NSS

Neutrosophic Sets and Systems, Vol. 60, 2023

University of New Mexico

Optimization of Single-valued Triangular Neutrosophic Fuzzy Travelling Salesman Problem

Subadhra Srinivas[1] and K. Prabakaran[2]

[1]Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chennai, India; ss0837@srmist.edu.in

[2]Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chennai, India; prabakak2@srmist.edu.in

[*]Correspondence: prabakak2@srmist.edu.in

Abstract. The travelling salesman problem(TSP) is a classic optimization puzzle, widely studied and celebrated for its significance in operations research, mathematics and computer science. It can also be described as an evolution from a mathematical curiosity to a problem that challenges the computation boundaries, sparks algorithmic innovation, and finds practical applications in various industries. The neutrosophic TSP(NTSP) extends the problem by introducing neutrosophy, handling indeterminacy and inconsistency with distances represented by neutrosophic numbers(NNs). The single-valued triangular fuzzy neutrosophic TSP(SVTFNTSP) goes a step further by incorporating both single-valued triangular fuzzy numbers(SVTFNs) and neutrosophy, representing distances with SVTFNNs. The single-valued triangular fuzzy neutrosophic numbers(SVTFNNs) provide a way to model uncertainty via triangular membership functions, offering a more nuanced representation of uncertain and vague distances. This arises the need to use them and enhances realism in solving complex real-world optimization problems. These extensions adapt the TSP to varying uncertain and vague data, ideal for intricate real-world optimization scenarios. This research article delves into the SVTFNTSP, expressed as a single-valued triangular fuzzy neutrosophic distance matrix(SVTFNDM) with SVTFNNs as its core elements, accounting for both uncertainty and imprecision. The investigation encompasses the formulation and examination of this specialized problem by incorporating a score function to assess defuzzification and optimality, alongside the utilization of a proposed systematic stepwise approach to efficiently ascertain optimal solutions. This approach is practically demonstrated through its application to real-world scenarios, effectively showcasing its feasibility and real-world relevance. Subsequently, through a rigorous comparative analysis with the established methodologies, the superior effectiveness and value of the proposed approach are highlighted, specifically in terms of minimizing total travelling costs. This reaffirms its potential as a robust solution for tackling the SVTFNTSP by underlining its practical utility and enhanced performance.

Keywords : Neutrosophic set, Neutrosophic number, Single-valued triangular fuzzy neutrosophic number, Single-valued triangular fuzzy neutrosophic distance matrix, Travelling salesman problem, Single-valued triangular fuzzy neutrosophic travelling salesman problem, Score function, Range, Optimal solution, Cycle.

1. **Introduction**

In our daily lives, we frequently encounter a variety of unclear, ambiguous, and inadequate situations. Thus, as an extension of classical sets that enables partial membership(awards a membership grade) for each element - Zadeh [1] developed the idea of fuzzy sets in 1965. The fuzzy set theory has had considerable success in many disciplines because of its capacity to handle uncertainty. Atanassov [2] presented the idea of intuitionistic fuzzy sets in 1983 as an extension of fuzzy sets that not only contains the membership grade but also the non-membership grade of each element due to certain of its constraints. Neutrosophic sets(NS) are an extension of intuitionistic fuzzy sets that incorporate the truth(T), indeterminacy(I), and falsity(F) membership grades for each element. The notion was first described by Smarandache [3] in 1995.

The Travelling Salesman Problem(TSP) is a mathematical challenge that has been studied for centuries, making it difficult to attribute its invention to a single individual. However, the problem gained formal recognition and attention in the 1800s and 1900s as mathematicians explored related concepts. The mathematician and computer scientist George Dantzig is often credited with formulating the TSP in its modern mathematical terms in the 1950s. He introduced the problem as a mathematical challenge and used it to illustrate the concept of linear programming. The problem's historical development involved the contributions of various mathematicians across different time periods. The TSP seeks the shortest route for a salesperson to visit cities once and return to the starting city, minimizing distance or cost. This optimizes route planning and algorithm advancement. The TSP holds profound significance as both a theoretical benchmark and a practical problem-solving tool. As a theoretical challenge, it embodies the complexities of optimization and serves as a yardstick for evaluating algorithmic innovations. In practical realms, the TSP finds applications in diverse fields like logistics, enhancing delivery routes, reducing transportation costs, and improving resource utilization. The problem's versatility underscores its significance in solving complex real-world optimization challenges across industries and domains. Neutrosophic numbers(NNs) hold significant value due to their ability to capture uncertainty, indeterminacy, and inconsistency in a structured manner. They find applications in decision-making, expert systems, and medical diagnoses. They enhance problem-solving by addressing imprecise or incomplete information, enabling more informed choices in diverse domains. Single-valued triangular fuzzy neutrosophic numbers(SVTFNNs) carry substantial importance by seamlessly integrating triangular fuzzy sets and neutrosophy. This fusion enhances the representation of uncertainty and indeterminacy in a comprehensive manner. These numbers find practical applications in decision analysis, risk assessment, and multi-criteria decision-making, where complex and uncertain information is prevalent. The ability of single-valued triangular fuzzy neutrosophic numbers to model both fuzziness and neutrosophy enhances the accuracy of real-world problem-solving, offering a versatile tool to navigate intricate situations and foster well-informed decisions. This ignites interest and fosters a drive to explore and experiment with the single-valued triangular fuzzy neutrosophic traveling salesman problem(SVTFNTSP).

The research landscape surrounding the Traveling Salesman Problem (TSP) and its extensions, such as the Neutrosophic TSP and the Single-Valued Triangular Fuzzy Neutrosophic TSP, has been vibrant and dynamic. Scholars have extensively explored the classic TSP, focusing on developing algorithms, heuristics, and metaheuristics to efficiently find near-optimal solutions for larger instances. The application of the neutrosophic idea is the subject of several recent research publications. Researchers have studied and analysed the issue of completing an assignment in a classical, fuzzy and intuitionistic fuzzy environment [4–8]. In 2019, Prabha and Vimala [9] used the branch and bound method to solve the triangular neutrosophic fuzzy assignment problem, which is demonstrated with an agricultural issue. Using the order relations method, Khalifa Abd El-Wahed et al. [10–12] were able to resolve the neutrosophic fuzzy assignment issue where the matrix elements are interval-valued trapezoidal neutrosophic fuzzy numbers, optimized neutrosophic complex programming using lexicographic order and resolved the interval-type fuzzy linear fractional programming problem in neutrosophic environment using a fuzzy mathematical programming approach respectively. Chakraborty et al [13] proposed a few de-neutrosophication techniques to tackle different forms of triangular fuzzy neutrosophic numbers and also explored on their applications to various fields. A new ranking function of triangular fuzzy neutrosophic numbers put forward by Das et al. [14] and applied to integer programming. Pranab et al. [15] aggregated of triangular fuzzy neutrosophic set information and extended its applications to multi-attribute decision-making. Broumi [16] handled the shortest path problem by using interval valued trapezoidal and triangular fuzzy neutrosophic numbers. The neutrosophic inventory backorder problem was examined by Mullai and Surya [17] and resolved using triangular fuzzy neutrosophic numbers. Smarandache [18] established the Delphi method for evaluating scientific research proposals in a neutrosophic environment. Researchers [19–22] have investigated diverse real-life issues under a neutrosophic environment and resolved the same with the use of single-valued triangular fuzzy neutrosophic matrix games and by developing different score functions for both ranking and turning the neutrosophic data into the appropriate crisp data. Subasri and Selvakumari [23, 24] used the ones assignment method and the branch and bound approach, to solve the travelling salesman problem in a neutrosophic environment utilising triangular and trapezoidal fuzzy distances respectively. S. Dhouib [25] used the Dhouib-Matrix-TSP1 Heuristic to optimise the traveling salesman problem for single-valued triangular fuzzy neutrosophic numbers. Broumi et al. [26] analyzed and answered the shortest path problem under triangular fuzzy neutrosophic environment. Abduallah et al. [27, 28] conducted detailed case studies on leveraging neutrosophic theory in appraisal decision framework and neutrosophic healthcare systems and worked toward sustainable emerging economics based on industry 5.0 and a responsive resilient supply chain based on industry 5.0 respectively. Maissam and Smarandache [29] explored about the use of neutrosophic methods of operation research in the management of corporate work. Uddin et al. [30] introduced a new extension to the intuitionistic fuzzy metric-like spaces. Saleem et al. [31] stablished a unique solution for the integral equations through the intuitionistic extended fuzzy b-metric-like spaces. Ishtiaq, Ahmed et al. [32–35] resolved the non-linear fractional differential equations and guarenteed the existence of some fixed point results in neutrosophic metric, orthogonal neutrosophic metric, generalized neutrosophic metric and neutrosophic metric-like spaces respectively.

Here comes a small discussion about the limitations along with the gaps of the existing algorithms related to this study, which has lead to the need for using the proposed algorithm for solving the single-valued triangular fuzzy neutrosophic traveling salesman problem (SVTFNTSP), its new features and the potential advantages it can offer over existing methods, including the differences between the proposed and the existing methodologies are as follows : Certainly, various algorithms have been proposed for solving the Traveling Salesman Problem (TSP), and each method comes with its own set of advantages and disadvantages. Some of the above mentioned methods involve exploring all possible permutations, leading to exponential time complexity. It becomes impractical for large TSP instances. Some optimization algorithms for TSP, can exhibit exponential growth in the number of nodes explored, which makes it slow for large instances. They can be complex and computationally intensive, particularly when dealing with more intricate TSP instances or constraints. A few methods rely on mass values associated with cities, which may not always be readily available or meaningful for real-world TSP applications. Some methods are based on the assumption that the TSP instance can be represented by a special matrix, which may not hold for all real-world scenarios. It limits its applicability. Many TSP algorithms, including those mentioned, can be sensitive to the initial solution or starting point, potentially leading to suboptimal results if a good initial solution is not found. Some of these methods may struggle with scalability when applied to large TSP instances due to their exponential nature or computational demands. Certain methods may be better suited to specific types of TSP instances and may not perform well on variations or extensions of the problem. Some methods may not guarantee finding the optimal solution but rather provide approximate solutions. For certain applications requiring exact solutions, this can be a limitation. Analyzing the computational complexity and convergence properties of these methods can be challenging, making it difficult to predict their performance in advance. Some TSP algorithms may not be easily parallelizable, limiting their ability to take advantage of modern multi-core processors and distributed computing environments.

