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Heptagonal Neutrosophic Topology

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Abstract. This article aims at developing the concept of heptagonal neutrosophic topology using heptagonal neutrosophic numbers. The heptagonal neutrosophic union,intersection and complement defined to compare the HNN. The interior, closure, exterior and boundary have introduced to discuss the properties of heptagonal neutrosophic topology and investigated to clarify the new concept and new possibilities in Heptagonal Neutrosophic Topological Space.

Keywords: Neutrosophic set, Heptagonal neutrosophic topology, neutrosophic interior, neutrosophic closure, neutrosophic exterior and neutrosophic boundary

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1. Introduction

Neutrosophic topological spaces have applications in various fields such as decision-making, computer science, and engineering, where the presence of indeterminate, vague, or uncertain information is prevalent. They provide a powerful tool for modeling and analyzing complex systems where classical topological spaces may not be sufficient. Subsequently after Zadeh's [22] introduction of the fuzzy set in the year 1965 with the membership function, the aforesaid fields are developed in various phases with many real life situations. The investigator focused their research in the above fields towards applications in practical problems with the help of intuitionistic fuzzy numbers with membership and non-membership values which was developed by Atanassov.K.T [8] in 1986.

There was a new finding between membership and non-membership values called indeterminacy and combined three values named as neutrosophic numbers which was introduced by Smarandache in 2005 [20]. After the introduction of neutrosophic numbers, investigators employ the concept of neutrosophic numbers and applied in various real life situations exclusively in topological spaces. Consequently, the neutrosophic topological spaces has been introduced by Salama.A.A and Alblowi.S.A in 2012 [4]. Lupia'nez [11–13] applied the neutrosophic concepts in topological spaces and developed a new research dimension in neutrosophic topological spaces.

The neutrosophic numbers from triangular to hexagonal have been published and have been documented their usage in actual life [17, 18]. In recent times (2021) Ali Hamza, Sara Farooq and Muhammad Rafaqat [7] presented Triangular neutrosophic topology. The topologies generated by triangular neutrosophic numbers were introduced by Kungumaraj.E and Narmatha.S [10] in 2022. In this article the extension work of [7] has been done and some of their properties have been investigated. This topological approach will be applied in network analysis, MCDM, image processing and topology optimization process.

This article incorporates five sections. The first section embraces the brief introduction, the second part encircles the preliminary definition and the results which are used in this article, the third section engrosses the main findings of Heptagonal Topological spaces and their properties, the fourth division comprehends the applications of third section which implies the continuous function and their properties of Heptagonal topological spaces. Finally the conclusion part contributes to expound the follow up work of this heptagonal topological space and applications of the same.

2. Preliminaries

Definition 2.1. Let X be a universe of discourse, A_N is a set disclosed in X. An element x from X is noted with respect to neutrosophic set as

$$A_N=\{< x; (\rho(x), \sigma(x), \omega(x)) >: x \in X\}$$

Where $\rho(x)$ is degree of truth membership, $\sigma(x)$ is degree of indeterminacy membership, $\omega(x)$ is degree of falsity membership. And $\rho(x), \sigma(x), \omega(x)$ are real standard or non standard subsets of $]0^-, 1^+[$. That is, There is no restrictions on the sum of $\rho(x), \sigma(x), \omega(x)$.

Definition 2.2. Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point S in S, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.
When S is continuous, a SVNS A can be written as A $=\int < T(x), I(x), F(x) > /x \in S$.
When S is discrete, a SVNS A can be written as A $=< T(x_i), I(x_i), F(x_i) > /x_i \in S$.

Definition 2.3. A Neutrosophic subset $\tilde{A}^N = (x, \mu_{\tilde{A}^N}(x), \nu_{\tilde{A}^N}(x), \vartheta_{\tilde{A}^N}(x)); x \in X$ of the real line R is called Neutrosophic number if the following conditions holds:
(i) There exist $x \in R$ such that $\mu_{\tilde{A}^N}(x) = 1$ and $\vartheta_{\tilde{A}^N}(x) = 0$

(ii) $\mu_{\tilde{A}^N}(x)$ is continuous function from $R \to [0,1]$ such that $0 \le \mu_{\tilde{A}^N}(x)+\nu_{\tilde{A}^N}(x)+\vartheta_{\tilde{A}^I}(x) \le 3$ for all $x \in X$

Definition 2.4. A Triangular Neutrosophic number $\tilde{A}^N$ is an Neutrosophic set in R with the following membership function $\mu_{\tilde{A}^N}(x)$, indetermiancy function$\nu_{\tilde{A}^N}(x)$ and non-membership function $\vartheta_{\tilde{A}^N}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, \ if \ a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, \ if \ a_2 \le x \le a_3 \\ 0, \ \text{otherwise} \end{cases} \qquad \nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^{"}}, \ if \ a_1^{'} \le x \le a_2 \\ \frac{x-a_2}{a_3^{"}-a_2}, \ if \ a_2 \le x \le a_3^{'} \\ 1, \ \text{otherwise} \end{cases} \qquad \vartheta_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^{'}}, \ if \ a_1^{'} \le x \le a_2 \\ \frac{x-a_2}{a_3^{'}-a_2}, \ if \ a_2 \le x \le a_3^{'} \\ 1, \ \text{otherwise} \end{cases}$$

where $a_1^{"} \le a_1{'} \le a_1 \le a_2 \le a_3 \le a_3^{'} \le_3$ " and $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$, or $\mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x)$, for all $x \in R$. This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3; a_1{'}, a_2, a_3{'})$.

Definition 2.5. Let (X,Y,$<,>$) be a dual pair, a dual topology on X is a locally convex topology τ so that

$$(X,Y)' \simeq Y$$

Here (X,Y)' denotes the continuous dual of (X,τ) and (X,Y)' $\simeq$ Y means that there is a linear isomorphism.

$$\Psi: Y \longrightarrow (X,Y)'.$$

Definition 2.6. Let $\tau \subseteq N(X)$ then τ is a neutrosophic topology on X if it satisfies the following conditions:

- $X, \phi \in \tau$
- The union and intersection of any number of neutrosophic sets in τ belongs to τ

The pair (X, τ) mentioned as neutrosophic topological space over X.

Definition 2.7. Let $\tau \subseteq N(X)$ be neutrosophic topological space over X then,

- ϕ and X as neutrosophic closed sets over X.
- The union and intersection of any two neutrosophic closed sets is a neutrosophic closed sets over X.

Definition 2.8. A heptagonal neutrosophic number S is defined and described as

$$S =< [(p, q, r, s, t, u, v); \mu], [(p', q', r', s', t', u', v'); \gamma], [(p'', q'', r'', s'', t'', u'', v''); \eta] >$$

where $\mu, \gamma, \eta \in [0, 1]$.The truth membership function ρ : R $\Rightarrow [0, \mu]$, the indeterminacy membership function σ : R$\Rightarrow [\gamma, 1]$, the falsity membership function ω : R $\Rightarrow [\eta, 1]$. Using ranking technique of heptagonal neutrosophic number is changed as,

$$\rho(x) = \frac{(p+q+r+s+t+u+v)}{7}$$
$$\sigma(x) = \frac{(p'+q'+r'+s'+t'+u'+v')}{7}$$
$$\omega(x) = \frac{(p''+q''+r''+s''+t''+u''+v'')}{7}$$

Heptagonal Neutrosophic Number Operations

(i) Inclusive: Let X be a non-empty set and A_{HN} and $B_{HN}$are NS of the form A_{HN}=$< x; \rho_{A_{HN}}(x), \sigma_{A_{HN}}(x), \omega_{A_{HN}}(x) >$, B_{HN}=$< x; \rho_{B_{HN}}(x), \sigma_{B_{HN}}(x), \omega_{_{HN}}(x) >$. Then their subsets may be defines as follows,

- $A_{HN} \subseteq B_{HN} \Rightarrow \rho_{A_{HN}}(x) \leq \rho_{B_{HN}}(x); \sigma_{A_{HN}}(x) \geq \sigma_{B_{HN}}(x); \omega_{A_{HN}}(x) \geq \omega_{B_{HN}}(x) \forall x \in$X.
- $B_{HN} \subseteq A_{HN} \Rightarrow \rho_{B_{HN}}(x) \leq \rho_{A_{HN}}(x); \sigma_{B_{HN}}(x) \geq \sigma_{A_{HN}}(x); \omega_{B_{HN}}(x) \geq \omega_{A_{HN}}(x) \forall x \in$X.

(ii)Equality: If $A_{HN} \subseteq B_{HN}$ and $B_{HN} \subseteq A_{HN}$ then A_{HN}=B_{HN} is called Equality of a neutrosophic sets.

