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Interval complex single-valued neutrosophic hypersoft set with Application in Decision Making

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Abstract. The interval complex single-valued neutrosophic hypersoft set (Ξ-set), together with its features and set-theoretic operations, is a new mathematical structure that is discussed in this article. For managing ambiguous and uncertain knowledge, the suggested structure integrates the interval complex single-valued neutrosophic set and hypersoft set. These two elements have already been regarded as trustworthy settings. The first component has the ability to manage information on interval and periodic types, while the second offers a multi-argument domain for concurrent consideration of numerous sub-attributes. The Ξ-set is used to aggregate these sets, allowing for the fusion of various qualities and any related uncertainty. The resultant aggregated sets, which take into account both the attribute values and the associated uncertainty, give a thorough representation of the decision aspect. To assist in decision-making, the method calculates how similar several options are to the optimum option using a distance-based similarity metric. By contrasting the combined sets of several options, the system determines the best option based on the specified selection criteria. Decision-makers can evaluate how changing attribute values may influence their choices using the suggested strategy's endorsement of sensitivity analysis. The efficacy of the recommended decision-support mechanism is demonstrated through a case study with a real-world choice dilemma. The results show how well the framework can handle ambiguity and uncertainty while providing decision-makers with meaningful insights and encouraging rational choices. Finally, the multi-attribute decision-support system based on aggregations of Ξ-set provides a reliable framework for dealing with difficult choice issues that are characterized by ambiguity and vagueness.

Keywords: complex fuzzy set; interval-valued fuzzy set; complex fuzzy soft set; complex intuitionistic fuzzy soft set; complex neutrosophic soft set; hypersoft set; complex fuzzy hypersoft set.

1. Introduction

In the context of an η-set environment with $\mathcal{IV}$ settings, this research article attempts to present the ideas of Ξ-set through the use of theoretical, axiomatic, graphical, and computational approaches. An algorithm is designed for $\mathbb{DSS}$ after conceptualizing the fundamental elementary conceptions of this structure. A real-world application is used to verify the proposed algorithm. The existing pertinent models are explored in detail using the proposed structure, and their generalization is elaborated under specific evaluation aspects. For addressing and modeling uncertainty, ambiguity, and imprecision in $\mathbb{DM}$ processes, $\mathbb{F}$-sets [1] offer a mathematical foundation. They provide more precise and robust analysis by providing more flexible and realistic modeling of real-world occurrences. Artificial intelligence, control systems, pattern recognition, and $\mathbb{DSS}$s are just a few of the areas where fuzzy sets are used. In order to describe complicated and structured uncertainty, a $\mathbb{CF}$-set [2] characterizes a particular feature of the object's uncertainty as a combination of $\mathscr{A}-$term and $\mathscr{P}-$term. Ramot et al. [3, 4] examined the novel idea of $\mathbb{CF}$-sets. The $\mathbb{CF}$-set offers a framework for mathematically expressing $\mathbb{M}_{fn}$ in a set in terms of a complex number. $\mathbb{CF}$-sets have been employed in a number of applications, such as control, pattern recognition, and $\mathbb{DM}$ [5]. $\mathbb{CF}$-sets may be used to simulate intricate connections between input characteristics and output labels in pattern recognition. When designing robust controllers for control, $\mathbb{CF}$-sets can be used to account for noisy and uncertain environments [6]. The $\mathbb{CIF}$-set [7, 8] enables modeling the ambiguous information that incorporates not only the $\mathbb{M}_{fn}$ but also the $\mathbb{N}_{fn}$ which are complex-valued functions. Rani and Garg [9] created $\mathbb{DMR}$ utilizing Hausdorff, Euclidean, and Hamming metrics and studied numerous desirable relations based on these measures [10]. They applied the concept in the $\mathbb{DM}$ process to these $\mathbb{DMR}$, especially in the fields of pattern recognition and medical diagnostics [11]. The complex-valued $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$ are all present in a $\mathcal{CN}$-set. Ali & Smarandache [12] discussed $\mathcal{CN}$-set along with its set theoretic operations and applied in $\mathbb{DM}$ [13]. An extension of the $\mathcal{CN}$-set known as the $\mathbb{IVCN}$-set [14] uses $\mathbb{IVC}$ entries to describe the $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$. Additional uncertainty attributes, such as the degree of vagueness and ambiguity, can be represented using the interval values. The $\mathbb{IVCN}$-sets have been used for a variety of tasks, including diagnosis, image processing, and $\mathbb{DM}$. The $\mathbb{IVCN}$-set has been used to simulate the decision-maker's level of confidence, uncertainty, and ambiguity with reference to various possibilities in the recruitment process [15]. The $\mathbb{IVCN}$-set has been used in image processing to represent the level of uncertainty involved in picture segmentation and recognition [16]. The contributions of scholars [17–19] are significant regarding the handling of uncertainties.

Molodtsov [20] developed $\mathbb{S}$-set theory as a method for handling uncertainty in data analysis and $\mathbb{DM}$. A crisp set that permits the insertion of ambiguous or speculative information is known as a $\mathbb{S}$-set in which each element is connected to a collection of parameters that may be used to symbolize various forms of uncertainty, including haziness, ambiguity, and inconsistent behavior [21,22]. The $\mathbb{S}$-sets have been employed in a wide range of disciplines, including machine learning, image processing, $\mathbb{DM}$, and data mining. The $\mathbb{S}$-sets have been applied to

$\mathbb{DM}$ to simulate human preferences and judgments in cases where the information is lacking or ambiguous. Babitha & Sunil [23] established the idea of $\mathbb{S}$-set relations and studied various related terminologies. Ali et al. [24] presented a number of novel operations and aggregation techniques on $\mathbb{S}$-sets. It has been demonstrated that these new strategies enhance the precision and efficacy of $\mathbb{DM}$ algorithms as as well as the efficiency of pattern recognition and clustering methods [25]. A hybrid notion known as $\mathbb{FS}$-set [26] contains the characteristics of both $\mathbb{F}$-sets and $\mathbb{S}$-sets. Application areas for the $\mathbb{FS}$-set idea include $\mathbb{DM}$ [27] in order to accommodate uncertainty and model inaccurate or incomplete data. The $\mathbb{FS}$-set-based $\mathbb{DM}$ techniques have been proven to be successful in enhancing $\mathbb{DM}$ accuracy and dependability [28, 29]. A hybrid idea known as $\mathbb{IFS}$-set [30] combines the qualities of $\mathbb{S}$-set and $\mathbb{IF}$-set, was presented as a generalisation of $\mathbb{IF}$-set and $\mathbb{S}$-set. By using level $\mathbb{S}$-sets of $\mathbb{IFS}$-sets and providing some illustrated instances, Jiang et al. [31] proposed an adaptable method to $\mathbb{DM}$. They discussed the weighted $\mathbb{IFS}$-sets and their potential use in $\mathbb{DM}$. A hybrid idea known as $\mathcal{NS}$-set contains the characteristics of both $\mathbb{S}$-set and $\mathcal{N}$-set. Maji [32] investigated the idea of a $\mathcal{N}$-set, applied it to $\mathbb{S}$-sets, and developed a $\mathcal{NS}$-set. He defined certain terms, performed some operations, and established some characteristics for the idea of $\mathcal{NS}$-set. In order to construct two $\mathcal{NS}$-sets, Deli & Broumi [33] defined a relation on $\mathcal{NS}$-sets and examined symmetric, transitive, and reflexive $\mathcal{NS}$ relations.

Das & Samanta [34] presented a description of the soft complex set and soft complex number and studied some of its fundamental aspects utilizing $\mathcal{F}$ numbers with the idea of $\mathbb{S}$-set along with the development of distinction and integration of $\mathcal{S}$ functions. The $\mathbb{CFS}$-sets were explored, and the aggregation operation in these sets was examined by Thirunavukarasu et al. [35]. They provided an example of prospective applications that illustrate how aggregation processes may be successfully used in numerous situations with uncertainties and periodicity. The idea of $\mathbb{CIFS}$-set presented by Kumar & Bajaj [36] allowed several parametrization techniques to tackle real-world issues involving $\mathbb{MCDM}$. As a combination of $\mathbb{CF}$-sets, $\mathcal{N}$-sets, and $\mathbb{S}$-sets, Smarandache et al. [37] presented the $\mathcal{CNS}$-set model with some of its fundamental set-theoretic operations. To illustrate the usefulness of this paradigm, a $\mathbb{DM}$ scenario incorporating ambiguous and subjective information was suggested.

1.1. *Research Gap and Motivation*

In the area of $\mathbb{DM}$ under uncertainty, η-set theory [38], a development of $\mathbb{S}$-set theory, has attracted interest. By enabling items to partially belong to distinct sets, it overcomes the drawbacks of conventional set theory. η-sets offer an adaptable framework for simulating ambiguous and uncertain information, enabling more sensible and reliable $\mathbb{DM}$ procedures. η-set applications have been studied in a variety of fields, including healthcare, finance, and environmental management. The literature emphasizes the usefulness and adaptability of η-sets in handling uncertainty, providing interesting directions for further study and real-world applications. Different hybrids with graphical settings [39–41], vague settings [42] and refined settings [43–45] were developed by researchers. However, the contributions of the researchers [46–49] are also worth noting regarding decision-making in hypersoft settings.

Decision-making and uncertainty modeling have both seen a considerable increase in interest in the idea of $\mathbb{IVFHS}$-sets [50]. The $\mathbb{IVFHS}$-sets offer an adaptable framework to deal with ambiguity and uncertainty in $\mathbb{DM}$. Numerous fields, including healthcare [51], banking, supply chain management, environmental assessment, and human resource management [52], have been the subject of research into these applications. According to the research, they are good at capturing and depicting ambiguous and imprecise information, empowering decision-makers to make well-informed decisions. The development of aggregation operations, similarity indices, and $\mathbb{DM}$ techniques based on $\mathbb{IVFHS}$-sets has been the subject of studies. According to the research, $\mathbb{IVFHS}$-sets are useful tools for handling difficult $\mathbb{DM}$ situations with uncertainty and ambiguity. Additional study is required to investigate their applicability in certain fields and to improve their computational efficiency.

The term "interval data" refers to situations in real life when data may be categorized as a set with values ranging from minimum to maximum (lower limits to upper bounds). Data may contain repeating values that correspond to specified parameters. Data repetition can be caused by a variety of sources. This sort of data is classified as periodic. There is currently no adequate model in the literature on fuzzy sets that deals with

(1) sub-attribute values in the form of $\mathbb{DAVS}$,
(2) data of the interval type, and
(3) PN-data, all at once.

The model Ξ-set is being characterized to satisfy the literary requirement. By using the $\mathbb{MAA}$-mapping, which uses the power set of the starting universe (a collection of $\mathbb{IF}$-sets or $\mathcal{N}$-sets) as its domain and maps it to the $\mathbb{CP}$ of the $\mathbb{DAVS}$, case (1) is addressed. Consideration of the lower and upper bounds of reported intervals is used to address scenario (2), whereas case (3) involves the inclusion of the $\mathscr{A}-$term and $\mathscr{P}-$terms into the Argand plane.

1.2. *Paper Layout*

The first section summarizes the literature review and study background of Ξ-set. In Section 2, some elementary notions from literature are discussed to understand the basic knowledge. In Section 3, the novel concept of Ξ-set is initiated along with the aggregation operations of Ξ-set. A $\mathbb{DSS}$ is developed in Section 4 for product selection based on the aggregation

TABLE 1. Abbreviation and notation table.

