: $\mathfrak{W} \rightarrow \mathfrak{D}$ [0,1].

Now, Define a refined minimum (briefly rmin) of two elements in $\mathfrak{D}$[0,1].

Define the symbols " $\leq$ ", " $\geq$ " & "$=$".

In case, if two elements in $\mathfrak{D}$ [0,1], then it expressed as $\mathfrak{D}_1$:=[a_1, b_1], $\mathfrak{D}_2$:=[a_2, b_2] $\in \mathfrak{D}$ [0,1].

Then, rmin $(\mathfrak{D}_1, \mathfrak{D}_2)$ = [min{ a_1, a_2}, min $\{b_1, b_2\}$], $\mathfrak{D}_1 \geq \mathfrak{D}_2$ iff $a_1 \geq a_2$, $b_1 \geq b_2$.

Similarly, there exist $\mathfrak{D}_1 \leq \mathfrak{D}_2$ & $\mathfrak{D}_1 = \mathfrak{D}_2$.

Definition 2.3.[4] Let $\bar{\varsigma}_1$ & $\bar{\varsigma}_2$ are two int_val fuzzy sets on $\mathfrak{W}$, then intersection $\bar{\varsigma}_1 \cap \bar{\varsigma}_2$ of $\bar{\varsigma}_1$ and $\bar{\varsigma}_2$ is referred as

$$(\bar{\varsigma}_1 \cap \bar{\varsigma}_2)(\epsilon) \geq \text{rmin}\{\bar{\varsigma}_1(\epsilon), \bar{\varsigma}_2(\epsilon)\}$$

Definition 2.4.[14] Let the neutrosophic fuzzy set ξ ={ϵ : $\varsigma_T(\epsilon), \varsigma_I(\epsilon), \varsigma_F(\epsilon)$ / $\epsilon \in \mathfrak{W}$ }, where $\varsigma_T, \varsigma_I, \varsigma_F$ are fuzzy sets in $\mathfrak{W}$, then it is denoted by $\varsigma_T(\epsilon)$ is true membership function, $\varsigma_I(\epsilon)$ is indeterminate membership function & $\varsigma_F(\epsilon)$ is false membership function.

Definition 2.5. The structure of $\bar{\xi}$ = {(ϵ : $\bar{\varsigma}_T(\epsilon)$, $\bar{\varsigma}_I(\epsilon), \bar{\varsigma}_F(\epsilon)$ / $\epsilon \in \mathfrak{W}$ } is referred to have int_val neutrosophic set in $\mathfrak{W}$, where $\bar{\varsigma}_{T,I,F}$: $\mathfrak{W} \rightarrow \mathfrak{D}$ [0,1], $\bar{\varsigma}_T(\epsilon)$ denotes truth int_val membership function, $\bar{\varsigma}_I(\epsilon)$ denotes indeterminate int_val membership function, $\bar{\varsigma}_F(\epsilon)$ denotes false int_val membership function.

Definition 2.6.[2] Suppose $\mathfrak{W}$ be the non-empty subset with the binary operation $*$ & a constant 0, then ($\mathfrak{W}$, $*$, 0) is Ẑ– algebra if,

i) $\epsilon * 0 = 0$

ii) $0 * \epsilon = \epsilon$

iii) $\epsilon * \epsilon = \epsilon$

iv) $\epsilon * \varrho = \varrho * \epsilon$, when $\epsilon \neq 0$ and $\varrho \neq 0$ $\forall$ $\epsilon, \varrho \in \mathfrak{W}$.

v)

Example 2.7. Let $\mathfrak{W}$= $\{0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ be the set defined on $\mathfrak{W}$ with the constant 0 and a binary operations $*$ by introucing cayley's table

$*$	0	ω_1	ω_2	ω_3	ω_4	ω_5
0	0	ω_1	ω_2	ω_3	ω_4	ω_5
ω_1	0	ω_1	ω_5	ω_4	ω_3	ω_2
ω_2	0	ω_5	ω_2	ω_1	ω_5	ω_4
ω_3	0	ω_4	ω_1	ω_3	ω_1	ω_2
ω_4	0	ω_3	ω_5	ω_1	ω_4	ω_3
ω_5	0	ω_2	ω_4	ω_2	ω_3	ω_5

Definition 2.8. If $\mathfrak{W}$ is non-empty subset in neutrosophic Ẑ-algebra, then it's defined by Ẑ-subalgebra of $\mathfrak{W}$,

$$(\epsilon * \varrho) \in \mathfrak{W}, \forall\ \epsilon, \varrho \in \mathfrak{W}.$$

Definition 2.9.[10] Let ($\mathfrak{W}$, $*$,0) be the Ẑ– algebra with the operation $*$ and constant 0 then the neutrosophic set $\xi = \{ \epsilon : \varsigma_T, \varsigma_I, \varsigma_F / \epsilon \in \mathfrak{W}\}$, is known to be neutrosophic Ẑ- subalgebra of $\mathfrak{W}$.

i) $\varsigma_T(\epsilon * \varrho) \geq \min\{\varsigma_T(\epsilon), \varsigma_T(\varrho)\}$

ii) $\varsigma_I(\epsilon * \varrho) \geq \min\{\varsigma_I(\epsilon), \varsigma_I(\varrho)\}$

iii) $\varsigma_F(\epsilon * \varrho) \leq \max\{\varsigma_F(\epsilon), \varsigma_F(\varrho)\}$

Definition 2.10.[13] Let ($\mathfrak{W}$, $*$,0) be Ẑ– algebra, then the fuzzy set ς in $\mathfrak{W}$ with membership function ξ_ς it is known as fuzzy Ẑ-subalgebra of a Ẑ-algebra $\mathfrak{W}$, if $\forall\, \epsilon, \varrho \in \mathfrak{W}$, if the following condition is satisfied

$$\xi_\varsigma(\epsilon * \varrho) \geq \min\{\xi_\varsigma(\epsilon), \xi_\varsigma(\varrho)\}$$

Definition 2.11. Let the Ẑ– algebra ($\mathfrak{W}$, $*$,0) and fuzzy set ς in $\mathfrak{W}$ with a membership function ξ_ς then it is named as Anti-fuzzy Ẑ-subalgebra of a Ẑ-algebra $\mathfrak{W}$, if $\forall\ \epsilon, \varrho \in \mathfrak{W}$, if the following condition is satisfied

$$\xi_\varsigma(\epsilon * \varrho) \leq \max\{\xi_\varsigma(\epsilon), \xi_\varsigma(\varrho)\}$$

Definition 2.12. Let ($\mathfrak{W}$, $*$,0) be a Ẑ– algebra then int_val fuzzy set $\bar{\xi}_\varsigma$ in $\mathfrak{W}$ is referred as an interval_valued fuzzy Ẑ-subalgebra of a Ẑ– algebra $\mathfrak{W}$, if

$$\bar{\xi}_\varsigma(\epsilon * \varrho) \geq \mathrm{rmin}\{\bar{\xi}_\varsigma(\epsilon), \bar{\xi}_\varsigma(\varrho)\}\ \forall\, \epsilon, \varrho \in \mathfrak{W}$$

Definition 2.13.[7] If U be the subset in universe $\mathfrak{W}$, then the rsup property of an int_val fuzzy set $\bar{\varsigma}$ is referred as $\bar{\varsigma}(\epsilon_0) = \underset{\epsilon \in U}{rsup}\,\bar{\varsigma}(\epsilon)$, if $\exists\, \epsilon, \epsilon_0 \in U$.

Definition 2.14. Let $\bar{\varsigma}$ be the int_val neutrosophic fuzzy set in any set of $\mathfrak{W}$ is known as rsup_rsup_rinf property, then the subset U of $\mathfrak{W}$ then $\exists\ \epsilon_0 \in$ U $\ni \bar{\varsigma}_T(\epsilon_0) = \underset{\epsilon \in U}{rsup}(\bar{\varsigma}_T(\epsilon))$, $\bar{\varsigma}_I(\epsilon_0) = \underset{\epsilon \in U}{rsup}(\bar{\varsigma}_I(\epsilon))$, $\bar{\varsigma}_F(\epsilon_0) = \underset{\epsilon \in U}{rinf}(\bar{\varsigma}_F(\epsilon))$.