The proposed methodology of this research article brings an innovative approach by incorporating the range(a measure of dispersion) to problem-solving. It might have the potential to discover high-quality solutions that outperform existing methods. They employ strategies that are not present in traditional approaches. It is designed to adapt to different problem characteristics and constraints. This adaptability can make it suitable for a wider range of SVTFNTSP instances without extensive customization. It can handle larger and more complex SVTFNTSP instances efficiently. This can be crucial for solving real-world problems of practical significance. It might achieve faster convergence to solutions, potentially reducing the overall computation time for solving SVTFNTSP instances. It has the potential to generalize to other related problems or domains beyond NTSP. This versatility can make them valuable for addressing a broader set of challenges. It is designed to be robust to variations in problem instances or data. They may be less sensitive to changes in input parameters, leading to more reliable performance. In rapidly evolving research fields, using the proposed mathodology can provide a competitive advantage by accessing the latest advancements in optimization and computational intelligence. While existing methods may have established parameter settings and heuristics, the proposed mathodology offers opportunities for customization to better fit the specific requirements and constraints of a given NTSP instance.

This research article explores the single-valued triangular fuzzy neutrosophic traveling salesman problem (SVTFNTSP), which is represented using a single-valued triangular fuzzy neutrosophic distance matrix (SVTFNDM) with SVTFNNs as its fundamental components. This formulation takes into account both uncertainty and imprecision. The study involves the development and investigation of this specialized problem, introducing a score function for defuzzification and optimality assessment. Additionally, it presents a systematic stepwise approach to efficiently determine optimal solutions. The practical applicability of this approach is demonstrated through its implementation in real-world scenarios, highlighting its feasibility and relevance. Furthermore, through a rigorous comparative analysis with established methodologies, the article emphasizes the superior effectiveness and value of the proposed approach, particularly in terms of minimizing total travel costs. This underscores its potential as a robust solution for addressing the SVTFNTSP, emphasizing its practical utility and enhanced performance. The following is how the paper is set up : The abstract and introduction are included in Section 1. We provide some fundamental definitions of a fuzzy set, an intuitionistic fuzzy set, a neutrosophic set and a fuzzy number with their respective examples in the Preliminaries section of Section 2. Neutrosophic number, properties of neutrosophic numbers, single-valued triangular fuzzy neutrosophic number, along with their corresponding examples, travelling salesman problem (TSP), mathematical formulation of TSP and a score function are some of the subjects covered in Section 3. The methodology for solving the single-valued triangular fuzzy neutrosophic travelling salesman issue is presented in Section 4 and comprises defuzzifying the neutrosophic data before using the suggested algorithm step-by-step to get the best answer. The proposed approach to addressing the "Travelling Salesman Problem" in a neutrosophic environment is illustrated in Section 5. Section 6 highlights some significant results and discussions. Section 7 concludes the research article.

2. Preliminaries

Definition 2.1. Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \longrightarrow [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A, for each $x \in X$.

$$A = \{(x, \mu_A(x)) : x \in X\}.$$

Example :

- Variable : Happiness.
- Fuzzy Sets : Unhappy, Neutral, Happy.
- Membership Function : $\mu(Unhappy) = 0.2$; $\mu(Neutral) = 0.5$; $\mu(Happy) = 0.9$.

Definition 2.2. Let X be a non empty set. An Intuitionistic fuzzy set A in X is of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where the functions μ_A, $\nu_A : X \longrightarrow [0, 1]$ define respectively the degree of membership and the degree of non-membership for every element $x \in X$ to the set A, which is a subset of X.

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the intuitionistic fuzzy set index or hesitation margin is the degree of indeterminacy of x in A where $\pi_A(x) \in [0, 1]$ i.e; $\pi_A : X \longrightarrow [0, 1]$ and $0 \leq \pi_A(x) \leq 1$ for every x∈X.

Example : IFS representing the taste "Spicy".

- Element : Dish X.
- Membership degree : $\mu(DishX) = 0.7$(Dish X is "somewhat" spicy).
- Non-membership degree : $\nu(DishX) = 0.3$(Dish X is "not very" not spicy).
- Hesitancy degree : $\pi(DishX) = 0.4$(There is moderate uncertainty in the classification).

Definition 2.3. Let X be a non empty set. A Neutrosophic set $A \in X$ is of the form $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$, where the functions $T_A, I_A, F_A : X \longrightarrow ^-]0, 1[^+$ define respectively the degree of truth membership, the degree of indeterminacy and the degree of falsity membership for every element $x \in X$ to the set A, which is a subset of X.

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

Example : Weather Condition. Suppose we want to describe the "cloudiness" of the sky using a neutrosophic fuzzy set. The set may have the following degrees of membership :

- Truth-membership : 0.7(70 percent cloudy).
- Indeterminacy-membership : 0.2(20 percent uncertain).
- Falsity-membership : 0.1(10 percent not cloudy).

Definition 2.4. The fuzzy set A defined on the set of real numbers is said to be a fuzzy number if A and its membership function $\mu_A(X)$ has the following properties :

(1) A is normal and convex.
(2) A is bounded.
(3) $\mu_A(X)$ is piece - wise continuous.

Example : Fuzzy Number for a Distance - A fuzzy number describing the distance between two cities in kilometers : (300, 350, 400). This indicates that the distance is most likely around 350 km, and there is some degree of membership for distances between 300 km and 400 km.

3. **Neutrosophic numbers and its properties**

3.1. *Neutrosophic numbers*

Definition 3.1. Let X be a non empty set. A Neutrosophic set $A \in X$ is of the form $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$, where the functions $T_A, I_A, F_A : X \longrightarrow [0,1]$ define respectively the degree of truth membership, the degree of indeterminacy membership and the degree of falsity membership for every element $x \in X$ to the set A, which is a subset of X.

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

If in particular, X has only one element, A is called a neutrosophic number, which can be denoted by, $A = (T_A(x), I_A(x), F_A(x))$.

Example : Neutrosophic Number for a Student's Performance Grade - Representing a neutrosophic fuzzy number for a student's performance grade in a subject : (7.2, 7.5, 7.8). This means that the student's grade is most likely around 7.5(truth-membership degree of 7.5), with a small level of indeterminacy(0.3) and a very low level of falsity(0.6).

3.1.1. *Properties of Neutrosophic numbers*

Let $A, B \in X$. Then their operations are defined as,

(1) $(T_A(x), I_A(x), F_A(x)) + (T_B(x), I_B(x), F_B(x)) = (T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x))$.
(2) $(T_A(x), I_A(x), F_A(x)).(T_B(x), I_B(x), F_B(x)) = (T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x)I_B(x), F_A(x) + F_B(x) - F_A(x)F_B(x))$.
(3) $k(T_A(x), I_A(x), F_A(x)) = (1 - (1 - T_A(x))k, I_A(x)k, F_A(x)k), (k \in R)$.
(4) $(T_A(x), I_A(x), F_A(x))k = (T_A(x)k, 1 - (1 - I_A(x))k, 1 - (1 - F_A(x))k)(k \in R)$.

3.1.2. *Single-valued triangular fuzzy neutrosophic number*

The single-valued triangular fuzzy neutrosophic number $a = ((a_1, a_2, a_3); \alpha_a, \beta_a, \gamma_a)$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy and falsehood membership functions are defined as follows, respectively

$$T_a(x) = \left\{ \begin{array}{rcl} \alpha_a(\frac{x-a_1}{a_2-a_1}) & \text{for} & a_1 \leq x \leq a_2 \\ \alpha_a & \text{for} & x = a_2 \\ \alpha_a(\frac{a_3-x}{a_3-a_2}) & \text{for} & a_2 < x \leq a_3 \\ 0 & \text{for} & otherwise \end{array} \right\}.$$

$$I_a(x) = \left\{ \begin{array}{rll} \frac{a_2 - x + \beta_a(x-a_1)}{a_2-a_1} & \text{for} & a_1 \le x \le a_2 \\ \beta_a & \text{for} & x = a_2 \\ \frac{x - a_2 + \beta_a(a_3-x)}{a_3-a_2} & \text{for} & a_2 < x \le a_3 \\ 1 & \text{for} & otherwise \end{array} \right\}.$$

$$F_a(x) = \left\{ \begin{array}{rll} \frac{a_2 - x + \gamma_a(x-a_1)}{a_2-a_1} & \text{for} & a_1 \le x \le a_2 \\ \gamma_a & \text{for} & x = a_2 \\ \frac{x - a_2 + \gamma_a(a_3-x)}{a_3-a_2} & \text{for} & a_2 < x \le a_3 \\ 1 & \text{for} & otherwise \end{array} \right\}.$$

where α_a, β_a, $\gamma_a \in$ [0,1], $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Example : Single-Valued Triangular Fuzzy Neutrosophic Number for Age - $A = ((6,7,8);1,0,0)$. This represents an individual's age, where the truth-membership degree α_a is 1, indicating that the person's age is exactly 7 years (value $a_2 = 7$). The indeterminacy-membership degree β_a and falsity-membership degree γ_a are both 0, indicating that there is no uncertainty or inconsistency associated with this age value.

