NSS
Neutrosophic Sets and Systems, Vol. 61, 2023
University of New Mexico

# Time for a New Player in Business Analytics: An MCDM Scheme Based on One-Dimensional Uncertain Linguistic Interval-Valued Neutrosophic Fuzzy Data

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**Abstract:** Pooling the opinions of the decision makers about premier alternative from a list of given choices is the most crucial phenomenon theory of decision making. The MSM operator is an effective approach and can identify the collective connection among various input viewpoints. So, tacking the fully benefit of MSM operator, this paper presents a new novel in depth investigation of MSM in context of Neutrosophic fuzzy variables. Initially we developed the structure of One-dimension uncertain linguistic interval valued neutrosophic fuzzy variables, deliberate their fundamental properties along with meaningful laws. Moreover, we encompass the MSM operator to the extent of one dimensional uncertain-linguistic Neutrosophic fuzzy variables and introduced some novel aggregations operators including one-dimensional uncertain linguistic interval-valued Neutrosophic fuzzy, weighted fuzzy and ordered weighted fuzzy McLaurin symmetric mean operator. The practical utilization of the proposed aggregation operators in business analytic are also included in the paper. In last comparison is made between current vs old studies which proved solidity and reliability of the new operator's vs existing operators in the literature.

**Keywords:** Linguistic numbers; Fuzzy variables; MSM; Aggregation operators; MCDM; Business analytic; Interval Valued fuzzy Data

**1. Introduction**

There exist imprecise, uncertain, and vague situations in real life, like in engineering, economics, trade, life, and social sciences. The techniques of classical mathematics are not fruitful to meet this problem. Therefore, a number of models, mainly including the theory of probability, fuzzy set [1] and its interval-valued extension [2], soft set (SS) [3], intuitionistic-fuzzy set [4], Pythagorean fuzzy-Set [5], linguistic term sets [6] and neutrosophic sets [7] with some of its modifications and extensions [8, 9,10] etc. have been presented to take in hand such situations.

In decision making strategies, indeterminacy is a substantial factor existent in theory of decision. In review forms, consider three assortments for sexual category: (1) Male (2) Female (3) Other. Therefore, the imprecision and uncertainty with indeterminacy cannot be described merely with the assistance intuitionistic and/or soft sets. Thus, Smarandache [7] coined the perception of neutrosophic sets (NSs). Beyond, NS comprises truth, false and indeterminacy membership functions. Currently, the research on the theory of NSs has been established vibrantly [11]. Later, NSs were hybridized with SSs to initiate neutrosophic soft set (NSS) [12]. Subsequently, several researchers have worked on this idea [12-16]. Wang et al. [17] commenced the conception of an IVN set and later Zhang et al. [18] applied this notion in theory of multiple attributes. Unlike IFS, centralized calculations are more adaptable as they have fewer limitations. Utilizing a neutrosophic set ensemble can effectively analyze uncertainty in datasets, particularly those of a significant scale. In addition, the concept behind this collection has developed and united with SSs to generate a new kind of set known as interval-valued neutrosophic set. Here are [19-25] also some significant characteristics of these sets when subjected to various algebraic operations.

For the sake of tracking down the finest alternative while solving MCDM issues and other related complications, one of the critical phases is aggregation whereby the views of the decision experts are required to be aggregated by using suitable technique of aggregation.

A straightforward way to understand this task is to see it as a MCDM issue, where the selection of suitable aggregation operators is crucial. The role of aggregation operators is like an appliance to pool the estimations of several decision makers into a collective one. In the decision-making area, there are many aggregation operators e.g., the arithmetic and geometric weighted average [26, 27]. Various strategies can be used in literature to combine numbers using different methodologies (Aggregation operators) in research areas. A method called intuitionistic fuzzy soft aggregation operator, developed by Garg and Arora [26], was created to combine information for decision making. Ye [27] introduced two innovative methods for combining data using trapezoidal intuitionistic fuzzy numbers. Awang et al. [28] and his colleagues proposed using the SVNWA operator as a method for combining information. Later, the notion of INWA is more explored and expanded by Huang et al. [29]. Zhang et al. [30] discussed the (INNWA) and Geometric-averaging operators. The primary emphasis of this paper is on resolving MCDM problems with heterogeneous priority levels for criteria, and it suggests employing neutrosophic uncertain linguistic factors to simplify the computation. The two eye catching operators included IVNWAA and IVNWGA was developed by Hussain et al. [31]. Aiwu et al., [32] put forward the IVNSGWA, a generalized weighted aggregation operator for neutrosophic aggregates incorporating interval values. Peng and Wang [33] introduced multiple aggregation operators for combining multi-valued neutral environments. Recently, Hamid et al. [34] presented multistage decision analysis in the framework of q-rung m-polar fuzzy setting. In [35], Naeem et al. employed generalized aggregation operators for medical diagnosis in Pythagorean fuzzy soft environment. Riaz et al. [36] made use of weighted aggregation operators for investment strategy using multipolar Pythagorean numbers. Ye [37] originated the idea of a (TNNWAA) and (TNNWGA) operator in the framework of trapezoidal Neutrosophic-number. Furthermore, a modified "TOPSIS" approach is introduced for handling trapezoidal opacification neutrosophic data. Jana and Pal [38] studied (SVNSWA) and (SVNSWGA) using single-valued neutrosophic soft environment.

Neutrosophic Frameworks with Applications can be explained in simpler terms as the use of a specific approach to analyze and solve problems. This approach considers three different perspectives - true, false, and indeterminate - and considers their interplay to come up with practical solutions. These frameworks can be applied in various fields and have practical uses in problem-solving and decision-making processes. Abdel et al. [39] discussed a strategy for managing rural water administration using a Coordinates Neutrosophic Territorial Administration Positioning Strategy. Hafeez et al. [40]. Developed the Neutrosophic MCDM Model to rank and evaluate different ways of managing healthcare waste to achieve better effectiveness and sustainability. Karak et al. [41] created a plan on how to solve transportation problems in a unique situation.

The MSM operator has gained the attention of the researchers working on decision making techniques from the past few years. Wang et al. [42] suggested single-Valued neutrosophic linguistic MSM aggregation operator by making use of operational laws of the underlying set. Wu et al. [43] presented some practical utilities of single valued neutrosophic linguistic set (SVNLS). In current, some generalized aggregation-operators have been efficaciously explored by many researchers and scholar across the globe. Some prominent operators are studied in [44-47]. Even though, the aggregation operators presented so far have far-flung applications in decision making, but sometimes there exist complex fuzzy information where these operators fail to work. According to the best knowledge of authors, no worth mentioning work has been done on the utility of one-dimensional uncertain linguistic interval-valued neutrosophic fuzzy variables.

To fill the gaps followings study is conducted and for smooth understanding of the notions presented: The basic ideas in Section-2 can be viewed as an introductory overview. In the very next section, operational laws, modified operational laws, expectancy, and some basic attributes of ODULIVNF variables are studied. Along with some novel operators meant for aggregation in this section have also been presented there. These operators include un-weighted, weighted, and ordered weighted one-dimensional uncertain linguistic interval-valued neutrosophic fuzzy MSM operator. An MCDM via proposed scheme as well as an algorithm is added in the section four. The same section also presents an MCDM model based upon one-dimension uncertain linguistic interval-valued neutrosophic fuzzy variables i.e., practical utility of the proposed aggregation operators in business analytics. Comparison study is of the current technique with some existing methodologies is also made part of the same section. Results and discussion are presented therein too. Finally, Section-5 concludes the article.

**Motivation:**

The MSM operator has gained attention of the researchers working on decision making techniques from last decade because of its valuable performance. In recent years, there have been new methods developed for integrating data that have proven to be effective. Amongst all, MSM operator is a prominent because this operator is empowered with the attribute of catching the conjoint association amongst the multiple input opinions. Even though, the aggregation operators based on MSM have far-flung applications in decision-making, but sometimes there exist complex fuzzy information where these operators fail to work. According to the best knowledge of authors, no worth mentioning work has been done on the utility of one-dimensional uncertain linguistic interval-valued neutrosophic fuzzy variables. To reduce such issues in the way of fuzzy theory and to fill this gap the following research is conducted.

**Novelty:**

The primary aim of this article is to meet this end by initiating the notion of one-dimensional uncertain linguistic interval-valued neutrosophic fuzzy variables. Accompanied by their various necessary characteristics. This model is equipped with the ability to take in hand uncertain statistics and moderate the complexity prevailing in the data. Operational laws modified operational laws, expectancy, and some basic attributes of these variables leaded us towards some novel MSM operators for aggregation. These operators include un-weighted, weighted, and ordered weighted one-dimensional uncertain linguistic interval-valued neutrosophic fuzzy MSM operators. The practical utility of the current aggregation operators is an influential in business analytics and other trade Markets.

## 2. Basic and Fundamental Concepts

From now onward, $X$ will represent a non-void universe, unless stated otherwise.

**Definition 2.1** [1] A collection of the form $T=\{(s,\mu_T(s)):s\in X\}$ with the map $\mu_T:T\rightarrow[0,1]$ declared the degree of belongness of elements to the set is called fuzzy set.

**Example:** Consider the reference set of students Y = {g1, g2, g3, g4}. Choose B = {(g1, 0.9) (g2, O.4) (g3, 0.8) (g4, 1)}. The set B indicates the degree of smartness. i.e., g1 is 0.9 and so on.

**Definition 2.2** [4] By an IFS, we mean an assemblage of the form $\tilde{M}=\{(\tilde{x},\tilde{\mu}_{\tilde{M}}(\tilde{x}),\tilde{v}_{\tilde{M}}(\tilde{x})):\tilde{x}\in\tilde{X}\}$. The maps $\tilde{v}_{\tilde{M}},\tilde{\mu}_{\tilde{M}}:\tilde{M}\rightarrow[0,1]$ with sum equal to 1 and acknowledged belongness and non-belongness to a set.

**Example:** Consider B to be an IFS with $v$B(y) = 0.2 and $\mu$B(y) = 0.5 then, $\mu$B($x$) = 0.3, $\pi$B($x$) = 0.7, and $\partial$B(y) = 0.18. $\mu$B(y) = 0.5 and $v$B(y) = 0.2 show the belongness and not belongness of object to IFS respectively.

**Definition 2.3** [7] A family of the form $\tilde{M}=\{(\tilde{x},\tilde{\eta}_{\tilde{M}}(\tilde{x}),v_{\tilde{M}}(\tilde{x}),\tilde{\mu}_{\tilde{M}}(\tilde{x})):\tilde{x}\in\tilde{X}\}$ is called a *neutrosophic set*. The maps $\tilde{\eta}_{\tilde{M}},\tilde{v}_{\tilde{M}},\tilde{\mu}_{\tilde{M}}:\tilde{T}\rightarrow\left]0^-,1^+\right[$ along with the constraint that their sum lies in $\left]0^-,3^+\right[$, and acknowledged value of truth, indeterminacy, and value of falsity in a set.

**Example:** Consider the triplet (0.3,0.1,0.2) ∈M, the degree of an in A is 0.3, 0.1, and 0.2 denotes the membership, indeterminacy, and non-membership respectively. Similarly, the element b (0.4,0.5,0.1) ∈M and the element C (0.1,0.2,0.7) ∈M with components sum equal to 1.

**Definition 2.4** [17] An object of the form

$$\tilde{M}=\left\{\left(\tilde{x},\mu_{\tilde{M}}\left(\tilde{x}\right)=\left[\mu_{\tilde{M}}^{L}\left(\tilde{x}\right),\mu_{\tilde{M}}^{U}\left(\tilde{x}\right)\right],\eta_{\tilde{M}}\left(s\right)=\left[\eta_{\tilde{M}}^{L}\left(\tilde{x}\right),\eta_{\tilde{M}}^{U}\left(\tilde{x}\right)\right],\nu_{\tilde{M}}\left(\tilde{x}\right)=\left[\nu_{\tilde{M}}^{L}\left(\tilde{x}\right),\nu_{\tilde{M}}^{U}\left(\tilde{x}\right)\right]\right):\tilde{x}\in\tilde{X}\right\}$$

is acknowledged as an interval-valued neutrosophic set. The maps $\eta_{\tilde{M}},\mu_{\tilde{M}},\nu_{\tilde{M}}:\tilde{M}\rightarrow\left]0^{-},1^{+}\right[$ along with the restriction known as "value of truth, indeterminacy and value of falsity, respectively".

**Example:** Consider M be an IVNS. The element m ((0.50-0.51), (0.10–0.15) ∪ [0.20–0.30], {0.20, 0.24, and 0.28}) ∈M. The membership is between 0.50-0.51 indeterminacy of m to M is between 0.10-0.15 or between 0.20-0.30 and non-membership is 0.20 or 0.24 or 0.28. limits don't exclude while actual approximation are not considered because of numerous sources.