(iii)Union and Intersection: Let X be a non empty set and A_{HN} and B_{HN} are in NS of the form A_{HN}=$< x; \rho_{A_{HN}}(x), \sigma_{A_{HN}}(x), \omega_{A_{HN}}(x) >$, B_{HN}=$< x; \rho_{B_{HN}}(x), \sigma_{B_{HN}}(x), \omega_{_{HN}}(x) >$, then $A_{HN} \cup B_{HN}$ and $A_{HN} \cap B_{HN}$ is defined as follows,

- $A_{HN} \cup B_{HN}$=$\{< x; (\rho_{A_{HN}}(x) \vee \rho_{B_{HN}}(x); \sigma_{A_{HN}}(x) \wedge \sigma_{B_{HN}}(x); \omega_{A_{HN}}(x) \wedge \omega_{B_{HN}}(x)) >: x \in X\}$
- $A_{HN} \cap B_{HN}$=$\{< x; (\rho_{A_{HN}}(x) \wedge \rho_{B_{HN}}(x); \sigma_{A_{HN}}(x) \vee \sigma_{B_{HN}}(x); \omega_{A_{HN}}(x) \vee \omega_{B_{HN}}(x)) >: x \in X\}$

(iv)Complement: Let A_{HN}=$< x; \rho_{A_{HN}}(x), \sigma_{A_{HN}}(x), \omega_{A_{HN}}(x) >$ in NS and complement of A^C_{HN} is defined as:

$$A^C_{HN} = \{< x; (\rho(x), \sigma(x), \omega(x)) >: x \in X\}$$

(v)Universal and Empty set: Let A_{HN}=$< x; \rho_{A_{HN}}(x), \sigma_{A_{HN}}(x), \omega_{A_{HN}}(x) >$ in NS and universal setI_A and empty set O_A of A_{HN} is defined as:

- I_A=$\{< x; 1, 0, 0 >: x \in X\}$
- O_A=$\{< x; 0, 1, 1 >: x \in X\}$

Example 2.9. Let A_{HN}, B_{HN} and C_{HN} are HNN and defined as follows,

$$A_{HN} = \{< x; (0{,}72, 0{,}41, 0{,}35, 0{,}81, 0{,}77, 0{,}73, 0{,}77), (0{,}83, 0{,}88, 0{,}93, 0{,}99, 0{,}96, 0{,}90, 0{,}94), (0{,}86, 0{,}99, 0{,}97, 0{,}93, 0{,}94, 0{,}91, 0{,}86) >, < y; (0{,}91, 0{,}32, 0{,}56, 0{,}48, 0{,}81, 0{,}72, 0{,}67), (0{,}78, 0{,}83, 0{,}21, 0{,}38, 0{,}56, 0{,}33, 0{,}98), (0{,}36, 0{,}86, 0{,}96, 0{,}32, 0{,}44, 0{,}56, 0{,}72) >\}$$

$$B_{HN} = \{< x; (0{,}96, 0{,}65, 0{,}73, 0{,}75, 0{,}83, 0{,}56, 0{,}54), (0{,}75, 0{,}95, 0{,}45, 0{,}38, 0{,}79, 0{,}57, 0{,}13), (0{,}59, 0{,}36, 0{,}68, 0{,}47, 0{,}36, 0{,}95, 0{,}44) >, < y; (0{,}38, 0{,}69, 0{,}88, 0{,}98, 0{,}77, 0{,}36, 0{,}98), (0{,}32, 0{,}72, 0{,}42, 0{,}62, 0{,}90, 0{,}22, 0{,}62), (0{,}42, 0{,}52, 0{,}62, 0 = 72, 0{,}36, 0{,}72, 0{,}61) >\}$$

$$C_{HN} = \{< x; (0{,}73, 0{,}74, 0{,}96, 0{,}34, 0{,}85, 0{,}89, 0{,}64), (0{,}46, 0{,}35, 0{,}25, 0{,}96, 0{,}36, 0{,}56, 0{,}16), (0{,}84, 0{,}85, 0{,}37, 0{,}57, 0{,}67, 0{,}22, 0{,}10) >, < y; (0{,}76, 0{,}72, 0{,}78, 0{,}62, 0{,}92, 0{,}56, 0{,}88), (0{,}38, 0{,}98, 0{,}22, 0{,}32, 0{,}54, 0{,}64, 0{,}31), (0{,}86, 0{,}96, 0{,}52, 0{,}22, 0{,}41, 0{,}51, 0{,}32) >\}$$

Using ranking technique by definition 2.4, We get

$$A_{HN} = \{< x; (0{,}65, 0{,}92, 0{,}92) >, < y; (0{,}64, 0{,}58, 0{,}60) >\}$$
$$B_{HN} = \{< x; (0{,}72, 0{,}57, 0{,}55) >, < y; (0{,}72, 0{,}54, 0{,}57) >\}$$
$$C_{HN} = \{< x; (0{,}74, 0{,}44, 0{,}52) >, < y; (0{,}75, 0{,}48, 0{,}53) >\}$$

From definition 2.4 we have,

$$\text{(i)} A_{HN} \subseteq B_{HN};\ B_{HN} \subseteq C_{HN} \Rightarrow A_{HN} \subseteq C_{HN}$$

$$A_{HN} \cup B_{HN} = \{< x; (0{,}65 \vee 0{,}72, 0{,}92 \wedge 0{,}57, 0{,}92 \wedge 0{,}55) >, < y; (0{,}64 \vee 0{,}72, 0{,}58 \wedge 0{,}54, 0{,}60 \wedge 0{,}57) >\}$$

$$A_{HN} \cup B_{HN} = \{< x; (0{,}72, 0{,}57, 0{,}55) >, < y; (0{,}72, 0{,}54, 0{,}57) >\}$$

Similarly,

$$B_{HN} \cup C_{HN} = \{< x; (0{,}74, 0{,}44, 0{,}52) >, < y; (0{,}75, 0{,}48, 0{,}53) >\}$$
$$A_{HN} \cup C_{HN} = \{< x; (0{,}74, 0{,}44, 0{,}52) >, < y; (0{,}75, 0{,}48, 0{,}53) >\}$$

$$(ii) A_{HN} \cap B_{HN} = \{< x; (0{,}65 \wedge 0{,}72, 0{,}92 \vee 0{,}57, 0{,}92 \vee 0{,}55) >, < y; (0{,}64 \wedge 0{,}72, 0{,}58 \vee 0{,}54, 0{,}60 \vee 0{,}57) >\}$$

$$A_{HN} \cap B_{HN} = \{< x; (0{,}65, 0{,}92, 0{,}92) >, < y; (0{,}64, 0{,}58, 0{,}60) >\}$$

Similarly,

$$B_{HN} \cap C_{HN} = \{< x; (0{,}72, 0{,}57, 0{,}55) >, < y; (0{,}72, 0{,}54, 0{,}57) >\}$$
$$A_{HN} \cap C_{HN} = \{< x; (0{,}65, 0{,}92, 0{,}92) >, < y; (0{,}64, 0{,}58, 0{,}60) >\}$$

$$(iii) A^C_{HN} = \{< x; (0{,}65, 1 - 0{,}92, 0{,}92) >, < y; (0{,}64, 1 - 0{,}58, 0{,}60) >\}$$
$$A^C_{HN} = \{< x; (0{,}92, 0{,}08, 0{,}65) >, < y; (0{,}60, 0{,}42, 0{,}64) >\}$$

Similarly

$$B^C_{HN} = \{< x; (0{,}55, 0{,}43, 0{,}72) >, < y; (0{,}57, 0{,}46, 0{,}72) >\}$$
$$C^C_{HN} = \{< x; (0{,}52, 0{,}56, 0{,}74) >, < y; (0{,}53, 0{,}52, 0{,}75) >\}$$

Theorem 2.10. *Let A_{HN},$B_{HN} \in N(X)$, then the following results are true*

1. $A_{HN} \cap A_{HN} = A_{HN}$ *and* $A_{HN} \cup A_{HN} = A_{HN}$
2. $A_{HN} \cap B_{HN} = B_{HN} \cap A_{HN}$ *and* $B_{HN} \cup A_{HN} = A_{HN} \cup B_{HN}$
3. $A_{HN} \cap \phi = \phi$ *and* $A_{HN} \cap X = A_{HN}$
4. $A_{HN} \cup \phi = A_{HN}$ *and* $A_{HN} \cup X = X$
5. $A_{HN} \cap (B_{HN} \cap C_{HN}) = (A_{HN} \cap B_{HN}) \cap C_{HN}$
6. $A_{HN} \cup (B_{HN} \cup C_{HN}) = (A_{HN} \cup B_{HN}) \cup C_{HN}$
7. $A_{HN} \cap (B_{HN} \cup C_{HN}) = (A_{HN} \cap B_{HN}) \cup (A_{HN} \cap C_{HN})$
8. $A_{HN} \cup (B_{HN} \cap C_{HN}) = (A_{HN} \cup B_{HN}) \cap (A_{HN} \cup C_{HN})$
9. $(A^C_{HN})^C = A_{HN}$
10. $A_{HN} \cup A^C_{HN} = X$ *and* $A_{HN} \cap A^C_{HN} = \phi$.

Proof: The results are obvious by the properties of HNN sets.

