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Pentapartitioned Neutrosophic Fuzzy Optimization Method for Multi-objective Reliability Optimization Problem

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Abstract: Fuzzy logic is an important mathematical tool that deals with uncertainty and imprecision in decision-making processes. The prevalent frameworks, known as neutrosophic sets, study the connection of neutralities with various ideational spectra in addition to generalizing concept of fuzzy sets. Using a penta-partitioned neutrosophic fuzzy environment, a novel optimization technique is proposed in this study. Proposed optimization technique is an expansion of fuzzy optimization, intuitionistic fuzzy optimization (IFO), single-valued neutrosophic optimization (NSO) and four valued neutrosophic optimization (FVNO). Here, the neutrosophic set's indeterminacy term is broken down into three components: contradiction (*C*), unknown (*U*), ignorance (*I*). To demonstrate the applicability and effectiveness of the suggested approach a numerical example is solved and the outcomes are contrasted with those of other methods already in use by cumulative percentage gap and sum of optimal values. Finally, a multi objective reliability optimization model of LCD display unit is solved by this method.

Keywords: Reliability; Neutrosophic set; Neutrosophic optimization; Pentapartitioned neutrosophic optimization.

1.Introduction

Recent years have seen a rise in interest in the topic of reliability optimization, which aims to enhance the performance and dependability of complex systems. Reliability optimization involves making decisions regarding system design, maintenance, and resource allocation to improve the system's ability to function effectively and consistently in various operating conditions. However, traditional reliability optimization approaches often encounter challenges when dealing with multiple conflicting objectives, such as maximizing system performance while minimizing costs or minimizing failure rates while maximizing system availability. In literature reliability optimization models are solved using various exact, heuristic and metaheuristic methods. For example, Misra [1] described usage of the integer programming technique before introducing [2] the use of the maximum principle and lagrange multipliers to solve reliability optimization problems. Sakawa [3] has presented multi-objective reliability allocation problem utilizing surrogate worth trading strategies to minimize system cost while maximize system reliability. A method using parametric programming was presented by Chern and Jan [4]. W.kuo, V.R.Prsad [5] solved system reliability optimization problem using some heuristic and metaheuristic algorithms Kuo et. al [6] presented some fundamental method and its application by solving reliability optimization model.

Uncertainty and ambiguity are very practical issue in real life mathematical problems. To address these challenges, researchers have explored the application of advanced mathematical tools and techniques, including fuzzy sets and optimization methods, to multi-objective reliability optimization problems. In 1965, Zadeh [7] invented the fuzzy set (FS). Fuzzy sets deals with one membership value in [0,1] ,but sometimes uncertainty is not properly expressed by single membership value. So, considering the membership as an interval in [0,1] the interval-valued fuzzy set (IVFS) [8] was invented. In some situations, only membership is insufficient to fully convey the uncertainty, so non-membership value also required to clarify the vagueness. That is outside the purview of FS and IVFS. Atanassov [9] first suggested intuitionistic fuzzy sets (IFS) in 1986, expands beyond the scope of IVFS and FS. IFS introduce the notion of considering the total of membership and non-membership values, ensuring that the sum $(\leq)1$.

In 1998, Smarandache [10] introduced neutrosophic sets (NSs) extending from FS, IFS, hesitant fuzzy sets, and IVFS, to handle uncertain information encountered in real-world situations. Neutrosophic sets serve as a valuable mathematical tool for addressing ambiguous and conflicting information. They consist of three independent components: truth, falsity, and indeterminacy membership. However, applying neutrosophic fuzzy sets in practical scenarios presents challenges due to the presence of both standard and non-standard intervals of membership values. To address these challenges, Wang et al. [11, 12] invented single-valued NSs and interval-valued NSs, enabling the application of NSs to real-world problems. In an effort to generalize neutrosophic sets further, F. Smarandache [13] introduced n-Valued neutrosophic logic through categorizing truth, indeterminacy, and falsity into n types. Subsequently, Freen et al. [14] defined four-valued neutrosophic set (FVNS) by refining the indeterminacy term into unknown and contradiction. Expanding on this concept, Mallik and Pramanik [15] introduced the penta-partitioned neutrosophic set, which splits the indeterminacy term into contradiction, unknown, and ignorance.

In a wide range of areas, optimization methods are crucial for addressing a variety of practical problems and decision-making problems. In last few decades fuzzy optimization [16] is very efficient tool as it deals with the ambiguity and uncertainty of real-life problems. Bellman and Zadeh [17] first introduced decisions, goals and constraints in fuzzy. As an extension of this work Zimmerman [18] introduced fuzzy programming method. To deal with the non-membership of an information Angelov [19] invented intuitionistic fuzzy optimization method. It's interesting to observe that there are several optimization problems that require a collection of membership grades rather than a single grade of membership since experts' estimates of the optimization's parameters vary significantly. Considering this a multi objective optimization problem (MOOP) is solved by Bharati [20] in hesitant fuzzy environment. To solve a MOOP, Sarkar et al. [21] applied the multi-objective neutrosophic optimization algorithm. Abdel-Basset et al. [22] introduced neutrosophic goal programming approach. The integer programming problem was proposed by Mohamed, Mai et al. [23] in triangular neutrosophic environment. Group decision-making problem was solved by Abdel-Basset et al. [24] utilizing triangular neutrosophic weighted aggregation operator. In an IVNSs framework, Garg [25] has presented a nonlinear programming method to solve MCDM problems.

This method provides a systematic approach for tackling decision-making challenges in uncertain and ambiguous environments. Recently, Freen et al. [14] proposed FVRNO method to solve MOOP and applied it to car-side impact and riser design problems. Recently, in various field the concept of Penta partitioned neutrosophic graph (PNG) is used to find the optimal path using Penta partitioned neutrosophic set. Quek, Shio Gai, et al. [26] used the concept of PNG to find the safest path of travel and stay to reduce the spread of COVID-19. Broumi, Said, et al [27] used PNG to solve MCDM problem. Das, Suman, et al [28,29] introduced single valued bipolar PNS and its application by solving MADM problem, and author also presented single valued PNG and solution strategy to MCDM problem.