Full name	Abbreviation	Full name	Abbreviation
Fuzzy set	$\mathbb{F}$-set	Interval-valued $\mathbb{F}$-set	$\mathbb{IVF}$-set
set of all $\mathbb{IVF}$-sets	$\mathscr{C}(\mathbb{U})$	Intuitionistic $\mathbb{F}$-set	$\mathbb{IF}$-set
Complex $\mathbb{IF}$-set	$\mathbb{CIF}$-set	Neutrosophic set	$\mathbb{N}$-set
Interval $\mathbb{N}$-set	$\mathbb{IN}$-set	Complex $\mathbb{F}$-set	$\mathbb{CF}$-set
Interval-valued $\mathbb{CIF}$-set	$\mathbb{IVCIF}$-set	Complex $\mathbb{SVN}$-set	$\mathbb{CSVN}$-set
Complex $\mathbb{SVNS}$-set	$\mathbb{CSVNS}$-set		
Interval $\mathbb{CSVN}$-set	$\mathbb{ICSVN}$-set	Soft set	$\mathbb{S}$-set
Fuzzy $\mathbb{S}$-set	$\mathbb{FS}$-set	Intuitionistic $\mathbb{FS}$-set	$\mathbb{IFS}$-set
Hypersoft set	η-set	Interval $\mathbb{CSVN}$-hypersoft set	Ξ-set
Universal Set	$\mathbb{U}$	Power set of $\mathbb{U}$	$\mathscr{P}(\mathbb{U})$
Single-argument approximate mapping	$\mathbb{SAA}$-mapping	Multi-argument approximate mapping	$\mathbb{MAA}$-mapping
Membership function	$\mathbb{M}_{fn}$	Non-membership function	$\mathbb{N}_{fn}$
Indeterminacy function	$\mathbb{I}_{fn}$	Approximate function	$\mathbb{A}_{fn}$
Amplitude term	$\mathscr{A}-$term	Phase term	$\mathscr{P}-$term
Periodic nature data term	PN-data	Interval valued data	IV-data
Cartesian product	$\mathbb{CP}$	Set of parameters	$\mathbb{SP}$
Disjoint attribute valued set	$\mathbb{DAVS}$	Interval-valued $\mathbb{CFS}$-set	$\mathbb{IVCFS}$-set
Notation	**Description**	**Notation**	**Description**
Unit closed interval	$\mathscr{I}$	ϖ	$[0, 2\pi]$
Collection of all sub-intervals of $\mathscr{I}$	$\Im(\mathscr{I})$		

of Ξ-set aided by the proposed algorithm, and illustrated with the help of a diagram. A comparative analysis of the proposed model with some selected modes has been provided in Section 5 to check its efficiency. Finally, Section 6 concludes the research work.

2. Preliminaries

In this section, Table 1 demonstrates the abbreviations and notations used in this research article.

Definition 2.1. [1] A $\mathbb{F}$-set $\mathbb{A}$ over $\mathbb{U}$ is characterized by a $\mathbb{M}_{fn}$: $\mathbb{A}_m$, where $\mathbb{A}_m : \mathbb{U} \to \mathscr{I}$ is given by $\mathbb{A} = \{(\check{\Upsilon}, \mathbb{A}_m(\check{\Upsilon})) | \check{\Upsilon} \in \mathbb{U}\}$, which assigns a real value within $\mathscr{I}$ to each $\check{\Upsilon} \in \mathbb{U}$ and $\mathbb{A}_m(\check{\Upsilon})$ is $\mathbb{M}_{fn}$ of $\check{\Upsilon} \in \mathbb{U}$.

Definition 2.2. [2] A $\mathbb{CF}$-set $\mathbb{E}$ over $\mathbb{U}$ can be written as $\mathbb{E} = \{(\check{\Upsilon}, \mathbb{E}_m(\check{\Upsilon})) : \check{\Upsilon} \in \mathbb{U}\} = \left\{\left(\check{\Upsilon}, A_m\left(\check{\Upsilon}\right) e^{iP_m(\check{\Upsilon})}\right) : \check{\Upsilon} \in \mathbb{U}\right\}$, where $\mathbb{E}_m$ represents $\mathbb{M}_{fn}$ of $\mathbb{E}$ with $A_m\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term and $P_m\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term and $i = \sqrt{-1}$.

Definition 2.3. [7,8] A $\mathbb{CIF}$-set $\mathbb{F}$ over $\mathbb{U}$ can be written as

$$\mathbb{F} = \{(\check{\Upsilon}, \mathbb{F}_m(\check{\Upsilon}), \mathbb{F}_n(\check{\Upsilon})) : \check{\Upsilon} \in \mathbb{U}\} = \left\{\left(\check{\Upsilon}, A_m\left(\check{\Upsilon}\right) e^{iP_m(\check{\Upsilon})}, A_n\left(\check{\Upsilon}\right) e^{iP_n(\check{\Upsilon})}\right) : \check{\Upsilon} \in \mathbb{U}\right\}$$

where $\mathbb{F}_m$ and $\mathbb{F}_n$ represents $\mathbb{M}_{fn}$ and $\mathbb{N}_{fn}$ of $\mathbb{F}$ with $A_m\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term and $P_m\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term of $\mathbb{M}_{fn}$ and $A_n\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term and $P_n\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term of $\mathbb{N}_{fn}$ such that $0 \leq \mathbb{F}_m + \mathbb{F}_n \leq 1$ and hesitancy grade $\mathbb{F}_h\left(\check{\Upsilon}\right) = 1 - \mathbb{F}_m\left(\check{\Upsilon}\right) - \mathbb{F}_n\left(\check{\Upsilon}\right)$.

Definition 2.4. [12] A $\mathbb{CSVN}$-set $\mathbb{G}$ over $\mathbb{U}$ can be written as

$$\mathbb{G} = \{(\check{\Upsilon}, \mathbb{G}_m(\check{\Upsilon}), \mathbb{G}_n(\check{\Upsilon}), \mathbb{G}_i(\check{\Upsilon})) : \check{\Upsilon} \in \mathbb{U}\} =$$

$$\left\{\left(\check{\Upsilon}, A_m\left(\check{\Upsilon}\right) e^{iP_m(\check{\Upsilon})}, A_n\left(\check{\Upsilon}\right) e^{iP_n(\check{\Upsilon})}, A_i\left(\check{\Upsilon}\right) e^{iP_i(\check{\Upsilon})}\right) : \check{\Upsilon} \in \mathbb{U}\right\}$$

where $\mathbb{G}_m$, $\mathbb{G}_n$ and $\mathbb{G}_i$ represents $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$ of $\mathbb{G}$ with $A_m\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term, $P_m\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term of $\mathbb{M}_{fn}$, $A_n\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term, $P_n\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term of $\mathbb{N}_{fn}$ and $A_i\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term, $P_i\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term of $\mathbb{I}_{fn}$ such that $0 \leq \mathbb{G}_m + \mathbb{G}_n + \mathbb{G}_i \leq 3$.

Definition 2.5. [20] A $\mathbb{S}$-set $(\mathbb{H}, \Delta)$ over $\mathbb{U}$ is a set of order pairs such that $\mathbb{H} : \Delta \rightarrow \mathscr{P}(\mathbb{U})$ is given by

$$(\mathbb{H}, \Delta) = \{(\delta, \mathbb{H}(\check{\Upsilon})) : \delta \in \Delta, \check{\Upsilon} \in \mathbb{U}, \mathbb{H}(\check{\Upsilon}) \in \mathscr{P}(\mathbb{U})\}.$$

Definition 2.6. [37] A set $(\mathbb{N}, \Delta)$ is called $\mathbb{CSVNS}$-set over $\mathbb{U}$ if $\mathbb{N}$ is a parameterized gathering of $\mathbb{CSVN}$-subsets of $\mathbb{U}$ and is given by $\mathbb{N} : \Delta \rightarrow \mathscr{P}(\mathbb{U})$ and is defined by

$$(\mathbb{N}, \Delta) = \left\{\left(\delta, \left\{\frac{\mathbb{N}_m(\check{\Upsilon}), \mathbb{N}_n(\check{\Upsilon}), \mathbb{N}_i(\check{\Upsilon})}{\check{\Upsilon}}\right\}\right) : \check{\Upsilon} \in \mathbb{U}, \delta \in \Delta\right\}$$

where $\mathbb{N}_m(\check{\Upsilon}) = A_m\left(\check{\Upsilon}\right) e^{iP_m(\check{\Upsilon})}$ represents the $\mathbb{M}_{fn}$ of $\mathbb{N}$ with $A_m\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term, $P_m\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term, $\mathbb{N}_n(\check{\Upsilon}) = A_n\left(\check{\Upsilon}\right) e^{iP_n(\check{\Upsilon})}$ represents the $\mathbb{N}_{fn}$ of $\mathbb{N}$ with $A_n\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term, $P_n\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term and $\mathbb{N}_i(\check{\Upsilon}) = A_i\left(\check{\Upsilon}\right) e^{iP_i(\check{\Upsilon})}$ represents the $\mathbb{I}_{fn}$ of $\mathbb{N}$ with $A_i\left(\check{\Upsilon}\right) \in \mathscr{I}$ as $\mathscr{A}-$term, $P_i\left(\check{\Upsilon}\right) \in \varpi$ as $\mathscr{P}-$term such that $0 \leq \mathbb{N}_m(\check{\Upsilon}) + \mathbb{N}_n(\check{\Upsilon}) + \mathbb{N}_i(\check{\Upsilon}) \leq 3$.

Definition 2.7. [38] $(\mathbb{O}, \Delta)$ is called η-set over $\mathbb{U}$ if $\mathbb{O} : \Delta \rightarrow \mathscr{P}(\mathbb{U})$ where $\Delta = \prod_{i=1}^{n} \Delta_i$ such that Δ_i are $\mathbb{DAVS}$ of sub-parameters, each set corresponding to a unique parameters $\delta \in \Delta$.

Definition 2.8. If $\mathcal{C}_{\mathcal{SVN}}(\mathbb{U})$ denotes the set containing all $\mathcal{SVN}$-subsets over $\mathbb{U}$ then $\mathcal{SVNHS}$-set $(\mathbb{R}, \Delta)$ is obtained when the mapping $\mathbb{O} : \Delta \rightarrow \mathscr{P}(\mathbb{U})$ in Definition 2.7 is replaced by $\mathbb{R} : \Delta \rightarrow C_{SVN}(\mathbb{U})$ and all other conditions of Definition 2.7 are remained valid.

Definition 2.9. [56] If $\mathcal{C}_{\mathcal{CSVN}}(\mathbb{U})$ represents the collection of all $\mathbb{CSVN}$-subsets over $\mathbb{U}$ then $\mathcal{CSVNHS}$-set $(\mathbb{V}, \Delta)$ is obtained when the mapping $\mathbb{O} : \Delta \rightarrow \mathscr{P}(\mathbb{U})$ in Definition 2.7 is replaced by $\mathbb{V} : \Delta \rightarrow \mathcal{C}_{\mathcal{CSVN}}(\mathbb{U})$ and all other conditions of Definition 2.7 are remained valid.

3. Interval complex single-valued neutrosophic hypersoft set (Ξ-set)

This section develops the fundamental theory of the Ξ-set.

Definition 3.1. An $\mathbb{ICSVN}$-set $\mathbb{G}_{\mathbb{I}}$ over $\mathbb{U}$ can be written as

$$\mathbb{G}_{\mathbb{I}} = \{(\check{\Upsilon}, \langle \mathbb{G}_{\mathbb{I}m}(\check{\Upsilon}), \mathbb{G}_{\mathbb{I}n}(\check{\Upsilon}), \mathbb{G}_{\mathbb{I}i}(\check{\Upsilon})\rangle) : \check{\Upsilon} \in \mathbb{U}\} =$$
$$\left\{\left(\check{\Upsilon}, A_m(\check{\Upsilon}) e^{iP_m(\check{\Upsilon})}, A_n(\check{\Upsilon}) e^{iP_n(\check{\Upsilon})}, A_i(\check{\Upsilon}) e^{iP_i(\check{\Upsilon})}\right) : \check{\Upsilon} \in \mathbb{U}\right\}.$$

where $\mathbb{G}_{\mathbb{I}m}$ represents $\mathbb{M}_{fn}$ of $\mathbb{G}_{\mathbb{I}}$ with $A_m(\check{\Upsilon}) \in \mathfrak{I}(\mathscr{I})$ as $\mathscr{A}-$term, $P_m(\check{\Upsilon}) \subseteq \varpi$ as $\mathscr{P}-$term, $\mathbb{G}_{\mathbb{I}n}$ represents $\mathbb{N}_{fn}$ with $A_n(\check{\Upsilon}) \in \mathfrak{I}(\mathscr{I})$ as $\mathscr{A}-$term, $P_n(\check{\Upsilon}) \subseteq \varpi$ as $\mathscr{P}-$term and $\mathbb{G}_{\mathbb{I}i}$ represents $\mathbb{I}_{fn}$ with $A_i(\check{\Upsilon}) \in \mathfrak{I}(\mathscr{I})$ as $\mathscr{A}-$term, $P_i(\check{\Upsilon}) \subseteq \varpi$ as $\mathscr{P}-$term and $0 \leq \inf \mathbb{G}_{\mathbb{I}m} + \inf \mathbb{G}_{\mathbb{I}n} + \inf \mathbb{G}_{\mathbb{I}i} \leq \sup \mathbb{G}_{\mathbb{I}m} + \sup \mathbb{G}_{\mathbb{I}n} + \sup \mathbb{G}_{\mathbb{I}i} \leq 3$.

