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Neutrosophic hybrid structures and neutrosophic hybrid matrices

Punniyamoorthy K[1], Vijayabalaji S[2,*], Raghavendra Rao A V[3] and Belide Shashidhar[4]

[1]Department of Mathematics, Rajalakshmi Engineering College (Autonomous), Thandalam, Chennai-602105, Tamilnadu, India.; drkpunniyam@gmail.com

[2]Department of Mathematics (S & H), University College of Engineering, Panruti (A Constituent College of Anna University, Chennai), Panruti-607106, Tamilnadu, India.; balaji1977harshini@gmail.com

[3]B V Raju Institute of Technology, Vishnupur, Narsapur, Medak District-502313, Telangana, India.; raghavendrarao.av@bvrit.ac.in

[4]shashidhar.belide@gmail.com

[*]Correspondence: balaji1977harshini@gmail.com

Abstract. Motivated by the theory of hybrid structures, our aim in this paper is to introduce the notion of hybrid matrices and bring out an application. Some operations on hybrid matrices are discussed. The notion of hybrid matrices is then generalized by introducing the novel idea of neutrosophic hybrid matrices. Some interesting operations and results on neutrosophic hybrid matrices are presented. As an application a multi-criteria decision making (MCDM) problem is presented together with an algorithm and example. The new method is compared with the existing one to exibit its efficiency.

Keywords: soft set; soft matrix; hybrid structure; hybrid matrix; neutrosophic hybrid structure; neutrosophic hybrid matrix

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1. Introduction

An innovative idea of soft set theory has been efficiently developed by Molodtsov [15]. This tool has wide scope of applications in several fields such as engineering, medicine, sciences and mathematical modelling. He identified that the classical and recent theories play vital role in the study of uncertainty. However with the rapidly growing quantity and type of uncertainties, these ideas have their own hurdles and drawbacks as given in Molodtsov [15]. Recent applications of soft sets, introduction to soft matrices and their developments can be viewed in the articles Çağman et al., Maji et al., Mondal et al., Vijayabalaji et al. ([5], [14], [16], [20]).

Neutrosophic set is a modern tool in mathematics extensively used for problems containing imprecise, indeterminant and inconsistent data. This novel idea was initiated by Smarandache [19]. This is a generalized concept of fuzzy set theory by Zadeh [23] and intuitionistic fuzzy set by Atanassov [3]. It is established that neutrosophic sets produce more accurate results than those obtained by using intuitionistic fuzzy sets or fuzzy sets.

Maji [12] has further generalized the new concept of neutrosophic set to neutrosophic soft set. The notion of neutrosophic soft matrix was developed by Deli et a.l [7]. The novelty of neutrosophic set is that it comprises of three various membership functions namely a truth, an indeterminacy and a falsity membership functions. Jun [10] applied soft set theory to BCK/BCI algebra. A remarkable theory of hybrid structure was by introduced Jun et al. [11]. The novelty of this structure is that it combines soft set with its grade. An algorithm to exhibit the application of neutrosophic hybrid matrix is also provided.

So far no systematic development has been made in the theory of hybrid matrix using hybrid structure. Our main motivation is to present the notion of hybrid matrices and study their properties. We then intend to generalize the idea of hybrid matrices to neutrosophic hybrid matrices using neutrosphic structure as a tool.

In this paper, some preliminaries about soft set, soft matrix, hybrid structure and some operations between two hybrid structures are provided in section 2. Also we define complement of a hybrid structure, cartesian product and hybrid relation between two hybrid structures and introduce the concept of the hybrid matrices and various types of hybrid matrices with suitable examples in section 3. We introduce various operations on hybrid matrices and some properties of hybrid matrices are also studied in section 4. In section 5 we define neutrosophic hybrid structure and its operations using inception of neutrosophic concepts like neutrosophic set, neutrosophic soft set and neutrosophic soft matrices. Section 6 defines the notion of neutrosophic hybrid matrices involving several operations and we study their properties with suitable examples. A MCDM problem based on neutrosophic hybrid matrix and a comparative analysis of our work with Maji's [13] work is also carried out in section 7.

2. **Preliminaries**

The basic ideas are presented below. For convenient let us represent $\mathcal{U}$ to be an universe set, $\mathbb{H}$ being a set of parameters and $\mathcal{P}(\mathcal{U})$ representing power set of $\mathcal{U}$ with $\mathbb{A} \subseteq \mathbb{H}$.

Definition 2.1. [15] A pair $(\mathcal{X}, \mathbb{A})$ is called soft set over $\mathcal{U}, \mathcal{X} : \mathbb{A} \longrightarrow \mathcal{P}(\mathcal{U})$.

Definition 2.2. [16] A representation of soft set in matrix form is called as soft matrix.

Definition 2.3. [11] $\mathcal{X}_\lambda = (\mathcal{X}, \lambda) : \mathbb{H} \longrightarrow \mathcal{P}(\mathcal{U}) \times \mathcal{I}, \varrho \longrightarrow (\mathcal{X}(\varrho), \lambda(\varrho))$ is called as hybrid structure where $\mathcal{X} : \mathbb{H} \longrightarrow \mathcal{P}(\mathcal{U}), \lambda : \mathbb{H} \longrightarrow \mathcal{I}$ are mappings and $\mathcal{I}$ is the unit interval $[0, 1]$.

Example 2.4. [11] Let $\mathcal{U} = \{\upsilon_1, \upsilon_2, \upsilon_3, \ldots, \upsilon_{10}\}$ be the universe set and $\mathbb{H} = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6\}$ be the set of parameters.

TABLE 1. Representation of the hybrid structure $\mathcal{X}_\lambda$

$\mathbb{H}$	$\mathcal{X}$	λ
ϱ_1	$x_1 = \{\upsilon_1, \upsilon_2\}$	0.2
ϱ_2	$x_2 = \{\upsilon_2, \upsilon_3, \upsilon_4, \upsilon_6\}$	0.4
ϱ_3	$x_3 = \{\upsilon_3, \upsilon_5, \upsilon_7\}$	0.1
ϱ_4	$x_4 = \{\upsilon_1, \upsilon_2, \upsilon_6, \upsilon_9\}$	0.9
ϱ_5	$x_5 = \{\upsilon_6, \upsilon_7\}$	0.6
ϱ_6	$x_6 = \{\upsilon_1, \upsilon_2, \upsilon_4\}$	0.8

Definition 2.5. [11] Let $\mathcal{X}_\lambda$ and $\mathcal{Y}_\gamma$ be hybrid structures in $\mathbb{H}$. Then $\mathcal{X}_\lambda \sqcap \mathcal{Y}_\gamma : \mathbb{H} \longrightarrow \mathcal{P}(\mathcal{U}) \times \mathcal{I}, \varrho \longrightarrow ((\mathcal{X}_\lambda \sqcap \mathcal{Y}_\gamma)(\varrho), (\lambda \vee \gamma)(\varrho))$ for all $\varrho \in \mathbb{H}$, is called as the hybrid intersection where

$\mathcal{X}_\lambda \sqcap \mathcal{Y}_\gamma : \mathbb{H} \longrightarrow \mathcal{P}(\mathcal{U}), \varrho \longrightarrow \mathcal{X}(\varrho) \sqcap \mathcal{Y}(\varrho)$

$\vee\gamma : \mathbb{H} \longrightarrow \mathcal{I}, \varrho \longrightarrow \curlyvee\{\lambda(\varrho), \gamma(\varrho)\}$.

Definition 2.6. [11] Let $\mathcal{X}_\lambda$ and $\mathcal{Y}_\gamma$ be hybrid structures in $\mathbb{H}$. Then $\mathcal{X}_\lambda \sqcup \mathcal{Y}_\gamma : \mathbb{H} \longrightarrow \mathcal{P}(\mathcal{U}) \times \mathcal{I}, \varrho \longrightarrow ((\mathcal{X}_\lambda \sqcup \mathcal{Y}_\gamma)(\varrho), (\lambda \wedge \gamma)(\varrho))$ for all $\varrho \in \mathbb{H}$, is called as the hybrid union where

$\mathcal{X}_\lambda \sqcup \mathcal{Y}_\gamma : \mathbb{H} \longrightarrow \mathcal{P}(\mathcal{U}), \varrho \rightarrow \mathcal{X}(\varrho) \sqcup \mathcal{Y}(\varrho)$

$\wedge\gamma : \mathbb{H} \longrightarrow \mathcal{I}, \varrho \longrightarrow \curlywedge\{\lambda(\varrho), \gamma(\varrho)\}$.

Definition 2.7. [7] A mapping $\mathcal{X}_\mathcal{N} : \mathbb{A} \longrightarrow \mathcal{N}(\mathcal{U})$ ia called as neutrosophic soft set over $\mathcal{U}$, $\mathcal{N}(\mathcal{U})$ being the set of all neutrosophic sets in $\mathcal{U}$.

Definition 2.8. [7] Matrix representation of the neutrosophic soft set is called as the neutrosophic soft matrix.

3. **Hybrid matrix and its types**

Inspired by the theory of soft matrices, we introduce the concept of hybrid matrix and its types. Before entering into the notion hybrid matrix we define the complement, cartesian product and relation on hybrid structure as follows.