In formation of a system design, reliability optimization is one of the crucial jobs. Finding the most effective way to raise system reliability in limited resource has always been the reliability engineer's main objective. There are several parameters in the MOOP that are constantly vague and ambiguous in nature for ambiguity in decision makers judgments. Fuzzy technique is used to analyze this in MOOP to manage such kind of nature. Fuzzy non-linear programming was utilized by Park [30] for the reliability apportionment problem of series system. Fuzzy global optimization reliability model was utilized by Ravi et al. [31]. To address the reliability optimization problem, Huang [32] suggested a multi-objective fuzzy optimization approach. Later, the intuitionistic fuzzy optimization approach [19] is used in a variety of study areas in reliability optimization problem. Mahapatra et al. [33] used IFO methods to solve reliability optimization model. To address the problem of multi-objective reliability optimization IFO method was applied in interval environments by Garg et al [34]. Islam and Kundu [35] applied NSO technique to solve the reliability optimization of LCD display unit. As far as known to us, there isn't a research paper in the literature that addresses how to solve a MOOP in a pentapartitioned neutrosophic environment.

In this article, a penta-partitioned neutrosophic fuzzy environment is used to suggest a multi-objective optimization technique. To show that the suggested strategy is effective a nonlinear MOOP is solved and the outcomes are compared against those of other techniques already in use. Also, this method is applied to solve the reliability optimization model of LCD display unit and the result is compared with four valued refined optimization method. Remaining part is arranged as: the definition of fuzzy set, its extension and properties are discussed in section 2. The Proposed penta-partitioned neutrosophic fuzzy optimization technique and computational algorithm is explained in section 3. In section 4 a numerical example is solved by developed method. Reliability model of LCD display unit is shown in section 5. In Section 6, results and discussion are presented. Finally, in section 7. conclusion and future works are discussed.

2. Preliminaries

Definition 1. Fuzzy set (FS) [7]

E be the universal set, then the FS $\tilde{F}$ on the set E is defined as $\tilde{F} = \{(e, \mu_{\tilde{F}}(e)) \mid e \in E\}$, where $\mu_{\tilde{F}}: E \to [0,1]$ is membership function on E.

Example: Consider set of number $E = \{1,2,3,4,5,6\}$, fuzzy set $\tilde{F}$ is number closed to 4. Then we can define $\tilde{F} = \{(1,0), (2,.2), (3,.6), (4,1), (5,.5), (6,.3)\}$.

Definition 2. Intuitionistic Fuzzy Set (IFS) [9]

E is the universal set, the IFS $\tilde{I}$ on E is the collection of order triplets $\tilde{I} = \{(e, \mu_{\tilde{I}}(e), \vartheta_{\tilde{I}}(e))| \ e \in E\}$, where $\mu_{\tilde{I}}, \vartheta_{\tilde{I}}: E \to [0,1]$ represent membership, non-membership function on E, $0 \le \mu_{\tilde{I}}(e) + \vartheta_{\tilde{I}}(e) \le 1$ for all $e \in E$.

Here the function $\pi_{\tilde{I}}(e) = (1 - \mu_{\tilde{I}}(e) - \vartheta_{\tilde{I}}(e))$ is the hesitancy degree for each $e \in E$.

Example:

If a company produce three products $E = \{p_1, p_2, p_3\}$, there be three opinions on these products, "good (membership)", "bad(non-membership)", "no idea (hesitancy)". Then the intuitionistic fuzzy set $\tilde{I} = \{(p_1, .6, .3), (p_2, .7, .25), (p_3, .8, .16)\}$. Here $\pi_{\tilde{I}}(p_1) = .1, \pi_{\tilde{I}}(p_2) = .05, \pi_{\tilde{I}}(p_1) = .04$.

Definition 3. Neutrosophic fuzzy set (NSs) [10]

E is the universal set. NSs on E is $\widetilde{N} = \{(e, T_{\widetilde{N}}(e), I_{\widetilde{N}}(e), F_{\widetilde{N}}(e))| \ e \in E\}$, here $T_{\widetilde{N}}(e), I_{\widetilde{N}}(e), F_{\widetilde{N}}(e)$ are subsets of $]0^-, 1^+[$ which represent truth, indeterminacy and falsity membership on E and $0^- \le SupT_{\widetilde{N}}(e) + SupI_{\widetilde{N}}(e) + SupF_{\widetilde{N}}(e) \le 3^+$ for all $e \in E$. In real life, the application of NS is difficult because the membership values are subsets of $]0^-, 1^+[$.

Definition 4. Single valued neutrosophic set (SVNSs) [11]

In SVNSs, for all $e \in E$ (universal set) the set $\widetilde{SN}$ is characterized by $T_{\widetilde{SN}}(e), I_{\widetilde{SN}}(e), F_{\widetilde{SN}}(e)$, each takes single value in $[0,1]$, $0 \le T_{\widetilde{SN}}(e) + I_{\widetilde{SN}}(e) + F_{\widetilde{SN}}(e) \le 3$.Where,

$$\widetilde{SN} = \{(e, T_{\widetilde{SN}}(e), I_{\widetilde{SN}}(e), F_{\widetilde{SN}}(e)): e \in E\}.$$

Example: Suppose a phone company launch a phone, customers may review the phone on the basis of $E = \{e_1 = price, e_2 = ram \ \ and \ \ storage, e_3 = battery \ \ service, e_4 = camera\}$. The customers opinion on each criterion be positive (truth degree), Indeterminate, negative (falsity degree). Then the set $\widetilde{SN}$ on E as : $\widetilde{SN} = \{(e_1, .7, .5, .4), (e_2, .5, .6, .3), (e_3, .3, .4, .8), (e_4, .8, .3, .4)\}$.