Theorem 2.11. *Let A_{HN},$B_{HN} \in N(X)$. Then*

1. $(\cup_{i \in I} A_{HN_i})^C = \cap_{i \in I} A^C_{HN_i}$
2. $(\cap_{i \in I} A_{HN_i})^C = \cup_{i \in I} A^C_{HN_i}$

Proof: (i)First verify $(\cup_{i \in I} A_{HN_i})^C \subseteq \cap_{i \in I} A^C_{HN_i}$. Let a $\in (\cup_{i \in I} A_{HN_i})^C$. Thus $a \notin \cup_{i \in I} A_{HN_i}$, so a cannot be in any of the sets A_{HN_i} i.e., for all i $\in$ I, we have $a \notin A_{HN_i}$, hence $a \in A^C_{HN_i}$ for all i $\in$ I. Thus $a \in \cap_{i \in I} A^C_{HN_i}$.Therefore, $(\cup_{i \in I} A_{HN_i})^C \subseteq \cap_{i \in I} A^C_{HN_i}$.
(ii)Now verify $\cap_{i \in I} A^C_{HN_i} \subseteq (\cup_{i \in I} A_{HN_i})^C$. Let $a \in \cap_{i \in I} A^C_{HN_i}$. Thus $a \in A^C_{HN_i}$ for all i$\in$I, hence $a \notin A_{HN_i}$ for all i$\in$I, so $a \notin \cup_{i \in I} A_{HN_i}$, hence $a \in (\cup_{i \in I} A_{HN_i})^C$.Therefore, $\cap_{i \in I} A^C_{HN_i} \subseteq (\cup_{i \in I} A_{HN_i})^C$.

Therefore, $(\cup_{i \in I} A_{HN_i})^C = (\cap_{i \in I} A_{HN_i})^C$.

Theorem 2.12. *Let A_{HN},$B_{HN} \in N(X)$. Then*

1. $B_{HN} \cap (\cup_{i\in I} A_{HN_i})$=$\cup_{i\in I}(B_{HN} \cap A_{HN_i})$
2. $B_{HN} \cup (\cap_{i\in I} A_{HN_i})$=$\cap_{i\in I}(B_{HN} \cup A_{HN_i})$

Proof: (i)Firstly we verify $B_{HN}\cap(\cup_{i\in I} A_{HN_i}) \subseteq \cup_{i\in I}(B_{HN}\cap A_{HN_i})$. If x $\in B_{HN}\cap(\cup_{i\in I} A_{HN_i})$, then x $\in B_{HN}$ and $x \in \cup_{i\in I} A_{HN_i}$. Then x $\in A_{HN_i}$ for some i∈I. Thus, $x \in B_{HN} \cap A_{HN_i}$. Hence, $x \in \cup_{i\in I}(B_{HN} \cap A_{HN_i})$. Therefore, $B_{HN} \cap (\cup_{i\in I} A_{HN_i}) \subseteq \cup_{i\in I}(B_{HN} \cap A_{HN_i})$.
(ii)Now verifying, $\cup_{i\in I}(B_{HN} \cap A_{HN_i}) \subseteq B_{HN} \cap (\cup_{i\in I} A_{HN_i})$. If $x \in \cup_{i\in I}(B_{HN} \cap A_{HN_i})$, then $x \in B_{HN} \cap A_{HN_i}$ for some i∈I. It follows that x $\in B_{HN}$ and x $\in \cup_{i\in I} A_{HN_i}$. Consequently, x $\in B_{HN} \cap (\cup_{i\in I} A_{HN_i})$. Therefore, $\cup_{i\in I}(B_{HN} \cap A_{HN_i}) \subseteq B_{HN} \cap (\cup_{i\in I} A_{HN_i})$. Therefore, $B_{HN} \cap (\cup_{i\in I} A_{HN_i})$=$\cup_{i\in I}(B_{HN} \cap A_{HN_i})$.

3. Heptagonal Neutrosophic topology and its Properties

Definition 3.1. Let X be a set. Let N(x) be a neutrosophic topology, τ be the collection of subsets of N(X) of X, then τ is a heptagonal neutrosophic topology on X, if it satisfy the following conditions;

- N(X) and $\phi \in \tau$
- Union of arbitrarily many elements of τ is an element of τ.
- Intersection of finite elements of τ is an element of τ.

Therefore the pair (X, τ) is a heptagonal neutrosophic topological space over X.
The set in τ are called HN - open set of X. The complement of HN - open set is called HN - closed set.

Example 3.2. Let X=$\{x, y\}$ and $A_{HN} \in$ N(X) then,

$$A_{HN} = \{< x; (0{,}72, 0{,}41, 0{,}35, 0{,}81, 0{,}77, 0{,}73, 0{,}77), (0{,}83, 0{,}88, 0{,}93, 0{,}99, 0{,}96, 0{,}90, 0{,}94), (0{,}86, 0{,}99, 0{,}97, 0{,}93, 0{,}94, 0{,}91, 0{,}86) >, < y; (0{,}91, 0{,}32, 0{,}56, 0{,}48, 0{,}81, 0{,}72, 0{,}67), (0{,}78, 0{,}83, 0{,}21, 0{,}38, 0{,}56, 0{,}33, 0{,}98), (0{,}36, 0{,}86, 0{,}96, 0{,}32, 0{,}44, 0{,}56, 0{,}72) >\}$$

By definition 2.4: We get
A_{HN}=$\{< x; (0{,}65, 0{,}92, 0{,}92) >, < y; (0{,}64, 0{,}58, 0{,}60) >\}$
Hence, τ=$\{\phi, X, A_{HN}\}$ is a heptagonal neutrosophic topology on X.

Example 3.3. Let X={x, y} and B_{HN}, C_{HN} ∈N(X) then,

B_{HN} ={$< x$; (0,96, 0,65, 0,73, 0,75, 0,83, 0,56, 0,54), (0,75, 0,95, 0,45, 0,38, 0,79, 0,57, 0,13), (0,59, 0,36, 0,68, 0,47, 0,36, 0,95, 0,44) $>, < y$; (0,38, 0,69, 0,88, 0,98, 0,77, 0,36, 0,98), (0,32, 0,72, 0,42, 0,62, 0,90, 0,22, 0,62), (0,42, 0,52, 0,62, 0 = 72, 0,36, 0,72, 0,61) $>$}

C_{HN} ={$< x$; (0,73, 0,74, 0,96, 0,34, 0,85, 0,89, 0,64), (0,46, 0,35, 0,25, 0,96, 0,36, 0,56, 0,16), (0,84, 0,85, 0,37, 0,57, 0,67, 0,22, 0,10) $>, < y$; (0,76, 0,72, 0,78, 0,62, 0,92, 0,56, 0,88), (0,38, 0,98, 0,22, 0,32, 0,54, 0,64, 0,31), (0,86, 0,96, 0,52, 0,22, 0,41, 0,51, 0,32) $>$}

By definition 2.4:, We get

$$B_{HN} = \{< x; (0{,}72, 0{,}57, 0{,}55) >, < y; (0{,}72, 0{,}54, 0{,}57) >\}$$

$$C_{HN} = \{< x; (0{,}74, 0{,}44, 0{,}52) >, < y; (0{,}75, 0{,}48, 0{,}53) >\}$$

Let (N(X),τ_1) and (N(X),τ_2) are heptagonal neutrosophic topological space. τ_1={ϕ, B_{HN}, X} and τ_2={ϕ, C_{HN}, X} is a heptagonal neutrosophic topology on X.
$\tau_1 \bigcap \tau_2$={ϕ, X, B_{HN}, C_{HN}} is not a heptagonal neutrosophic topology on X because $B_{HN} \bigcup C_{HN} \notin \tau_1 \bigcap \tau_2$.Whereas,$\tau$={$\phi, X, B_{HN}, C_{HN}, B_{HN} \bigcup C_{HN}, B_{HN} \bigcap C_{HN}$} is a heptagonal neutrosophic topology on X.

Remark: Let (X,τ) be a heptagonal neutrosophic topological space(HNTS). Then (X,τ)^C is the dual topology, whose elements are A^C_{HN} for $A_{HN} \in$ (X,τ). Any open set in τ is known as heptagonal neutrosophic open set(HNOs). Any closed set in τ is known as heptagonal neutrosophic closed set(HNCs) iff it's complement is heptagonal neutrosophic open set.

Definition 3.4. The heptagonal neutrosophic interior and Heptagonal neutrosophic closure are given by,

- HNint(A_{HN})=$\bigcup$\{O_{HN}/O_{HN} is a HNOs $\in$ X where $O_{HN} \subseteq A_{HN}$\} and it is the largest HN-open subset of A_{HN}.
- HNcl(B_{HN})=$\bigcap$\{J_{HN}/J_{HN} is a HNCs $\in$ X where $J_{HN} \subseteq B_{HN}$\} and it is the smallest HN-closed set containing B_{HN}.