Definition 3.1. $\mathcal{X}^c_\lambda = (\mathcal{X}^c, \lambda^c) : \mathbb{H}^c \rightarrow \mathcal{P}(\mathcal{U}) \times \mathcal{I}, \varrho \rightarrow (\mathcal{X}^c(\varrho), \lambda^c(\varrho))$ is called the complement of a hybrid structure where $\mathcal{X}^c(\varrho) = \mathcal{U} - \mathcal{X}(\varrho)$ and $\lambda^c(\varrho) = 1 - \lambda(\varrho)$ for all $\varrho \in \neg\mathbb{H}$.

Definition 3.2. Let $\mathcal{X}_\lambda$ and $\mathcal{Y}_\gamma$ be hybrid structures in $\mathbb{H}$. The cartesian product of $\mathcal{X}_\lambda$ and $\mathcal{Y}_\gamma$is:

$$\mathcal{X}_\lambda \times \mathcal{Y}_\gamma = \{\{(\theta,\eta) : \theta \in \mathcal{X}(\varrho), \eta \in \mathcal{Y}(\varrho)\}, \min\{\lambda(\varrho), \gamma(\varrho)\}\}, \text{ for all } \varrho \in \mathbb{H}.$$

Definition 3.3. Given two hybrid structures $\mathcal{X}_\lambda$ and $\mathcal{Y}_\gamma$ in $\mathbb{H}$, then the hybrid relation between $\mathcal{X}_\lambda$ and $\mathcal{Y}_\gamma$is:

$$\mathcal{R} = \{\{(\theta,\eta) : \theta \in \mathcal{X}(\varrho), \eta \in \mathcal{Y}(\varrho)\}, \min\{\lambda(\varrho), \gamma(\varrho)\}\} \subset \mathcal{X}_\lambda \times \mathcal{Y}_\gamma, \text{ for all } \varrho \in \mathbb{H}.$$

Definition 3.4. The hybrid matrix over $(\mathcal{X}_\lambda, \mathbb{H})$ is defined by $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{X}_\lambda, \mathbb{H})] = \mathfrak{M}\left[(\mathcal{X}(\varrho), \lambda(\varrho))\right] = (\mathfrak{m}_{ij})_{m\times n}$, for some $\varrho \in \mathbb{H}$. That is, a hybrid matrix is a matrix whose elements are the elements of the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$.

That is, $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{X}_\lambda, \mathbb{H})] = \begin{bmatrix} (x_1,0.2) & (x_3,0.1) & (x_4,0.9) \\ (x_2,0.4) & (x_6,0.8) & (x_1,0.2) \\ (x_3,0.1) & (x_1,0.2) & (x_5,0.6) \\ (x_6,0.8) & (x_2,0.4) & (x_3,0.1) \end{bmatrix}$.

Definition 3.5. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{X}_\lambda, \mathbb{H})] = \mathfrak{M}\left[(\mathcal{X}(\varrho), \lambda(\varrho))\right]$ be a hybrid matrix over a hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$. Then the zero hybrid matrix is $[\mathfrak{M}_{\mathfrak{H}}] = [0]$ if $\mathcal{X}(\varrho) = \phi, \lambda(\varrho) = 0$, for all $\varrho \in \mathbb{H}$. That is $[\mathfrak{M}_{\mathfrak{H}}] = (\mathfrak{m}_{ij})_{m\times n} = \mathfrak{M}[(\phi, 0)] \forall$ i and j.

Definition 3.6. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{X}_\lambda, \mathbb{H})] = \mathfrak{M}\left[(\mathcal{X}(\varrho), \lambda(\varrho))\right]$ be a hybrid matrix over a hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$. Then the universe hybrid matrix is $[\mathfrak{M}_{\mathfrak{H}}] = [\mathcal{U}]$ if $\mathcal{X}(\alpha) = \mathcal{U}, \lambda(\varrho) = 1$, for all $\varrho \in \mathbb{H}$. That is $[\mathfrak{M}_{\mathfrak{H}}] = (\mathfrak{m}_{ij})_{m\times n} = \mathfrak{M}[(\mathcal{U}, 1)] \forall$ i and j.

Definition 3.7. A hybrid row matrix is a matrix with single row.

Example 3.8. An example of hybrid row matrix is $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (x_1,0.2) & (x_3,0.1) & (x_4,0.9) \end{bmatrix}$.

Definition 3.9. A hybrid column matrix is a matrix with single column.

Example 3.10. An example of hybrid column matrix is $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (x_1,0.2) \\ (x_2,0.4) \\ (x_3,0.1) \\ (x_6,0.8) \end{bmatrix}$.

Definition 3.11. A hybrid matrix with equal number of rows and columns is called hybrid square matrix.

Example 3.12. An example of hybrid square matrix is $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (x_1,0.2) & (x_3,0.1) & (x_4,0.9) \\ (x_2,0.4) & (x_6,0.8) & (x_1,0.2) \\ (x_3,0.1) & (x_1,0.2) & (x_5,0.6) \end{bmatrix}$.

4. Operations on hybrid matrices

Some interesting operations on hybrid matrices are presented in this section.

Definition 4.1. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$ be two hybrid matrices of same order over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$.
Then the AND operation of two hybrid matrices is given below.

$$[\mathfrak{M}_{\mathfrak{H}}]AND[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] \curlywedge [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = [\mathfrak{L}_{\mathfrak{H}}],$$

where $[\mathfrak{L}_{\mathfrak{H}}] = (\mathfrak{l}_{ij})_{m\times n} = [\mathfrak{L}(\mathcal{R}_\nu, \mathbb{H})] = [\mathfrak{L}\left(\mathcal{R}(\varrho), \nu(\varrho)\right)]$
$= [\mathfrak{L}\left(\mathcal{R}(\varrho) = \mathcal{P}(\varrho) \curlywedge \mathcal{Q}(\varrho), \nu(\varrho) = \max\{\eta(\varrho), \gamma(\varrho)\}\right)]$, for some $\varrho \in \mathbb{H}$.

Example 4.2. Let $\mathcal{U} = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}$ be the universe set and $\mathbb{H} = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$ be the set of parameters.
Let $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_1, \upsilon_2\}, 0.2) & (\{\upsilon_3, \upsilon_4, \upsilon_5\}, 0.1) \\ (\{upsilon_1, \upsilon_4, \upsilon_5\}, 0.4) & (\{\upsilon_2, \upsilon_3\}, 0.8) \end{bmatrix}$
and $[\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_3, \upsilon_5\}, 0.1) & (\{\upsilon_1, \upsilon_3\}, 0.6) \\ (\{\upsilon_1, \upsilon_2, \upsilon_3\}, 0.9) & (\{\upsilon_2, \upsilon_3, \upsilon_4\}, 0.4) \end{bmatrix}$.
Then $[\mathfrak{M}_{\mathfrak{H}}]AND[\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} (\phi, 0.2) & (\{\upsilon_3\}, 0.6) \\ (\{\upsilon_1\}, 0.9) & (\{\upsilon_2, \upsilon_3\}, 0.8) \end{bmatrix}$.

Definition 4.3. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$ be two hybrid matrices of same order over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$.
Then the OR operation of two hybrid matrices is given below.

$$[\mathfrak{M}_{\mathfrak{H}}]OR[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] \curlyvee [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = [\mathfrak{L}_{\mathfrak{H}}],$$

where $[\mathfrak{L}_{\mathfrak{H}}] = (\mathfrak{l}_{ij})_{m\times n} = [\mathfrak{L}(\mathcal{R}_\nu, \mathbb{H})] = [\mathfrak{L}\left((\mathcal{R}(\varrho), \nu(\varrho)\right)]$
$= [\mathfrak{L}\left(\mathcal{R}(\varrho) = \mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho), \nu(\varrho) = \min\{\eta(\varrho), \gamma(\varrho)\}\right)]$, for some $\varrho \in \mathbb{H}$.

Example 4.4. Let $\mathcal{U} = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}$ be the universe set and $\mathbb{H} = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$ be the set of parameters.
Let $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_1, \upsilon_2\}, 0.2) & (\{\upsilon_3, \upsilon_4, \upsilon_5\}, 0.1) \\ (\{\upsilon_1, \upsilon_4, \upsilon_5\}, 0.4) & (\{\upsilon_2, \upsilon_3\}, 0.8) \end{bmatrix}$
and $[\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_3, \upsilon_5\}, 0.1) & (\{\upsilon_1, \upsilon_3\}, 0.6) \\ (\{\upsilon_1, \upsilon_2, \upsilon_3\}, 0.9) & (\{\upsilon_2, \upsilon_3, \upsilon_4\}, 0.4) \end{bmatrix}$.
Then $[\mathfrak{M}_{\mathfrak{H}}]OR[\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_5\}, 0.1) & (\{\upsilon_1, \upsilon_3, \upsilon_4, \upsilon_5\}, 0.1) \\ (\mathcal{U}, 0.4) & (\{\upsilon_2, \upsilon_3, \upsilon_4\}, 0.4) \end{bmatrix}$.

Definition 4.5. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ be a hybrid matrix over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$. Then $[\mathfrak{M}_{\mathfrak{H}}]^c = \left(\mathfrak{M}^c_{ij}\right)_{m\times n} = [\mathfrak{M}\left(\mathcal{U} - \mathcal{P}(\varrho), 1 - \eta(\varrho)\right)]$,for some $\varrho \in \mathbb{H}$, is called the complement of a hybrid matrix.

