NSS
Neutrosophic Sets and Systems, Vol. 60, 2023
University of New Mexico

Properties of neutrosophic $\varkappa$-ideals in subtraction semigroups

G. Muhiuddin[1], B. Elavarasan [2], K. Porselvi[3] and D. Al-Kadi[4]
[1] Department of Mathematics, University of Tabuk, P.O. Box-741, Tabuk-71491, Saudi Arabia.
[1] E-mail:chishtygm@gmail.com.
[2,3]Department of Mathematics, Karunya Institute of Technology and Sciences, Coimbatore - 641 114, India.
[2]E-mail:belavarasan@gmail.com; [3] E-mail:porselvi94@yahoo.co.in.
[4] Department of Mathematics and Statistic, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia. [4]E-mail: d.alkadi@tu.edu.sa
*Correspondence: belavarasan@gmail.com

Abstract. Our aim is to explore the idea of neutrosophic $\mathfrak{N}-$ideals in near-subtraction semigroups in this article and obtain some outcomes that are equivalent to them. We also illustrate the notion of a neutrosophic $\varkappa-$ intersection. Additionally, in a near-subtraction semigroup, we examine the term homomorphism of a neutrosophic $\varkappa-$ structure and establish some conclusions based on a homomorphic neutrosophic $\varkappa-$ structure preimage of a neutrosophic $\varkappa-$ left (respectively, right) ideal.
Keywords: Semigroups; Subtraction semigroups; neutrosophic $\varkappa-$structures, neutrosophic $\varkappa-$ ideals, homomorphism.

---

1. Introduction

In [26], Schein investigated the systems of the type $(\Sigma, \circ, \setminus)$, where Σ is a family of functions closed under the composition $\circ$ of functions (and therefore $(\Sigma, \circ)$ is a function semigroup) and the set theoretic subtraction $\setminus$ (and therefore $(\Sigma, \setminus)$ is a subtraction algebra). In [29], Zelinka examined Schein's suggestion for the multiplication structure and discovered a method for resolving a challenge in a kind of subtraction algebra, namely atomic subtraction algebras. In subtraction algebras [11], Jun et al. proposed the idea of ideals by examining the characterisation of ideals. In [10], Jun et al. explored the ideals produced by a set and its associated outcomes. Dheena et al. [1], formed the ideas of near-subtraction semigroups as well as strongly regular near-subtraction semigroups. They found an equivalent assertion for a near-subtraction semigroup to be strongly regular.

Zadeh [30] developed the idea that a fuzzy subset φ of a set K is a map from K into $[0, 1]$. Since then, this concept has been effectively used in a range of applications, including image processing, control systems, engineering, robotics, industrial automation, and optimisation.

In subtraction algebras, Lee et al. [14] established the term fuzzy ideal and made some assertions that a fuzzy set is to be a fuzzy ideal. Prince Williams [28] coined the terms fuzzy ideals and fuzzy intersection in near-subtraction semigroups and homomorphic fuzzy images and preimages of a near-subtraction semigroup.

In [16], Molodtsov introduced a concept, namely the soft set $(F, \Im)$, which is a mapping from $\Im$ into the power set of $\mathbb{U}$ given a base universe set $\mathbb{U}$ and the gathering of attributes $\Im$. Jun et al. [12] extended Molodtsov's concept to hybrid structures, a concept that is similar to the theories of soft and fuzzy sets, and proved a number of hybrid structure attributes for a gathering of parameter values over a base universe set. The authors further explored the ideas of hybrid subalgebras, and hybrid fields based on this approach. Several authors produced hybrid concepts in a variety of algebraic structures (See [2–5, 15, 17, 18, 20–23]).

Smarandache came up with neutrophophic sets as a way to deal with the constant unpredictability. It makes intuitionistic fuzzy sets as well as fuzzy sets more broad. Neutrosophic sets can be described by these three things: their membership functions for indeterminacy (I), falsity (F), and truth (T). These sets can be used in a lot of different ways to deal with the problems that come from unclear information. A neutrosophic set can tell the difference between membership functions that are absolute and those that are relative. Smarandache used these collections for non-standard analyses like sports choices (losing, tying, and winning), control theory, decision-making theory, and so on. This area has been studied by several authors(See [8, 9, 27]).

Khan et al. examined ϵ-neutrosophic $\varkappa$-subsemigroup and a semigroup in [13]. Elavarasan et al. [6] examined the idea of neutrosophic $\varkappa$-ideals in semigroups. Elavarasan et al. presented neutrosophic filters and bi-filters in a semigroup and examined their properties in [7]. Muhiuddin et al. provided the definitions and characteristics of neutrosophic $\varkappa$-interior ideals as well as neutrosophic $\varkappa$- ideals in ordered semigroups in [19].

Porselvi et al. proposed neutrosophic $\varkappa$-interior ideal structure as well as neutrosophic $\varkappa$-simple in semigroups in [25], and they obtained comparable statements for the two types of structures. Porselvi et al. [24] described numerous characteristics of a neutrosophic $\varkappa$-bi-ideal structure in a semigroup and showed that when a semigroup is regular left duo, both a neutrosophic $\varkappa$-right ideal and a neutrosophic $\varkappa$-bi-ideal are identical. They discussed analogous claims for the regular semigroup with regard to the neutrosophic $\varkappa$-product.

This article explores the idea of neutrosophic $\varkappa-$ideal in near-subtraction semigroups and its associated characteristics. Additionally, we provide examples of a neutrosophic $\varkappa$-left ideal that is not a neutrosophic $\varkappa$-right ideal and vice versa. Moreover, we examine and discuss the neutrosophic $\varkappa$-image, neutrosophic $\varkappa$-intersection, and neutrosophic $\varkappa$-preimage of a near-subtraction semigroup using homomorphism.

2. Preliminaries of subtraction semigroups

We compile some basic definitions for near-subtraction semigroups in this portion, which we will use in the next section.

Definition 2.1. [26] A set $\Im(\neq \emptyset)$ with the binary operation " $-$ " that fulfils the below assertions is referred to as a subtraction algebra. $\forall q_0, l_0, i_0 \in \Im$,

(i) $q_0 - (l_0 - q_0) = q_0$.

(ii) $q_0 - (q_0 - l_0) = l_0 - (l_0 - q_0)$.

(iii) $(q_0 - l_0) - i_0 = (q_0 - i_0) - l_0$.

The following are some characteristics of a subtraction algebra:

(i) $q_0 - 0 = q_0$ and $0 - q_0 = 0$.

(ii) $(q_0 - l_0) - q_0 = 0$.

(iii) $(q_0 - l_0) - l_0 = q_0 - l_0$.

(iv) $(q_0 - l_0) - (l_0 - q_0) = q_0 - l_0$, where $0 = q_0 - q_0$ is an element that is independent on the choice of $q_0 \in \Im$.

Definition 2.2. [29] A set $\Im(\neq \emptyset)$ with the binary operations " $-$ " and "." that satisfies the following requirements is referred to as a subtraction semigroup:

(i) $(\Im, -)$ and $(\Im, .)$ are a subtraction algebra and a semigroup, respectively.

(ii) $l_0(l_1 - l_2) = l_0l_1 - l_0l_2$ and $(l_0 - l_1)l_2 = l_0l_2 - l_1l_2$ $\forall l_0, l_1, l_2 \in \Im$.

