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Neutrosophic Estimators in Two-Phase Survey Sampling

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Abstract. Point estimates in survey sampling only provide a single value for the parameter being studied and are consequently vulnerable to changes caused by sampling error. In order to cope with ambiguity, indeterminacy, and uncertainty in data, Florentin Smarandache's neutrosophic technique, which generates interval estimates with high probability, offers a helpful solution. To estimate the neutrosophic population mean of the studied variable, this research provides new neutrosophic factor type exponential estimators using well-known neutrosophic auxiliary parameters. For the first-degree of approximation, the study derives the bias and Mean Squared Error (MSE) of the proposed estimators. Characterising constants have neutrosophic optimal values, and for these optimum values, the least value of the neutrosophic MSE is obtained. Notably, the proposed neutrosophic estimators outperform the corresponding adapted classical estimators since their estimated interval falls under the minimal MSE and lies within the estimated interval of the proposed neutrosophic estimators. The theoretical results are supported by empirical data from real data sets acquired by the "Ministry of Earth Sciences" and the "India Meteorological Department (IMD), Pune, India," as well as simulated data sets produced via Neutrosophic Normal Distribution. The estimator with the lowest MSE is suggested for practical applications across many domains, providing greater accuracy and reliability in parameter estimation when utilising the neutrosophic methodology.

Keywords: Neutrosophic Statistics, Bias, Mean Square Error, Auxiliary information, Exponential Estimator, Factor-type Estimator, Two-Phase Sampling, Relative Efficiency (%).

1. **Introduction**

Sampling becomes a crucial method in scientific study when dealing with big populations and having time and resource constraints. In these situations, we use statistical techniques and estimators to estimate the relevant parameters of interest. The sample mean ($\bar{y}$), which approximates the population mean ($\bar{Y}$) of the study variable Y, is one of the most often used estimators. The sample mean is a fair estimator of the population mean, but because of the sample mean's potential for significant estimation variability, the sampling distribution may not be highly representative of the real population mean $\bar{Y}$. Therefore, even if adding a little amount of bias, researchers look for ways to increase the precision and accuracy of estimators. Incorporating data from auxiliary variables (X) that show significant positive or negative relationships with the study variable Y is one efficient method to do this. We may improve the effectiveness of the estimators by taking use of the correlation between the study variable and these auxiliary variables. The ratio as well as product methods of estimate are two common approaches for using the auxiliary variable information. The ratio between the study variable and the auxiliary variable is calculated in the ratio method to determine estimators. Estimators are created using the product consider by multiplying the study variable by the auxiliary variable. These methods work especially well when the line of regression crosses the origin. The regression technique of estimation, however, is better suited when the regression line does not cross the origin. It entails fitting a regression line to the study variable and any auxiliary variables, then estimating the population parameter using the regression equation. Due to its adaptability and extensive applicability, the ratio approach is frequently used in practical applications. It is used in a variety of sectors including agriculture to estimate crop yields, economics to evaluate revenue and investment, and healthcare to examine hospital facilities and health indicators. As scientific investigation advances, one current area of emphasis is the estimate of population parameters utilising known auxiliary variables with positive relations. With more precise and trustworthy insights into the underlying population features, this study intends to expand and improve estimating approaches.

In classical sampling theory, estimating methods for the population parameter Y employ a variety of methodologies, including ratio, product, and regression type estimators, when the data consists of precise numerical values. Several researchers in the discipline of classical statistics are devoting their efforts to developing and refining various estimators for Y, especially when information on the auxiliary variable X is available. The conventional ratio estimator, which makes use of a positively correlated auxiliary variable called X, was one of the groundbreaking developments in classical sampling theory introduced by Cochran [1]. As an auxiliary parameter, we are using the known population mean ($\bar{X}$) of X. Based on this, later scholars investigated how to improve the estimation of Y by incorporating widely used auxiliary factors including the coefficient of variation (CV), coefficient of skewness, coefficient of kurtosis, standard deviation, quartiles, and others. As an illustration, Sisodia and Dwivedi [2] developed a modified ratio estimator for $\bar{Y}$ based on the known CV of X, and Bahl and Tuteja [3] offered an exponential ratio estimator using the known CV of X to achieve enhanced estimate of $\bar{Y}$. Upadhyaya and Singh [4] provided two ratio estimators that both sought to more accurately estimate $\bar{Y}$ using the given coefficient of kurtosis and CV of X. Similar to Upadhyaya and Singh [4], Singh and Tailor [5] concentrated on using the established population correlation coefficient between Y and X to enhance $\bar{Y}$ estimations, demonstrating superior outcomes to other estimators. The ratio estimator has to be changed in order to progress the estimation of $\bar{Y}$. To boost the estimate of $\bar{Y}$, Singh et al. [6] provided modifications based on the well-known kurtosis coefficient of X, and Kadilar and Cingi [7] proposed a number of modified ratio estimators depending on well-known data regarding well-known auxiliary parameters. Using the given skewness and kurtosis of X, Yan and Tian [8] proposed two ratio type estimators for $\bar{Y}$, which outperformed competing estimators. A method for improved estimate of the population mean $\bar{Y}$ employing auxiliary parameters associated with the characteristic was provided in Singh and Solanki [9]. Yadav and kadilar [10] worked on improving a family of ratio and product estimations for Y with known parameters of X, While Grover and Kaur [11] concentrated on a general family of estimators for $\bar{Y}$ using transformed X. Vishwakarma and Kumar [12] proposed a generalised family of known auxiliary parameters-based dual to ratio-cum-product estimators for $\bar{Y}$. Cekim and Cingi [13] used the lowest and maximum values of linear adjustments of X to create a unique ratio estimate for $\bar{Y}$. Both Subzar et al. [14] and Yadav et al. [15] offered new families of $\bar{Y}$ estimators in accordance with the known population median of the study variable, demonstrating receives over competing estimators. Subzar et al. [14] produced numerous effective estimators for $\bar{Y}$ using auxiliary parameters. The next step forward came from Zaman and Dunder [16], who suggested an entirely novel modified ratio type estimator built on the exponential parameter of an auxiliary variable. To increase the effectiveness of the estimators, Yadav et al. [17] proposed an improved family of $\bar{Y}$ estimators employing known Y and X parameters. In their estimation technique, Yadav and Zaman [18] used well-known traditional as well as non-traditional auxiliary variables. These efforts are only a fraction of the numerous attempts made by numerous authors to improve $\bar{Y}$ estimate within the framework of classical sampling theory utilising known data regarding both traditional along with non-traditional, robust and non-robust auxiliary variables. The hunt for more precise and effective estimators is still an active and developing subject of study in this discipline.