Example 4.6. Let $\mathcal{U} = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}$ be the universe set and $\mathbb{H} = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$ be the set of parameters.

Let $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_1, \upsilon_2\}, 0.2) & (\{\upsilon_3, \upsilon_4, \upsilon_5\}, 0.1) \\ (\{\upsilon_1, \upsilon_4, \upsilon_5\}, 0.4) & (\{\upsilon_2, \upsilon_3\}, 0.8) \end{bmatrix}$

and $[\mathfrak{M}_{\mathfrak{H}}]^c = \begin{bmatrix} (\{\upsilon_3, \upsilon_4, \upsilon_5\}, 0.8) & (\{\upsilon_1, \upsilon_2\}, 0.9) \\ (\{\upsilon_2, \upsilon_3\}, 0.6) & (\{\upsilon_1, \upsilon_4, \upsilon_5\}, 0.2) \end{bmatrix}$.

Definition 4.7. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$ be two hybrid matrices over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$.
Then the union operation of two hybrid matrices is given below.

$$[\mathfrak{M}_{\mathfrak{H}}] \sqcup [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] \sqcup [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = [\mathfrak{L}_{\mathfrak{H}}] = (\mathfrak{l}_{ij}).$$

Remark 4.8. $\mathfrak{l}_{ij} = \sqcup_\varrho \mathcal{R}_\nu(\varrho) = \sqcup_\varrho (\mathcal{R}(\varrho), \nu(\varrho))$, where ϱ being the parameter which is common of the i^{th} row of $[\mathfrak{M}_{\mathfrak{H}}]$ and j^{th} column of $[\mathfrak{N}_{\mathfrak{H}}]$ and $\nu(\varrho) = \min\{\eta(\varrho), \gamma(\varrho)\}$.

Example 4.9. Let $\mathcal{U} = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}$ be the universe set and $\mathbb{H} = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5\}$ be the set of parameters.

Let $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_2, \upsilon_3\}, 0.8) & (\{\upsilon_4, \upsilon_5\}, 0.1) & (\{\upsilon_2, \upsilon_3, \upsilon_4\}, 0.6) \\ (\{\upsilon_1, \upsilon_3, \upsilon_4\}, 0.4) & (\{\upsilon_2, \upsilon_3\}, 0.8) & (\{\upsilon_1, \upsilon_2, \upsilon_3\}, 0.1) \\ (\{\upsilon_2, \upsilon_3, \upsilon_4\}, 0.6) & (\{\upsilon_1\}, 0.7) & (\{\upsilon_1, \upsilon_2\}, 0.2) \end{bmatrix}$

and $[\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_3, \upsilon_5\}, 0.1) & (\{\upsilon_1, \upsilon_3\}, 0.6) & (\{\upsilon_2\}, 0.5) \\ (\{\upsilon_2, \upsilon_3, \upsilon_4\}, 0.6) & (\{\upsilon, \upsilon_2, \upsilon_3\}, 0.1) & (\{\upsilon_2, \upsilon_3\}, 0.8) \\ (\{\upsilon_4, \upsilon_5\}, 0.1) & (\{\upsilon_2, \upsilon_3\}, 0.8) & (\{\upsilon_1\}, 0.7) \end{bmatrix}$.

The union of $[\mathfrak{M}_{\mathfrak{H}}]$ and $[\mathfrak{N}_{\mathfrak{H}}]$ is given by,

$[\mathfrak{M}_{\mathfrak{H}}] \sqcup [\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}, 0.1) & (\{\upsilon_2, \upsilon_3\}, 0.8) & (\{\upsilon_2, \upsilon_3\}, 0.8) \\ \phi & (\{\upsilon_1, \upsilon_2, \upsilon_3\}, 0.1) & (\{\upsilon_2, \upsilon_3\}, 0.8) \\ (\{\upsilon_2, \upsilon_3, \upsilon_4\}, 0.6) & \phi & (\{\upsilon_1\}, 0.7) \end{bmatrix}$.

Definition 4.10. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$ be two hybrid matrices over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$.
Then the intersection operation of two hybrid matrices is given below.

$$[\mathfrak{M}_{\mathfrak{H}}] \sqcap [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] \sqcap [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = [\mathfrak{L}_{\mathfrak{H}}] = (\mathfrak{l}_{ij}).$$

Remark 4.11. $\mathfrak{l}_{ij} = \sqcap_\varrho \mathcal{R}_\nu(\varrho) = \sqcap_\varrho (\mathcal{R}(\varrho), \nu(\varrho))$, where ϱ being the parameter which is common to the i^{th} row of $[\mathfrak{M}_{\mathfrak{H}}]$ and j^{th} column of $[\mathfrak{N}_{\mathfrak{H}}]$ and $\nu(\varrho) = \max\{\eta(\varrho), \gamma(\varrho)\}$.

Example 4.12. Let $\mathcal{U} = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}$ be the universe set and $\mathbb{H} = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ be the set of parameters.

Let $[\mathfrak{M}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_2,\upsilon_3\},0.8) & (\{\upsilon_4,\upsilon_5\},0.1) & (\{\upsilon_2,\upsilon_3,\upsilon_4\},0.6) \\ (\{\upsilon_1,\upsilon_3,\upsilon_4\},0.4) & (\{\upsilon_2,\upsilon_3\},0.8) & (\{\upsilon_1,\upsilon_2,\upsilon_3\},0.1) \\ (\{\upsilon_2,\upsilon_3,\upsilon_4\},0.6) & (\{\upsilon_1\},0.7) & (\{\upsilon_1,\upsilon_2\},0.2) \end{bmatrix}$

and $[\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} (\{\upsilon_3,\upsilon_5\},0.1) & (\{\upsilon_1,\upsilon_3\},0.6) & (\{\upsilon_2\},0.5) \\ (\{\upsilon_2,\upsilon_3,\upsilon_4\},0.6) & (\{\upsilon_1,\upsilon_2,\upsilon_3\},0.1) & (\{\upsilon_2,\upsilon_3\},0.8) \\ (\{\upsilon_4,\upsilon_5\},0.1) & (\{\upsilon_2,\upsilon_3\},0.8) & (\{\upsilon_1\},0.7) \end{bmatrix}$.

The intersection of $[\mathfrak{M}_{\mathfrak{H}}]$ and $[\mathfrak{N}_{\mathfrak{H}}]$ is given by,

$$[\mathfrak{M}_{\mathfrak{H}}] \sqcap [\mathfrak{N}_{\mathfrak{H}}] = \begin{bmatrix} \phi & \phi & \phi \\ \phi & (\{\upsilon_3\},0.8) & \phi \\ \phi & \phi & \phi \end{bmatrix}.$$

Theorem 4.13. *Let $[\mathfrak{M}_{\mathfrak{H}}]$ and $[\mathfrak{N}_{\mathfrak{H}}]$ be two hybrid matrices of same order over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$. Then the following results related to the operations hold.*

(1) $[\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]$
(2) $[\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}_{\mathfrak{H}}] \curlywedge [\mathfrak{M}_{\mathfrak{H}}]$
(3) $([\mathfrak{M}_{\mathfrak{H}}]^c)^c = [\mathfrak{M}_{\mathfrak{H}}]$
(4) $([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}])^c = [\mathfrak{M}_{\mathfrak{H}}]^c \curlywedge [\mathfrak{N}_{\mathfrak{H}}]^c$
(5) $([\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}])^c = [\mathfrak{M}_{\mathfrak{H}}]^c \curlyvee [\mathfrak{N}_{\mathfrak{H}}]^c$.

Proof. Let $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$

(1) $[\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]$

$$\begin{aligned} [\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}] &= [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] \curlyvee [\mathfrak{N}(\mathcal{Q}_\gamma, \mathbb{H})] \\ &= [\mathfrak{M}\left(\mathcal{P}(\varrho), \eta(\varrho)\right)] \curlyvee [\mathfrak{N}\left(\mathcal{Q}(\varrho), \gamma(\varrho)\right)] \\ &= [\mathfrak{L}\left(\mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho), \min\{\eta(\varrho), \gamma(\varrho)\}\right)] \\ &= [\mathfrak{L}\left(\mathcal{Q}(\varrho) \curlyvee \mathcal{P}(\varrho), \min\{\eta(\varrho), \gamma(\varrho)\}\right)] \\ &= [\mathfrak{N}\left(\mathcal{Q}(\varrho), \gamma(\varrho)\right)] \curlyvee [\mathfrak{M}\left(\mathcal{P}(\varrho), \eta(\varrho)\right)] \\ &= [\mathfrak{N}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]. \end{aligned}$$

(2) $[\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}_{\mathfrak{H}}] \curlywedge [\mathfrak{M}_{\mathfrak{H}}]$
Proof is similar to (1).