The Single-valued triangular fuzzy neutrosophic number is expressed using the following Figure 1 :

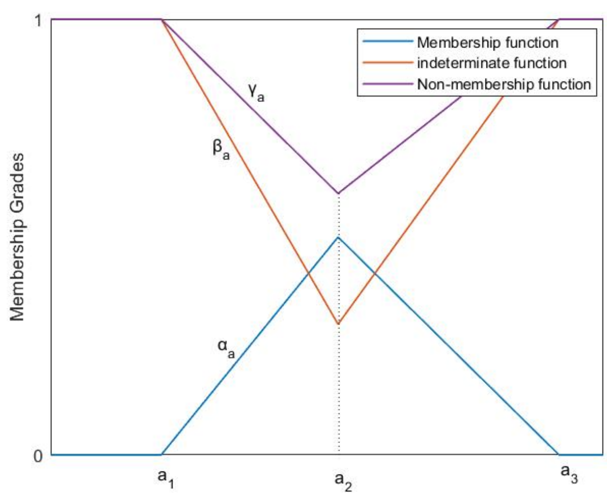

FIGURE 1. Single-valued triangular fuzzy neutrosophic number :

3.2. *Travelling salesman problem(TSP)*

3.2.1. *Description of TSP*

The Traveling salesman problem is a well-known algorithmic problem which consists of a salesman and a set of destinations or points. This problem refers to the challenge of determining the shortest yet effective route for a travelling salesman to visit a list of specific destinations. The salesman has to visit each set of destinations starting from a particular one and returning to the same. His main objective is to find the shortest route from a set of different routes to minimize the total travel cost or the total distance travelled.

3.2.2. *Mathematical formulation of TSP*

The TSP can be formulated mathematically as follows:

$\sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}\ p_{ij},$

$\sum_{j=1}^{n} p_{ij} = 1,\ i = 1, 2, ...n$

$\sum_{i=1}^{n} p_{ij} = 1,\ j = 1, 2, ...n$

$p_{ij} = 0$ or 1, $i = 1, 2, ...n$ and $j = 1, 2, ...n$,

where, p_{ij} is a binary variable(if destination i and destination j are not connected, then, $p_{ij} = 0$, else $p_{ij} = 1$) and d_{ij} denotes the distance between destination i and destination j.

3.2.3. *Score function*

The score function as in [26] to be utilized for converting the neutrosophic data of the NTSP into crisp data is as follows :

$$S(a) = \frac{1}{12}((a_1 + 2a_2 + a_3)(2 + \alpha_a - \beta_a - \gamma_a)), \tag{1}$$

where, $a = ((a_1, a_2, a_3); \alpha_a, \beta_a, \gamma_a)$ is a single-valued triangular neutrosophic number, α_a, β_a, $\gamma_a \in$ [0,1], $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

4. **Methodology for solving Neutrosohic Travelling salesman problem**

4.1. *Defuzzification of the Neutrosophic data*

The TSP is represented in the form of a matrix called the single-valued triangular fuzzy neutrosophic distance matrix, given by,

$$S = \begin{pmatrix} \infty & s_{12} & . & . & . & s_{1n} \\ s_{21} & \infty & . & . & . & s_{2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ s_{n1} & s_{n2} & . & . & . & \infty \end{pmatrix}$$

Here, all its elements are single-valued triangular fuzzy neutrosophic numbers of the form, $a = ((a_1, a_2, a_3); \alpha_a, \beta_a, \gamma_a)$. Using the above score function (1)

$$S(a) = \tfrac{1}{12}((a_1 + 2a_2 + a_3)(2 + \alpha_a - \beta_a - \gamma_a)),$$

each element(single-valued triangular fuzzy neutrosophic number) of the NTSP matrix is defuzzified, hence being converted into their respective crisp numbers.

4.2. *The suggested method for solving the single-valued triangular fuzzy neutrosophic travelling salesman problem :*

The following are the steps involved in the proposed method for solving the single-valued triangular fuzzy neutrosophic travelling salesman problem(SVTFNTSP) :

Step 1 : In the single-valued triangular fuzzy neutrosophic distance matrix(SVTFNDM), the first step is to convert all the single-valued triangular fuzzy neutrosophic data into their corresponding crisp data using the specified score function (1) as mentioned above.

Step 2 : The next step is to calculate the range value of every column of the single-valued triangular fuzzy neutrosophic distance matrix, utilizing the formula, Range = Highest value - Least value and place it below the corresponding columns.

Step 3 : The next task is to find the highest range value from all the range values calculated and select the corresponding column.

Step 4 : Now, choose the least value from the column selected and divide all the remaining entries of the matrix by the selected value. Having performed these few steps, would create certain number of ones in the matrix.

Step 5 : Try choosing exactly one 1 from each row and column. If we are able to do so, the optimal soution(OS) is obtained. If not, draw lines such that all the 1's are covered. Choose the minimum element from the uncovered elements, divide all these elements by the same, multiply the same at the intersection of the drawn lines and the other elements under the drawn line remain the same.

Step 6 : Now, again try selecting exactly one 1 in each row and column. If possible, we can move forward towards the optimal solution. Orelse, repeat step 5 till we are able to choose exactly one 1 in each row and column.

Step 7 : Finally, after being able to select exactly one 1 in each row and column, since it is a TSP, we should check whether the travelling schedule forms a cycle.,i.e., starting from city 1, the schedule should take us through all cities and return back to city 1 itself. If that is the case, we have got the crisp travelling schedule(CTS) along with the total minimal crisp travelling cost(TMCTC) or the total minimal crisp distance travelled(TMCDT) for the given single-valued triangular fuzzy neutrosophic travelling salesman problem, which will themselves serve as the crisp optimal solution(COS) and the crisp optimal travelling cost(COTC) or the crisp optimal distance travelled(CODT) respectively. Orelse, exchange any two rows(cities), one being the city to which the schedule moves from the first city(denoted by city x) and the other row(city), having the highest average among all the other rows, except city 1 and city x(this city is denoted by city y). Hence, exchanging these two cities x and y, would result in the crisp travelling schedule(CTS) along with the total minimal crisp travelling cost(TMCTC) or the total minimal crisp distance travelled(TMCDT) for the given single-valued triangular fuzzy neutrosophic travelling salesman problem, which will themselves serve as the crisp optimal solution(COS) and the crisp optimal travelling cost(COTC) or the crisp optimal distance travelled(CODT) respectively(The main objective of the salesman being to find the shortest route from a set of different routes, thereby minimizing the total travel cost or the total distance travelled).

5. Illustrations for the suggested methodology :

5.1. *Illustration 1 :*

Consider the following symmetric TSP in the form of a single-valued triangular fuzzy neutrosophic distance matrix [25],

$$S = \begin{pmatrix} \infty & s_{12} & s_{13} & s_{14} \\ s_{21} & \infty & s_{23} & s_{24} \\ s_{31} & s_{32} & \infty & s_{34} \\ s_{41} & s_{42} & s_{43} & \infty \end{pmatrix}$$

where its elements(single-valued triangular fuzzy neutrosophic numbers) are as follows :
$s_{12} = s_{21} = \langle (4, 6, 10) ; 0.8, 0.4, 0.2\rangle$; $s_{13} = s_{31} = \langle (2, 5, 9) ; 0.7, 0.6, 0.3\rangle$;
$s_{14} = s_{41} = \langle (4, 7, 9) ; 0.6, 0.6, 0.3\rangle$; $s_{23} = s_{32} = \langle (1, 5, 8) ; 0.8, 0.5, 0.2\rangle$;
$s_{24} = s_{42} = \langle (2, 7, 9) ; 0.8, 0.5, 0.4\rangle$; $s_{34} = s_{43} = \langle (1, 5, 10) ; 0.8, 0.3, 0.1\rangle$.

Step 1 : Using the above specified score function (1),

$$S(a) = \frac{1}{12}((a_1 + 2a_2 + a_3)(2 + \alpha_a - \beta_a - \gamma_a)),$$

converting the given neutrosophic data into their corresponding crisp data, we obtain,
$S(s_{12}) = S(s_{21}) = \frac{26\times2.2}{12} = \frac{57.2}{12} = 4.7667$; $S(s_{13}) = S(s_{31}) = \frac{21\times1.8}{12} = \frac{37.8}{12} = 3.15$;

$S(s_{14}) = S(s_{41}) = \frac{27\times1.7}{12} = \frac{45.9}{12} = 3.825$; $S(s_{23}) = S(s_{32}) = \frac{19\times2.1}{12} = \frac{39.9}{12} = 3.325$;

$S(s_{24}) = S(s_{42}) = \frac{25\times1.9}{12} = \frac{47.5}{12} = 3.9583$; $S(s_{34}) = S(s_{43}) = \frac{21\times2.4}{12} = \frac{50.4}{12} = 4.2.$

The finally obtained crisp equivalent TSP matrix is given by,

$$S = \begin{pmatrix} \infty & 4.7667 & 3.15 & 3.825 \\ 4.7667 & \infty & 3.325 & 3.9583 \\ 3.15 & 3.325 & \infty & 4.2 \\ 3.825 & 3.9583 & 4.2 & \infty \\ \end{pmatrix}$$

Step 2 : The next step is to calculate the range value of every column of the single-valued triangular fuzzy neutrosophic distance matrix, utilizing the formula, Range = Highest value - Least value. The required values for the columns 1 - 4 are found to be 1.6167, 1.4417, 1.05 and 0.375 respectively.

Step 3 : Now, the highest range value from all the range values calculated is 1.6167 and the corresponding column is selected, which is found to be column 1.