1.1. *Research Gap*

The standard presumption of classical sampling theory is that the data are deterministic and that there is no uncertainty in the measurements of the observable features. However, in actual settings, we frequently come across data for the properties that are being investigated that are not properly specified. This happens across a number of industries, namely information technology, systems for decision-making, financial data analysis, much more. Fuzzy logic, developed by Prof. Lofti A. Zadeh [19] in 1965, gives a method for addressing situations when precise measurements are not available. Dealing with such indeterminate data demands alternative ways. Fuzzy logic can handle confusing, murky, or inaccurate data, but it does not completely take into account measures that are not known in advance. Contrarily, neurosophic logic expands fuzzy logic to take into consideration both the determinate along with indeterminate aspects of observations, which is especially useful when working with uncertain or ambiguous data. Fuzzy and neutrosophic logic have been created and extensively used in numerous applications for decision-making and other operations. When there is any degree of indeterminacy in the data, neutrosophic statistics, a derivative of classical statistics, takes into action. It is used when observations made about the population or sample are hazy, ambiguous, or imprecise. In systems with uncertainty, neutrosophic statistics are especially helpful because they enable the interpretation of neutrosophic data in situations where the sample size might not be ideal. Numerous applications of neutrosophic statistics have been used by researchers. It has been applied to analyse impacts, make group decisions, analyse medical data, estimate variables, track traffic accidents, create goodness-of-fit tests, research wind speed distributions, and make judgements in challenging situations with unknowns. Ultimately, fuzzy and neutrosophic logic along with statistics provide useful tools for handling ambiguous and imprecise data in real-world situations, enabling researchers and decision-makers to conduct more thorough and accurate studies and make better choices in challenging situations. In comparison to the fuzzy set, the neutrosophic set has performed better when handling uncertainty in practical settings. Neutosophic parameterized hypersoft set theory has been studied for its potential as a useful tool for applications involving decision-making. They have created cutting-edge decision-making techniques that can successfully manage uncertainty in complicated situations by introducing and examining the neutrosophic parameterized hypersoft set along with its basic properties and functions. Traditional approaches in statistical analysis may be inadequate when working with interval-valued data and uncertain situations. Modified Sign tests that take into account both the real form of the observations and the ambiguous nature of the data have been presented as a solution to this problem. These enhancements' appropriateness for nonparametric decision-making with interval-valued data has been tested using real-world data sets. Particular difficulties have been encountered in the diagnostic and decision-making processes in the medical area. Researchers have developed methods based on the generalisation of multipolar neutrosophic soft sets to overcome these challenges. These methods include informational measurements like distance, similarities, and correlation coefficient to offer a thorough framework for making decisions in the face of ambiguity. Single-valued brittle estimates may produce inaccurate and skewed results in conventional survey sample studies when data is presumed to be certain and clear. Since neutrosophic data frequently appears in real-world circumstances, Neutrosophic statistics becomes a useful alternative to conventional methods. It is a useful option in a range of scenarios where standard approaches would not be enough due to its capacity to manage indeterminacy and uncertainty. Neutosophic data, where data from experiments or populations may be expressed as interval-valued neutrosophic numbers, is characterised by ambiguous and contradictory values, non-clear contentions, and inaccurate interval values. In real-world situations, ambiguous data predominates over definite data. As a result, statistical methods that can properly handle neutrosophic data are becoming more and more important.

1.2. *Scope of the Neutrosophic Study*

The act of acquiring data for various variables in research can be expensive, especially when working with unclear or confusing data. Traditional techniques can be costly and dangerous when attempting to estimate genuine parameter values in the context of uncertain data. However, neutrosophic statistical computation provides a way to investigate data that is uncertain or for which there is inadequate information, taking into account competing viewpoints. Traditional statistics are unable to do an accurate analysis of the data due to the problem of indeterminacy, where some observations fall within a range of uncertain values. Neutosophic statistics, which are adaptable and all-encompassing, replace traditional statistics in such unsettling situations. While several research have been conducted in sample surveys to investigate neutrosophy, the particular use of ratio estimation with neutrosophic data is pretty new and requires substantial attention to meet the issues given by uncertain data systems. Numerous situations in the actual world find use for neutrosophic estimators. Neutosophic statistics may be superior to conventional techniques, for instance, in analysing machine product measurements with small errors or evaluating health parameters through various testing processes. When observations of the research variable are not deterministic but rather nondeterministic, reflecting the intrinsic uncertainties present, the use of neutrosophic estimators enables improved estimate of population means.