(3) $([\mathfrak{M}_{\mathfrak{H}}]^c)^c = [\mathfrak{M}_{\mathfrak{H}}]$
Since, $[\mathfrak{M}_{\mathfrak{H}}]^c = \left(\mathfrak{m}_{ij}^c\right) = [\mathfrak{M}\left(\mathcal{U} - \mathcal{P}(\varrho), 1 - \eta(\varrho)\right)]$

$$\begin{aligned} ([\mathfrak{M}_{\mathfrak{H}}]^c)^c &= [\mathfrak{M}\left(\mathcal{U} - \{\mathcal{U} - \mathcal{P}(\varrho)\}, 1 - \{1 - \eta(\varrho)\}\right)] \\ &= [\mathfrak{M}\left(\mathcal{P}(\varrho), \eta(\varrho)\right)] \\ &= [\mathfrak{M}(\mathcal{P}_\eta, \mathbb{H})] \\ &= [\mathfrak{M}_{\mathfrak{H}}]. \end{aligned}$$

(4) $([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}])^c = [\mathfrak{M}_{\mathfrak{H}}]^c \curlywedge [\mathfrak{N}_{\mathfrak{H}}]^c$

Since, $[\mathfrak{M}_{\mathfrak{H}}]^c = \left(\mathfrak{m}^c_{ij}\right) = [\mathfrak{M}\,(\mathcal{U} - \mathcal{P}(\varrho), 1 - \eta(\varrho))]$

and $[\mathfrak{N}_{\mathfrak{H}}]^c = \left(\mathfrak{n}^c_{ij}\right) = [\mathfrak{N}\,(\mathcal{U} - \mathcal{Q}(\varrho), 1 - \gamma(\varrho))]$

$$\begin{aligned}
([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}])^c &= ([\mathfrak{L}\,(\mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho), \min\{\eta(\varrho), \gamma(\varrho)\})])^c \\
&= [\mathfrak{L}\,(\mathcal{U} - \{\mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho)\}, \max\{1 - \eta(\varrho), 1 - \gamma(\varrho)\})] \\
&= [\mathfrak{L}\,(\{\mathcal{U} - \mathcal{P}(\varrho)\} \curlywedge \{\mathcal{U} - \mathcal{Q}(\varrho)\}, \max\{1 - \eta(\varrho), 1 - \gamma(\varrho)\})] \\
&= [\mathfrak{M}\,(\mathcal{U} - \mathcal{P}(\varrho), 1 - \eta(\varrho))] \curlywedge [\mathfrak{N}\,(\mathcal{U} - \mathcal{Q}(\varrho), 1 - \gamma(\varrho))] \\
&= [\mathfrak{M}_{\mathfrak{H}}]^c \curlywedge [\mathfrak{N}_{\mathfrak{H}}]^c.
\end{aligned}$$

(5) $([\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}])^c = [\mathfrak{M}_{\mathfrak{H}}]^c \curlyvee [\mathfrak{N}_{\mathfrak{H}}]^c$

Proof is similar to (4).

Theorem 4.14. *Let $[\mathfrak{L}_{\mathfrak{H}}], [\mathfrak{M}_{\mathfrak{H}}]$ and $[\mathfrak{N}_{\mathfrak{H}}]$ be three hybrid matrices of same order over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$. Then the following results related to the operations hold.*

(1) $([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]) \curlyvee [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{L}_{\mathfrak{H}}] \curlyvee ([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}])$
(2) $([\mathfrak{L}_{\mathfrak{H}}] \curlywedge [\mathfrak{M}_{\mathfrak{H}}]) \curlywedge [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{L}_{\mathfrak{H}}] \curlywedge ([\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}])$
(3) $[\mathfrak{L}_{\mathfrak{H}}] \curlyvee ([\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}]) = ([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]) \curlywedge ([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}])$
(4) $[\mathfrak{L}_{\mathfrak{H}}] \curlywedge ([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}]) = ([\mathfrak{L}_{\mathfrak{H}}] \curlywedge [\mathfrak{M}_{\mathfrak{H}}]) \curlyvee ([\mathfrak{L}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}])$.

Proof. Let $[\mathfrak{L}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\nu, \mathbb{H})] = (\mathfrak{l}_{ij})_{m\times n}$, $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{Q}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}(\mathcal{R}_\gamma, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$

(1) $([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]) \curlyvee [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{L}_{\mathfrak{H}}] \curlyvee ([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}])$

$$\begin{aligned}
([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]) \curlyvee [\mathfrak{N}_{\mathfrak{H}}] &= ([\mathfrak{L}\,(\mathcal{P}(\varrho), \nu(\varrho))] \curlyvee [\mathfrak{M}\,(\mathcal{Q}(\varrho), \eta(\varrho))]) \curlyvee [\mathfrak{N}\,(\mathcal{R}(\varrho), \gamma(\varrho))] \\
&= ([\mathfrak{S}\,(\mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho), \min\{\nu(\varrho), \eta(\varrho)\})]) \curlyvee [\mathfrak{N}\,(\mathcal{R}(\varrho), \gamma(\varrho))] \\
&= [\mathfrak{T}\,(\{\mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho)\} \curlyvee \mathcal{R}(\varrho), \min\{\nu(\varrho), \eta(\varrho), \gamma(\varrho)\})] \\
&= [\mathfrak{T}\,(\mathcal{P}(\varrho) \curlyvee \{\mathcal{Q}(\varrho) \curlyvee \mathcal{R}\varrho)\}, \min\{\nu(\varrho), \eta(\varrho), \gamma(\varrho)\})] \\
&= [\mathfrak{L}\,(\mathcal{P}(\varrho), \nu(\varrho))] \curlyvee ([\mathfrak{S}\,(\mathcal{Q}(\varrho) \curlyvee \mathcal{R}(\varrho), \min\{\eta(\varrho), \gamma(\varrho)\})]) \\
&= [\mathfrak{L}_{\mathfrak{H}}] \curlyvee ([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}]).
\end{aligned}$$

(2) $([\mathfrak{L}_{\mathfrak{H}}] \curlywedge [\mathfrak{M}_{\mathfrak{H}}]) \curlywedge [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{L}_{\mathfrak{H}}] \curlywedge ([\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}])$

Proof is similar to (1).

(3) $[\mathfrak{L}_{\mathfrak{H}}] \curlyvee ([\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}]) = ([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]) \curlywedge ([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}])$

$$\begin{aligned}
LHS &= [\mathfrak{L}_{\mathfrak{H}}] \curlyvee ([\mathfrak{M}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}]) \\
&= [\mathfrak{L}(\mathcal{P}(\varrho), \nu(\varrho))] \curlyvee ([\mathfrak{M}(\mathcal{Q}(\varrho), \eta(\varrho))] \curlywedge [\mathfrak{N}(\mathcal{R}(\varrho), \gamma(\varrho))]) \\
&= [\mathfrak{L}(\mathcal{P}(\varrho), \nu(\varrho))] \curlyvee [\mathfrak{S}(\mathcal{Q}(\varrho) \curlywedge \mathcal{R}(\varrho), \max\{\eta(\varrho), \gamma(\varrho)\})] \\
&= [\mathfrak{T}(\mathcal{P}(\varrho) \curlyvee \{\mathcal{Q}(\varrho) \curlywedge \mathcal{R}(\varrho)\}, \min\{\nu(\varrho), \max\{\eta(\varrho), \gamma(\varrho)\}\})] \\
&= [\mathfrak{T}((\mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho)) \curlywedge (\mathcal{P}(\varrho) \curlyvee \mathcal{R}(\varrho)), \max\{\min\{\nu(\varrho), \eta(\varrho)\}, \min\{\eta(\varrho), \gamma(\varrho)\}\})] \\
&= [\mathfrak{S}((\mathcal{P}(\varrho) \curlyvee \mathcal{Q}(\varrho)), \min\{\nu(\varrho), \eta(\varrho)\})] \curlywedge [\mathfrak{W}((\mathcal{P}(\varrho) \curlyvee \mathcal{R}(\varrho)), \min\{\eta(\varrho), \gamma(\varrho)\})] \\
&= ([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{M}_{\mathfrak{H}}]) \curlywedge ([\mathfrak{L}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}]).
\end{aligned}$$

(4) $[\mathfrak{L}_{\mathfrak{H}}] \curlywedge ([\mathfrak{M}_{\mathfrak{H}}] \curlyvee [\mathfrak{N}_{\mathfrak{H}}]) = ([\mathfrak{L}_{\mathfrak{H}}] \curlywedge [\mathfrak{M}_{\mathfrak{H}}]) \curlyvee ([\mathfrak{L}_{\mathfrak{H}}] \curlywedge [\mathfrak{N}_{\mathfrak{H}}])$

Proof is similar to (3).

Theorem 4.15. *Let $[\mathfrak{L}_{\mathfrak{H}}], [\mathfrak{M}_{\mathfrak{H}}]$ and $[\mathfrak{N}_{\mathfrak{H}}]$ be three hybrid matrices over the hybrid structure $(\mathcal{X}_\lambda, \mathbb{H})$. Then the following results related to the operations hold.*

(1) $([\mathfrak{L}_{\mathfrak{H}}] \sqcup [\mathfrak{M}_{\mathfrak{H}}]) \sqcup [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{L}_{\mathfrak{H}}] \sqcup ([\mathfrak{M}_{\mathfrak{H}}] \sqcup [\mathfrak{N}_{\mathfrak{H}}])$

(2) $([\mathfrak{L}_{\mathfrak{H}}] \sqcap [\mathfrak{M}_{\mathfrak{H}}]) \sqcap [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{L}_{\mathfrak{H}}] \sqcap ([\mathfrak{M}_{\mathfrak{H}}] \sqcap [\mathfrak{N}_{\mathfrak{H}}])$.