Step 4 : Next, we choose the least value 3.15 from the column 1 selected as follows :

$$S = \begin{pmatrix} \infty & 4.7667 & 3.15 & 3.825 \\ 4.7667 & \infty & 3.325 & 3.9583 \\ \boxed{3.15} & 3.325 & \infty & 4.2 \\ 3.825 & 3.9583 & 4.2 & \infty \\ \end{pmatrix}$$

Then, we divide all the remaining entries of the matrix by this chosen least value. Having performed these few steps, would create certain number of ones in the matrix, which is shown in the following table :

$$S = \begin{pmatrix} \infty & 1.51 & 1 & 1.22 \\ 1.51 & \infty & 1.06 & 1.26 \\ 1 & 1.06 & \infty & 1.33 \\ 1.22 & 1.26 & 1.33 & \infty \\ \end{pmatrix}$$

The steps having been performed so far has created quite a few number of 1's.

Step 5 : Let us try selecting exactly one 1 in each row and column, as shown below :

$$S = \begin{pmatrix} \infty & 1.51 & \boxed{1} & 1.22 \\ 1.51 & \infty & 1.06 & 1.26 \\ \boxed{1} & 1.06 & \infty & 1.33 \\ 1.22 & 1.26 & 1.33 & \infty \\ \end{pmatrix}$$

We find that it is not possible from the above matrix. Hence in order to reach the optimal solution, we draw lines by covering all the 1's in the matrix, which covers row 1 and column 1. The least element among all the uncovered elements is found to be 1.06. We divide all these uncovered elements by 1.06, multiply the same at the intersection of the drawn lines and the other elements under the drawn line remain the same. The resulting matrix is,

$$S = \begin{pmatrix} \infty & 1.51 & 1 & 1.22 \\ 1.51 & \infty & 1 & 1.19 \\ 1 & 1 & \infty & 1.25 \\ 1.22 & 1.19 & 1.25 & \infty \\ \end{pmatrix}$$

Step 6 : Now, let us again try selecting exactly one 1 in each row and column, which is shown below :

$$S = \begin{pmatrix} \infty & 1.51 & \boxed{1} & 1.22 \\ 1.51 & \infty & 1 & 1.19 \\ \boxed{1} & 1 & \infty & 1.25 \\ 1.22 & 1.19 & 1.25 & \infty \\ \end{pmatrix}$$

Hence, from the above matrix, we find that it is not possible to select exactly one 1 in each row and column. Thus, we cannot move forward towards the optimal solution. So, there is need to repeat step 5. Hence, again we draw lines by covering all the 1's in the matrix, which covers row 3 and column 3. The least element among all the uncovered elements is found to be 1.19. We divide all these uncovered elements by 1.19, multiply the same at the intersection of the drawn lines and the other elements under the drawn line remain the same. The resulting matrix is,

$$S = \begin{pmatrix} \infty & 1.27 & 1 & 1.03 \\ 1.27 & \infty & 1 & 1 \\ 1 & 1 & \infty & 1.25 \\ 1.03 & 1 & 1.25 & \infty \\ \end{pmatrix}$$

At this stage, we are able to select exactly one 1 in each row and column, which is shown below :

$$S = \begin{pmatrix} \infty & 1.27 & \boxed{1} & 1.03 \\ 1.27 & \infty & 1 & \boxed{1} \\ \boxed{1} & 1 & \infty & 1.25 \\ 1.03 & \boxed{1} & 1.25 & \infty \\ \end{pmatrix}$$

Hence, we can move forward towards the optimal solution. So, there is no need to repeat step 5.

Step 7 : The crisp travelling schedule(CTS) here is, $1 \to 3, 2 \to 4, 3 \to 1$ and $4 \to 2$. Thus, we find that this schedule is not a cycle. Since city 1 moves forward to city 3, city x is chosen to be city 3. On calculating the average of all the rows(cities), city 4 is found to have the highest average and hence is considered as city y. On exchanging cities 3 and 4(i.e.,cities x and y), we obtain the cycle(CTS) - $1 \to 3 \to 2 \to 4 \to 1$, which itself serves as the crisp optimal travelling schedule(COTS), where the resulting matrix is,

$$S = \begin{pmatrix} \infty & 1.27 & \boxed{1} & 1.03 \\ 1.27 & \infty & 1 & \boxed{1} \\ 1.03 & \boxed{1} & 1.25 & \infty \\ \boxed{1} & 1 & \infty & 1.25 \end{pmatrix}$$

The total minimal crisp travelling cost(TMCTC) = Rs.(3.15 + 3.325 + 3.9583 + 3.825) = Rs.14.2583, which itself serves as the crisp optimal travelling cost(COTC).

5.2. *Illustration 2 :*

Consider the following symmetric TSP in the form of a single-valued triangular fuzzy neutrosophic distance matrix [25],

$$S = \begin{pmatrix} \infty & s_{12} & s_{13} & s_{14} & s_{15} \\ s_{21} & \infty & s_{23} & s_{24} & s_{25} \\ s_{31} & s_{32} & \infty & s_{34} & s_{35} \\ s_{41} & s_{42} & s_{43} & \infty & s_{45} \\ s_{51} & s_{52} & s_{53} & s_{54} & \infty \end{pmatrix}$$

where its elements(single-valued triangular fuzzy neutrosophic numbers) are as follows :

$s_{12} = s_{21} = \langle(1, 9, 20)\,; 0.9, 0.4, 0.1\rangle$; $s_{13} = s_{31} = \langle(2, 9, 25)\,; 0.8, 0.5, 0.1\rangle$;
$s_{14} = s_{41} = \langle(5, 7, 9)\,; 0.9, 0.7, 0.1\rangle$; $s_{15} = s_{51} = \langle(2, 9, 19)\,; 0.4, 0.5, 0.3\rangle$;
$s_{23} = s_{32} = \langle(3, 9, 14)\,; 0.7, 0.3, 0.3\rangle$; $s_{24} = s_{42} = \langle(5, 8, 13)\,; 0.6, 0.2, 0.4\rangle$;
$s_{25} = s_{52} = \langle(7, 9, 18)\,; 0.4, 0.1, 0.1\rangle$; $s_{34} = s_{43} = \langle(4, 8, 17)\,; 0.8, 0.5, 0.2\rangle$;
$s_{35} = s_{53} = \langle(5, 9, 15)\,; 0.9, 0.6, 0.1\rangle$; $s_{45} = s_{54} = \langle(1, 9, 16)\,; 0.7, 0.4, 0.3\rangle$.

Step 1 : Using the above mentioned score function (1),

$$S(A) = \tfrac{1}{5}((a_1 + a_2 + a_3) - (\alpha_a + \beta_a + \gamma_a)),$$

converting the given neutrosophic data into their corresponding crisp data, we obtain, $S(s_{12}) = S(s_{21}) = \frac{39\times 2.4}{12} = \frac{93.6}{12} = 7.8$; $S(s_{13}) = S(s_{31}) = \frac{45\times 2.2}{12} = \frac{99}{12} = 8.25$;

$S(s_{14}) = S(s_{41}) = \frac{28\times 2.1}{12} = \frac{58.8}{12} = 4.9$; $S(s_{15}) = S(s_{51}) = \frac{39\times 1.7}{12} = \frac{66.3}{12} = 5.525$;

$S(s_{23}) = S(s_{32}) = \frac{35\times 2.1}{12} = \frac{73.5}{12} = 6.125$; $S(s_{24}) = S(s_{42}) = \frac{34\times 2}{12} = \frac{68}{12} = 5.6667$;

$S(s_{25}) = S(s_{52}) = \frac{43\times 2.2}{12} = \frac{94.6}{12} = 7.8833$; $S(s_{34}) = S(s_{43}) = \frac{37\times 2.1}{12} = \frac{77.7}{12} = 6.475$;

$S(s_{35}) = S(s_{53}) = \frac{38\times 2.2}{12} = \frac{83.6}{12} = 6.9667$; $S(s_{45}) = S(s_{54}) = \frac{35\times 2}{12} = \frac{70}{12} = 5.8333.$

The finally obtained crisp equivalent TSP matrix is given by,

$$S = \begin{pmatrix} \infty & 7.8 & 8.25 & 4.9 & 5.525 \\ 7.8 & \infty & 6.125 & 5.6667 & 7.8833 \\ 8.25 & 6.125 & \infty & 6.475 & 6.9667 \\ 4.9 & 5.6667 & 6.475 & \infty & 5.8333 \\ 5.525 & 7.8833 & 6.9667 & 5.8333 & \infty \end{pmatrix}$$

Step 2 : The next step is to calculate the range value of every column of the single-valued triangular fuzzy neutrosophic distance matrix, utilizing the formula, Range = Highest value - Least value. The required values for the columns 1 - 5 are found to be 3.35, 2.2166, 2.125, 1.575 and 2.3583 respectively.

Step 3 : Now, the highest range value, from all the range values calculated is 3.35 and the corresponding column is selected, which is found to be column 1.

Step 4 : Next, we choose the least value, 4.9 from the column 1 selected, as follows :

$$S = \begin{pmatrix} \infty & 7.8 & 8.25 & 4.9 & 5.525 \\ 7.8 & \infty & 6.125 & 5.6667 & 7.8833 \\ 8.25 & 6.125 & \infty & 6.475 & 6.9667 \\ \boxed{4.9} & 5.6667 & 6.475 & \infty & 5.8333 \\ 5.525 & 7.8833 & 6.9667 & 5.8333 & \infty \end{pmatrix}$$

Then, we divide all the remaining entries of the matrix by this chosen least value. Having performed these few steps, would create certain number of ones in the matrix, which is shown in the following table :

$$S = \begin{pmatrix} \infty & 1.5918 & 1.6837 & 1 & 1.1276 \\ 1.5918 & \infty & 1.25 & 1.1565 & 1.6088 \\ 1.6837 & 1.25 & \infty & 1.3214 & 1.4218 \\ 1 & 1.1565 & 1.3214 & \infty & 1.1905 \\ 1.1276 & 1.6088 & 1.4218 & 1.1905 & \infty \end{pmatrix}$$

The steps having been performed so far has created quite a few number of 1's.