Definition 2.15. Let ($\mathfrak{W}$, $*$,0) and ($\mathfrak{W}'$, $*'$, $0'$) be two Ẑ– algebra, then the mapping from h: ($\mathfrak{W}$, $*$,0) $\rightarrow$ ($\mathfrak{W}'$, $*'$,$0'$) is known as Ẑ– homomorphism of Ẑ– algebra if

$$\mathrm{h}\,(\epsilon * \varrho) = \mathrm{h}\,(\epsilon) *' \mathrm{h}\,(\varrho)$$

Definition 2.16. Let $\bar{\bar{\xi}} = \{\ \epsilon,\ \bar{\varsigma}_{T,I,F}(\epsilon)\ /\ \epsilon \in \mathfrak{W}\ \}$ be the neutrosophic set in $\hat{Z}$ and f maps from $\mathfrak{W} \rightarrow \mathfrak{Y}$, image of $\bar{\bar{\xi}}$ under f, f($\bar{\bar{\xi}}$) represented to be $\{\mathrm{f}_{rsup}(\bar{\varsigma}_T),\ \mathrm{f}_{rsup}(\bar{\varsigma}_I),\ \mathrm{f}_{rinf}(\bar{\varsigma}_F),\ \epsilon \in \mathfrak{Y}\}$

i) $$\mathrm{f}_{rsup}(\bar{\varsigma}_T)\ (\varrho) = \begin{cases} \underset{\epsilon\, \in \mathrm{f}^{-1}(\varrho)}{rsup}\ \bar{\varsigma}_T(\epsilon) & if\ \mathrm{f}^{-1}(\varrho) \neq \phi \\ 1 & otherwise \end{cases}$$

ii) $$\mathrm{f}_{rsup}(\bar{\varsigma}_I)(\varrho) = \begin{cases} \underset{\epsilon\, \in \mathrm{f}^{-1}(\varrho)}{rsup}\ \bar{\varsigma}_I(\epsilon) & if\ \mathrm{f}^{-1}(\varrho) \neq \phi \\ 1 & otherwise \end{cases}$$

iii) $$\mathrm{f}_{rinf}(\bar{\varsigma}_F)(\varrho) = \begin{cases} \underset{\epsilon\, \in \mathrm{f}^{-1}(\varrho)}{rinf}\ \bar{\varsigma}_F(\epsilon) & if\ \mathrm{f}^{-1}(\varrho) \neq \phi \\ 0 & otherwise \end{cases}$$

Definition 2.17. If $\mathrm{f}: \mathfrak{W} \rightarrow \mathfrak{Y}$ is a function. Let $\bar{\varsigma}_{T_1,I_1,F_1}$ & $\bar{\varsigma}_{T_2,I_2,F_2}$ be two int_val neutrosophic set in $\mathfrak{W}$ & $\mathfrak{Y}$ respectively, then inverse image of f is represented as f^{-1} $(\bar{\varsigma}_{T_2,I_2,F_2})$ = { ϵ, f^{-1} $(\bar{\varsigma}_{T_2}(\epsilon))$, f^{-1} $(\bar{\varsigma}_{I_2}(\epsilon))$, f^{-1} $(\bar{\varsigma}_{F_2}(\epsilon))$, / $\epsilon \in \mathfrak{W}$} such that $\mathrm{f}^{-1}(\bar{\varsigma}_{T_2})\ \mathrm{f}\ (\epsilon) = \bar{\varsigma}_{T_2}(\mathrm{f}(\epsilon))$, $\mathrm{f}^{-1}(\bar{\varsigma}_{I_2})\ \mathrm{f}\ (\epsilon) = \bar{\varsigma}_{I_2}(\mathrm{f}(\epsilon))$, $\mathrm{f}^{-1}(\bar{\varsigma}_{F_2})\ \mathrm{f}\ (\epsilon) = \bar{\varsigma}_{F_2}(\mathrm{f}(\epsilon))$.

Definition 2.18.[13] Let h be an $\hat{Z}$-endomorphism of int_val neutrosophic $\hat{Z}$-algebras and $\bar{\bar{\xi}} = \{\epsilon, \bar{\varsigma}_{T,I,F}(\epsilon)\ /\ \epsilon \in \mathfrak{W}\}$ be the neutrosophic set in $\mathfrak{W}$, then define a new fuzzy set $\bar{\varsigma}^h$ in $\mathfrak{W}$, as

$\bar{\bar{\xi}}_{\bar{\varsigma}^h}\ (\epsilon) = \bar{\bar{\xi}}_{\bar{\varsigma}}(\mathrm{h}(\epsilon))\ \forall\ \epsilon \in \mathfrak{W}$.

Definition 2.19.[14] If $\bar{\varsigma}_\xi$ and $\bar{\varsigma}_\zeta$ are the two int_val fuzzy sets of $\mathfrak{W}$, then the cartesian product $\bar{\varsigma}_\xi \times \bar{\varsigma}_\zeta :\ \mathfrak{W} \times \mathfrak{W} \rightarrow \mathcal{D}$ [0,1] is defined as

$$(\bar{\varsigma}_\xi \times \bar{\varsigma}_\zeta)(\ \epsilon, \varrho) = \mathrm{rmin}\{\bar{\varsigma}_\xi\ (\epsilon),\ \bar{\varsigma}_\zeta\ (\varrho)\}\ \forall\ \epsilon \in \mathfrak{W}.$$

3. Interval-valued neutrosophic in Ẑ-algebra

This section describes the definitions on val neutrosophic in Ẑ-algebra in detail.

Definition 3.1. Let ($\mathfrak{W}$, $*$,0) be $\hat{Z}$-algebra. The int_val neutrosophic set $\bar{\xi}$ = { ϵ: $\bar{\varsigma}_T, \bar{\varsigma}_I, \bar{\varsigma}_F(\epsilon)/\epsilon \in \mathfrak{W}$} in $\mathfrak{W}$ is known as int_val neutrosophic $\hat{Z}$-algebra of $\mathfrak{W}$, if satisfies the below condition

i) $\bar{\varsigma}_T(\epsilon * \varrho) \geq rmin\,\{\bar{\varsigma}_T(\epsilon),\ \bar{\varsigma}_T(\varrho)\}$

ii) $\bar{\varsigma}_I(\epsilon * \varrho) \geq\ rmin\,\{\bar{\varsigma}_I(\epsilon),\ \bar{\varsigma}_I(\varrho)\}$

iii) $\bar{\varsigma}_F(\epsilon * \varrho) \leq rmax\,\{\bar{\varsigma}_F(\epsilon),\ \bar{\varsigma}_F(\varrho)\}$

Example 3.2. Consider the example 2.2

$$\bar{\varsigma}_{T,I,F} = \begin{cases} [0.4,0.8] & \epsilon = 0\ when\ (\epsilon = 0, \varrho \neq 0)\ or\ (\epsilon \neq 0, \varrho = 0) \\ [0.3,0.7] & \epsilon = \omega_1, \omega_2 \\ [0.2,0.6] & \epsilon = \ \omega_4, \omega_5 \end{cases}$$

$$= \{[0.1,0.5] \quad \epsilon = \omega_3$$

Hence, the above example satisfies the condition of int_val neutrosophic of Ẑ-algebra.

Theorem 3.3. Intersection of two int_val neutrosophic Ẑ-algebra of $\mathfrak{W}$ is again an interval-valued neutrosophic Ẑ-algebra of $\mathfrak{W}$.

Proof: Let $\bar{\varsigma}_{T_1,I_1,F_1}$ and $\bar{\varsigma}_{T_2,I_2,F_2}$ are the two neutrosophic int_val neutrosophic Ẑ-algebra of $\mathfrak{W}$. Then,

$$(\bar{\varsigma}_{T_1} \cap \bar{\varsigma}_{T_2})(\epsilon * \varrho) \geq \text{rmin}\{\bar{\varsigma}_{T_1}(\epsilon * \varrho), \bar{\varsigma}_{T_2}(\epsilon * \varrho)\}$$

$$= \{\text{rmin}\{\bar{\varsigma}_{T_1}(\epsilon), \bar{\varsigma}_{T_1}(\varrho)\}, \text{rmin}\{\bar{\varsigma}_{T_2}(\epsilon), \bar{\varsigma}_{T_2}(\varrho)\}\}$$

$$= \{\text{rmin}\{\bar{\varsigma}_{T_1}(\epsilon), \bar{\varsigma}_{T_2}(\epsilon)\}, \text{rmin}\{\bar{\varsigma}_{T_1}(\varrho), \bar{\varsigma}_{T_2}(\varrho)\}\}$$

$$= \text{rmin}\{(\bar{\varsigma}_{T_1 \cap T_2})(\epsilon), (\bar{\varsigma}_{T_1 \cap T_2})(\varrho)\}$$

$$\therefore (\bar{\varsigma}_{T_1} \cap \bar{\varsigma}_{T_2})(\epsilon * \varrho) \geq \text{rmin}\{(\bar{\varsigma}_{T_1 \cap T_2})(\epsilon), (\bar{\varsigma}_{T_1 \cap T_2})(\varrho)\}$$