**Definition 2.5** [6] Assume that $S_{[0,h]}$ is a continuous linguistic-Term set (CLTs). A LIF set is defined as $\left\{\left(\hat{a},\hat{S}_{\hat{\alpha}}\left(\hat{a}\right),\hat{S}_{\hat{\beta}}\left(\hat{a}\right)\right):\hat{a}\in\hat{M};\hat{S}_{\hat{\alpha}},\hat{S}_{\hat{\beta}}\in\hat{S}_{[0,r]}\right\}$ where $\hat{\alpha}+\hat{\beta}\in\left[0,r\right]$ . $\hat{S}_{\alpha}$ and $\hat{S}_{\beta}$ are "Linguistic membership" and "non-membership values". The quantity $\hat{S}_{-\left(\hat{\alpha}+\hat{\beta}-r\right)}$ is acknowledged as degree of indeterminacy. The doublet $\left(\hat{S}_{\hat{\alpha}},\hat{S}_{\hat{\beta}}\right)$ is reckoned as linguistic intuitionistic fuzzy number (LIFN).

**Example:** Consider finite discrete ordered LT values by $\hat{S}=\{\hat{s}_0,\hat{s}_1,...,\hat{s}_r\}$, where $r$ is the even. E.g., for $r$=4, then the chosen linguistic term $\tilde{S}$ with following corresponding semantics is expressed as follows: "
$$\hat{S}=\begin{Bmatrix}\hat{s}_0,\\ \hat{s}_1,\\ \hat{s}_2,\\ \hat{s}_3\end{Bmatrix}=\begin{Bmatrix}\hat{s}_0(Low),\\ \hat{s}_1(Slightly-low),\\ \hat{s}_2(Medium),\\ \hat{s}_3(Slightly-high)\end{Bmatrix}$$

**3. One-dimension uncertain linguistic Interval-valued neutrosophic fuzzy variables, operational laws, and their basic properties**

**Definition 3.1** $\widehat{S}=\left\langle\begin{matrix}[\widehat{s}_l,\widehat{s}_m],\\ [\widehat{s}_n,\widehat{s}_o],\\ [\widehat{s}_p,\widehat{s}_q]\end{matrix}\right\rangle$; $\widehat{S}$ is said to be one-dimension uncertain linguistic neutrosophic fuzzy variable with following condition. $\left\langle\widehat{s}_l,\widehat{s}_m,\widehat{s}_n,\widehat{s}_o,\widehat{s}_p,\widehat{s}_q\in\widehat{S}\right\rangle$ while $\langle l\le m\ ;n\le o \text{ and } p\le q\rangle$ whereas $\left\langle\widehat{s}_l,\widehat{s}_n,\widehat{s}_p\right\rangle$ and $\left\langle\widehat{s}_m,\widehat{s}_o,\widehat{s}_q\right\rangle$ are the lower and upper limits.

**Definition 3.2** If $\tilde{S}_1=\left\langle\begin{matrix}[\tilde{s}_{l_1},\tilde{s}_{m_1}],\\ [\tilde{s}_{n_1},\tilde{s}_{o_1}],\\ [\tilde{s}_{p_1},\tilde{s}_{q_1}]\end{matrix}\right\rangle$ and $\tilde{S}_2=\left\langle\begin{matrix}[\tilde{s}_{l_2},\tilde{s}_{m_2}],\\ [\tilde{s}_{n_2},\tilde{s}_{o_2}],\\ [\tilde{s}_{p_2},\tilde{s}_{q_2}]\end{matrix}\right\rangle$; Then operational rules for them are defined as fallows.

**1:**
$$\begin{aligned}\tilde{S}_1\oplus\tilde{S}_2&=\left([\tilde{s}_{l_1},\tilde{s}_{m_1}],[\tilde{s}_{n_1},\tilde{s}_{o_1}],[\tilde{s}_{p_1},\tilde{s}_{q_1}]\right)\oplus\left([\tilde{s}_{l_2},\tilde{s}_{m_2}],[\tilde{s}_{n_2},\tilde{s}_{o_2}],[\tilde{s}_{p_2},\tilde{s}_{q_2}]\right)\\&=\left([s_{l_1+l_2},s_{m_1+m_2}],[s_{n_1+n_2},s_{o_1+o_2}],[s_{p_1+p_2},s_{q_1+q_2}]\right)\end{aligned}\quad(1)$$

**2:**
$$\begin{aligned}\tilde{S}_1\otimes\tilde{S}_2&=\left([\tilde{s}_{l_1},\tilde{s}_{m_1}],[\tilde{s}_{n_1},\tilde{s}_{o_1}],[\tilde{s}_{p_1},\tilde{s}_{q_1}]\right)\otimes\left([\tilde{s}_{l_2},\tilde{s}_{m_2}],[\tilde{s}_{n_2},\tilde{s}_{o_2}],[\tilde{s}_{p_2},\tilde{s}_{q_2}]\right)\\&=\left([s_{l_1\times l_2},s_{m_1\times m_2}],[s_{n_1\times n_2},s_{o_1\times o_2}],[s_{p_1\times p_2},s_{q_1\times q_2}]\right)\end{aligned}\quad(2)$$

**3:** $\frac{\tilde{S}_1}{\tilde{S}_2} = \frac{\left(\left[\tilde{s}_{l_1}, \tilde{s}_{m_1}\right], \left[\tilde{s}_{n_1}, \tilde{s}_{o_1}\right], \left[\tilde{s}_{p_1}, \tilde{s}_{q_1}\right]\right)}{\left(\left[\tilde{s}_{l_2}, \tilde{s}_{m_2}\right], \left[\tilde{s}_{n_2}, \tilde{s}_{o_2}\right], \left[\tilde{s}_{p_2}, \tilde{s}_{q_2}\right]\right)} = \left(\left[\tilde{s}_{\frac{l_1}{l_2}}, \tilde{s}_{\frac{m_1}{m_2}}\right], \left[\tilde{s}_{\frac{n_1}{n_2}}, \tilde{s}_{\frac{o_1}{o_2}}\right], \left[\tilde{s}_{\frac{p_1}{p_2}}, \tilde{s}_{\frac{q_1}{q_2}}\right]\right)$ (3)

**4:** $k \otimes \tilde{S}_1 = k\tilde{S}_1 = \left[\tilde{s}_{l_1}, \tilde{s}_{m_1}\right], \left[\tilde{s}_{n_1}, \tilde{s}_{o_1}\right], \left[\tilde{s}_{p_1}, \tilde{s}_{q_1}\right] = \left(\left[\tilde{s}_{kl_1}, \tilde{s}_{km_1}\right], \left[\tilde{s}_{kn_1}, \tilde{s}_{ko_1}\right], \left[\tilde{s}_{kp_1}, \tilde{s}_{kq_1}\right]\right);\ k \geq 0$ (4)

**5:** $\tilde{S}_1^k = \left(\left[\tilde{s}_{l_1}, \tilde{s}_{m_1}\right], \left[\tilde{s}_{n_1}, \tilde{s}_{o_1}\right], \left[\tilde{s}_{p_1}, \tilde{s}_{q_1}\right]\right)^k = \left(\left[\tilde{s}_{l_1^k}, \tilde{s}_{m_1^k}\right], \left[\tilde{s}_{n_1^k}, \tilde{s}_{o_1^k}\right], \left[\tilde{s}_{p_1^k}, \tilde{s}_{q_1^k}\right]\right);\ k \geq 0$ (5)

**Definition 3.3** Consider $\tilde{S} = \left\langle \begin{matrix} \left[\tilde{s}_l, \tilde{s}_m\right], \\ \left[\tilde{s}_n, \tilde{s}_o\right], \\ \left[\tilde{s}_p, \tilde{s}_q\right] \end{matrix} \right\rangle$; expectation of $\tilde{S}$ is stated as by eq#6.

$$E\left(\tilde{S}_1\right) = \frac{l+m}{6(\alpha-1)} \times \frac{n+o}{6(\beta-1)} \times \frac{p+q}{6(\gamma-1)} \quad (6)$$

**Definition 3.4** Consider

$$\tilde{S}_1 = \left\langle \begin{matrix} \left[\tilde{s}_{l_1}, \tilde{s}_{m_1}\right], \\ \left[\tilde{s}_{n_1}, \tilde{s}_{o_1}\right], \\ \left[\tilde{s}_{p_1}, \tilde{s}_{q_1}\right] \end{matrix} \right\rangle \text{ and } \tilde{S}_2 = \left\langle \begin{matrix} \left[\tilde{s}_{l_2}, \tilde{s}_{m_2}\right], \\ \left[\tilde{s}_{n_2}, \tilde{s}_{o_2}\right], \\ \left[\tilde{s}_{p_2}, \tilde{s}_{q_2}\right] \end{matrix} \right\rangle;$$

as any two ODULNFVs. Then $\tilde{E}\left(\tilde{s}_1\right) \leq \tilde{E}\left(\tilde{s}_2\right)$ indicates expectancy of first number is lesser than expectancy of second number that is $\left\langle \tilde{S}_1 \leq \tilde{s}_2 \right\rangle$ or vice versa.

**Definition 3.5** Let $\hat{s}_1 = \left\langle \begin{matrix} \left[\tilde{s}_{l_1}, \tilde{s}_{m_1}\right], \\ \left[\tilde{s}_{n_1}, \tilde{s}_{o_1}\right], \\ \left[\tilde{s}_{p_1}, \tilde{s}_{q_1}\right] \end{matrix} \right\rangle$ and $\hat{s}_2 = \left\langle \begin{matrix} \left[\tilde{s}_{l_2}, \tilde{s}_{m_2}\right], \\ \left[\tilde{s}_{n_2}, \tilde{s}_{o_2}\right], \\ \left[\tilde{s}_{p_2}, \tilde{s}_{q_2}\right] \end{matrix} \right\rangle$ be any two ONDLNFVs. Then, for three scalars $\left\langle \varphi, \dot{\varphi}\, and\, \ddot{\varphi} > 0 \right\rangle$ the following results hold: noted that $\langle\ \rangle$ is just a notation.

1) $\left\langle \hat{s}_{\hat{p}} \oplus \hat{s}_{\hat{q}} \right\rangle = \left\langle \hat{s}_{\hat{q}} \oplus \hat{s}_{\hat{p}} \right\rangle$ (7)

2) $\left\langle \hat{s}_{\hat{p}} \otimes \hat{s}_{\hat{q}} \right\rangle = \left\langle \hat{s}_{\hat{q}} \otimes \hat{s}_{\hat{p}} \right\rangle$ (8)

3) $\left\langle \varphi\left(\hat{s}_{\hat{p}} \oplus \hat{s}_{\hat{q}}\right) \right\rangle = \left\langle \varphi\hat{s}_{\hat{q}} \oplus \varphi\hat{s}_{\hat{p}} \right\rangle$ (9)

4) $\left\langle \varphi\hat{s}_{\hat{p}} \oplus \dot{\varphi}\hat{s}_{\hat{p}} \right\rangle = \left\langle \left(\varphi + \dot{\varphi}\right)\hat{s}_{\hat{p}} \right\rangle$ (10)

5) $\left\langle \left(\hat{s}_{\tilde{p}}\right)^{\varphi} \otimes \left(\hat{s}_{\tilde{p}}\right)^{\varphi} \right\rangle = \left\langle \left(\hat{s}_{\tilde{p}} \otimes \hat{s}_{\tilde{p}}\right)^{\varphi} \right\rangle$ (11)

6) $\left\langle \hat{s}_{\tilde{p}}^{\varphi} \otimes \hat{s}_{\tilde{p}}^{\dot{\varphi}} \right\rangle = \left\langle \hat{s}_{\tilde{p}}^{\varphi+\dot{\varphi}} \right\rangle$ (12)

### 3.1 One-dimension uncertain linguistic interval-valued neutrosophic fuzzy Maclaurin symmetric mean operators.

The MSM was originally developed by Maclaurin. MSM supports decision making by merging and evaluating information about various alternatives and their associations. MSM helps make decisions by combining information and thus widely used as a most beneficial trick to capture the "interrelationship among the multi-input values". By taking the fully command over MSM operator with utilizing the concept of ODULNFVs with its meaningful properties, now in this section we built up some new novel aggregation operators i.e., ODULIVNFMSM, ODULIVNWFMSM and ODULIVNOWFMSM.

**Definition 3.1.1** consider $\tilde{S}_r = \left\langle \begin{matrix} \left[s_{\tilde{a}_i}, s_{\tilde{b}_i}\right], \\ \left[s_{\tilde{c}_i}, s_{\tilde{d}_i}\right], \\ \left[s_{\tilde{e}_i}, s_{\tilde{f}_i}\right] \end{matrix} \right\rangle$ such that $\mathrm{r} = [1:1:n]$ ; be a non-empty collection of ODULNF variable. The w be a vector of

$\tilde{S}_r; \mathrm{r} = [1:1:n]$ with $w_t \in [0,1], t = 1, 2, ..., n;$ and $\sum_{t=1}^{n} w_t = 1.$ Then the defining relation below

$$\mathrm{ODULIVNWFMSM}^{(k)}\left(\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, ..., \tilde{S}_n\right) = \left( \frac{\otimes_{1 \le t_1 < ... < t_k \le n} \left( \oplus_{r=1}^{k} \left( W_{t_r} \otimes \tilde{S}_{t_r} \right) \right)}{C_n^k} \right)^{\frac{1}{k}}; \quad (13)$$

Is called to be a WODULNFMSM operator. Where k tuples combination of {1…, n} of t is $\left(t_1, t_2, t_3, ..., t_k\right)$ and $C_n^k$ denotes the binomial coefficient.