Step 5 : Let us try selecting exactly one 1 in each row and column, as shown below :

$$S = \begin{pmatrix} \infty & 1.5918 & 1.6837 & \boxed{1} & 1.1276 \\ 1.5918 & \infty & 1.25 & 1.1565 & 1.6088 \\ 1.6837 & 1.25 & \infty & 1.3214 & 1.4218 \\ \boxed{1} & 1.1565 & 1.3214 & \infty & 1.1905 \\ 1.1276 & 1.6088 & 1.4218 & 1.1905 & \infty \end{pmatrix}$$

We find that it is not possible from the above matrix. Hence in order to reach the optimal solution, we draw lines by covering all the 1's in the matrix, which covers row 1 and column 1. The least element among all the uncovered elements is found to be 1.1565. We divide all these uncovered elements by 1.1565, multiply the same at the intersection of the drawn lines and the other elements under the drawn line remain the same. The resulting matrix is,

$$S = \begin{pmatrix} \infty & 1.5918 & 1.6837 & 1 & 1.1276 \\ 1.5918 & \infty & 1.0808 & 1 & 1.3911 \\ 1.6837 & 1.0808 & \infty & 1.4259 & 1.2294 \\ 1 & 1 & 1.4259 & \infty & 1.0294 \\ 1.1276 & 1.3911 & 1.2294 & 1.0294 & \infty \end{pmatrix}$$

Step 6 : Now, let us again try selecting exactly one 1 in each row and column, which is shown below :

$$S = \begin{pmatrix} \infty & 1.5918 & 1.6837 & \boxed{1} & 1.1276 \\ 1.5918 & \infty & 1.0808 & 1 & 1.3911 \\ 1.6837 & 1.0808 & \infty & 1.4259 & 1.2294 \\ \boxed{1} & 1 & 1.4259 & \infty & 1.0294 \\ 1.1276 & 1.3911 & 1.2294 & 1.0294 & \infty \end{pmatrix}$$

Hence, from the above matrix, we find that it is not possible to select exactly one 1 in each row and column. Thus, we cannot move forward towards the optimal solution. So, there is need to repeat step 5. Hence, again we draw lines by covering all the 1's in the matrix, which covers row 4 and column 4. The least element among all the uncovered elements is found to be 1.0808. We divide all these uncovered elements by 1.0808, multiply the same at the intersection of the drawn lines and the other elements under the drawn line remain the same. The resulting matrix is,

$$S = \begin{pmatrix} \infty & 1.4728 & 1.5578 & 1 & 1.0433 \\ 1.4728 & \infty & 1 & 1 & 1.2871 \\ 1.5578 & 1 & \infty & 1.4259 & 1.1375 \\ 1 & 1 & 1.4259 & \infty & 1.0294 \\ 1.0433 & 1.2871 & 1.1375 & 1.0294 & \infty \end{pmatrix}$$

At this stage, we are not able to select exactly one 1 in each row and column, which is shown below :

$$S = \begin{pmatrix} \infty & 1.4728 & 1.5578 & \boxed{1} & 1.0433 \\ 1.4728 & \infty & \boxed{1} & 1 & 1.2871 \\ 1.5578 & \boxed{1} & \infty & 1.4259 & 1.1375 \\ \boxed{1} & 1 & 1.4259 & \infty & 1.0294 \\ 1.0433 & 1.2871 & 1.1375 & 1.0294 & \infty \end{pmatrix}$$

Hence, we cannot move forward towards the optimal solution. So, there is need to repeat step 5. Thus, again repeating step 5 for a few number of times, we obtain the resulting matrix, where we are able to select exactly one 1 in each row and column, which is shown below :

$$S = \begin{pmatrix} \infty & 1.5161 & 1.4931 & \boxed{1} & 1 \\ 1.5161 & \infty & \boxed{1} & 1.0433 & 1.2871 \\ 1.4931 & \boxed{1} & \infty & 1.3852 & 1.0592 \\ 1 & 1.0433 & 1.3852 & \infty & \boxed{1} \\ \boxed{1} & 1.2871 & 1.0592 & 1 & \infty \end{pmatrix}$$

Hence, we can move forward towards the optimal solution. So, there is no need to repeat step 5.

Step 7 : The crisp travelling schedule(CTS) here is, $1 \to 4, 2 \to 3, 3 \to 2, 4 \to 5$ and $5 \to 1$. Thus, we find that this schedule is not a cycle. Since city 1 moves forward to city 4, city x is chosen to be city 4. On calculating the average of all the rows(cities), city 3 is found to have the highest average and hence is considered as city y. On exchanging cities 3 and 4(i.e.,cities x and y), we obtain the cycle(CTS) - $1 \to 4 \to 2 \to 3 \to 5 \to 1$, which is the crisp optimal travelling schedule(COTS), where the resulting matrix is,

$$S = \begin{pmatrix} \infty & 1.5161 & 1.4931 & \boxed{1} & 1 \\ 1.5161 & \infty & \boxed{1} & 1.0433 & 1.2871 \\ 1 & 1.0433 & 1.3852 & \infty & \boxed{1} \\ 1.4931 & \boxed{1} & \infty & 1.3852 & 1.0592 \\ \boxed{1} & 1.2871 & 1.0592 & 1 & \infty \end{pmatrix}$$

The total minimal crisp travelling cost(TMCTC) = Rs.(4.9 + 5.6667 + 6.125 + 6.9667 + 5.525) = Rs.29.1834, which itself serves as the crisp optimal travelling cost(COTC).

5.3. *Illustration 3 :*

Consider the following elements(single-valued triangular fuzzy neutrosophic numbers)
$s_{12} = s_{21} = \langle (2, 8, 18) ; 0.8, 0.3, 0.2\rangle$; $s_{13} = s_{31} = \langle (1, 9, 24) ; 0.9, 0.6, 0.2\rangle$;
$s_{14} = s_{41} = \langle (4, 9, 15) ; 0.6, 0.5, 0.2\rangle$; $s_{15} = s_{51} = \langle (3, 6, 13) ; 0.6, 0.3, 0.3\rangle$;
$s_{23} = s_{32} = \langle (6, 9, 19) ; 0.9, 0.4, 0.1\rangle$; $s_{24} = s_{42} = \langle (1, 7, 12) ; 0.9, 0.1, 0.2\rangle$;
$s_{25} = s_{52} = \langle (5, 9, 18) ; 0.9, 0.8, 0.2\rangle$; $s_{34} = s_{43} = \langle (3, 8, 23) ; 0.7, 0.1, 0.1\rangle$;
$s_{35} = s_{53} = \langle (2, 8, 32) ; 0.6, 0.5, 0.4\rangle$; $s_{45} = s_{54} = \langle (2, 5, 11) ; 0.9, 0.3, 0.1\rangle$.

of the symmetric TSP in the form of a single-valued triangular fuzzy neutrosophic distance matrix [25],

$$S = \begin{pmatrix} \infty & s_{12} & s_{13} & s_{14} & s_{15} \\ s_{21} & \infty & s_{23} & s_{24} & s_{25} \\ s_{31} & s_{32} & \infty & s_{34} & s_{35} \\ s_{41} & s_{42} & s_{43} & \infty & s_{45} \\ s_{51} & s_{52} & s_{53} & s_{54} & \infty \end{pmatrix}$$

Step 1 : Using the above specified score function (1),

$$S(a) = \frac{1}{12}((a_1 + 2a_2 + a_3)(2 + \alpha_a - \beta_a - \gamma_a)),$$

converting the given neutrosophic data into their corresponding crisp data, we obtain, $S(s_{12}) = S(s_{21}) = \frac{36\times2.3}{12} = \frac{82.8}{12} = 6.9$; $S(s_{13}) = S(s_{31}) = \frac{43\times2.1}{12} = \frac{90.3}{12} = 7.525$;

$S(s_{14}) = S(s_{41}) = \frac{37\times1.9}{12} = \frac{70.3}{12} = 5.8583$; $S(s_{15}) = S(s_{51}) = \frac{28\times2}{12} = \frac{56}{12} = 4.6667$;

$S(s_{23}) = S(s_{32}) = \frac{43\times2.4}{12} = \frac{103.2}{12} = 8.6$; $S(s_{24}) = S(s_{42}) = \frac{27\times2.6}{12} = \frac{70.2}{12} = 5.85$;

$S(s_{25}) = S(s_{52}) = \frac{42\times2.5}{12} = \frac{105}{12} = 8.75$; $S(s_{34}) = S(s_{43}) = \frac{50\times1.7}{12} = \frac{85}{12} = 7.0833$;

$S(s_{35}) = S(s_{53}) = \frac{23\times2.5}{12} = \frac{57.5}{12} = 4.7917$; $S(s_{45}) = S(s_{54}) = \frac{41\times1.9}{12} = \frac{77.9}{12} = 6.4917$.