Definition 5. Four-valued neutrosophic set (FVNS) [14]

By splitting indeterminacy in two ways, there are two types of FVNS. For one of such FVNS, indeterminacy is split into unknown (U) and contradiction (C), where $C = T \wedge F$. The values of T, U, C and F are the function from E to $[0,1]$ and $0 \le T_{\widetilde{FN}}(e) + U_{\widetilde{FN}}(e) + C_{\widetilde{FN}}(e) + F_{\widetilde{FN}}(e) \le 4$. Thus, this type of FVNS is

$$\widetilde{FN} = \{(e, T_{\widetilde{FN}}(e), I_{\widetilde{FN}}(e), C_{\widetilde{FN}}(e) F_{\widetilde{FN}}(e)): e \in E\}.$$

For another type of FVNS, here the indeterminacy split into two parts, Ignorance (G) and contradiction (C), where $C = T \wedge F$ and $G = T \vee F$. The values of T, G, C and F are the function from E to $[0,1]$ and $0 \le T_{\widetilde{FN}}(e) + C_{\widetilde{FN}}(e) + G_{\widetilde{FN}}(e) + F_{\widetilde{FN}}(e) \le 4$. Thus, this type of FVNS is

$$\widetilde{FN} = \{(e, T_{\widetilde{FN}}(e), C_{\widetilde{FN}}(e), G_{\widetilde{FN}}(e) F_{\widetilde{FN}}(e)): e \in E\}.$$

Example of FVNS:

Consider a criterion set $E = \{e_1, e_2, e_3, e_4\}$. There be four types of opinion for each criterion, such as "truth, contradiction, unknown, falsity" or "truth, ignorance, contradiction, falsity", where each degree in $[0,1]$. Then we can construct FVNS as

$$X = \frac{\langle 0.6, 0.3, 0.5, 0.4 \rangle}{e_1} + \frac{\langle 0.5, 0.3, 0.7, 0.4 \rangle}{e_2} + \frac{\langle 0.7, 0.3, 0.4, 0.2 \rangle}{e_3} + \frac{\langle 0.8, 0.3, 0.2, 0.1 \rangle}{e_4}$$

Definition 6: Penta-partitioned neutrosophic set (PNS) [15]

PNS was defined by Rama Mallick and Surapati Pramanik using the concepts of n-valued neutrosophic set. Here indeterminacy divided into ignorance, contradiction, and unknown (U, G, C). This is how PNS is defined:

E be a universal set. PNS, $\widetilde{PN}$ over E is the combination of Truth($T_{\widetilde{PN}}$), unknown($U_{\widetilde{PN}}$),ignorance ($G_{\widetilde{PN}}$), contradiction($C_{\widetilde{PN}}$), falsity($F_{\widetilde{PN}}$) memberships which are in [0,1]for all $e \in E$ and

$0 \leq T_{\widetilde{PN}}(e) + C_{\widetilde{PN}}(e) + G_{\widetilde{PN}}(e) + U_{\widetilde{PN}}(e) + F_{\widetilde{PN}}(e) \leq 5.$

2.1 ***Basic properties***

Definition 7. [15] P_1 and P_2 be two PNSs over E then $P_1 \subseteq P_2$ iff $T_{P_1}(e) \leq T_{P_2}(e), C_{P_1}(e) \leq C_{P_2}(e), G_{P_1}(e) \geq G_{P_2}(e), U_{P_1}(e) \geq U_{P_2}(e)$ and $F_{P_1}(e) \geq F_{P_2}(e)$ for all $e \in E$.

Definition 8. [15] The complement of PNS P is denoted by P^c and is defined by:

$P = \{(T_P(e), C_P(e), G_P(e), U_P(e), F_P(e)) | e \in E\}$, then

$$P^c = \{(F_P(e), U_P(e), 1 - G_P(e), C_P(e), T_P(e)) \mid e \in E\}$$

i.e, $T_{P^c}(e) = F_P(e), C_{P^c}(e) = U_P(e), G_{P^c}(e) = 1 - G_P(e), U_{P^c}(e) = C_P(e), F_{P^c}(e) = T_P(e)$ for all $e \in E$

Definition 9. [15] P_1 and P_2 be two PNSs. Then $P_1 \cup P_2$ and $P_1 \cap P_2$ is defined by:

$$P_1 \cup P_2 = \left\{ \left(\begin{matrix} \max\left(T_{P_1}(e), T_{P_2}(e)\right), \max\left(C_{P_1}(e), C_{P_2}(e)\right), \min\left(G_{P_1}(e), G_{P_2}(e)\right), \\ \min\left(U_{P_1}(e), U_{P_2}(e)\right), \min\ \left(F_{P_1}(e), F_{P_2}(e)\right) \end{matrix} \right) \middle| e \in E \right\}$$

$$P_1 \cap P_2 = \left\{ \left(\begin{matrix} \min\left(T_{P_1}(e), T_{P_2}(e)\right), \min\left(C_{P_1}(e), C_{P_2}(e)\right), \max\left(G_{P_1}(e), G_{P_2}(e)\right), \\ \max\left(U_{P_1}(e), U_{P_2}(e)\right), \max\ \left(F_{P_1}(e), F_{P_2}(e)\right) \end{matrix} \right) \middle| e \in E \right\}$$

2.2. ***Example of PNS:***

Suppose a company have manufactured a car. The quality of the car is determined by some domain experts over the set of criterions $E = \{e_1, e_2, e_3\}$, where e_1=reliability, e_2= fuel consumption, e_3= cost. The question to the domain experts is "is the car is good?". There may be the five types of degrees of opinions in [0,1] under each category, which are "good", "contradictory", "Ignorance", "Unknown", "Bad". P and Q two PNSs, which are opinion of two experts on W, are defined by:

$$P = \frac{\langle 0.6,0.3,0.3,0.5,0.4 \rangle}{e_1} + \frac{\langle 0.5,0.3,0.7,0.4,0.2 \rangle}{e_2} + \frac{\langle 0.7,0.3,0.4,0.2,0.2 \rangle}{e_3}$$
$$Q = \frac{\langle 0.4,0.7,0.3,0.5,0.6 \rangle}{e_1} + \frac{\langle 0.2,0.8,0.3,0.5,0.7 \rangle}{e_2} + \frac{\langle 0.3,0.6,0.8,0.5,0.6 \rangle}{e_3}.$$

Then we have,

$$P^C = \langle 0.4,0.5,0.7,0.3,0.6 \rangle / e_1 + \langle 0.2,0.4,0.3,0.3,0.5 \rangle / e_2 + \langle 0.2,0.2,0.6,0.3,0.7 \rangle / e_3$$
$$P \cup Q = \langle 0.6,0.7,0.3,0.5,0.4 \rangle / e_1 + \langle 0.5,0.8,0.3,0.4,0.2 \rangle / e_2 + \langle 0.7,0.6,0.4,0.2,0.2 \rangle / e_3$$
$$P \cap Q = \langle 0.4,0.3,0.3,0.5,0.6 \rangle / e_1 + \langle 0.2,0.3,0.7,0.5,0.7 \rangle / e_2 + \langle 0.3,0.3,0.8,0.5,0.6 \rangle / e_3$$