**Theorem 3.1.2.** Consider $\tilde{S}_r = \left(\left[s_{\tilde{a}_r}, s_{\tilde{b}_r}\right], \left[s_{\tilde{c}_r}, s_{\tilde{d}_r}\right], \left[s_{\tilde{e}_r}, s_{\tilde{f}_r}\right]\right)$; $\mathrm{r} = [1:1:n]$ ; be a non-void assemblage of ODULNF variables. "w" is representing the weights of $\tilde{S}_r$; $\mathrm{r} = [1:1:n]$ with $w_t \in [0,1], \quad t = 1, 2, ..., n;$ and $\sum_{t=1}^{n} w_t = 1$. Then the aggregated value is also a ODULNF variable and can be obtained as follow.

$$\mathrm{ODULIVNFMSM}^{(k)}\left(\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, ..., \tilde{S}_n\right) =$$

$$
=\left(\begin{array}{l}
\left\langle \tilde{S}_{\left(\frac{\sum_{1\le t_1<\dots<t_k\le n}\prod_{r=1}^{k} w_{t_r}\tilde{a}_{t_r}}{C_n^k}\right)^{\left(\frac{1}{k}\right)}}, \tilde{S}_{\left(\frac{\sum_{1\le j_1<\dots<j_k\le n}\prod_{i=1}^{k} w_{t_r}\tilde{b}_{t_r}}{C_n^k}\right)^{\left(\frac{1}{k}\right)}} \right\rangle, \\
\left\langle \begin{array}{l}
\tilde{S}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod\left(1\le t_1\le t_2\le\dots\le t_{k\le} n\right)\left(1-\frac{\prod_{r=1}^{n}\frac{1}{(\hat{\beta}-1)^{-1}}\left[1-\left(1-\frac{c_{t_r}}{\hat{\beta}-1}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\left(\frac{1}{k}\right)}}, \tilde{S}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod\left(1\le t_1\le t_2\le\dots\le t_{k\le} n\right)\left(1-\frac{\prod_{r=1}^{n}\frac{1}{(\hat{\beta}-1)^{-1}}\left[1-\left(1-\frac{d_{t_r}}{\hat{\beta}-1}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\left(\frac{1}{k}\right)}}
\end{array} \right\rangle, \\
\left\langle \begin{array}{l}
\tilde{S}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod\left(1\le t_1\le t_2\le\dots\le t_{k\le} n\right)\left(1-\frac{\prod_{r=1}^{n}\frac{1}{(\hat{\beta}-1)^{-1}}\left[1-\left(1-\frac{e_{t_r}}{\hat{\beta}-1}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\left(\frac{1}{k}\right)}}, \tilde{S}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod\left(1\le t_1\le t_2\le\dots\le t_{k\le} n\right)\left(1-\frac{\prod_{r=1}^{n}\frac{1}{(\hat{\beta}-1)^{-1}}\left[1-\left(1-\frac{f_{t_r}}{\hat{\beta}-1}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\left(\frac{1}{k}\right)}}
\end{array} \right\rangle
\end{array}\right) \tag{14}
$$

**Proof:**

In accordance with the operational rules of ODULIVNF variable

$$\oplus_{r=1}^{k}\tilde{S}_{t_r}=\begin{pmatrix}\left\langle \tilde{S}_{\prod_{r=1}^{n}a_{t_r}},\tilde{S}_{\prod_{r=1}^{n}b_{t_r}}\right\rangle,\\ \left\langle \dfrac{\tilde{S}_{\prod_{r=1}^{n}c_{t_r}}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)},\dfrac{\tilde{S}_{\prod_{r=1}^{n}d_{t_r}}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}\right\rangle,\\ \left\langle \dfrac{\tilde{S}_{\prod_{r=1}^{n}d_{t_r}}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)},\dfrac{\tilde{S}_{\prod_{t=1}^{n}e_{t_r}}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}\right\rangle\end{pmatrix}$$

$$\oplus_{r=1}^{k}w_{t_r}\tilde{S}_{t_r}=\begin{pmatrix}\left\langle \tilde{S}_{\left(\prod_{r=1}^{n}w_{t_r}a_{t_r}\right)},\tilde{S}_{\left(\prod_{r=1}^{n}w_{t_r}b_{t_r}\right)}\right\rangle,\\ \left\langle \dfrac{\tilde{S}_{\left(\prod_{r=1}^{n}w_{t_r}c_{t_r}\right)}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)},\dfrac{\tilde{S}_{\left(\prod_{r=1}^{n}w_{t_r}d_{t_r}\right)}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}\right\rangle,\\ \left\langle \dfrac{\tilde{S}_{\left(\prod_{r=1}^{n}w_{t_r}d_{t_r}\right)}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)},\dfrac{\tilde{S}_{\left(\prod_{r=1}^{n}w_{t_r}e_{t_r}\right)}}{\left(\dfrac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}\right\rangle\end{pmatrix}$$

And

$$\underset{\left(1\le t_1\le t_2\le\ldots\le t_n\right)}{\bigoplus}\left(\oplus_{r=1}^{k} w_{t_r}\tilde{S}_{t_r}\right)=\left(\begin{array}{c}\left\langle \tilde{S}_{\sum_{\left(1\le t_1\le t_2\le\ldots\le t_n\right)}\prod_{r=1}^{n} w_{t_r}a_{t_r}},\tilde{S}_{\sum_{\left(1\le t_1\le t_2\le\ldots\le t_n\right)}\prod_{r=1}^{n} w_{t_r}b_{t_r}}\right\rangle,\\ \left\langle \tilde{S}_{\frac{\frac{1}{\left(\hat{\beta}-1\right)^{-1}}\left(1-\prod_{\left(1\le t_1\le t_2\le\ldots\le t_n\right)}\left(1-\frac{\prod_{r=1}^{n} w_{t_r}c_{t_r}}{\left(\hat{\beta}-1\right)}\right)\right)}{\left(\frac{1}{\left(\hat{\beta}-1\right)^{1-k}}\right)}},\tilde{S}_{\frac{\frac{1}{\left(\hat{\beta}-1\right)^{-1}}\left(1-\prod_{\left(1\le t_1\le t_2\le\ldots\le t_n\right)}\left(1-\frac{\prod_{r=1}^{n} w_{t_r}d_{t_r}}{\left(\hat{\beta}-1\right)}\right)\right)}{\left(\frac{1}{\left(\hat{\beta}-1\right)^{1-k}}\right)}}\right\rangle,\\ \left\langle \tilde{S}_{\frac{\frac{1}{\left(\hat{\beta}-1\right)^{-1}}\left(1-\prod_{1\le t_1\le t_2\le\ldots\le t_n}\left(1-\frac{\prod_{r=1}^{n} w_{t_r}e_{t_r}}{\left(\hat{\beta}-1\right)}\right)\right)}{\left(\frac{1}{\left(\hat{\beta}-1\right)^{1-k}}\right)}},\tilde{S}_{\frac{\frac{1}{\left(\hat{\beta}-1\right)^{-1}}\left(1-\prod_{1\le t_1\le t_2\le\ldots\le t_n}\left(1-\frac{\prod_{t=1}^{n} w_{t_r}f_{t_r}}{\left(\hat{\beta}-1\right)}\right)\right)}{\left(\frac{1}{\left(\hat{\beta}-1\right)^{1-k}}\right)}}\right\rangle\end{array}\right)$$

Then we obtain $\frac{1}{C_n^k}\left(\underset{\left(1\le t_1\le t_2\le\ldots\le t_n\right)}{\bigoplus}\left(\oplus_{r=1}^{k} w_{t_r}\tilde{S}_{t_r}\right)\right)=\left(\frac{\underset{\left(1\le t_1\le t_2\le\ldots\le t_n\right)}{\bigoplus}\left(\oplus_{r=1}^{k} w_{t_r}\tilde{S}_{t_r}\right)}{C_n^k}\right)^{\frac{1}{k}}$; Therefore

$$\text{ODULIVNFMSM}^{(k)}\left(\tilde{S}_1,\tilde{S}_2,\tilde{S}_3,\ldots,\tilde{S}_n\right)=\left(\frac{\underset{\left(1\le t_1\le t_2\le\ldots\le t_{k\le}n\right)}{\bigoplus}\left(\oplus_{r=1}^{k} w_{t_r}\tilde{S}_{t_r}\right)}{C_n^k}\right)^{\frac{1}{k}}=$$

$$
= \left(
\begin{array}{l}
\left\langle \tilde{s}_{\left( \frac{\sum_{\left(1 \le t_1 \le t_2 \le \ldots \le t_k \le n\right)} \prod_{r=1}^{k} w_{t_r} \tilde{a}_{t_r}}{C_n^k} \right)^{\left(\frac{1}{k}\right)}}, \tilde{s}_{\left( \frac{\sum_{\left(1 \le t_1 \le t_2 \le \ldots \le t_k \le n\right)} \prod_{r=1}^{k} w_{t_r} \tilde{b}_{t_r}}{C_n^k} \right)^{\left(\frac{1}{k}\right)}} \right\rangle \\
\left\langle \tilde{s}_{\frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left( 1 - \left[ \Pi\left(1 \le t_1 \le t_2 \le \ldots \le t_k \le n\right) \left( 1 - \frac{\prod_{r=1}^{n} \frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left[ 1 - \left( 1 - \frac{c_{t_r}}{\hat{\beta}-1} \right)^{w_{t_r}} \right]}{\left( \frac{1}{\left(\hat{\beta}-1\right)^{-k}} \right)} \right) \right]^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}}, \tilde{s}_{\frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left( 1 - \left[ \Pi\left(1 \le t_1 \le t_2 \le \ldots \le t_k \le n\right) \left( 1 - \frac{\prod_{r=1}^{n} \frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left[ 1 - \left( 1 - \frac{d_{t_r}}{\hat{\beta}-1} \right)^{w_{t_r}} \right]}{\left( \frac{1}{\left(\hat{\beta}-1\right)^{-k}} \right)} \right) \right]^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\rangle, \\
\left\langle \tilde{s}_{\frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left( 1 - \left[ \Pi\left(1 \le t_1 \le t_2 \le \ldots \le t_k \le n\right) \left( 1 - \frac{\prod_{r=1}^{n} \frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left[ 1 - \left( 1 - \frac{e_{t_r}}{\hat{\beta}-1} \right)^{w_{t_r}} \right]}{\left( \frac{1}{\left(\hat{\beta}-1\right)^{-k}} \right)} \right) \right]^{\left(\frac{1}{C_n^k}\right)} \right)^{\frac{1}{k}}}, \tilde{s}_{\frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left( 1 - \left[ \Pi\left(1 \le t_1 \le t_2 \le \ldots \le t_k \le n\right) \left( 1 - \frac{\prod_{r=1}^{n} \frac{1}{\left(\hat{\beta}-1\right)^{-1}} \left[ 1 - \left( 1 - \frac{e_{t_r}}{\hat{\beta}-1} \right)^{w_{t_r}} \right]}{\left( \frac{1}{\left(\hat{\beta}-1\right)^{-k}} \right)} \right) \right]^{\left(\frac{1}{C_n^k}\right)} \right)^{\frac{1}{k}}} \right\rangle,
\end{array}
\right)
$$

**Theorem 3.1.3. Idempotency**

Consider $\breve{S}_r = \left\langle \begin{array}{c} \left[\breve{s}_{\tilde{a}_r}, \breve{s}_{\tilde{b}_r}\right], \\ \left[\breve{s}_{\tilde{c}_r}, \breve{s}_{\tilde{d}_r}\right], \\ \left[\breve{s}_{\tilde{e}_r}, \breve{s}_{\tilde{f}_r}\right] \end{array} \right\rangle$ such that $\langle \mathrm{r} = 1, 2, 3, ..., n \rangle$ ; be a collection of ODULIVNF variable

then $ODULIVNFMSM^{(k)}\left(\breve{s}, \breve{s}, \breve{s}, ..., \breve{s}\right) = \breve{S}$.