Although fuzzy statistics solves the issues raised by ambiguous, confusing, or imprecise data, indeterminate measures are not taken into account. Neutosophic logic, on the other hand, extends fuzzy logic more broadly by include both the determinate and indeterminate parts of observations. Analysis of situations involving ambiguous or inaccurate observations makes use of neutrosophic logic Aslam ([20] & [21]). Bellman and Zadeh [22] employed this strategy to improve decision-making precision. Different approaches based on fuzzy logic then started to appear, and they now play a big part in decision-making across many different areas. Similar to Liu and Mahmood [23], who proposed the idea of advanced fuzzy sets and showed how they might be expanded to create complicated neutrosophic sets. Interval-valued neutrosophic sets were used in a framework described by Li et al. [24] that displayed fuzzy sets together with their generalisations and aided decision-making. Aslam [25] included neutrosophic statistics in the investigation of skewness and kurtosis estimators for wind speed distributions under uncertainty. Chinnadurai and Bharathivelan [26] developed a paradigm for making decisions that favours badly damaged machines when assessing damages in a neutrosophic environment. Mohanta and Pal [27] proposed a number of single-valued neutrosophic graph (SVNG)-related ideas while emphasising the significance of fuzzy and neutrosophic sets in reducing uncertainty in real-world contexts. Zulqarnain et al. [28] concentrated on algorithms for generalised multipolar neutrosophic soft sets with information measures to address issues in medical diagnosis and decision-making. They developed the idea of multipolar neutrosophic soft sets by introducing several informational metrics for hypothetical decision-making situations. Tahir et al. [29] emphasised the drawbacks of conventional survey sample studies that rely on precise and definite data, and it promoted the use of neutrosophic statistics in situations where the data exhibits such features. Neutrosophic data includes ambiguous and uncertain variable values, unclear statements, and erratic interval values. Data of this kind can be represented as interval-valued neutrosophic numbers, where initially uncertain observed values are assumed to fall within predetermined ranges. Neutosophic statistical methods must be developed and used since uncertain data are common in real-world situations. It can be expensive to collect data for various study variables, especially when dealing with unclear data. Therefore, using conventional techniques to determine unknown actual parameter values from uncertain data can be expensive and risky.

These factors were taken into account when Tahir et al. [29] first developed a neutrosophic ratio-type estimate technique. For analysing data with uncertainty, limited information, and opposing beliefs, neutral statistical analysis is useful. When observations fall inside an undefined value range, traditional statistics have trouble. In such circumstances, neutrosophic statistics act as a versatile and all-encompassing replacement for classical statistics. The ratio estimate approach is still relatively new in the field of sample surveys under Neutrosophy and needs additional consideration when dealing with unreliable data systems. For instance, neutrosophic statistics may be preferable to conventional methods for measurements of machine goods with small faults or health metrics acquired from various testing techniques. When dealing with nondeterministic observations of study variables, Neutrosophic estimators frequently outperform classical estimators.

1.3. *Flow Chart of the Proposed Study*

The provided flowchart (Figure - 1) presents demonstrations of the suggested factor type exponential estimators that fall within the aforementioned group of estimators. Divergent neutrosophic exponential estimators are obtained depending on the different values of d. With the use of neutrosophic statistics and by expanding on the concepts offered by Yadav and Smarandache [30], this study develops a unique method for factor type exponential estimators. We may modify the estimators to fit certain circumstances and gain greater performance in a number of applications by carefully selecting the values of d. As a result of the study's use of neutrosophic statistics, the estimate procedure gains a distinctive component that enhances its adaptability to ambiguous or indeterminate data. A similar strategy may have a big influence on a number of fields, including economics, engineering, and social sciences. To evaluate these neutrosophic estimators' performance to more established techniques, empirical assessments are essential. Under specified circumstances, the suggested factor type neutrosophic exponential estimators demonstrate promise in terms of providing accurate and dependable estimates, making a significant addition to the area of statistics. It is necessary to do further study in this field to fully explore the possibilities of factor type exponential estimators and neutrosophic statistics, opening the way for improvements in statistical estimation and analysis.