3. Proposed penta-partitioned neutrosophic fuzzy optimization technique

If we take a look at a multi-objective optimization problem (MOOP),

$$\text{Minimize } \{Z_i(w)\} \qquad i = 1, \dots, m.$$

Subject to

$$f_j(w) \le b_j \qquad j = 1, \dots, n. \qquad (1)$$
$$w \ge 0$$

where $Z_i(w)$ are m objectives, $f_j(w)$ are the n constraints, w are decision variables, and m and n presents number of objectives and constraints respectively. $\tilde{D}$ is the decision set, which combines penta-partitioned neurotrophic goals $(\tilde{O}_i)$ and constraints $(\tilde{L}_j)$, is defined by:

$$\tilde{D} = \left(\cap_{i=1}^{m} \tilde{O}_i\right) \cap \left(\cap_{j=1}^{n} \tilde{L}_j\right) = \{w, T_{\tilde{D}}(w), C_{\tilde{D}}(w), G_{\tilde{D}}(w), U_{\tilde{D}}(w), F_{\tilde{D}}(w))\}$$

Where $w \in W$.

$$T_{\tilde{D}}(w) = \min\{T_{\tilde{O}_1}(w), T_{\tilde{O}_2}(w), \dots, T_{\tilde{O}_m}(w); T_{\tilde{L}_1}(w), T_{\tilde{L}_2}(w), \dots, T_{\tilde{L}_n}(w)\} = A$$
$$C_{\tilde{D}}(w) = \min\{C_{\tilde{O}_1}(w), C_{\tilde{O}_2}(w), \dots, C_{\tilde{O}_m}(w); C_{\tilde{L}_1}(w), C_{\tilde{L}_2}(w), \dots, C_{\tilde{L}_n}(w)\} = B$$
$$G_{\tilde{D}}(w) = \max\{G_{\tilde{O}_1}(w), G_{\tilde{O}_2}(w), \dots, G_{\tilde{O}_m}(w); G_{\tilde{L}_1}(w), G_{\tilde{L}_2}(w), \dots, G_{\tilde{L}_n}(w)\} = C$$
$$U_{\tilde{D}}(w) = \max\{U_{\tilde{O}_1}(w), U_{\tilde{O}_2}(w), \dots, U_{\tilde{O}_m}(w); U_{\tilde{L}_1}(w), U_{\tilde{L}_2}(w), \dots, U_{\tilde{L}_n}(w)\} = D$$
$$F_{\tilde{D}}(w) = \max\{F_{\tilde{O}_1}(w), F_{\tilde{O}_2}(w), \dots, F_{\tilde{O}_m}(w); F_{\tilde{L}_1}(w), F_{\tilde{L}_2}(w), \dots, F_{\tilde{L}_n}(w)\} = E$$

Where $T_{\tilde{D}}, C_{\tilde{D}}, G_{\tilde{D}}$, $U_{\tilde{D}}$ and $F_{\tilde{D}}$ presents the truth, contradiction, ignorance, unknown and falsity degree of membership of penta-partitioned neutrosophic decision set, respectively. Now using PNO, the above problem (1) is reformulated into a MOOP as:

$Max\ A, \quad Max\ B, \quad Min\ C, \quad Min\ D, \quad Min\ E.$

Subject to,

$$T_{\tilde{O}_i}(w) \ge A, \quad T_{\tilde{L}_j}(w) \ge A$$
$$C_{\tilde{O}_i}(w) \ge B, \quad C_{\tilde{L}_j}(w) \ge B$$
$$G_{\tilde{O}_i}(w) \le C, \quad G_{\tilde{L}_j}(w) \le C$$
$$U_{\tilde{O}_i}(w) \le D, \quad U_{\tilde{L}_j}(w) \le D$$
$$F_{\tilde{O}_i}(w) \le E, \quad F_{\tilde{L}_j}(w) \le E$$
$$A \ge B, A \ge C, A \ge D, A \ge E$$
$$0 \le A + B + C + D + E \le 5$$
$$A, B, C, D, E \in [0,1], \ i = 1, \dots, m$$
$$f_j(w) \le b_j, \quad w \ge 0, \quad j = 1, \dots, n. \qquad (2)$$

Computational method:

Step 1: Each objective function is solved individually ignoring the others subject to the constraints.

Step 2: Determine the value of other objective functions at the point where the best value of the individual objective function occurs.

Step 3: Using above two steps, construct pay-off matrix:

$$\begin{bmatrix} Z_1^*(w_1) & Z_2(w_1) & \cdots & Z_m(w_1) \\ Z_1(w_2) & Z_2^*(w_2) & \cdots & Z_m(w_2) \\ \vdots & \vdots & \ddots & \vdots \\ Z_1(w_m) & Z_2(w_m) & \cdots & Z_m^*(w_m) \end{bmatrix}.$$

Step 4: Find lower bound L_m^T, upper bound U_m^T of truth membership of each $Z_m(w)$ by,

$\mathrm{U}_m^T = max\{Z_m(w_i)\}$ and $\mathrm{L}_m^T = min\{Z_m(w_i)\}$, $i = 1,2, \dots, m$.

Lower bound L_m^C and upper bound U_m^C for contradiction membership of objective functions $Z_m(w)$ are,

$$\mathrm{L}_m^C = \mathrm{L}_m^T \quad \text{and} \quad \mathrm{U}_m^C = \mathrm{L}_m^T + q_m(\mathrm{U}_m^T - \mathrm{L}_m^T),$$

lower bound L_m^G , Upper bound U_m^G for Ignorance membership of objectives $Z_m(w)$ are

$$\mathrm{U}_m^G = \mathrm{U}_m^T \quad \text{and} \quad \mathrm{L}_m^G = \mathrm{L}_m^T + r_m(\mathrm{U}_m^T - \mathrm{L}_m^T),$$

The upper bounds U_m^U and lower bounds L_m^U for unknown membership function of objectives are,

$$\mathrm{U}_m^U = \mathrm{U}_m^T \quad \text{and} \quad \mathrm{L}_m^U = \mathrm{L}_m^T + s_m(U_m^T - \mathrm{L}_m^T),$$

The upper bounds U_m^F and lower bounds L_m^F of falsity membership function of objectives are,

$$\mathrm{U}_m^F = \mathrm{U}_m^T \quad \text{and} \quad \mathrm{L}_m^F = \mathrm{L}_m^T + t_m(U_m^T - \mathrm{L}_m^T),$$

where $q_m, r_m, s_m, t_m \in (0,1)$.