**Proof:**

Since must write $ODULIVNFMSM^{(k)}\left(\breve{s}, \breve{s}, \breve{s}, ..., \breve{s}\right) = \breve{S}$

$$= \left[ \begin{array}{c} \left\langle \breve{s}_{\left(\frac{\sum_{(1\le t_1<...<t_k\le n)}\Pi_{i=1}^{k}\tilde{a}_{t_r}}{C_n^k}\right)^{\frac{1}{k}}}, \breve{s}_{\left(\frac{\sum_{(1\le t_1<...<t_k\le n)}\Pi_{i=1}^{k}\tilde{b}_{t_r}}{C_n^k}\right)^{\frac{1}{k}}} \right\rangle, \\ \left\langle \breve{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\Pi_{\left(1\le t_1<...<t_k\le n\right)}\left(1-\frac{\Pi_{i=1}^{n}\tilde{c}_{t_r}}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)\right]^{\left(\frac{1}{C_n^k}\right)}\right)^{\frac{1}{k}}}, \breve{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\Pi_{\left(1\le t_1<...<t_k\le n\right)}\left(1-\frac{\Pi_{i=1}^{n}\tilde{d}_{t_r}}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)\right]^{\left(\frac{1}{C_n^k}\right)}\right)^{\frac{1}{k}}} \right\rangle, \\ \left\langle \breve{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\Pi_{\left(1\le t_1<...<t_k\le n\right)}\left(1-\frac{\Pi_{i=1}^{n}\tilde{e}_{t_r}}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)\right]^{\left(\frac{1}{C_n^k}\right)}\right)^{\frac{1}{k}}}, \breve{s}_{\frac{1}{(q-1)^{-1}}\left(1-\left[\Pi_{\left(1\le t_1<...<t_k\le n\right)}\left(1-\frac{\Pi_{i=1}^{n}\tilde{f}_{t_r}}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)\right]^{\left(\frac{1}{C_n^k}\right)}\right)^{\frac{1}{k}}} \right\rangle \end{array} \right] \quad (16)$$

$$
= \left[ \begin{array}{l}
\left\langle \breve{S}_{\left(\frac{C_n^k \tilde{a}^k}{C_n^k}\right)^{\frac{1}{k}}}, \breve{S}_{\left(\frac{C_n^k \tilde{b}^k}{C_n^k}\right)^{\frac{1}{k}}} \right\rangle, \\
\left\langle \breve{S}_{\frac{1}{\left(\hat{\beta}-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{c}^k}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)^{\frac{1}{C_n^k}}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}, \breve{S}_{\frac{1}{\left(\hat{\beta}-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{d}^k}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)^{\frac{1}{C_n^k}}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}} \right\rangle, \\
\left\langle \breve{S}_{\frac{1}{\left(\hat{\beta}-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{e}^k}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)^{\frac{1}{C_n^k}}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}, \breve{S}_{\frac{1}{\left(q-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{f}^k}{\frac{1}{(\hat{\beta}-1)^{-k}}}\right)^{\frac{1}{C_n^k}}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}} \right\rangle
\end{array} \right]
$$

$$= \left( \begin{array}{l} \left\langle \breve{S}_{\left(\tilde{a}^k\right)^{\frac{1}{k}}}, \breve{S}_{\left(\tilde{b}^k\right)^{\frac{1}{k}}} \right\rangle, \\ \left\langle \breve{S}_{\frac{1}{\left(\widehat{\beta}-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{c}^k}{\frac{1}{(\widehat{\beta}-1)^{-k}}}\right)\right]\right)^{\frac{1}{k}}}, \breve{S}_{\frac{1}{\left(\widehat{\beta}-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{d}^k}{\frac{1}{(\widehat{\beta}-1)^{-k}}}\right)\right]\right)^{\frac{1}{k}}} \right\rangle, \\ \left[ \breve{S}_{\frac{1}{\left(\widehat{\beta}-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{e}^k}{\frac{1}{(\widehat{\beta}-1)^{-k}}}\right)\right]\right)^{\frac{1}{k}}}, \breve{S}_{\frac{1}{\left(\widehat{\beta}-1\right)^{-1}}\left(1-\left[\left(1-\frac{\tilde{f}^k}{\frac{1}{(\widehat{\beta}-1)^{-k}}}\right)\right]\right)^{\frac{1}{k}}} \right] \end{array} \right)$$

$$==\left\langle \begin{matrix} \left[ s_{\tilde{a}}, s_{\tilde{b}} \right], \\ \left[ s_{\tilde{c}}, s_{\tilde{d}} \right], \\ \left[ s_{\tilde{e}}, s_{\tilde{f}} \right] \end{matrix} \right\rangle$$

Which is required proof.

**Theorem 3.1.4: Commutativity**

**Let us consider** $(\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n)$ is any permutation of $(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n)$

$$ODULIVNFMSM^{(k)}(\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n) = ODULIVNFMSM^{(k)}(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n)$$

**Proof:**

Since $ODULIVNFMSM^{(k)}(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n) = \left( \frac{\sum_{(1 \le t_1 < ... < t_k \le n)} \Pi_{r=1}^{k} \tilde{s}_{t_r}}{C_n^k} \right)^{\frac{1}{k}} = \left( \frac{\sum_{(1 \le t_1 < ... < t_k \le n)} \Pi_{r=1}^{k} \hat{s}_{t_r}}{C_n^k} \right)^{\frac{1}{k}} =$

$ODULIVNFMSM^{(k)}(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n)$. Which is required result.

**Theorem 3.1.5: Monotonicity**

If $\breve{a}_{t_r} \ge a_{t_r}, \breve{b}_{t_r} \ge b_{t_r}, \breve{c}_{t_r} \ge c_{t_r}, \breve{d}_{t_r} \ge d_{t_r}, \breve{e}_{t_r} \ge e_{t_r}$ and $\breve{f}_{t_r} \ge f_{t_r}$. for all r,

$$ODULIVNFMSM^{(k)}(\breve{s}_1, \breve{s}_2, \breve{s}_3, ..., \breve{s}_n) = ODULIVNFMSM^{(k)}(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n)$$

**Proof:**

Since $\mathrm{ODULIVNFMSM}^{(k)}(\breve{s}_1, \breve{s}_2, \breve{s}_3, ..., \breve{s}_n) = \left( \frac{\sum_{(1 \le t_1 < ... < t_k \le n)} \Pi_{r=1}^{k} \breve{s}_{t_r}}{C_n^k} \right)^{\frac{1}{k}}$

$$
= \left[ \begin{array}{l}
\left\langle \tilde{s}_{\left(\frac{\sum_{(1\le t_1<\ldots<t_k\le n)}\prod_{r=1}^{k}\breve{a}}{C_n^k}\right)^{\frac{1}{k}}}, \tilde{s}_{\left(\frac{\sum_{(1\le t_1<\ldots<t_k\le n)}\prod_{r=1}^{k}\breve{b}}{C_n^k}\right)^{\frac{1}{k}}} \right\rangle \\
\left\langle \tilde{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\breve{c}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}, \tilde{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\breve{d}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}} \right\rangle, \\
\left\langle \tilde{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\breve{e}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}, \tilde{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\breve{f}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}} \right\rangle
\end{array} \right] \quad (17)
$$

If $\breve{a}_{t_r} \geq a_{t_r}, \breve{b}_{t_r} \geq b_{t_r}, \breve{c}_{t_r} \geq c_{t_r}, \breve{d}_{t_r} \geq d_{t_r}, \breve{e}_{t_r} \geq e_{t_r}$ and $\breve{f}_{t_r} \geq f_{t_r}$.

Since $\left\langle \breve{a}_{t_r} \geq a_{t_r} \right\rangle$, then

$$\left(\frac{1}{C_n^k}\left(\sum_{(1\leq t_1<...<t_k\leq n)}\Pi_{r=1}^k \breve{a}_{t_r}\right)\right)^{\frac{1}{k}} \geq \left(\frac{1}{C_n^k}\left(\sum_{(1\leq t_1<...<t_k\leq n)}\Pi_{r=1}^k a_{t_r}\right)\right)^{\frac{1}{k}}$$

Same for $\left\langle \breve{b}_{t_r} \geq b_{t_r} \right\rangle$ we must write,

$$\left(\frac{1}{C_n^k}\sum_{1\leq t_1<...<t_k\leq n}\Pi_{r=1}^k \breve{b}_{t_r}\right)^{\frac{1}{k}} \geq \left(\frac{1}{C_n^k}\sum_{1\leq t_1<...<t_k\leq n}\Pi_{r=1}^k b_{t_r}\right)^{\frac{1}{k}}$$

Since $\left\langle \breve{c}_{t_r} \geq c_{t_r} \right\rangle$, then $\left\langle \frac{\left(\Pi_{r=1}^n \breve{c}_{t_r}\right)}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)} \right\rangle \geq \left\langle \frac{\left(\Pi_{r=1}^n c_{t_r}\right)}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)} \right\rangle$ and

$$\left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n \breve{c}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}} \leq \left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n c}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}$$

$$\Rightarrow 1-\left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n \breve{c}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}} \geq 1-\left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n c}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}$$

Then

$$\left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n \breve{c}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}} \right\rangle \geq \left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n c}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}} \right\rangle$$

Similarly, for $\breve{d}_{t_r} \geq d_{j_i}, \breve{e}_{t_r} \geq e_{t_r}$ and $\breve{f}_{t_r} \geq f_{t_r}$

$$\left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n \breve{d}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}} \right\rangle \geq \left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\leq t_1<...<t_k\leq n)}\left(1-\frac{\Pi_{r=1}^n d}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}} \right\rangle$$

$$\left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\Pi_{r=1}^{n}\breve{e}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right\rangle \ge \left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\Pi_{r=1}^{n}e}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right\rangle$$

$$\left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\Pi_{r=1}^{n}\breve{f}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right\rangle \ge \left\langle \left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{(1\le t_1<\ldots<t_k\le n)}\left(1-\frac{\Pi_{r=1}^{n}f}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right\rangle$$

Using # 3.3, we have

$$E\left(\tilde{s}_r\right)=\left\langle \frac{\left(\breve{a}_r+\breve{b}_r\right)}{6(p-1)}\times\frac{\left(\tilde{c}_r+\breve{d}_r\right)}{6(q-1)}\times\frac{\left(\breve{e}_r+\breve{f}_r\right)}{6(r-1)}\right\rangle \tag{18}$$

And

$$E\left(\dot{s}_r\right)=\left\langle \frac{\left(a_r+b_r\right)}{6(p-1)}\times\frac{\left(c_r+d_r\right)}{6(q-1)}\times\frac{\left(e_r+f_r\right)}{6(r-1)}\right\rangle \tag{19}$$

And then by definition 3.4 we can write $E\left(\tilde{s}_r\right)\ge E\left(\dot{s}_r\right)$. So, finally we have

$$ODULIVNFMSM^{(k)}\left(\breve{s}_1,\breve{s}_2,\breve{s}_3,\ldots,\breve{s}_n\right)=ODULIVNFMSM^{(k)}\left(\hat{s}_1,\hat{s}_2,\ldots,\hat{s}_n\right).$$

Which is the required result.

**Theorem 3.1.6: Boundedness**

If $\begin{bmatrix}\left\langle \breve{s}^-=\min\left(\breve{s}_1^-,\breve{s}_2^-,\ldots,\breve{s}_n^-\right)\right\rangle \\ \left\langle \breve{s}^+=\max\left(\breve{s}_1^+,\breve{s}_2^+,\ldots,\breve{s}_n^+\right)\right\rangle\end{bmatrix}$, then $\breve{S}^-\le ODULNFMSM^{(k)}\left(\breve{s}_1,\breve{s}_2,\ldots,\breve{s}_n\right)\le\breve{S}^+$

**Proof:**

Suppose $\begin{bmatrix}\left\langle \breve{s}^-=\min\left(\breve{s}_1^-,\breve{s}_2^-,\ldots,\breve{s}_n^-\right)\right\rangle \\ \left\langle \breve{s}^+=\max\left(\breve{s}_1^+,\breve{s}_2^+,\ldots,\breve{s}_n^+\right)\right\rangle\end{bmatrix}$. According to the above theorem, we have

$ODULIVNFMSM^{(k)}\left(\breve{s}^-,\breve{s}^-,\ldots,\breve{s}^-\right)\le ODULIVNFMSM^{(k)}\left(\breve{s},\breve{s},\ldots,\breve{s}\right)\le ODULIVNFMSM^{(k)}\left(\breve{S}^+,\breve{S}^+,\ldots,\breve{S}^+\right)$ Now, according to theorem 3.1.3 we have,

Thus,

$$s^-\le ODULNFMSM^{(k)}\left(\tilde{s}_1,\tilde{s}_2,\tilde{s}_3,\ldots,\tilde{s}_n\right)\le s^+.$$

Which is required result.