The finally obtained crisp equivalent TSP matrix is given by,

$$S = \begin{pmatrix} \infty & 6.9 & 7.525 & 5.8583 & 4.6667 \\ 6.9 & \infty & 8.6 & 5.85 & 6.4917 \\ 7.525 & 8.6 & \infty & 8.75 & 7.0833 \\ 5.8583 & 5.85 & 8.75 & \infty & 4.7917 \\ 4.6667 & 6.4917 & 7.0833 & 4.7917 & \infty \end{pmatrix}$$

Step 2 : The next step is to calculate the range value of every column of the single-valued triangular fuzzy neutrosophic distance matrix, utilizing the formula, Range = Highest value - Least value. The required values for the columns 1 - 5 are found to be 2.8583, 2.75, 1.6667, 3.9583 and 2.4166 respectively.

Step 3 : Now, the highest range value, from all the range values calculated is 3.9583 and the corresponding column is selected, which is found to be column 4.

Step 4 : Next, we choose the least value, 4.7917 from the column 4 selected, as follows :

$$S = \begin{pmatrix} \infty & 6.9 & 7.525 & 5.8583 & 4.6667 \\ 6.9 & \infty & 8.6 & 5.85 & 6.4917 \\ 7.525 & 8.6 & \infty & 8.75 & 7.0833 \\ 5.8583 & 5.85 & 8.75 & \infty & 4.7917 \\ 4.6667 & 6.4917 & 7.0833 & \boxed{4.7917} & \infty \end{pmatrix}$$

Then, we divide all the remaining entries of the matrix by this chosen least value. Having performed these few steps, would create certain number of ones in the matrix, which is shown in the following matrix :

$$S = \begin{pmatrix} \infty & 1.44 & 1.5704 & 1.2226 & 0.9739 \\ 1.44 & \infty & 1.7948 & 1.2209 & 1.3548 \\ 1.5704 & 1.7948 & \infty & 1.8261 & 1.4782 \\ 1.2226 & 1.2209 & 1.8261 & \infty & 1 \\ 0.9739 & 1.3548 & 1.4782 & 1 & \infty \end{pmatrix}$$

The steps having been performed so far has created quite a few number of 1's.

Step 5 : Let us try selecting exactly one 1 in each row and column, as shown below :

$$S = \begin{pmatrix} \infty & 1.44 & 1.5704 & 1.2226 & 0.9739 \\ 1.44 & \infty & 1.7948 & 1.2209 & 1.3548 \\ 1.5704 & 1.7948 & \infty & 1.8261 & 1.4782 \\ 1.2226 & 1.2209 & 1.8261 & \infty & \boxed{1} \\ 0.9739 & 1.3548 & 1.4782 & \boxed{1} & \infty \end{pmatrix}$$

We find that it is not possible from the above matrix. Hence in order to reach the optimal solution, we draw lines by covering all the 1's in the matrix, which covers row 4 and column 4. The least element among all the uncovered elements is found to be 1.2209. We divide all these uncovered elements by 1.2209, multiply the same at the intersection of the drawn lines and the other elements under the drawn line remain the same. The resulting matrix is,

$$S = \begin{pmatrix} \infty & 1.1795 & 1.2863 & 1.0014 & 0.9739 \\ 1.1795 & \infty & 1.4701 & 1 & 1.3548 \\ 1.2863 & 1.4701 & \infty & 1.4957 & 1.4782 \\ 1.0014 & 1 & 1.4957 & \infty & 1 \\ 0.9739 & 1.3548 & 1.4782 & 1 & \infty \end{pmatrix}$$

Step 6 : Now, let us again try selecting exactly one 1 in each row and column, which is shown below :

$$S = \begin{pmatrix} \infty & 1.1795 & 1.2863 & 1.0014 & 0.9739 \\ 1.1795 & \infty & 1.4701 & \boxed{1} & 1.3548 \\ 1.2863 & 1.4701 & \infty & 1.4957 & 1.4782 \\ 1.0014 & \boxed{1} & 1.4957 & \infty & 1 \\ 0.9739 & 1.3548 & 1.4782 & 1 & \infty \end{pmatrix}$$

Hence, from the above matrix, we find that it is not possible to select exactly one 1 in each row and column. Thus, we cannot move forward towards the optimal solution. So, there is need to repeat step 5. Hence, again repeating step 5 for a few number of times, we obtain the resulting matrix, where we are able to select exactly one 1 in each row and column, which is shown below :

$$S = \begin{pmatrix} \infty & 1.2111 & \boxed{1} & 1.1444 & 1 \\ 1.2111 & \infty & 1 & \boxed{1} & 1.2172 \\ 1 & \boxed{1} & \infty & 1.1324 & 1.0054 \\ 1.1444 & 1 & 1.1324 & \infty & \boxed{1} \\ \boxed{1} & 1.2172 & 1.0054 & 1 & \infty \end{pmatrix}$$

Hence, we can move forward towards the optimal solution. So, there is no need to repeat step 5.

Step 7 : The crisp travelling schedule(CTS) here is, $1 \to 3, 2 \to 4, 3 \to 2, 4 \to 5$ and $5 \to 1$. Since we find that this schedule is a cycle(CTS), $1 \to 3 \to 2 \to 4 \to 5 \to 1$, there is no need to exchange the cities and hence this itself serves as the crisp optimal travelling schedule(COTS), where the resulting matrix is the same as that obtained in the previous step which is,

$$S = \begin{pmatrix} \infty & 1.2111 & \boxed{1} & 1.1444 & 1 \\ 1.2111 & \infty & 1 & \boxed{1} & 1.2172 \\ 1 & \boxed{1} & \infty & 1.1324 & 1.0054 \\ 1.1444 & 1 & 1.1324 & \infty & \boxed{1} \\ \boxed{1} & 1.2172 & 1.0054 & 1 & \infty \\ \end{pmatrix}$$

The total minimal crisp travelling cost(TMCTC) = Rs.(7.525 + 8.6 + 5.85 + 4.7917 + 4.6667) = Rs.31.4334, which itself serves as the crisp optimal travelling cost(COTC).

Remark : We observe that the TTC for illustrations 1, 2 and 3 obtained here using the proposed method(PM) differ from those of the corresponding illustrations acquired using the Dhouib-Matrix-TSP1(DM-TSP1) heuristic in [25]. The following section discusses further insights and justifications on the SVTFNTSP that was taken into account in this study.

6. **Results and Discussions :**

The following tables 1, 2 and figures 2, 3, 4 and 5 provide the solutions of the SVTFNTSP obtained using the proposed approach and a comparison of the solutions of the proposed approach here to solve the SVTFNTSP, with an existing method(EM : Dhouib-Matrix-TSP1(DM-TSP1) heuristic) as in [25]. Some of the significant results are shown in these tables and figures.

TABLE 1. Solutions of the SVTFNTSP obtained using the proposed method :

Illustrations	**CTS**	**TMCTC**
Illustration 1	$1 \to 3 \to 2 \to 4 \to 1$	Rs.14.2583
Illustration 2	$1 \to 4 \to 2 \to 3 \to 5 \to 1$	Rs.29.1834
Illustration 3	$1 \to 3 \to 2 \to 4 \to 5 \to 1$	Rs.31.4334

- The table 1 showcases the crisp travelling schedule(CTS), the total minimal crisp travelling cost(TMCTC) of all the above considered three illustrations, obtained using the proposed method.

Table 2. Comparison of the solutions of the SVTFNTSP obtained using the proposed approach, with an existing method as in [25] :

Illustrations	CTS of the EM	CTS of the PM	TMCTC of the EM	TMCTC of the PM
Illustration 1	$1 \to 4 \to 2 \to 3 \to 1$	$1 \to 3 \to 2 \to 4 \to 1$	Rs.14.27	Rs.14.2583
Illustration 2	$1 \to 5 \to 3 \to 2 \to 4 \to 1$	$1 \to 4 \to 2 \to 3 \to 5 \to 1$	Rs.29.20	Rs.29.1834
Illustration 3	$1 \to 3 \to 2 \to 4 \to 5 \to 1$	$1 \to 3 \to 2 \to 4 \to 5 \to 1$	Rs.31.44	Rs.31.4334

- The table 2 compares the CTS and TMCTC calculated using the proposed method(PM) with those of the three illustrations, expressed as real-world problems taken into consideration above, using an existing method(EM : Dhouib-Matrix-TSP1(DM-TSP1) heuristic) as in [25].
- Thus, the above comparison ensures that for the given SVTFNTSPs, the CTS and the TMCTC found by applying the proposed method itself serve as the COTS and the COTC, respectively.
- The same above conclusions can be drawn from the figure 2 which provides an overview of the solutions(COTCs) to the three SVTFNTSPs that were previously taken into consideration and solved using the previously mentioned existing method with that of the same three problems using the proposed method. It does so by demonstrating a sizable amount of variation in the values of the solutions, through the usage of the proposed method.

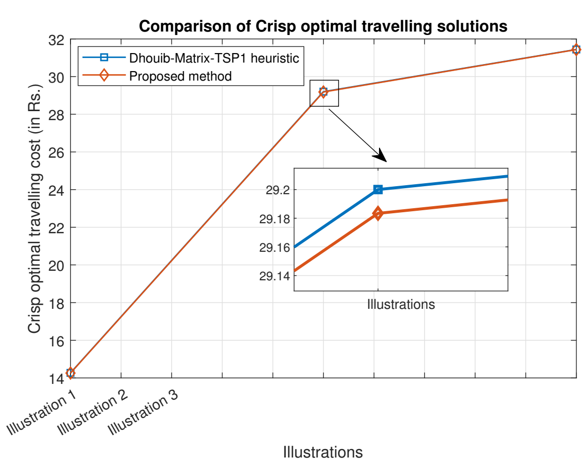

Figure 2. An overview of the optimal solutions determined by Dhouib-Matrix-TSP1 heuristic and the proposed approach :

- Figure 2 also shows that the solutions to the aforementioned three SVTFNTSPs obtained using the proposed method(represented by the orange line) are better(in terms of TMCTC/COTC) than those obtained using the other existing method(represented by the blue line) as in [25]. It also presents an even more clear picture of how the solution of the SVTFNTSPs under consideration using the proposed method(represented by the orange line) is better than that of the same SVTFNTSPs using the other existing method(represented by the blue line) as in [25], by providing just a glimpse of an enlarged version of the sample with a considerable amount of variation in the total minimal crisp travelling costs(TMCTC/COTC).