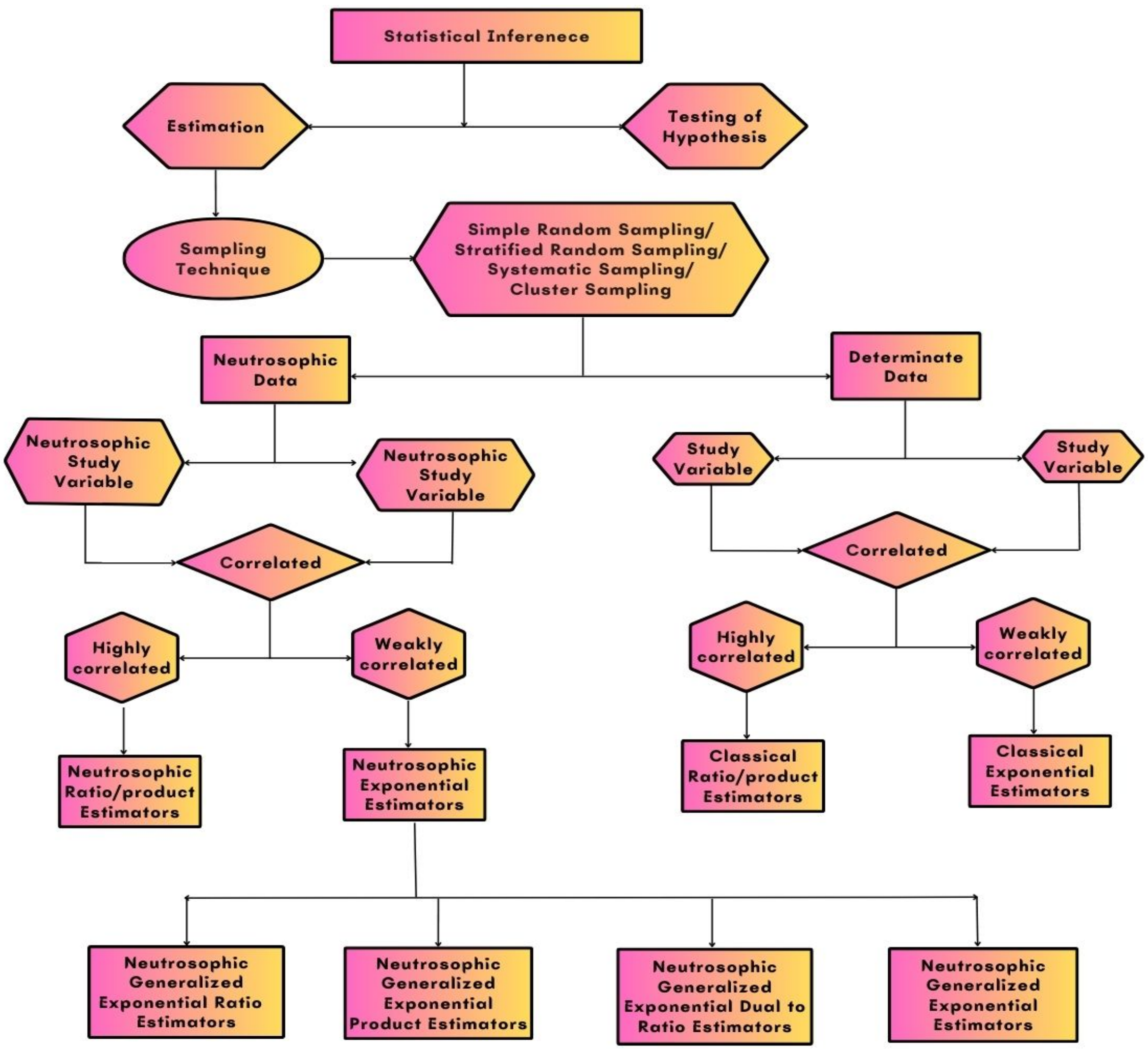

FIGURE 1. Flow Chart of the Proposed Study In Neutrosophic Framework.

1.4. *Observations and Terminology in Neutrosophic Statistics*

One of the main observations inside the neutrosophic environment is the use of quantitative neutrosophic data, where a number may live within an unknown interval $[N\Phi_L, N\Phi_U]$. Interval values of neutrosophic numbers can be stated in numerous ways. Neutosophic interval values are specifically specified in Yadav and Smarandache [30] as $Z_{(N\Phi)} = Z_{(N\Phi_L)} + Z_{(N\Phi_U)}I_{(N\Phi)}$, where $I_{(N\Phi)} \in [I_{(N\Phi_L)}, I_{(N\Phi_U)}]$. For the considered neutrosophic data, we use the same notations as Yadav and Smarandache [30], which take the interval form as $Z_{(N\Phi)} \in [Z_{(N\Phi_L)}, Z_{(N\Phi_U)}]$, where $Z_{(N\Phi_L)}$ and $Z_{(N\Phi_U)}$ indicate the respective Lower and Upper values of the neutrosophic variable $Z_{(N\Phi_L, N\Phi_U)}$. We use the simple random sampling without replacement (SRSWOR) approach to extract a neutrosophic random sample of size $n_{(N\Phi)} \in [n_{(N\Phi_L)}, n_{(N\Phi_U)}]$ from the aforementioned population under the assumption that the neutrosophic population consists of $N_{(N\Phi)} \in [N_{(N\Phi_L)}, N_{(N\Phi_U)}]$ unique units. For the neutrosophic data under discussion, each observation on the i^{th} unit of the sample for the study variable is designated as $Y_{(N\Phi)} \in [Y_{(N\Phi_L)}, Y_{(N\Phi_U)}]$, and the secondary variable $X_{(N\Phi)} \in [X_{(N\Phi_L)}, X_{(N\Phi_U)}]$. we define $\bar{Y}_{(N\Phi)} = \frac{1}{N}\sum_{i=1}^{N_{(N\Phi)}} Y_{i_{(N\Phi)}}$ and $\bar{X}_{(N\Phi)} = \frac{1}{N}\sum_{i=1}^{N_{(N\Phi)}} X_{i_{(N\Phi)}}$ as the population means for the neutrosophic variables $Y_{(N\Phi)}$ and $X_{(N\Phi)}$, respectively, acting as the overall averages of the neutrosophic data set. The neutrosophic study variable $Y_{(N\Phi)}$ has a sample mean of $\bar{y}_{(N\Phi)} = \frac{1}{n}\sum_{i=1}^{n_{(N\Phi)}} y_{i_{(N\Phi)}}$ and $X_{(N\Phi)}$, has a sample mean of $\bar{x}_{(N\Phi)} = \frac{1}{N}\sum_{i=1}^{n_{(N\Phi)}} x_{i_{(N\Phi)}}$. Additionally, The neutrosophic coefficients of variation for $Y_{(N\Phi)}$ and $X_{(N\Phi)}$ are given as $C_{y_{(N\Phi)}}$ and $C_{x_{(N\Phi)}}$, respectively. Additionally, the correlation coefficient between the neutrosophic variables $Y_{(N\Phi)}$ and $X_{(N\Phi)}$ denoted as $\rho_{y_{(N\Phi)}x_{(N\Phi)}}$. The neutrosophic coefficients of skewness and kurtosis for the neutrosophic variable $X_{(N\Phi)}$ are calculated as $\beta_{1(x_{(N\Phi)})}$ and $\beta_{2(x_{(N\Phi)})}$, respectively.