Similarly, $(\bar{\varsigma}_{I_1} \cap \bar{\varsigma}_{I_2})(\epsilon * \varrho) \geq \text{rmin}\{(\bar{\varsigma}_{I_1 \cap I_2})(\epsilon), (\bar{\varsigma}_{I_1 \cap I_2})(\varrho)\}$

$$(\bar{\varsigma}_{F_1} \cap \bar{\varsigma}_{F_2})(\epsilon * \varrho) \leq \text{rmax}\{\bar{\varsigma}_{F_1}(\epsilon * \varrho), \bar{\varsigma}_{F_2}(\epsilon * \varrho)\}$$

$$= \{\text{rmax}\{\bar{\varsigma}_{F_1}(\epsilon), \bar{\varsigma}_{F_1}(\varrho)\}, \text{rmax}\{\bar{\varsigma}_{F_2}(\epsilon), \bar{\varsigma}_{F_2}(\varrho)\}\}$$

$$= \{\text{rmax}\{\bar{\varsigma}_{F_1}(\epsilon), \bar{\varsigma}_{F_2}(\epsilon)\}, \text{rmax}\{\bar{\varsigma}_{F_1}(\varrho), \bar{\varsigma}_{F_2}(\varrho)\}\}$$

$$= \text{rmax}\{(\bar{\varsigma}_{F_1 \cap F_2})(\epsilon), (\bar{\varsigma}_{F_1 \cap F_2})(\varrho)\}$$

$$\therefore (\bar{\varsigma}_{F_1} \cap \bar{\varsigma}_{F_2})(\epsilon * \varrho) \leq \text{rmax}\{(\bar{\varsigma}_{F_1 \cap F_2})(\epsilon), (\bar{\varsigma}_{F_1 \cap F_2})(\varrho)\}$$

Hence $\bar{\varsigma}_{T_1,I_1,F_1}$ and $\bar{\varsigma}_{T_2,I_2,F_2}$ is an int_val neutrosophic Ẑ-algebra of $\mathfrak{W}$.

Theorem 3.4. Intersection of any set of int_val neutrosophic of Ẑ-algebra of $\mathfrak{W}$ is again an int_val neutrosophic of Ẑ-algebra of $\mathfrak{W}$.

Lemma 3.5. If $\bar{\xi}$ be an int_val neutrosophic Ẑ-subalgebra of $\mathfrak{W}$, then

i) $\bar{\varsigma}_T(0) \geq \bar{\varsigma}_T(\epsilon)$, $\bar{\varsigma}_I(0) \geq \bar{\varsigma}_I(\epsilon)$, & $\bar{\varsigma}_F(0) \leq \bar{\varsigma}_F(\epsilon)$ $\forall$ $\epsilon \in \mathfrak{W}$.

ii) $\bar{\varsigma}_T(0) \geq \bar{\varsigma}_T(\epsilon) \geq \bar{\varsigma}_T(\epsilon^*)$, $\bar{\varsigma}_I(0) \geq \bar{\varsigma}_I(\epsilon) \geq \bar{\varsigma}_I(\epsilon^*)$, $\bar{\varsigma}_F(0) \leq \bar{\varsigma}_F(\epsilon) \leq \bar{\varsigma}_F(\epsilon^*)$, where $\epsilon^* = 0 * \epsilon$

Proof: For any $\epsilon \in \mathfrak{W}$,

i) $\bar{\varsigma}_T(0) = [\varsigma_T^L(0), \varsigma_T^U(0)]$

$$\geq [\varsigma_T^L(\epsilon), \varsigma_T^U(\epsilon)]$$

$$= \bar{\varsigma}_T(\epsilon)$$

Likewise, $\bar{\varsigma}_I(0) = [\varsigma_I^L(0), \varsigma_I^U(0)]$

$$\geq [\varsigma_I^L(\epsilon), \varsigma_I^U(\epsilon)]$$

$$= \bar{\varsigma}_I(\epsilon)$$

$\bar{\varsigma}_F(0) = [\varsigma_F^L(0), \varsigma_F^U(0)]$

$$\leq [\varsigma_F^L(\epsilon), \varsigma_F^U(\epsilon)]$$

$= \bar{\varsigma}_F(\epsilon)$

ii) Also, for $\epsilon \in \mathfrak{W}$

$\bar{\varsigma}_T(\epsilon^*) = [\varsigma_T^L(\epsilon^*), \varsigma_T^U(\epsilon^*)]$

$= [\varsigma_T^L(0 * \epsilon), \varsigma_T^U(0 * \epsilon)]$

$= [\min(\varsigma_T^L(0), \varsigma_T^U(\epsilon), \min((\varsigma_T^L(0), \varsigma_T^U(\epsilon))]$

$\bar{\varsigma}_T(\epsilon^*) \geq [\varsigma_T^L(\epsilon), \varsigma_T^U(\epsilon)]$

$= \bar{\varsigma}_T(\epsilon)$

$\bar{\varsigma}_I(\epsilon^*) = [\varsigma_I^L(\epsilon^*), \varsigma_I^U(\epsilon^*)]$

$= [\varsigma_I^L(0 * \epsilon), \varsigma_I^U(0 * \epsilon)]$

$= [\min(\varsigma_I^L(0), \varsigma_I^U(\epsilon), \min((\varsigma_I^L(0), \varsigma_I^U(\epsilon))]$

$\bar{\varsigma}_I(\epsilon^*) \geq [\varsigma_I^L(\epsilon), \varsigma_I^U(\epsilon)]$

$= \bar{\varsigma}_I(\epsilon)$

$\bar{\varsigma}_F(\epsilon^*) = [\varsigma_F^L(\epsilon^*), \varsigma_F^U(\epsilon^*)]$

$= [\varsigma_F^L(0 * \epsilon), \varsigma_F^U(0 * \epsilon)]$

$= [\max(\varsigma_F^L(0), \varsigma_F^U(\epsilon), \max((\varsigma_F^L(0), \varsigma_F^U(\epsilon))]$

$\therefore$ $\bar{\varsigma}_F(\epsilon^*) \leq [\varsigma_F^L(\epsilon), \varsigma_F^U(\epsilon)]$

$= \bar{\varsigma}_F(\epsilon)$

Theorem 3.6. If there is a sequence $\{\epsilon_n\}$ in $\mathfrak{W}$, such that $\lim_{n\to\infty} \bar{\varsigma}_T(\epsilon_n) = [1,1]$, $\lim_{n\to\infty} \bar{\varsigma}_I(\epsilon_n) = [1,1]$, $\lim_{n\to\infty} \bar{\varsigma}_F(\epsilon_n) = [0,0]$. Let $\bar{\xi}$ be an int_val neutrosophic Ẑ-subalgebra of $\mathfrak{W}$, then $\bar{\varsigma}_T(0) = [1,1]$, $\bar{\varsigma}_I(0) = [1,1]$, $\bar{\varsigma}_F(0) = [0,0]$.

Proof: Let, $\bar{\varsigma}_T(0) \geq \bar{\varsigma}_T(\epsilon)$, for all $\epsilon \in \mathfrak{W}$,

$\bar{\varsigma}_T(0) \geq \bar{\varsigma}_T(\epsilon_n)$

Similarly, $\bar{\varsigma}_I(0) \geq \bar{\varsigma}_I(\epsilon_n)$ & $\bar{\varsigma}_F(0) \leq \bar{\varsigma}_F(\epsilon_n)$ $\forall$ $n \geq 0$

Thus, $[1,1] \geq \bar{\varsigma}_T(0) \geq \lim_{n\to\infty} \bar{\varsigma}_T(\epsilon_n) = [1,1]$

$\Longrightarrow \bar{\varsigma}_T(0) = [1,1]$

Similarly, [1,1] $\geq \bar{\varsigma}_I(0) \geq \lim_{n\to\infty} \bar{\varsigma}_I(\epsilon_n)$ = [1,1]

$$\Longrightarrow \bar{\varsigma}_I(0) = [1,1]$$

Likewise, [1,1] $\leq \bar{\varsigma}_F(0) \leq \lim_{n\to\infty} \bar{\varsigma}_F(\epsilon_n)$ = [0,0]

$$\Longrightarrow \bar{\varsigma}_F(0) = [0,0]$$

Theorem 3.7. Let $\bar{\bar{\xi}}$= {ϵ: $\bar{\varsigma}_T(\epsilon)$, $\bar{\varsigma}_I(\epsilon)$, $\bar{\varsigma}_F(\epsilon)$ $\forall\ \epsilon \in \mathfrak{W}$ } such that $[\varsigma_T^L, \varsigma_T^U]$, $[\varsigma_I^L, \varsigma_I^U]$ are fuzzy Ẑ -subalgebra & $[\varsigma_F^L, \varsigma_F^U]$ is anti-fuzzy Ẑ-subalgebra of $\mathfrak{W}$, then $\bar{\bar{\xi}}$= { ϵ: $\bar{\varsigma}_T(\epsilon)$, $\bar{\varsigma}_I(\epsilon)$, $\bar{\varsigma}_F(\epsilon) \forall\ \epsilon \in \mathfrak{W}$} is an int_val neutrosophic Ẑ-subalgebra of $\mathfrak{W}$.