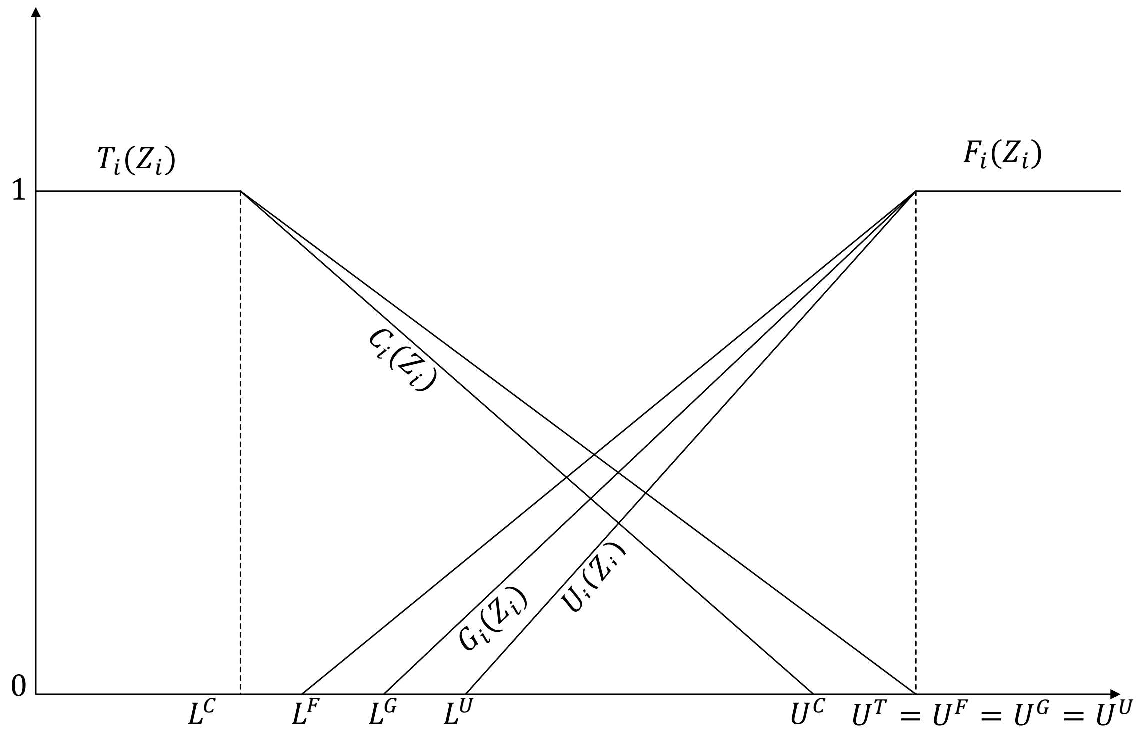

Figure1. membership functions of the objective functions

Step 5: In this step, truth, contradiction, ignorance, unknown, falsity membership functions are:

$$T_m(Z_m(w)) = \begin{cases} 1 & Z_m(w) \leq \mathrm{L}_m^T \\ \dfrac{\mathrm{U}_m^T - Z_m(w)}{\mathrm{U}_m^T - \mathrm{L}_m^T} & \mathrm{L}_m^T \leq Z_m(w) \leq \mathrm{U}_m^T \\ 0 & Z_m(w) \geq \mathrm{U}_m^T \end{cases}$$

$$C_m(Z_m(w)) = \begin{cases} 1 & Z_m(w) \leq \mathrm{L}_m^C \\ \dfrac{\mathrm{U}_m^C - Z_m(w)}{\mathrm{U}_m^U - \mathrm{L}_m^C} & \mathrm{L}_m^C \leq Z_m(w) \leq \mathrm{U}_m^C \\ 0 & Z_m(w) \geq \mathrm{U}_m^U \end{cases}$$

$$G_m(Z_m(w)) = \begin{cases} 0 & Z_m(w) \leq \mathrm{L}_m^G \\ \dfrac{Z_m(w) - \mathrm{L}_m^G}{\mathrm{U}_m^G - \mathrm{L}_m^G} & \mathrm{L}_m^G \leq Z_m(w) \leq \mathrm{U}_m^G \\ 1 & Z_m(w) \geq \mathrm{U}_m^G \end{cases}$$

$$U_m\big(Z_m(w)\big) = \begin{cases} 0 & Z_m(w) \leq \mathrm{L}_m^U \\ \dfrac{Z_m(w) - \mathrm{L}_m^U}{\mathrm{U}_m^U - \mathrm{L}_m^U} & \mathrm{L}_m^U \leq Z_m(w) \leq \mathrm{U}_m^U \\ 1 & Z_m(w) \geq \mathrm{U}_m^U \end{cases}$$

$$F_m(Z_m(w)) = \begin{cases} 0 & Z_m(w) \leq \mathrm{L}_m^F \\ \dfrac{Z_m(w) - \mathrm{L}_m^F}{\mathrm{U}_m^F - \mathrm{L}_m^F} & \mathrm{L}_m^F \leq Z_m(w) \leq \mathrm{U}_m^F \\ 1 & Z_m(w) \geq \mathrm{U}_m^F \end{cases}$$

Step 6: Now PNO method for MOOP is presented by max-min method as:

Max $(A + B - C - D - E)$,

Subject to

$$T_m\big(Z_m(w)\big) \geq A$$
$$C_m\big(Z_m(w)\big) \geq B$$
$$G_m\big(Z_m(w)\big) \leq C$$
$$U_m\big(Z_m(w)\big) \leq D$$
$$F_m\big(Z_m(w)\big) \leq E$$

$f_j(w) \leq b_j$, $w \geq$

0, $j = 1, \ldots, n$

with, $0 \leq A + B + C + D + E \leq 5$,

$A \geq B, A \geq C, A \geq D, A \geq E$, $A, B, C, D, E \in [0,1]$

(3)