Theorem 3.5. *If X be a set. Let (N(X),τ) is a HN topological space over X and A_{HN}, B_{HN} ∈N(X) then,*

1. *HNint(ϕ)=ϕ and HNint(X)=N(X)*
2. *HNint(A_{HN}) $\subseteq A_{HN}$*
3. *A_{HN} is HN open if and only if A_{HN}=HNint(A_{HN})*

4. *HNint(HNint(A_{HN}))=HNint(A_{HN})*
5. $A_{HN} \subseteq B_{HN} \Rightarrow$ *HNint(A_{HN})⊆HNint(B_{HN})*
6. *HNint(A_{HN})∪HNint(B_{HN})⊆HNint($A_{HN} \cup B_{HN}$)*
7. *HNint(A_{HN})∩HNint(B_{HN})=HNint($A_{HN} \cap B_{HN}$)*

Proof: i) Since ϕ and N(X) are HN-open, then HNint(ϕ)=ϕ and HNint(X)=N(X).
ii) From the definition of heptagonal neutrosophic interior, HNint(A_{HN})⊆ A_{HN}
iii) If A_{HN} is HN-open set over X, then A_{HN} is the largest HN-open set containing A. So, A_{HN}=HNint(A_{HN}).
Conversely, If A_{HN}=HNint(A_{HN}), then A_{HN} is the largest HN-open set containing A_{HN} and hence A_{HN} is HN-open.
iv) As HNint(A_{HN}) is open set, then HNint(HNint(A_{HN}))=HNint(A_{HN}).
v) When $A_{HN} \subseteq B_{HN}$, Also we know that, HNint(A_{HN})⊆ $A_{HN} \subseteq B_{HN}$. As HNint(A_{HN}) is a HN-subset of B_{HN}. So, HNint(A_{HN})⊆HNint(B_{HN}).
vi) It is obvious that, $A_{HN} \subseteq A_{HN} \cup B_{HN}$ and $B_{HN} \subseteq A_{HN} \cup B_{HN}$. From v),
HNint(A_{HN})⊆HNint($A_{HN} \cup B_{HN}$) and HNint(B_{HN})⊆HNint($A_{HN} \cup B_{HN}$)
⇒ HNint(A_{HN})∪HNint(B_{HN})⊆HNint($A_{HN} \cup B_{HN}$).
vii) It is obvious that $A_{HN} \cap B_{HN} \subseteq A_{HN}$ and $A_{HN} \cap B_{HN} \subseteq A_{HN}$. From v) HNint($A_{HN} \cap B_{HN}$)⊆HNint($A_{HN}$) and HNint($A_{HN} \cap B_{HN}$)⊆HNint($A_{HN}$) Also HNint($A_{HN}$)=$A_{HN}$ and HNint(B_{HN})=B_{HN}. Therefore, HNint(A_{HN})∩HNint(B_{HN})⊆ $A_{HN} \cap B_{HN}$
⇒HNint(A_{HN})∩HNint(B_{HN})=HNint($A_{HN} \cap B_{HN}$).

Example 3.6. Let X={x,y} and $A_{HN}, B_{HN}, C_{HN} \in$ N(X) then,

$$A_{HN} = \{< x; (0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6), (0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6), (0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6, 0{,}6) >, < y; (0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8), (0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8), (0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8, 0{,}8) >\}$$

$$B_{HN} = \{< x; (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9) >, < y; (0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1), (0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1), (0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1, 0{,}1) >\}$$

$$C_{HN} = \{< x; (0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2), (0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2), (0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2, 0{,}2) >, < y; (0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4), (0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4), (0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4) >\}$$

By definition 2.4: we get

$$A_{HN} = \{< x; (0{,}6, 0{,}6, 0{,}6) >, < y; (0{,}8, 0{,}8, 0{,}8) >\}$$
$$B_{HN} = \{< x; (0{,}9, 0{,}9, 0{,}9) >, < y; (0{,}1, 0{,}1, 0{,}1) >\}$$
$$C_{HN} = \{< x; (0{,}2, 0{,}2, 0{,}2) >, < y; (0{,}4, 0{,}4, 0{,}4) >\}$$

HNint(B_{HN})=ϕ and HNint(C_{HN})=ϕ
Since HNint($B_{HN} \cup C_{HN}$)=ϕ therefore, HNint(B_{HN})$\cup C_{HN}$)=ϕ therefore
HNint(B_{HN})$\cup$HNint(C_{HN})$\subseteq$HNint($B_{HN} \cup C_{HN}$).

Theorem 3.7. *If X be a set. Let (N(X),τ) is a HN topological space over X and A_{HN}, B_{HN} $\in$N(X) then,*

1. *HNcl(ϕ)=ϕ and HNcl(X)=N(X)*
2. *A_{HN} $\subseteq$HNcl(A_{HN})*
3. *A_{HN} is HN closed if and only if A_{HN}=HNcl(A_{HN})*
4. *HNcl(HNcl(A_{HN}))=HNcl(A_{HN})*
5. *$A_{HN} \subseteq B_{HN} \Rightarrow$ HNcl(A_{HN})$\subseteq$HNcl(B_{HN})*
6. *HNcl($A_{HN} \cup B_{HN}$)=HNcl(A_{HN})$\cup$HNint(B_{HN})*
7. *HNcl($A_{HN} \cap B_{HN}$)$\subseteq$HNcl(A_{HN})$\cup$HNcl(B_{HN})*

Proof: i) If A_{HN} is HN-closed then A_{HN}=HNcl(A_{HN}). Also is ϕ and X are HN-closed, then HNcl(ϕ)=ϕ and HNcl(X)=X.
ii) From the definition of HN-closure. It is obvious from the definition that A_{HN} $\subseteq$HNcl(A_{HN}).
iii) If A_{HN} is HN-closed set over X, then A_{HN} contains A_{HN} and that itself a HN-closed set over X. Then A_{HN} is the smallest HN-closed set containing A_{HN}. So, A_{HN}=HNcl(A_{HN}).
Conversely, If A_{HN}=HNcl(A_{HN}), then A_{HN} is the smallest HN-closed set containing A_{HN} and hence A_{HN} is HN-closed.
iv) From above, As A_{HN} is closed, then A_{HN}=HNcl(A_{HN}). As HNcl(A_{HN}) is open set, then HNcl(HNcl(A_{HN}))=HNcl(A_{HN})
v) When $A_{HN} \subseteq B_{HN}$, Since B_{HN} $\subseteq$HNcl(B_{HN}) $\Rightarrow$ A_{HN} $\subseteq$HNcl(B_{HN}) That is HNcl(B_{HN}) is a HN-closed set contains A_{HN}. But HNcl(A_{HN}) is the smallest HN-closed set contain A_{HN}. Thus, HNcl(A_{HN})$\subseteq$HNcl(B_{HN}).
vi),vii) is obvious

Example 3.8. Let X={x,y} and A_{HN}, B_{HN} ∈N(X) then,

$$A_{HN} =\{< x; (0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4), (0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4), (0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4, 0{,}4) >, < y; (0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7), (0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7), (0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7, 0{,}7) >\}$$

$$B_{HN} =\{< x; (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5), (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5), (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5) >, < y; (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9) >\}$$

By definition 2.4: we have

$$A_{HN} =\{< x; (0{,}4, 0{,}4, 0{,}4) >, < y; (0{,}7, 0{,}7, 0{,}7) >\}$$
$$B_{HN} =\{< x; (0{,}5, 0{,}5, 0{,}5) >, < y; (0{,}9, 0{,}9, 0{,}9) >\}$$

Then we have,

$$A_{HN} \cup B_{HN} =\{< x; (0{,}5, 0{,}4, 0{,}4) >, < y; (0{,}9, 0{,}7, 0{,}7) >\}$$
$$A_{HN} \cap B_{HN} =\{< x; (0{,}4, 0{,}5, 0{,}5) >, < y; (0{,}7, 0{,}9, 0{,}9) >\}$$

Consider, τ={ϕ,X,$A_{HN}, B_{HN}, A_{HN} \cup B_{HN}, A_{HN} \cap B_{HN}$} is a HN topology. After taking complements,
τ={X,ϕ,A^C_{HN},B^C_{HN},$(A_{HN} \cup B_{HN})^C$,$(A_{HN} \cap B_{HN})^C$}.Where,

$$A^C_{HN} =\{< x; (0{,}4, 0{,}6, 0{,}4) >, < y; (0{,}7, 0{,}3, 0{,}7) >\}$$
$$B^C_{HN} =\{< x; (0{,}5, 0{,}5, 0{,}5) >, < y; (0{,}9, 0{,}1, 0{,}9) >\}$$
$$(A_{HN} \cup B_{HN})^C =\{< x; (0{,}4, 0{,}6, 0{,}5) >, < y; (0{,}7, 0{,}3, 0{,}9) >\}$$
$$(A_{HN} \cap B_{HN})^C =\{< x; (0{,}5, 0{,}5, 0{,}4) >, < y; (0{,}9, 0{,}1, 0{,}7) >\}$$
$$HNcl(A_{HN}) =X$$

$$HNcl(B_{HN}) =X$$
$$A^C_{HN} \cap B^C_{HN} =\{< x; (0{,}4, 0{,}6, 0{,}5) >, < y; (0{,}7, 0{,}3, 0{,}9) >\}$$
$$HNcl(A_{HN} \cap B_{HN}) =(A_{HN} \cup B_{HN})^C$$
$$HNcl(A_{HN} \cap B_{HN}) \subseteq HNcl(A_{HN}) \cap HNcl(B_{HN}).$$

Definition 3.9. Let A_{HN} be a subset of a heptagonal neutrosophic topological space (N(X),τ). A point x∈ A^C_{HN} is said to be an exterior point of A if there exists an open set U containing x such that, U∈ A^C_{HN}. It is denoted by HNext(A_{HN}) and defined as:

$$\text{HNext}(A_{HN})=\{\bigcup B; B \subseteq \tau, B \in X - A\}$$

Theorem 3.10. *If X be a set. Let (N(X),τ) is a HN topological space over X and* A_{HN}, B_{HN} *∈N(X) then*