Proof: For any ϵ, $\varrho \in \mathfrak{W}$, then

$$\begin{aligned}
\bar{\varsigma}_T(\epsilon * \varrho) &= [\varsigma_T^L(\epsilon * \varrho), \varsigma_T^U(\epsilon * \varrho)] \\
&\geq [\min\{\varsigma_T^L(\epsilon), \varsigma_T^L(\varrho)\}, \min\{\varsigma_T^U(\epsilon), \varsigma_T^U(\varrho)\}] \\
&= \min\{[\varsigma_T^L(\epsilon), \varsigma_T^U(\epsilon)], [\varsigma_T^L(\varrho), \varsigma_T^U(\varrho)]\} \\
&= \mathrm{rmin}\{\bar{\varsigma}_T(\epsilon), \bar{\varsigma}_T(\varrho)\}
\end{aligned}$$

$$\therefore\ \bar{\varsigma}_T(\epsilon * \varrho) \geq \mathrm{rmin}\{\bar{\varsigma}_T(\epsilon), \bar{\varsigma}_T(\varrho)\}$$

Similarly, $\bar{\varsigma}_I(\epsilon * \varrho) \geq \mathrm{rmin}\{\bar{\varsigma}_I(\epsilon), \bar{\varsigma}_I(\varrho)\}$

Hence, $\bar{\varsigma}_T$, $\bar{\varsigma}_I$ are fuzzy Ẑ-subalgebra of $\mathfrak{W}$.

$$\begin{aligned}
\bar{\varsigma}_F(\epsilon * \varrho) &= [\varsigma_F^L(\epsilon * \varrho), \varsigma_F^U(\epsilon * \varrho)] \\
&\leq [\max\{\varsigma_F^L(\epsilon), \varsigma_F^L(\varrho), \max\{\varsigma_F^U(\epsilon), \varsigma_F^U(\varrho)\}] \\
&= \max\{[\varsigma_F^L(\epsilon), \varsigma_F^U(\epsilon)], [\varsigma_F^L(\varrho), \varsigma_F^U(\varrho)]\} \\
&= \mathrm{rmax}\{\bar{\varsigma}_F(\epsilon), \bar{\varsigma}_F(\varrho)\}
\end{aligned}$$

$$\bar{\varsigma}_F(\epsilon * \varrho) \leq \mathrm{rmax}\{\bar{\varsigma}_F(\epsilon), \bar{\varsigma}_F(\varrho)\}$$

Hence, $\bar{\varsigma}_F$ is Anti-fuzzy Ẑ-subalgebra of $\mathfrak{W}$.

$\therefore$ $\bar{\bar{\xi}}$= {ϵ: $\bar{\varsigma}_T(\epsilon)$, $\bar{\varsigma}_I(\epsilon)$, $\bar{\varsigma}_F(\epsilon)$ $\forall\ \epsilon \in \mathfrak{W}$ } is an int_val neutrosophic Ẑ-subalgebra of $\mathfrak{W}$.

Theorem 3.8. If $\bar{\bar{\xi}}$= {ϵ: $\bar{\varsigma}_T(\epsilon)$, $\bar{\varsigma}_I(\epsilon)$, $\bar{\varsigma}_F(\epsilon)$ $\forall\ \epsilon \in \mathfrak{W}$ } is an int_val neutrosophic Ẑ-subalgebra of $\mathfrak{W}$, then the sets

$$\mathfrak{W}_{\bar{\varsigma}_T} = \{\ \epsilon \in \mathfrak{W}\ /\ \bar{\varsigma}_T(\epsilon) = \bar{\varsigma}_T(\bar{0})\}$$

$$\mathfrak{W}_{\bar{\varsigma}_I} = \{\ \epsilon \in \mathfrak{W}\ /\ \bar{\varsigma}_I(\epsilon) = \bar{\varsigma}_I(\bar{0})\}$$

$$\mathfrak{W}_{\bar{\varsigma}_F} = \{ \epsilon \in \mathfrak{W} \ / \ \bar{\varsigma}_F(\epsilon) = \bar{\varsigma}_F(\bar{0})\}$$ are Ẑ -subalgebra of $\mathfrak{W}$.

Proof: For $\epsilon, \varrho \in \mathfrak{W}_{\bar{\varsigma}_T}$, then

$$\bar{\varsigma}_T(\epsilon) = \bar{\varsigma}_T(\bar{0}) = \bar{\varsigma}_T(\varrho)$$

Now, $\bar{\varsigma}_T(\epsilon * \varrho) \geq \text{rmin}\{\bar{\varsigma}_T(\epsilon) * \bar{\varsigma}_T(\varrho)\}$

$= \text{rmin}\{\bar{\varsigma}_T(\bar{0}), \bar{\varsigma}_T(\bar{0})\}$

$= \bar{\varsigma}_T(\bar{0})$

$\therefore \ \bar{\varsigma}_T(\epsilon * \varrho) \geq \bar{\varsigma}_T(\bar{0})$

Similarly, $\bar{\varsigma}_I(\epsilon * \varrho) \geq \bar{\varsigma}_I(\bar{0})$

$\bar{\varsigma}_F(\epsilon * \varrho) \leq \text{rmax}\{\bar{\varsigma}_F(\epsilon) * \bar{\varsigma}_F(\varrho)\}$

$= \text{rmax}\{\bar{\varsigma}_F(\bar{0}), \bar{\varsigma}_F(\bar{0})\}$

$= \bar{\varsigma}_F(\bar{0})$

$\therefore \ \bar{\varsigma}_F(\epsilon * \varrho) \leq \bar{\varsigma}_F(\bar{0})$

Hence, $\epsilon * \varrho \in \mathfrak{W}_{\bar{\varsigma}_{T,I,F}}$ is Ẑ -subalgebra of $\mathfrak{W}$.

Theorem 3.9. Given $(\mathfrak{W}, *, 0)$ & $(\mathfrak{W}', *', 0)$ be the two Ẑ-algebras & f: $\mathfrak{W} \to \mathfrak{Y}$ is homomorphism of Ẑ - algebras. If $\bar{\xi}$ is an int_val neutrosophic Ẑ-algebra of $\mathfrak{W}$, defined by f $(\bar{\varsigma}_{T,I,F}) = \{\epsilon, (\bar{\varsigma}_{T,I,F})(\epsilon) = \bar{\varsigma}_{T,I,F}(\text{f}(\epsilon))\}$ then f $(\bar{\varsigma}_{T,I,F})$ is an int_val neutrosophic Ẑ– subalgebra of $\mathfrak{W}$.

Proof: Given, $\epsilon, \varrho \in \mathfrak{W}$

$(\bar{\varsigma}_{T_f})(\epsilon * \varrho) = \bar{\varsigma}_T(\text{f}(\epsilon * \varrho))$

$= \bar{\varsigma}_T(\text{f}(\epsilon) * \text{f}(\varrho))$

$\geq \text{rmin}\{\bar{\varsigma}_T(\text{f}(\epsilon), \bar{\varsigma}_T(\text{f}(\varrho)\}$

$(\bar{\varsigma}_{T_f}) \geq \text{rmin}\{(\bar{\varsigma}_{T_f})(\epsilon), (\bar{\varsigma}_{T_f})(\varrho)\}$

Similarly, $(\bar{\varsigma}_{I_f}) \geq \text{rmin}\{(\bar{\varsigma}_{I_f})(\epsilon), (\bar{\varsigma}_{I_f})(\varrho)\}$

$(\bar{\varsigma}_{F_f})(\epsilon * \varrho) = \bar{\varsigma}_F(\text{f}(\epsilon * \varrho))$

$= \bar{\varsigma}_F(\text{f}(\epsilon) * \text{f}(\varrho))$

$\leq \text{rmax}\{\bar{\varsigma}_F(\text{f}(\epsilon)), \bar{\varsigma}_F(\text{f}(\varrho)\}$

$(\bar{\varsigma}_{F_f}) \leq \text{rmax}\{(\bar{\varsigma}_{F_f})(\epsilon), (\bar{\varsigma}_{F_f})(\varrho)\}$

Therefore, f $(\bar{\varsigma}_{T,I,F})$ is an int_val neutrosophic Ẑ– subalgebra of $\mathfrak{W}'$.