Definition 2.3. [29] A set $\Im(\neq \emptyset)$ with the binary operations " $-$ " and "." that satisfy the following requirements is referred to as a near-subtraction semigroup (NSS for short):

(i) $(\Im, -)$ and $(\Im, .)$ are a subtraction algebra and a semigroup, respectively.

(ii) $(l_0 - l_1)l_2 = l_0l_2 - l_1l_2$ $\forall l_0, l_1, l_2 \in \Im$.

Clearly $0l_0 = 0$ $\forall l_0 \in \Im$.

Hereafter, $\Im$ represents the near-subtraction semigroup.

Definition 2.4. If $l_0 - l_1 \in L$ whenever $l_0, l_1 \in L$, then a subset $L(\neq \emptyset)$ of $\Im$ is said to be a subalgebra of $\Im$.

Definition 2.5. Let $(\Im, -, .)$ be a NSS. A subset $\Re(\neq \emptyset)$ of $\Im$ is referred as

(i) a right ideal whenever $\Re$ is a subalgebra of $(\Im, -)$ and $\Re\Im \subseteq \Re$.

(ii) a left ideal whenever $\Re$ is a subalgebra of $(\Im, -)$ and $p_1c_1 - p_1(w_1 - c_1) \in \Re$ $\forall p_1, w_1 \in \Im; c_1 \in \Re$.

(iii) an ideal whenever $\Re$ is both a right and a left ideal.

3. Preliminaries of Neutrosophic $\varkappa$- structures

This portions outlines the basic ideas of neutrosophic $\varkappa$-structures of $\Im$, which are essential for the sequel.

For a set $Q(\neq \emptyset)$, $\mathcal{F}(Q,\mathbb{I}^-)$ is the family of functions with negative-values from a set Q to $\mathbb{I}^-$, where $\mathbb{I}^- = [-1,0]$. An element $k_1 \in \mathcal{F}(Q,\mathbb{I}^-)$ is known as a $\varkappa$-function on Q and $\varkappa$-structure denotes (Q,k_1) of X.

Definition 3.1. [12] For a set $Q(\neq \emptyset)$, a *neutrosophic $\varkappa$- structure* of Q is described as below:

$$Q_M := \frac{Q}{(T_M,I_M,F_M)} = \left\{\frac{v_0}{(T_M(v_0),I_M(v_0),F_M(v_0))} \ : \ v_0 \in Q\right\},$$

where T_M on Q means the negative truth membership function, I_M on Q means the negative indeterminacy membership function and F_M on Q means the negative false membership function.

Note 3.2. *Q_M satisfies the requirement:* $-3 \leq T_M(b_1) + I_M(b_1) + F_M(b_1) \leq 0 \ \forall b_1 \in Q$.

Definition 3.3. [13] For a set $Q(\neq \emptyset)$, let $Q_J := \frac{Q}{(T_J,I_J,F_J)}$ and $Q_V := \frac{Q}{(T_V,I_V,F_V)}$,

(i) Q_J is defined as a *neutrosophic $\varkappa$-substructure* of Q_V, represented by $Q_J \subseteq Q_V$, if it fulfils the below criteria: for any $z_0 \in Q$,

$$T_J(z_0) \geq T_V(z_0), I_J(z_0) \leq I_V(z_0), F_J(z_0) \geq F_V(z_0).$$

If $Q_J \subseteq Q_V$ and $Q_V \subseteq Q_J$, then $Q_J = Q_V$.

(ii) The intersection of Q_J and Q_V is a neutrosophic $\varkappa$-structure over Q and is defined as follows: $Q_J \cap Q_V = Q_{J\cap V} = (Q; T_{J\cap V}, I_{J\cap V}, F_{J\cap V})$, where

$$\begin{aligned}(T_J \cap T_V)(h_0) =& T_{J\cap V}(h_0) = T_J(h_0) \vee T_V(h_0),\\ (I_J \cap I_V)(h_0) =& I_{J\cap V}(h_0) = I_J(h_0) \wedge I_V(h_0),\\ (F_J \cap F_V)(h_0) =& F_{J\cap V}(h_0) = F_J(h_0) \vee F_V(h_0) \ for\ any\ h_0 \in Q.\end{aligned}$$

Definition 3.4. For $V_0 \subseteq Q \neq \emptyset$, consider the neutrosophic $\varkappa$-structure

$$\chi_{V_0}(Q_D) = \frac{Q}{(\chi_V(T)_D, \chi_V(I)_D, \chi_V(F)_D)},$$

where

$$\chi_{V_0}(T)_D : Q \to \mathbb{I}^-, \ j_1 \to \begin{cases} -1 & if\ j_1 \in V_0 \\ 0 & if\ j_1 \notin V_0, \end{cases}$$

$$\chi_{V_0}(I)_D : Q \to \mathbb{I}^-, \ j_1 \to \begin{cases} 0 & if\ j_1 \in V_0 \\ -1 & if\ j_1 \notin V_0, \end{cases}$$

$$\chi_{V_0}(F)_D : Q \to \mathbb{I}^-, \ j_1 \to \begin{cases} -1 & if\ j_1 \in V_0 \\ 0 & if\ j_1 \notin V_0, \end{cases}$$

which is described as the *characteristic neutrosophic $\varkappa$-structure* of V_0 over Q.

Definition 3.5. [12] For a nonempty set Q, let $Q_N = \frac{Q}{(T_N, I_N, F_N)}$ and $\eth, \varphi, \Theta \in \mathbb{I}^-$ with $-3 \leq \eth + \varphi + \Theta \leq 0$. Consider the following sets:

$T_N^{\eth} = \{c_1 \in Q \mid T_N(c_1) \leq \eth\}, I_N^{\varphi} = \{c_1 \in Q \mid I_N(c_1) \geq \varphi\}, F_N^{\Theta} = \{c_1 \in Q | F_N(c_1) \leq \Theta\}$. Then the set $Q_N(\eth, \varphi, \Theta) = \{c_1 \in Q | T_N(c_1) \leq \eth, I_N(c_1) \geq \varphi, F_N(c_1) \leq \Theta\}$ is referred as a $(\eth, \varphi, \Theta)$-*level set* of Q_N. Note that $Q_N(\eth, \varphi, \Theta) = T_N^{\eth} \cap I_N^{\varphi} \cap F_N^{\Theta}$.

4. Neutrosophic $\varkappa$-ideals in subtraction semigroups

The idea of neutrosophic $\varkappa-$ ideals in near-subtraction is defined in this portion. We also develop a case where a neutrosophic $\varkappa-$ right ideal is not a neutrosophic $\varkappa-$ left ideal, and vice versa, and we describe certain properties of a neutrosophic $\varkappa-$ structure's homomorphism in a near-subtraction semigroup.

Definition 4.1. A neutrosophic $\varkappa$-structure $\Im_B = \frac{\Im}{(T_B, I_B, F_B)}$ of $\Im$ is defined as a *neutrosophic $\varkappa$-ideal* of $\Im$ if it meets the below axioms:

(i) $(\forall g_0, l_0 \in \Im) \begin{pmatrix} T_B(g_0 - l_0) \leq T_B(g_0) \vee T_B(l_0) \\ I_B(g_0 - l_0) \geq I_B(g_0) \wedge I_B(l_0) \\ F_B(g_0 - l_0) \leq F_B(g_0) \vee F_B(l_0) \end{pmatrix}$.

(ii) $(\forall s_0, j_0, l_0 \in \Im) \begin{pmatrix} T_B(s_0 l_0 - s_0(j_0 - l_0)) \leq T_B(l_0) \\ I_B(s_0 l_0 - s_0(j_0 - l_0)) \geq I_B(l_0) \\ F_B(s_0 l_0 - s_0(j_0 - l_0)) \leq F_B(l_0) \end{pmatrix}$.