Definition 3.2. Consider two $\mathbb{ICSVN}$-sets

$$\mathbb{G}_{\mathbb{I}}{}^1 = \left\{\left(\check{\Upsilon}, \mathbb{G}_{\mathbb{I}m}^1(\check{\Upsilon}), \mathbb{G}_{\mathbb{I}n}^1(\check{\Upsilon}), \mathbb{G}_{\mathbb{I}i}^1(\check{\Upsilon})\right) : \check{\Upsilon} \in \mathbb{U}\right\}$$

and

$$\mathbb{G}_{\mathbb{I}}{}^2 = \left\{\left(\check{\Upsilon}, \mathbb{G}_{\mathbb{I}m}^2(\check{\Upsilon}), \mathbb{G}_{\mathbb{I}n}^2(\check{\Upsilon}), \mathbb{G}_{\mathbb{I}i}^2(\check{\Upsilon})\right) : \check{\Upsilon} \in \mathbb{U}\right\}$$

having respective $\mathbb{M}_{fn}$: $\mathbb{G}_{\mathbb{I}m}^1(\check{\Upsilon}) = A_m^1(\check{\Upsilon}) e^{iP_m{}^1(\check{\Upsilon})}$, $\mathbb{G}_{\mathbb{I}m}^2(\check{\Upsilon}) = A_m^2(\check{\Upsilon}) e^{iP_m{}^2(\check{\Upsilon})}$, $\mathbb{N}_{fn}$: $\mathbb{G}_{\mathbb{I}n}^1(\check{\Upsilon}) = A_n^1(\check{\Upsilon}) e^{iP_n{}^1(\check{\Upsilon})}$, $\mathbb{G}_{\mathbb{I}n}^2(\check{\Upsilon}) = A_n^2(\check{\Upsilon}) e^{iP_n{}^2(\check{\Upsilon})}$ and $\mathbb{I}_{fn}$: $\mathbb{G}_{\mathbb{I}i}^1(\check{\Upsilon}) = A_i^1(\check{\Upsilon}) e^{iP_i{}^1(\check{\Upsilon})}$, $\mathbb{G}_{\mathbb{I}i}^2(\check{\Upsilon}) = A_i^2(\check{\Upsilon}) e^{iP_i{}^2(\check{\Upsilon})}$.

(1). The *union* of $\mathbb{G}_{\mathbb{I}}{}^1$ and $\mathbb{G}_{\mathbb{I}}{}^2$ is again an $\mathbb{ICSVN}$-set $\mathbb{G}_{\mathbb{I}}{}^3 = \mathbb{G}_{\mathbb{I}}{}^1 \cup \mathbb{G}_{\mathbb{I}}{}^2$, where its $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $I_{fn} \forall \check{\Upsilon} \in \mathbb{U}$ can be given by

$$\mathbb{G}_{\mathbb{I}m}^3(\check{\Upsilon}) = A_m^3(\check{\Upsilon}) e^{iP_m{}^3(\check{\Upsilon})} = \begin{bmatrix} \max\left(\inf A_m^1(\check{\Upsilon}), \inf A_m^2(\check{\Upsilon})\right), \\ \max\left(\sup A_m^1(\check{\Upsilon}), \sup A_m^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_m{}^1(\check{\Upsilon}), \inf P_m{}^2(\check{\Upsilon})\right), \\ \max\left(\sup P_m{}^1(\check{\Upsilon}), \sup P_m{}^2(\check{\Upsilon})\right) \end{bmatrix}}.$$

$$\mathbb{G}_{\mathbb{I}n}^3(\check{\Upsilon}) = A_n^3(\check{\Upsilon}) e^{iP_n{}^3(\check{\Upsilon})} = \begin{bmatrix} \min\left(\inf A_n^1(\check{\Upsilon}), \inf A_n^2(\check{\Upsilon})\right), \\ \min\left(\sup A_n^1(\check{\Upsilon}), \sup A_n^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_n{}^1(\check{\Upsilon}), \inf P_n{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_n{}^1(\check{\Upsilon}), \sup P_n{}^2(\check{\Upsilon})\right) \end{bmatrix}}$$

$$\mathbb{G}_{\mathbb{I}i}^3(\check{\Upsilon}) = A_i^3(\check{\Upsilon}) e^{iP_i{}^3(\check{\Upsilon})} = \begin{bmatrix} \min\left(\inf A_i^1(\check{\Upsilon}), \inf A_i^2(\check{\Upsilon})\right), \\ \min\left(\sup A_i^1(\check{\Upsilon}), \sup A_i^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_i{}^1(\check{\Upsilon}), \inf P_i{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_i{}^1(\check{\Upsilon}), \sup P_i{}^2(\check{\Upsilon})\right) \end{bmatrix}}.$$

(2). The *intersection* of $\mathbb{G}_{\mathbb{I}}{}^1$ and $\mathbb{G}_{\mathbb{I}}{}^2$ is again an $\mathbb{IVCIF}$-set $\mathbb{G}_{\mathbb{I}}{}^4 = \mathbb{G}_{\mathbb{I}}{}^1 \cap \mathbb{G}_{\mathbb{I}}{}^2$, where its $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $I_{fn} \forall \check{\Upsilon} \in \mathbb{U}$ can be given by

$$\mathbb{G}_{\mathbb{I}m}^4(\check{\Upsilon}) = A_m^4(\check{\Upsilon}) e^{iP_m{}^4(\check{\Upsilon})} = \begin{bmatrix} \min\left(\inf A_m^1(\check{\Upsilon}), \inf A_m^2(\check{\Upsilon})\right), \\ \min\left(\sup A_m^1(\check{\Upsilon}), \sup A_m^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_m{}^1(\check{\Upsilon}), \inf P_m{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_m{}^1(\check{\Upsilon}), \sup P_m{}^2(\check{\Upsilon})\right) \end{bmatrix}}.$$

$$\mathbb{G}_{\mathbb{I}n}^4(\check{\Upsilon}) = A_n^4(\check{\Upsilon}) e^{iP_n{}^4(\check{\Upsilon})} = \begin{bmatrix} \max\left(\inf A_n^1(\check{\Upsilon}), \inf A_n^2(\check{\Upsilon})\right), \\ \max\left(\sup A_n^1(\check{\Upsilon}), \sup A_n^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_n{}^1(\check{\Upsilon}), \inf P_n{}^2(\check{\Upsilon})\right), \\ \max\left(\sup P_n{}^1(\check{\Upsilon}), \sup P_n{}^2(\check{\Upsilon})\right) \end{bmatrix}}.$$

$$\mathbb{G}_{\mathbb{I}_i}^4(\check{\curlyvee}) = A_i^4(\check{\curlyvee})\, e^{iP_i^4(\check{\curlyvee})} = \begin{bmatrix} \max\left(\inf A_i^1(\check{\curlyvee}), \inf A_i^2(\check{\curlyvee})\right), \\ \max\left(\sup A_i^1(\check{\curlyvee}), \sup A_i^2(\check{\curlyvee})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_i^1(\check{\curlyvee}), \inf P_i^2(\check{\curlyvee})\right), \\ \max\left(\sup P_i^1(\check{\curlyvee}), \sup P_i^2(\check{\curlyvee})\right) \end{bmatrix}}.$$

(3). The *complement* of $\mathbb{G_I}^1$ denoted by

$$(\mathbb{G_I}^1)^c = \left\{\left(\check{\curlyvee}, {\mathbb{G}_{\mathbb{I}_m}^1}^c(\check{\curlyvee}), {\mathbb{G}_{\mathbb{I}_n}^1}^c(\check{\curlyvee})\right) : \check{\curlyvee} \in \mathbb{U}\right\}$$

where
${\mathbb{G}_{\mathbb{I}_m}^1}^c(\check{\curlyvee}) = \mathbb{G}_{\mathbb{I}_n}^1(\check{\curlyvee})$,
${\mathbb{G}_{\mathbb{I}_n}^1}^c(\check{\curlyvee}) = \mathbb{G}_{\mathbb{I}_m}^1(\check{\curlyvee})$ and
${\mathbb{G}_{\mathbb{I}_i}^1}^c = \left[1 - A_i^{1^+}(\check{\curlyvee}), 1 - A_i^{1^-}(\check{\curlyvee})\right] e^{i\left[2\pi - P_i^{1^+}(\check{\curlyvee}), 2\pi - P_i^{1^-}(\check{\curlyvee})\right]}$.

Definition 3.3. A set $(\mathbb{N_I}, \Delta)$ is called $\mathcal{ICSVNS}$-set over $\mathbb{U}$ if $\mathbb{N_I}$ is a parameterized gathering of $\mathcal{IVCSVN}$-subsets of $\mathbb{U}$ and is given by $\mathbb{N_I} : \Delta \to \mathscr{P}(\mathbb{U})$ and is defined by

$$(\mathbb{N_I}, \Delta) = \left\{\left(\delta, \left\{\frac{\mathbb{N}_{\mathbb{I}m}(\check{\curlyvee}), \mathbb{N}_{\mathbb{I}n}(\check{\curlyvee}), \mathbb{N}_{\mathbb{I}i}(\check{\curlyvee})}{\check{\curlyvee}}\right\}\right) : \check{\curlyvee} \in \mathbb{U}, \delta \in \Delta\right\}$$

where $\mathbb{N}_{\mathbb{I}m}(\check{\curlyvee}) = A_m(\check{\curlyvee})\, e^{iP_m(\check{\curlyvee})}$ represents the $\mathbb{M}_{fn}$ of $\mathbb{N_I}$ with $A_m(\check{\curlyvee}) \in \mathfrak{I}(\mathscr{I})$ as $\mathscr{A}-$term, $P_m(\check{\curlyvee}) \subseteq \varpi$ as $\mathscr{P}-$term, $\mathbb{N}_{\mathbb{I}n}(\check{\curlyvee}) = A_n(\check{\curlyvee})\, e^{iP_n(\check{\curlyvee})}$ represents the $\mathbb{N}_{fn}$ of $\mathbb{N_I}$ with $A_n(\check{\curlyvee}) \in \mathfrak{I}(\mathscr{I})$ as $\mathscr{A}-$term, $P_n(\check{\curlyvee}) \subseteq \varpi$ as $\mathscr{P}-$term and $\mathbb{N}_{\mathbb{I}i}(\check{\curlyvee}) = A_i(\check{\curlyvee})\, e^{iP_i(\check{\curlyvee})}$ represents the $\mathbb{I}_{fn}$ of $\mathbb{N_I}$ with $A_i(\check{\curlyvee}) \in \mathfrak{I}(\mathscr{I})$ as $\mathscr{A}-$term, $P_i(\check{\curlyvee}) \subseteq \varpi$ as $\mathscr{P}-$term such that $0 \leq \inf \mathbb{N}_{\mathbb{I}m}(\check{\curlyvee}) + \inf \mathbb{N}_{\mathbb{I}n}(\check{\curlyvee}) + \inf \mathbb{N}_{\mathbb{I}i}(\check{\curlyvee}) \leq \sup \mathbb{N}_{\mathbb{I}m}(\check{\curlyvee}) + \sup \mathbb{N}_{\mathbb{I}n}(\check{\curlyvee}) + \sup \mathbb{N}_{\mathbb{I}i}(\check{\curlyvee}) \leq 3$.

