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Some Results for Compatible maps on Neutrosophic Metric Spaces

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Abstract. The neutrosophic theory has been effectively used to address uncertainty and ambiguity. Neutrosophic Metric Space (NMS) was introduced by Krisci and Simsek in 2020. Following that, several kinds of compatible maps and their characteristics were investigated in the context of Intuitionistic fuzzy metric spaces and fuzzy metric spaces. In this paper, the author introduce the notion of compatible maps of type (α) and type (β) in neutrosophic metric space. For this purpose, four non-comparable mappings are used to prove the basic results. Furthermore, we prove several common fixed points results for compatible maps of type (α) and type (β) in neutrosophic metric space and provide a non-trivial examples.

Keywords: Fixed point; Neutrosophic metric Space; Compatible maps.

1. Introduction

The concept of metric spaces and the Banach contraction principle serve as the foundation of fixed point theory. The openness of metric space attracts a huge number of academics to the axiomatic interpretation. Following Zadeh's [29] introduction of the idea of fuzzy sets (FSs), many academics offered a variety of generalisations for classical structures. The idea of Fuzzy Metric Space (FMS) was first put forth in 1975 by Kramosil and Michalek [14]. Later, George and Veeramani [6] redefined the concept of FMS. Following then, several researchers looked at the FMS characteristics and produced numerous fixed point results. Intuitionistic Fuzzy Sets(IFSs) was introduced by Atanassov [1] with the concept of non - membership to

FSs. Park [22] defined Intuitionistic Fuzzy Metric Space (IFMS) from the concept of IFSs and given some fixed point results. In FMS and IFMS various fixed point theorems has been proved by Alaca et al [2]. Grabiec [20] gave fuzzy interpretation of Banach and Edelstein fixed point theorems in the sense of Kramosil and Michalek. Weakly commuting maps in metric spaces were first proposed by Sessa [24], who started the trend of enhancing commutativity in fixed point theorems. Jungck [24] soon enlarged this concept to compatible maps. Smarandache [25, 26] established the new idea called Neutrosophic logic and Neutrosophic Set (NS) in 1998. In general, the ideas of FS and IFS deal with degrees of membership and non-membership, respectively. By incorporating a degree of indeterminacy, the neutrosophic set generalises fuzzy and intuitionistic fuzzy sets. Hence several researchers have made studies on the concept of neutrosophic set. Parimala Mani et al. [8, 9]obtained decision making applications form Neutrosophic Support Soft Topological Spaces. Sahin et al. [23]studied adequacy of online education using Hausdorff Measures based on neutrosophic quadruple sets. Recently, Sahin and Kargin [19] obtained neutrosophic triplet metric spaces and neutrosophic triplet normed spaces. Kirisci and Simsek [15] established the concept of neutrosophic metric spaces (NMSs) that deals with membership, non-membership and naturalness functions and derived various fixed point theorems for neutrosophic metric space. Sowndrarajan and Jeyaraman et al. [12,27] studied Banach and Edelstein contraction fixed point results for neutrosophic metric space.
In this manuscript, we introduce the notion of compatible maps of type (α) and type (β) in neutrosophic metric space. We also establish fixed point results by using four mappings and obtain a non trivial example

2. **Preliminaries**

Definition 2.1 [26] Let Σ be a non-empty fixed set. A Neutrosophic Set N in Σ is a collection of elements in the form $N = \{\langle a, \xi_N(a), \varrho_N(a), \nu_N(a)\rangle : a \in \Sigma\}$ where the functions $\xi_N(a), \varrho_N(a)$ and $\nu_N(a)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element a $\in$ N to the set Σ.

Definition 2.2 [10] A continuous t - norm (CTN) is a function $\star : [0,1] \times [0,1] \to [0,1]$ that satisfies the following conditions;
For all $\varrho_1, \varrho_2, \varrho_3, \varrho_4 \in$ [0, 1]
(i) $\varrho_1 \star 1 = \varrho_1$;
(ii) If $\varrho_1 \leq \varrho_3$ and $\varrho_2 \leq \varrho_4$ then $\varrho_1 \star \varrho_2 \leq \varrho_3 \star \varrho_4$;
(iii) $\star$ is continuous;
(iv) $\star$ is commutative and associative.

Definition 2.3 [10] A continuous t - co norm (CTC) is a function $\diamond : [0,1] \times [0,1] \to [0,1]$ that satisfies the following conditions;

For all $\varrho_1, \varrho_2, \varrho_3, \varrho_4 \in [0, 1]$

(i) $\varrho_1 \diamond 0 = \varrho_1$;

(ii) If $\varrho_1 \leq \varrho_3$ and $\varrho_2 \leq \varrho_4$ then $\varrho_1 \diamond \varrho_2 \leq \varrho_3 \diamond \varrho_4$;

(iii) $\diamond$ is continuous;

(iv) $\diamond$ is commutative and associative.

3. Neutrosophic Metric Spaces

In this section, we define basic concepts of neutrosophic metric space and prove various properties of the space with suitable examples.

Definition 3.1 [27] A 6 - tuple $(\Sigma, \Lambda, \aleph, \daleth, \star, \diamond)$ is called Neutrosophic Metric Space(NMS), if Σ is an arbitrary non empty set, $\star$ is a neutrosophic CTN, $\diamond$ is a neutrosophic CTC and $\Lambda, \aleph, \daleth$ are neutrosophic sets on $\Sigma^2 \times \mathbb{R}^+$ satisfying the following conditions:

For all $\varrho, \varsigma, \omega \in \Sigma, \vartheta \in \mathbb{R}^+$

(i) $0 \leq \Lambda(\varrho, \varsigma, \vartheta) \leq 1;\ 0 \leq \aleph(\varrho, \varsigma, \vartheta) \leq 1;\ 0 \leq \daleth(\varrho, \varsigma, \vartheta) \leq 1;$

(ii) $\Lambda(\varrho, \varsigma, \vartheta) + \aleph(\varrho, \varsigma, \vartheta) + \daleth(\varrho, \varsigma, \vartheta) \leq 3;$

(iii) $\Lambda(\varrho, \varsigma, \vartheta) = 1$ if and only if $\varrho = \varsigma$;

(iv) $\Lambda(\varrho, \varsigma, \vartheta) = \Lambda(\varsigma, \varrho, \vartheta)$ for $\vartheta > 0$;

(v) $\Lambda(\varrho, \varsigma, \vartheta) \star \Lambda(\varsigma, \varrho, \mu) \leq \Lambda(\varrho, \omega, \vartheta + \mu)$, for all $\vartheta, \mu > 0$;

(vi) $\Lambda(\varrho, \varsigma, .) : [0, \infty) \to [0, 1]$ is neutrosophic continuous ;

(vii) $lim_{\vartheta \to \infty} \Lambda(\varrho, \varsigma, \vartheta) = 1$ for all $\vartheta > 0$;

(viii) $\aleph(\varrho, \varsigma, \vartheta) = 0$ if and only if $\varrho = \varsigma$;

(ix) $\aleph(\varrho, \varsigma, \vartheta) = \aleph(\varsigma, \varrho, \vartheta)$ for $\vartheta > 0$;

(x) $\aleph(\varrho, \varsigma, \vartheta) \diamond \aleph(\varrho, \omega, \mu) \geq \aleph(\varrho, \omega, \vartheta + \mu)$, for all $\vartheta, \mu > 0$;