(iii) $(\forall l_0, q_0 \in \Im) \begin{pmatrix} T_B(l_0 q_0) \leq T_B(l_0) \\ I_B(l_0 q_0) \geq I_B(l_0) \\ F_B(l_0 q_0) \leq F_B(l_0) \end{pmatrix}$.

Note that $\Im_B$ of $\Im$ is a *neutrosophic $\varkappa-$left ideal* when (i) and (ii) are hold, and $\Im_B$ of $\Im$ is a *neutrosophic $\varkappa-$right ideal* when (i) and (iii) are hold.

Notation 1. *Let $\Im$ be a NSS. Then we use the below notations:*

(i) $\mathscr{N}_{\mathfrak{I}}(\Im)$ is the gathering of all neutrosophic $\varkappa-$ ideals of $\Im$.

(ii) $\mathscr{N}_{\mathfrak{R}}(\Im)$ is the gathering of all neutrosophic $\varkappa-$ right ideals of $\Im$.

(iii) $\mathscr{N}_{\mathfrak{L}}(\Im)$ is the gathering of all neutrosophic $\varkappa-$ left ideals of $\Im$.

Here are a few examples of neutrosophic $\varkappa$-ideals.

Example 4.2. Let $\Im = \{0, i_0, p_0\}$ be a set with two operations " $-$ " and "." that are represented by the below tables:

-	0	i_0	p_0
0	0	0	0
i_0	i_0	0	i_0
p_0	p_0	p_0	0

.	0	i_0	p_0
0	0	0	0
i_0	0	i_0	0
p_0	i_0	0	p_0

Then $(\Im, -, .)$ is a NSS. Define a neutrosophic $\varkappa$-structure $\Im_N := \{\frac{0}{(w,l,w_1)}, \frac{i_0}{(r,k,r_1)}, \frac{p_0}{(y,v,y_1)}\}$ of $\Im$ for $v, k, l, w, w_1, r, r_1, y, y_1 \in [-1, 0]$.

(i) If $y > r = w; v < k = l$ and $y_1 > r_1 = w_1$, then $\Im_N \in \mathscr{N}_{\mathfrak{I}}(\Im)$.

(ii) If $y = r > w; k = v < l$ and $y_1 = r_1 > w_1$, then $\Im_N \in \mathscr{N}_{\mathfrak{R}}(\Im)$, but $\Im_N \notin \mathscr{N}_{\mathfrak{L}}(\Im)$ as $T_N(p_0.0-p_0(p_0-0)) = T_N(i_0) = r \nleq w = T_N(0); I_N(p_0.0-p_0(p_0-0)) = I_N(i_0) = k \ngeq l = I_N(0)$ and $F_N(p_0.0 - p_0(p_0 - 0)) = F_N(i_0) = r_1 \nleq w_1 = F_N(0)$.

(iii) If $r > y > w; k < v < l$ and $r_1 > y_1 > w_1$, then $\Im_N$ is neither in $\mathscr{N}_{\mathfrak{R}}(\Im)$ nor in $\mathscr{N}_{\mathfrak{L}}(\Im)$ as $T_N(p_0.0 - p_0(i_0 - 0)) = T_N(i_0) = r \nleq w = T_N(0)$, $I_N(p_0.0 - p_0(i_0 - 0)) = I_N(i_0) = k \ngeq l = I_N(0)$, $F_N(p_0.0 - p_0(i_0 - 0)) = F_N(i_0) = r_1 \nleq w_1 = F_N(0)$ and $T_N(p_0.0) = T_N(i_0) = r \nleq y = T_N(p_0)$, $I_N(p_0.0) = I_N(i_0) = k \ngeq v = I_N(p_0)$, $F_N(p_0.0) = F_N(i_0) = r_1 \nleq y_1 = F_N(p_0)$. But it fulfils the assertion (i) of Definition 4.1.

Example 4.3. Let $\Im = \{0, r, l, k\}$ be a set with two operations " − " and "." are given by

-	0	r	l	k
0	0	0	0	0
r	r	0	k	l
l	l	0	0	l
k	k	0	k	0

.	0	r	l	k
0	0	0	0	0
r	0	r	l	k
l	0	0	0	0
k	0	r	l	k

Then $(\Im, -, .)$ is a NSS. For $p, w, n, m, m_1, y, y_1, s, s_1 \in [-1, 0]$, define a neutrosophic $\varkappa$-structure $\Im_N := \{\frac{0}{(m,p,m_1)}, \frac{r}{(y,w,y_1)}, \frac{l}{(s,n,s_1)}, \frac{k}{(s,n,s_1)}\}$ of $\Im$. If $s > y > m, n < w < p$ and $s_1 > y_1 > m_1$, then $\Im_N \in \mathscr{N}_{\mathfrak{L}}(\Im)$, but $\Im_N \notin \mathscr{N}_{\mathfrak{R}}(\Im)$ as $T_N(r.l) = T_N(l) = s \nleq y = T_N(r)$, $I_N(r.l) = I_N(l) = n \ngeq w = I_N(r)$ and $F_N(r.l) = F_N(l) = s_1 \nleq y_1 = F_N(r)$.

Theorem 4.4. *For $\Im_N = \frac{\Im}{(T_N, I_N, F_N)}$, the listed assertions are equivalent:*

(i) For any $\varrho, \lambda, \nu \in \mathbb{I}^-$, $\Im_N(\varrho, \lambda, \nu)(\neq \phi)$ of $\Im$ is a left(right) ideal,

(ii) $\Im_N \in \mathscr{N}_{\mathfrak{L}}(\Im)$ $(\mathscr{N}_{\mathfrak{R}}(\Im))$.

Proof: *(i)* ⇒ *(ii)* Let $c, z \in \Im$. Then $T_N(c) = q_1; F_N(c) = r_1; I_N(c) = t_1$ and $T_N(z) = q_2; F_N(z) = r_2; I_N(z) = t_2$, for some $q_1, q_2, t_1, t_2, r_1, r_2 \in \mathbb{I}^-$.

If $q = max\{q_1, q_2\}; t = min\{t_1, t_2\}$ and $r = max\{r_1, r_2\}$, then $T_N(c) \leq q; I_N(c) \geq t; F_N(c) \leq r$ and $T_N(z) \leq q; I_N(z) \geq t; F_N(z) \leq r$, so $c, z \in \Im_N(q, t, r)$. By assumption, we get $c - z \in \Im_N(q, t, r)$ which implies $T_N(c-z) \leq q = T_N(c) \vee T_N(z)$; $I_N(c-z) \geq t = I_N(c) \wedge I_N(z)$; $F_N(c-z) \leq r = F_N(c) \vee F_N(z)$.

For any $n_0, v \in \Im$, we have $n_0c-n_0(v-c) \in \Im_N(q_1, t_1, r_1)$ which implies $T_N(n_0c-n_0(v-c)) \leq q_1 = T_n(c), I_N(n_0c-n_0(v-c)) \geq t_1 = I_N(c), F_N(n_0c-n_0(v-c)) \leq r_1 = F_N(c)$. So $\Im_N \in \mathscr{N}_{\mathfrak{L}}(\Im)$.