1. *HNext(ϕ)=X*
2. *HNext(X)=ϕ*
3. *HNext*(A_{HN})$\subseteq A^C$=*X-*A_{HN} *for any* $A_{HN} \subseteq X$
4. $A_{HN} \subseteq B_{HN} \Rightarrow$ *HNext*(B_{HN})$\subseteq$*HNext*(A_{HN})
5. *HNint*(A_{HN})$\subseteq$*HNext(HNext*(A_{HN}))
6. *HNext*($A_{HN} \cup B_{HN}$)=*HNext*(A_{HN})∩*HNext*(B_{HN})
7. *HNext*($A_{HN} \cap B_{HN}$)=*HNext*(A_{HN})∪*HNext*(B_{HN})

Proof: i) HNext(ϕ)=HNint(X-ϕ)=X.

ii) HNext(X)=HNint(X-X)=ϕ.

iii) HNext(A_{HN})=int(A^C_{HN})⊆ A^C_{HN}. Since HNint(A_{HN})⊆ A_{HN}.

iv) If $A_{HN} \subseteq B_{HN}$, Then, HNext(B_{HN})=HNint(B^C_{HN}) Also we know that, $A_{HN} \subseteq B_{HN} \Rightarrow B^C_{HN} \subseteq A^C_{HN}$.Also, HNint($B^C_{HN}$)⊆HNint($A^C_{HN}$)

(i) implies, HNint(B_{HN})=HNint(B^C_{HN})⊆HNint(A^C_{HN}) ⊆HNext(A_{HN})

$\Rightarrow$ HNint(B_{HN})⊆HNint(A_{HN})

v) From(iii), HNext(A_{HN})⊆ A^C_{HN}

HNint(A^C_{HN})⊆HNext(HNext(A_{HN}))

HNint(A^C_{HN})⊆HNext(HNext(A_{HN}))

HNint(A_{HN})⊆HNext(HNext(A_{HN}))

vi) HNext($A_{HN} \cup B_{HN}$) =HNint($A_{HN} \cup B_{HN}$)^C

=HNint($A^C_{HN} \cap B^C_{HN}$)

=HNint(A^C_{HN})∩ HNint(B^C_{HN})

HNext($A_{HN} \cup B_{HN}$)=HNext(A_{HN})∩HNext(B_{HN})

vii) HNext($A_{HN} \cap B_{HN}$) =HNint($A_{HN} \cap B_{HN}$)^C

=HNint($A^C_{HN} \cup B^C_{HN}$)

=HNint(A^C_{HN})∪HNint(B^C_{HN})

HNext($A_{HN} \cap B_{HN}$)=HNext(A_{HN})∪HNext(B_{HN})

Definition 3.11. Let A_{HN} be a subset of a heptagonal neutrosophic topological space X and a point x∈X is said to be boundary point of A_{HN} if each open set containing at x intersects both A_{HN} and A^C_{HN}. It is denoted by HNFr(A_{HN}) and defined as:

$$\text{HNFr}(A_{HN})=\text{HNcl}(A_{HN})\bigcap\text{HNcl}(A_{HN})^C \text{ or}$$
$$\text{HNFr}(A_{HN})=\text{HNcl}(A_{HN})\text{-HNint}(A_{HN}) \text{ or}$$
$$\text{HNFr}(A_{HN})=\text{X-}\{\text{HNint}(A_{HN})\bigcup\text{HNext}(A_{HN})\}$$

Remark: The boundary point is also known as boundary point. the set of all boundary point of a set A_{HN} is called the boundary of A_{HN} or the boundary of A_{HN}, which is denoted by HNfr(A_{HN}). Since by the definition, each boundary point of A_{HN} is also a boundary point of A^C_{HN} ad vice versa, so the boundary of A_{HN} is same as that of A^C_{HN} , i.e. HNfr(A_{HN})=HNfr(A^C_{HN}).

Theorem 3.12. *If A_{HN} is a subset of a HN topological space over X and then the following statements of boundary holds:*

1. *HNcl(X-A_{HN})=X-HNint(A_{HN})*
2. *HNfr(A_{HN})=HNcl(A_{HN})$\cap$ HNint(X-A_{HN})*
3. *HNfr(A_{HN}) is closed*
4. *HNfr(A_{HN})=HNfr(X-A_{HN})*
5. *HNfr(A_{HN})$\cap$ HNint(A_{HN})=ϕ*
6. *HNfr(HNint(A_{HN}))$\subseteq$ HNfr(A_{HN})*
7. *(HNfr(A_{HN}))^C=HNext(A_{HN})$\cup$HNint(A_{HN})*
8. *HNcl(A_{HN})=HNint(A_{HN})$\cup$ HNfr(A_{HN})*

Proof: i) let x$\in$ HNcl(X-A_{HN}) then x is the closure of X-A_{HN}. Then for every U$\in$ τ with x$\in$U, we have that; U$\cap$(X-A_{HN})=ϕ.
So there does not exist a open neighborhood of x that is fully contained in A_{HN}. This x$\notin$ HNint(A_{HN}) i.e., x$\in$(X- HNint(A_{HN})) so, HNcl(X-A_{HN})$\subseteq$X-HNint(A_{HN})
Now, let x$\in$(X- HNint(A_{HN})). Then x$\notin$ HNint(A_{HN}). So for ever open neighborhood U of x, we have that U $\nsubseteq$ A_{HN}. So U$\cap$(X-A_{HN})$\neq$0 for every open neighborhood U of x.Thus x$\in$HNcl(X-A_{HN}) so HNcl(X-A_{HN})$\supseteq$X-HNint(A_{HN})
Therefore, HNcl(X-A_{HN})=X-HNint(A_{HN})
ii) by definition we have HNfr(A_{HN})=HNcl(A_{HN})$\cap$HNint(A_{HN})
Or equivalently, HNfr(A_{HN})=HNcl(A_{HN})$\cap$(X- HNint(A_{HN})
From(i), HNfr(A_{HN})=HNcl(A_{HN})$\cap$HNcl(X-A_{HN})
iii) from2 HNfr(A_{HN}) can be written as as intersection of two closed sets and so HNfr(A_{HN}) is closed.
iv) From(ii), HNfr(A_{HN})=HNcl(A_{HN})$\cap$ HNcl(X-A_{HN}) Since, X-(X-A_{HN})=A_{HN}, also bt the proposition that: HNfr(X-A_{HN})=HNcl(X-A_{HN})$\cap$ HNcl(X-(X-A_{HN})) HNfr(X-A_{HN})= HNcl(X-A_{HN})$\cap$ HNcl(A_{HN})
Comparing, $\Rightarrow$ HNfr(A_{HN})=HNfr(X-A_{HN}).
v) and vi) is obvious
vii) A_{HN} $\in$ N(X).Then,
(HNfr(A_{HN}))^C=(HNcl(A_{HN})$\cap$HNfr(A_{HN}))^C
(HNfr(A_{HN}))^C=(HNcl(A_{HN}))^C$\cup$(HNfr(A_{HN}))^C

$(\text{HNfr}(A_{HN}))^C$=$(\text{HNcl}(A_{HN}))^C \cup (\text{HNint}(A_{HN}))^C$
$(\text{HNfr}(A_{HN}))^C$=$(\text{HNext}(A_{HN})) \cup (\text{HNfr}(A_{HN}))$.
viii) $A_{HN} \in$ N(X).Then, by definition and remark
HNint(A_{HN})∪HNfr(A_{HN})=HNint(A_{HN})∪(HNcl(A_{HN})∩HNfr(A_{HN}))
HNint(A_{HN})∪HNfr(A_{HN})=HNint(A_{HN})∪(HNcl(A_{HN})∩(HNint(A_{HN})∪HNfr(A_{HN}))
HNint(A_{HN})∪HNfr(A_{HN})=HNcl(A_{HN})∩$(\text{HNint}(A_{HN}) \cup \text{HNint}(A_{HN}))^C$
HNint(A_{HN})∪HNfr(A_{HN})=HNcl(A_{HN})∩X
HNint(A_{HN})∪HNfr(A_{HN})=HNcl(A_{HN}).

4. Applications of Heptagonal Neutrosophic Topology

Definition 4.1. Let X_{HN} and Y_{HN} are the non-void sets and f: $X_{HN} \longrightarrow Y_{HN}$ be a function, then

1. If A_{HN}=$\{\langle x, [\rho_{A_{HN}}(x), \sigma_{A_{HN}}(x), \omega_{A_{HN}}(x)]\rangle; x \in X_{HN}\}$ is a HN set in X_{HN}, then the image of A_{HN} under $f(A_{HN})$ is denoted by,
   $f(A_{HN})$=$\{\langle y, [f(\rho_{A_{HN}}(y)), f(\sigma_{A_{HN}}(y)), f(\omega_{A_{HN}}(y))]\rangle; y \in Y_{HN}\}$.
2. If B_{HN}=$\{\langle x, [\rho_{A_{HN}}(x), \sigma_{AHN}(x), \omega_{A_{HN}}(x)]\rangle; x \in X_{HN}\}$ is a HN set in X_{HN}, then the inverse-image of B_{HN} under $f^{-1}(B_{HN})$ is denoted by,
   $f^{-1}(B_{HN})$=$\{\langle x, [f^{-1}(\rho_{A_{HN}}(x)), f^{-1}(\sigma_{A_{HN}}(x)), f^{-1}(\omega_{A_{HN}}(x))]\rangle; x \in X_{HN}\}$.