(xi) $\aleph(\varrho, \varsigma, .) : [0, \infty) \to [0, 1]$ is neutrosophic continuous ;

(xii) $lim_{\vartheta \to \infty} \aleph(\varrho, \varsigma, \vartheta) = 0$ for all $\vartheta > 0$;

(xiii) $\daleth(\varrho, \varsigma, \vartheta) = 0$ if and only if $\varrho = \varsigma$;

(xiv) $\daleth(\varrho, \varsigma, \vartheta) = \daleth(\varsigma, \varrho, \vartheta)$ for $\vartheta > 0$;

(xv) $\daleth(\varrho, \varsigma, \vartheta) \diamond \daleth(\varrho, \omega, \mu) \geq \daleth(\varrho, \omega, \vartheta + \mu)$, for all $\vartheta, \mu > 0$;

(xvi) $\daleth(\varrho, \varsigma, .) : [0, \infty) \to [0, 1]$ is neutrosophic continuous ;

(xvii) $lim_{\vartheta \to \infty} \daleth(\varrho, \varsigma, \vartheta) = 0$ for all $\vartheta > 0$;

(xviii) If $\vartheta > 0$ then $\Lambda(\varrho, \varsigma, \vartheta) = 0, \aleph(\varrho, \varsigma, \vartheta) = 1, \daleth(\varrho, \varsigma, \vartheta) = 1$.

Then $(\Lambda, \aleph, \daleth)$ is called neutrosophic metric on Σ. The functons $\Lambda, \aleph$ and $\daleth$ denote degree of closedness, neturalness and non - closedness between ϱ and ς with respect to ϑ respectively.

Example 3.2 [27] Let (Σ, d) be a metric space. $\Lambda, \aleph, \beth : \Sigma^2 \times \mathbb{R}^+ \to [0,1]$ defined by

$$\Lambda(\varrho,\varsigma,\vartheta) = \frac{\vartheta}{\vartheta + d(\varrho,\varsigma)}; \quad \aleph(\varrho,\varsigma,\vartheta) = \frac{d(\varrho,\varsigma)}{\vartheta + d(\varrho,\varsigma)}; \quad \beth(\varrho,\varsigma,\vartheta) = \frac{d(\varrho,\varsigma)}{\vartheta}$$

for all $\varrho, \varsigma \in \Sigma$ and $\vartheta > 0$. where $\varrho \star \varsigma = min\{\varrho, \varsigma\}$ and $\varrho \diamond \varsigma = max\{\varrho, \varsigma\}$. Then $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ is called NMS induced by a standard neutrosophic metric.

Definition 3.3 Let $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ be neutrosophic metric space. Then

(a) $\{\varrho_n\}$ in Σ is converging to a point $\varrho \in \Sigma$ if for each $\vartheta > 0$

$$lim_{n\to\infty}\Lambda(\varrho_n, \varrho, \vartheta) = 1; \quad lim_{n\to\infty}\aleph(\varrho_n, \varrho, \vartheta) = 0; \quad lim_{n\to\infty}\beth(\varrho_n, \varrho, \vartheta) = 0.$$

(b) $\{\varrho_n\}$ in Σ is called a Cauchy if for each $\epsilon > 0$ and $\vartheta > 0$ there exist $n \in \mathbb{N}$ such that $\Lambda(\varrho_{n+p}, \varrho_n, \vartheta) = 1$; $\aleph(\varrho_{n+p}, \varrho_n, \vartheta) = 0$; $\beth(\varrho_{n+p}, \varrho_n, \vartheta) = 0$.

(c) $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ is said to be complete NMS if every Cauchy sequence is convergence in it.

Lemma 3.4 Let $\{\varrho_n\}$ be a sequence in a NMS $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$. If there exist a number $k \in (0, 1)$such that for all $\varrho, \varsigma \in \Lambda$ and $\vartheta > 0$

$$\begin{aligned} \Lambda(\varrho_{n+2}, \varrho_{n+1}, k\vartheta) &\geq \Lambda(\varrho_{n+1}, \varrho_n, k\vartheta), \\ \aleph(\varrho_{n+2}, \varrho_{n+1}, k\vartheta) &\leq \aleph(\varrho_{n+1}, \varrho_n, k\vartheta), \\ \beth(\varrho_{n+2}, \varrho_{n+1}, k\vartheta) &\leq \beth(\varrho_{n+1}, \varrho_n, k\vartheta) \end{aligned} \tag{1}$$

for all $\vartheta > 0$ and $n = 1, 2, 3 \cdots$, then $\{\varrho_n\}$ is a Cauchy sequence in Λ

Proof. By Mathematical induction, we have

$$\begin{aligned} \Lambda(\varrho_{n+2}, \varrho_{n+1}, \vartheta) &\geq \Lambda(\varrho_2, \varrho_1, \frac{\vartheta}{k^n}), \\ \aleph(\varrho_{n+2}, \varrho_{n+1}, \vartheta) &\leq \aleph(\varrho_2, \varrho_1, \frac{\vartheta}{k^n}), \\ \beth(\varrho_{n+2}, \varrho_{n+1}, \vartheta) &\leq \beth(\varrho_2, \varrho_1, \frac{\vartheta}{k^n}) \end{aligned} \tag{2}$$

for all $\vartheta > 0$ and $n = 1, 2, \ldots$.

$$\begin{aligned} \Lambda(\varrho_n, \varrho_{n+p}, \vartheta) &\geq \Lambda(\varrho_1, \varrho_2, \frac{\vartheta}{pk^{n-1}}) \star \cdots \star \Lambda(\varrho_1, \varrho_2, \frac{\vartheta}{pk^{n+p-2}}), \\ \aleph(\varrho_n, \varrho_{n+p}, \vartheta) &\leq \aleph(\varrho_1, \varrho_2, \frac{\vartheta}{pk^{n-1}}) \diamond \cdots \diamond \aleph(\varrho_1, \varrho_2, \frac{\vartheta}{pk^{n+p-2}}), \\ \beth(\varrho_n, \varrho_{n+p}, \vartheta) &\leq \beth(\varrho_1, \varrho_2, \frac{\vartheta}{pk^{n-1}}) \diamond \cdots \diamond \beth(\varrho_1, \varrho_2, \frac{\vartheta}{pk^{n+p-2}}). \end{aligned} \tag{3}$$

Therefore, from equation(1),we have

$$\begin{aligned} \lim_{n\to\infty} \Lambda(\varrho_1, \varrho_{n+p}, \vartheta) &\geq 1 \star 1 \star \cdots \star 1 \geq 1, \\ \lim_{n\to\infty} \aleph(\varrho_1, \varrho_{n+p}, \vartheta) &\leq 0 \star 0 \diamond \cdots \diamond 0 \leq 0 \\ \lim_{n\to\infty} \beth(\varrho_1, \varrho_{n+p}, \vartheta) &\leq 0 \star 0 \diamond \cdots \diamond 0 \leq 0 \end{aligned} \tag{4}$$

which implies that $\{\varrho_n\}$ is a Cauchy sequence in Λ. □

Definition 3.5 Let Φ and Ψ be two mappings from neutrosophic metric space Σ into itself. The mappings are said to be compatible if

$$\lim_{n\to\infty} \Lambda(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 1, \ \lim_{n\to\infty} \aleph(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0, \ \lim_{n\to\infty} \beth(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0. \tag{5}$$

for all $\vartheta > 0$ whenever $\{\varrho_n\} \subset \Lambda$ such that $lim_{n\to\infty}\Phi(\varrho_n) = lim_{n\to\infty}\Psi(\varrho_n) = \varrho$ for some $\varrho \in \Sigma$.