Also, for $r \in \Im$, we have $cr \in \Im_N(q_1, t_1, r_1)$ which implies $T_N(cr) \leq q_1 = T_N(c); I_N(cr) \geq t_1 = I_N(c); F_N(cr) \leq r_1 = F_N(c)$. So $\Im_N \in \mathscr{N}_{\mathfrak{R}}(\Im)$.

(ii) ⇒ *(i)* Let $q, z \in \Im_N(\varrho, \lambda, \nu)$. Then $T_N(q - z) \leq T_N(q) \vee T_N(z) \leq \varrho$; $I_N(q - z) \geq I_N(q) \wedge I_N(z) \geq \lambda$ and $F_N(q - z) \leq F_N(q) \vee F_N(z) \leq \nu$ which imply $q - z \in \Im_N(\varrho, \lambda, \nu)$.

Also, $T_N(qz) \leq T_N(q) \leq \varrho$; $I_N(qz) \geq I_N(q) \geq \lambda$ and $F_N(qz) \leq F_N(q) \leq \nu$ imply that $qz \in \Im_N(\varrho, \lambda, \nu)$. So $\Im_N(\varrho, \lambda, \nu)$ of $\Im$ is a right ideal.

For $l \in \Im_N(\varrho, \lambda, \nu)$ and $s, j \in \Im$, we have $T_N(sl - s(j-l)) \leq T_N(l) = \varrho; I_N(sl - s(j-l)) \geq I_N(l) = \lambda$ and $F_N(sl - s(j-l)) \leq F_n(l) = \nu$ which imply $sl - s(j-l) \in \Im_N(\varrho, \lambda, \nu)$.

So, $\Im_N(\varrho, \lambda, \nu)$ of $\Im$ is a left ideal.

We have the succeeding corollary as a outcome of the Theorem 4.4.

Corollary 4.5. *For $\emptyset \neq D \subseteq \Im$, a neutrosophic $\varkappa$- structure $\Im_N = \frac{\Im}{(T_N, I_N, F_N)}$ of $\Im$ is characterized as below: For $g_1, l_1, \omega_1, t_1, s_1, v_1 \in [-1, 0]$,*

$$T_N(y_0) := \begin{cases} g_1 & if\ y_0 \in D \\ l_1 & otherwise \end{cases}; \quad I_N(y_0) := \begin{cases} \omega_1 & if\ y_0 \in D \\ t_1 & otherwise, \end{cases}; \quad F_N(y_0) := \begin{cases} s_1 & if\ y_0 \in D \\ v_1 & otherwise, \end{cases}$$

where $g_1 < l_1; \omega_1 > t_1$ and $s_1 < v_1$ in $[-1, 0]$, the mentioned below statements are equivalent:

(i) $\Im_N \in \mathscr{N}_{\mathfrak{L}}(\Im)(\mathscr{N}_{\mathfrak{R}}(\Im))$,

(ii) D of $\Im$ is a left(right) ideal.

Corollary 4.6. *For $\emptyset \neq L \subseteq \Im$ and $\Im_N = \frac{\Im}{(T_N, I_N, F_N)}$, the listed below statements are equivalent:*

(i) $\chi_L(\Im_N) \in \mathscr{N}_{\mathfrak{L}}(\Im)(\mathscr{N}_{\mathfrak{R}}(\Im))$,

(ii) L of $\Im$ is a left(right) ideal.

Theorem 4.7. *Let $\Im_N = \frac{\Im}{(T_N, I_N, F_N)} \in \mathscr{N}_{\mathfrak{L}}(\Im)(\mathscr{N}_{\mathfrak{R}}(\Im))$. Then the sets $T_N^0 = \{c_1 \in Q \mid T_N(c_1) = T_N(0)\}, I_N^0 = \{c_1 \in Q \mid I_N(c_1) = I_N(0)\}, F_N^0 = \{c_1 \in Q | F_N(c_1) = F_N(0)\}$ of $\Im$ are left (right) ideals.*

Proof: For $l_0, w_0 \in T_N^0 \cap I_N^0 \cap F_N^0$, we have $T_N(l_0 - w_0) \leq T_N(l_0) \vee T_N(w_0) = T_N(0)$, $I_N(l_0 - w_0) \geq I_N(l_0) \wedge I_N(w_0) = I_N(0)$ and $F_N(l_0 - w_0) \leq F_N(l_0) \vee F_N(w_0) = F_N(0)$. So $l_0 - w_0 \in T_N^0 \cap I_N^0 \cap F_N^0$.

For $s \in \Im$, we have $T_N(sl_0 - s(w_0 - l_0)) \leq T_N(l_0) = T_N(0)$, $I_N(sl_0 - s(w_0 - l_0)) \geq I_N(l_0) = I_N(0)$ and $F_N(sl_0 - s(w_0 - l_0)) \leq F_N(l_0) = F_N(0)$. So $sl_0 - s(w_0 - l_0) \in T_N^0 \cap I_N^0 \cap F_N^0$.

Therefore T_N^0, I_N^0 and F_N^0 are left ideals.

Theorem 4.8. *Let $\Im_J := \frac{\Im}{(T_J, I_J, F_J)}$ and $\Im_W := \frac{\Im}{(T_W, I_W, F_W)}$ be the neutrosophic $\varkappa$-structures in $\Im$. If $\Im_J, \Im_W \in \mathscr{N}_{\mathfrak{L}}(\Im)(\mathscr{N}_{\mathfrak{R}}(\Im))$, then $\Im_J \cap \Im_W \in \mathscr{N}_{\mathfrak{L}}(\Im)(\mathscr{N}_{\mathfrak{R}}(\Im))$.*

Proof: Let $w_1, f_1 \in \Im$. Then

$$\begin{aligned} T_{J\cap W}(f_1 - w_1) &= (T_J \cap T_W)(f_1 - w_1) \\ &= T_J(f_1 - w_1) \vee T_W(f_1 - w_1) \\ &\leq \{T_J(f_1) \vee T_J(w_1)\} \vee \{T_W(f_1) \vee T_W(w_1)\} \\ &= (T_J \cap T_W)(f_1) \vee (T_J \cap T_W)(w_1) = T_{J\cap W}(f_1) \vee T_{J\cap W}(w_1), \end{aligned}$$

$$\begin{aligned}
I_{J\cap W}(f_1-w_1) &= (I_J\cap I_W)(f_1-w_1)\\
&= I_J(f_1-w_1)\wedge I_W(f_1-w_1)\\
&\geq \{I_J(f_1)\wedge I_J(w_1)\}\wedge\{I_W(f_1)\wedge I_W(w_1)\}\\
&= (I_J\cap I_W)(f_1)\wedge(I_J\cap I_W)(w_1) = I_{J\cap W}(f_1)\wedge I_{J\cap W}(w_1),\\
F_{J\cap W}(f_1-w_1) &= (F_J\cap F_W)(f_1-w_1)\\
&= F_J(f_1-w_1)\vee F_W(f_1-w_1)\\
&\leq \{F_J(f_1)\vee F_J(w_1)\}\vee\{F_W(f_1)\vee F_W(w_1)\}\\
&= (F_J\cap F_W)(f_1)\vee(F_J\cap F_W)(w_1) = F_{J\cap W}(f_1)\vee F_{J\cap W}(w_1).
\end{aligned}$$