Proof. Let $[\mathfrak{L}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{P}_\nu, \mathbb{H})] = (\mathfrak{l}_{ij})$, $[\mathfrak{M}_{\mathfrak{H}}] = [\mathfrak{M}(\mathcal{Q}_\eta, \mathbb{H})] = (\mathfrak{m}_{ij})$ and $[\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{N}(\mathcal{R}_\gamma, \mathbb{H})] = (\mathfrak{n}_{ij})$

(1) $([\mathfrak{L}_{\mathfrak{H}}] \sqcup [\mathfrak{M}_{\mathfrak{H}}]) \sqcup [\mathfrak{N}_{\mathfrak{H}}] = [\mathfrak{L}_{\mathfrak{H}}] \sqcup ([\mathfrak{M}_{\mathfrak{H}}] \sqcup [\mathfrak{N}_{\mathfrak{H}}])$

Let $[\mathfrak{L}_{\mathfrak{H}}] \sqcup [\mathfrak{M}_{\mathfrak{H}}] = (\mathfrak{r}_{ij})$, then

$$\mathfrak{r}_{ij} = \sqcup_\varrho \mathcal{S}_\tau(\varrho) = \sqcup_\varrho (\mathcal{S}(\varrho), \tau(\varrho))$$

where ϱ being the parameter which is common of the i^{th} row of $[\mathfrak{L}_{\mathfrak{H}}]$ and j^{th} column of $[\mathfrak{M}_{\mathfrak{H}}]$ and $\tau(\varrho) = \min?\{\nu(\varrho), \eta(\varrho)\}$.

Also, let $([\mathfrak{L}_{\mathfrak{H}}] \sqcup [\mathfrak{M}_{\mathfrak{H}}]) \sqcup [\mathfrak{N}_{\mathfrak{H}}] = (\mathfrak{t}_{ij})$

$$\mathfrak{t}_{ij} = \sqcup_\varrho \mathcal{T}_\theta(\varrho) = \sqcup_\varrho (\mathcal{T}(\varrho), \theta(\varrho))$$

where ϱ being the parameter which is common of the i^{th} row of $[\mathfrak{L}_{\mathfrak{H}}] \sqcup [\mathfrak{M}_{\mathfrak{H}}]$ and j^{th} column of $[\mathfrak{N}_{\mathfrak{H}}]$ and $\theta(\varrho) = \min?\{\tau(\varrho), \gamma(\varrho)\}$.

Clearly, the common parameters of i^{th} row of $[\mathfrak{L}_{\mathfrak{H}}] \sqcup [\mathfrak{M}_{\mathfrak{H}}]$ are the parameters of i^{th} row of $[\mathfrak{L}_{\mathfrak{H}}]$.

$$\mathfrak{t}_{ij} = \sqcup_\varrho \mathcal{T}_\theta(\varrho) = \sqcup_\varrho (\mathcal{T}(\varrho), \theta(\varrho))$$

where ϱ being the parameter which is common of the i^{th} row of $[\mathfrak{L}_{\mathfrak{H}}]$ and j^{th} column of $[\mathfrak{N}_{\mathfrak{H}}]$ and $\theta(\varrho) = \min\{\nu(\varrho), \eta(\varrho), \gamma(\varrho)\}$.

Again let $[\mathfrak{M}_{\mathfrak{H}}] \sqcup [\mathfrak{N}_{\mathfrak{H}}] = (\mathfrak{w}_{ij})$, then

$$\mathfrak{w}_{ij} = \sqcup_\beta \mathcal{S}_\tau(\beta) = \sqcup_\beta (\mathcal{S}(\beta), \tau(\beta))$$

where β being the parameter which is common of the i^{th} row of $[\mathfrak{M}_\mathfrak{H}]$ and j^{th} column of $[\mathfrak{N}_\mathfrak{H}]$ and $\tau(\beta) = \min\{\eta(\beta), \gamma(\beta)\}$.

Also, let $[\mathfrak{L}_\mathfrak{H}] \sqcup ([\mathfrak{M}_\mathfrak{H}] \sqcup [\mathfrak{N}_\mathfrak{H}]) = (\mathfrak{u}_{ij})$, then

$$\mathfrak{u}_{ij} = \sqcup_\beta \mathcal{T}_\theta(\beta) = \sqcup_\beta (\mathcal{T}(\beta), \theta(\beta))$$

where β being the parameter which is common of the i^{th} row of $[\mathfrak{L}_\mathfrak{H}]$ and j^{th} column of $[\mathfrak{M}_\mathfrak{H}] \sqcup [\mathfrak{N}_\mathfrak{H}]$ and $\theta(\beta) = \min\{\nu(\beta), \tau(\beta)\}$.

Since the common parameters of j^th column of $[\mathfrak{M}_\mathfrak{H}] \sqcup [\mathfrak{N}_\mathfrak{H}]$ are the parameters of j^{th} column of $[\mathfrak{N}_\mathfrak{H}]$.

$$\mathfrak{u}_{ij} = \sqcup_\beta \mathcal{S}_\theta(\beta) = \sqcup_\beta (\mathcal{S}(\beta), \theta(\beta))$$

where β being the parameter which is common of the i^{th} row of $[\mathfrak{L}_\mathfrak{H}]$ and j^{th} column of $[\mathfrak{N}_\mathfrak{H}]$ and $\theta(\beta) = \min\{\nu(\beta), \eta(\beta), \gamma(\beta)\}$.

Thus, $\mathfrak{s}_{ij} = \mathfrak{u}_{ij}$.

That is, $([\mathfrak{L}_\mathfrak{H}] \sqcup [\mathfrak{M}_\mathfrak{H}]) \sqcup [\mathfrak{N}_\mathfrak{H}] = [\mathfrak{L}_\mathfrak{H}] \sqcup ([\mathfrak{M}_\mathfrak{H}] \sqcup [\mathfrak{N}_\mathfrak{H}])$.

(2) $([\mathfrak{L}_\mathfrak{H}] \sqcap [\mathfrak{M}_\mathfrak{H}]) \sqcap [\mathfrak{N}_\mathfrak{H}] = [\mathfrak{L}_\mathfrak{H}] \sqcap ([\mathfrak{M}_\mathfrak{H}] \sqcap [\mathfrak{N}_\mathfrak{H}])$

Proof is similar to (1).

5. Neutrosophic hybrid structure and its operations

We define the neutrosophic hybrid structure as a generalization of hybrid structure in this section . We study several operations on neutrosophic hybrid structure with necessary examples.

Definition 5.1. $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda} = (\mathcal{X}_{\widehat{\mathcal{N}}}, \lambda) : \mathbb{H} \longrightarrow \mathcal{N}(\mathcal{U}) \times \mathcal{I}$,
$\varrho \longrightarrow (< \varrho, (\mathcal{T}_\mathbb{H}(\varrho), \mathcal{I}_\mathbb{H}(\varrho), \mathcal{F}_\mathbb{H}(\varrho)) >, \lambda(\varrho))$ is called as the neutrosophic hybrid structure where $\mathcal{X}_{\widehat{\mathcal{N}}} : \mathbb{H} \longrightarrow \mathcal{N}(\mathcal{U}), \lambda : \mathbb{H} \longrightarrow \mathcal{I}$ are mappings and $\mathcal{I}$ is the unit interval $[0, 1]$.

Example 5.2. Let $\mathcal{U} = \{\upsilon_1, \upsilon_2, \upsilon_3\}$ be the universe set and $\mathbb{H} = \{\varrho_1, \varrho_2, \varrho_3\}$ be the set of parameters. Then

$$\mathcal{X}_{\widehat{\mathcal{N}}}(\varrho_1) = \{(< \upsilon_1, (0.2, 0.6, 0.5) >, 0.4), (< \upsilon_2, (0.3, 0.5, 0.8) >, 0.2), (< \upsilon_3, (0.8, 0.3, 0.6) >, 0.7)\}$$

$$\mathcal{X}_{\widehat{\mathcal{N}}}(\varrho_2) = \{(< \upsilon_1, (0.2, 0.6, 0.3) >, 0.1), (< \upsilon_2, (0.2, 0.5, 0.1) >, 0.6), (< \upsilon_3, (0.9, 0.8, 0.4) >, 0.3)\}$$

$$\mathcal{X}_{\widehat{\mathcal{N}}}(\varrho_3) = \{(< \upsilon_1, (0.3, 0.4, 0.5) >, 0.9), (< \upsilon_2, (0.3, 0.7, 0.1) >, 0.3), (< \upsilon_3, (0.5, 0.4, 0.2) >, 0.4)\}$$

Definition 5.3. Let $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}$ and $\mathcal{Y}_{\widehat{\mathcal{N}}_\gamma}$ be two neutrosophic hybrid structures in $\mathbb{H}$. Then their neutrosophic hybrid intersection is $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda} \sqcap \mathcal{Y}_{\widehat{\mathcal{N}}_\gamma} = \mathcal{K}_{\widehat{\mathcal{N}}_\nu}$ where
$\mathcal{K}_{\widehat{\mathcal{N}}_\nu}(\varrho) = \left(< \varrho, \left(\mathcal{T}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho)\right) >, \nu(\varrho) = \curlyvee\{\lambda(\varrho), \gamma(\varrho)\}\right),$

$\mathcal{T}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho) = \curlywedge\{\mathcal{T}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{Y}_{\widehat{\mathcal{N}}}}(\varrho)\}, \mathcal{I}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho) = \curlyvee\{\mathcal{I}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{Y}_{\widehat{\mathcal{N}}}}(\varrho)\}$ and
$\mathcal{F}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho) = \curlyvee\{\mathcal{F}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{Y}_{\widehat{\mathcal{N}}}}(\varrho)\}$ for all $\varrho \in \mathbb{H}$.