This equivalent to:

Max $(A + B - C - D - E)$

Subject to

$$Z_m(w) + (\mathrm{U}_m^T - \mathrm{L}_m^T) \cdot A \le \mathrm{U}_m^T$$
$$Z_m(w) + (\mathrm{U}_m^C - \mathrm{L}_m^C) \cdot B \le \mathrm{U}_m^C$$
$$Z_m(w) - (\mathrm{U}_m^G - \mathrm{L}_m^G) \cdot C \le \mathrm{L}_m^G$$
$$Z_m(w) - (\mathrm{U}_m^U - \mathrm{L}_m^U) \cdot D \le \mathrm{L}_m^U$$
$$Z_m(w) - (\mathrm{U}_m^F - \mathrm{L}_m^F) \cdot E \le \mathrm{L}_m^F$$
$$f_j(w) \le b_j, \quad w \ge 0, \quad j = 1, \dots, n$$

For all m objectives

$0 \le A + B + C + D + E \le 5,$ $\quad A \ge B, A \ge C, A \ge D, A \ge E$ $\quad A, B, C, D, E \in [0,1]$

(4)

4. Numerical example [14]

Consider the following MOOP:

$$\mathrm{Min}\, Z_1(x_1, x_2) = x_1^{-1} x_2^{-2},$$
$$\mathrm{Min}\, Z_2(x_1, x_2) = 2x_1^{-2} x_2^{-3},$$

Subject to $\quad x_1 + x_2 \le 1.$

$$x_1, x_2 \ge 0 \qquad (5)$$

	Z_1	Z_2
X^1	6.75	60.78
X^2	6.94	57.87

Step 1: Solving the above objective functions individually ignoring other objective subject to the constraint, we get the optimal values $Z_1^*(X^1) = 6.75$ at the point $X^1 = (.33, .67)$ and $Z_2^*(X^2) = 57.87$ at the point $X^2 = (.4, .6)$.

Step 2: At the point of optimal the values of other objectives have calculated. Here $Z_1(X^2) = 6.94$ and $Z_2(X^1) = 60.78$.

Step 3: The pay-off matrix is:

Step 4: Calculate upper and lower bound of membership functions corresponding to each objective function: $\mathrm{L}_1^T = 6.75,\ \mathrm{U}_1^T = 6.94$

$$\mathrm{L}_1^C = 6.75, \quad \mathrm{U}_1^C = 6.75 + 0.19 \times q_1 = 6.902$$
$$\mathrm{L}_1^G = 6.75 + 0.19 \times r_1 = 6.7975, \quad \mathrm{U}_1^G = 6.94$$
$$\mathrm{L}_1^U = 6.75 + 0.19 \times s_1 = 6.807, \quad \mathrm{U}_1^U = 6.94$$
$$\mathrm{L}_1^F = 6.75 + 0.19 \times t_1 = 6.788, \quad \mathrm{U}_1^F = 6.94$$

$$\mathrm{L}_2^T = 57.87, \ \mathrm{U}_2^T = 60.78$$
$$\mathrm{L}_2^C = 57.87, \quad \mathrm{U}_2^C = 57.87 + 2.91 \times q_2 = 60.489$$
$$\mathrm{L}_2^G = 57.87 + 2.91 \times r_2 = 58.3065, \quad \mathrm{U}_2^G = 60.78$$
$$\mathrm{L}_2^U = 57.87 + 2.91 \times s_2 = 58.452, \quad \mathrm{U}_2^U = 60.78$$
$$\mathrm{L}_2^F = 57.87 + 2.91 \times t_2 = 58.161, \quad \mathrm{U}_2^F = 60.78$$

where $q_1 = 0.800,\ r_1 = 0.250,\ s_1 = 0.300,\ t_1 = 0.200,\ q_2 = 0.900,\ r_2 = 0.150,\ s_2 = 0.200,\ t_2 = 0.100$.

Step 5: Now, membership functions of $T, C, U, G, and\ F$ can be defined as:

$$T_1(x_1{}^{-1}x_2^{-2}) = \begin{cases} 1 & x_1^{-1}x_2^{-2} \le 6.75 \\ \dfrac{6.94 - x_1^{-1}x_2^{-2}}{6.94 - 6.75} & 6.75 \le x_1^{-1}x_2^{-2} \le 6.94 \\ 0 & x_1^{-1}x_2^{-2} \ge 6.94 \end{cases}$$

$$T_2(2x_1^{-2}x_2^{-3}) = \begin{cases} 1 & 2x_1^{-2}x_2^{-3} \le 57.87 \\ \dfrac{60.78 - 2x_1^{-2}x_2^{-3}}{60.78 - 57.87} & 57.87 \le 2x_1^{-2}x_2^{-3} \le 60.78 \\ 0 & x_1^{-1}x_2{}^{-2} \ge 60.78 \end{cases}$$

$$C_1(x_1{}^{-1}x_2^{-2}) = \begin{cases} 1 & x_1^{-1}x_2^{-2} \le 6.75 \\ \dfrac{6.902 - x_1^{-1}x_2^{-2}}{6.94 - 6.75} & 6.75 \le x_1^{-1}x_2^{-2} \le 6.902 \\ 0 & x_1^{-1}x_2^{-2} \ge 6.902 \end{cases}$$

$$C_2(2x_1^{-2}x_2^{-3}) = \begin{cases} 1 & 2x_1^{-2}x_2^{-3} \le 57.87 \\ \dfrac{60.489 - 2x_1^{-2}x_2^{-3}}{60.489 - 57.87} & 57.87 \le 2x_1^{-2}x_2^{-3} \le 60.489 \\ 0 & x_1^{-1}x_2{}^{-2} \ge 60.78 \end{cases}$$

$$G_1(x_1{}^{-1}x_2^{-2}) = \begin{cases} 0 & x_1{}^{-1}x_2^{-2} \le 6.7975 \\ \dfrac{x_1{}^{-1}x_2^{-2} - 6.7975}{6.94 - 6.7975} & 6.7975 \le x_1{}^{-1}x_2^{-2} \le 6.94 \\ 1 & x_1{}^{-1}x_2^{-2} \ge 6.94 \end{cases}$$