For $s_1\in\Im$, we have

$$\begin{aligned}
T_{J\cap W}(s_1w_1-s_1(f_1-w_1)) &= (T_J\cap T_W)(s_1w_1-s_1(f_1-w_1))\\
&= T_J(s_1w_1-s_1(f_1-w_1))\vee T_W(s_1w_1-s_1(f_1-w_1))\\
&\leq T_J(w_1)\vee T_W(w_1) = (T_J\cap T_W)(w_1),\\
I_{J\cap W}(s_1w_1-s_1(f_1-w_1)) &= (I_J\cap I_W)(s_1w_1-s_1(f_1-w_1))\\
&= I_J(s_1w_1-s_1(f_1-w_1))\wedge I_W(s_1w_1-s_1(f_1-w_1))\\
&\geq I_J(w_1)\wedge I_W(w_1) = (I_J\cap I_W)(w_1),\\
F_{J\cap W}(s_1w_1-s_1(f_1-w_1)) &= (F_J\cap F_W)(s_1w_1-s_1(f_1-w_1))\\
&= F_J(s_1w_1-s_1(f_1-w_1))\vee F_W(s_1w_1-s_1(f_1-w_1))\\
&\leq F_J(w_1)\vee F_W(w_1) = (F_J\cap F_W)(w_1).
\end{aligned}$$

So, $\Im_J\cap\Im_W\in\mathscr{N}_{\mathfrak{L}}(\Im)$.

Hereafter, the symbols $\Im$ and $\Im'$ denote the near-subtraction semigroups.

Definition 4.9. A homomorphism ξ of $\Im$ into $\Im'$ such that $\xi(w_1-a_1)=\xi(w_1)-\xi(a_1)$ and $\xi(w_1a_1)=\xi(w_1)\xi(a_1)\ \forall w_1,a_1\in\Im$ is defined.

Definition 4.10. Consider a mapping $\Omega:\mathbb{N}\to\mathbb{M}$, where $\mathbb{N},\mathbb{M}\neq\{\phi\}$. Suppose $\mathbb{M}_S:=\frac{\mathbb{M}}{(T_S,I_S,F_S)}$ over $\mathbb{M}$ is a neutrosophic $\varkappa$-structure. Then, under Ω, the preimage of $\mathbb{M}_S$ is described as a neutrosophic $\varkappa$-structure $\Omega^{-1}(\mathbb{M}_S)=\frac{\mathbb{N}}{(\Omega^{-1}(T_S),\Omega^{-1}(I_S),\Omega^{-1}(F_S))}$ over $\mathbb{N}$, where $\Omega^{-1}(T_S)(l_0)=T_S(\Omega(l_0))$, $\Omega^{-1}(I_S)(l_0)=I_S(\Omega(l_0))$ and $\Omega^{-1}(F_S)(l_0)=F_S(\Omega(l_0))$ for all $l_0\in\mathbb{N}$.

Theorem 4.11. *Let $\Omega:\Im\to\Im'$ be a homomorphism of NSS. If $\Im'_S\in\mathscr{N}_{\mathfrak{I}}(\Im')$, where $\Im'_S:=\frac{\Im'}{(T_S,I_S,F_S)}$, then $\Omega^{-1}(\Im'_S)\in\mathscr{N}_{\mathfrak{I}}(\Im)$.*

Proof: Let $k_0,g_0\in\Im$. Then

$$\begin{aligned}
\Omega^{-1}(T_S)(k_0-g_0) &= T_S(\Omega(k_0-g_0)) = T_S(\Omega(k_0)-\Omega(g_0))\\
&\leq T_S(\Omega(k_0))\vee T_S(\Omega(g_0)) = \Omega^{-1}(T_S)(k_0)\vee\Omega^{-1}(T_S)(g_0),
\end{aligned}$$

$$\Omega^{-1}(I_S)(k_0 - g_0) = I_S(\Omega(k_0 - g_0)) = I_S(\Omega(k_0) - \Omega(g_0))$$
$$\geq I_S(\Omega(k_0)) \wedge I_S(\Omega(g_0)) = \Omega^{-1}(I_S)(k_0) \wedge \Omega^{-1}(I_S)(g_0),$$
$$\Omega^{-1}(F_S)(k_0 - g_0) = F_S(\Omega(k_0 - g_0)) = F_S(\Omega(k_0) - \Omega(g_0))$$
$$\leq F_S(\Omega(k_0)) \vee F_S(\Omega(g_0)) = \Omega^{-1}(F_S)(k_0) \vee \Omega^{-1}(F_S)(g_0).$$

Let $q_0 \in \Im$. Then

$$\begin{aligned}
\Omega^{-1}(T_S)(q_0k_0 - q_0(g_0 - k_0)) &= T_S(\Omega(q_0k_0 - q_0(g_0 - k_0))) \\
&= T_S(\Omega(q_0k_0) - \Omega(q_0(g_0 - k_0))) \\
&= T_S(\Omega(q_0)\Omega(k_0) - \Omega(q_0)(\Omega(g_0) - \Omega(k_0))) \\
&\leq T_S(\Omega(k_0)) = \Omega^{-1}(T_S)(k_0), \\
\Omega^{-1}(I_S)(q_0k_0 - q_0(g_0 - k_0)) &= I_S(\Omega(q_0k_0 - q_0(g_0 - k_0))) \\
&= I_S(\Omega(q_0k_0) - \Omega(q_0(g_0 - k_0))) \\
&= I_S(\Omega(q_0)\Omega(k_0) - \Omega(q_0)(\Omega(g_0) - \Omega(k_0))) \\
&\geq I_S(\Omega(k_0)) = \Omega^{-1}(I_S)(k_0), \\
\Omega^{-1}(F_S)(q_0k_0 - q_0(g_0 - k_0)) &= F_S(\Omega(q_0k_0 - q_0(g_0 - k_0))) \\
&= F_S(\Omega(q_0k_0) - \Omega(q_0(g_0 - k_0))) \\
&= F_S(\Omega(q_0)\Omega(k_0) - \Omega(q_0)(\Omega(g_0) - \Omega(k_0))) \\
&\leq F_S(\Omega(k_0)) = \Omega^{-1}(F_S)(k_0).
\end{aligned}$$

Also,

$$\Omega^{-1}(T_S)(k_0g_0) = T_S(\Omega(k_0g_0) = T_S(\Omega(k_0)\Omega(g_0)) \leq T_S(\Omega(k_0)) = \Omega^{-1}(T_S)(k_0),$$
$$\Omega^{-1}(I_S)(k_0g_0) = I_S(\Omega(k_0g_0) = I_S(\Omega(k_0)\Omega(g_0)) \geq I_S(\Omega(k_0)) = \Omega^{-1}(I_S)(k_0),$$
$$\Omega^{-1}(F_S)(k_0g_0) = F_S(\Omega(k_0g_0) = F_S(\Omega(k_0)\Omega(g_0)) \leq F_S(\Omega(k_0)) = \Omega^{-1}(F_S)(k_0).$$

So, $\Omega^{-1}(\Im'_S) \in \mathscr{N}_{\mathfrak{I}}(\Im)$.