4. Homomorphism of Interval-valued (int_val) Neutrosophic Ẑ– Subalgebra

In this section, will look at some methods for investigating results on int_val neutrosophic Ẑ - subalgebra homomorphism.

Theorem. 4.1. If f : $\mathfrak{W} \rightarrow \mathfrak{Y}$ is the homomorphism of $\hat{Z}$ - algebra. If $\bar{\varsigma}_{T,I,F}$ be the int_val neutrosophic $\hat{Z}$ -subalgebra of $\mathfrak{Y}$, then $f^{-1}(\bar{\varsigma}_{T,I,F}) = \{(f^{-1}(\bar{\varsigma}_T), f^{-1}(\bar{\varsigma}_I), f^{-1}(\bar{\varsigma}_F) / \epsilon \in \mathfrak{W}\}$ is also the int_val neutrosophic $\hat{Z}$ - subalgebra of $\mathfrak{Y}$, where $f^{-1}(\bar{\varsigma}_T(\epsilon)) = \bar{\varsigma}_T f(\epsilon)$, $f^{-1}(\bar{\varsigma}_I(\epsilon)) = \bar{\varsigma}_I f(\epsilon)$, $f^{-1}(\bar{\varsigma}_F(\epsilon)) = \bar{\varsigma}_F f(\epsilon)$, for every $\epsilon \in \mathfrak{W}$.

Proof: Given $\bar{\varsigma}_{T,I,F}$ be the int_val neutrosophic $\hat{Z}$ -subalgebra of $\mathfrak{Y}$,

Let $\epsilon, \varrho \in \mathfrak{W}$

Then, $f^{-1}(\bar{\varsigma}_T(\epsilon * \varrho) = \bar{\varsigma}_T f(\epsilon * \varrho)$

$$= \bar{\varsigma}_T(f(\epsilon) * \bar{\varsigma}_T f(\varrho))$$
$$\geq \text{rmin}\{\bar{\varsigma}_T(f(\epsilon)), (\bar{\varsigma}_T(f(\varrho))\}$$
$$\geq \text{rmin}\{f(\bar{\varsigma}_T(\epsilon)), f(\bar{\varsigma}_T(\varrho))\}$$
$$= \text{rmin}\{f^{-1}(\bar{\varsigma}_T(\epsilon)) , f^{-1}(\bar{\varsigma}_T(\varrho))\}$$

$$f^{-1}(\bar{\varsigma}_T(\epsilon * \varrho) \geq \min\{f^{-1}(\bar{\varsigma}_T(\epsilon)) , f^{-1}(\bar{\varsigma}_T(\varrho))\}$$

Similarly, $f^{-1}(\bar{\varsigma}_I)(\epsilon * \varrho) \geq \text{rmin}\{ f^{-1}(\bar{\varsigma}_I(\epsilon)) , f^{-1}(\bar{\varsigma}_I(\varrho))\}$

$$f^{-1}(\bar{\varsigma}_F)(\epsilon * \varrho) = \bar{\varsigma}_F(f(\epsilon * \varrho))$$
$$= \bar{\varsigma}_F(f(\epsilon) * f(\varrho))$$
$$\leq \text{rmax}\{ \bar{\varsigma}_F(f(\epsilon)), \bar{\varsigma}_F(f(\varrho))\}$$
$$= \text{rmax}\{f^{-1}(\bar{\varsigma}_F(\epsilon)) , f^{-1}(\bar{\varsigma}_F(\varrho))\}$$

$$f^{-1}(\bar{\varsigma}_F)(\epsilon * \varrho) \leq \text{rmax}\{f^{-1}(\bar{\varsigma}_F(\epsilon)) , f^{-1}(\bar{\varsigma}_F(\varrho))\}$$

$\therefore f^{-1}(\bar{\varsigma}_{T,I,F}) = \{(f^{-1}(\bar{\varsigma}_T), f^{-1}(\bar{\varsigma}_I), f^{-1}(\bar{\varsigma}_F)\}$ is an int_val neutrosophic $\hat{Z}$- subalgebra of $\mathfrak{Y}$.

Theorem 4.2. If $f : \mathfrak{W} \rightarrow \mathfrak{Y}$ is the homomorphism from $\hat{Z}$– algebra $\mathfrak{W}$ *to* $\mathfrak{Y}$. If $\bar{\bar{\xi}} = (\bar{\varsigma}_{T,I,F})$ be the int_val neutrosophic $\hat{Z}$– algebra of $\mathfrak{W}$, then the image of $f(\bar{\bar{\xi}}) = \{ \epsilon , f_{rsup}(\bar{\varsigma}_T)\, f_{rsup}(\bar{\varsigma}_I), f_{rinf}(\bar{\varsigma}_F) / \epsilon \in \mathfrak{W} \}$ of $\bar{\bar{\xi}}$ under f is also the int_val neutrosophic $\hat{Z}$– subalgebra of $\mathfrak{Y}$.

Proof: Let $\bar{\bar{\xi}} = (\bar{\varsigma}_{T,I,F})$ be the int_val neutrosophic $\hat{Z}$– subalgebra of $\mathfrak{W}$, let $\varrho_1, \varrho_2 \in \mathfrak{Y}$.

We know that, $\epsilon_1 * \epsilon_2 / \epsilon_1 \in f^{-1}(\varrho_1)\ \&\ \epsilon_2 \in f^{-1}(\varrho_2)\} \subseteq \{ \epsilon \in \mathfrak{W} / \epsilon \in f^{-1}(\varrho_1 * \varrho_2)$

Now,

$$f_{rsup}(\bar{\varsigma}_T)((\varrho_1 * \varrho_2) = \text{rsup}\{(\bar{\varsigma}_T) / \epsilon \in f^{-1}(\varrho_1 * \varrho_2)\}$$

$$= \text{rsup}\{(\bar{\varsigma}_T)\, \epsilon_1 * \epsilon_2 / \epsilon_1 \in f^{-1}(\varrho_1)\ \&\ \epsilon_2 \in f^{-1}(\varrho_2)\}$$

$$\geq \text{rsup}\{\text{rmin}\{\bar{\varsigma}_T(\epsilon_1), \bar{\varsigma}_T(\epsilon_2) / \epsilon_1 \in f^{-1}(\varrho_1)\ \&\ \epsilon_2 \in f^{-1}(\varrho_2)\}$$

$$= \text{rmin}\{\text{rsup}\{\bar{\varsigma}_T(\epsilon_1) / \epsilon_1 \in f^{-1}(\varrho_1), \bar{\varsigma}_T(\epsilon_2) / \epsilon_2 \in f^{-1}(\varrho_2)\}\}$$

$$f_{rsup}(\bar{\varsigma}_T)((\varrho_1 * \varrho_2) \geq \text{rmin}\{f_{\text{rsup}}(\bar{\varsigma}_T(\varrho_1)), f_{\text{rsup}}(\bar{\varsigma}_T(\varrho_2))\}$$