Definition 3.6 Let Φ and Ψ be two mappings from NMS Σ into itself. The mappings are said to be compatible maps of type(α) if

$$\begin{aligned} &\lim_{n\to\infty} \Lambda(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 1 \quad and \quad \lim_{n\to\infty} \Lambda(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \vartheta) = 1, \\ &\lim_{n\to\infty} \aleph(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 0 \quad and \quad \lim_{n\to\infty} \aleph(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \vartheta) = 0, \\ &\lim_{n\to\infty} \beth(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 0 \quad and \quad \lim_{n\to\infty} \beth(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \vartheta) = 0. \end{aligned}$$

for all $\vartheta > 0$ whenever $\{\varrho_n\} \subset \Lambda$ such that $lim_{n\to\infty}\Phi(\varrho_n) = lim_{n\to\infty}\Psi(\varrho_n) = \varrho$ for some $\varrho \in \Sigma$.

Definition 3.7 Let Φ and Ψ be two mappings from NMS Σ into itself. The mappings are said to be compatible maps of type(β) if for all $\vartheta > 0$

$$\lim_{n\to\infty} \Lambda(\Phi\Phi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 1, \ \lim_{n\to\infty} \aleph(\Phi\Phi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 0, \ \lim_{n\to\infty} \beth(\Phi\Phi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 0.$$

for all $\vartheta > 0$ whenever $\{\varrho_n\} \subset \Lambda$ such that $lim_{n\to\infty}\Phi(\varrho_n) = lim_{n\to\infty}\Psi(\varrho_n) = \varrho$ for some $\varrho \in \Sigma$.

Proposition 3.8 Let Σ be a NMS and Φ, Ψ be continuous mapping from Σ into itself. Then Φ and Ψ be compatible if and only if they are compatible of type(α).

Proof: Let $\{\varrho_n\} \subset \Lambda$ such that $lim_{n\to\infty}\Phi(\varrho_n) = lim_{n\to\infty}\Psi(\varrho_n) = \varrho$ for some $\varrho \in \Lambda$. Since Φ is continuous, we have $lim_{n\to\infty}\Phi\Phi(\varrho_n) = lim_{n\to\infty}\Phi\Psi(\varrho_n) = \Phi\Psi$. Also, since Φ, Ψ are compatible,

$$\lim_{n\to\infty} \Lambda(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 1, \ \lim_{n\to\infty} \aleph(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0, \ \lim_{n\to\infty} \beth(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0.$$

for all $\vartheta > 0$. From the inequality,

$$\Lambda(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) \geq \Lambda(\Phi\Phi(\varrho_n), \Phi\Psi(\varrho_n), \frac{\vartheta}{2}) \star \Lambda(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \frac{\vartheta}{2}),$$
$$\aleph(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) \leq \aleph(\Phi\Phi(\varrho_n), \Phi\Psi(\varrho_n), \frac{\vartheta}{2}) \diamond \aleph(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \frac{\vartheta}{2}),$$
$$\beth(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) \leq \beth(\Phi\Phi(\varrho_n), \Phi\Psi(\varrho_n), \frac{\vartheta}{2}) \diamond \beth(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \frac{\vartheta}{2}).$$

Therefore,

$$\lim_{n\to\infty} \Lambda(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 1, \ \lim_{n\to\infty} \aleph(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0, \ \lim_{n\to\infty} \beth(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0.$$

Also we get,

$$\lim_{n\to\infty} \Lambda(\Psi\Psi(\varrho_n), \Phi\Psi(\varrho_n), \vartheta) = 1, \ \lim_{n\to\infty} \aleph(\Psi\Psi(\varrho_n), \Phi\Psi(\varrho_n), \vartheta) = 0, \ \lim_{n\to\infty} \beth(\Psi\Psi(\varrho_n), \Phi\Psi(\varrho_n), \vartheta) = 0.$$

Hence Φ and Ψ are compatible of type α.

Conversely, Let $\{\varrho_n\} \subset \Lambda$ such that $\lim_{n\to\infty} \Phi(\varrho_n) = \lim_{n\to\infty} \Phi(\varrho_n) = \varrho$ for some $\varrho \in \Lambda$.

Since Ψ is also continuous, we have

$$\lim_{n\to\infty} \Psi\Phi(\varrho_n) = \lim_{n\to\infty} \Psi\Psi(\varrho_n) = \Psi\varrho$$

Since Φ and Ψ are compatible of type (α), we get

$$\Lambda(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \frac{\vartheta}{2}) = \Lambda(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \frac{\vartheta}{2}) = 1,$$
$$\aleph(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \frac{\vartheta}{2}) = \aleph(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \frac{\vartheta}{2}) = 0,$$
$$\beth(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \frac{\vartheta}{2}) = \beth(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \frac{\vartheta}{2}) = 0$$

for all $\vartheta > 0$. Thus from the inequality,

$$\Lambda(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) \geq \Lambda(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \frac{\vartheta}{2}) \star \Lambda(\Psi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \frac{\vartheta}{2}),$$
$$\aleph(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) \leq \aleph(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \frac{\vartheta}{2}) \diamond \aleph(\Psi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \frac{\vartheta}{2}),$$
$$\beth(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) \leq \beth(\Phi\Phi(\varrho_n), \Phi\Psi(\varrho_n), \frac{\vartheta}{2}) \diamond \beth(\Phi\Psi(\varrho_n), \Psi\Phi(\varrho_n), \frac{\vartheta}{2})$$

Therefore

$$\lim_{n\to\infty} \Lambda(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 1, \ \lim_{n\to\infty} \aleph(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0, \ \lim_{n\to\infty} \beth(\Phi\Phi(\varrho_n), \Psi\Phi(\varrho_n), \vartheta) = 0.$$

Hence Φ and Ψ are compatible maps. □

Proposition 3.9 Let $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ be a NMS and Φ, Ψ be self mappings from Σ into itself. If Φ, Ψ are compatible maps of type (α) and $\Phi(\varrho) = \Psi(\varrho)$ for some $\varrho \in \Sigma$, then $\Phi\Psi(\varrho) = \Psi\Psi(\varrho) = \Psi\Phi(\varrho) = \Phi\Phi(\varrho)$

*Proof:*Let $\{\varrho_n\} \subset \Sigma$ defined by $\lim_{n\to\infty} \varrho_n = \varrho$ for some $\varrho \in \Sigma$ and $n = 1, 2, \dots$ and $\Phi(\varrho) = \Psi(\varrho)$. Then we have

$$\lim_{n\to\infty} \Phi(\varrho_n) = \lim_{n\to\infty} \Psi(\varrho_n) = \Phi(\varrho) = \Psi(\varrho).$$

Since, Φ, Ψ are compatible of type(α), we get

$$\begin{aligned}
\Lambda(\Phi\Psi(\varrho), \Psi\Psi(\varrho), \vartheta) &= \lim_{n\to\infty} \Lambda(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 1,\\
\aleph(\Phi\Psi(\varrho), \Psi\Psi(\varrho), \vartheta) &= \lim_{n\to\infty} \aleph(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 0,\\
\beth(\Phi\Psi(\varrho), \Psi\Psi(\varrho), \vartheta) &= \lim_{n\to\infty} \beth(\Phi\Psi(\varrho_n), \Psi\Psi(\varrho_n), \vartheta) = 0.
\end{aligned}$$