Definition 5.4. Let $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}$ and $\mathcal{Y}_{\widehat{\mathcal{N}}_\gamma}$ be two neutrosophic hybrid structures in $\mathbb{H}$. Then their neutrosophic hybrid union is $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda} \sqcup \mathcal{Y}_{\widehat{\mathcal{N}}_\gamma} = \mathcal{K}_{\widehat{\mathcal{N}}_\nu}$ where
$\mathcal{K}_{\widehat{\mathcal{N}}_\nu}(\varrho) = \Big(< \varrho, \Big(\mathcal{T}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho)\Big) >, \nu(\varrho) = \rfloor\sqcap\nabla\updownarrow\dagger\sqsupseteq\rceil\lceil\}\rceil\{\lambda(\varrho), \gamma(\varrho)\}\Big)$,
$\mathcal{T}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho) = \curlyvee\{\mathcal{T}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{Y}_{\widehat{\mathcal{N}}}}(\varrho)\}, \mathcal{I}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho) = \curlywedge\{\mathcal{I}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{Y}_{\widehat{\mathcal{N}}}}(\varrho)\}$ and
$\mathcal{F}_{\mathcal{K}_{\widehat{\mathcal{N}}}}(\varrho) = \curlywedge\{\mathcal{F}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{Y}_{\widehat{\mathcal{N}}}}(\varrho)\}$ for all $\varrho \in \mathbb{H}$.

Definition 5.5. $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}{}^c(\varrho) = \Big(< \varrho, \Big(\mathcal{F}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho)\Big) >, 1 - \lambda(\varrho)\Big)$, for all $\varrho \in \neg\mathbb{H}$ is called the complement of a neutrosophic hybrid structure.

Definition 5.6. Let $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}$ and $\mathcal{Y}_{\widehat{\mathcal{N}}_\gamma}$ be two neutrosophic hybrid structures in $\mathbb{H}$. The cartesian product of $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}$ and $\mathcal{Y}_{\widehat{\mathcal{N}}_\gamma}$ is:
$\mathcal{X}_{\widehat{\mathcal{N}}_\lambda} \times \mathcal{Y}_{\widehat{\mathcal{N}}_\gamma} = \big\{\{(\theta, \eta) : \theta \in \mathcal{X}_{\widehat{\mathcal{N}}}(\varrho), \eta \in \mathcal{Y}_{\widehat{\mathcal{N}}}(\varrho)\}, \min\{\lambda(\varrho), \gamma(\varrho)\}\big\}$, for all $\varrho \in \mathbb{H}$.

Definition 5.7. Let $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}$ and $\mathcal{Y}_{\widehat{\mathcal{N}}_\gamma}$ be two neutrosophic hybrid structures in $\mathbb{H}$ over $\mathcal{N}(\mathcal{U})$. Then the neutrosophic hybrid relation of $\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}$ and $\mathcal{Y}_{\widehat{\mathcal{N}}_\gamma}$ is:
$\mathcal{R} = \big\{\{(\theta, \eta) : \theta \in \mathcal{X}_{\widehat{\mathcal{N}}}(\varrho), \eta \in \mathcal{Y}_{\widehat{\mathcal{N}}}(\varrho)\}, \min\{\lambda(\varrho), \gamma(\varrho)\}\big\} \subset \mathcal{X}_{\widehat{\mathcal{N}}_\lambda} \times \mathcal{Y}_{\widehat{\mathcal{N}}_\gamma}$, for all $\varrho \in \mathbb{H}$.

6. Neutrosophic hybrid matrix and its properties

In this section we define the neutrosophic hybrid matrix as a generalization of hybrid matrix. We also provide various types of neutrosophic hybrid matrices. Some interesting operations on neutrosophic hybrid matrices are also given. For convenience the following notations are used in this section,
$\max\{\mathcal{T}_{\mathcal{A}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{B}_{\widehat{\mathcal{N}}}}(\varrho)\} = \mathcal{T}_{\vee(\mathcal{A},\mathcal{B})}(\varrho); \min\{\mathcal{T}_{\mathcal{A}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{B}_{\widehat{\mathcal{N}}}}(\varrho)\} = \mathcal{T}_{\wedge(\mathcal{A},\mathcal{B})}(\varrho)$;
$\max\{\mathcal{I}_{\mathcal{A}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{B}_{\widehat{\mathcal{N}}}}(\varrho)\} = \mathcal{I}_{\vee(\mathcal{A},\mathcal{B})}(\varrho); \min\{\mathcal{I}_{\mathcal{A}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{B}_{\widehat{\mathcal{N}}}}(\varrho)\} = \mathcal{I}_{\wedge(\mathcal{A},\mathcal{B})}(\varrho)$;
$\max\{\mathcal{F}_{\mathcal{A}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{B}_{\widehat{\mathcal{N}}}}(\varrho)\} = \mathcal{F}_{\vee(\mathcal{A},\mathcal{B})}(\varrho); \min\{\mathcal{F}_{\mathcal{A}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{B}_{\widehat{\mathcal{N}}}}(\varrho)\} = \mathcal{F}_{\wedge(\mathcal{A},\mathcal{B})}(\varrho)$.

Definition 6.1. Let $\Big(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\Big)$ be neutrosophic hybrid structure defined over $\mathcal{N}(\mathcal{U})$. Then the neutrosophic hybrid matrix over $(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H})$ is defined by $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H})] = \mathfrak{M}\left[\left(\mathcal{X}_{\widehat{\mathcal{N}}}(\varrho), \lambda(\varrho)\right)\right] = (\mathfrak{m}_{ij})_{m\times n}$, for some $\varrho \in \mathbb{H}$. In other words a neutrosophic hybrid matrix is a matrix whose elements are the elements of the neutrosophic hybrid structure $\Big(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\Big)$. That is,

$$[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H})] = \begin{bmatrix} ((0.2, 0.6, 0.5), 0.4) & ((0.3, 0.5, 0.8), 0.2) & ((0.8, 0.3, 0.6), 0.7) \\ ((0.2, 0.6, 0.3), 0.1) & ((0.2, 0.5, 0.1), 0.6) & ((0.9, 0.8, 0.4), 0.3) \\ ((0.3, 0.4, 0.5), 0.9) & ((0.3, 0.7, 0.1), 0.3) & ((0.5, 0.4, 0.2), 0.4) \end{bmatrix}.$$

Definition 6.2. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H})] = \mathfrak{M}\left[\left(\mathcal{X}_{\widehat{\mathcal{N}}}(\varrho), \lambda(\varrho)\right)\right]$ be a neutrosophic hybrid matrix over a neutrosophic hybrid structure $\Big(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\Big)$. Then the zero neutrosophic hybrid matrix is $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = (\mathfrak{m}_{ij})_{m\times n} = \mathfrak{M}[((0,1,1),0)]$ for all i and j.

That is, $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0,1,1),0) & ((0,1,1),0) \\ ((0,1,1),0) & ((0,1,1),0) \end{bmatrix}$.

Definition 6.3. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H})] = \mathfrak{M}\left[\left(\mathcal{X}_{\widehat{\mathcal{N}}}(\varrho), \lambda(\varrho)\right)\right]$ be a neutrosophic hybrid matrix over a neutrosophic hybrid structure $\left(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\right)$. Then the universe neutrosophic hybrid matrix is $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = (\mathfrak{m}_{ij})_{m\times n} = \mathfrak{M}[((1,0,0),1)]$ for all i and j.

That is, $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((1,0,0),1) & ((1,0,0),1) \\ ((1,0,0),1) & ((1,0,0),1) \end{bmatrix}$.

Definition 6.4. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ be a neutrosophic hybrid matrix over a hybrid neutrosophic structure $\left(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\right)$ with respect to a universe $\mathcal{N}(\mathcal{U})$. The complement of a neutrosophic hybrid matrix is

$[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c = (\mathfrak{m}_{ij}^c)_{m\times n} = \left[\mathfrak{M}\left(\left(\mathcal{F}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{X}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\lambda(\varrho)\right)\right]$, for all $\varrho \in \neg\mathbb{H}$.

Example 6.5. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0.3,0.5,0.9),0.2) & ((0.6,0.7,0.2),0.7) \\ ((0.8,0.3,0.1),0.8) & ((0.1,0.5,0.8),0.1) \end{bmatrix}$ be a neutrosophic hybrid matrix. Then the complement of a neutrosophic hybrid matrix is

$[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c = \begin{bmatrix} ((0.9,0.5,0.3),0.8) & ((0.2,0.7,0.6),0.3) \\ ((0.1,0.3,0.8),0.2) & ((0.8,0.5,0.1),0.9) \end{bmatrix}$.