$$G_2\left(2x_1{}^{-2}x_2^{-3}\right) = \begin{cases} 0 & 2x_1^{-2}x_2^{-3} \le 58.3065 \\ \dfrac{2x_1^{-2}x_2^{-3} - 58.3065}{60.78 - 58.3065} & 58.3065 \le 2x_1^{-2}x_2^{-3} \le 60.78 \\ 1 & 2_1^{-2}x_2^{-3} \ge 60.78 \end{cases}$$

$$U_1(x_1{}^{-1}x_2^{-2}) = \begin{cases} 0 & x_1{}^{-1}x_2^{-2} \le 6.807 \\ \dfrac{x_1{}^{-1}x_2^{-2} - 6.807}{6.94 - 6.807} & 6.807 \le x_1{}^{-1}x_2^{-2} \le 6.94 \\ 1 & x_1{}^{-1}x_2^{-2} \ge 6.94 \end{cases}$$

$$U_2(2x_1{}^{-2}x_2^{-3}) = \begin{cases} 0 & 2x_1^{-2}x_2^{-3} \le 58.452 \\ \dfrac{2x_1^{-2}x_2^{-3} - 58.452}{60.78 - 58.452} & 58.452 \le 2x_1^{-2}x_2^{-3} \le 60.78 \\ 1 & 2_1^{-2}x_2^{-3} \ge 60.78 \end{cases}$$

$$F_1(x_1{}^{-1}x_2^{-2}) = \begin{cases} 0 & x_1{}^{-1}x_2^{-2} \le 6.788 \\ \dfrac{x_1{}^{-1}x_2^{-2} - 6.788}{6.94 - 6.788} & 6.788 \le x_1{}^{-1}x_2^{-2} \le 6.94 \\ 1 & x_1{}^{-1}x_2^{-2} \ge 6.94 \end{cases}$$

$$F_2(2x_1{}^{-2}x_2^{-3}) = \begin{cases} 0 & 2x_1^{-2}x_2^{-3} \le 58.161 \\ \dfrac{2x_1^{-2}x_2^{-3} - 58.161}{60.78 - 58.161} & 58.161 \le 2x_1^{-2}x_2^{-3} \le 60.78 \\ 1 & 2_1^{-2}x_2^{-3} \ge 60.78 \end{cases}$$

Step 6: The above problem in PNS is now

$$\text{Max} \;\; (A + B - C - D - E)$$

Subject to

$$x_1^{-1}x_2^{-2} + (0.19)A \le 6.94$$
$$2x_1^{-2}x_2^{-3} + (2.19)A \le 60.78$$
$$x_1^{-1}x_2^{-2} + (0.152)B \le 6.902$$
$$2x_1^{-2}x_2^{-3} + (2.619)B \le 60.489$$
$$x_1^{-1}x_2^{-2} - (0.1425)C \le 6.7975$$
$$2x_1^{-2}x_2^{-3} - (2.4735)C \le 58.3065$$
$$x_1^{-1}x_2^{-2} - (0.133)D \le 6.807$$
$$2x_1^{-2}x_2^{-3} - (2.328)D \le 58.452$$
$$x_1^{-1}x_2^{-2} - (0.152)E \le 6.788$$
$$2x_1^{-2}x_2^{-3} - (2.619)E \le 58.161$$
$$x_1 + x_2 \le 1$$
$$0 \le A, B, C, D, E \le 1 \;\; and \;\; A \ge B, C, D, E.$$
$$x_1, x_2 \ge 0$$

(6)

The outcomes of the suggested approach and comparison with alternative approaches, IFO, NSO and FVRNO using LINGO software are shown by table 1 and table 2 in results and discussion section.

5. Application of proposed method on multi-objective reliability optimization model [35]

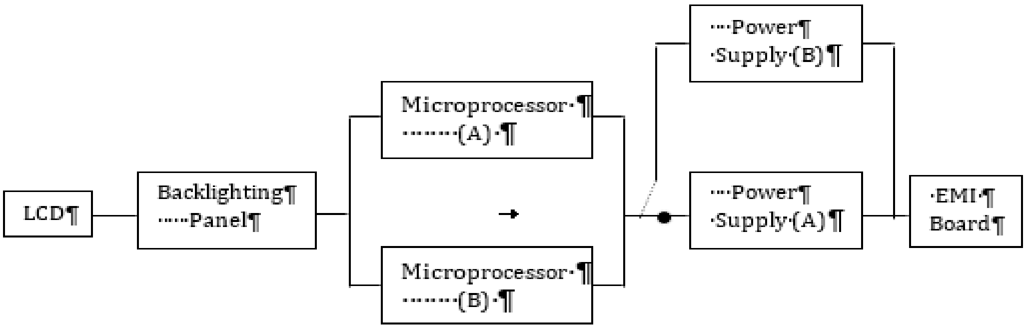

Figure2. LCD display unit

The multi-objective reliability optimization model of the LCD display unit is as follows:

$$\text{Max}\, R(\text{r}) = \text{r}_1\left(r_2^{10} + 10r_2^9(1-r_2)\right)(1-(1-r_3)^2)\left(r_4 + r_4 \ln\left(\frac{1}{r_4}\right)\right)\text{r}_5$$

$$\text{Min}\, C(\text{r}) = \sum_{j=1}^{5} c_j \left[\tan\left(\frac{\pi}{2} r_j\right)\right]^{a_j}$$

$$\text{Subject to, } V(r) = \sum_{j=1}^{5} v_j r_j^{b_j} \leq V_{max}$$

$$0.5 \leq r_j \leq 1 \qquad j = 1,2,\dots,5 \qquad (7)$$

This problem (7) is equivalent to:

Min $R'(r) = 1 - R(r)$ and Min $C(r)$, subject to same constraints as above. (8)

6. Results and discussion

Table1. results for problem (1) by sum of optimal objective values.