Definition 3.4. Consider two $\mathcal{ICSVNS}$-sets $(\mathbb{N_I}^1, \Delta_1)$ and $(\mathbb{N_I}^2, \Delta_2)$ having respective $\mathbb{M}_{fn}$: $\mathbb{N}_{\mathbb{I}m}^1 = A_m^1(\check{\curlyvee})\, e^{iP_m^1(\check{\curlyvee})}$, $\mathbb{N}_{\mathbb{I}m}^2 = A_m^2(\check{\curlyvee})\, e^{iP_m^2(\check{\curlyvee})}$, $\mathbb{N}_{fn}$: $\mathbb{N}_{\mathbb{I}n}^1 = A_n^1(\check{\curlyvee})\, e^{iP_n^1(\check{\curlyvee})}$, $\mathbb{N}_{\mathbb{I}n}^2 = A_n^2(\check{\curlyvee})\, e^{iP_n^2(\check{\curlyvee})}$ and $\mathbb{I}_{fn}$: $\mathbb{N}_{\mathbb{I}i}^1 = A_i^1(\check{\curlyvee})\, e^{iP_i^1(\check{\curlyvee})}$, $\mathbb{N}_{\mathbb{I}i}^2 = A_i^2(\check{\curlyvee})\, e^{iP_i^2(\check{\curlyvee})}$

(1) The *union* of $(\mathbb{N_I}^1, \Delta_1)$ and $(\mathbb{N_I}^2, \Delta_2)$ is again an $\mathcal{ICSVNS}$-set $(\mathbb{N_I}^3, \Delta_3) = (\mathbb{N_I}^1, \Delta_1) \cup (\mathbb{N_I}^2, \Delta_2)$, where $\Delta_3 = \Delta_1 \cup \Delta_2$, for all $\delta \in \Delta_3$, $\check{\curlyvee} \in \mathbb{U}$, and its $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$ are defined as

$$\mathbb{N}_{\mathbb{I}m}^3(\check{\curlyvee}) = \begin{cases} \mathbb{N}_{\mathbb{I}m}^1(\check{\curlyvee}) & if\ \delta \in \Delta_1 \backslash \Delta_2 \\ \mathbb{N}_{\mathbb{I}m}^2(\check{\curlyvee}) & if\ \delta \in \Delta_2 \backslash \Delta_1 \\ \begin{bmatrix} \max\left(\inf A_m^1(\check{\curlyvee}), \inf A_m^2(\check{\curlyvee})\right), \\ \max\left(\sup A_m^1(\check{\curlyvee}), \sup A_m^2(\check{\curlyvee})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_m^1(\check{\curlyvee}), \inf P_m^2(\check{\curlyvee})\right), \\ \max\left(\sup P_m^1(\check{\curlyvee}), \sup P_m^2(\check{\curlyvee})\right) \end{bmatrix}} & if\ \delta \in \Delta_1 \cap \Delta_2 \end{cases}$$

$$\mathbb{N}_{\mathbb{I}n}^3(\check{\curlyvee}) = \begin{cases} \mathbb{N}_{\mathbb{I}n}^1(\check{\curlyvee}) & if\ \delta \in \Delta_1 \backslash \Delta_2 \\ \mathbb{N}_{\mathbb{I}n}^2(\check{\curlyvee}) & if\ \delta \in \Delta_2 \backslash \Delta_1 \\ \begin{bmatrix} \min\left(\inf A_n^1(\check{\curlyvee}), \inf A_n^2(\check{\curlyvee})\right), \\ \min\left(\sup A_n^1(\check{\curlyvee}), \sup A_n^2(\check{\curlyvee})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_n^1(\check{\curlyvee}), \inf P_n^2(\check{\curlyvee})\right), \\ \min\left(\sup P_n^1(\check{\curlyvee}), \sup P_n^2(\check{\curlyvee})\right) \end{bmatrix}} & if\ \delta \in \Delta_1 \cap \Delta_2 \end{cases}$$

$$\mathbb{N}_{\mathbb{I}_i}^{3}(\check{\Upsilon}) = \begin{cases} \mathbb{N}_{\mathbb{I}_i}^{1}(\check{\Upsilon}) & if\ \delta \in \Delta_1 \backslash \Delta_2 \\ \mathbb{N}_{\mathbb{I}_i}^{2}(\check{\Upsilon}) & if\ \delta \in \Delta_2 \backslash \Delta_1 \\ \begin{bmatrix} \min\left(\inf A_i^1(\check{\Upsilon}), \inf A_i^2(\check{\Upsilon})\right), \\ \min\left(\sup A_i^1(\check{\Upsilon}), \sup A_i^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_i{}^1(\check{\Upsilon}), \inf P_i{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_i{}^1(\check{\Upsilon}), \sup P_i{}^2(\check{\Upsilon})\right) \end{bmatrix}} & if\ \delta \in \Delta_1 \cap \Delta_2 \end{cases}$$

(2) The *restricted union* of $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)$ and $(\mathbb{N}_{\mathbb{I}}{}^2, \Delta_2)$ denoted by $(\mathbb{N}_{\mathbb{I}}{}^4, \Delta_4) = (\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1) \cup_R (\mathbb{N}_{\mathbb{I}}{}^2, \Delta_2)$, where $\Delta_4 = \Delta_1 \cap \Delta_2$, for all $\delta \in \Delta_4$, $\check{\Upsilon} \in \mathbb{U}$, its $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$ are defined as

$$\mathbb{N}_{\mathbb{I}_m}^{4}(\check{\Upsilon}) = \begin{bmatrix} \max\left(\inf A_m^1(\check{\Upsilon}), \inf A_m^2(\check{\Upsilon})\right), \\ \max\left(\sup A_m^1(\check{\Upsilon}), \sup A_m^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_m{}^1(\check{\Upsilon}), \inf P_m{}^2(\check{\Upsilon})\right), \\ \max\left(\sup P_m{}^1(\check{\Upsilon}), \sup P_m{}^2(\check{\Upsilon})\right) \end{bmatrix}}$$

$$\mathbb{N}_{\mathbb{I}_n}^{4}(\check{\Upsilon}) = \begin{bmatrix} \min\left(\inf A_n^1(\check{\Upsilon}), \inf A_n^2(\check{\Upsilon})\right), \\ \min\left(\sup A_n^1(\check{\Upsilon}), \sup A_n^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_n{}^1(\check{\Upsilon}), \inf P_n{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_n{}^1(\check{\Upsilon}), \sup P_n{}^2(\check{\Upsilon})\right) \end{bmatrix}}$$

$$\mathbb{N}_{\mathbb{I}_i}^{4}(\check{\Upsilon}) = \begin{bmatrix} \min\left(\inf A_i^1(\check{\Upsilon}), \inf A_i^2(\check{\Upsilon})\right), \\ \min\left(\sup A_i^1(\check{\Upsilon}), \sup A_i^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_i{}^1(\check{\Upsilon}), \inf P_i{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_i{}^1(\check{\Upsilon}), \sup P_i{}^2(\check{\Upsilon})\right) \end{bmatrix}}$$

(3) The *intersection* of $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)$ and $(\mathbb{N}_{\mathbb{I}}{}^2, \Delta_2)$ denoted by $(\mathbb{N}_{\mathbb{I}}{}^5, \Delta_5) = (\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1) \cap (\mathbb{N}_{\mathbb{I}}{}^2, \Delta_2)$, where $\Delta_5 = \Delta_1 \cap \Delta_2$, for all $\delta \in \Delta_5$, $\check{\Upsilon} \in \mathbb{U}$ its $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$ is defined as

$$\mathbb{N}_{\mathbb{I}_m}^{5}(\check{\Upsilon}) = \begin{bmatrix} \min\left(\inf A_m^1(\check{\Upsilon}), \inf A_m^2(\check{\Upsilon})\right), \\ \min\left(\sup A_m^1(\check{\Upsilon}), \sup A_m^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_m{}^1(\check{\Upsilon}), \inf P_m{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_m{}^1(\check{\Upsilon}), \sup P_m{}^2(\check{\Upsilon})\right) \end{bmatrix}}$$

$$\mathbb{N}_{\mathbb{I}_n}^{5}(\check{\Upsilon}) = \begin{bmatrix} \max\left(\inf A_n^1(\check{\Upsilon}), \inf A_n^2(\check{\Upsilon})\right), \\ \max\left(\sup A_n^1(\check{\Upsilon}), \sup A_n^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_n{}^1(\check{\Upsilon}), \inf P_n{}^2(\check{\Upsilon})\right), \\ \max\left(\sup P_n{}^1(\check{\Upsilon}), \sup P_n{}^2(\check{\Upsilon})\right) \end{bmatrix}}$$

$$\mathbb{N}_{\mathbb{I}_i}^{5}(\check{\Upsilon}) = \begin{bmatrix} \max\left(\inf A_i^1(\check{\Upsilon}), \inf A_i^2(\check{\Upsilon})\right), \\ \max\left(\sup A_i^1(\check{\Upsilon}), \sup A_i^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_i{}^1(\check{\Upsilon}), \inf P_i{}^2(\check{\Upsilon})\right), \\ \max\left(\sup P_i{}^1(\check{\Upsilon}), \sup P_i{}^2(\check{\Upsilon})\right) \end{bmatrix}}$$

(4) The *extended intersection* of $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)$ and $(\mathbb{N}_{\mathbb{I}}{}^2, \Delta_2)$ denoted by $(\mathbb{N}_{\mathbb{I}}{}^6, \Delta_6) = (\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1) \cap_E (\mathbb{N}_{\mathbb{I}}{}^2, \Delta_2)$, where $\Delta_6 = \Delta_1 \cup \Delta_2$, for all $\delta \in \Delta_6$, $\check{\Upsilon} \in \mathbb{U}$, its $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$ are defined as

$$\mathbb{N}_{\mathbb{I}_m}^{6}(\check{\Upsilon}) = \begin{cases} \mathbb{N}_{\mathbb{I}_m}^{1}(\check{\Upsilon}) & if\ \delta \in \Delta_1 \backslash \Delta_2 \\ \mathbb{N}_{\mathbb{I}_m}^{2}(\check{\Upsilon}) & if\ \delta \in \Delta_2 \backslash \Delta_1 \\ \begin{bmatrix} \min\left(\inf A_m^1(\check{\Upsilon}), \inf A_m^2(\check{\Upsilon})\right), \\ \min\left(\sup A_m^1(\check{\Upsilon}), \sup A_m^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \min\left(\inf P_m{}^1(\check{\Upsilon}), \inf P_m{}^2(\check{\Upsilon})\right), \\ \min\left(\sup P_m{}^1(\check{\Upsilon}), \sup P_m{}^2(\check{\Upsilon})\right) \end{bmatrix}} & if\ \delta \in \Delta_1 \cap \Delta_2 \end{cases}$$

$$\mathbb{N}_{\mathbb{I}_n}^{6}(\check{\Upsilon}) = \begin{cases} \mathbb{N}_{\mathbb{I}_n}^{1}(\check{\Upsilon}) & if\ \delta \in \Delta_1 \backslash \Delta_2 \\ \mathbb{N}_{\mathbb{I}_n}^{2}(\check{\Upsilon}) & if\ \delta \in \Delta_2 \backslash \Delta_1 \\ \begin{bmatrix} \max\left(\inf A_n^1(\check{\Upsilon}), \inf A_n^2(\check{\Upsilon})\right), \\ \max\left(\sup A_n^1(\check{\Upsilon}), \sup A_n^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_n{}^1(\check{\Upsilon}), \inf P_n{}^2(\check{\Upsilon})\right), \\ \max\left(\sup P_n{}^1(\check{\Upsilon}), \sup P_n{}^2(\check{\Upsilon})\right) \end{bmatrix}} & if\ \delta \in \Delta_1 \cap \Delta_2 \end{cases}$$

$$\mathbb{N}_{\mathbb{I}_i}^{6}(\check{\Upsilon}) = \begin{cases} \mathbb{N}_{\mathbb{I}_i}^{1}(\check{\Upsilon}) & if\ \delta \in \Delta_1 \backslash \Delta_2 \\ \mathbb{N}_{\mathbb{I}_i}^{2}(\check{\Upsilon}) & if\ \delta \in \Delta_2 \backslash \Delta_1 \\ \begin{bmatrix} \max\left(\inf A_i^1(\check{\Upsilon}), \inf A_i^2(\check{\Upsilon})\right), \\ \max\left(\sup A_i^1(\check{\Upsilon}), \sup A_i^2(\check{\Upsilon})\right) \end{bmatrix} e^{i\begin{bmatrix} \max\left(\inf P_i{}^1(\check{\Upsilon}), \inf P_i{}^2(\check{\Upsilon})\right), \\ \max\left(\sup P_i{}^1(\check{\Upsilon}), \sup P_i{}^2(\check{\Upsilon})\right) \end{bmatrix}} & if\ \delta \in \Delta_1 \cap \Delta_2 \end{cases}$$