Definition 6.6. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{P}_{\widehat{\mathcal{N}}_\eta}, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathbb{H}}}] = [\mathfrak{N}(\mathcal{Q}_{\widehat{\mathcal{N}}_\gamma}, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$ be two neutrosophic hybrid matrices of same order over the neutrosophic hybrid structure $\left(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\right)$.

Then the union operation of two neutrosophic hybrid matrices is:

$[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \left[\mathfrak{L}\left(\left(\mathcal{T}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho)\right), \min\{\eta(\varrho), \gamma(\varrho)\}\right)\right]$

where $\mathcal{T}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho) = \mathcal{T}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{I}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho) = \mathcal{I}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho)$ and $\mathcal{F}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho) = \mathcal{F}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho)$.

Example 6.7. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0.3,0.5,0.9),0.2) & ((0.6,0.7,0.2),0.7) \\ ((0.8,0.3,0.1),0.8) & ((0.1,0.5,0.8),0.1) \end{bmatrix}$ and

$[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0.6,0.2,0.5),0.4) & ((0.3,0.2,0.5),0.2) \\ ((0.3,0.7,0.2),0.3) & ((0.5,0.8,0.9),0.8) \end{bmatrix}$ be two neutrosophic hybrid matrices.

Then, $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0.6,0.2,0.5),0.2) & ((0.6,0.2,0.2),0.2) \\ ((0.8,0.3,0.1),0.3) & ((0.5,0.5,0.8),0.1) \end{bmatrix}$.

Definition 6.8. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{P}_{\widehat{\mathcal{N}}_\eta}, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathbb{H}}}] = [\mathfrak{N}(\mathcal{Q}_{\widehat{\mathcal{N}}_\gamma}, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$ be two neutrosophic hybrid matrices of same order over the neutrosophic hybrid structure $\left(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\right)$.

Then the intersection operation of two neutrosophic hybrid matrices is:

$[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \left[\mathfrak{L}\left(\left(\mathcal{T}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho)\right), \max\{\eta(\varrho), \gamma(\varrho)\}\right)\right]$

where $\mathcal{T}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho) = \mathcal{T}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{I}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho) = \mathcal{I}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho)$ and $\mathcal{F}_{\mathcal{L}_{\widehat{\mathcal{N}}}}(\varrho) = \mathcal{F}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho)$.

Example 6.9. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0.3, 0.5, 0.9), 0.2) & ((0.6, 0.7, 0.2), 0.7) \\ ((0.8, 0.3, 0.1), 0.8) & ((0.1, 0.5, 0.8), 0.1) \end{bmatrix}$ and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0.6, 0.2, 0.5), 0.4) & ((0.3, 0.2, 0.5), 0.2) \\ ((0.3, 0.7, 0.2), 0.3) & ((0.5, 0.8, 0.9), 0.8) \end{bmatrix}$ be two neutrosophic hybrid matrices. Then, $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = \begin{bmatrix} ((0.3, 0.5, 0.9), 0.4) & ((0.3, 0.7, 0.5), 0.7) \\ ((0.3, 0.7, 0.2), 0.8) & ((0.1, 0.8, 0.9), 0.8) \end{bmatrix}$.

Theorem 6.10. *Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$ and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$ be two neutrosophic hybrid matrices of same order over the hybrid structure $\left(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\right)$. Then the following results related to the operations hold.*

(1) $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$
(2) $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$
(3) $\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c\right)^c = [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$
(4) $\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)^c = [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c$
(5) $\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)^c = [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c$.

Proof. Let $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{P}_{\widehat{\mathcal{N}}_\eta}, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{N}(\mathcal{Q}_{\widehat{\mathcal{N}}_\gamma}, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$

(1) $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$

$$\begin{aligned}
[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] &= \left[\mathfrak{M}\left(\left(\mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), \eta(\varrho)\right)\right] \\
&\quad \sqcup \left[\mathfrak{N}\left(\left(\mathcal{T}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho)\right), \gamma(\varrho)\right)\right] \\
&= \left[\mathfrak{L}\left(\left(\mathcal{T}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{I}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{F}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho)\right), \min\{\eta(\varrho), \gamma(\varrho)\}\right)\right] \\
&= \left[\mathfrak{L}\left(\left(\mathcal{T}_{\vee(\mathcal{Q},\mathcal{P})}(\varrho), \mathcal{I}_{\wedge(\mathcal{Q},\mathcal{P})}(\varrho), \mathcal{F}_{\wedge(\mathcal{Q},\mathcal{P})}(\varrho)\right), \min\{\eta(\varrho), \gamma(\varrho)\}\right)\right] \\
&= \left[\mathfrak{N}\left(\left(\mathcal{T}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\alpha)\right), \gamma(\varrho)\right)\right] \\
&\quad \sqcup \left[\mathfrak{M}\left(\left(\mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), \eta(\alpha)\right)\right] \\
&= [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}].
\end{aligned}$$

(2) $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$
Proof is similar to (1).

(3) $\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c\right)^c = [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$
Since $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c = (\mathfrak{m}^c_{ij})_{m\times n} = \left[\mathfrak{M}\left(\left(\mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\eta(\varrho)\right)\right]$

$$\begin{aligned}
\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathbb{H}}}]^c\right)^c &= \left[\mathfrak{M}\left(\left(\mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\eta(\varrho)\right)\right]^c \\
&= \left[\mathfrak{M}\left(\left(\mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\{1-\eta(\varrho)\}\right)\right] \\
&= \left[\mathfrak{M}\left(\left(\mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), \eta(\varrho)\right)\right] \\
&= [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}].
\end{aligned}$$

(4) $\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup \mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)^c = [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c$

Since $[\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c = (\mathfrak{m}^c_{ij})_{m\times n} = \left[\mathfrak{M}\left(\left(\mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\eta(\varrho)\right)\right]$

and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c = (\mathfrak{n}^c_{ij})_{m\times n} = \left[\mathfrak{N}\left(\left(\mathcal{F}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\gamma(\varrho)\right)\right]$

$$\begin{aligned}
\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)^c &= \left[\mathfrak{L}\left(\left(\mathcal{T}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{I}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{F}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho)\right), \min\{\eta(\varrho), \gamma(\varrho)\}\right)\right]^c \\
&= \left[\mathfrak{L}\left(\left(\mathcal{F}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{I}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{T}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho)\right), \max\{1-\eta(\varrho), 1-\gamma(\varrho)\}\right)\right] \\
&= \left[\mathfrak{M}\left(\left(\mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\eta(\varrho)\right)\right] \\
&\quad \sqcap \left[\mathfrak{N}\left(\left(\mathcal{F}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{T}_{\mathcal{Q}_{\widehat{\mathcal{N}}}}(\varrho)\right), 1-\gamma(\varrho)\right)\right] \\
&= [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathbb{H}}}]^c \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c.
\end{aligned}$$

(5) $\left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)^c = [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]^c$

Proof is similar to (4).

Theorem 6.11. *Let $[\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}], [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$ and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]$ be three hybrid matrices of same order over the hybrid structure $\left(\mathcal{X}_{\widehat{\mathcal{N}}_\lambda}, \mathbb{H}\right)$. Then the following results related to the operations hold.*

(1) $\left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$

(2) $\left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$

(3) $[\mathfrak{L}\widehat{\mathcal{N}}_{\mathfrak{H}}] \sqcup \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) = \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcap \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$

(4) $[\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) = \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$.

Proof. Let $[\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{L}(\mathcal{P}_{\widehat{\mathcal{N}}_\nu}, \mathbb{H})] = (\mathfrak{l}_{ij})_{m\times n}, [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{M}(\mathcal{Q}_{\widehat{\mathcal{N}}_\eta}, \mathbb{H})] = (\mathfrak{m}_{ij})_{m\times n}$ and $[\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{N}(\mathcal{R}_{\widehat{\mathcal{N}}_\gamma}, \mathbb{H})] = (\mathfrak{n}_{ij})_{m\times n}$

(1) $\left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$

$$\begin{aligned}
LHS &= \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathbb{H}}}]\right) \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \\
&= \left[\mathfrak{S}\left(\left(\mathcal{T}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho), \ddot{\mathcal{I}}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{F}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho)\right), \min\{\nu(\varrho), \eta(\varrho)\}\right)\right] \\
&\quad \sqcup \left[\mathfrak{N}\left(\left(\mathcal{T}_{\mathcal{R}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{R}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{R}_{\widehat{\mathcal{N}}}}(\varrho)\right), \gamma(\varrho)\right)\right] \\
&= \left[\mathfrak{V}\left(\left(\mathcal{T}_{\vee(\mathcal{P},\mathcal{Q},\mathcal{R})}(\varrho), \mathcal{I}_{\wedge(\mathcal{P},\mathcal{Q},\mathcal{R})}(\varrho), \mathcal{F}_{\wedge(\mathcal{P},\mathcal{Q},\mathcal{R})}(\varrho)\right), \min\{\nu(\varrho), \eta(\varrho), \gamma(\varrho)\}\right)\right] \\
&= \left[\mathfrak{M}\left(\left(\mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), \nu(\varrho)\right)\right] \\
&\quad \sqcup \left[\mathfrak{S}\left(\left(\mathcal{T}_{\vee(\mathcal{Q},\mathcal{R})}(\varrho), \mathcal{I}_{\wedge(\mathcal{Q},\mathcal{R})}(\varrho), \mathcal{F}_{\wedge(\mathcal{Q},\mathcal{R})}(\varrho)\right), \min\{\eta(\varrho), \gamma(\varrho)\}\right)\right] \\
&= [\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right).
\end{aligned}$$

(2) $\left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] = [\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$

Proof is similar to (1).