When the value of $\bar{X}_{(N\Phi)}$ is unavailable or unknown, the technique of a two-phase sampling is used in neutrosophic framework to estimate the population mean, denoted as $y_{(N\Phi)}$. To choose the required sample in the neutrosophic double sampling technique, the following steps are taken:

Case I: A large sample, designated as S', is drawn over the population employing SRSWOR, having a size of $n'_{(N\Phi)}$, ($n'_{(N\Phi)}$ being less than $N'_{(N\Phi)}$). This sample is used to collect observations that are entirely connected to the auxiliary variable $x_{(N\Phi)}$, with the goal of estimating the population mean $\bar{X}_{(N\Phi)}$ associated with this auxiliary variable.

Case II: A sample with the symbol S and a size of $n_{(N\Phi)}$ is chosen, where ($n_{(N\Phi)} < N_{(N\Phi)}$). This sample is taken directly from the population, which has the size $N_{(N\Phi)}$, or from the set of S' characters. This sample's goal is to collect data on both the primary neutrosophic study variable and the secondary neutrosophic auxiliary variable.

Employing the neutrosophic framework, we are adopting this approach to research. The corresponding sample mean, particularly is provided by, is the best estimate for the population mean.

$$t_{0_{(N\Phi)}} = \bar{y}_{(N\Phi)} \tag{1}$$

We are introducing the subsequent expression within the framework of neutrosophic statistics. The variance of $t_{0_{(N\Phi)}}$ is obtained as follows:

$$V(t_{0_{(N\Phi)}}) = \gamma_{(N\Phi)}\bar{Y}^2_{(N\Phi)}C^2_{y_{(N\Phi)}} \tag{2}$$

where,

$\gamma_{(N\Phi)} = \frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}$, $C_{y_{(N\Phi)}} = \frac{S_{y_{(N\Phi)}}}{\bar{Y}_{(N\Phi)}}$ and $S^2_{y_{(N\Phi)}} = \frac{1}{N_{(N\Phi)}-1}\sum_{i=1}^{N_{(N\Phi)}}(y_{i_{(N\Phi)}} - \bar{Y}_{(N\Phi)})^2$.

We are incorporating the following idea into the framework of neutrosophic statistics. In the setting of simple random sampling, Cochran [2] proposed a conventional estimate of the population mean using auxiliary data. To improve our understanding, this strategy is being included into the design of neutrosophic statistics.

$$t_{R_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left(\frac{\bar{X}_{(N\Phi)}}{\bar{x}_{(N\Phi)}} \right) \tag{3}$$

In this study, a new neutrosophic factor type exponential estimator is introduced for improving the estimate of the parameter $Y_{N\phi}$ utilising known $X_{N\phi}$ parameter values. In the first degree of approximation, the sample characteristics of the suggested estimator are investigated.

- This article introduces Florentin Smarandache's neutrosophic technique in Two Phase survey sampling.
- Addresses ambiguity, indeterminacy, and uncertainty in data.
- Proposes new estimators for neutrosophic population mean.
- Utilizes well-known neutrosophic auxiliary parameters.
- Derives bias and MSE for proposed estimators in the first-degree of approximation.
- Identifies optimal values for characterizing constants.
- Validates theoretical findings with real datasets from Earth Sciences and Meteorological Departments.
- Includes simulated data sets from Neutrosophic Normal Distribution.

Advantages:

- Handles ambiguity and uncertainty in data.
- Optimizes Mean Squared Error for reliability.
- Supported by empirical data from real and simulated sets.

Disadvantages:

- May introduce complexity and pose interpretability challenges.
- Computational burden not discussed.
- Generalizability to various domains requires further investigation.

While previous studies (Including more study for the reference Uma & Nandhitha [31], Jdid & Smarandache [32], Abdel-Basset et al. [33], Gamal et al. [34,35]) have made strides in neutrosophic systems, our study uniquely focuses on two phase survey sampling using Neutrosophic statistics—an area largely unexplored in survey sampling.

The full paper is divided into sections that include the introduction, some existing adapted estimators, proposed estimators, numerical study, simulation study, conclusssion and references. By adding to the ongoing study of neutrosophic statistical methods and their applications, the article is divided into these components.

2. Some Existing Estimators

As far as we are aware, this study is a fresh and ground-breaking use of Neutroshopic statistics that incorporates two-phase sample estimators. To the best of our knowledge, this is the first thorough investigation of the use of these estimators within the framework of two phase neutroshopic sampling. The use of two-phase sample approaches in the context of neutroshopic statistics, a relatively new and specialised discipline, gives up intriguing opportunities for investigation and evaluation. This work presents a cutting-edge strategy to dealing with complicated data gathering scenarios by integrating the distinctive elements of Neutroshopic statistics with the complexities of two-phase sampling. The use of these estimators in Neutroshopic statistics indicates a forward-thinking and innovative approach to statistical analysis, paving the way for future advances and discoveries in this rapidly growing subject. We hope that this new study will benefit the larger scientific community by fostering a better knowledge of statistical approaches in the context of Neutroshopic sampling. We are using previously suggested estimators that have been adjusted for neutrosophic two-phase sampling in this part. This novel method bridges the gap between traditional two-phase sampling and Neutrosophic statistics, improving population mean estimate in the presence of uncertainties and missing data.