Definition 4.12. Consider a onto map $\Omega : \mathbb{N} \to \mathbb{M}$, where $\mathbb{N}, \mathbb{M} \neq \{\phi\}$. Suppose $\mathbb{N}_{\mathscr{B}} := \frac{\mathbb{N}}{(T_{\mathscr{B}}, I_{\mathscr{B}}, F_{\mathscr{B}})}$ over $\mathbb{N}$ is a neutrosophic $\varkappa$-structure. Then, under Ω, the image of $\mathbb{N}_{\mathscr{B}}$ is described as a neutrosophic $\varkappa$-structure

$$\Omega(\mathbb{N}_{\mathscr{B}}) = \frac{\mathbb{M}}{(\Omega(T_{\mathscr{B}}), \Omega(I_{\mathscr{B}}), \Omega(F_{\mathscr{B}}))}$$

over $\mathbb{M}$, where, for all $y_2 \in \mathbb{M}$,

$$\Omega(T_{\mathscr{B}})(y_2) = \bigwedge_{y_1 \in \Omega^{-1}(y_2)} T_{\mathscr{B}}(y_1),$$
$$\Omega(I_{\mathscr{B}})(y_2) = \bigvee_{y_1 \in \Omega^{-1}(y_2)} I_{\mathscr{B}}(y_1),$$
$$\Omega(F_{\mathscr{B}})(y_2) = \bigwedge_{y_1 \in \Omega^{-1}(y_2)} F_{\mathscr{B}}(y_1).$$

Theorem 4.13. *Let $\xi : \Im \to \Im'$ be an onto homomorphism of NSS and $\Im'_{\mathscr{Z}} := \frac{\Im'}{(T_{\mathscr{Z}}, I_{\mathscr{Z}}, F_{\mathscr{Z}})}$ is a neutrosophic $\varkappa$-structure of $\Im'$. If $\xi^{-1}(\Im'_{\mathscr{Z}}) \in \mathscr{N}_{\mathfrak{I}}(\Im)$, then $\Im'_{\mathscr{Z}} \in \mathscr{N}_{\mathfrak{I}}(\Im')$.*

Proof: Let $v'_0, r'_0 \in \Im'$. Then $\exists\ v_0, r_0 \in \Im$ such that $\xi(v_0) = v'_0$ and $\xi(r_0) = r'_0$. Now,

$$\begin{aligned}
T_{\mathscr{Z}}(v'_0 - r'_0) = T_{\mathscr{Z}}(\xi(v_0) - \xi(r_0)) = T_{\mathscr{Z}}(\xi(v_0 - r_0)) &= \xi^{-1}(T_{\mathscr{Z}})(v_0 - r_0)\\
&\le \xi^{-1}(T_{\mathscr{Z}})(v_0) \vee \xi^{-1}(T_{\mathscr{Z}})(r_0)\\
&= T_{\mathscr{Z}}(\xi(v_0)) \vee T_{\mathscr{Z}}(\xi(r_0))\\
&= T_{\mathscr{Z}}(v'_0) \vee T_{\mathscr{Z}}(r'_0),\\
I_{\mathscr{Z}}(v'_0 - r'_0) = I_{\mathscr{Z}}(\xi(v_0) - \xi(r_0)) = I_{\mathscr{Z}}(\xi(v_0 - r_0)) &= \xi^{-1}(I_{\mathscr{Z}})(v_0 - r_0)\\
&\ge \xi^{-1}(I_{\mathscr{Z}})(v_0) \wedge \xi^{-1}(I_{\mathscr{Z}})(r_0)\\
&= I_{\mathscr{Z}}(\xi(v_0)) \wedge I_{\mathscr{Z}}(\xi(r_0))\\
&= I_{\mathscr{Z}}(v'_0) \wedge I_{\mathscr{Z}}(r'_0),\\
F_{\mathscr{Z}}(v'_0 - r'_0) = F_{\mathscr{Z}}(\xi(v_0) - \xi(r_0)) = F_{\mathscr{Z}}(\xi(v_0 - r_0)) &= \xi^{-1}(F_{\mathscr{Z}})(v_0 - r_0)\\
&\le \xi^{-1}(F_{\mathscr{Z}})(v_0) \vee \xi^{-1}(F_{\mathscr{Z}})(r_0)\\
&= F_{\mathscr{Z}}(\xi(v_0)) \vee F_{\mathscr{Z}}(\xi(r_0))\\
&= F_{\mathscr{Z}}(v'_0) \vee F_{\mathscr{Z}}(r'_0).
\end{aligned}$$

Let $s'_0 \in \Im'$. Then $\exists s \in \Im$ such that $\xi(s) = s'_0$. Now

$$\begin{aligned}
T_{\mathscr{Z}}(s'_0 v'_0 - s'_0(r'_0 - v'_0)) &= T_{\mathscr{Z}}(\xi(s)\xi(v_0) - \xi(s)(\xi(r_0) - \xi(v_0)))\\
&= T_{\mathscr{Z}}(\xi(sv_0) - \xi(s)\xi(r_0 - v_0))\\
&= T_{\mathscr{Z}}(\xi(sv_0) - \xi(s(r_0 - v_0)))\\
&= T_{\mathscr{Z}}(\xi(sv_0 - s(r_0 - v_0)))\\
&= \xi^{-1}(T_{\mathscr{Z}})(sv_0 - s(r_0 - v_0)) \le \xi^{-1}(T_{\mathscr{Z}})(v_0) = T_{\mathscr{Z}}(\xi(v_0)) = T_{\mathscr{Z}}(v'_0),\\
I_{\mathscr{Z}}(s'_0 v'_0 - s'_0(r'_0 - v'_0)) &= I_{\mathscr{Z}}(\xi(s)\xi(v_0) - \xi(s)(\xi(r_0) - \xi(v_0)))\\
&= I_{\mathscr{Z}}(\xi(sv_0) - \xi(s)\xi(r_0 - v_0))\\
&= I_{\mathscr{Z}}(\xi(sv_0) - \xi(s(r_0 - v_0)))\\
&= I_{\mathscr{Z}}(\xi(sv_0 - s(r_0 - v_0)))\\
&= \xi^{-1}(I_{\mathscr{Z}})(sv_0 - s(r_0 - v_0)) \ge \xi^{-1}(I_{\mathscr{Z}})(v_0) = I_{\mathscr{Z}}(\xi(v_0)) = I_{\mathscr{Z}}(v'_0),\\
F_{\mathscr{Z}}(s'_0 v'_0 - s'_0(r'_0 - v'_0)) &= F_{\mathscr{Z}}(\xi(s)\xi(v_0) - \xi(s)(\xi(r_0) - \xi(v_0)))\\
&= F_{\mathscr{Z}}(\xi(sv_0) - \xi(s)\xi(r_0 - v_0))\\
&= F_{\mathscr{Z}}(\xi(sv_0) - \xi(s(r_0 - v_0)))\\
&= F_{\mathscr{Z}}(\xi(sv_0 - s(r_0 - v_0)))\\
&= \xi^{-1}(F_{\mathscr{Z}})(sv_0 - s(r_0 - v_0)) \le \xi^{-1}(F_{\mathscr{Z}})(v_0) = F_{\mathscr{Z}}(\xi(v_0)) = F_{\mathscr{Z}}(v'_0).
\end{aligned}$$

Also,

$$T_{\mathscr{Z}}(v_0'r_0') = T_{\mathscr{Z}}(\xi(v_0r_0)) = \xi^{-1}(T_{\mathscr{Z}})(v_0r_0) \leq \xi^{-1}(T_{\mathscr{Z}})(v_0) = T_{\mathscr{Z}}(\xi(v_0)) = T_{\mathscr{Z}}(v_0'),$$
$$I_{\mathscr{Z}}(v_0'r_0') = I_{\mathscr{Z}}(\xi(v_0r_0)) = \xi^{-1}(I_{\mathscr{Z}})(v_0r_0) \geq \xi^{-1}(I_{\mathscr{Z}})(v_0) = I_{\mathscr{Z}}(\xi(v_0)) = I_{\mathscr{Z}}(v_0'),$$
$$F_{\mathscr{Z}}(v_0'r_0') = F_{\mathscr{Z}}(\xi(v_0r_0)) = \xi^{-1}(F_{\mathscr{Z}})(v_0r_0) \leq \xi^{-1}(F_{\mathscr{Z}})(v_0) = F_{\mathscr{Z}}(\xi(v_0)) = F_{\mathscr{Z}}(v_0').$$

So, $\Im'_{\mathscr{Z}} \in \mathscr{N}_{\mathfrak{J}}(\Im')$.