- Knowing that a few classical methods are frequently used to test the optimality of the given TSP and provide better solutions than the other methods, we can infer from the figure 2 that the proposed method also seems to fulfil the same purpose (thereby providing the best possible solution (or) making the total crisp travelling costs as minimal as possible) by giving lower values for the TMCTC of the three SVTFNTSPs taken into consideration in this article, when compared to the Hungarian method thus making it as the COTS along with the COTC.

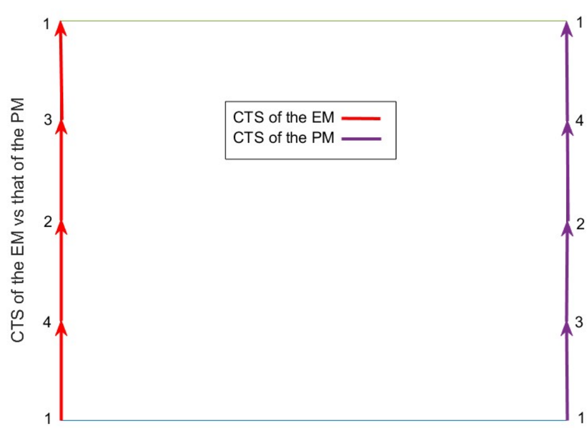

Figure 3. A comparison of the CTS(COTS) of illustration 1 using the existing method as in [25] with that of the proposed approach :

- The figure 3 presents a glimpse of the comparison of the (CTS/COTS) of illustration 1 using an existing method($1 \to 4 \to 2 \to 3 \to 1$)(represented by red arrow line) as in [25] and the proposed method($1 \to 3 \to 2 \to 4 \to 1$)(represented by violet arrow line) of the SVTFNTSP taken into consideration respectively.
- The figure 4 presents a glimpse of the comparison of the (CTS/COTS) of illustration 2 using an existing method($1 \to 5 \to 3 \to 2 \to 4 \to 1$)(represented by red arrow line) as in [25] and the proposed method($1 \to 4 \to 2 \to 3 \to 5 \to 1$)(represented by violet arrow line) of the SVTFNTSP taken into consideration respectively.

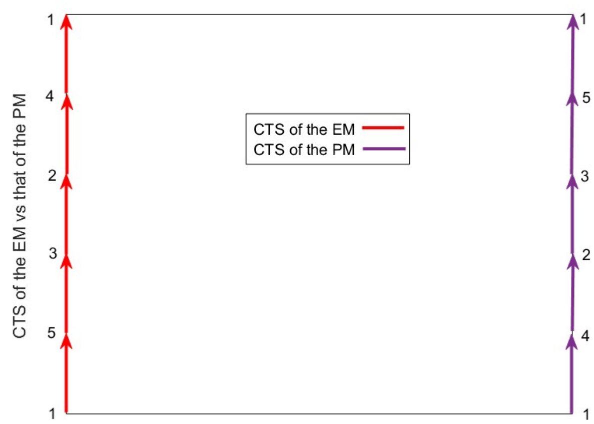

FIGURE 4. A comparison of the CTS(COTS) of illustration 2 using the existing method as in [25] with that of the proposed approach :

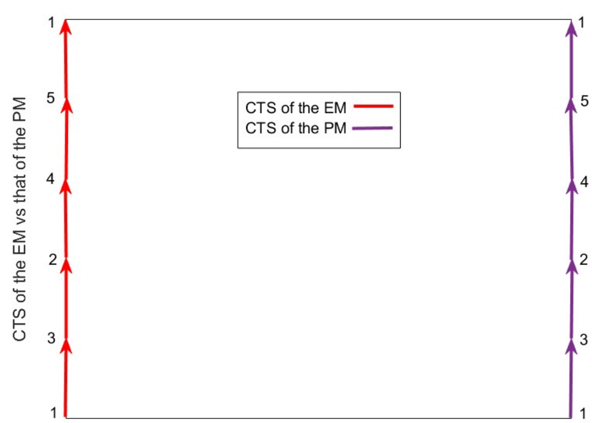

FIGURE 5. A comparison of the CTS(COTS) of illustration 3 using the existing method as in [25] with that of the proposed approach :

- The figure 5 presents a glimpse of the comparison of the (CTS/COTS) of illustration 3 using an existing method($1 \to 3 \to 2 \to 4 \to 5 \to 1$)(represented by red arrow line) as in [25] and the proposed method($1 \to 3 \to 2 \to 4 \to 5 \to 1$)(represented by violet arrow line) of the SVTFNTSP taken into consideration respectively.
- Exploring the single-valued triangular fuzzy neutrosophic travelling salesman problem(SVTFNTSP) comes with several limitations. Firstly, the SVTFNTSP deals with highly uncertain and ambiguous data, making it a complex problem to model and solve accurately. The lack of precise and standardized mathematical representations

for triangular fuzzy neutrosophic data can hinder the development of robust optimization algorithms. Additionally, as SVTFNTSP is an extension of the classic traveling salesman problem(TSP), it inherits its NP-hard complexity, meaning that finding an optimal solution within a reasonable timeframe can be computationally infeasible for large-scale instances. This limitation poses challenges in real-world applications where the problem size can be substantial. Moreover, the availability of real-world data in the form of single-valued triangular fuzzy neutrosophic numbers can be scarce, leading to difficulties in validating and benchmarking proposed algorithms. The lack of well-established benchmarks and standardized datasets makes it challenging to assess the performance of different approaches effectively. Furthermore, the SVTFNTSP introduces additional computational overhead compared to solving the traditional TSP, which can limit its practical applicability. Despite its potential to model uncertainty more accurately, the SVTFNTSP remains an area of ongoing research with open challenges in algorithm development and real-world implementation.

- Having incorporated the range(a measure of dispersion) in the proposed methodology for solving the single-valued triangular fuzzy neutrosophic traveling salesman problem (SVTFNTSP) can offer some potential advantages as follows : The single-valued triangular fuzzy neutrosophic sets in SVTFNTSP are designed to represent indeterminacy and uncertainty in the problem. The range provides a simple way to quantify the spread or variability of neutrosophic values, which can help capture the degree of uncertainty associated with each data point or city in the SVTFNTSP instance. The range is a straightforward measure to calculate and understand. It can be easily computed for neutrosophic values without the need for complex mathematical operations, making it accessible to a wide range of users. The range can be used to create visual representations of neutrosophic data, such as scatter plots or graphs, which can help analysts and decision-makers gain insights into the distribution of data points and associated uncertainty. In some cases, decision-makers may have preferences for solutions with lower or higher ranges. For example, they may prioritize routes with less variability in travel times. In such cases, the range can be used as an additional criterion for evaluating and ranking solutions. The range can be used to compare the dispersion of neutrosophic values across different cities or nodes in the NTSP. This can aid in identifying cities with higher or lower levels of uncertainty, which may influence route planning or decision-making. Analyzing how changes in neutrosophic values affect the range can help assess the sensitivity of the NTSP solution to variations in data, which can be valuable in robust decision-making. The range can complement other measures of dispersion and central tendency, such as standard deviation and mean, providing a more comprehensive view of the data distribution and uncertainty.

7. Conclusion

The SVTFNTSP presents a significant expansion of the traditional TSP, incorporating uncertainty and ambiguity through SVTFNS. This innovative framework empowers decision-makers to more effectively address complex real-world situations by portraying uncertain data more realistically. By taking into account various perspectives, it aids in a better comprehension of the preferences of decision-makers, ultimately enhancing the problem-solving process. Researchers have devised innovative algorithms and methodologies to tackle the intricate nature of single-valued triangular fuzzy neutrosophic data and resolve the same. They have explored a range of optimization techniques, heuristic approaches, and metaheuristics to achieve efficient solutions and enhance computational efficiency. Consequently, this research article earnestly endeavors to assist in this regard by investigating the characteristics, types, and resolutions of SVTFNTSP, introducing a novel method for its resolution. To demonstrate its efficiency and significance, the proposed approach is juxtaposed with specific classical methods. The proposed method consistently demonstrates its effectiveness, advantages, and potential through comparative analyses against alternative methods. It offers reductions in overall travelling costs, optimal solutions, and computational simplification while maintaining solution quality. By effectively leveraging SVTFNS, it adeptly captures uncertainty and ambiguity, yielding assignment solutions that approach optimality. Its ability to strike a balance between accuracy and computational efficiency makes it the preferred choice for real-world problem-solving, particularly in scenarios demanding time and resource optimization. This reduction in computational complexity and overall travelling cost enhances its applicability to larger and more complex SVTFNTSP instances, spanning diverse domains such as supply chain management, transportation, and logistics, where cost-effective solutions hold paramount importance. Notwithstanding its merits, ongoing research endeavors to fine-tune algorithms and address scalability issues, broadening its scope to fully realize its potential. Overall, SVTFNTSP offers a valuable and versatile approach for addressing uncertainty in decision-making, making a substantial contribution to real-world problem-solving across a spectrum of domains while advancing the field of decision theory and optimization techniques.