### 3.2 Some special k-based feature of ODULNFMSM operator

**(1):** By taking k=1, the ODULIVNFMSM operator took the form of ODULIVNF arithmetic operator.

$$ODULIVNFMSM^{(1)}\left(\tilde{s}_1,\tilde{s}_2,\tilde{s}_3,...,\tilde{s}_n\right)=$$

$$\left[\begin{array}{l}\left\langle \tilde{S}_{\frac{\sum_{i=1}^{n}\tilde{a}_i}{n}},\tilde{S}_{\frac{\sum_{i=1}^{n}\tilde{b}_i}{n}}\right\rangle,\\ \left\langle \tilde{S}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{j=1}^{n}\left(1-\frac{\tilde{c}_i}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]\right)^{1/n}},\tilde{S}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{j=1}^{n}\left(1-\frac{\breve{d}_i}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]\right)^{1/n}}\right\rangle,\\ \left\langle \tilde{S}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{j=1}^{n}\left(1-\frac{\breve{e}_i}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]\right)^{1/n}},\tilde{S}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi_{j=1}^{n}\left(1-\frac{\breve{f}_i}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]\right)^{1/n}}\right\rangle\end{array}\right] \quad (20)$$

**(2):** By taking k=2, the ODULNFMSM operator took the form of ODULIVNFB operator.

$$\mathrm{ODULIVNFMSM}^{(1)}\left(\tilde{S}_1,\tilde{S}_2,\tilde{S}_3,\ldots,\tilde{S}_n\right)=$$

$$\left[\begin{array}{l}\left\langle \tilde{s}_{\left(\frac{\sum_{(1\le j_1<\ldots<j_k\le n)}\prod_{i=1}^{k}\tilde{a}_{j_i}}{C_n^2}\right)^{\frac{1}{2}}}, \tilde{s}_{\left(\frac{\sum_{(1\le j_1<\ldots<j_k\le n)}\prod_{i=1}^{k}\tilde{b}_{j_i}}{C_n^2}\right)^{\frac{1}{2}}}\right\rangle, \\ \left\langle \tilde{s}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\prod_{(1\le j_1<\ldots<j_k\le n)}\left(1-\frac{\prod_{i=1}^{n}\tilde{c}_{j_i}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^2}}\right)^{\frac{1}{2}}}, \tilde{s}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\prod_{(1\le j_1<\ldots<j_k\le n)}\left(1-\frac{\prod_{i=1}^{n}\tilde{d}_{j_i}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^2}}\right)^{\frac{1}{2}}}\right\rangle, \\ \left\langle \tilde{s}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\prod_{(1\le j_1<\ldots<j_k\le n)}\left(1-\frac{\prod_{i=1}^{n}\tilde{e}_{j_i}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^2}}\right)^{\frac{1}{2}}}, \tilde{s}_{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\prod_{(1\le j_1<\ldots<j_k\le n)}\left(1-\frac{\prod_{i=1}^{n}\tilde{f}_{j_i}}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^2}}\right)^{\frac{1}{2}}}\right\rangle \end{array}\right]$$

(21)

$$=\mathrm{ODULIVNFMSM}^{(2)}\left(\tilde{s}_1,\tilde{s}_2,\tilde{s}_3,\ldots,\tilde{s}_n\right).$$

**(3):** By taking k=n, the ODULNFMSM operator took the form of ODULNFM operator (p=1, q=1).

$$\text{ODULIVNFMSM}^{(n)}\left(\tilde{s}_1,\tilde{s}_2,\tilde{s}_3,...,\tilde{s}_n\right)=\begin{bmatrix}\left\langle \tilde{s}_{\left(\Pi_{i=1}^{k}\tilde{a}_{ji}\right)^{\frac{1}{n}}},\tilde{s}_{\left(\Pi_{i=1}^{k}\tilde{b}_{ji}\right)^{\frac{1}{n}}}\right\rangle, \\ \left\langle \tilde{s}_{\left(\Pi_{i=1}^{k}\tilde{c}_{ji}\right)^{\frac{1}{n}}},\tilde{s}_{\left(\Pi_{i=1}^{k}\tilde{d}_{ji}\right)^{\frac{1}{n}}}\right\rangle, \\ \left\langle \tilde{s}_{\left(\Pi_{i=1}^{k}\tilde{e}_{ji}\right)^{\frac{1}{n}}},\tilde{s}_{\left(\Pi_{i=1}^{k}\tilde{f}_{ji}\right)^{\frac{1}{n}}}\right\rangle\end{bmatrix}$$

### 3.3 One-dimension uncertain linguistic interval valued neutrosophic weighted fuzzy MSM aggregation operator (ODULIVNWFMSM)

**Def: 3.3.1.** Consider $\tilde{S}_r=\left\langle\begin{matrix}\left[s_{\tilde{a}_r},s_{\tilde{b}_r}\right], \\ \left[s_{\tilde{c}_r},s_{\tilde{d}_r}\right], \\ \left[s_{\tilde{e}_r},s_{\tilde{f}_r}\right]\end{matrix}\right\rangle$ such that $\text{r}=[1:1:n]$; be a non-empty collection of ODULIVNF variable and w denoting the weight values that is.

$\tilde{S}_r$; with $w_t\in[0,1], t=1,2,...,n;$ and $\sum_{t=1}^{n}w_t=1.$

If $$\text{WODULNFMSM}^{(k)}\left(\tilde{S}_1,\tilde{S}_2,\tilde{S}_3,...,\tilde{S}_n\right)=\left(\frac{\otimes_{1\le t_1<...<t_k\le n}\left(\oplus_{r=1}^{k}\left(W_{t_r}\otimes\tilde{S}_{t_r}\right)\right)}{C_n^k}\right)^{\frac{1}{k}} \tag{22}$$

Then ODULIVNFMSM is said to be ODULIVNWFMSM operator. Where k tuples combination of 1,…,n of t is $\left(t_1,t_2,t_3,...,t_k\right)$ and $C_n^k$ denotes the binomial coefficient.

**Theorem 3.3.2.** Consider $\tilde{S}_r=\left\langle\begin{matrix}\left[s_{\tilde{a}_r},s_{\tilde{b}_r}\right], \\ \left[s_{\tilde{c}_r},s_{\tilde{d}_r}\right], \\ \left[s_{\tilde{e}_r},s_{\tilde{f}_r}\right]\end{matrix}\right\rangle$ such that $\text{r}=[1:1:n]$; be a non-empty collection of ODULIVNF variable and $w$ denoting the Weight values of $\tilde{S}_r$; $w_t\in[0,1], t=1,2,...,n;$ and $\sum_{t=1}^{n}w_t=1.$ Then ODULIVNFN can be obtained as fallows.

$$ODULIVNWFMSM^{(k)}\left(\tilde{S}_1,\tilde{S}_2,\tilde{S}_3,...,\tilde{S}_n\right)=$$

$$
=\left(\begin{array}{l}
\left\langle \tilde{S}\left(\frac{\Sigma\left(1\le t_1<\ldots<t_k\le n\right)\Pi_{r=1}^{k} w_{t_r}\tilde{a}_{t_r}}{C_n^k}\right)^{\frac{1}{k}},\tilde{S}\left(\frac{\Sigma\left(1\le j_1<\ldots<j_k\le n\right)\Pi_{i=1}^{k} w_{t_r}\tilde{b}_{t_r}}{C_n^k}\right)^{\frac{1}{k}}\right\rangle, \\
\left\langle \tilde{S}\left(\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi\left(1\le t_1\le t_2\le\ldots\le t_{k}\le n\right)\left(1-\frac{\Pi_{r=1}^{n}\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left[1-\left(1-\frac{c_{t_r}}{\left(\hat{\beta}-1\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)\right)^{\frac{1}{k}},\tilde{S}\left(\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi\left(1\le t_1\le t_2\le\ldots\le t_{k}\le n\right)\left(1-\frac{\Pi_{r=1}^{n}\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left[1-\left(1-\frac{d_{t_r}}{\left(\hat{\beta}-1\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)\right)^{\frac{1}{k}}\right\rangle, \\
\left\langle \tilde{S}\left(\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi\left(1\le t_1\le t_2\le\ldots\le t_{k}\le n\right)\left(1-\frac{\Pi_{r=1}^{n}\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left[1-\left(1-\frac{e_{t_r}}{\left(\hat{\beta}-1\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)\right)^{\frac{1}{k}},\tilde{S}\left(\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\left[\Pi\left(1\le t_1\le t_2\le\ldots\le t_{k}\le n\right)\left(1-\frac{\Pi_{r=1}^{n}\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left[1-\left(1-\frac{f_{t_r}}{\left(\hat{\beta}-1\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)\right)^{\frac{1}{k}}\right\rangle
\end{array}\right) \tag{23}
$$

**Proof:**

From the point of view of operational rules of ODULIVNF Variables, we must write

$$\oplus_{r=1}^{k}\left(\tilde{S}_{t_r}\right)=\left(\begin{array}{l}\left\langle \tilde{s}_{\prod_{r=1}^{n} a_{t_r}}, \tilde{s}_{\prod_{r=1}^{n} b_{t_r}} \right\rangle, \\ \left\langle \frac{\tilde{s}_{\prod_{r=1}^{n} c_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}, \frac{\tilde{s}_{\prod_{r=1}^{n} d_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)} \right\rangle, \\ \left\langle \frac{\tilde{s}_{\prod_{r=1}^{n} e_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}, \frac{\tilde{s}_{\prod_{t=1}^{n} f_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)} \right\rangle \end{array}\right)$$

$$\oplus_{r=1}^{k}\left(w_{t_r}\tilde{S}_{t_r}\right)=\left(\begin{array}{c}\left\langle \tilde{s}_{\prod_{r=1}^{n} w_{t_r} a_{t_r}}, \tilde{s}_{\prod_{r=1}^{n} w_{t_r} b_{t_r}} \right\rangle, \\ \left\langle \frac{\tilde{s}_{\prod_{r=1}^{n} w_{t_r} c_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}, \frac{\tilde{s}_{\prod_{r=1}^{n} w_{t_r} d_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)} \right\rangle, \\ \left\langle \frac{\tilde{s}_{\prod_{r=1}^{n} w_{t_r} e_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)}, \frac{\tilde{s}_{\prod_{r=1}^{n} w_{t_r} f_{t_r}}}{\left(\frac{1}{\left(\widehat{\beta}-1\right)^{1-k}}\right)} \right\rangle \end{array}\right)$$

And

$$\bigoplus_{(1\le t_1\le t_2\le \ldots \le t_n)}\left(\oplus_{r=1}^{k}\left(w_{t_r}\tilde{S}_{t_r}\right)\right)=$$

$$\left(\begin{array}{c}\left\langle \tilde{s}_{\sum_{(1\le t_1\le t_2\le\ldots\le t_n)}\prod_{r=1}^{n} w_{t_r} a_{t_r}}, \tilde{s}_{\sum_{(1\le t_1\le t_2\le\ldots\le t_n)}\prod_{r=1}^{n} w_{t_r} b_{t_r}} \right\rangle, \\ \left\langle \tilde{s}_{\frac{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\prod_{(1\le t_1\le t_2\le\ldots\le t_n)}\left(1-\frac{\prod_{r=1}^{n} w_{t_r} c_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)\right)}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}}, \tilde{s}_{\frac{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\prod_{(1\le t_1\le t_2\le\ldots\le t_n)}\left(1-\frac{\prod_{r=1}^{n} w_{t_r} d_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)\right)}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}} \right\rangle, \\ \left\langle \tilde{s}_{\frac{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\prod_{(1\le t_1\le t_2\le\ldots\le t_n)}\left(1-\frac{\prod_{r=1}^{n} w_{t_r} e_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)\right)}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}}, \tilde{s}_{\frac{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\prod_{(1\le t_1\le t_2\le\ldots\le t_n)}\left(1-\frac{\prod_{t=1}^{n} w_{t_r} f_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)\right)}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}} \right\rangle \end{array}\right)$$

Then we obtain

$$\frac{1}{C_n^k}\left(\bigoplus_{1\le t_1\le t_2\le\ldots\le t_n}\left(\oplus_{r=1}^{k}\left(w_{t_r}\tilde{S}_{t_r}\right)\right)\right)=\left(\frac{\bigoplus_{1\le t_1\le t_2\le\ldots\le t_n}\left(\oplus_{r=1}^{k}\left(w_{t_r}\tilde{S}_{t_r}\right)\right)}{C_n^k}\right)^{\frac{1}{k}}$$

$$= \left( \begin{array}{l} \left\langle \tilde{s}_{\frac{\sum_{(1\le t_1\le t_2\le \ldots \le t_k\le n)} \prod_{r=1}^{n} w_{t_r} a_{t_r}}{C_n^k}}, \tilde{s}_{\frac{\sum_{(1\le t_1\le t_2\le \ldots \le t_k\le n)} \prod_{r=1}^{n} w_{t_r} b_{t_r}}{C_n^k}} \right\rangle, \\ \left\langle \tilde{s}_{\frac{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\prod_{(1\le t_1\le t_2\le \ldots \le t_k\le n)}\left[1-\frac{\prod_{r=1}^{n} w_{t_r} c_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}}, \tilde{s}_{\frac{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\prod_{(1\le t_1\le t_2\le \ldots \le t_k\le n)}\left[1-\frac{\prod_{r=1}^{n} w_{t_r} d_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}} \right\rangle, \\ \left\langle \tilde{s}_{\frac{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\prod_{1\le t_1\le t_2\le \ldots \le t_k\le n}\left[1-\frac{\prod_{r=1}^{n} w_{t_r} e_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}}, \tilde{s}_{\frac{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left(1-\prod_{(1\le t_1\le t_2\le \ldots \le t_k\le n)}\left[1-\frac{\prod_{i=1}^{n} w_{t_r} e_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}{\left(\frac{1}{(\hat{\beta}-1)^{1-k}}\right)}} \right\rangle \end{array} \right)$$

Therefore

$$\text{ODULIVNWFMSM}^{(k)}\left(\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \ldots, \tilde{S}_n\right) = \left( \frac{\bigoplus_{(1\le t_1 \ldots \le t_{k\le} n)} \left(\oplus_{r=1}^{k}\left(w_{t_r} \tilde{S}_{t_r}\right)\right)}{C_n^k} \right)^{\frac{1}{k}}$$

$$
\left(
\begin{array}{l}
\left\langle \tilde{s}_{\left(\frac{\sum_{(1\le t_1\le t_2\le\ldots\le t_k\le n)}\prod_{r=1}^{k} w_{t_r}\tilde{a}_{t_r}}{C_n^k}\right)^{\frac{1}{k}}}, \tilde{s}_{\left(\frac{\sum_{(1\le t_1\le t_2\le\ldots\le t_k\le n)}\prod_{r=1}^{k} w_{t_r}\tilde{b}_{t_r}}{C_n^k}\right)^{\frac{1}{k}}} \right\rangle, \\
= \left\langle
\begin{array}{l}
\tilde{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod_{(1\le t_1\le t_2\le\ldots\le t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left[1-\left(1-\frac{c_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}, \tilde{s}_{\frac{1}{(q-1)^{-1}}\left(1-\left[\prod_{(1\le t_1\le t_2\le\ldots\le t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)\left[1-\left(1-\frac{d_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}} \\
\tilde{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod_{(1\le t_1\le t_2\le\ldots\le t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\frac{1}{(\hat{\beta}-1)^{-1}}\left[1-\left(1-\frac{e_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}, \tilde{s}_{\frac{1}{(\hat{\beta}-1)^{-1}}\left(1-\left[\prod_{(1\le t_1\le t_2\le\ldots\le t_k\le n)}\left(1-\frac{\prod_{r=1}^{n}\frac{1}{(\hat{\beta}-1)^{-1}}\left[1-\left(1-\frac{f_{t_r}}{\left(\frac{1}{(\hat{\beta}-1)^{-1}}\right)}\right)^{w_{t_r}}\right]}{\left(\frac{1}{(\hat{\beta}-1)^{-k}}\right)}\right)\right]^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}
\end{array}
\right\rangle
\end{array}
\right)
$$

The other desirable properties may be easily proved on same lines as stated above.