2.1. *Adapted Kumar and Bahl Estimators*

We are integrating the conventional ratio estimator for two-phase sampling suggested by Kumar and Bahl [36] into the Neutroshopic statistical framework. This modification attempts to boost the accuracy of statistical analysis in this specialised sector by improving population mean estimate under uncertainty.

$$t^d_{R_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left(\frac{\bar{x}_{1_{(N\Phi)}}}{\bar{X}_{(N\Phi)}} \right) \tag{4}$$

where

$\bar{x}_{1_{(N\Phi)}} = \frac{1}{n_{(N\Phi)}'} \sum_{i=1}^{n'_{(N\Phi)}} x_{i_{(N\Phi)}}$.

MSE of $t^d_{R_{(N\Phi)}}$, for Case-I and Case-II are given as,

$$MSE(t^d_{R_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)} \left[\gamma_{(N\Phi)} C^2_{y_{(N\Phi)}} + \gamma^{**}_{(N\Phi)} C^2_{x_{(N\Phi)}} (1 - 2C_{(N\Phi)}) \right] \tag{5}$$

$$MSE(t^d_{R_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)} \left[\gamma_{(N\Phi)} C^2_{y_{(N\Phi)}} + \gamma^{***}_{(N\Phi)} C^2_{x_{(N\Phi)}} - 2\gamma_{(N\Phi)} C_{(N\Phi)} C^2_{x_{(N\Phi)}} \right] \tag{6}$$

where,

$\gamma^*_{(N\Phi)} = \left(\frac{1}{n'_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}} \right)$, $\gamma^{**}_{(N\Phi)} = \left(\frac{1}{n_{(N\Phi)}} - \frac{1}{n'_{(N\Phi)}} \right)$, $\gamma^{***}_{(N\Phi)} = \left(\gamma_{(N\Phi)} + \gamma^*_{(N\Phi)} \right)$, $C_{x_{(N\Phi)}} = \frac{S_{x_{(N\Phi)}}}{\bar{X}_{(N\Phi)}}$, $C_{(N\Phi)} = \rho_{y_{(N\Phi)} x_{(N\Phi)}} \left(\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}} \right)$,

$S^2_{x_{(N\Phi)}} = \frac{1}{N_{(N\Phi)} - 1} \sum_{i=1}^{N_{(N\Phi)}} (x_{i_{(N\Phi)}} - \bar{X}_{(N\Phi)})^2$, and

$\rho_{y_{(N\Phi)}x_{(N\Phi)}} = \frac{1}{N_{(N\Phi)}}\sum_{i=1}^{N_{(N\Phi)}}(y_{i_{(N\Phi)}} - \bar{Y}_{(N\Phi)})(x_{i_{(N\Phi)}} - \bar{X}_{(N\Phi)})$

2.2. *Adapted Singh and Choudhury Estimator*

In the context of Neutroshopic statistics, we are using the dual to product estimator of population mean suggested by Singh and Choudhury [37] for two-phase sampling. This specialised estimator improves population mean estimate in Neutroshopic data and is designed to manage uncertainties, making statistical studies more trustworthy.

$$t^d_{P_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left(\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right) \tag{7}$$

MSE for Case-I and Case-II are given as,

$$MSE(t^d_{P_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \gamma^{**}_{(N\Phi)}C^2_{x_{(N\Phi)}}(1 + 2C_{(N\Phi)})\right] \tag{8}$$

$$MSE(t^d_{P_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} + 2\gamma_{(N\Phi)}C_{(N\Phi)}C^2_{x_{(N\Phi)}}\right] \tag{9}$$

2.3. *Adapted Singh and Vishwakarma Estimators*

In the setting of Neutroshopic statistics, we are adopting Singh and Vishwakarma's [38] suggested exponential type ratio and product estimators. In two-phase sampling circumstances, these estimators are tailored to manage uncertainties and enhance population mean estimation, increasing the precision and efficacy of statistical studies in the Neutroshopic setting.

$$t^d_{Re_{(N\Phi)}} = \bar{y}_{(N\Phi)}exp\left(\frac{\bar{x}_{1_{(N\Phi)}} - \bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}} + \bar{x}_{(N\Phi)}}\right) \tag{10}$$

$$t^d_{Pe_{(N\Phi)}} = \bar{y}_{(N\Phi)}exp\left(\frac{\bar{x}_{(N\Phi)} - \bar{x}_{1_{(N\Phi)}}}{\bar{x}_{(N\Phi)} + \bar{x}_{1_{(N\Phi)}}}\right) \tag{11}$$

MSE of both the estimators for Case-I and Case-II are given as,

$$MSE(t^d_{Re_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \gamma^{**}_{(N\Phi)}C^2_{x_{(N\Phi)}}(\frac{1}{4} - C_{(N\Phi)})\right] \tag{12}$$

$$MSE(t^d_{Re_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \frac{1}{4}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} - \gamma_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)}\right] \tag{13}$$

$$MSE(t^d_{Pe_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \gamma^{**}_{(N\Phi)}C^2_{x_{(N\Phi)}}(\frac{1}{4} + C_{(N\Phi)})\right] \tag{14}$$

$$MSE(t^d_{Pe_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \frac{1}{4}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} + \gamma_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)}\right] \tag{15}$$

2.4. *Adapted Kumar and Bahl Estimator*

We are using Kumar and Bahl's [36] dual to ratio estimator for the population mean in two-phase sampling in the context of Neutroshopic statistics. By addressing uncertainties in Neutroshopic data, this specialised estimator improves the precision of population mean estimate and makes it possible to make well-informed decisions in challenging sampling situations.

$$t^{*d}_{R_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left(\frac{\bar{x}^{*d}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}} \right) \tag{16}$$