Optimization Methods	Optimal decision variables (x_1^*, x_2^*)	Optimal value of objectives (Z_1^*, Z_2^*)	Sum of the optimal objective values $Z = (Z_1^* + Z_2^*)$
IFO	$x_1^* = .3659009$, $x_2^* = .6356811$	$Z_1^* = 6.797078$ $Z_2^* = 58.79110$	$Z = 65.588178$
NSO	$x_1^* = .3635224$, $x_2^* = .6364776$	$Z_1^* = 6.790513$ $Z_2^* = 58.68732$	$Z = 65.477833$
FVRNO	$x_1^* = .365902$, $x_2^* = .634098$	$Z_1^* = 6.797081071$ $Z_2^* = 58.59104971$	$Z = 65.3881308$
PNO (proposed method)	$x_1^* = .3688571$, $x_2^* = .6311429$	$Z_1^* = 6.8059139$ $Z_2^* = 58.4696639$	$Z = 65.2755778$

In Table (1), we have shown that sum of the optimal objective values by IFO is 65.588178, by NSO is 65.477833, by FVRNO is 65.3881308. Here by the proposed method, the same is 65.2755778. since the both objectives of problem (5) are minimization type, we can conclude the proposed method is better.

Percentage gap= $\left|\frac{Achieved\ \ value-Best\ \ Value}{Achieved\ \ value}\right| \times 100\%$

Table 2. results for problem (1) by percentage gap.

Optimization Methods	Percentage gap of Z_1^*	Percentage gap of Z_2^*	Total percentage gap
IFO	0.0965856	0.546742789	0.643328389
NSO	0	0.370874151	0.370874151
FVRNO	0.0966308	0.207174663	0.303805463
PNO	0.2262870	0	0.2262870

From Table 2, we have shown that the total percentage gap by IFO, NSO, FVRNO and PNO are 0.643328389, 0.370874151, 0.303805463, 0.2262870 respectively. So, the developed method is better in the view of percentage gap. Graphical presentation of the results of problem (5) by total optimal values of objectives and total percentage gap are presented in Figure 3 and Figure 4.

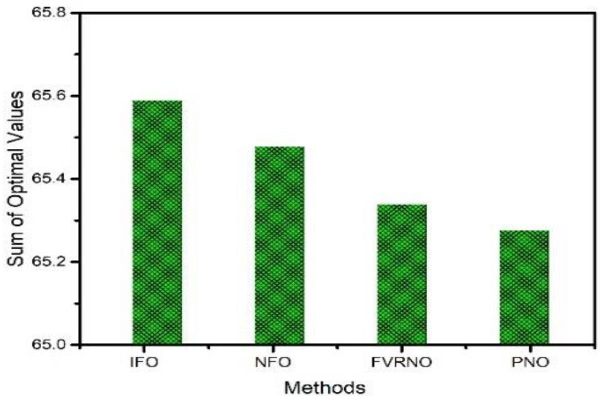

Figure 3. Comparison of the developed method with other by total optimal values

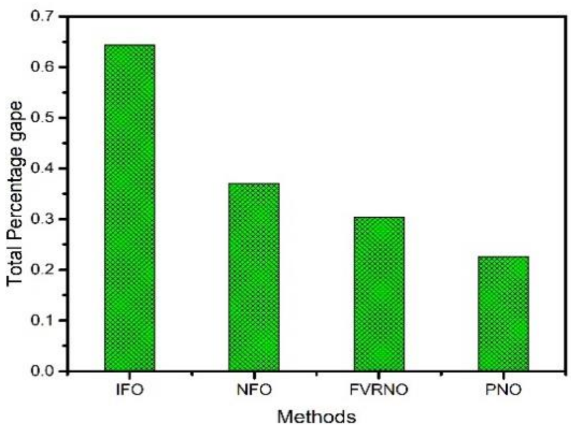

Figure 4. Comparison of the developed method with other by total percentage gap

Table 3. Data used for the reliability optimization model.

c_1	c_2	c_3	c_4	c_5	v_1	v_2	v_3	v_4	v_5	$a_j(\forall j)$	$b_j(\forall j)$	$V_{\max}$
28	30	32	35	29	6	4.5	3.75	3.5	7	0.4	1	24

Pay-off matrix is:

	R'	C
R^1	0.01478132	5196.368
R^2	0.9982949	154

Table 4. Optimal solutions by FVRNO and PNO methods

Methods	r_1	r_2	r_3	r_4	r_5	R^*	C^*
FVRNO	0.9762522	0.9809396	0.9096640	0.8776727	0.9756593	0.923499	470.4295
PNO	0.9725617	0.9791286	0.9015574	0.8669358	0.9718783	0.91108	449.5225

Table 5. Efficiency of the proposed method by total percentage gap

Methods	percentage gap of R^*	percentage gap of C^*	Total percentage gap
FVRNO	0	4.4442366	4.442366
PNO	1.36310752	0	1.36310752

In Table 4the outcomes of the suggested approach (PNO) and FVRNO solving the problem (7) are shown. The comparison of the result is presented by percentage gap in table 5. From table 5, we can conclude that the result obtained by PNO method is better than FVRNO. The graphical presentation of the result of percentage gap is presented by Figure 5.

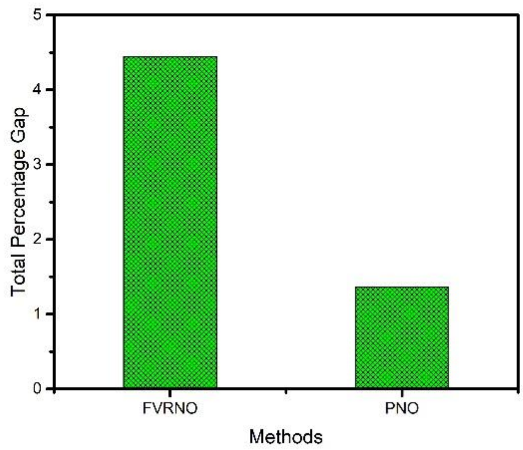

Figure 5. Comparison of the proposed method with other by total percentage gap

7. Conclusion and future directions

Using a penta-partitioned neutrosophic fuzzy environment, we have suggested a new computational approach in this article. A well-known example is solved to show the efficiency of developed method and the results are compared with other existing methods such as IFO, NFO, FVRNO by sum of optimal values and total percentage gap in table 1 and table 2. We have also applied this method to solve multi objective reliability optimization model (LCD display unit) by maximizing the system reliability and minimizing system cost and the results are compared with FVRNO by total percentage gap in table 5. We could deduce from the results that the suggested approach is effective and more flexible than those already in use.

In future, we can apply this method to inventory model, transportation problem, portfolio selection model etc. considering various fuzzy parameters.

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