Definition 4.14. A neutrosophic $\varkappa$- structure $\Im_{\mathscr{B}} := \frac{\Im}{(T_{\mathscr{B}}, I_{\mathscr{B}}, F_{\mathscr{B}})}$ is defined to fulfils the sup property in $\Im$ if $\forall\, S \subseteq \Im$, $\exists\, l_0 \in S : T_{\mathscr{B}}(l_0) = \bigwedge_{l \in S} T_{\mathscr{B}}(l);\ I_{\mathscr{B}}(l_0) = \bigvee_{l \in S} I_{\mathscr{B}}(l);\ F_{\mathscr{B}}(l_0) = \bigwedge_{l \in S} F_{\mathscr{B}}(l)$.

Proposition 4.15. *A homomorphic image of a neutrosophic $\varkappa$-ideal having sup property is a neutrosophic $\varkappa$-ideal.*

Proof: Let $\varrho : \Im \to \Im'$ be a homomorphism of NSS and let $\Im_{\mathscr{Z}} := \frac{\Im}{(T_{\mathscr{Z}}, I_{\mathscr{Z}}, F_{\mathscr{Z}})}$ of $\Im$ be a neutrosophic $\varkappa$-ideal having sup property.

Suppose $\varrho(b), \varrho(w) \in \Im'$ and let $b_0 \in \varrho^{-1}(\varrho(b))$ and $w_0 \in \varrho^{-1}(\varrho(w))$ be such that

$$T_{\mathscr{Z}}(b_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} T_{\mathscr{Z}}(k_0), \quad I_{\mathscr{Z}}(b_0) = \bigvee_{k_0 \in \varrho^{-1}(\varrho(b))} I_{\mathscr{Z}}(k_0), \quad F_{\mathscr{Z}}(b_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} F_{\mathscr{Z}}(k_0),$$
$$T_{\mathscr{Z}}(w_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} T_{\mathscr{Z}}(k_0), \quad I_{\mathscr{Z}}(w_0) = \bigvee_{k_0 \in \varrho^{-1}(\varrho(b))} I_{\mathscr{Z}}(k_0), \quad F_{\mathscr{Z}}(w_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} F_{\mathscr{Z}}(k_0).$$

Then

$$\begin{aligned}
\varrho(T_{\mathscr{Z}})(\varrho(b) - \varrho(w)) &= \bigwedge_{z \in \varrho^{-1}(\varrho(b) - \varrho(w))} T_{\mathscr{Z}}(z) \leq T_{\mathscr{Z}}(b_0) \vee T_{\mathscr{Z}}(w_0) \\
&= \left(\bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} T_{\mathscr{Z}}(k_0) \right) \vee \left(\bigwedge_{k_0 \in \varrho^{-1}(\varrho(w))} T_{\mathscr{Z}}(k_0) \right) \\
&= \varrho(T_{\mathscr{Z}})(\varrho(b)) \vee \varrho(T_{\mathscr{Z}})(\varrho(w)), \\
\varrho(I_{\mathscr{Z}})(\varrho(b) - \varrho(w)) &= \bigvee_{z \in \varrho^{-1}(\varrho(b) - \varrho(w))} I_{\mathscr{Z}}(z) \geq I_{\mathscr{Z}}(b_0) \wedge I_{\mathscr{Z}}(w_0) \\
&= \left(\bigvee_{k_0 \in \varrho^{-1}(\varrho(b))} I_{\mathscr{Z}}(k_0) \right) \wedge \left(\bigvee_{k_0 \in \varrho^{-1}(\varrho(w))} I_{\mathscr{Z}}(k_0) \right) \\
&= \varrho(I_{\mathscr{Z}})(\varrho(b)) \wedge \varrho(I_{\mathscr{Z}})(\varrho(w)), \\
\varrho(F_{\mathscr{Z}})(\varrho(b) - \varrho(w)) &= \bigwedge_{z \in \varrho^{-1}(\varrho(b) - \varrho(w))} F_{\mathscr{Z}}(z) \leq F_{\mathscr{Z}}(b_0) \vee F_{\mathscr{Z}}(w_0) \\
&= \left(\bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} F_{\mathscr{Z}}(k_0) \right) \vee \left(\bigwedge_{k_0 \in \varrho^{-1}(\varrho(w))} F_{\mathscr{Z}}(k_0) \right) \\
&= \varrho(F_{\mathscr{Z}})(\varrho(b)) \vee \varrho(F_{\mathscr{Z}})(\varrho(w)).
\end{aligned}$$

Given $\varrho(s) \in \Im'$ and let $s_0 \in \varrho^{-1}(\varrho(s))$. Then

$$\begin{aligned}
\varrho(T_{\mathscr{Z}})(\varrho(s)\varrho(b) - \varrho(s)(\varrho(w) - \varrho(b))) &= \bigwedge_{z \in \varrho^{-1}(\varrho(s)\varrho(b) - \varrho(s)(\varrho(w) - \varrho(b)))} T_{\mathscr{Z}}(z) \\
&\leq T_{\mathscr{Z}}(b_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} T_{\mathscr{Z}}(k_0) = \varrho(T_{\mathscr{Z}})(\varrho(b)), \\
\varrho(I_{\mathscr{Z}})(\varrho(s)\varrho(b) - \varrho(s)(\varrho(w) - \varrho(b))) &= \bigvee_{z \in \varrho^{-1}(\varrho(s)\varrho(b) - \varrho(s)(\varrho(w) - \varrho(b)))} I_{\mathscr{Z}}(z) \\
&\geq I_{\mathscr{Z}}(b_0) = \bigvee_{k_0 \in \varrho^{-1}(\varrho(b))} I_{\mathscr{Z}}(k_0) = \varrho(I_{\mathscr{Z}})(\varrho(b)), \\
\varrho(F_{\mathscr{Z}})(\varrho(s)\varrho(b) - \varrho(s)(\varrho(w) - \varrho(b))) &= \bigwedge_{z \in \varrho^{-1}(\varrho(s)\varrho(b) - \varrho(s)(\varrho(w) - \varrho(b)))} F_{\mathscr{Z}}(z) \\
&\leq F_{\mathscr{Z}}(b_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} F_{\mathscr{Z}}(k_0) = \varrho(F_{\mathscr{Z}})(\varrho(b)).
\end{aligned}$$

Also,

$$\begin{aligned}
\varrho(T_{\mathscr{Z}})(\varrho(b)\varrho(w)) &= \bigwedge_{z \in \varrho^{-1}(\varrho(b)\varrho(w))} T_{\mathscr{Z}}(z) \leq T_{\mathscr{Z}}(b_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} T_{\mathscr{Z}}(k_0) = \varrho(T_{\mathscr{Z}})(\varrho(b)), \\
\varrho(I_{\mathscr{Z}})(\varrho(b)\varrho(w)) &= \bigvee_{z \in \varrho^{-1}(\varrho(b)\varrho(w))} I_{\mathscr{Z}}(z) \geq I_{\mathscr{Z}}(b_0) = \bigvee_{k_0 \in \varrho^{-1}(\varrho(b))} I_{\mathscr{Z}}(k_0) = \varrho(I_{\mathscr{Z}})(\varrho(b)), \\
\varrho(F_{\mathscr{Z}})(\varrho(b)\varrho(w)) &= \bigwedge_{z \in \varrho^{-1}(\varrho(b)\varrho(w))} F_{\mathscr{Z}}(z) \leq F_{\mathscr{Z}}(b_0) = \bigwedge_{k_0 \in \varrho^{-1}(\varrho(b))} F_{\mathscr{Z}}(k_0) = \varrho(F_{\mathscr{Z}})(\varrho(b)).
\end{aligned}$$

Hence $\varrho(\Im_{\mathscr{Z}})$ is a neutrosophic $\varkappa$-ideal of $\varrho(\Im)$.