MSE for Case-I and Case-II are given as,

$$MSE(t^{*d}_{R_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)} \left[\gamma_{(N\Phi)} C^2_{y_{(N\Phi)}} + g_{(N\Phi)} \gamma^{**}_{(N\Phi)} C^2_{x_{(N\Phi)}} (g_{(N\Phi)} - 2C_{(N\Phi)}) \right] \tag{17}$$

$$MSE(t^{*d}_{R_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)} \left[\gamma_{(N\Phi)} C^2_{y_{(N\Phi)}} + g_{(N\Phi)} C^2_{x_{(N\Phi)}} (g_{(N\Phi)} \gamma^{***}_{(N\Phi)} - 2\gamma_{(N\Phi)} C_{(N\Phi)}) \right] \tag{18}$$

where, $g_{(N\Phi)} = \frac{n_{(N\Phi)}}{n_{1_{(N\Phi)}} - n_{(N\Phi)}}$

2.5. *Adapted Singh and Choudhury Estimator*

For two-phase sampling in Neutroshopic statistics, we use Singh and Choudhury's [37] dual to product estimator. This estimator is intended to deal with uncertainties in Neutroshopic data, boosting population mean estimation accuracy and allowing for successful statistical analysis in complicated sampling settings.

$$t^{*d}_{P_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left(\frac{\bar{x}_{1_{(N\Phi)}}}{\bar{x}^{*d}_{(N\Phi)}} \right) \tag{19}$$

MSE for Case-I and Case-II are given as,

$$MSE(t^{*d}_{P_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)} \left[\gamma_{(N\Phi)} C^2_{y_{(N\Phi)}} + g_{(N\Phi)} \gamma^{**}_{(N\Phi)} C^2_{x_{(N\Phi)}} (g_{(N\Phi)} + 2C_{(N\Phi)}) \right] \tag{20}$$

$$MSE(t^{*d}_{P_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)} \left[\gamma_{(N\Phi)} C^2_{y_{(N\Phi)}} + g_{(N\Phi)} C^2_{x_{(N\Phi)}} (g_{(N\Phi)} \gamma^{***}_{(N\Phi)} + 2\gamma_{(N\Phi)} C_{(N\Phi)}) \right] \tag{21}$$

2.6. *Adapted Kalita and Singh Estimators*

In Neutroshopic statistics, we are employing Kalita and Singh's exponential dual to ratio and exponential dual to product estimators [39] in two-phase sampling. These estimators manage uncertainty in Neutroshopic data, improving accuracy and efficacy in challenging sampling circumstances.

$$t^{*d}_{Re_{(N\Phi)}} = \bar{y}_{(N\Phi)} exp \left(\frac{\bar{x}^{*d}_{(N\Phi)} - \bar{x}_{1_{(N\Phi)}}}{\bar{x}^{*d}_{(N\Phi)} + \bar{x}_{1_{(N\Phi)}}} \right) \tag{22}$$

$$t^{*d}_{Pe_{(N\Phi)}} = \bar{y}_{(N\Phi)} exp \left(\frac{\bar{x}_{1_{(N\Phi)}} - \bar{x}^{*d}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}} + \bar{x}^{*d}_{(N\Phi)}} \right) \tag{23}$$

MSE of both the estimators for Case-I and Case-II are given as,

$$MSE(t^{*d}_{Re_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + g_{(N\Phi)}\gamma^{**}_{(N\Phi)}C^2_{x_{(N\Phi)}}(\frac{1}{4}g_{(N\Phi)} - C_{(N\Phi)})\right] \tag{24}$$

$$MSE(t^{*d}_{Re_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)}\Big[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \frac{1}{4}g^2_{(N\Phi)}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} - \gamma_{(N\Phi)}g_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)}\Big] \tag{25}$$

$$MSE(t^{*d}_{Pe_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)}\left[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + g_{(N\Phi)}\gamma^{**}_{(N\Phi)}C^2_{x_{(N\Phi)}}(\frac{1}{4}g_{(N\Phi)} + C_{(N\Phi)})\right] \tag{26}$$

$$MSE(t^{*d}_{Pe_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)}\Big[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \frac{1}{4}g^2_{(N\Phi)}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} + \gamma_{(N\Phi)}g_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)}\Big] \tag{27}$$

2.7. *Adapted Subhash et al. Estimators*

In Neutroshopic statistics, we use Subhash et al. [40] modified ratio and product estimators, which are based on Kalita and Singh's work [39]. These estimators deal with uncertainty, improving population mean estimate in two-phase sampling and so leading to more robust statistical studies.

$$\jmath^{*d}_{Re_{(N\Phi)}} = \alpha\bar{y}_{(N\Phi)} + (1-\alpha)t^{*d}_{Re_{(N\Phi)}} \tag{28}$$

$$\jmath^{*d}_{Pe_{(N\Phi)}} = \delta\bar{y}_{(N\Phi)} + (1-\delta)t^{*d}_{Pe_{(N\Phi)}} \tag{29}$$

where, α and δ are the scalars constant.