Definition 4.2. A map f: $X_{HN} \longrightarrow Y_{HN}$ is called as heptagonal neutosophic continuous function if the inverse image $f^{-1}(A_{HN})$ of each heptagonal neutosophic open set A_{HN} is the heptagonal neutrosophic open in X_{HN}.

Definition 4.3. A map f: $X_{HN} \longrightarrow Y_{HN}$ is called as heptagonal neutosophic continuous function if the inverse image $f^{-1}(A_{HN})$ of each heptagonal neutrosophic closed set A_{HN} is the heptagonal neutrosophic closed in X_{HN}.

Theorem 4.4. *Let X and Y be a set. Let A_{HN} $\{A_{HN_i}$:i∈ I$\}$ be heptagonal neutrosophic set in X_{HN} and Let B_{HN} $\{B_{HN_i}$:i∈ I$\}$ be heptagonal neutrosophic set in Y_{HN} anf f: $X_{HN} \longrightarrow Y_{HN}$. Then,*

1. $A_{HN_1} \subseteq A_{HN_2} \Longleftrightarrow f(A_{HN_1}) \subseteq f(A_{HN_2})$
2. $B_{HN_1} \subseteq B_{HN_2} \Longleftrightarrow f^{-1}(B_{HN_1}) \subseteq f^{-1}(B_{HN_2})$
3. $A_{HN} \subseteq f^{-1}(f(A_{HN}))$ *and if f is injective, then* A_{HN}=$f^{-1}(f(A_{HN}))$
4. $f^{-1}(f(B_{HN})) \subseteq B_{HN}$ *and if f is surjective, then* $f^{-1}(f(B_{HN})) = B_{HN}$
5. $f^{-1}(\cup B_{HN_i})$=$\cup f^{-1}(B_{HN_i})$ *and* $f^{-1}(\cap B_{HN_i})$=$\cap f^{-1}(B_{HN_i})$
6. $f^{-1}(\cup A_{HN_i})$=$\cup f^{-1}(A_{HN_i})$ *and* $f^{-1}(\cap A_{HN_i}) \subseteq \cap f^{-1}(A_{HN_i})$ *and if f is injective, then* $f^{-1}(\cap A_{HN_i})$=$\cap f^{-1}(A_{HN_i})$
7. $f^{-1}(1_{HN})$=1_{HN} *and* $f^{-1}(0_{HN})$=0_{HN}

8. *$f(1_{HN})$=1_{HN} and $f(0_{HN})$=0_{HN} if f is injective.*

Proof: *The proof is obvious from the basic properties.*

Example 4.5. Let X_{HN}=$\{x, y\}$ and Y_{HN}=$\{x, y\}$and B_{HN}, C_{HN}, D_{HN} ∈N(X) then,

B_{HN} =$\{\langle x; (0{,}96, 0{,}65, 0{,}73, 0{,}75, 0{,}83, 0{,}56, 0{,}54), (0{,}75, 0{,}95, 0{,}45, 0{,}38, 0{,}79, 0{,}57, 0{,}13), (0{,}59, 0{,}36, 0{,}68, 0{,}47, 0{,}36, 0{,}95, 0{,}44)\rangle, \langle y; (0{,}38, 0{,}69, 0{,}88, 0{,}98, 0{,}77, 0{,}36, 0{,}98), (0{,}32, 0{,}72, 0{,}42, 0{,}62, 0{,}90, 0{,}22, 0{,}62), (0{,}42, 0{,}52, 0{,}62, 0{,}72, 0{,}36, 0{,}72, 0{,}61)\rangle\}$

C_{HN} =$\{\langle x; (0{,}73, 0{,}74, 0{,}96, 0{,}34, 0{,}85, 0{,}89, 0{,}64), (0{,}46, 0{,}35, 0{,}25, 0{,}96, 0{,}36, 0{,}56, 0{,}16), (0{,}84, 0{,}85, 0{,}37, 0{,}57, 0{,}67, 0{,}22, 0{,}10) >, \langle y; (0{,}76, 0{,}72, 0{,}78, 0{,}62, 0{,}92, 0{,}56, 0{,}88), (0{,}38, 0{,}98, 0{,}22, 0{,}32, 0{,}54, 0{,}64, 0{,}31), (0{,}86, 0{,}96, 0{,}52, 0{,}22, 0{,}41, 0{,}51, 0{,}32)\rangle\}$

D_{HN} =$\{\langle x; (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5), (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5), (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5)\rangle, \langle y; (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9)\rangle\}$

By definition 2.10:, We get

$$B_{HN} =\{\langle(0{,}72, 0{,}57, 0{,}55)\rangle, \langle(0{,}72, 0{,}54, 0{,}57)\rangle\}$$

$$C_{HN} =\{\langle(0{,}74, 0{,}44, 0{,}52)\rangle, \langle(0{,}75, 0{,}48, 0{,}53)\rangle\}$$

$$D_{HN} =\{\langle x; (0{,}5, 0{,}5, 0{,}5)\rangle, \langle y; (0{,}9, 0{,}9, 0{,}9)\rangle\}$$

Then the family E_{HN}=$\{0_{HN}, 1_{HN}, B_{HN}\}$ is a heptagonal neutrosophic topology on X_{HN} and F_{HN}=$\{0_{HN}, 1_{HN}, C_{HN}\}$ is a heptagonal neutrosophic topology on Y_{HN}.

Thus (X_{HN}, B_{HN}) and (Y_{HN}, C_{HN}) are heptagonal neutrosophic toplogical spaces.

Define $f : (X_{HN}, B_{HN}) \longrightarrow (Y_{HN}, C_{HN})$ as f(x)=y, f(y)=x and f(z)=z.

Then, f is heptagonal neutrosophic continuous function.

Theorem 4.6. *Let f:$X_{HN} \longrightarrow Y_{HN}$ be a single valued HN function, whereX_{HN} and Y_{HN} are HN topological spaces. Then the following statements are equivalent:*

1. The function f is HN continuous.
2. The inverse image of each HN open set in Y_{HN} is HN open in X_{HN}.

Proof: (i)⟹(ii):

Firstly, assume that f:$X_{HN} \longrightarrow Y_{HN}$ is HN continuous. Let A_{HN} be HN open in Y_{HN}. Then A^C_{HN} is HN closed in Y_{HN}. Since Y_{HN} is HN continuous $f^{-1}(A^C_{HN})$ is HN closed in X_{HN}. But $f^{-1}(A^C_{HN})$=X-$f^{-1}(B_{HN})$. Thus $X_{HN} - f^{-1}(B_{HN})$ is HN closed in X_{HN} and we have that

$f^{-1}(A_{HN})$ is HN open in X. Therefore, (i)$\Longrightarrow$(ii).

(ii)$\Longrightarrow$(i):

Conversely, we assume that the inverse image of each HN open set in Y_{HN} is HN open in X_{HN}. Let B_{HN} be any HN closed set in Y_{HN}. Then B_{HN}^{C} is HN open in V. By our assumption, $f^{-1}(B_{HN}^{C})$ is HN open in X_{HN}. But then, $f^{-1}(B_{HN}^{C})$=$X_{HN} - f^{-1}(B_{HN})$. Then $X_{HN} - f^{-1}(B_{HN})$ is HN open in X_{HN} and also $f^{-1}(B_{HN})$ is HN closed in X_{HN}. Therefore f is HN continuous. Hence, (ii)$\Longrightarrow$(i). Therefore (i) and(ii) are equivalent.

Theorem 4.7. *A mapping f:$X_{HN} \longrightarrow Y_{HN}$ is heptagonal neutrosophic continuous iff the inverse image of every heptagonal neutrosophic closed set in Y_{HN} is heptagonal neutrosophic closed in X_{HN}.*

Proof: Firstly we assume that f is a HN continuous. Let A_{HN} be a heptagonal neutrosophic closed set in Y_{HN}. Then A_{HN}^{C} is open in Y_{HN}. By our assumption, f is HN continuous function, $f^{-1}(A_{HN})$ is HN open in X_{HN}. But then, $f^{-1}(A_{HN}^{C})$=$X_{HN} - f^{-1}(A_{HN})$.

Therefore, $f^{-1}(A_{HN})$ is heptagonal neutrosophic closed in X_{HN}.

Conversely, assume the pre image of every heptagonal neutrosophic closed set in Y_{HN} is heptagonal neutrosophic closed in X_{HN}. Let B_{HN} be a HN open set in Y_{HN}, then B_{HN}^{C} is HN closed in Y_{HN}. By hypothesis that, $f^{-1}(B_{HN}^{C})$=$X_{HN} - f^{-1}(B_{HN})$ is HN closed in X_{HN} and so $f^{-1}(B_{HN})$ is HN open in X_{HN}.

Therefore, f is heptagonal neutrosophic continuous.

Theorem 4.8. *A mapping :$X_{HN} \longrightarrow Y_{HN}$ is heptagonal neutrosophic continuous if and only if f(HNcl(A_{HN}))⊂HNcl(f(A_{HN})) for every subset A_{HN} of X_{HN}.* ***Proof:*** Firstly. We assume that f is HN continuous. Let A_{HN} be any subset of X_{HN}. Then HNcl(f(A_{HN})) is a HN closed set in X_{HN}. Since by our assumption f is HN continuous, $f^{-1}(HNcl(f(A_{HN})))$ is HN closed in X_{HN} and it contains A_{HN}. By the definition of HN closure, HNcl(A_{HN}) is the intersection of all HN closed sets containing A_{HN}. Therefore, HNcl(A_{HN})⊆ $f^{-1}(HNcl(f(A_{HN})))$.