(5) The *complement* of $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)$ denoted by $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)^c = (\mathbb{N}_{\mathbb{I}}{}^{1^c}, \neg\Delta_1)$ such that $\mathbb{N}_{\mathbb{I}}{}^{1^c} : \neg\Delta_1 \to \mathscr{P}(\Bbbk)$ is given by $\mathbb{M}_{fn}$:$\mathbb{N}_{\mathbb{I}_m}^{1^c}(\check{\Upsilon}) = \mathbb{N}_{\mathbb{I}_n}^{1}(\check{\Upsilon})$, $\mathbb{N}_{fn}$: $\mathbb{N}_{\mathbb{I}_n}^{1^c}(\check{\Upsilon}) = \mathbb{N}_{\mathbb{I}_m}^{1}(\check{\Upsilon})$ and $\mathbb{I}_{fn}$: $\mathbb{N}_{\mathbb{I}_i}^{1^c}(\check{\Upsilon}) = \left[1 - \sup \mathbb{N}_{\mathbb{I}_i}^{1}(\check{\Upsilon}), 1 - \inf \mathbb{N}_{\mathbb{I}_i}^{1}(\check{\Upsilon})\right]$

(6) The *relative complement* of $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)$ denoted by $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)^r$ where $(\mathbb{N}_{\mathbb{I}}{}^1, \Delta_1)^r = (\mathbb{N}_{\mathbb{I}}{}^{1^r}, \Delta_1)$ such that $\mathbb{N}_{\mathbb{I}}{}^{1^r} : \Delta_1 \to \mathscr{P}(\Bbbk)$ is given by $\mathbb{M}_{fn}$:$\mathbb{N}_{\mathbb{I}_m}^{1^r}(\check{\Upsilon}) = \mathbb{N}_{\mathbb{I}_n}^{1}(\check{\Upsilon})$, $\mathbb{N}_{fn}$: $\mathbb{N}_{\mathbb{I}_n}^{1^r}(\check{\Upsilon}) = \mathbb{N}_{\mathbb{I}_m}^{1}(\check{\Upsilon})$ and $\mathbb{I}_{fn}$: $\mathbb{N}_{\mathbb{I}_i}^{1^r}(\check{\Upsilon}) = \left[1 - \sup \mathbb{N}_{\mathbb{I}_i}^{1}(\check{\Upsilon}), 1 - \inf \mathbb{N}_{\mathbb{I}_i}^{1}(\check{\Upsilon})\right]$

Definition 3.5. Let $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, \ldots.., \mathcal{Y}_n$ are $\mathbb{DAVS}$ of n distinct attributes $\zeta_1, \zeta_2, \zeta_3, \ldots.., \zeta_n$ respectively for $n \geq 1, \mathcal{Y} = \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_3 \times \ldots.. \times \mathcal{Y}_n$ and $\Delta(\underline{\delta})$ be a $\mathcal{ICSVNS}$-set defined over $\Bbbk \forall \underline{\delta} = (\beta_1, \beta_2, \beta_3, \ldots.., \beta_n) \in \mathcal{Y}$. Then, the Ξ-set, denoted by $\Omega_{\mathcal{Y}} = (\Delta, \mathcal{Y})$, over $\Bbbk$ is given as

$$\Omega_{\mathcal{Y}} = \{(\underline{\delta}, \Delta(\underline{\delta})) : \underline{\delta} \in \mathcal{Y}, \Delta(\underline{\delta}) \in C_{IV}(\Bbbk)\},$$

where $\Delta : \mathcal{Y} \to C_{IV}(\Bbbk), \Delta(\underline{\delta}) = \emptyset\ if\ \underline{\delta} \notin \mathcal{Y}$ is an $\mathcal{ICSVN}$ $\mathbb{A}_{fn}$ of $\Omega_{\mathcal{Y}}$ and $\Delta(\underline{\delta}) = \langle [\overleftarrow{\Delta}_1(\underline{\delta}), \overrightarrow{\Delta}_1(\underline{\delta})], [\overleftarrow{\Delta}_2(\underline{\delta}), \overrightarrow{\Delta}_2(\underline{\delta})], [\overleftarrow{\Delta}_3(\underline{\delta}), \overrightarrow{\Delta}_3(\underline{\delta})] \rangle$ with lower bounds and upper bounds of $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$ and $\mathbb{I}_{fn}$ are described as folow

(a) $(\overleftarrow{\Delta}_1(\underline{\delta}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}, \overrightarrow{\Delta}_1(\underline{\delta}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}})$ for the $\mathbb{M}_{fn}$ of $\Omega_{\mathcal{Y}}$

(b) $(\overleftarrow{\Delta}_2(\underline{\delta}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}, \overrightarrow{\Delta}_2(\underline{\delta}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}})$ for the $\mathbb{N}_{fn}$ of $\Omega_{\mathcal{Y}}$

(c) $(\overleftarrow{\Delta}_3(\underline{\delta}) = \overleftarrow{\gamma} e^{i\overleftarrow{\theta}}, \overrightarrow{\Delta}_3(\underline{\delta}) = \overrightarrow{\gamma} e^{i\overrightarrow{\theta}})$ for the $\mathbb{I}_{fn}$ of $\Omega_{\mathcal{Y}}$ and $\Delta(\underline{\delta})$ is known as $\underline{\delta}$-member of Ξ-set $\forall \underline{\delta} \in \mathcal{Y}$.

Note: $\uplus_{IVCNHS}$ denotes the collection of all Ξ-sets.

Definition 3.6. The *complement* of Ξ-set $(\Delta, \mathcal{Y})$, denoted by $(\Delta, \mathcal{Y})^c$ is stated as

$$(\Delta, \mathcal{Y})^c = \{(\check{\ltimes}, (\Delta(\check{\ltimes}))^c) : \check{\ltimes} \in \mathcal{Y}, (\Delta(\check{\ltimes}))^c \in C_{IV}(\Bbbk)\}$$

where the $\mathscr{A}-$term and $\mathscr{P}-$terms of the $\mathbb{M}_{fn}$ $(\Delta(\check{\ltimes}))^c$ are given by
$(\overleftarrow{\gamma}_{\mathcal{Y}}(\check{\ltimes}))^c = 1 - \overleftarrow{\gamma}_{\mathcal{Y}}(\check{\ltimes})$, $(\overrightarrow{\gamma}_{\mathcal{Y}}(\check{\ltimes}))^c = 1 - \overrightarrow{\gamma}_{\mathcal{Y}}(\check{\ltimes})$ and
$(\overleftarrow{\theta}_{\mathcal{Y}}(\check{\ltimes}))^c = 2\pi - \overleftarrow{\theta}_{\mathcal{Y}}(\check{\ltimes})$, $(\overrightarrow{\theta}_{\mathcal{Y}}(\check{\ltimes}))^c = 2\pi - \overrightarrow{\theta}_{\mathcal{Y}}(\check{\ltimes})$ respectively.

TABLE 2. Tabular Representation of $\S_\Lambda$.

$\S_\Lambda$	$\check{\ltimes}_1$	$\check{\ltimes}_2$	...	$\check{\ltimes}_r$
$\check{\Upsilon}_1$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_1), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_1), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_1) \end{pmatrix}$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_1), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_1), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_1) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_1), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_1), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_1) \end{pmatrix}$
$\check{\Upsilon}_2$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_2), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_2), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_2) \end{pmatrix}$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_2), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_2), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_2) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_2), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_2), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_2) \end{pmatrix}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\check{\Upsilon}_m$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_m), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_m), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_1)}(\check{\Upsilon}_m) \end{pmatrix}$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_m), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_m), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_2)}(\check{\Upsilon}_m) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_m), \\ \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_m), \\ \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_r)}(\check{\Upsilon}_m) \end{pmatrix}$

Now the aggregation procedures and their conclusive systems for the Ξ-set are established in the form of $\mathcal{CSVNHS}$-set and its cardinal set that results in an aggregate $\mathcal{F}-$set with fuzzy-like features. The terms $\Lambda, \mathfrak{E}, \S_\Lambda$and $\uplus_{ICSVNHS}$are are consistent with definition 3.5. The aggregation operations developed in this research article are modified versions of aggregations discussed in [62].

Definition 3.7. Let $\S_\Lambda \in \uplus_{IVCNHS}$. Assume that $\mathbb{U} = \{\check{\Upsilon}_1, \check{\Upsilon}_2, ..., \check{\Upsilon}_m\}$ and $\mathfrak{E} = \{\mathcal{L}_1, \mathcal{L}_2, ..., \mathcal{L}_n\}$ with $\mathcal{L}_1 = \{e_{11}, e_{12}, ..., e_{1n}\}, \mathcal{L}_2 = \{e_{21}, e_{22}, ..., e_{2n}\}, ..., \mathcal{L}_n = \{e_{n1}, e_{n2}, ..., e_{nn}\}$ and $\Lambda = \mathcal{L}_1 \times \mathcal{L}_2 \times ..... \times \mathcal{L}_n = \{\check{\ltimes}_1, \check{\ltimes}_2, ..., \check{\ltimes}_n, ..., \check{\ltimes}_{n^n} = \check{\ltimes}_r\}$, each $\check{\ltimes}_i$ is n-tuple element of Λ and $|\Lambda| = r = n^n$ then $\S_\Lambda$ can be presented in the following tabular notation (see Table 2). Where $\aleph^1_{\mathbb{X}_\Lambda(x)}, \aleph^2_{\mathbb{X}_\Lambda(x)}$ and $\aleph^2_{\mathbb{X}_\Lambda(x)}$ are $\mathbb{M}_{fn}$, $\mathbb{I}_{fn}$ and $\mathbb{N}_{fn}$ of $\mathbb{X}_\Lambda$ respectively with $\mathcal{IVN}$ values. If $\alpha_{ij} = (\aleph^1_{\mathbb{X}_\Lambda(\check{\ltimes}_j)}(\check{\Upsilon}_i), \aleph^2_{\mathbb{X}_\Lambda(\check{\ltimes}_j)}(\check{\Upsilon}_i), \aleph^3_{\mathbb{X}_\Lambda(\check{\ltimes}_j)}(\check{\Upsilon}_i))$, for $i = \mathscr{N}_1^m$ and $j = \mathscr{N}_1^r$ then Ξ-set $\S_\Lambda$ is specifically identified by a matrix,

$$[\alpha_{ij}] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1r} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mr} \end{bmatrix}$$

is called an $m \times r$ Ξ-set matrix..

Definition 3.8. If $\S_\Lambda \in \uplus_{ICSVNHS}$ then cardinal set of $\S_\Lambda$ is defined as

$$\|\S_\Lambda\| = \left\{ (\aleph^1_{\|\S_\Lambda\|}(\check{\ltimes}), \aleph^2_{\|\S_\Lambda\|}(\check{\ltimes}), \aleph^3_{\|\S_\Lambda\|}(\check{\ltimes}))/\check{\ltimes} : \check{\ltimes} \in \Lambda \right\},$$

where $\aleph^1_{\|\S_\Lambda\|}, \aleph^2_{\|\S_\Lambda\|}, \aleph^3_{\|\S_\Lambda\|} : \Lambda \to [0,1]$ are $\mathbb{M}_{fn}$, $\mathbb{I}_{fn}$ and $\mathbb{N}_{fn}$ of $\|\S_\Lambda\|$ with

$$\aleph^1_{\|\S_\Lambda\|}(\check{\ltimes}), \aleph^2_{\|\S_\Lambda\|}(\check{\ltimes}), \aleph^3_{\|\S_\Lambda\|}(\check{\ltimes}) = \frac{|\mathbb{X}_\Lambda(\check{\ltimes})|}{|\mathbb{U}|}$$

TABLE 3. Tabular Representation of $\|\S_\Lambda\|$.