(3) $[\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) = \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcap \left([\mathfrak{L}\_\widehat{\mathcal{N}}_{\mathfrak{H}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$

$$\begin{aligned}
&[\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)\\
&= \left[\mathfrak{M}\left(\left(\mathcal{T}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{I}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho), \mathcal{F}_{\mathcal{P}_{\widehat{\mathcal{N}}}}(\varrho)\right), \nu(\varrho)\right)\right]\\
&\quad \sqcup \left[\mathfrak{S}\left(\left(\mathcal{T}_{\wedge(\mathcal{Q},\mathcal{R})}(\varrho), \mathcal{I}_{\vee(\mathcal{Q},\mathcal{R})}(\varrho), \mathcal{F}_{\vee(\mathcal{Q},\mathcal{R})}(\varrho)\right), \max\{\eta(\varrho), \gamma(\varrho)\}\right)\right]\\
&= \left[\mathfrak{Z}\left(\left(\mathcal{T}_{\vee\{\mathcal{P},\wedge(\mathcal{Q},\mathcal{R})\}}, \mathcal{I}_{\wedge\{\mathcal{P},\vee(\mathcal{Q},\mathcal{R})\}}, \mathcal{F}_{\wedge\{\mathcal{P},\vee(\mathcal{Q},\mathcal{R})\}}\right), \min\{\nu(\varrho), \max\{\eta(\varrho), \gamma(\varrho)\}\}\right)\right]\\
&= \left[\mathfrak{V}\left(\left(\mathcal{T}_{\vee(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{I}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho), \mathcal{F}_{\wedge(\mathcal{P},\mathcal{Q})}(\varrho)\right), \min\{\nu(\varrho), \eta(\varrho)\}\right)\right]\\
&\quad \left[\mathfrak{W}\left(\left(\mathcal{T}_{\vee(\mathcal{P},\mathcal{R})}(\varrho), \mathcal{I}_{\wedge(\mathcal{P},\mathcal{R})}(\varrho), \mathcal{F}_{\wedge(\mathcal{P},\mathcal{R})}(\varrho)\right), \min\{\nu(\varrho), \eta(\varrho)\}\right)\right]\\
&= \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcap \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right).
\end{aligned}$$

(4) $[\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap \left([\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcup [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) = \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{M}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right) \sqcup \left([\mathfrak{L}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}] \sqcap [\mathfrak{N}_{\widehat{\mathcal{N}}_{\mathfrak{H}}}]\right)$
Proof is similar to (3).

7. MCDM based on neutrosophic hybrid matrix

This section starts with an algorithm for solving a multi-criteria decision making problem based on neutrosophic hybrid matrices using the notion of comparison matrices. The algorithm is described by a suitable example.

Definition 7.1. Comparison matrix is a matrix whose rows are the different groups $g_1, g_2, \ldots, g_n$ and the columns are the parameters $\varrho_1, \varrho_2, \ldots, \varrho_n$.
The elements of the matrix are calculated by $c_{ij} = (s_{ij} = a + b - c, w_{ij} = d)$, where a, b, c and d are integers calculated as how many times $T_{h_i}(e_j)$ exceeds or equal to $T_{h_k}(e_j), I_{h_i}(e_j)$ exceeds or equal to $I_{h_k}(e_j), F_{h_i}(e_j)$ exceeds or equal to $F_{h_k}(e_j)$and $w_{h_i}(e_j)$ exceeds or equal to $w_{h_k}(e_j)$ for $h_j \neq h_k, \forall h_k \in \mathcal{U}$, respectively.

Definition 7.2. The score of an object g_i is $S_i = \sum_j s_{ij}$. The weightage of an object g_i is $W_i = \sum_j w_{ij}$.

Development in technology is aimed at betterment of life style of people worldwide. Especially technological developments have mixed effects on the study habits and attitudes of student, both good and adverse, that is support and distraction result as a consequence of technological development. We try to analyze the impact of technology on students life using the following algorithm as an MCDM based on neutrosophic hybrid matrices.

7.1. *Algorithm*

The steps of the algorithm for decision making using the construction of a comparison matrix are given below.

Step 1: Identify the possible subsets of the parameter set and neutrosophic hybrid set.

Step 2: Find the neutrosophic hybrid matrix.

Step 3: Compute the comparison matrix of the neutrosophic hybrid matrix.

Step 4: Compute the score S_i? and weightage W_i of g_i.

Also find $S_k = \max?S_i$ and $W_k = \max?W_i$.

Step 5: Determine the result, if the scores are equal we consider the weightage.

Example 7.3. We analyze the study habits and attitudes of the student groups from the particular city using the above algorithm.

Step 1: Let $\mathcal{U} = \{g_1, g_2, g_3, g_4, g_5\}$ be the set of group of students. Consider the parameters as changes in student study habits and attitudes like maximum, average and minimum change. That is the parameter set is given by

$\mathbb{H} = \{\varrho_1 = \text{maximum change}, \varrho_2 = \text{average change}, \varrho_3 = \text{minimum change}\}$.

Step 2: Consider the neutrosophic hybrid matrix whose rows are the different group of students $\{g_1, g_2, g_3, g_4, g_5\}$ and the columns are the parameters $\varrho_1, \varrho_2, \varrho_3$.

$$[\mathfrak{M}_{\widehat{\mathcal{N}_{\mathfrak{H}}}}] = \begin{bmatrix} ((0.2, 0.6, 0.5), 0.4) & ((0.3, 0.9, 0.2), 0.1) & ((0.3, 0.4, 0.5), 0.6) \\ ((0.8, 0.3, 0.9), 0.1) & ((0.9, 0.8, 0.4), 0.3) & ((0.1, 0.4, 0.2), 0.4) \\ ((0.2, 0.1, 0.5), 0.5) & ((0.7, 0.6, 0.9), 0.2) & ((0.3, 0.7, 0.1), 0.3) \\ ((0.7, 0.4, 0.3), 1) & ((0.3, 0.5, 0.8), 0.2) & ((0.3, 0.4, 0.6), 0.5) \\ ((0.5, 0.4, 0.2), 0.4) & ((0.2, 0.5, 0.5), 0) & ((0.4, 0.6, 0.8), 0.7) \end{bmatrix}.$$

Step 3: The comparison matrix of the above neutrosophic hybrid matrix is

$$[c_{ij}] = \begin{bmatrix} (2, 3) & (6, 1) & (1, 3) \\ (1, 1) & (6, 4) & (1, 1) \\ (-2, 4) & (1, 3) & (5, 0) \\ (4, 1) & (0, 3) & (0, 2) \\ (4, 3) & (-1, 0) & (3, 4) \end{bmatrix}.$$

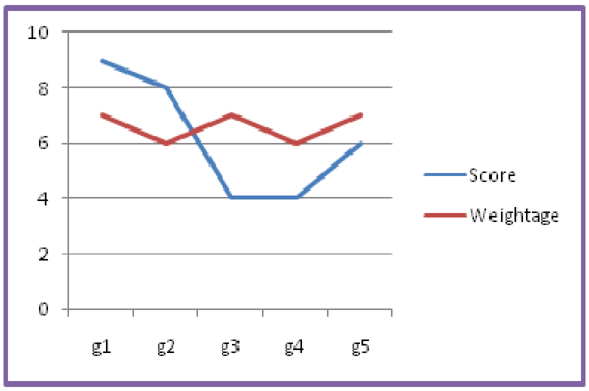
Step 4:

Now we compute the score and weightage for each group g_i,

Table 2. Representation of the score and weightage for each group

$\mathbb{H}$	**Score** (S_i)	**Weightage** (W_i)
g_1	9	7
g_2	8	6
g_3	4	7
g_4	4	6
g_5	6	7

The graphical representation of the score and weightage for each group g_i,



Step 5: The maximum score is secured by group 1. That is the group 1 of the students almost adopt the usage of technology. So this group of students have maximum changes in study habit and attitude. Group 3 and group 4 secured minimum score. But weightage of group 4 is less than group 3. So the students of group 4 have minimum changes in study habit and attitude. Rest of the groups are average changes in study habit and attitude.

We compare our result with that of Maji [13]. Both methods give the same scores for each group. In Maji's [13] method the decision becomes random where more than one group have equal scores. This difficulty is overcome in our method using weights in neutrosophic hybrid matrices. This facilitates for choice of better group among the ones with identical score.

8. **Conclusion**

The new notions of hybrid matrices and neutrosophic hybrid matrices are introduced and some of their theoretical properties are studied. We have also developed an algorithm for solving a MCDM problem using neutrosophic hybrid matrices. As future research direction we contemplate to provide more methods for solving multiple criteria decision making (MCDM) problems based on hybrid matrices and neutrosophic hybrid matrices.

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