Similarly, $f_{rsup}(\bar{\varsigma}_I)(\varrho_1 * \varrho_2) \geq \text{rmin}\{f_{\text{rsup}}(\bar{\varsigma}_I(\varrho_1)), f_{\text{rsup}}(\bar{\varsigma}_I(\varrho_2))$

$$\mathrm{f}_{rinf}(\bar{\varsigma}_F)\,(\varrho_1 * \varrho_2) = \mathrm{rinf}\ \{\bar{\varsigma}_F(\epsilon)/\ \epsilon \in \mathrm{f}^{-1}(\varrho_1 * \varrho_2)\}$$

$$\leq \mathrm{rinf}\,\{(\bar{\varsigma}_F)\ \epsilon_1 * \epsilon_2 / \epsilon_1 \in \mathrm{f}^{-1}(\varrho_1)\ \&\ \epsilon_2 \in \mathrm{f}^{-1}(\varrho_2)\}$$

$$\leq \mathrm{rinf}\{\mathrm{rmax}\{\bar{\varsigma}_F(\epsilon_1),\ \varsigma_F(\epsilon_2)/\ \epsilon_1 \in \mathrm{f}^{-1}(\varrho_1)\ \&\ \epsilon_2 \in \mathrm{f}^{-1}(\varrho_2)\}$$

$$= \mathrm{rmax}\,\{\mathrm{rinf}\,\{\bar{\varsigma}_F(\epsilon_1)/\ \epsilon_1 \in \mathrm{f}^{-1}(\varrho_1), \bar{\varsigma}_F(\epsilon_2)/\ \epsilon_2 \in \mathrm{f}^{-1}(\varrho_2)\}\}$$

$$= \mathrm{rmax}\,\{\mathrm{f}_{\mathrm{rinf}}(\bar{\varsigma}_F(\varrho_1)),\ \mathrm{f}_{\mathrm{rinf}}(\bar{\varsigma}_F(\varrho_2))\}$$

Hence, $\mathrm{f}_{\mathrm{rinf}}(\bar{\varsigma}_F)\,(\varrho_1 * \varrho_2) \leq \mathrm{rmax}\,\{\mathrm{f}_{\mathrm{rinf}}(\bar{\varsigma}_F(\varrho_1)),\ \mathrm{f}_{\mathrm{rinf}}(\bar{\varsigma}_F(\varrho_2))\}$

Theorem 4.3. Suppose $\mathrm{f}: \mathfrak{W} \to \mathfrak{Y}$ is the homomorphism of Ẑ– algebra. If $\bar{\xi} = \{\epsilon: \mathrm{f}\,(\bar{\varsigma}_{T,I,F})(\epsilon)\ /\ \epsilon \in \mathfrak{W}\}$ be an int_val neutrosophic Ẑ– algebra of $\mathfrak{W}$, then its pre-image $of\ \mathrm{f}^{-1}(\bar{\xi}) = \{\epsilon: f^{-1}(\bar{\varsigma}_{T,I,F})/\epsilon \in \mathfrak{W}\}$ of $\bar{\xi}$ under f is also an int_val neutrosophic Ẑ– subalgebra in $\mathfrak{W}$.

Proof:

$$\mathrm{f}^{-1}(\bar{\varsigma}_T)(\epsilon * \varrho) = \bar{\varsigma}_T(\mathrm{f}\,(\epsilon * \varrho))$$

$$= \bar{\varsigma}_T(\mathrm{f}\,(\epsilon) * \mathrm{f}\,(\varrho))$$

$$\geq \mathrm{rmin}\{\bar{\varsigma}_T(\mathrm{f}(\epsilon)), \bar{\varsigma}_T(\mathrm{f}(\varrho)\}$$

$$= \mathrm{rmin}\,\{\mathrm{f}^{-1}(\bar{\varsigma}_T)(\epsilon), \mathrm{f}^{-1}(\bar{\varsigma}_T)\,(\varrho)\}$$

$$\therefore\ \mathrm{f}^{-1}(\bar{\varsigma}_T)(\epsilon * \varrho) \geq \mathrm{rmin}\,\{\mathrm{f}^{-1}(\bar{\varsigma}_T)(\epsilon), \mathrm{f}^{-1}(\bar{\varsigma}_T)(\varrho)\}$$

Similarly, $\mathrm{f}^{-1}(\bar{\varsigma}_I)(\epsilon * \varrho) \geq \mathrm{rmin}\,\{\mathrm{f}^{-1}(\bar{\varsigma}_I)(\epsilon), \mathrm{f}^{-1}(\bar{\varsigma}_I)(\varrho)\}$

$$\mathrm{f}^{-1}(\bar{\varsigma}_F)(\epsilon * \varrho) = \bar{\varsigma}_F(\mathrm{f}\,(\epsilon * \varrho))$$

$$= \bar{\varsigma}_F(\mathrm{f}\,(\epsilon) * \mathrm{f}\,(\varrho))$$

$$\leq rmax\{\bar{\varsigma}_F(\mathrm{f}(\epsilon)), \bar{\varsigma}_F(\mathrm{f}(\varrho))\}$$

$$= \mathrm{rmax}\,\{\mathrm{f}^{-1}(\bar{\varsigma}_F)(\epsilon), \mathrm{f}^{-1}(\bar{\varsigma}_F)(\varrho)\}$$

$$\mathrm{f}^{-1}(\bar{\varsigma}_F)(\epsilon * \varrho) \leq \mathrm{rmax}\,\{\mathrm{f}^{-1}(\bar{\varsigma}_F)(\epsilon), \mathrm{f}^{-1}(\bar{\varsigma}_F)(\varrho)\}$$

$\therefore\ \mathrm{f}^{-1}(\bar{\xi}) = \{\epsilon,\ \mathrm{f}^{-1}(\bar{\varsigma}_{T,I,F})/\epsilon \in \mathfrak{W}\}$ of $\bar{\xi}$ under f is the int_val neutrosophic Ẑ– subalgebra of $\mathfrak{W}$.

Theorem 4.4. If h is a Ẑ-endomorphism of Ẑ –algebra $(\mathfrak{W}, *, 0)$. If $\bar{\xi} = \{\epsilon : \bar{\varsigma}_{T,I,F}\ /\ \epsilon \in \mathfrak{W}\}$ be an int_val neutrosophic Ẑ-subalgebra of $\mathfrak{W}$, then $\bar{\xi}^h = \{\ \epsilon : \bar{\xi}^h_{T,I,F}\ /\ \epsilon \in \mathfrak{W}\}$ is also an int_val neutrosophic Ẑ-subalgebra of $\mathfrak{W}$.

Proof: Given, h be an $\hat{Z}$-endomorphism of $\hat{Z}$ −algebra $(\mathfrak{W}, *, 0)$.

Let $\bar{\xi}$ be the int_val neutrosophic $\hat{Z}$-subalgebra of $\mathfrak{W}$,

Let, $\epsilon, \varrho \in \mathfrak{W}$, then

$$\bar{\xi}_{T^h}(\epsilon * \varrho) = \bar{\xi}_T(\text{h}(\epsilon * \varrho)$$

$$= \bar{\xi}_T(\text{h}(\epsilon) * \text{h}(\varrho))$$

$$\geq \text{rmin}\{\bar{\xi}_T(\text{h}(\epsilon), \bar{\xi}_T(\text{h}(\varrho)\}$$

$$\bar{\xi}_{T^h}(\epsilon * \varrho) \geq \text{rmin}\{\bar{\xi}_{T^h}(\epsilon), \bar{\xi}_{T^h}(\varrho)\}$$

Similarly, $\bar{\xi}_{I^h}(\epsilon * \varrho) \geq \text{rmin}\{\bar{\xi}_{I^h}(\epsilon), \bar{\xi}_{I^h}(\varrho)\}$

$$\bar{\xi}_{F^h}(\epsilon * \varrho) = \bar{\xi}_F(\text{h}(\epsilon * \varrho)$$

$$= \bar{\xi}_F(\text{h}(\epsilon) * \text{h}(\varrho))$$

$$\leq \text{rmax}\{\bar{\xi}_F(\text{h}(\epsilon), \bar{\xi}_F(\text{h}(\varrho)\}$$

$$\bar{\xi}_{F^h}(\epsilon * \varrho) \leq \text{rmax}\{\bar{\xi}_{F^h}(\epsilon), \bar{\xi}_{F^h}(\varrho)\}$$

Hence, $\bar{\xi}^h$ is also an int_val neutrosophic $\hat{Z}$-subalgebra of $\mathfrak{W}$.

Theorem 4.4. Suppose J is the subset of $\mathfrak{W}$. An int_val neutrosophic set $\bar{\xi} = \{\epsilon : \bar{\varsigma}_{T,I,F} \ / \ \epsilon \in \mathfrak{W}\}$ such that $\bar{\varsigma}_{T,I} = \begin{cases} \bar{t} & \epsilon \in J \\ \bar{s} & \epsilon \notin J \end{cases}$, $\bar{\varsigma}_F = \begin{cases} \bar{\alpha} & \epsilon \in J \\ \bar{\beta} & \epsilon \notin J \end{cases}$ where $\bar{t}, \bar{s}, \bar{\alpha}, \bar{\beta} \in \mathfrak{D}\,[0,1]$ with $\bar{t} \geq \bar{s}$, $\bar{\alpha} \leq \bar{\beta}$, Then the int_val neutrosophic set $\bar{\xi} = \{\epsilon : \bar{\varsigma}_T, \bar{\varsigma}_I, \bar{\varsigma}_F \ / \ \epsilon \in \mathfrak{W}\}$ is an int_val neutrosophic of $\hat{Z}$-algebra of $\mathfrak{W}$.