Λ	$\check{\ltimes}_1$	$\check{\ltimes}_2$	$\cdots$	$\check{\ltimes}_r$
$\aleph_{\|\S_\Lambda\|}$	$\begin{pmatrix} \aleph^1_{\|\S_\Lambda\|}(\check{\ltimes}_1), \\ \aleph^2_{\|\S_\Lambda\|}(\check{\ltimes}_1), \\ \aleph^3_{\|\S_\Lambda\|}(\check{\ltimes}_1) \end{pmatrix}$	$\begin{pmatrix} \aleph^1_{\|\S_\Lambda\|}(\check{\ltimes}_2), \\ \aleph^2_{\|\S_\Lambda\|}(\check{\ltimes}_2), \\ \aleph^3_{\|\S_\Lambda\|}(\check{\ltimes}_2) \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \aleph^1_{\|\S_\Lambda\|}(\check{\ltimes}_r), \\ \aleph^2_{\|\S_\Lambda\|}(\check{\ltimes}_r), \\ \aleph^3_{\|\S_\Lambda\|}(\check{\ltimes}_r) \end{pmatrix}$

respectively. These have $\mathbb{ISVN}$ values.

Note: The collection of all cardinal sets of Ξ-sets is denoted by $\|C_{icsvnhs}(\mathbb{U})\|$ such that $\|C_{icsvnhs}(\mathbb{U})\| \subseteq \mathbb{ISVN}(\Lambda)$.

Definition 3.9. Assume $\S_\Lambda \in C_{icsvnhs}(\mathbb{U})$, $\|\S_\Lambda\| \in \|C_{icsvnhs}(\mathbb{U})\|$ and $\mathfrak{C}$ as in Definition 3.5, then Table 3 represents $\|\S_\Lambda\|$.

If $\alpha_{1j} = (\aleph^1_{\|\S_\Lambda\|}(\check{\ltimes}_j), \aleph^2_{\|\S_\Lambda\|}(\check{\ltimes}_j), \aleph^3_{\|\S_\Lambda\|}(\check{\ltimes}_j))$, for $j = \mathcal{N}_1^r$ then the following matrix represents the cardinal set $\|\S_\Lambda\|$,

$$[\alpha_{ij}]_{1\times r} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1r} \end{bmatrix}$$

and is called *cardinal matrix* of $\|\S_\Lambda\|$.

Definition 3.10. Let $\S_\Lambda \in C_{icsvnhs}(\mathbb{U})$ and $\|\S_\Lambda\| \in \|C_{icsvnhs}(\mathbb{U})\|$. Then Ξ-aggregation operator is defined as

$$\overbrace{\S_\Lambda} = A_{icifhs}\left(\|\S_\Lambda\|, \S_\Lambda\right)$$

where

$$A_{icsvnhs} : \|C_{icsvnhs}(\mathbb{U})\| \times C_{icifhs}(\mathbb{U}) \to \mathbb{F}(\mathbb{U}).$$

$\overbrace{\S_\Lambda}$ is called the aggregate $\mathcal{F}-$set of Ξ-set $\S_\Lambda$.
Its $\mathbb{M}_{fn}$ is given as

$$\aleph_{\overbrace{\S_\Lambda}} : \mathbb{U} \to [0,1]$$

with

$$\aleph_{\overbrace{\S_\Lambda}}(\nu) = \frac{1}{|\Lambda|} \sum_{\check{\ltimes} \in \Lambda} \aleph_{Card(\S_\Lambda)}(\check{\ltimes}) \aleph_{Card(\mathbb{X}_\Lambda)}(\nu).$$

Definition 3.11. Let $\S_\Lambda \in C_{icsvnhs}(\mathbb{U})$ and $\overbrace{\S_\Lambda}$ be its aggregate $\mathcal{F}-$set. Assume $\mathbb{U} = \{\check{\curlyvee}_1, \check{\curlyvee}_2, ..., \check{\curlyvee}_m\}$, then $\overbrace{\S_\Lambda}$ can be presented as

$$\begin{bmatrix} \S_\Lambda & \vdots & \aleph_{\underbrace{}_{\S_\Lambda}} \\ \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots\cdots \\ \check{\Upsilon}_1 & \vdots & \aleph_{\underbrace{}_{\S_\Lambda}}(\check{\Upsilon}_1) \\ \check{\Upsilon}_2 & \vdots & \aleph_{\underbrace{}_{\S_\Lambda}}(\check{\Upsilon}_2) \\ \vdots & \vdots & \vdots \\ \check{\Upsilon}_m & \vdots & \aleph_{\underbrace{}_{\S_\Lambda}}(\check{\Upsilon}_m) \end{bmatrix}$$

If $\alpha_{i1} = \aleph_{\underbrace{}_{\S_\Lambda}}(\check{\Upsilon}_i)$ for $i = \mathbb{N}_1^m$ then $\overbrace{\S_\Lambda}$ is represented by the matrix,

$$[\alpha_{i1}]_{m\times 1} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix}$$

which is called *aggregate matrix* of $\overbrace{\S_\Lambda}$ over $\mathbb{U}$.

4. Decision support system based on aggregation of Ξ-set

In light of the definitions provided in previous subsection, an algorithm is now described in this section to facilitate the $\mathbb{DSS}$, and the supplied method will be validated with the aid of an example from a real-world scenario.

Algorithm 4.1. The brief description of algorithm 4.1 is displayed in Figure 1.

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Algorithm : $\mathcal{DS}$ Algorithm Based on Aggregations of Ξ-set

==

▷ **Start**

▷ **Input Stage**:

———1. Assume $\mathbb{U}$ as sample space

———2. Assume $\mathfrak{E}$ as $\mathbb{SP}$

———3. Classify $\mathbb{SP}$ into $\mathbb{DAVS}$ $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, ..., \mathcal{L}_n$

▷ **Construction Stage**:

———4. $\Lambda = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times ... \times \mathcal{L}_n$

———5. Construct Ξ-set $\mathbb{X}_\Lambda$ over $\mathbb{U}$, in compliance with Definition 3.5,

▷ **Computation Stage**:

———6. Determine $\parallel \S_\Lambda \parallel$ for $\mathscr{A}-$term and $\mathscr{P}-$term employing Definition 3.8,

———7. Determine $\overbrace{\S_\Lambda}$ for $\mathscr{A}-$term and $\mathscr{P}-$term employing Definition 3.10,

———8. Determine $\aleph_{\underbrace{}_{\S_\Lambda}}(\nu)$ employing Definition 3.10,

▷ **Output Stage**:

———9. Figure out the best alternative by max modulus of $\aleph_{\widehat{\S_\Lambda}}(\nu)$ employing Definition 3.11.

▷**End**

==

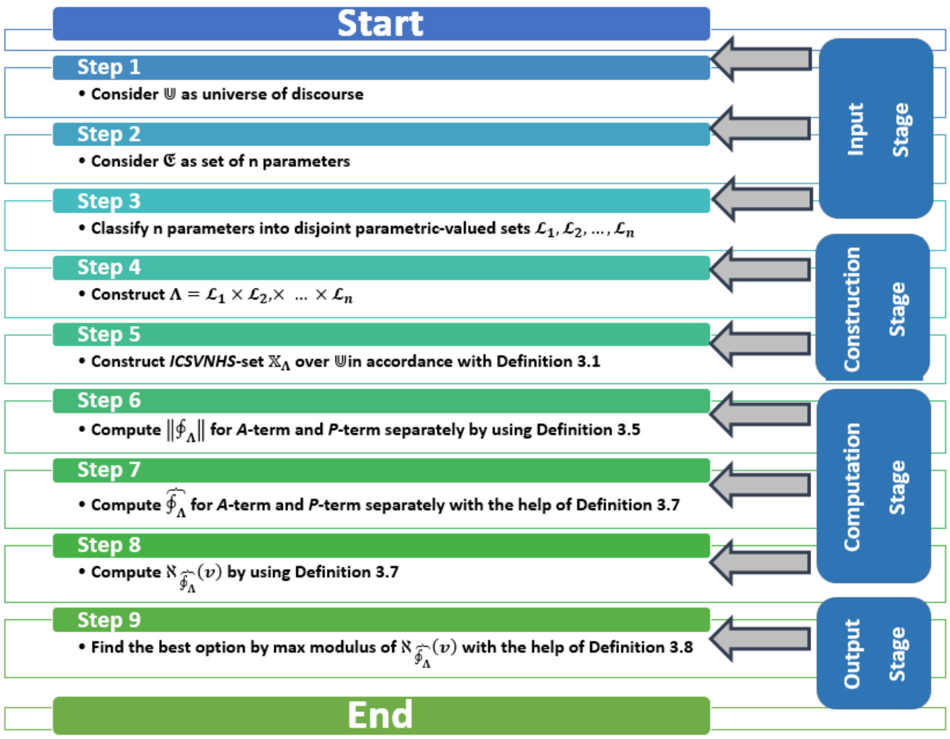

FIGURE 1. $\mathcal{DS}$ Algorithm Based on Aggregations of Ξ-set.

The following real-life example is used to illustrate algorithm:

4.1. *Decision support system based on aggregation of* Ξ*-set*

In this section, a real-world scenario of product selection is discussed based on the aggregation operation of Ξ-set.

Example 4.2. Suppose a person wants to purchase an LED TV from the market. He consults an expert, says Mr. "P" for the feathers that are necessary to take into consideration while buying a TV. To provide a satisfying viewing experience, numerous elements should be considered when choosing an LED TV's features. Here are some essential characteristics (attributes) that Mr. P should take into account:

Screen Size The viewing experience on an LED TV is significantly influenced by the screen size. To choose the right screen size, take into account the room's available area as well as the viewing distance. A more immersive watching experience is often provided by larger displays, but it's essential to make sure the TV is comfortable in the specified space. There are many screen sizes available on the market, but Mr. P preferred 32-inch and 42-inch sizes over others.

Display Technology Although LCD, OLED, and QLED are some of the numerous panel types that are available, LED TVs use LED backlighting technology. Each technology has benefits and disadvantages. While QLED delivers rich colors and high brightness levels, OLED offers great image quality with deep blacks and broad viewing angles. Although less expensive, LCD displays may have contrast and viewing angle restrictions. Mr. P preferred QLED over others.

Resolution The degree of clarity and detail in the material presented on TV depends on its resolution. Full HD (1920x1080 pixels), 4K Ultra HD (3840x2160 pixels), and 8K Ultra HD (7680x4320 pixels) are popular resolutions. In general, higher resolutions provide images that are more realistic and detailed, but the availability of 4K or 8K material should also be taken into account. Due to the unavailability of 4K and 8K, HD is taken into consideration by Mr. P.

Refresh Rate The number of times per second that the TV changes the image on the screen is referred to as the refresh rate. A higher refresh rate, such as 120Hz or 240Hz, helps reduce motion blur in fast-paced situations or sports by allowing for better motion handling. However, a normal refresh rate of 60Hz or 120Hz is generally enough for everyday viewing reasons, so 60Hz and 120Hz refresh rates are preferred.

HDR (High Dynamic Range) The contrast and color accuracy of the presented information are improved with HDR technology. Find TVs that can display HDR content in formats like HDR10, Dolby Vision, or HLG (Hybrid Log-Gamma). Wider color gamuts and more accurate highlights and shadows are possible with HDR-compatible TVs, making for a picture that is more vivid and realistic. Mr. P ignored the HDR attribute.

Smart Features Nowadays, many LED TVs include smart capabilities that provide users access to applications, streaming services, and web surfing. When assessing a TV's smart features, take into account the user interface, the availability of apps, and the simplicity of navigation. Mr. P preferred the LED's having smart features over others.

Connectivity Options Make sure the TV has enough connectors for connecting your gadgets, such as HDMI ports for connecting game consoles, Blu-ray players, or sound systems. For versatility, Mr. P takes into account the LED's accessibility to USB ports, Wi-Fi, Ethernet, and Bluetooth connectivity.

Sound Quality The overall satisfaction is greatly influenced by both the auditory experience and the visual experience, which are both essential. Think about the TV's built-in speakers, or see whether it includes audio-enhancing features like DTS or Dolby Atmos. Mr. P preferred LED's with built-in speakers over others.

Energy Efficiency In general, LED TVs are energy-efficient, but to lower long-term running expenses and environmental effects, it is important to evaluate the energy consumption and energy-saving features of the TV. Mr. P ignored this attribute.