Therefore $\Phi\Psi(\varrho) = \Psi\Psi(\varrho)$. Also, we have $\Psi\Phi(\varrho) = \Phi\Phi(\varrho)$.
Since $\Phi(\varrho) = \Psi(\varrho)$, $\Psi\Psi(\varrho) = \Phi\Psi(\varrho)$. Hence $\Phi\Psi(\varrho) = \Psi\Psi(\varrho) = \Psi\Phi(\varrho) = \Phi\Phi(\varrho)$. □

Proposition 3.10 Let $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ be a NMS and Φ, Ψ be two self maps from Σ into itself. If Φ, Ψ are compatible maps of type $(\alpha)$and $\{\varrho_n\} \subset \Sigma$ such that
$\lim_{n\to\infty} \Phi(\varrho_n) = \lim_{n\to\infty} \Psi(\varrho_n) = \varrho$ for some $\varrho \in \Sigma$, then

(i) $\lim_{n\to\infty} \Psi\Phi(\varrho_n) = \Phi\varrho$ if Φ is continuous at $\varrho \in \Sigma$,

(ii) $\Phi\Psi(\varrho) = \Psi\Phi(\varrho)$ and $\Phi(\varrho) = \Psi(\varrho)$ if Φ, Ψ are continuous at $\varrho \in \Sigma$.

Proof:(i) Since Φ is continuous at ϱ and $\lim_{n\to\infty} \Phi(\varrho_n) = \varrho$, $\lim_{n\to\infty} \Phi\Phi(\varrho_n) = \Phi\varrho$. Also we have Φ, Ψ are compatible maps of ype (α), Then

$$\lim_{n\to\infty} \Lambda(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \vartheta) = 1,\ \lim_{n\to\infty} \aleph(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \vartheta) = 0,\ \lim_{n\to\infty} \beth(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \vartheta) = 0.$$

for all $\vartheta > 0$. From the definition (3.1),

$$\begin{aligned}
\lim_{n\to\infty} \Lambda(\Psi\Phi(\varrho_n), \Phi(\varrho), \vartheta) &\geq \lim_{n\to\infty} \Lambda(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \frac{\vartheta}{2}) \star \Lambda(\Phi\Phi(\varrho_n), \Phi(\varrho), \frac{\vartheta}{2}) \geq 1,\\
\lim_{n\to\infty} \aleph(\Psi\Phi(\varrho_n), \Phi(\varrho), \vartheta) &\leq \lim_{n\to\infty} \aleph(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \frac{\vartheta}{2}) \diamond \aleph(\Phi\Phi(\varrho_n), \Phi(\varrho), \frac{\vartheta}{2}) \leq 0,\\
\lim_{n\to\infty} \beth(\Psi\Phi(\varrho_n), \Phi(\varrho), \vartheta) &\leq \lim_{n\to\infty} \beth(\Psi\Phi(\varrho_n), \Phi\Phi(\varrho_n), \frac{\vartheta}{2}) \diamond \beth(\Phi\Phi(\varrho_n), \Phi(\varrho), \frac{\vartheta}{2}) \leq 0.
\end{aligned}$$

Hence $\lim_{n\to\infty} \Psi\Phi(\varrho_n) = \Phi(\varrho)$.
(ii) we have $\lim_{n\to\infty} \Phi(\varrho_n) = \lim_{n\to\infty} \Psi(\varrho_n) = \varrho$. and Φ, Ψ are continuous at $\varrho \in \Sigma$. From the result(i) we have, $\lim_{n\to\infty} \Phi\Psi(\varrho_n) = \Phi(\varrho)$ and $\lim_{n\to\infty} \Psi\Phi(\varrho_n) = \Psi(\varrho)$. Since the limit is always unique, so we obtain $\Phi(\varrho) = \Psi(\varrho)$. By Proposition(3.9), Hence, we prove that $\Phi\Psi(\varrho) = \Psi\Phi(\varrho)$. □

Example 3.11 Let $\Sigma = [0, \infty)$ be a metric d which is defined by $d(\varrho, \varsigma) = |\varrho - \varsigma|$, where $\star$ and $\diamond$ defined by $a \star b = min\{a, b\}, \quad a \diamond b = max\{a, b\}$. we define $(\Lambda, \aleph, \beth)$ by

$$\Lambda(\varrho, \varsigma, \vartheta) = \left(exp\left(\frac{d(\varrho, \varsigma)}{\vartheta}\right)\right)^{-1},$$

$$\aleph(\varrho, \varsigma, \vartheta) = \frac{exp\left(\frac{d(\varrho,\varsigma)}{\vartheta}\right) - 1}{exp\left(\frac{d(\varrho,\varsigma)}{\vartheta}\right)},$$

$$\beth(\varrho, \varsigma, \vartheta) = exp\left(\frac{d(\varrho, \varsigma)}{\vartheta}\right).$$

for all $\varrho, \varsigma \in \Sigma$ and $\vartheta > 0$. Then $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ is a NMS. Let Φ, Ψ be defined by

$$\Phi(\varrho) = \begin{cases} 1, & \text{if for all } \ \varrho \in [0, 1] \\ 1 + \varrho, & \text{if for all } \ \varrho \in (1, \infty) \end{cases}$$

$$\Psi(\varrho) = \begin{cases} 1, & \text{if for all } \ \varrho \in [0, 1] \\ 1 + \varrho, & \text{if for all } \ \varrho \in [0, 1) \end{cases}$$

Let $\{\varrho_n\}$ be a sequence in Σ such that $\lim_{n\to\infty} \Phi\varrho_n = \lim_{n\to\infty} \Psi\varrho_n = \omega$. From the definition of Φ, Ψ, ϱ and $\lim_{n\to\infty} \varrho_n = 0$. Since Φ, Ψ are discontinuous at $\varrho = 1$, Therefore (Φ, Ψ) are compatible maps of type (β).

4. **Main Results**

In this section, we present some interesting concepts such as compatible maps of of type (α) and type (β) in neutrosophic metric space with suitable examples. Also we prove some fixed point theorems using compatible mapping of type (α).