### 4. MCDM via OULIVNWFMSM operator

For tactful evaluation, consider a finite set $A$ of alternatives D be a set of decision makers values and $\lambda$ represent a weight vector made by DMs. $D_\mu$ obtained by $\lambda_\mu \in [0,1]$ $for\,\mu$=1,2,...,p; while weight sum is equal to 1 that is $\sum_{\mu=1}^{p} \lambda_\mu = 1$. The attributes set is C with $\omega = (\omega_{1,} \omega_2, ...., \omega_p)^T$. The $\omega_j \in [0,1]$ $and \sum_{j=1}^{n} \omega_j = 1 with\ j = 1, 2, ..., n.$ Thus $S^\mu = [s_{i_j}^\mu]_{m\times n}$ ; $\mu = 1, 2, ..., p$

$S^\mu = [s_{i_j}^\mu]_{m\times n}$ ; $\mu = 1, 2, ..., p$ is a decision matrix and looks $S_{i_j}^\mu = \left( \left[ s_{a_{i_j}^\mu}, s_{b_{i_j}^\mu} \right], \left[ s_{c_{i_j}^\mu}, s_{d_{i_j}^\mu} \right], \left[ s_{e_{i_j}^\mu}, s_{f_{i_j}^\mu} \right] \right)$ gives the evaluation value alternatives because of attributes values. The process can be shortly demonstrated by algorithm with flow chart.

### 4.1 ALGORITHM

The following are the necessary and sufficient steps for evaluating numerical data.

**Step-1:** Initially we calculate the ODULIVN fuzzy decision-matrix".

**Step-2:** The Normalization of attributes will be sort out if required e.g., if they are different types i.e., benefit or cost .

**Step-3:** We aggregate the fuzzy informative data of each decision maker by proposed operator.

**Step-4:** In third step we calculate the expectancy of each ODULIVNF variable using $E(\tilde{S}_1) = \left( \frac{l+m}{6(\alpha-1)} \times \frac{n+o}{6(\beta-1)} \times \frac{p+q}{6(\gamma-1)} \right)$

**Step-5:** In last step we rank the chosen alternatives by adopting the expectancy criteria stated in definition 3.4.

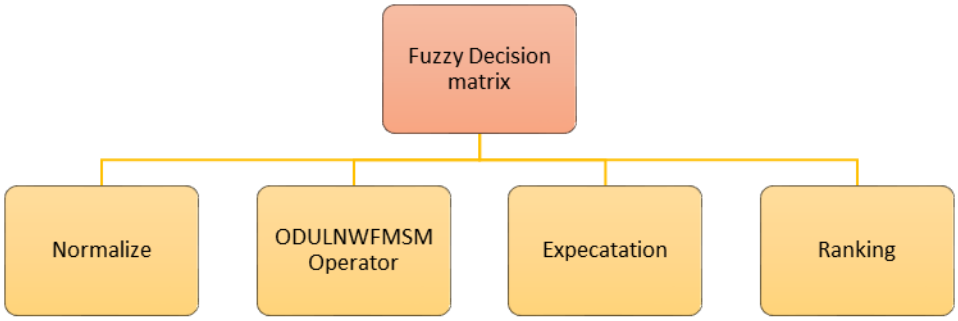


**Figure-1: Flow chart of proposed mechanism**

### 4.2 Practical utility of the proposed aggregation operators in business analytics

Business analytics (BA) is an area that drives concrete, data-driven modifications in a business. Indeed, it is a tool a business group requires to take accurate decisions that are probable to influence the whole organization for they assist in improving lucrativeness. BA is a real-world utility of arithmetical analysis that emphasizes on providing practicable commendations. BA specialists engage in how to use the perceptions they derive from the data. Their objective is to draw concrete conclusions pertaining to a business by finding answers to specific queries about why things happened (past analysis of the happening), what will happen (forecasting) and what should be done (recommending necessary measures to be taken essentially). The experts of BA pool the fields of administration and business accompanied by the techniques of information technology that are successfully employed in this field. The business feature involves a preeminent knowledge of the business in addition to the practical inhibitions that subsist. The analytical part comprises a clear and flawless perception of the data handling using information technology techniques whose combination certainly bridges the gap amongst administration and technology.

Exploring the existing data, the business analytics give valuable suggestions to tackle hindrances and improve businesses. Many take-away restaurants and fast-food companies around the globe have been successfully implementing BA to enhance their business that leads to reasonable increase in profits and expansion of their business. By keeping an eye on how engaged the drive-thru is, these businesses can boost their effectualness in the course of prime times of their business. When the que becomes over-crowded, the digital order boards change. They start highlighting those products which can be readied and offered expeditiously. When there is less traffic, employees can suggest items that are more expensive (having higher margins) and take more time in preparation. Other sorts of BA applications perform more than merely responding to the prevailing situation. These methodologies give a helping hand to businesses anticipate which customers are least probable to come again. In such a case, they can then focus on promotions and advertisement to such customers to lift the rate of retention.

**4.3 Example:**

Assume that a fast-food restaurant wants to expand its business by increasing its customers. The restaurant works from 2:00 PM to 2:00 AM. The restaurant hires the services of BA to flourish its business and make it more lucrative. For this purpose, the restaurant assigns the task of deciding at which time slots the restaurant should offer which package of fast food to gain and retain maximum customers. The time slots available are in table-1 and available packages are shown in table-2. noted that slots will represent the alternatives and packages will show attributes values in numerical computation.

| Slots | Time distribution | |
|---|---|---|
| | startup | end up |
| S-1 | 2-PM | 4-PM |
| S-2 | 4-PM | 6-PM |
| S-3 | 6- PM | 6-PM |
| S-4 | 6-PM | 8-PM |
| S-5 | 8-PM | 10-PM |
| S-6 | 10-PM | 12-PM |

**Table-1**

Suppose that there are six packages available.

| | |
|---|---|
| P1 | Package-1 |
| P2 | Package-2 |
| P3 | Package-1 |
| P4 | Package-1 |
| P5 | Package-5 |
| P6 | Package-6 |

**Table-2**

**The evaluation steps by the proposed method**

| *Alternatives /Attributes* | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| $\tilde{A}_1$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_2,s_4]\\ [s_1,s_4]\\ [s_2,s_3]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_2,s_3]\\ [s_1,s_5]\\ [s_2,s_3]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_2]\\ [s_4,s_5]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ |
| $\tilde{A}_2$ | $\begin{pmatrix}[s_1,s_2]\\ [s_4,s_5]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_2,s_4]\\ [s_1,s_4]\\ [s_2,s_3]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ |
| $\tilde{A}_3$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_2]\\ [s_4,s_5]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_2,s_4]\\ [s_1,s_4]\\ [s_2,s_3]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ |
| $\tilde{A}_4$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_2]\\ [s_4,s_5]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ |
| $\tilde{A}_5$ | $\begin{pmatrix}[s_1,s_2]\\ [s_4,s_5]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ |
| $\tilde{A}_6$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_2,s_4]\\ [s_1,s_4]\\ [s_2,s_3]\end{pmatrix}$ | $\begin{pmatrix}[s_1,s_3]\\ [s_3,s_5]\\ [s_2,s_4]\end{pmatrix}$ | $\begin{pmatrix}[s_4,s_6]\\ [s_2,s_5]\\ [s_1,s_6]\end{pmatrix}$ | $\begin{pmatrix}[s_5,s_6]\\ [s_1,s_2]\\ [s_3,s_4]\end{pmatrix}$ |

**Table-3**

| | | | | | |
|---|---|---|---|---|---|
| $\begin{pmatrix}[s_{0.3210},s_{0.4203}]\\ [s_{0.3213},s_{1.4312}]\\ [s_{0.2642},s_{1.0204}]\end{pmatrix}$ | $\begin{pmatrix}[s_{1.0231},s_{1.4323}]\\ [s_{0.2393},s_{1.1125}]\\ [s_{0.3542},s_{1.5304}]\end{pmatrix}$ | $\begin{pmatrix}[s_{1.2791},s_{1.9433}]\\ [s_{0.9823},s_{1.3625}]\\ [s_{0.3352},s_{0.7634}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.1013},s_{0.4002}]\\ [s_{0.4913},s_{1.0391}]\\ [s_{1.2622},s_{1.9214}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.0231},s_{0.1322}]\\ [s_{0.2991},s_{0.6021}]\\ [s_{0.8572},s_{1.5201}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.2701},s_{0.3433}]\\ [s_{1.3121},s_{1.5625}]\\ [s_{0.6752},s_{0.8694}]\end{pmatrix}$ |
| $\begin{pmatrix}[s_{1.9210},s_{1.9243}]\\ [s_{0.9243},s_{0.9742}]\\ [s_{1.2132},s_{1.5204}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.0311},s_{0.5353}]\\ [s_{0.0333},s_{0.1721}]\\ [s_{0.2507},s_{0.6314}]\end{pmatrix}$ | $\begin{pmatrix}[s_{1.0071},s_{1.0233}]\\ [s_{0.7403},s_{1.3155}]\\ [s_{0.2132},s_{0.6534}]\end{pmatrix}$ | $\begin{pmatrix}[s_{1.0248},s_{1.9463}]\\ [s_{0.2213},s_{1.2071}]\\ [s_{0.2034},s_{1.1264}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.0301},s_{0.5823}]\\ [s_{0.5631},s_{0.6741}]\\ [s_{0.5436},s_{1.0014}]\end{pmatrix}$ | $\begin{pmatrix}[s_{1.4829},s_{1.8213}]\\ [s_{0.7403},s_{1.3895}]\\ [s_{0.2082},s_{1.6545}]\end{pmatrix}$ |
| $\begin{pmatrix}[s_{1.5213},s_{1.6243}]\\ [s_{0.3219},s_{0.4710}]\\ [s_{0.0142},s_{0.1294}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.0235},s_{1.0393}]\\ [s_{1.0303},s_{1.1985}]\\ [s_{0.5102},s_{0.5397}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.6531},s_{1.3106}]\\ [s_{0.6433},s_{0.9891}]\\ [s_{0.2317},s_{0.7994}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.0413},s_{1.0244}]\\ [s_{0.3419},s_{0.7360}]\\ [s_{0.0102},s_{0.5936}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.5615},s_{0.7390}]\\ [s_{0.3751},s_{0.5721}]\\ [s_{1.5462},s_{1.5017}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.5329},s_{1.0986}]\\ [s_{0.2671},s_{1.3891}]\\ [s_{1.4317},s_{1.0207}]\end{pmatrix}$ |

**Table-2**

| | | | | | |
|---|---|---|---|---|---|
| $\begin{pmatrix}[s_{0.3241},s_{0.5378}]\\ [s_{0.1090},s_{1.4218}]\\ [s_{0.2902},s_{1.3284}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.0192},s_{1.3109}]\\ [s_{1.1343},s_{1.9340}]\\ [s_{0.2132},s_{1.5004}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.3081},s_{0.6597}]\\ [s_{0.3219},s_{1.8710}]\\ [s_{0.0142},s_{0.1004}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.5219},s_{1.8243}]\\ [s_{0.3219},s_{0.8719}]\\ [s_{0.4173},s_{1.6494}]\end{pmatrix}$ | $\begin{pmatrix}[s_{1.0815},s_{1.4203}]\\ [s_{0.2381},s_{0.4710}]\\ [s_{1.0102},s_{1.1291}]\end{pmatrix}$ | $\begin{pmatrix}[s_{0.8213},s_{1.3462}]\\ [s_{0.1210},s_{0.6745}]\\ [s_{1.4523},s_{1.5274}]\end{pmatrix}$ |

**Table -4**

| $\tilde{A}_i; i=1,2,\ldots,6$ | $\tilde{A}_2$ | $\tilde{A}_2$ | $\tilde{A}_3$ | $\tilde{A}_4$ | $\tilde{A}_5$ | $\tilde{A}_6$ |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| *Expected value* | 0.1161 | 0.1149 | 0.1151 | 0.1156 | 0.1152 | 0.1159 |

**Table-5**

**Ranking based on proposed scheme**

$$A_1 \succ A_6 \succ A_4 \succ A_5 \succ A_3 \succ A_2$$

**Table-6**

## 4.4 Comparative study based on proposed-model vs. existing-models.

A comparative study is presented for validation, feasibility, and effectiveness of the propound operator.