By considering these attributes, one can make an informed decision when buying an LED TV that meets their specific preferences and viewing requirements. There are four types of LED's that are available in market that fulfill the above preferences, so they form the set: $\mathbb{U} = \{\breve{\curlyvee}_1, \breve{\curlyvee}_2, \breve{\curlyvee}_3, \breve{\curlyvee}_4\}$. The expert Mr. P considers a $\mathbb{SP}$, $\mathfrak{E} = \{e_1, e_2, ..., \{e_7\}$. For i = 1, 2,..., 7, where the attributes e_i stand for "screen size", "display technology", "resolution", "refresh rate", "smart features", "connectivity options", and "sound quality", respectively Corresponding to each attribute, the $\mathbb{DAVS}$ are: $\mathcal{L}_1 = \{e_{11}, e_{12}\}$; $\mathcal{L}_2 = \{e_{21}\}$; $\mathcal{L}_3 = \{e_{31}\}$; $\mathcal{L}_4 = \{e_{41}, e_{42}\}$; $\mathcal{L}_5 = \{e_{51}\}$; $\mathcal{L}_6 = \{e_{61}\}$ and $\mathcal{L}_7 = \{e_{71}\}$. Then the set $\Lambda = \mathcal{L}_1 \times \mathcal{L}_2 \times ... \times \mathcal{L}_7 = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ where each λ_i is a 7-tuple. We construct Ξ-sets $\psi_\Lambda(\lambda_1), \psi_\Lambda(\lambda_2), \psi_\Lambda(\lambda_3), \psi_\Lambda(\lambda_4)$ are defined as,

$$\psi_\Lambda(\lambda_1) = \left\{ \begin{array}{c} \frac{([0.2,0.5],[0.3,0.4],[0.0,0.1])e^{i([0.2,0.8],[0.1,0.3],[0.2,0.3])\pi}}{\breve{\curlyvee}_1}, \frac{([0.0,0.2],[0.1,0.3],[0.3,0.5])e^{i([0.2,0.3],[0.1,0.4],[0.1,0.3])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.0,0.2],[0.2,0.4],[0.0,0.2])e^{i([0.1,0.4],[0.1,0.4],[0.0,0.2])\pi}}{\breve{\curlyvee}_3}, \frac{([0.1,0.4],[0.4,0.5],[0.0,0.1])e^{i([0.1,0.3],[0.2,0.4],[0.2,0.3])\pi}}{\breve{\curlyvee}_4} \end{array} \right\},$$

$$\psi_\Lambda(\lambda_2) = \left\{ \begin{array}{c} \frac{([0.2,0.3],[0.2,0.5],[0.1,0.2])e^{i([0.0,0.2],[0.1,0.4],[0.1,0.2])\pi}}{\breve{\curlyvee}_1}, \frac{([0.1,0.3],[0.2,0.5],[0.0,0.1])e^{i([0.1,0.3],[0.3,0.4],[0.1,0.2])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.0,0.1],[0.1,0.3],[0.4,0.5])e^{i([0.2,0.4],[0.1,0.3],[0.0,0.2])\pi}}{\breve{\curlyvee}_3}, \frac{([0.1,0.3],[0.1,0.4],[0.1,0.3])e^{i([0.1,0.3],[0.2,0.3],[0.2,0.4])\pi}}{\breve{\curlyvee}_4} \end{array} \right\},$$

$$\psi_\Lambda(\lambda_3) = \left\{ \begin{array}{c} \frac{([0.2,0.4],[0.1,0.3],[0.1,0.3])e^{i([0.0,0.2],[0.1,0.5],[0.1,0.3])\pi}}{\breve{\curlyvee}_1}, \frac{([0.2,0.3],[0.0,0.3],[0.1,0.4])e^{i([0.2,0.4],[0.1,0.4],[0.1,0.2])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.0,0.2],[0.2,0.3],[0.1,0.5])e^{i([0.1,0.3],[0.2,0.4],[0.1,0.3])\pi}}{\breve{\curlyvee}_3}, \frac{([0.0,0.2],[0.1,0.3],[0.1,0.3])e^{i([0.1,0.3],[0.2,0.6],[0.0,0.1])\pi}}{\breve{\curlyvee}_4} \end{array} \right\},$$

$$\psi_\Lambda(\lambda_4) = \left\{ \begin{array}{c} \frac{([0.2,0.3],[0.1,0.4],[0.1,0.3])e^{i([0.0,0.2],[0.2,0.5],[0.1,0.2])\pi}}{\breve{\curlyvee}_1}, \frac{([0.1,0.5],[0.2,0.3],[0.0,0.1])e^{i([0.1,0.3],[0.3,0.4],[0.1,0.3])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.1,0.3],[0.1,0.4],[0.1,0.3])e^{i([0.2,0.3],[0.1,0.3],[0.1,0.3])\pi}}{\breve{\curlyvee}_3}, \frac{([0.1,0.2],[0.2,0.5],[0.1,0.2])e^{i([0.1,0.3],[0.2,0.3],[0.0,0.3])\pi}}{\breve{\curlyvee}_4} \end{array} \right\}.$$

Step 1: Ξ-set $\mathbb{X}_\Lambda$ is written as,

$$\mathbb{X}_\Lambda = \left\{ \begin{array}{c} \left(\lambda_1, \left\{ \begin{array}{c} \frac{([0.2,0.5],[0.3,0.4],[0.0,0.1])e^{i([0.2,0.8],[0.1,0.3],[0.2,0.3])\pi}}{\breve{\curlyvee}_1}, \frac{([0.0,0.2],[0.1,0.3],[0.3,0.5])e^{i([0.2,0.3],[0.1,0.4],[0.1,0.3])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.0,0.2],[0.2,0.4],[0.0,0.2])e^{i([0.1,0.4],[0.1,0.4],[0.0,0.2])\pi}}{\breve{\curlyvee}_3}, \frac{([0.1,0.4],[0.4,0.5],[0.0,0.1])e^{i([0.1,0.3],[0.2,0.4],[0.2,0.3])\pi}}{\breve{\curlyvee}_4} \end{array} \right\}\right), \\ \left(\lambda_2, \left\{ \begin{array}{c} \frac{([0.2,0.3],[0.2,0.5],[0.1,0.2])e^{i([0.0,0.2],[0.1,0.4],[0.1,0.2])\pi}}{\breve{\curlyvee}_1}, \frac{([0.1,0.3],[0.2,0.5],[0.0,0.1])e^{i([0.1,0.3],[0.3,0.4],[0.1,0.2])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.0,0.1],[0.1,0.3],[0.4,0.5])e^{i([0.2,0.4],[0.1,0.3],[0.0,0.2])\pi}}{\breve{\curlyvee}_3}, \frac{([0.1,0.3],[0.1,0.4],[0.1,0.3])e^{i([0.1,0.3],[0.2,0.3],[0.2,0.4])\pi}}{\breve{\curlyvee}_4} \end{array} \right\}\right), \\ \left(\lambda_3, \left\{ \begin{array}{c} \frac{([0.2,0.4],[0.1,0.3],[0.1,0.3])e^{i([0.0,0.2],[0.1,0.5],[0.1,0.3])\pi}}{\breve{\curlyvee}_1}, \frac{([0.2,0.3],[0.0,0.3],[0.1,0.4])e^{i([0.2,0.4],[0.1,0.4],[0.1,0.2])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.0,0.2],[0.2,0.3],[0.1,0.5])e^{i([0.1,0.3],[0.2,0.4],[0.1,0.3])\pi}}{\breve{\curlyvee}_3}, \frac{([0.0,0.2],[0.1,0.3],[0.1,0.3])e^{i([0.1,0.3],[0.2,0.6],[0.0,0.1])\pi}}{\breve{\curlyvee}_4} \end{array} \right\}\right), \\ \left(\lambda_4, \left\{ \begin{array}{c} \frac{([0.2,0.3],[0.1,0.4],[0.1,0.3])e^{i([0.0,0.2],[0.2,0.5],[0.1,0.2])\pi}}{\breve{\curlyvee}_1}, \frac{([0.1,0.5],[0.2,0.3],[0.0,0.1])e^{i([0.1,0.3],[0.3,0.4],[0.1,0.3])\pi}}{\breve{\curlyvee}_2}, \\ \frac{([0.1,0.3],[0.1,0.4],[0.1,0.3])e^{i([0.2,0.3],[0.1,0.3],[0.1,0.3])\pi}}{\breve{\curlyvee}_3}, \frac{([0.1,0.2],[0.2,0.5],[0.1,0.2])e^{i([0.1,0.3],[0.2,0.3],[0.0,0.3])\pi}}{\breve{\curlyvee}_4} \end{array} \right\}\right) \end{array} \right\}$$

Step 2: The cardinal is computed as,

$\parallel \mathbb{X}_\Lambda \parallel (A-term) =$

$$\left\{ \begin{array}{c} ([0.075,0.325],[0.250,0.400],[0.075,0.225])/\lambda_1,([0.100,0.250],[0.150,0.425],[0.150,0.275])/\lambda_2, \\ ([0.100,0.275],[0.100,0.300],[0.100,0.375])/\lambda_3,([0.125,0.325],[0.150,0.400],[0.075,0.225])/\lambda_4 \end{array} \right\}$$

$\parallel \mathbb{X}_\Lambda \parallel (P-term) =$

$$\left\{ \begin{array}{c} ([0.150,0.450],[0.125,0.375],[0.125,0.275])/\lambda_1,([0.100,0.300],[0.175,0.350],[0.100,0.250])/\lambda_2, \\ ([0.100,0.300],[0.150,0.475],[0.075,0.225])/\lambda_3,([0.100,0.275],[0.200,0.375],[0.075,0.275])/\lambda_4 \end{array} \right\}$$

Step 3: The set $\widehat{\mathbb{X}_\Lambda}$ can be established as,

$\widehat{\mathbb{X}_\Lambda}(A-term) =$

$$\frac{1}{4}\begin{bmatrix} [0.2,0.5],[0.3,0.4],[0.0,0.1] & [0.2,0.3],[0.2,0.5],[0.1,0.2] & [0.2,0.4],[0.1,0.3],[0.1,0.3] & [0.2,0.3],[0.1,0.4],[0.1,0.3] \\ [0.0,0.2],[0.1,0.3],[0.3,0.5] & [0.1,0.3],[0.2,0.5],[0.0,0.1] & [0.2,0.3],[0.0,0.3],[0.1,0.4] & [0.1,0.5],[0.2,0.3],[0.0,0.1] \\ [0.0,0.2],[0.2,0.4],[0.0,0.2] & [0.0,0.1],[0.1,0.3],[0.4,0.5] & [0.0,0.2],[0.2,0.3],[0.1,0.5] & [0.1,0.3],[0.1,0.4],[0.1,0.3] \\ [0.1,0.4],[0.4,0.5],[0.0,0.1] & [0.1,0.3],[0.1,0.4],[0.1,0.3] & [0.0,0.2],[0.1,0.3],[0.1,0.3] & [0.1,0.2],[0.2,0.5],[0.1,0.2] \end{bmatrix}$$

$$\times \begin{bmatrix} [0.075,0.325],[0.250,0.400],[0.075,0.225] \\ [0.100,0.250],[0.150,0.425],[0.150,0.275] \\ [0.100,0.275],[0.100,0.300],[0.100,0.375] \\ [0.125,0.325],[0.150,0.400],[0.075,0.225] \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0.2 & 0.0 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.2 & 0.3 \\ 0.0 & 0.4 & 0.1 & 0.1 \\ 0.0 & 0.1 & 0.0 & 0.1 \end{bmatrix} \begin{bmatrix} 0.000 \\ 0.050 \\ 0.075 \\ 0.010 \end{bmatrix} = \begin{bmatrix} 0.004000 \\ 0.005750 \\ 0.007125 \\ 0.001500 \end{bmatrix}$$