Theorem 4.1 Let $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ be a complete neutrosophic metric space with $\vartheta \star \vartheta \geq \vartheta$, $\vartheta \diamond \vartheta \leq \vartheta$ for all $\vartheta \in [0, 1]$ and satisfy the condition (1). Let $\Phi, \Psi, \Omega, \Lambda$ and Γ be mappings from Σ into itself such that

(i) $\Gamma(\Sigma) \subset \Phi\Psi(\Sigma)$, $\Gamma(\Sigma) \subset \Omega\Lambda(\Sigma)$;

(ii) There exists $k \in (0, 1)$ such that for all $\varrho, \varsigma \in \Sigma$, $\beta \in (0, 2)$ and $\vartheta > 0$

$$\begin{aligned}\Lambda(\Gamma\varrho, \Gamma\varsigma, k\vartheta) &\geq \Lambda(\Phi\Psi\varrho, \Gamma\varrho, \vartheta) \star \Lambda(\Omega\Gamma\varsigma, \Gamma\varsigma, \vartheta) \star \Lambda(\Omega\Gamma\varsigma, \Gamma\varrho, \beta\vartheta) \\ &\star \Lambda(\Phi\Psi\varrho, \Gamma\varsigma, (2 - \beta)\vartheta) \star \Lambda(\Phi\Psi\varrho, \Omega\Gamma\varsigma, \vartheta), \\ \aleph(\Gamma\varrho, \Gamma\varsigma, k\vartheta) &\leq \aleph(\Phi\Psi\varrho, \Gamma\varrho, \vartheta) \diamond \aleph(\Omega\Gamma\varsigma, \Gamma\varsigma, \vartheta) \diamond \aleph(\Omega\Gamma\varsigma, \Gamma\varrho, \beta\vartheta) \\ &\diamond \aleph(\Phi\Psi\varrho, \Gamma\varsigma, (2 - \beta)\vartheta) \diamond \aleph(\Phi\Psi\varrho, \Omega\Gamma\varsigma, \vartheta), \\ \beth(\Gamma\varrho, \Gamma\varsigma, k\vartheta) &\leq \beth(\Phi\Psi\varrho, \Gamma\varrho, \vartheta) \diamond \beth(\Omega\Gamma\varsigma, \Gamma\varsigma, \vartheta) \diamond \beth(\Omega\Gamma\varsigma, \Gamma\varrho, \beta\vartheta) \\ &\diamond \beth(\Phi\Psi\varrho, \Gamma\varsigma, (2 - \beta)\vartheta) \diamond \beth(\Phi\Psi\varrho, \Omega\Gamma\varsigma, \vartheta).\end{aligned}$$

(iii) $\Gamma\Psi = \Psi\Gamma$, $\Gamma\Lambda = \Lambda\Gamma$, $\Phi\Psi = \Psi\Phi$ and $\Omega\Lambda = \Lambda\Omega$,

(iv) Φ and Ψ are continuous,

(v) Γ and $\Phi\Psi$ are compatible of type (α),

(vi) $\Lambda(\varrho, \Omega\Gamma\varrho, \vartheta) \geq \Lambda(\varrho, \Phi\Psi\varrho, \vartheta)$, $\aleph(\varrho, \Omega\Gamma\varrho, \vartheta) \leq \aleph(\varrho, \Phi\Psi\varrho, \vartheta)$, $\beth(\varrho, \Omega\Gamma\varrho, \vartheta) \leq \beth(\varrho, \Phi\Psi\varrho, \vartheta)$ for all $\varrho \in \Sigma$ and $\vartheta > 0$.

Then $\Phi, \Psi, \Omega, \Lambda$ and Γ have a common fixed point in Σ.

Proof: Since $\Gamma(\Sigma) \subset \Phi\Psi(\Sigma)$ for fixed $\varrho_0 \in \Lambda$, we choose a point $\varrho_1 \in \Lambda$ such that $\Gamma\varrho_0 = \Phi\Psi\varrho_1$. Since $\Gamma(\Sigma) \subset \Omega\Lambda(\Lambda)$, we take $\varrho_2 \in \Lambda$ for this point ϱ_1 such that $\Phi\varrho_1 = \Omega\Lambda\varrho_2$. Consider a sequence $\{\varsigma_n\} \subset \Lambda$, bu mathematical induction,

$$\varsigma_{2n} = \Gamma\varrho_{2n} = \Phi\Psi\varrho_{2n+1},\ \varsigma_{2n+1} = \Gamma\varrho_{2n+1} = \Phi\Psi\varrho_{2n+2}$$

for $n = 1, 2, \ldots$. From (ii)we have

$$\begin{aligned}
\Lambda(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) = \Lambda(\Gamma\varrho_{2n+1}, \Gamma\varrho_{2n+2}, k\vartheta) &\geq \Lambda(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta) \star \Lambda(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta) \\
&\star \Lambda(\varsigma_{2n+1}, \varsigma_{2n+1}, \vartheta) \star \Lambda(\varsigma_{2n}, \varsigma_{2n+2}, (1+q)\vartheta) \\
&\star \Lambda(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta), \\
\aleph(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) = \aleph(\Gamma\varrho_{2n+1}, \Gamma\varrho_{2n+2}, k\vartheta) &\leq \aleph(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta) \diamond \aleph(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta) \\
&\diamond \aleph(\varsigma_{2n+1}, \varsigma_{2n+1}, \vartheta) \diamond \aleph(\varsigma_{2n}, \varsigma_{2n+2}, (1+q)\vartheta) \qquad (6) \\
&\diamond \aleph(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta), \\
\beth(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) = \beth(\Gamma\varrho_{2n+1}, \Gamma\varrho_{2n+2}, k\vartheta) &\leq \beth(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta) \diamond \beth(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta) \\
&\diamond \beth(\varsigma_{2n+1}, \varsigma_{2n+1}, \vartheta) \diamond \beth(\varsigma_{2n}, \varsigma_{2n+2}, (1+q)\vartheta) \\
&\diamond \beth(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta)
\end{aligned}$$

for all $\vartheta > 0$ and $\beta = 1$ - q with q $\in (0, 1)$.

Since $\star, \diamond$ are continuous also $\Lambda(\varrho, \varsigma, .), \aleph(\varrho, \varsigma, .)$ and $\beth(\varrho, \varsigma, .)$ are continuous, let $q \to 1$ in the above equation, we get

$$\begin{aligned}
\Lambda(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) &\geq \Lambda(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta) \star \Lambda(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta), \\
\aleph(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) &\leq \aleph(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta) \diamond \aleph(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta), \qquad (7) \\
\beth(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) &\leq \beth(\varsigma_{2n}, \varsigma_{2n+1}, \vartheta) \diamond \beth(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta)
\end{aligned}$$

Also we have

$$\begin{aligned}
\Lambda(\varsigma_{2n+2}, \varsigma_{2n+3}, k\vartheta) &\geq \Lambda(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta) \star \Lambda(\varsigma_{2n+2}, \varsigma_{2n+3}, \vartheta), \\
\aleph(\varsigma_{2n+2}, \varsigma_{2n+3}, k\vartheta) &\leq \aleph(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta) \diamond \aleph(\varsigma_{2n+2}, \varsigma_{2n+3}, \vartheta), \qquad (8) \\
\beth(\varsigma_{2n+2}, \varsigma_{2n+3}, k\vartheta) &\leq \beth(\varsigma_{2n+1}, \varsigma_{2n+2}, \vartheta) \diamond \beth(\varsigma_{2n+2}, \varsigma_{2n+3}, \vartheta).
\end{aligned}$$