| Modeling tool | 2DULWBOWA Operator | W2DULMSM Operator | FLIOWA Operator |
|---|---|---|---|
| Alternatives with expected values | $A_1$ 0.8231<br>$A_2$ 0.9001<br>$A_3$ 0.3443<br>$A_4$ 0.2367<br>$A_5$ 0.7002<br>$A_6$ 0.7691 | $A_1$ 0.6199<br>$A_2$ 0.3183<br>$A_3$ 0.7115<br>$A_4$ 0.5146<br>$A_5$ 0.9201<br>$A_6$ 0.2117 | $A_1$ 0.5359<br>$A_2$ 0.7738<br>$A_3$ 0.4932<br>$A_4$ 0.8823<br>$A_5$ 0.7032<br>$A_6$ 0.6786 |
| Ranking | $A_2>A_1>A_6>A_5>A_3>A_4$ | $A_5>A_3>A_1>A_4>A_2>A_6>$ | $A_4>A_2>A_5>A_6>A_1>A_3$ |

**Table-7**

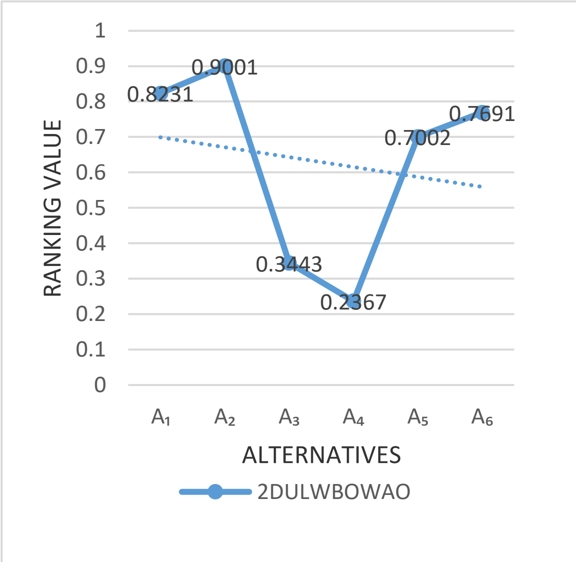


**Figure 2: MCDM based on 2DULWBOWA operator.**

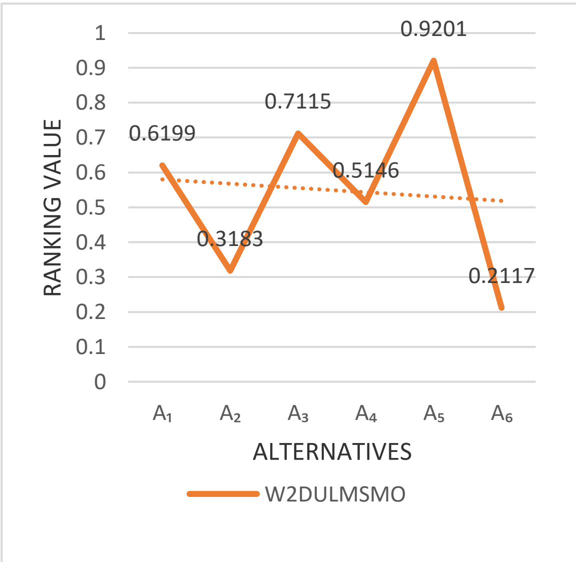


**Figure-3: MCDM based on W2DULMSM operator.**

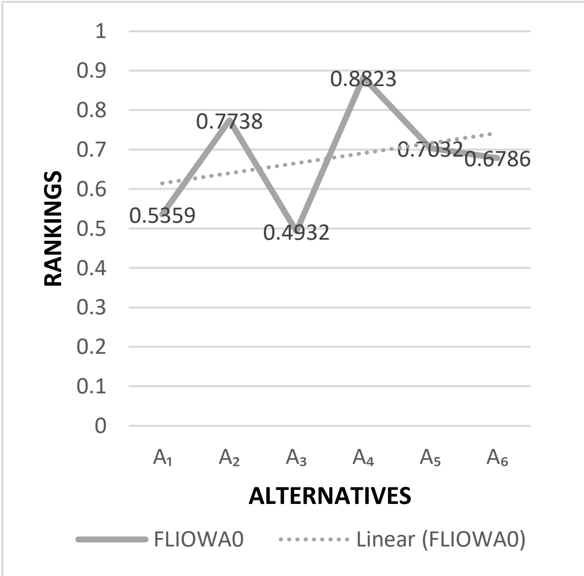


**Figure-4: MCDM based on FLIOWA operator.**

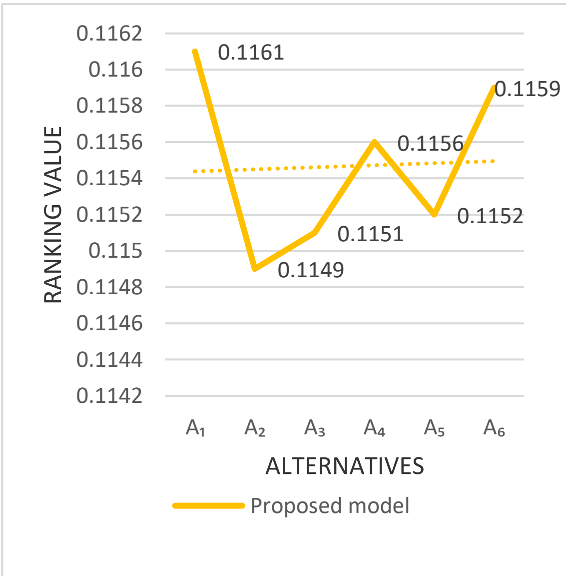


**Figure-5: MCDM based on proposed scheme.**

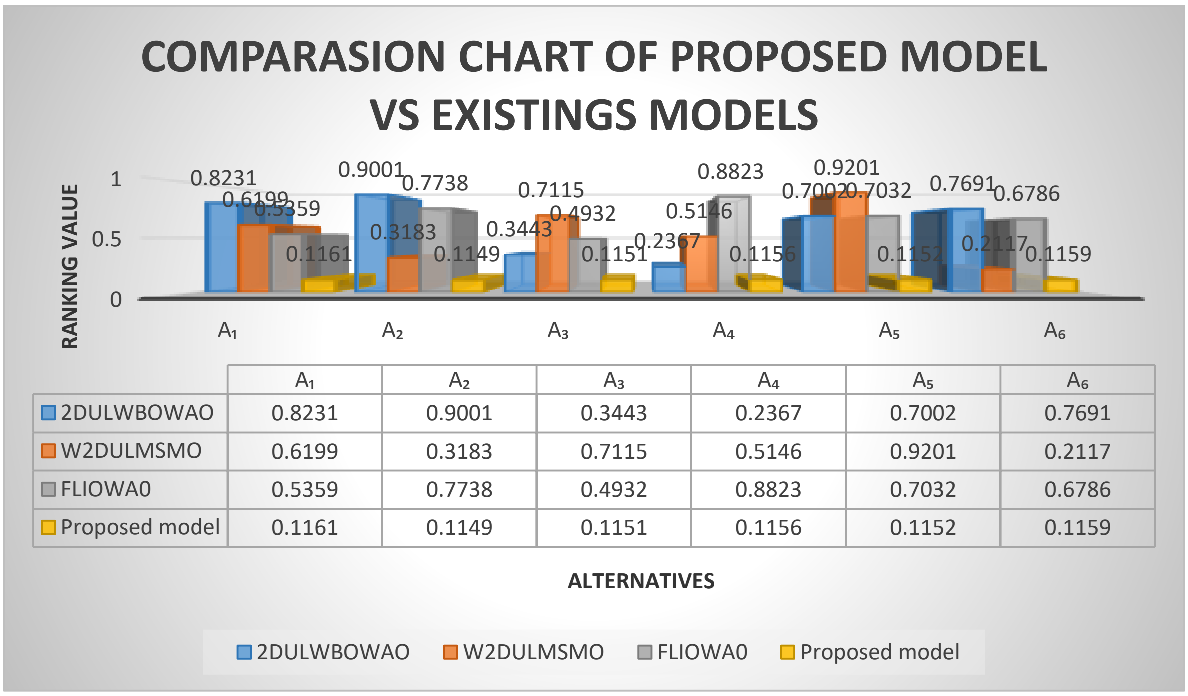


**Figure-6: Comparatively Study based on proposed model vs. existing models.**

### 4.5 Result and Discussion

No doubt, there exist hundreds of aggregations tools which play a vital role and to be considered very useful in theory of multiple decision. The proposed operator would be very beneficial and fruitful in the same area of interest because of its extra features and capabilities. In numerically analysis we examined that the results obtained by proposed MAGDM scheme are more flexible, reliable, and valuable as compared to all the MCDM strategies and tools which are already designed for the same set of modeling and plotting the data. To evaluate the new method's effectiveness, we experimented with it in comparison to three previously developed methods on a specific example. These methods are the 2DULWBOWAO method by Liu and Shi [50], the W2DULMSMO method by Chu and Liu [51], and the FLIOWAO method by Liu et al. [52]. To make things easy, we assigned values to certain variables: $k$ is set as 1 or 2, while $p$ and $q$ are both set as 1. Using these values, we determined the final rankings for the options. The outcomes of these methods are displayed in Figure-6 together with Table-7. From Table-7, we can tell that the options we chose have a significant difference in ranking. They are spread out and separated by large margins, and not very correct and seem unsuitable. However, the values obtained by the proposed method are very close and more correct and suitable. So, these values are like human thinking as well. The method advocated in this research proves to be productive. Which notify that proposed model has the high level of accuracy and validity. To comprehend the advantages of the proposed technique, these points will provide further explanation.

1. These two methods share the same operational guidelines as suggested by Liu and Shi [50], although they exhibit enhanced accuracy. While the new method considers the relationships between various elements, the old method overlooks such connections. The suggested method implies that the value of $k$ can reflect an individual's willingness to take risks.
2. In comparison to the W2DULMSMO method proposed by Liu et al. [52], these two methods could incorporate the relationship between input data/chosen values. However, the suggested approach can think about how all the input arguments are connected to each other, whereas the approach suggested by Chu and Liu [51] can only think about how two input arguments are connected to each other. Furthermore, our suggested method uses the latest operational rules with precise actions, whereas Chu and Liu's method [51] only use the old-fashioned

operational-rule. Clearly, the suggested technique is more adaptable and inclusive in solving the multi-input data problems compared to the method proposed by Chu and Liu [51].

3. These two approaches share the same rules as the FLIOWAO method introduced by Liu et al. [52], and they both account for the correlation between input arguments. Nevertheless, the recently developed technique enables an analysis of the relationships among multiple arguments. The method proposed by Liu et al. [52] is designed to specifically address the relationship between two arguments, which serves as a specific case within the overall application of the innovative approach.

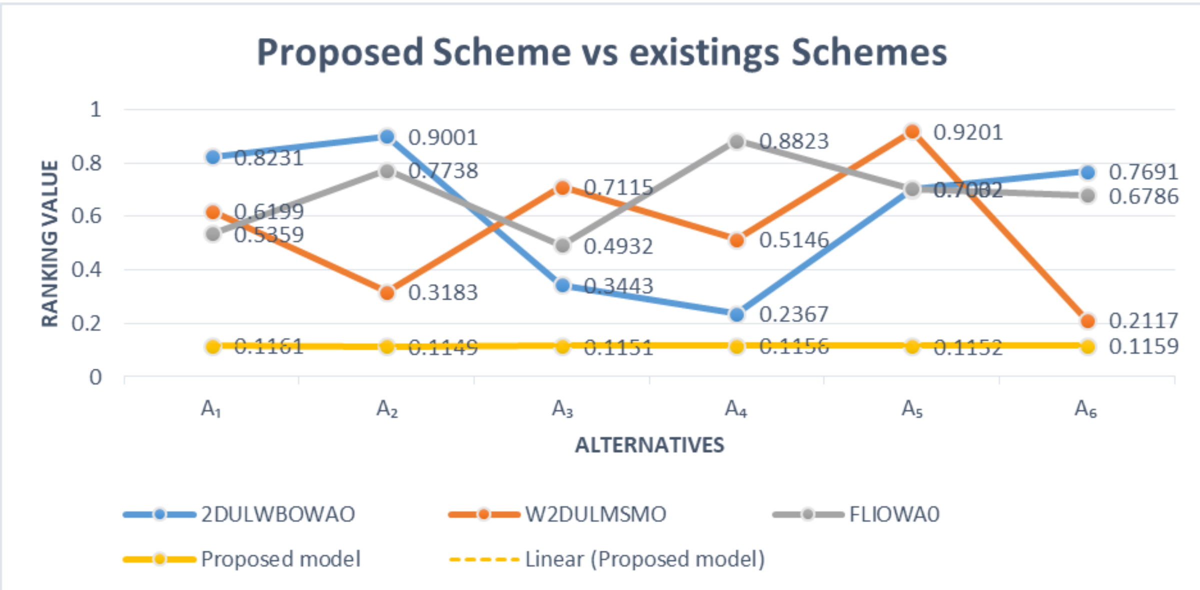


**Figure-7: Numerical Data for Proposed model vs. existing schemes**

## 5. Conclusions

The complexities of considering multiple criteria make decision-making an interesting and crucial component of modern group dynamics. Descriptive terms (linguistic variables) make it easier for decision makers to display assessment values. Decision makers utilize linguistic variables to incorporate their personal judgments regarding the probabilities of specific outcomes. Well-formulated and clearly defined linguistic variables can eliminate any potential gaps in information and enhance the accuracy of the process. In current research paper, we stated the idea of one-dimensional-uncertain-linguistic-neutrosophic-fuzzy variable. We extracted their modified operational laws, basic properties and stated the expectation of these variables. Moreover, For the sake of tracking down the finest alternative while solving multi-input data values (MCDM) problems and other utilities, one of the critical phases is aggregation whereby the suggestion of the decision expert is required to be calculated by using suitable multi-input techniques/operators. We developed some new novel aggregation operators to overcome the power of rapidly growing of convolution, complication, and the vagueness of the socioeconomic environment time to time. Moreover, one dimensional uncertain linguistic interval valued neutrosophic fuzzy information was successfully aggregated by proposed aggregation-operators. Furthermore, the use of a particular numerical example illustrates the strength and validity of the exhibit approach in facilitating group decision-making. Graphically representation for evaluated data based on proposed scheme is also added for easy understanding. Conclusively, comparative study based on proposed model vs. existing-models was managed by the help of decision-experts.