Proof: For $\epsilon, \varrho \in$ J

i) $\bar{\varsigma}_T(\epsilon) = \bar{t} = \bar{\varsigma}_T(\varrho)$

$$\Longrightarrow \bar{\varsigma}_T(\epsilon * \varrho) \geq \text{rmin}\{\bar{\varsigma}_T(\epsilon), \bar{\varsigma}_T(\varrho)\}$$

$$= \text{rmin}\{\bar{t}, \bar{t}\}$$

$$\bar{\varsigma}_T(\epsilon * \varrho) = \bar{t}$$

ii) $\bar{\varsigma}_I(\epsilon) = \bar{t} = \bar{\varsigma}_I(\varrho)$

$$\Longrightarrow \bar{\varsigma}_I(\epsilon * \varrho) \geq \text{rmin}\{\bar{\varsigma}_I(\epsilon), \bar{\varsigma}_I(\varrho)\}$$

$= \text{rmin}\ \{\bar{t}, \bar{t}\}$

$\bar{\varsigma}_I(\epsilon * \varrho) = \bar{t}$

iii) For, ϵ, $\varrho \in$ J

$\bar{\varsigma}_F(\epsilon) = \bar{\alpha} = \bar{\varsigma}_F(\varrho)$

$\Longrightarrow \bar{\varsigma}_F(\epsilon * \varrho) \leq \text{rmax}\ \{\bar{\varsigma}_F(\epsilon), \bar{\varsigma}_F(\varrho)\}$

$= \text{rmax}\ \{\bar{\alpha}, \bar{\alpha}\}$

$\bar{\varsigma}_F(\epsilon * \varrho) = \bar{\alpha}$

Hence, $\bar{\varsigma}_{T,I,F}$ is an int_val neutrosophic Ẑ-algebra of 𝔚.

Theorem 4.5. Let f: 𝔚 → 𝔜 be the homomorphism of Ẑ-algebra. If $\bar{\varsigma}_{T,I,F}$ is the int_val neutrosophic Ẑ-algebra of 𝔚, with the rsup_rsup_rinf property & kerf ⊆ $\mathfrak{W}\bar{\varsigma}_{T,I,F}$ then the image of th set $\bar{\xi}$ ={ϵ : $\bar{\varsigma}_{T,I,F}$ / $\epsilon \in$ 𝔚 } , f($\bar{\varsigma}_{T,I,F}$) is also an int_val neutrosophic Ẑ-algebra of 𝔜.

Proof:

i) Let $f(\epsilon_1) = \varrho_1$, $f(\epsilon_2) = \varrho_2$

$f(\bar{\varsigma}_T)(\varrho_1 * \varrho_2) = \text{rsup}\ \{\bar{\varsigma}_T(\epsilon_1 * \epsilon_2) : \epsilon \in f^{-1}(\varrho_1 * \varrho_2)$

$\geq \text{rsup}\ \{\ \bar{\varsigma}_T(\epsilon_1 * \epsilon_2) : \epsilon_1 \in f^{-1}(\varrho_1)\ \&\ \epsilon_2 \in f^{-1}\ (\varrho_2)\}$

$\geq \text{rsup}\ \{\text{rmin}\{\ \bar{\varsigma}_T(\epsilon_1),\ \bar{\varsigma}_T\ (\epsilon_2)\},\ \epsilon_1 \in f^{-1}(\varrho_1)\ \&\epsilon_2 \in f^{-1}(\ \varrho_2)\}$

$\geq \text{rmin}\ \{\text{rsup}\ \{\bar{\varsigma}_T(\epsilon_1) \colon \epsilon_1 \in f^{-1}(\varrho_1)\}, \text{rsup}\ \{\bar{\varsigma}_T\ (\epsilon_2) \colon \&\ \epsilon_2 \in f^{-1}\ (\varrho_2)\}\}$

$= \text{rmin}\ \{\ \underset{\epsilon_1 \in f^{-1}(\varrho)}{\text{rsup}}\ \{\bar{\varsigma}_T(\epsilon_1)\},\ \underset{\epsilon_2 \in f^{-1}(\varrho)}{\text{rsup}}\ \{\bar{\varsigma}_T(\epsilon_2)\}\}$

$= \text{rmin}\{\ f(\bar{\varsigma}_T)(\varrho_1), f(\bar{\varsigma}_T)(\varrho_2)\}$

ii) Similarly, $f\ (\bar{\varsigma}_I)(\varrho_1 * \varrho_2) \geq \text{rmin}\{\ f(\bar{\varsigma}_I)(\varrho_1), f(\bar{\varsigma}_I)(\varrho_2)\}$

iii) Let $f\ (\epsilon_1) = \varrho_1$, $f\ (\epsilon_2) = \varrho_2$

$f\ (\bar{\varsigma}_F)(\varrho_1 * \varrho_2) = \text{rinf}\ \{\bar{\varsigma}_F(\epsilon_1 * \epsilon_2) : \epsilon \in f^{-1}(\varrho_1 * \varrho_2)$

$\leq \text{rinf}\ \{\ \bar{\varsigma}_F(\epsilon_1 * \epsilon_2) : \epsilon_1 \in f^{-1}(\varrho_1)\ \&\ \epsilon_2 \in f^{-1}\ (\varrho_2)\}$

$\leq \text{rinf}\ \{\text{rmax}\{\ \bar{\varsigma}_F(\epsilon_1),\ \bar{\varsigma}_T\ (\epsilon_2)\},\ \epsilon_1 \in f^{-1}(\varrho_1)\ \&\ \epsilon_2 \in f^{-1}(\ \varrho_2)\}$

$\leq$rmax {rinf {$\bar{\varsigma}_F(\epsilon_1)$: $\epsilon_1 \in$ f$^{-1}(\varrho_1)$}, rinf {$\bar{\varsigma}_F$ (ϵ_2): & $\epsilon_2 \in$ f^{-1} (ϱ_2)}}

=rmax { $\underset{\epsilon_1 \in f^{-1}(\varrho)}{\text{rinf}}\{\bar{\varsigma}_F(\epsilon_1)\}$, $\underset{\epsilon_2 \in f^{-1}(\varrho)}{\text{rinf}}\{\bar{\varsigma}_F(\epsilon_2)\}$}

=rmax{f$(\bar{\varsigma}_F)(\varrho_1)$,f$(\bar{\varsigma}_F)(\varrho_2)$}

Hence, f $(\bar{\varsigma}_{T,I,F})$ is an int_val neutrosophic Ẑ-algebra of $\mathfrak{Y}$.

5. Product of Interval-valued(int_val**) neutrosophic Ẑ-algebra**

The section that follows the cartesian product of two int_val neutrosophic Ẑ-algebras $\bar{\xi} \times \bar{\zeta}$ of $\mathfrak{W}$ & $\mathfrak{Y}$ respectively.

Definition5.1. Let $\bar{\xi} = \{\epsilon, \bar{\varsigma}_{T,I,F}(\epsilon) / \epsilon \in \mathfrak{W}\}$ and $\bar{\zeta} = \{\varrho, \bar{\varsigma}_{T,I,F}(\varrho) / \varrho \in \mathfrak{Y}\}$ be two int_val neutrosophic sets of $\mathfrak{W}$ & $\mathfrak{Y}$ respectively. Then the cartesian product of $\bar{\xi}$ & $\bar{\zeta}$ is referred as $\bar{\xi}$ x $\bar{\zeta}$ then it is defined to be $\bar{\xi}$x $\bar{\zeta}$ = {(ϵ, ϱ), $\bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}(\epsilon,\varrho), \bar{\varsigma}_{I_{\bar{\xi}\times\bar{\zeta}}}(\epsilon,\varrho), \bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}(\epsilon,\varrho)/(\epsilon \times \varrho) \in \bar{\xi} \times \bar{\zeta}$} where $\bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}: \epsilon \times \varrho \to \mathfrak{D}$ [0,1] ; $\bar{\varsigma}_{I_{\bar{\xi}\times\bar{\zeta}}}: \epsilon \times \varrho \to \mathfrak{D}$ [0,1] and $\bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}: \epsilon \times \varrho \to \mathfrak{D}$ [0,1]. $\bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}$ = rmin {$\bar{\varsigma}_{T_{\bar{\xi}}}(\epsilon)$, $\bar{\varsigma}_{T_{\bar{\zeta}}}(\varrho)$}; $\bar{\varsigma}_{I_{\bar{\xi}\times\bar{\zeta}}}$ = rmin {$\bar{\varsigma}_{I_{\bar{\xi}}}(\epsilon)$, $\bar{\varsigma}_{I_{\bar{\zeta}}}(\varrho)$}; $\bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}$ = rmax {$\bar{\varsigma}_{F_{\bar{\xi}}}(\epsilon)$, $\bar{\varsigma}_{F_{\bar{\zeta}}}(\varrho)$}.