Therefore, f(HNcl(A_{HN}))⊂(HNcl(f(A_{HN}))).

Conversely, assume that f(HNcl(A_{HN}))⊂(HNcl(f(A_{HN}))). Let B_{HN} is HN closed in Y_{HN}, f(HNcl($f^{-1}(B_{HN})$))⊆HNcl(B_{HN}). Thus we have, HNcl($f^{-1}(B_{HN})$) $\subseteq f(^{-1}HNcl(B_{HN}))=$ $f^{-1}(B_{HN})$. But we know that $f^{-1}(B_{HN})$ ⊆HNcl($f^{-1}(B_{HN})$). Which then implies that, HNcl($f^{-1}(B_{HN})$)=$f^{-1}(B_{HN})$. Therefore $f^{-1}(B_{HN})$ is HN closed set in X_{HN} for every HN closed set B_{HN} in Y_{HN}.Then by the definition of HN continuity function,

f is heptagonal neutrosophic continuous.

Theorem 4.9. Let (X,τ_X) and (Y,τ_Y) be a heptagonal neutrosopic topological space and let f:$X_{HN} \longrightarrow Y_{HN}$ be the mapping. Then the following statements are equivalent.

1. f is HN continuous map.
2. For each subset $A_{HN} \subseteq X_{HN}$, we have $f(\overline{A}) \subseteq \overline{f(A)}$.
3. For every HN closed subset $B_{HN} \subseteq Y_{HN}$, then the set $f^{-1}(B_{HN})$ is HN closed in X_{HN}.
4. For each x$\in$ X_{HN} and each $B_{HN} \in \tau_Y$ containing f(x), there is some $U_{HN} \in \tau_X$ containing x and such that $f(U_H) \subseteq B_{HN}$.

Proof: We prove the above statements as follows: (i) implies (ii), (ii) implies (iii), (iii) implies (iv) and finally (i) implies (iv).

(i)$\Rightarrow$(ii): Assume that f is a HN continuous mapping. Let $A_{HN} \subseteq X_{HN}$ be a subset. For each x$\in$ $\overline{A_{HN}}$ we have to show that f(x)$\in$ $\overline{f(A_{HN})}$. Fix for such x and letting $B_{HN} \in \tau_Y$ be any HN open subset containing f(x). Since by oue assumption, f is HN continuous, the subset U_{HN}=$f^{-1}(B_{HN})$ is an HN open subsets that contains the element x. Note that $U_{HN} \cap A_{HN} \neq \varnothing$, therefore there exists y$\in$ $A_{HN} \cap u_{HN}$ and f(y)$\in$ $B_{HN} \cap f(A_{HN})$. Since every HN open subset containing f(x) intersects f(A_{HN}) nontrivially,

$$f(\overline{A})HN) \subseteq \overline{f(A_{HN})}.$$

(ii)$\Rightarrow$(iii): Assume that for subset $A_{HN} \subseteq X_{HN}$, we have $f(\overline{A}) \subseteq \overline{f(A)}$. Let $B_{HN} \subseteq Y_{HN}$ be a HN closed subset and let A_{HN}=$f^{-1}(B_{HN})$. We need to show that A_{HN}=$\overline{A_{HN}}$(more specifically that $\overline{A_{HN}} \subseteq A_{HN}$, the opposite containment is always true). So fix that x$\in$ $\overline{A_{HN}}$. Then,

$$\text{f(x)}\in f(\overline{A})HN) \subseteq \overline{f(A_{HN})} \subseteq \overline{B_{HN}}=B_{HN}.$$

That is, f(x)$\in$ B_{HN}. Or in other words x$\in$ $f-1(B_{HN})$=A_{HN} as required.

(iii)$\Rightarrow$(iv): Assume that, for every HN closed subset $B_{HN} \subseteq Y_{HN}$, then the set $f^{-1}(B_{HN})$ is HN closed in X_{HN}. Suppose the pre-images of HN closed sets are HN closed. Fix x$\in$ X_{HN}, and an HN open set $B_{HN} \in \tau_Y$ containing f(x). Then $Y_{HN} - B_{HN}$ is HN closed and hence $f^{-1}(Y_{HN} - B_{HN})$ a HN closed sunset of X_{HN} by our assumption and it does not contains x. But then the complement of this set, $X_{HN} - f^{-1}(Y_{HN} - B_{HN})$, is the HN open and does contains x. So let us fix the HN open set U_{HN} such that,

$$\text{x}\in U_{HN} \subseteq X_{HN} - f^{-1}(Y_{HN} - B_{HN}).$$

Then we have, $f(U_{HN}) \subseteq f(X_{HN} - f^{-1}(Y_{HN} - B_{HN})$=$f(X_{HN}) - (Y_{HN} - B_{HN}) \subseteq B_{HN}$,

$$f(U_{HN}) \subseteq B_{HN} \text{ as required.}$$

(i)$\Rightarrow$(iv):Assuming that, f is HN continuous map. Let x$\in$ X_{HN} and let $B_{HN} \in \tau_Y$ containing f(x). Then the set U_{HN}=$f^{-1}(B_{HN})$ is a HN open subset containing x. Conversely, assume that (iv) holds. Let $B_{HN} \in \tau_Y$ and let x$\in$ $f^{-1}(B_{HN})$. Then f(x)$\in$ B_{HN} and by the hypothesis there exists some $U_{HN_x} \in \tau_Y$ containing x and such that $f(U_{HN_x}) \subseteq B_{HN}$. Thus $U_{HN_x} \subset f^{-1}(B_{HN})$. It follows that $f^{-1}(B_{HN})$=$\bigcup_{x\in f^{-1}(B_{HN})} U_{HN_x}$, which is then the element of τ_X.

Theorem 4.10. A mapping f:$X_{HN} \longrightarrow Y_{HN}$ is heptagonal neutrosophic open function if and only if f(HNint(A_{HN}))⊂HNint(f(A_{HN})) for every subset A_{HN} of X_{HN}. **Proof:** Firstly we assume that, f:$X_{HN} \longrightarrow Y_{HN}$ is heptagonal neutrosophic open function and A_{HN} be a heptagonal neutosophic subset of X_{HN}. Clearly we can see that HNint(A_{HN}) is an HN open set in X_{HN} and HNint(A_{HN}) $\subseteq A_{HN}$. Since by our assumption f is a HN open function, so f(HNint(A_{HN})) is a HN open set in X_{HN}. And f(HNint(A_{HN}))$\subseteq f(A_{HN})$. Since each HN open set is a HN open set and $HNint(f(A_{HN}))$ is the largest HNopen set containing $f(A_{HN})$, so that $HNint(f(A_{HN}))$ is the largest HN open set contained in $f(A_{HN})$. Therefore ,

f(HNint(A_{HN}))⊂HNint(f(A_{HN})) for each HN subset A_{HN} of X_{HN}.

Conversely assume that, f(HNint(A_{HN}))⊂HNint(f(A_{HN})) for every subset A_{HN} of X_{HN}. Let B_{HN} be an HN set in X_{HN}. Therefore, HNint(B_{HN})=B_{HN}. By the hypothesis we have that, f(HNint(B_{HN}))⊂HNint(f(B_{HN})). Which implies that f(B_H)⊆HNint(f(B_H)). Also we have that HNint(f(B_H))⊆ f(B_H). Therefore f(B_H)=HNint(f(B_H)).That is, f(B_{HN}) is the HN open set in X_{HN}. Hence for every HN open set in X_{HN}, f(B_{HN})is the HN open set in X_{HN}. Therefore f is the HN open function.

Example 4.11. Let X_{HN}=$\{x, y\}$ and B_{HN}, C_{HN}, D_{HN} ∈N(X) then,

$$B_{HN} = \{\langle x; (0{,}96, 0{,}65, 0{,}73, 0{,}75, 0{,}83, 0{,}56, 0{,}54), (0{,}75, 0{,}95, 0{,}45, 0{,}38, 0{,}79, 0{,}57, 0{,}13), (0{,}59, 0{,}36, 0{,}68, 0{,}47, 0{,}36, 0{,}95, 0{,}44)\rangle, \langle y; (0{,}38, 0{,}69, 0{,}88, 0{,}98, 0{,}77, 0{,}36, 0{,}98), (0{,}32, 0{,}72, 0{,}42, 0{,}62, 0{,}90, 0{,}22, 0{,}62), (0{,}42, 0{,}52, 0{,}62, 0{,}72, 0{,}36, 0{,}72, 0{,}61)\rangle\}$$

$$C_{HN} = \{\langle x; (0{,}73, 0{,}74, 0{,}96, 0{,}34, 0{,}85, 0{,}89, 0{,}64), (0{,}46, 0{,}35, 0{,}25, 0{,}96, 0{,}36, 0{,}56, 0{,}16), (0{,}84, 0{,}85, 0{,}37, 0{,}57, 0{,}67, 0{,}22, 0{,}10) >, \langle y; (0{,}76, 0{,}72, 0{,}78, 0{,}62, 0{,}92, 0{,}56, 0{,}88), (0{,}38, 0{,}98, 0{,}22, 0{,}32, 0{,}54, 0{,}64, 0{,}31), (0{,}86, 0{,}96, 0{,}52, 0{,}22, 0{,}41, 0{,}51, 0{,}32)\rangle\}$$

$$D_{HN} = \{\langle x; (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5), (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5), (0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5, 0{,}5)\rangle, \langle y; (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9), (0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9, 0{,}9)\rangle\}$$