From equation (7) and (8)

$$\Lambda(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) \geq \Lambda(\varsigma_n, \varsigma_{n+1}, \vartheta) \star \Lambda(\varsigma_{n+1}, \varsigma_{n+2}, \vartheta),$$
$$\aleph(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) \leq \aleph(\varsigma_n, \varsigma_{n+1}, \vartheta) \diamond \aleph(\varsigma_{n+1}, \varsigma_{n+2}, \vartheta),$$
$$\beth(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) \leq \beth(\varsigma_n, \varsigma_{n+1}, \vartheta) \diamond \beth(\varsigma_{n+1}, \varsigma_{n+2}, \vartheta).$$

for $n = 1, 2, \ldots$. Then for positive integers n and p,

$$\Lambda(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) \geq \Lambda(\varsigma_n, \varsigma_{n+1}, \vartheta) \star \Lambda(\varsigma_{n+1}, \varsigma_{n+2}, \frac{\vartheta}{k^p}),$$
$$\aleph(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) \leq \aleph(\varsigma_n, \varsigma_{n+1}, \vartheta) \diamond \aleph(\varsigma_{n+1}, \varsigma_{n+2}, \frac{\vartheta}{k^p}),$$
$$\beth(\varsigma_{2n+1}, \varsigma_{2n+2}, k\vartheta) \leq \beth(\varsigma_n, \varsigma_{n+1}, \vartheta) \diamond \beth(\varsigma_{n+1}, \varsigma_{n+2}, \frac{\vartheta}{k^p}).$$

Since

$$lim_{n\to\infty}\Lambda(\varsigma_{n+1}, \varsigma_{n+2}, k\vartheta) = 1,\ lim_{n\to\infty}\aleph(\varsigma_{n+1}, \varsigma_{n+2}, k\vartheta) = 0, lim_{n\to\infty}\beth(\varsigma_{n+1}, \varsigma_{n+2}, k\vartheta) = 0,$$

we have

$$\Lambda(\varsigma_{n+1}, \varsigma_{n+2}, k\vartheta) \geq \Lambda(\varsigma_n, \varsigma_{n+1}, \vartheta),$$
$$\aleph(\varsigma_{n+1}, \varsigma_{n+2}, k\vartheta) \leq \aleph(\varsigma_n, \varsigma_{n+1}, \vartheta),$$
$$\beth(\varsigma_{n+1}, \varsigma_{n+2}, k\vartheta) \leq \beth(\varsigma_n, \varsigma_{n+1}, \vartheta).$$

By lemma(3.4), Since Σ is complete, so $\{\varsigma_n\}$ is a Cauchy sequence which is converges to a point $\varrho \in \Sigma$. Also $\{\Gamma\varrho_n\}, \{\Phi\Psi\varrho_{2n+1}\}, \{\Omega\Lambda\varrho_{2n+2}\}$ are subsequences of $\{\varsigma_n\}$, $\lim_{n\to\infty} \Gamma\varrho_n = \varrho = \lim_{n\to\infty} \Phi\Psi\varrho_{2n+1} = \lim_{n\to\infty} \Omega\Lambda\varrho_{2n+2}$. Also, since Φ, Ψ are continuous and $\Gamma\Phi\Psi$ are compatible of type (α), by proposition (3.9), we have $\lim_{n\to\infty} \Gamma\Phi\Psi(\varrho_{2n+1}) = \Phi\Psi\varrho$ and $\lim_{n\to\infty}(\Phi\Psi)^2\varrho_{2n+1} = \Phi\Psi\varrho$. By(ii) with $\beta = 1$, we obtain

$$\begin{aligned}
\Lambda(\Gamma\Phi\Psi\varrho_{2n+1}, \Gamma\varrho_{2n+2}, k\vartheta) \geq{}& \Lambda((\Phi\Psi)^2\varrho_{2n+1}, \Gamma\Phi\Psi\varrho_{2n+1}, \vartheta) \star \Lambda(\Omega\Lambda\varrho_{2n+2}, \Gamma\varrho_{2n+2}, \vartheta)\\
&\star \Lambda(\Omega\Gamma\varrho_{2n+2}, \Gamma\Phi\Psi\varrho, \vartheta) \star \Lambda((\Phi\Psi)^2\varrho_{2n+1}, \Gamma\Phi\varrho_{2n+2}, \vartheta)\\
&\star \Lambda((\Phi\Psi)^2\varrho_{2n+1}, \Omega\Lambda\varrho_{2n+2}, \vartheta),\\
\aleph(\Gamma\Phi\Psi\varrho_{2n+1}, \Gamma\varrho_{2n+2}, k\vartheta) \leq{}& \aleph((\Phi\Psi)^2\varrho_{2n+1}, \Gamma\Phi\Psi\varrho_{2n+1}, \vartheta) \diamond \aleph(\Omega\Lambda\varrho_{2n+2}, \Gamma\varrho_{2n+2}, \vartheta)\\
&\diamond \aleph(\Omega\Gamma\varrho_{2n+2}, \Gamma\Phi\Psi\varrho, \vartheta) \diamond \aleph((\Phi\Psi)^2\varrho_{2n+1}, \Gamma\Phi\varrho_{2n+2}, \vartheta)\\
&\diamond \aleph((\Phi\Psi)^2\varrho_{2n+1}, \Omega\Lambda\varrho_{2n+2}, \vartheta),\\
\beth(\Gamma\Phi\Psi\varrho_{2n+1}, \Gamma\varrho_{2n+2}, k\vartheta) \leq{}& \beth((\Phi\Psi)^2\varrho_{2n+1}, \Gamma\Phi\Psi\varrho_{2n+1}, \vartheta) \diamond \beth(\Omega\Lambda\varrho_{2n+2}, \Gamma\varrho_{2n+2}, \vartheta)\\
&\diamond \beth(\Omega\Gamma\varrho_{2n+2}, \Gamma\Phi\Psi\varrho, \vartheta) \diamond \beth((\Phi\Psi)^2\varrho_{2n+1}, \Gamma\Phi\varrho_{2n+2}, \vartheta)\\
&\diamond \beth((\Phi\Psi)^2\varrho_{2n+1}, \Omega\Lambda\varrho_{2n+2}, \vartheta),
\end{aligned}$$

which implies that

$$\begin{aligned}
\Lambda(\Phi\Psi\varrho,\varrho,k\vartheta) &= lim_{n\to\infty}\Lambda(\Gamma\Phi\Psi\varrho_{2n+1},\Gamma\varrho_{2n+2},k\vartheta)\\
&\geq 1\star 1\star\Lambda(\varrho,\Phi\Psi\varrho,\vartheta)\star\Lambda(\Phi\Psi\varrho,\varrho,\vartheta)\star\Lambda(\Phi\Psi\varrho,\varrho,\vartheta),\\
\aleph(\Phi\Psi\varrho,\varrho,k\vartheta) &= lim_{n\to\infty}\aleph(\Gamma\Phi\Psi\varrho_{2n+1},\Gamma\varrho_{2n+2},k\vartheta)\\
&\leq 0\diamond 0\diamond\aleph(\varrho,\Phi\Psi\varrho,\vartheta)\diamond\aleph(\Phi\Psi\varrho,\varrho,\vartheta)\diamond\aleph(\Phi\Psi\varrho,\varrho,\vartheta),\\
\beth(\Phi\Psi\varrho,\varrho,k\vartheta) &= lim_{n\to\infty}\beth(\Gamma\Phi\Psi\varrho_{2n+1},\Gamma\varrho_{2n+2},k\vartheta)\\
&\leq 0\diamond 0\diamond\beth(\varrho,\Phi\Psi\varrho,\vartheta)\diamond\beth(\Phi\Psi\varrho,\varrho,\vartheta)\diamond\beth(\Phi\Psi\varrho,\varrho,\vartheta.
\end{aligned}$$