Theorem 5.2. If $\bar{\xi}$ and $\bar{\zeta}$ be two int_val neutrosophic Ẑ-algebra of $\mathfrak{W}$ &$\mathfrak{Y}$ respectively, then $\bar{\xi} \times \bar{\zeta}$ is an int_val neutrosophic Ẑ-algebra of $\mathfrak{W}$ & $\mathfrak{Y}$.

Proof: Let $\bar{\xi}$={ϵ, $\bar{\varsigma}_{T_{\bar{\xi}}}(\epsilon)$, $\bar{\varsigma}_{I_{\bar{\xi}}}(\epsilon)$, $\bar{\varsigma}_{F_{\bar{\xi}}}(\epsilon)/\epsilon \in \mathfrak{W}$ } & $\bar{\zeta}$ = { ϱ, $\bar{\varsigma}_{T_{\bar{\zeta}}}(\varrho)$, $\bar{\varsigma}_{I_{\bar{\zeta}}}(\varrho)$, $\bar{\varsigma}_{F_{\bar{\zeta}}}(\varrho)/\varrho \in \mathfrak{Y}$ } be two int_val neutrosophic sets of $\mathfrak{W}$ &$\mathfrak{Y}$.

Take $\epsilon = (\epsilon_1, \varrho_1)$ and $\varrho = (\epsilon_2, \varrho_2)$

i) $\bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}(\epsilon * \varrho) = \bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}((\epsilon_1, \varrho_1) * (\epsilon_2, \varrho_2))$

$= \bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}((\epsilon_1 * \varrho_1), (\epsilon_2 * \varrho_2))$

= rmin{$\bar{\varsigma}_{T_{\bar{\xi}}}(\epsilon_1 * \varrho_1)$, $\bar{\varsigma}_{T_{\bar{\zeta}}}(\epsilon_2 * \varrho_2)$}

$\geq$ rmin {rmin {$\bar{\varsigma}_{T_{\bar{\xi}}}(\epsilon_1)$, $\bar{\varsigma}_{T_{\bar{\xi}}}(\epsilon_2)$,rmin{$\bar{\varsigma}_{T_{\bar{\zeta}}}(\varrho_1)$, $\bar{\varsigma}_{T_{\bar{\zeta}}}(\varrho_2)$}}

=rmin{rmin($\bar{\varsigma}_{T_{\bar{\xi}}}(\epsilon_1)$, $\bar{\varsigma}_{T_{\bar{\zeta}}}(\varrho_1)$},rmin{$\bar{\varsigma}_{T_{\bar{\xi}}}(\epsilon_2)$, $\bar{\varsigma}_{T_{\bar{\zeta}}}(\varrho_2)$}}

$=\text{rmin}\{\bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}(\epsilon),\ \bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}(\varrho)\}$

$\geq\text{rmin}\ \{\bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}(\epsilon), \bar{\varsigma}_{T_{\bar{\xi}\times\bar{\zeta}}}(\varrho)\}$

ii) Similarly, $\bar{\varsigma}_{I_{\bar{\xi}\times\bar{\zeta}}}(\epsilon * \varrho) \geq\text{rmin}\ \{\bar{\varsigma}_{I_{\bar{\xi}\times\bar{\zeta}}}(\epsilon), \bar{\varsigma}_{I_{\bar{\xi}\times\bar{\zeta}}}(\varrho)\}$

iii) $\bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}(\epsilon * \varrho) = \bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}((\epsilon_1, \varrho_1) * (\epsilon_2, \varrho_2))$

$= \bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}((\epsilon_1 * \varrho_1), (\epsilon_2 * \varrho_2))$

$= \text{rmax}\{\bar{\varsigma}_{F_{\bar{\xi}}}(\epsilon_1 * \varrho_1),\ \bar{\varsigma}_{F_{\bar{\zeta}}}(\epsilon_2 * \varrho_2)\}$

$\leq\text{rmax}\ \{\text{rmax}\ \{\bar{\varsigma}_{F_{\bar{\xi}}}(\epsilon_1),\ \bar{\varsigma}_{F_{\bar{\xi}}}(\epsilon_2), \text{rmax}\{\bar{\varsigma}_{F_{\bar{\zeta}}}(\varrho_1),\ \bar{\varsigma}_{F_{\bar{\zeta}}}(\varrho_2)\}\}$

$=\text{rmax}\{\text{rmin}(\bar{\varsigma}_{F_{\bar{\xi}}}(\epsilon_1),\ \bar{\varsigma}_{F_{\bar{\zeta}}}(\varrho_1)\}, \text{rmax}\{\bar{\varsigma}_{F_{\bar{\xi}}}(\epsilon_2),\ \bar{\varsigma}_{F_{\bar{\zeta}}}(\varrho_2)\}\}$

$=\text{rmax}\ \{\bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}(\epsilon),\ \bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}(\varrho)\}$

$\leq \text{rmax}\ \{\bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}(\epsilon), \bar{\varsigma}_{F_{\bar{\xi}\times\bar{\zeta}}}(\varrho)\}$

Hence, $\bar{\xi} \times \bar{\zeta}$ is an int_val neutrosophic Ẑ-algebra of $\mathfrak{W}$ & $\mathfrak{Y}$.

Theorem 5.3. If $\bar{\xi}_i = \{\epsilon \in \mathfrak{W}_i / \bar{\varsigma}_{T_{\bar{\xi}_i}}(\epsilon),\ \bar{\varsigma}_{I_{\bar{\xi}_i}}(\epsilon), \bar{\varsigma}_{F_{\bar{\xi}_i}}(\epsilon)\}$ be an int_val neutrosophic Ẑ-algebra of $\mathfrak{W}_i$ respectively, then $\prod_{i=1}^{n} \bar{\xi}_i$ is also an int_val neutrosophic Ẑ-algebra of $\prod_{i=1}^{n} \mathfrak{W}_i$.

Proof:

The Induction process on theorem 5.2.

i) $$\prod_{i=1}^{n} \bar{\varsigma}_{T_{\bar{\xi}_i}}(\epsilon_i * \varrho_i) \geq \text{rmin}\ \{\prod_{i=1}^{n} \bar{\varsigma}_{T_{\bar{\xi}_i}}(\epsilon_i), \prod_{i=1}^{n} \bar{\varsigma}_{T_{\bar{\xi}_i}}(\varrho_i)\}$$

ii) Similarly, $$\prod_{i=1}^{n} \bar{\varsigma}_{I_{\bar{\xi}_i}}(\epsilon_i) \geq\text{rmin}\ \{\prod_{i=1}^{n} \bar{\varsigma}_{I_{\bar{\xi}_i}}(\epsilon_i), \prod_{i=1}^{n} \bar{\varsigma}_{I_{\bar{\xi}_i}}(\varrho_i)\}$$

iii) $$\prod_{i=1}^{n} \bar{\varsigma}_{F_{\bar{\xi}_i}}(\epsilon_i * \varrho_i) \leq \text{rmax}\ \{\prod_{i=1}^{n} \bar{\varsigma}_{F_{\bar{\xi}_i}}(\epsilon_i), \prod_{i=1}^{n} \bar{\varsigma}_{F_{\bar{\xi}_i}}(\varrho_i)\}$$

5. Conclusions

The application of Int val neutrosophic Z-algebra marks a significant advancement in dealing with uncertainty and indeterminate information within various domains. Through its incorporation of interval-valued neutrosophic sets, int val neutrosophic Z-algebra provides a flexible framework for representing and manipulating information that encompasses not only truth and falsity but also the degree of indeterminacy present in real-world scenarios. This work deals about int_val neutrosophic in $\hat{Z}$-algebra using a binary operation $*$ and some of its properties and algebraic structures are also presented. In future, this work may extend to any type of algebra in many ways. This will be used in multiple types of fuzzy sets and their different extensions like int_val intuitionistic neutrosophic $\hat{Z}$ -algebra, cubic neutrosophic in $\hat{Z}$ -algebra.

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