5. **Conclusion**

We defined and examined neutrosophic $\varkappa-$ ideals in near-subtraction semigroups in this article. We formed ideals for a neutrosophic $\varkappa-$ ideal in a near-subtraction semigroup, and we also obtained various aspects of the neutrosophic $\varkappa-$ image as well as the neutrosophic $\varkappa-$ preimage of a near-subtraction semigroup using homomorphism mapping. In our future research work, we will explore the notion of a neutrosophic $\varkappa-$ prime ideal and its related properties in near-subtraction semigroups using the ideas and findings presented in this paper.

Acknowledgments

The authors express their sincere thanks to the referees for valuable comments and suggestions which improve the paper a lot.

References

1. Dheena, P.; Satheesh kumar, G. On strongly regular near-subtraction semigroups. Commun. Korean Math. Soc. 2007, 22(3), 323–330.
2. Elavarasan, B.; Porselvi, K.; Jun, Y.B. Hybrid generalized bi-ideals in semigroups. International Journal of Mathematics and Computer Science 2019, 14(3), 601–612.

3. Elavarasan, B.; Jun, Y.B. Hybrid ideals in semirings. Advances in Mathematics: Scientific Journal 2020, 9(3), 1349–1357.
4. Elavarasan, B.; Muhiuddin, G.; Porselvi, K.; Jun, Y. B. Hybrid structures applied to ideals in near-rings. Complex & Intell Syst. 2021, 7(3), 1489–1498.
5. Elavarasan, B.; Jun, Y. B. Regularity of semigroups in terms of hybrid ideals and hybrid bi-ideals. Kragujev. J. Math. 2022, 46(6), 857–864.
6. Elavarasan, B.; Smarandache, F.; Jun, Y. B. Neutrosophic $\varkappa-$ideals in semigroups. Neutrosophic Sets and Systems 2019, 28, 274 –280.
7. Elavarasan, B.; Porselvi, K.; Smarandache, F.; Jun, Y. B.; Muhiuddin, G. Neutrosophic N-filters in semigroups. Neutrosophic Sets and Systems 2022, 50, 515 –531.
8. Hezam, I. M. An Intelligent Decision Support Model for Optimal Selection of Machine Tool under Uncertainty: Recent Trends. Neutrosophic Systems with Applications 2023, 3, 35 – 44.
9. Jdid, M.; Smarandache, F.; Broumi, S. Inspection Assignment Form for Product Quality Control Using Neutrosophic Logic. Neutrosophic Systems with Applications 2023 , 1, 4–13.
10. Jun, Y. B.; Kim, H. S. On ideals in subtraction algebras. Sci. Math. Jpn. 2007, 65(1), 129–134.
11. Jun, Y. B.; Kim, H. S.; Roh, E. H. Ideal theory of subtraction algebras. Sci. Math. Jpn. 2005, 61(3), 459–464.
12. Jun, Y. B.; Song, S. Z.; Muhiuddin, G. Hybrid structures and applications. Annals of communications in Mathematics. 2018, 1(1), 11–25.
13. Khan, M.; Anis, S.; Smarandache, F.; Jun, Y. B. Neutrosophic $\varkappa$-structures and their applications in semigroups. Annals of Fuzzy Mathematics and Informatics 2017, 14(6), 583 – 598.
14. Lee, K. J.; Park, C. H. Some questions on fuzzifications of ideals in subtraction algebras. Commun. Korean Math. Soc. 2007, 22(3), 359–363.
15. Meenakshi, S.; Muhiuddin, G.; Elavarasan, B.; Al-Kadi, D. Hybrid ideals in near-subtraction semigroups. AIMS Mathematics 2022, 7(7), 13493–13507.
16. Molodtsov, D. Soft set theory-first results. Comput. Math. Appl. 1999, 37, 19–31.
17. Muhiuddin, G.; Grace John, J. C.; Elavarasan, B.; Jun, Y. B.; Porselvi, K. Hybrid structures applied to modules over semirings. J. Intell. Fuzzy Syst. 2022, 42(3), 2521–2531.
18. Muhiuddin, G.; Catherine Grace John, J.; Elavarasan, B.; Porselvi, K.; Al-Kadi, D. Properties of k-hybrid ideals in ternary semiring. J Intell Fuzzy Syst. 2022 , 42(6), 5799–5807.
19. Muhiuddin, G.; Porselvi, K.; Elavarasan, B.; Al-Kadi, D. Neutrosophic N-Structures in Ordered Semigroups. Computer Modeling in Engineering and Sciences 2022, 130(3).
20. Porselvi, K.; Elavarasan, B. On hybrid interior ideals in semigroups. Probl. Anal. Issues Anal. 2019, 8(26)(3), 137 –146.
21. Porselvi, K.; Elavarasan, B.; Jun, Y. B. Hybrid Interior Ideals in Ordered Semigroups. New Math. Nat. Comput. 2022, 18(1), 1 –8.
22. Porselvi, K.; Muhiuddin, G.; Elavarasan, B.; Assiry, A. Hybrid Nil Radical of a Ring. Symmetry 2022, 14(7), 1367.
23. Porselvi, K.; Muhiuddin, G.; Elavarasan, B.; Jun, Y. B.; Catherine Grace John, J. Hybrid ideals in an AG-groupoid. New Math. Nat. Comput. 2023, 19(1), 289 – 305.
24. Porselvi, K.; Elavarasan, B.; Smarandache, F.; Jun, Y. B. Neutrosophic $\varkappa$-bi-ideals in semigroups. Neutrosophic Sets and Systems 2020, 35, 422–434.
25. Porselvi, K.; Elavarasan, B.; Smarandache, F. Neutrosophic $\varkappa$-interior ideals in semigroups. Neutrosophic Sets and Systems 2020, 36, 70 – 80.
26. Schein, B.M. Difference Semigroups. Commun. Algebra. 1992, 20(8), 2153–2169.
27. Sleem, A.; Mostafa, N.; Elhenawy, I. Neutrosophic CRITIC MCDM Methodology for Ranking Factors and Needs of Customers in Product's Target Demographic in Virtual Reality Metaverse. Neutrosophic Systems with Applications 2023, 2, 55– 65.
28. Williams, D. R. P. Fuzzy ideals in near-subtraction semigroups. International scholarly and scientific research and innovation 2008, 2(7), 625–632.
29. Zelinka, B. Subtraction Semigroups. Math. Bohem. 1995 ,120(4), 445–447.
30. Zadeh, L. A. Fuzzy sets. Information and Control 1965, 8, 338–353.

Received: July 1, 2023. Accepted: Nov 19, 2023