By definition 2.10:, We get

$$B_{HN} = \{\langle(0{,}72, 0{,}57, 0{,}55)\rangle, \langle(0{,}72, 0{,}54, 0{,}57)\rangle\}$$

$$C_{HN} = \{\langle(0{,}74, 0{,}44, 0{,}52)\rangle, \langle(0{,}75, 0{,}48, 0{,}53)\rangle\}$$

$$D_{HN} = \{\langle x; (0{,}5, 0{,}5, 0{,}5)\rangle, \langle y; (0{,}9, 0{,}9, 0{,}9)\rangle\}$$

Then the family E_{HN}={$0_{HN}, 1_{HN}, B_{HN}$} , F_{HN}={$0_{HN}, 1_{HN}, C_{HN}$} and G_{HN}={$0_{HN}, 1_{HN}, D_{HN}$} .Thus (X_{HN}, E_{HN}), (X_{HN}, F_{HN}), (X_{HN}, G_{HN}) are heptagonal neutrosophic topological spaces.
Define $f : (X_{HN}, E_{HN}) \longrightarrow (X_{HN}, F_{HN})$ as f(x)=y, f(y)=x and f(z)=z.
Define $g : (X_{HN}, F_{HN}) \longrightarrow (X_{HN}, G_{HN})$ as g(x)=y, g(y)=z and g(z)=y.
clearly f and g are heptagonal neutrosophic continuous. But gof is not heptagonal neutrosophic continuous. For 1-D is heptagonal neutrosophic closed in (X_{HN}, G_{HN}). $f^{-1}(g^{-1}(1-D))$ is not heptagonal neutrosophic closed in (X_{HN}, E_{HN}). gof is not heptagonal neutrosophic continuous.

Theorem 4.12. A mapping f:$X_{HN} \longrightarrow Y_{HN}$ is heptagonal neutrosophic bijective function. Then the following statements are equivalent:

1. f is HN continuous function.
2. f is HN closed function.
3. f is HN open function.

Proof: (i)$\Longrightarrow$ (ii):
Firslty, assume that, f is HN continuous function, Let A_{HN} be any arbitrary HN closed set in X_{HN}. Then A^C_{HN} is an HN open set in X_{HN}. Since each HN open set is an HN open set, so A^C_{HN} is the Hn open set in X_{HN}. Since f is a bijective function, so that f(A^C_{HN})=f$(A_{HN})^C$ is an HN open set in X_{HN}. Hence f(A_{HN}) is an HN closed set in X_{HN}. Therefore, for each HN closed set in X_{HN}, then f(A_{HN}) is a HN closed set in X_{HN}.
$\Longrightarrow$ f is HN closed function

(ii)$\Longrightarrow$ (iii):
Firstly, assume that, f is HN closed function, Let B_{HN} be any arbitrary HN closed set in X_{HN}. Then B^C_{HN} is an HN closed set in X_{HN}. Since f is a HN closed function, so that f(B^C_{HN})=f$(B_{HN})^C$ is an HN closed set in X_{HN}. Hence f(B_{HN}) is an HN open set in X_{HN}. Therefore, for each HN open set in X_{HN}, then f(A_{HN}) is a HN open set in X_{HN}.
$\Longrightarrow$ f is HN open function.

(iii)$\Longrightarrow$(i):
Firstly, assume that, f is a HN open function. Let C_{HN} be any arbitrary HN open set in Y_{HN}. Then C_{HN} is an HN open set in Y_{HN}. Since each HN open set is an HN open set, so C_{HN} is the HN open set in Y_{HN}. Since f is a bijective function, so that $f^{-1}(C_{HN})$ is an HN open set in Y_{HN}. Again since each HN open set is an HN open set, so $f^{-1}(C_{HN})$ is the HN open set in Y_{HN}. Therefore, for each HN closed set in Y_{HN}, then $f^{-1}((A_{HN}))$ is a HN open set in Y_{HN}.
$\Longrightarrow$ f is HN continuous function.

Example 4.13. Let H_{HN}={x,y}, A_{HN}, B_{HN} and C_{HN} ∈N(X) are defined as follows,

$$A_{HN} =\{\langle x;(0{,}72,0{,}41,0{,}35,0{,}81,0{,}77,0{,}73,0{,}77),(0{,}83,0{,}88,0{,}93,0{,}99,0{,}96,0{,}90, 0{,}94),(0{,}86,0{,}99,0{,}97,0{,}93,0{,}94,0{,}91,0{,}86)\rangle,\langle y;(0{,}91,0{,}32,0{,}56,0{,}48,0{,}81, 0{,}72,0{,}67),(0{,}78,0{,}83,0{,}21,0{,}38,0{,}56,0{,}33,0{,}98),(0{,}36,0{,}86,0{,}96,0{,}32,0{,}44, 0{,}56,0{,}72)\rangle\}$$

$$B_{HN} =\{\langle x;(0{,}96,0{,}65,0{,}73,0{,}75,0{,}83,0{,}56,0{,}54),(0{,}75,0{,}95,0{,}45,0{,}38,0{,}79,0{,}57, 0{,}13),(0{,}59,0{,}36,0{,}68,0{,}47,0{,}36,0{,}95,0{,}44)\rangle,\langle y;(0{,}38,0{,}69,0{,}88,0{,}98,0{,}77, 0{,}36,0{,}98),(0{,}32,0{,}72,0{,}42,0{,}62,0{,}90,0{,}22,0{,}62),(0{,}42,0{,}52,0{,}62,0=72,0{,}36, 0{,}72,0{,}61)\rangle\}$$

$$C_{HN} =\{\langle x;(0{,}73,0{,}74,0{,}96,0{,}34,0{,}85,0{,}89,0{,}64),(0{,}46,0{,}35,0{,}25,0{,}96,0{,}36,0{,}56, 0{,}16),(0{,}84,0{,}85,0{,}37,0{,}57,0{,}67,0{,}22,0{,}10)\rangle,\langle y;(0{,}76,0{,}72,0{,}78,0{,}62,0{,}92, 0{,}56,0{,}88),(0{,}38,0{,}98,0{,}22,0{,}32,0{,}54,0{,}64,0{,}31),(0{,}86,0{,}96,0{,}52,0{,}22,0{,}41, 0{,}51,0{,}32)\rangle\}$$

Using De-neutosophication technique:$\frac{(p+q+r+s+t+u+v)}{7}$, We get

$$A_{HN} =\{\langle x;(0{,}65,0{,}92,0{,}92)\rangle,\langle y;(0{,}64,0{,}58,0{,}60)\rangle\}$$
$$B_{HN} =\{\langle x;(0{,}72,0{,}57,0{,}55)\rangle,\langle y;(0{,}72,0{,}54,0{,}57)\rangle\}$$
$$C_{HN} =\{\langle x;(0{,}74,0{,}44,0{,}52)\rangle,\langle y;(0{,}75,0{,}48,0{,}53)\rangle\}$$

Then the family E_{HN}=$\{0_{HN},1_{HN},A_{HN},B_{HN}\}$ and F_{HN}=$\{0_{HN},1_{HN},C_{HN}\}$ are heptagonal neutrosophic topologies on X_{HN}.

Thus (X_{HN},E_{HN}) and (X_{HN},F_{HN}),are heptagonal neutrosophic top0logical spaces.

Define $f:(X_{HN},E_{HN})\longrightarrow(X_{HN},F_{HN})$ as f(x)=y, f(y)=x and f(z)=x.

clearly f is heptagonal neutrosophic continuous. But f is not strongly heptagonal neutrosophic continuous. Since,

D_{HN}=$\{\langle x;(0{,}74,0{,}44,0{,}59)\rangle,\langle y;(0{,}5,0{,}48,0{,}53)\rangle\}$ is an heptagonal neutrosophic open set in (X_{HN},F_{HN}), $f^{-1}(D_{HN})$ is not heptagonal neutrosophic open in (X_{HN},E_{HN}).

5. Conclusions

In this current article, we have introduced heptagonal neutrosophic topology in neutrosophic environments with the help of ranking technique of Heptagonal numbers. Also the Heptagonal neutrosophic set operations are introduced with suitable examples. The Heptagonal neutrosophic interior and closure concepts are also explained to strengthen the HN topology. The theorems and properties of open sets and closed sets of HN topologies are explained with related examples. Further there is a scope to introduce continuous functions, connectedness and compactness based on HN topological spaces. Additionally, Topological Spaces and Bipartite Graph are used in conjunction with the Heptagonal Intuitionistic Fuzzy Number (HIFN) in [16] to solve the Intuitionistic Fuzzy Transportation Problems. Heptagonal Neutrosophic topological spaces can also be used in place of topological spaces and Neutrosophic Heptagonal Numbers can be used as an alternative to HIFN to solve the Neutrosophic Transportation Problems, which is one of the examples of applications of the concepts discussed in this article. We have further planned to expand Multi Criteria Decision Making (MCDM) to discover or select the best answer from the existing with the aid of Neutrosophic soft matrix.

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