Hence, by lemma (3.4), $\Phi\Psi\varrho=\varrho$. Also, by(vi), since $\Lambda(\varrho,\Omega\Gamma\varrho,\vartheta)\geq\Lambda(\varrho,\Phi\Psi\varrho,\vartheta)=1$ and $\aleph(\varrho,\Omega\Gamma\varrho,\vartheta)\leq\aleph(\varrho,\Phi\Psi\varrho,\vartheta)=0$ and $\beth(\varrho,\Omega\Gamma\varrho,\vartheta)\leq\beth(\varrho,\Phi\Psi\varrho,\vartheta)=0$ for all $\vartheta>0$, we get $\Omega\Lambda\varrho=\varrho$. By(ii) with $\beta=1$, we have

$$\begin{aligned}
\Lambda(\Gamma\Phi\Psi\varrho,\Gamma\varrho,k\vartheta) &\geq \Lambda((\Phi\Psi)^2\varrho_{2n+1},\Gamma\Phi\Psi\varrho_{2n+1},\vartheta)\star\Lambda(\Omega\Lambda\varrho,\Gamma\varrho,\vartheta)\\
&\star\Lambda(\Omega\Gamma\varrho,\Gamma\Phi\Psi\varrho_{2n+1},\vartheta)\star\Lambda((\Phi\Psi)^2\varrho_{2n+1},\Gamma\varrho,\vartheta)\\
&\star\Lambda((\Phi\Psi)^2\varrho_{2n+1},\Omega\Lambda\varrho,\vartheta),\\
\aleph(\Gamma\Phi\Psi\varrho,\Gamma\varrho,k\vartheta) &\leq \aleph((\Phi\Psi)^2\varrho_{2n+1},\Gamma\Phi\Psi\varrho_{2n+1},\vartheta)\diamond\aleph(\Omega\Lambda\varrho,\Gamma\varrho,\vartheta)\\
&\diamond\Lambda(\Omega\Gamma\varrho,\Gamma\Phi\Psi\varrho_{2n+1},\vartheta)\diamond\aleph((\Phi\Psi)^2\varrho_{2n+1},\Gamma\varrho,\vartheta)\\
&\diamond\aleph((\Phi\Psi)^2\varrho_{2n+1},\Omega\Lambda\varrho,\vartheta),\\
\beth(\Gamma\Phi\Psi\varrho,\Gamma\varrho,k\vartheta) &\leq \beth((\Phi\Psi)^2\varrho_{2n+1},\Gamma\Phi\Psi\varrho_{2n+1},\vartheta)\diamond\beth(\Omega\Lambda\varrho,\Gamma\varrho,\vartheta)\\
&\diamond\beth(\Omega\Gamma\varrho,\Gamma\Phi\Psi\varrho_{2n+1},\vartheta)\diamond\beth((\Phi\Psi)^2\varrho_{2n+1},\Gamma\varrho,\vartheta)\\
&\diamond\beth((\Phi\Psi)^2\varrho_{2n+1},\Omega\Lambda\varrho,\vartheta).
\end{aligned}$$

Thus

$$\begin{aligned}
\Lambda(\Phi\Psi\varrho,\Gamma\varrho,k\vartheta) &= lim_{n\to\infty}\Lambda(\Gamma\Phi\Psi\varrho_{2n+1},\Gamma\varrho,k\vartheta)\\
&\geq 1\star 1\star 1\star\Lambda(\Phi\Psi\varrho,\Gamma\varrho,\vartheta)\star 1\\
&\geq \Lambda(\Phi\Psi\varrho,\Gamma\varrho,k\vartheta),\\
\aleph(\Phi\Psi\varrho,\Gamma\varrho,k\vartheta) &= lim_{n\to\infty}\aleph(\Gamma\Phi\Psi\varrho_{2n+1},\Gamma\varrho,k\vartheta)\\
&\leq 0\diamond 0\diamond 0\diamond\aleph(\Phi\Psi\varrho,\Gamma\varrho,\vartheta)\diamond 0\\
&\leq \aleph(\Phi\Psi\varrho,\Gamma\varrho,k\vartheta),\\
\beth(\Phi\Psi\varrho,\Gamma\varrho,k\vartheta) &= lim_{n\to\infty}\beth(\Gamma\Phi\Psi\varrho_{2n+1},\Gamma\varrho,k\vartheta)\\
&\leq 0\diamond 0\diamond 0\diamond\beth(\Phi\Psi\varrho,\Gamma\varrho,\vartheta)\diamond 0\\
&\leq \beth(\Phi\Psi\varrho,\Gamma\varrho,k\vartheta).
\end{aligned}$$

By using the by Lemma (3.4), we get $\Phi\Psi\varrho = \Gamma\varrho = \varrho$. Now we will prove that $\Psi\varrho = \varrho$. By(ii) with $\beta = 1$ and (iii), we obtain

$$\Lambda(\Psi\varrho, \varrho, k\vartheta) = \Lambda(\Psi\Gamma\varrho, \Gamma\varrho, \vartheta) = \Lambda(\Gamma\Psi\varrho, \Gamma, k\vartheta) \geq \Lambda(\Phi\Phi\varrho, \Gamma\Psi, \vartheta) \star \Lambda(\Omega\Lambda\varrho, \Gamma\varrho, \vartheta) \star \Lambda(\Omega\Gamma\varrho, \Gamma\Psi\varrho, \vartheta) \star \Lambda(\Phi\Phi\varrho, \Gamma\varrho, \vartheta) \star \Lambda(\Phi\Phi\varrho, \Omega\Gamma\varrho, \vartheta),$$

$$\aleph(\Psi\varrho, \varrho, k\vartheta) = \Lambda(\Psi\Gamma\varrho, \Gamma\varrho, \vartheta) = \aleph(\Gamma\Psi\varrho, \Gamma, k\vartheta) \leq \aleph(\Phi\Phi\varrho, \Gamma\Psi, \vartheta) \diamond \aleph(\Omega\Lambda\varrho, \Gamma\varrho, \vartheta) \diamond \aleph(\Omega\Gamma\varrho, \Gamma\Psi\varrho, \vartheta) \diamond \aleph(\Phi\Phi\varrho, \Gamma\varrho, \vartheta) \diamond \aleph(\Phi\Phi\varrho, \Omega\Gamma\varrho, \vartheta),$$

$$\beth(\Psi\varrho, \varrho, k\vartheta) = \beth(\Psi\Gamma\varrho, \Gamma\varrho, \vartheta) = \beth(\Gamma\Psi\varrho, \Gamma, k\vartheta) \leq \beth(\Phi\Phi\varrho, \Gamma\Psi, \vartheta) \diamond \beth(\Omega\Lambda\varrho, \Gamma\varrho, \vartheta) \diamond \beth(\Omega\Gamma\varrho, \Gamma\Psi\varrho, \vartheta) \diamond \beth(\Phi\Phi\varrho, \Gamma\varrho, \vartheta) \diamond \beth(\Phi\Phi\varrho, \Omega\Gamma\varrho, \vartheta).$$