## Future work

In the future, using our identified ideas, the research work is expected to be significantly expanded, some beneficial methodological extensions and other useful applications. The current tool can be considered very effective and powerful in a broad field of decision-making methods, such as Average probability, data information and moving data. Implementing the suggested method in combination with other decision-making methods can enhance the functionality of various financial models.

## Conflicts of Interest

The authors announce that no conflicts of interest exist.

## Author Contributions

Each author participated equally to the writing and editing of the paper. The paper was examined by each author.

**Funding :** No funding is received for the presented work.

## Data availability

This article has all the data that were created or evaluated during this investigation.

## References:


1. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338-353.
2. Gorzałczany, M. B. (1987). A method of inference in approximate reasoning based on interval- valued fuzzy sets. Fuzzy Sets and Systems, 21(1), 1-17. https://doi.org/10.1016/0165-0114(87)90148-5
3. Molodstov, D. (1999). Soft set theory-first results. Computers & Mathematics with applications, 37, 19-31.
4. Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1), 87-96.
5. Peng, X., & Yang, Y. (2015). Some results for Pythagorean fuzzy sets. International Journal of Intelligent Systems, 30(11), 1133-1160. https://doi.org/10.1002/int.21738

6. Gou, X., & Xu, Z. (2016). Novel basic operational laws for linguistic terms, hesitant fuzzy linguistic term sets and probabilistic linguistic term sets. Information Sciences, 372, 407-427. https://doi.org/10.1016/j.ins.2016.08.034
7. Smarandache, F. (1995). Neutrosophic set: A generalization of the Intuitionistic Fuzzy Set, Neutrosophic Prob Set, Logic Amer.
8. Alkhazaleh, S., & Hazaymeh, A. A. (2017). n-valued refined neutrosophic soft sets and their applications in decision making problems and medical diagnosis. Journal of Artificial Intelligence and Soft Computing Research, 8(1), 79-86. DOI: https://doi.org/10.1515/jaiscr-2018-0005
9. Broumi, S., Deli, I., & Smarandache, F. (2015). N-valued interval neutrosophic sets and their application in medical diagnosis. Critical Review, Center for Mathematics of Uncertainty, Creighton University, Omaha, NE, USA, 10, 45-69. https://digitalrepository.unm.edu/math_fsp
10. Broumi, S., Deli, I., & Smarandache, F. (2014). Interval valued neutrosophic parameterized soft set theory and its decision making. Journal of New Results in Science, 3(7), 58-71.
11. Jiang, W., Zhong, Y., & Deng, X. (2018). A neutrosophic set based fault diagnosis method based on multi-stage fault template data. Symmetry, 10(8), 346. https://doi.org/10.3390/sym10080346
12. Maji, P. K. (2013). Neutrosophic soft set. Annals of Fuzzy Mathematics and Informatics, 5(1), 157-168.
13. Mukherjee, A., & Sarkar, S. (2014). Several similarity measures of neutrosophic soft sets and its application in real life problems. Annals of Pure and Applied Mathematics, 7(1), 1-6.
14. Naeem, K., Riaz, M., and Afzal, D. (2020). Fuzzy neutrosophic soft σ-algebra and fuzzy neutrosophic soft measure with applications, Journal of Intelligent and Fuzzy Systems, 39(1), 277-287. DOI: 10.3233/JIFS-191062
15. Şahin, R., & Küçük, A. (2014). On similarity and entropy of neutrosophic soft sets. Journal of intelligent & fuzzy systems, 27(5), 2417-2430. DOI: 10.3233/IFS-141211
16. Sumathi, I. R., & Arockiarani, I. (2016). Cosine similarity measures of neutrosophic soft set. Annals of Fuzzy Mathematics and Informatics, 12(5), 669-678.
17. Wang, H., Smarandache, F., Sunderraman, R., & Zhang, Y. Q. (2005). Interval neutrosophic sets and logic: theory and applications in computing: Theory and applications in computing (Vol. 5). Infinite Study.

18. Zhang, H., Wang, J., & Chen, X. (2016). An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets. Neural Computing and Applications, 27(3), 615-627. https://doi.org/10.1007/s00521-015-1882-3
19. Broumi, S., & Smarandache, F. (2013). Correlation coefficient of interval neutrosophic set. In Applied mechanics and materials (Vol. 436, pp. 511-517). Trans Tech Publications Ltd.
20. Broumi, S., & Smarandache, F. (2014). Cosine similarity measure of interval valued neutrosophic sets. Infinite Study. DOI: 10.6084/M9.FIGSHARE.1502645
21. Broumi, S., & Smarandache, F. (2015). New operations on interval neutrosophic sets. Journal of new theory, (1), 24-37.
22. Deli, I., Broumi, S., & Smarandache, F. (2014). Neutrosophic soft multi-set theory and its decision making. Neutrosophic Sets and Systems, 5, 65-76.
23. Naeem, K., and Davvaz, B. (2022) Information measures for MADM under m-polar neutrosophic environment. Granular Computing, 8(3), 597-616. DOI: 10.1007/s41066-022-00340-3
24. Siraj, A., Fatima, T., Afzal, D., Naeem, K., and Karaaslan, F. (2022). Pythagorean m-polar fuzzy neutrosophic topology with applications, Neutrosophic Sets and Systems, 48, 251-290. DOI: 10.5281/zenodo.6041514
25. Majumdar, P., & Samanta, S. K. (2014). On similarity and entropy of neutrosophic sets. Journal of Intelligent & Fuzzy Systems, 26(3), 1245-1252. DOI: 10.3233/IFS-130810
26. Garg, H., & Arora, R. (2018). Bonferroni mean aggregation operators under intuitionistic fuzzy soft set environment and their applications to decision-making. Journal of the Operational Research Society, 69(11), 1711-1724. https://doi.org/10.1080/01605682.2017.1409159
27. Ye, J. (2014). Prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets and their application to multicriteria decision-making. Neural Computing and Applications, 25(6), 1447-1454. https://doi.org/10.1007/s00521-014-1635-8
28. Awang, A., Aizam, N. A. H., & Abdullah, L. (2019). An integrated decision-making method based on neutrosophic numbers for investigating factors of coastal erosion. Symmetry, 11(3), 328. https://doi.org/10.3390/sym11030328

29. Huang, Y. H., Wei, G. W., & Wei, C. (2017). VIKOR method for interval neutrosophic multiple attribute group decision-making. Information, 8(4), 144. https://doi.org/10.3390/info8040144
30. Zhang, H. Y., Wang, J. Q., & Chen, X. H. (2014). Interval neutrosophic sets and their application in multicriteria decision making problems. The Scientific World Journal, 2014. https://doi.org/10.1155/2014/645953
31. Hussain, S. A. I., Mondal, S. P., & Mandal, U. K. (2019). VIKOR method for decision making problems in interval valued neutrosophic environment. In Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets (pp. 587-602). Springer, Cham.
32. Aiwu, Z., Jianguo, D., & Hongjun, G. (2015). Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator. Journal of Intelligent & Fuzzy Systems, 29(6), 2697-2706. DOI: 10.3233/IFS-151973
33. Peng, J. J., & Wang, J. Q. (2015). Multi-valued neutrosophic sets and its application in multi-criteria decision-making problems. Neutrosophic Sets and Systems, 10(1).
34. Hamid, T., Riaz, M., & Naeem, K. (2022). A study on weighted aggregation operators for q-rung orthopair m-polar fuzzy set with utility to multistage decision analysis. International Journal of Intelligent Systems, 37(9), 6354-6387. DOI: 10.1002/int.22847
35. Naeem, K., Riaz, M., and Karaaslan, F. (2021). A mathematical approach to medical diagnosis via Pythagorean fuzzy soft TOPSIS, VIKOR and generalized aggregation operators, Complex and Intelligent Systems, 7(5), 2783-2795. DOI: 10.1007/s40747-021-00458-y
36. Riaz, M., Naeem, K., Chinram, R., & Iampan, C. (2021). Pythagorean m-polar fuzzy weighted aggregation operators and algorithm for the investment strategic decision making. Journal of Mathematics, Article ID 6644994. DOI: 10.1155/2021/6644994
37. Ye, J. (2015). Trapezoidal neutrosophic set and its application to multiple attribute decision-making. Neural Computing and Applications, 26(5), 1157-1166. https://doi.org/10.1007/s00521-014-1787-6
38. Jana, C., & Pal, M. (2019). A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making. Symmetry, 11(1), 110. https://doi.org/10.3390/sym11010110

39. Abdel-Monem, A., Nabeeh, N. A., & Abouhawwash, M. (2023). An Integrated Neutrosophic Regional Management Ranking Method for Agricultural Water Management. *Neutrosophic Systems with Applications*, *1*, 22-28.
40. Abdelhafeez, A., Mohamed, H. K., & Khalil, N. A. (2023). Rank and Analysis Several Solutions of Healthcare Waste to Achieve Cost Effectiveness and Sustainability Using Neutrosophic MCDM Model. *Neutrosophic Systems with Applications*, *2*, 25-37.
41. Karak, M., Mahata, A., Rong, M., Mukherjee, S., Mondal, S. P., Broumi, S., & Roy, B. (2023). A Solution Technique of Transportation Problem in Neutrosophic Environment. *Neutrosophic Systems with Applications*, *3*, 17-34.
42. Wang, J. Q., Yang, Y., & Li, L. (2018). Multi-criteria decision-making method based on single-valued neutrosophic linguistic Maclaurin symmetric mean operators. Neural Computing and Applications, 30(5), 1529-1547. https://doi.org/10.1007/s00521-016-2747-0
43. Wu, Q., Wu, P., Zhou, L., Chen, H., & Guan, X. (2018). Some new Hamacher aggregation operators under single-valued neutrosophic 2-tuple linguistic environment and their applications to multi-attribute group decision making. Computers & Industrial Engineering, 116, 144-162. https://doi.org/10.1016/j.cie.2017.12.024
44. Garg, H., & Arora, R. (2021). Generalized Maclaurin symmetric mean aggregation operators based on Archimedean t-norm of the intuitionistic fuzzy soft set information. Artificial Intelligence Review, 54(4), 3173-3213.
45. Ju, Y., Liu, X., & Ju, D. (2016). Some new intuitionistic linguistic aggregation operators based on Maclaurin symmetric mean and their applications to multiple attribute group decision making. Soft Computing, 20(11), 4521-4548. https://doi.org/10.1007/s00500-015-1761-y
46. Liu, P., Li, Y., & Zhang, M. (2019). Some Maclaurin symmetric mean aggregation operators based on two-dimensional uncertain linguistic information and their application to decision making. Neural Computing and Applications, 31(8), 4305-4318. https://doi.org/10.1007/s00521-018-3350-3
47. Teng, F., Liu, Z., & Liu, P. (2018). Some power Maclaurin symmetric mean aggregation operators based on Pythagorean fuzzy linguistic numbers and their application to group decision making. International Journal of Intelligent Systems, 33(9), 1949-1985. https://doi.org/10.1002/int.22005

48. Wang, H., Smarandache, F., Sunderraman, R., & Zhang, Y. Q. (2005). Interval neutrosophic sets and logic: theory and applications in computing: Theory and applications in computing (Vol. 5). Infinite Study.
49. Zhang, H. (2014). Linguistic intuitionistic fuzzy sets and application in MAGDM. Journal of Applied Mathematics, 2014. https://doi.org/10.1155/2014/432092
50. Liu, P., & Shi, L. (2017). Some neutrosophic uncertain linguistic number Heronian mean operators and their application to multi-attribute group decision making. *Neural Computing and Applications*, *28*, 1079-1093.
51. Chu, Y., & Liu, P. (2015). Some two-dimensional uncertain linguistic Heronian mean operators and their application in multiple-attribute decision making. *Neural Computing and Applications*, *26*, 1461-1480.
52. Liu, P., Liu, J., & Chen, S. M. (2018). Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making. *Journal of the Operational Research Society*, *69*(1), 1-24.

Received: Aug 9, 2023. Accepted: Dec. 19, 2023