$\widehat{\mathbb{X}_\Lambda}(P-term) =$

$$\frac{1}{4}\begin{bmatrix} [0.2,0.8],[0.1,0.3],[0.2,0.3] & [0.0,0.2],[0.1,0.4],[0.1,0.2] & [0.0,0.2],[0.1,0.5],[0.1,0.3] & [0.0,0.2],[0.2,0.5],[0.1,0.2] \\ [0.2,0.3],[0.1,0.4],[0.1,0.3] & [0.1,0.3],[0.3,0.4],[0.1,0.2] & [0.2,0.4],[0.1,0.4],[0.1,0.2] & [0.1,0.3],[0.3,0.4],[0.1,0.3] \\ [0.1,0.4],[0.1,0.4],[0.0,0.2] & [0.2,0.4],[0.1,0.3],[0.0,0.2] & [0.1,0.3],[0.2,0.4],[0.1,0.3] & [0.2,0.3],[0.1,0.3],[0.1,0.3] \\ [0.1,0.3],[0.2,0.4],[0.2,0.3] & [0.1,0.3],[0.2,0.3],[0.2,0.4] & [0.1,0.3],[0.2,0.6],[0.0,0.1] & [0.1,0.3],[0.2,0.3],[0.0,0.3] \end{bmatrix}$$

$$\times \begin{bmatrix} [0.150,0.450],[0.125,0.375],[0.125,0.275] \\ [0.100,0.300],[0.175,0.350],[0.100,0.250] \\ [0.100,0.300],[0.150,0.475],[0.075,0.225] \\ [0.100,0.275],[0.200,0.375],[0.075,0.275] \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 0.5 & 0.0 & 0.0 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 \\ 0.3 & 0.3 & 0.0 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0.200 \\ 0.025 \\ 0.075 \\ 0.000 \end{bmatrix} = \begin{bmatrix} 0.025000 \\ 0.009375 \\ 0.016875 \\ 0.007500 \end{bmatrix}$$

$$\widehat{\mathbb{X}_\Lambda} = \left\{ 0.004000e^{i0.025000\pi}/\check{\Upsilon}_1, 0.005750e^{i0.009375\pi}/\check{\Upsilon}_2, 0.007125e^{i0.016875\pi}/\check{\Upsilon}_3, 0.001500e^{i0.007500\pi}/\check{\Upsilon}_4 \right\}$$

Assume the modulus value of $\max\left(\aleph_{\overbrace{\mathbb{X}_\Lambda}}\right)$

$= \max\{0.004101260482/\check{\Upsilon}_1, 0.005804159727/\check{\Upsilon}_2, 0.007246254583/\check{\Upsilon}_3, 0.001511292293/\check{\Upsilon}_4\}$

$= 0.007246254583/\check{\Upsilon}_3$. This means that the LED $\check{\Upsilon}_3$ may be recommended by Mr. P for purchase.

5. Discussion and comparative analysis

Different $\mathbb{DM}$ algorithmic techniques have already been explored in the literature by [35–37, 55–59, 61] that were based on hybridized complex set architectures with $\mathcal{F}-$set,$\mathcal{IF}-$set, and $\mathcal{SVN}-$set under $\mathcal{S}-$set environment. The lack of several crucial characteristics has a negative impact on the process of $\mathbb{DM}$. For instance, considering "screen size," "screen resolution," "refresh rate," e.t.c., as only attributes in a scenario based on product selection is insufficient because these indicators may have different values (parameters) and sub-values (sub-parameters). It is much more appropriate to further classify these parameters into their $\mathbb{DAVS}$, as we have done in Example 4.2. The aforementioned current $\mathbb{DM}$ models are insufficient for $\mathcal{IV}$ data or $\mathcal{MAA}-$mapping, however, the shortcomings of these models have been solved in the suggested model. By taking into account $\mathcal{MAA}-$mapping, the $\mathbb{DM}$ process will become more dependable and trustworthy. In Table 4, a comparison analysis of proposed model with relevant existing models has been carried out. The Table 4 makes it abundantly clear that our proposed structure, Ξ-set is more flexible and generalized than existing relevant models for the reason that these models [35–37, 55–59, 61] are customized for their particular cases by excluding certain or all features among $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$, $\mathbb{I}_{fn}$, $\mathbb{SAA}$-mapping, $\mathbb{MAA}$-mapping, $PN-$data and $IV-$data. The visual illustration of this generalization of our suggested structure is shown in Figure 2.

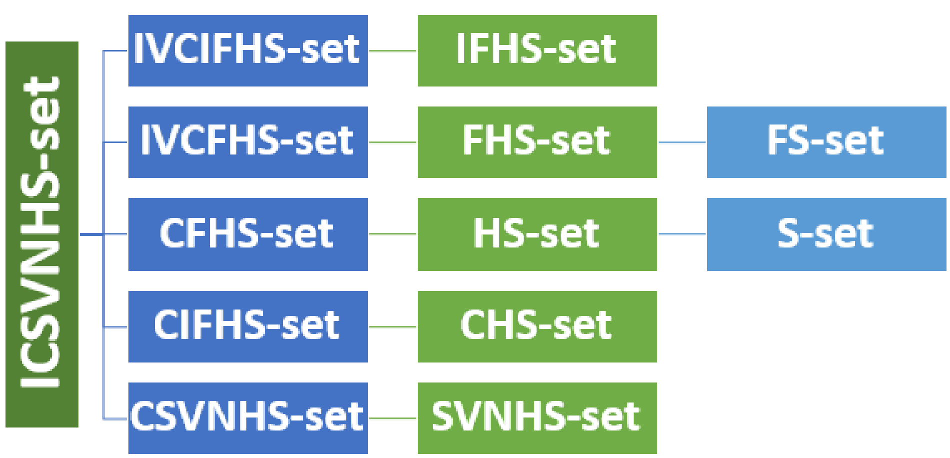

FIGURE 2. Generalization of Proposed Structure

TABLE 4. Comparison analysis of proposed model with some existing relevant models

Authors	Structure	$\mathbb{A}_{fn}$	Remarks
Thirunavukarasu et al. [35]	$\mathbb{CFS}$-set	$\mathbb{SAA}$-mapping	Insufficient for $\mathcal{IV}$ data, $\mathbb{N}_{fn}$, $\mathbb{I}_{fn}$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$.
Fan et al. [58]	$\mathbb{IVCFS}$-set	$\mathbb{SAA}$-mapping	Shows inadequacy for $\mathbb{N}_{fn}$, $\mathbb{I}_{fn}$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Selvachandran et al. [61]	$\mathbb{IVCFS}$-set	$\mathbb{SAA}$-mapping	Insufficient for $\mathbb{N}_{fn}$, $\mathbb{I}_{fn}$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Kumar et al. [36]	$\mathbb{CIFS}$-set	$\mathbb{SAA}$-mapping	Insufficient for $\mathcal{IV}$ data, $\mathbb{I}_{fn}$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Ali et al. [55]	$\mathbb{CIFS}$-set	$\mathbb{SAA}$-mapping	Insufficient for $\mathcal{IV}$ data, $\mathbb{I}_{fn}$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Khan et al. [59]	$\mathbb{CIFS}$-set	$\mathbb{SAA}$-mapping	Insufficient for $\mathcal{IV}$ data, $\mathbb{I}_{fn}$ and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Smarandache et al. [37]	$\mathbb{CSVNS}$-set	$\mathbb{SAA}$-mapping	Insufficient for $\mathcal{IV}$ data and partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Al-Sharqi et al. [57]	$\mathcal{ICSVNS}$-set	$\mathbb{SAA}$-mapping	Shows inadequacy for partitioning $\mathbb{SP}$ to $\mathbb{DAVS}$
Rahman et al. [56]	$\mathcal{CFHS}$-set	$\mathbb{MAA}$-mapping	Insufficient for $\mathcal{IV}$ data, $\mathbb{N}_{fn}$, $\mathbb{I}_{fn}$.
Rahman et al. [56]	$\mathcal{CIFHS}$-set	$\mathbb{MAA}$-mapping	Insufficient for $\mathcal{IV}$ data and $\mathbb{I}_{fn}$.
Rahman et al. [56]	$\mathcal{CSVNHS}$-set	$\mathbb{MAA}$-mapping	Insufficient for $\mathcal{IV}$ data.
Rahman et al. [60]	$\mathcal{IVCFHS}$-set	$\mathbb{MAA}$-mapping	Insufficient for $\mathbb{N}_{fn}$, $\mathbb{I}_{fn}$.
Proposed Structure	Ξ-set	$\mathbb{MAA}$-mapping	Addresses the restrictions and faults of preceding structures.

5.1. *Merits of proposed Study*

The following are some advantages of the proposed study that are mentioned in this subsection:

(i) The proposed method utilized the Ξ-set concepts to address current $\mathbb{DM}$ difficulties. As a result, this model has enormous potential in the realistic portrayal of computational invasions. The offered approach enables investigators to handle a real-world situation where the periodicity of data in the form of intervals has to be addressed.

(ii) Due to the suggested structure's emphasis on a thorough examination of qualities (sub-attributes) rather than a narrow focus on those traits (attributes), the $\mathbb{DM}$ process is improved, adaptable, and more dependable.

(iii) It discusses the features and qualities of the current relevant structures, i.e., $\mathbb{IVCFHS}$-set, $\mathbb{CFHS}$-set, $\mathbb{CIFHS}$-set, $\mathbb{CSVNHS}$-set, $\mathbb{IVCFS}$-set, $\mathbb{IVCIFS}$-set, $\mathbb{IVCNS}$-set, $\mathbb{CFS}$-set, $\mathbb{CIFS}$-set, $\mathbb{CNS}$-set, etc., so it is not unreasonable to call it the generalized form of all these structures.

TABLE 5. Comparison with existing models under appropriate features

Authors	Structure	$\mathbb{M}_{fn}$	$\mathbb{N}_{fn}$	$\mathbb{I}_{fn}$	$\mathbb{SAA}$-mapping	$\mathbb{MAA}$-mapping	PN- data	IV data
Ali et al. [55]	$\mathbb{CIFS}$-set	✓	✓	×	✓	×	✓	×
Al-Sharqi et al. [57]	$\mathbb{ICSVNS}$-set	✓	✓	✓	✓	×	✓	✓
Fan et al. [58]	$\mathbb{IVCFS}$-set	✓	×	×	✓	×	✓	✓
Khan et al. [59]	$\mathbb{CIFS}$-set	✓	✓	×	✓	×	✓	×
Kumar et al. [36]	$\mathbb{CIFS}$-set	✓	✓	×	✓	×	✓	×
Smarandache et al. [37]	$\mathbb{CSVNS}$-set	✓	✓	✓	✓	×	✓	×
Selvachandran et al. [61]	$\mathbb{IVCFS}$-set	✓	×	×	✓	×	✓	✓
Thirunavukarasu et al. [35]	$\mathbb{CFS}$-set	✓	×	×	✓	×	✓	×
Rahman et al. [56]	$\mathbb{CFHS}$-set	✓	×	×	✓	✓	✓	×
Rahman et al. [56]	$\mathbb{CIFHS}$-set	✓	✓	×	✓	✓	✓	×
Rahman et al. [56]	$\mathbb{CSVNHS}$-set	✓	✓	✓	✓	✓	✓	×
Rahman et al. [60]	$\mathbb{IVCFHS}$-set	✓	×	×	✓	✓	✓	✓
Proposed Structure	Ξ-set	✓	✓	✓	✓	✓	✓	✓

Tables 4 and 5 make it simple to determine the benefit of the proposed study. Table 4 demonstrates the main features of the study. Table 5 demonstrates the dominant features, including $\mathbb{M}_{fn}$, $\mathbb{N}_{fn}$, $\mathbb{I}_{fn}$, $\mathbb{SAA}$-mapping, $\mathbb{MAA}$-mapping, PN data, and IV data of the proposed study.

6. **Conclusion**

This article discusses a novel theoretical framework, the interval complex single-valued neutrosophic hypersoft set (Xi-set), along with its characteristics and set-theoretic operations. The recommended structure blends the interval complex single-valued neutrosophic set and hypersoft set to regulate unclear and unsure knowledge. These two components are already recognized for their dependable settings. While the second provides a multi-argument domain for the concurrent assessment of several sub-attributes, the first component can manage data on intervals and periodic types. The set theoretic operations, including complement, difference, union, and intersection of the Ξ-set, are also described. It has been designed to use aggregate matrices, cardinal sets, aggregate $\mathbb{F}$-sets, and aggregate matrices as aggregation operators. A $\mathbb{DM}$ technique that is based on aggregation operators of the Ξ-set has been suggested. To assess the model's flexibility and validity, the suggested structure and its $\mathbb{DSS}$ in a real-world scenario have been compared with some previously published relevant research. The present work has explored the conceptual basis for a generalized model, that is, Ξ-set, to deal with $\mathbb{DM}$ real-life situations by using hypothetical data. The authors have pledged to present multiple instance reports based on the Xi-set using actual data. It is feasible to extend hybrid set structures more broadly by including expert sets, prospective fuzzy-set-like models, fuzzy-set-like parameterized families, and algebraic structures.

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