Therefore, we get $\Psi\varrho = \varrho$. Since $\Phi\Psi\varrho = \varrho$, hence $\Phi\varrho = \varrho$. Next we show that $\Lambda\varrho = \varrho$. By(ii) with $\beta = 1$ and (iii), we get

$$\Lambda(\Lambda\varrho, \varrho, k\vartheta) = \Lambda(\Lambda\Gamma\varrho, \Gamma\varrho, k\vartheta) = \Lambda(\Gamma\varrho, \Lambda\Gamma\varrho, k\vartheta) = 1 \star 1 \star \Lambda(\Lambda\varrho, \varrho, \vartheta) \star \Lambda(\varrho, \Gamma\varrho, \vartheta) \star \Lambda(\varrho, \Gamma\varrho, \vartheta),$$

$$\aleph(\Lambda\varrho, \varrho, k\vartheta) = \aleph(\Lambda\Gamma\varrho, \Gamma\varrho, k\vartheta) = \aleph(\Gamma\varrho, \Lambda\Gamma\varrho, k\vartheta) = 0 \diamond 0 \diamond \aleph(\Lambda\varrho, \varrho, \vartheta) \diamond \aleph(\varrho, \Gamma\varrho, \vartheta) \diamond \aleph(\varrho, \Gamma\varrho, \vartheta),$$

$$\beth(\Lambda\varrho, \varrho, k\vartheta) = \beth(\Lambda\Gamma\varrho, \Gamma\varrho, k\vartheta) = \beth(\Gamma\varrho, \Lambda\Gamma\varrho, k\vartheta) = 0 \diamond 0 \diamond \beth(\Lambda\varrho, \varrho, \vartheta) \diamond \beth(\varrho, \Gamma\varrho, \vartheta) \diamond \beth(\varrho, \Gamma\varrho, \vartheta).$$

which implies that $\vartheta\varrho = \varrho$. Since $\Omega\Lambda\varrho = \varrho$, we have $\Omega\varrho = \Omega\Lambda\varrho = \varrho$. Hence, we get $\Phi\varrho = \Psi\varrho = \Omega\varrho = \Lambda\varrho = \Omega\varrho = \varrho$, that is ϱ is a common fixed point of $\Phi, \Psi, \Omega, \Lambda$ and Γ. Uniqueness of the fixed point ϱ follows from (ii). Hence ϱ is unique common fixed point of the five mappings $\Phi, \Psi, \Omega, \Lambda$ and Γ. □

Corollary 4.2 Let Σ be a complete neutrosophic metric space with $\vartheta \star \vartheta \geq \vartheta$, $\vartheta \diamond \vartheta \leq \vartheta$ for all $\vartheta \in [0, 1]$. Let Φ, Ψ and Γ be mappings from Σ into itself such that

(i) $\Gamma(\Sigma) \subset \Phi(\Sigma)$, $\Gamma(\Sigma) \subset \Omega(\Sigma)$;

(ii) There exists $k \in (0, 1)$ such that for all $\varrho, \varsigma \in \Sigma$, $\beta \in (0, 2)$ and $\vartheta > 0$

$$\Lambda(\Gamma\varrho, \Gamma\varsigma, k\vartheta) \geq \Lambda(\Phi\varrho, \Gamma\varrho, \vartheta) \star \Lambda(\Omega\varsigma, \Gamma\varsigma, \vartheta) \star \Lambda(\Phi\varrho, \Omega\varsigma, \beta\vartheta) \star \Lambda(\Phi\varrho, \Gamma\varsigma, (2-\beta)\vartheta) \star \Lambda(\Omega\varsigma, \Gamma\varrho, \vartheta),$$

$$\aleph(\Gamma\varrho, \Gamma\varsigma, k\vartheta) \leq \aleph(\Phi\varrho, \Gamma\varrho, \vartheta) \diamond \aleph(\Omega\varsigma, \Gamma\varsigma, \vartheta) \diamond \aleph(\Phi\varrho, \Omega\varsigma, \beta\vartheta) \diamond \aleph(\Phi\varrho, \Gamma\varsigma, (2-\beta)\vartheta) \diamond \aleph(\Omega\varsigma, \Gamma\varrho, \vartheta),$$

$$\beth(\Gamma\varrho, \Gamma\varsigma, k\vartheta) \leq \beth(\Phi\varrho, \Gamma\varrho, \vartheta) \diamond \beth(\Omega\varsigma, \Gamma\varsigma, \vartheta) \diamond \beth(\Phi\varrho, \Omega\varsigma, \beta\vartheta) \diamond \beth(\Phi\varrho, \Gamma\varsigma, (2-\beta)\vartheta) \diamond \beth(\Omega\varsigma, \Gamma\varrho, \vartheta).$$

(iii) Φ is continuous,

(iv) Γ and Φ are compatible of type (α),

(vi) $\Lambda(\varrho, \Omega\varrho, \vartheta) \geq \Lambda(\varrho, \Phi\varrho, \vartheta)$, $\aleph(\varrho, \Omega\varrho, \vartheta) \leq \aleph(\varrho, \Phi\varrho, \vartheta)$, $\beth(\varrho, \Omega\varrho, \vartheta) \leq \beth(\varrho, \Phi\varrho, \vartheta)$. for all $\varrho \in \Sigma$ and $\vartheta > 0$.

Then Φ, Ω and Γ have a common fixed point in Σ.

Proof. Suppose I_X be the identity mapping on Σ. We prove this corollary by using theorem (4.1) with $\Psi = \Gamma = I_X$. □

Example 4.3 Let $\Sigma = \{\frac{1}{n}; n \in \mathbb{N}\} \cup \{0\}$ be a metric defined by $d(\varrho, \varsigma) = |\varrho - \varsigma|$. For all $\varrho, \varsigma \in \Sigma$ and $\vartheta \in (0, \infty)$, define

$$\Lambda(\varrho, \varsigma, \vartheta) = \frac{\vartheta}{\vartheta + |\varrho - \varsigma|}; \quad \aleph(\varrho, \varsigma, \vartheta) = \frac{|\varrho - \varsigma|}{\vartheta + |\varrho - \varsigma|}; \quad \beth(\varrho, \varsigma, \vartheta) = \frac{|\varrho - \varsigma|}{\vartheta}$$

Clearly $(\Sigma, \Lambda, \aleph, \beth, \star, \diamond)$ is a complete neutrosophic metric space on Σ. Here $\star$ is defined by $\varrho \star \varsigma = min\{\varrho, \varsigma\}$ and $\diamond$ is defined as $\varrho \diamond \varsigma = max\{\varrho, \varsigma\}$ respectively.

Let $\Phi, \Psi, \Omega, \Lambda$ and Γ is defined by

$$\Phi(\varrho) = \frac{\varrho}{4},\ \Psi(\varrho) = \frac{\varrho}{6}, \Omega(\varrho) = \frac{\varrho}{2}, \Lambda(\varrho) = \frac{\varrho}{3},\ \Gamma(\varrho) = \frac{\varrho}{36}.$$

Then we have $\Gamma(\Sigma) \subset \Phi(\Sigma)$, $\Gamma(\Sigma) \subset \Omega(\Sigma)$; It is evident that $\Phi, \Psi, \Omega, \Lambda$ and Γ are continuous. Also the all conditions of Theorem(4.1) has been satisfied. Hence 0 is a unique fixed point of $\Phi, \Psi, \Omega, \Lambda$ and Γ .

5. Conclusion:

In this paper, we establish a novel concept termed Neutrosophic Metric Space (NMS) and investigate its many features. In the context of NMS, compatible maps of type (α) and type (β) definitions are defined, and various fixed point results are proven for five mappings. In addition, we provided several instances to support our findings. Additionally, neutrosophic normed space, neutrosophic triplet b-metric space, and neutrosophic triplet bipolar metric spaces can all be included in the concept of compatible mappings.

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