8. Future work

Future work intends to explore the application of this proposed technique to handle the multi-objective travelling salesman problem and to provide decision-makers in logistics, transportation, and related domains with a robust and effective tool for solving real-world

SVTFNTSP instances, recognizing the considerable ongoing research in this field. The expected outcome of this future work is that, this approach would have the potential to significantly improve route planning and delivery services for sales personnel, resource allocation, portfolio optimization, environmental planning, project scheduling and cost optimization while considering the intricate nature of uncertain data. It will bring forth a range of advantages, a broad scope of applications, and diverse uses. This method offers the advantage of optimizing decision-making by efficiently balancing multiple conflicting objectives, making it ideal for scenarios where objectives include minimizing travel distance, cost, and time while maximizing customer satisfaction or other pertinent criteria. Its scope extends across industries such as supply chain management, tourism, manufacturing, urban planning, logistics and telecommunications, among others. This innovative approach holds the potential to revolutionize decision-making processes, delivering more robust, efficient, and balanced solutions for complex multi-objective scenarios in various domains. It would contribute to advancing the field of decision-making under uncertainty and further demonstrate the versatility and applicability of single-valued triangular fuzzy neutrosophic techniques in solving practical problems. Such an endeavor would augment the findings and benefit the domain of single-valued triangular fuzzy neutrosophic research.

References

[1] Zadeh,L.A., Fuzzy sets, *Inf. Control* (1965), 8, pp. 338-353.

[2] Atanassov,K.T., Intuitionistic fuzzy sets, *Fuzzy Sets Syst* (1986), 20(1), pp. 87-96.

[3] Smarandache,F., Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, *Int. J. Pure Appl. Math.* (2005), 24, pp. 287-297.

[4] Mehmet Merkepci,Mohammad Abobala, Security Model for Encrypting Uncertain Rational Data Units Based on Refined Neutrosophic Integers Fusion and El Gamal Algorithm, Fusion: Practice and Applications, Vol. 10 , No. 2 , (2023) : 35-41 (Doi : https://doi.org/10.54216/FPA.100203)

[5] Esakkiammal,T., and Murugesan,R., Mass - A New Ones Assignment Method for Finding Optimal Solution of Assignment Problems, *Turkish Journal of Computer and Mathematics Education* (2021), 12(10), pp. 2737-2744.

[6] Mehmet Merkepci,Mohammad Abobala,Ali Allouf, The Applications of Fusion Neutrosophic Number Theory in Public Key Cryptography and the Improvement of RSA Algorithm, Fusion: Practice and Applications, Vol. 10 , No. 2 , (2023) : 69-74 (Doi : https://doi.org/10.54216/FPA.100206)

[7] Lone,M.A., Mir,S., Ismail,Y., and Majid,R., Intuitionistic Fuzzy Assignment Problem : An Application in Agriculture, *Asian Journal of Agricultural Extension, Economics and Sociology* (2017), 15(4), pp. 2320-7027.

[8] Srinivas,B., and Ganesan.G., A Method for solving intuitionistic fuzzy assignment problem using Branch and Bound method, *International Journal of Engineering Technology, Management and Applied Sciences* (2015), 3(2).

[9] Krishna Prabha,S., and Vimala,S., Neutrosophic Assignment Problem via BnB Algorithm, *Advances in Algebra and Analysis, Trends in Mathematics* (2019), pp. 323-330 .

[10] Khalifa Abd El-Wahed,H., and Kumar,P., A Novel Method for Neutrosophic Assignment Problem by using Interval-Valued Trapezoidal Neutrosophic Number, *Neutrosophic Sets and Systems* (2020), 36, pp. 24-36, (Doi: https://doi.org/10.5281/zenodo.4065363).

[11] Khalifa Abd El-Wahed,H., Kumar,P., and Smarandache,F., On optimizing neutrosophic complex programming using lexicographic order, *Infinite Study* (2020).

[12] Khalifa Abd El-Wahed,H., and Kumar,P., Interval-type fuzzy linear fractional programming problem in neutrosophic environment: A fuzzy mathematical programming approach, *Neutrosophic Sets and Systems* (2021), 47, pp. 38-49.

[13] Chakraborty,A., Mondal,S.P., Ahmadian,A., Senu,N., Alam,S., and Salahshour,S., Different forms of triangular neutrosophic numbers, de-neutrosophication techniques and their applications, *Symmetry* (2018), 10(8), pp. 327.

[14] Das, Sapan Kumar, and Edalatpanah,S.A., A new ranking function of triangular neutrosophic number and its application in integer programming, *International Journal of Neutrosophic Science 4.2* (2020), pp. 82-92.

[15] Pranab Biswas., Surapati Pramanik., and Bibhas,C.Giri., Aggregation of triangular fuzzy neutrosophic set information and its applications to multi-attribute decision-making, *Neutrosophic Sets and Systems* (2016), 12.

[16] Broumi,S., Nagarajan,D., Bakali,A., Talea,M., Smarandache,F., and Lathamaheswari,M., The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment, *Complex and Intelligent Systems* (2019), 5, pp. 391-402.

[17] Mullai,M., and Surya,R., Neutrosophic Inventory Backorder Problem Using Triangular Neutrosophic Numbers, *Neutrosophic Sets and Systems* (2020), 31, p. 148-155.

[18] Smarandache,F., Ricardo,J.E., Caballero,E.G., Leyva Vázquez,M.Y., and Hernández,N.B., Delphi method for evaluating scientific research proposals in a neutrosophic environment, *Neutrosophic Sets and Systems* (2020).

[19] Nemis García Arias., Edwin Prado Calderón., Leonardo Rosillo Abarca., and Sol David Lopezdomínguez Rivas., Neutrosophic Games Applied to Political Situations, *Neutrosophic Sets and Systems* (2020), 37.

[20] Roberto Carlos Jiménez Martínez., César Elías Paucar Paucar., José Ignacio Cruz Arboleda., Miguel Ángel Guambo Llerena., and Erick González Caballero., Neutrosophic Matrix Games to Solve Project Management Conflicts, *Neutrosophic Sets and Systems* (2021), 44.

[21] Pramanik,S., and Roy,T.K., Neutrosophic game theoretic approach to indo-pak conflict over jammu-kashmir, *Neutrosophic Sets and Systems* (2014), 2, pp. 82-101.

[22] Ye,J., Trapezoidal neutrosophic set and its application to multiple attribute decision-making, *Neural Computing and Applications* (2015), 26, pp. 1157-1166.

[23] Subasri,S., and Selvakumari,K., Solving Neutrosophic Travelling Salesaman Problem in Triangular Fuzzy Number Using Ones Assignment Method, *Eurasian Journal of Analytical Chemistry* (2018), 13, pp. 285-291.

[24] Subasri,S., and Selvakumari,K., Neutrosophic Travelling Salesman Problem in Trapezoidal Fuzzy number using Branch and Bound Technique, *Journal of Physics : Conference Series* (2019).

[25] Dhouib,S., Optimization of Travelling Salesman Problem on Single Valued Triangular Neutrosophic Number using Dhouib-Matrix-TSP1 Heuristic, *International Journal of Engineering* (2021), 34(12), pp. 2642-2647.

[26] Said Broumi., Assia Bakali., Mohamed Talea., Florentin Smarandache and Luige Vladareanu., Shortest Path Problem Under Triangular Fuzzy Neutrosophic Information, *10th International Conference on Software, Knowledge, Information Management and Applications (SKIMA)* (2016).

[27] Mona Mohamed and Abduallah Gamal, Toward Sustainable Emerging Economics based on Industry 5.0 : Leveraging Neutrosophic Theory in Appraisal Decision Framework, *Neutrosophic Systems with Applications* (2023), 1, pp. 14-21

[28] Abduallah Gamal and Amal F.Abd El-Gawad , Mohamed Abouhawwash, Towards a Responsive Resilient Supply Chain based on Industry 5.0 : A Case Study in Healthcare Systems, *Neutrosophic Systems with Applications* (2023), 2, pp. 8–24,

[29] Maissam Jdid and Florentin Smarandache, The Use of Neutrosophic Methods of Operation Research in the Management of Corporate Work, *Neutrosophic Systems with Applications* (2023), 3, pp. 1–16

[30] Uddin,F., Ishtiaq,U., Javed,K., Aiadi,S.S., Arshad,M., Souayah,N., and Mlaiki,N., A New Extension to the Intuitionistic Fuzzy Metric-like Spaces, *Symmetry 2022* (2022), 14, pp. 1400.

[31] Saleem,N., Javed,K., Uddin,F., Ishtiaq,U., Ahmed,K., Abdeljawad,T., and Alqudah,M.A., Unique Solution of Integral Equations via Intuitionistic Extended Fuzzy b-Metric-Like Spaces, *CMES-Computer Modeling in Engineering and Sciences* (2023), 135(1).

[32] Ali,U., Alyousef,H.A., Ishtiaq,U., Ahmed,K., and Ali,S., Solving nonlinear fractional differential equations for contractive and weakly compatible mappings in neutrosophic metric spaces, *Journal of Function Spaces 2022* (2022).

[33] Ishtiaq,U., Javed,K., Uddin,F., Sen,M.D.L., Ahmed,K., and Ali,M.U., Fixed point results in orthogonal neutrosophic metric spaces, *Complexity 2021*, (2021), pp. 1-18.

[34] Riaz,M., Ishtiaq,U., Park,C., Ahmad,K., and Uddin,F., Some fixed point results for xi-chainable neutrosophic and generalized neutrosophic cone metric spaces with application, *AIMS MATHEMATICS* (2022), 7(8), pp. 14756-14784.

[35] Din,F.U., Javed,K., Ishtiaq,U., Ahmed,K., Arshad,M., and Park,C., Existence of fixed point results in neutrosophic metric-like spaces, *AIMS MATHEMATICS* (2022), 7(9), pp. 17105-17122.

Received: July 7, 2023. Accepted: Nov 18, 2023