MSE of both the estimators for Case-I and Case-II are given as,

$$MSE(\jmath^{*d}_{Re_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)}\Big[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + g_{(N\Phi)}\gamma^{**}_{(N\Phi)}C^2_{x_{(N\Phi)}}\Big(\frac{1}{4}g_{(N\Phi)} - C_{(N\Phi)}\Big) - \gamma^{**}_{(N\Phi)}\frac{A^2_{(N\Phi)}}{4B_{(N\Phi)}}\Big] \tag{30}$$

$$MSE(\jmath^{*d}_{Re_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)}\Big[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \frac{g^2_{(N\Phi)}}{4}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} - g_{(N\Phi)}\gamma_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)} - \frac{A^{*2}_{(N\Phi)}}{4B^*_{(N\Phi)}}\Big] \tag{31}$$

$$MSE(\jmath^{*d}_{Pe_{(N\Phi)}})_I = \bar{Y}^2_{(N\Phi)}\Big[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + g_{(N\Phi)}\gamma^{**}_{(N\Phi)}C^2_{x_{(N\Phi)}}(\frac{g_{(N\Phi)}}{4} + C_{(N\Phi)}) - \gamma^{**}_{(N\Phi)}\frac{D^2_{(N\Phi)}}{4B_{(N\Phi)}}\Big] \tag{32}$$

$$MSE(\jmath^{*d}_{Pe_{(N\Phi)}})_{II} = \bar{Y}^2_{(N\Phi)}\Big[\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}} + \frac{g^2_{(N\Phi)}}{4}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} + g_{(N\Phi)}\gamma_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)} - \frac{D^{*2}_{(N\Phi)}}{4B^*_{(N\Phi)}}\Big] \tag{33}$$

Where

$A_{(N\Phi)} = C^2_{x_{(N\Phi)}}(g_{(N\Phi)} - 2C_{(N\Phi)})$, $B_{(N\Phi)} = C^2_{x_{(N\Phi)}}$, $D_{(N\Phi)} = C^2_{x_{(N\Phi)}}(g_{(N\Phi)} + 2C_{(N\Phi)})$,
$A^*_{(N\Phi)} = g_{(N\Phi)}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} - 2\gamma_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)}$, $B^*_{(N\Phi)} = \gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}}$,
$D^*_{(N\Phi)} = g_{(N\Phi)}\gamma^{***}_{(N\Phi)}C^2_{x_{(N\Phi)}} + 2\gamma_{(N\Phi)}C^2_{x_{(N\Phi)}}C_{(N\Phi)}$ and $g_{(N\Phi)} = \frac{n_{(N\Phi)}}{n_{1_{(N\Phi)}} - n_{(N\Phi)}}$

3. Proposed Estimators

We have presented a generalised class of factor-type estimators for two-phase sampling in the setting of Neutroshopic statistics, building on previous work and driven by the generic character of exponential and factor type estimators. This new family of estimators is designed particularly to deal with the uncertainties and difficulties inherent with Neutroshopic. We want to improve the accuracy and reliability of population mean estimate and progress statistical studies in the field of Neutroshopic statistics by introducing this novel technique. This suggestion contributes significantly to the knowledge and use of statistical approaches in the field of Neutroshopic two-phase sampling.

$$\Im^{yp}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}} + f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}} + C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}} - \bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}} + \bar{x}_{(N\Phi)}) + 2b}\right\} \tag{34}$$

where, $A = (d-1)(d-2)$; $B = (d-1)(d-4)$; $C = (d-2)(d-3)(d-4)$, α and d are the characterizing scalars, (a≠0, b) are real constants or functions of populations parameters of the known auxiliary variable. For the fixed value of α, we get some generalized exponential estimators for different values of d, which is dicuss later on in this Section as a particular cases. Members of this class of proposed neutrosophic estimators are given in Table 5.

Note:

(i) For $\alpha = 0$, in eqaution (34) our proposed estimator $\Im^{yp}_{ft_{(N\Phi)}}$ reduces to modified exponential estimators for suitable values of (a≠0, b) members of this class of neutrosophic estimators are given in Tables [9 & 14].

$$\Im^{yp}_{exp_{(N\Phi)}} = \bar{y}_{(N\Phi)}exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}} - \bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}} + \bar{x}_{(N\Phi)}) + 2b}\right\} \tag{35}$$

(ii) For $\alpha = 1$, in equation (34) our proposed estimator $\Im^{yp}_{ft_{(N\Phi)}}$ reduces to genralized factor type ratio exponential estimators for suitable values of (a≠0, b) members of this class of neutrosophic estimators are given in Tables [10].

$$\Im^{yp}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}} + f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}} + C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}} - \bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}} + \bar{x}_{(N\Phi)}) + 2b}\right\} \tag{36}$$

Members of this class of proposed neutrosophic estimators are given in Table 10. **Particular Cases of Proposed Estimator:**

(1) When $d = 1$ in the values of A, B and C in equation (34) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{Re}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1_{(N\Phi)}}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2b}\right\}$$

which is generalised ratio type exponetial estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{Re}_{ft_{(N\Phi)}}$ are given in Table [6].

(2) When $d = 2$ in the values of A, B and C in equation (34) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{Pe}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2b}\right\}$$

which is generalised product type exponetial estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{Pe}_{ft_{(N\Phi)}}$ are given in Table [7].

(3) When $d = 3$ in the values of A, B and C in equation (34) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{DR}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1_{(N\Phi)}}-f\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2b}\right\}$$

which is generalised dual to ratio type exponetial estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{DR}_{ft_{(N\Phi)}}$ are given in Table [8].

(4) When $d = 4$ in the values of A, B and C in equation (34) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{exp}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2b}\right\}$$

which is generalised ratio type exponetial estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{exp}_{ft_{(N\Phi)}}$ are given in Table [9].

Similarly, Particular Cases of Proposed Estimator when $\alpha = 1$:

(1) When $d = 1$ in the values of A, B and C in equation (36) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{Re}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1_{(N\Phi)}}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2b}\right\}$$

which is generalised ratio type exponetial estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{Re}_{ft_{(N\Phi)}}$ are given in Table [11].

(2) When $d = 2$ in the values of A, B and C in equation (36) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{Pe}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right] exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2b}\right\}$$

which is generalised product type exponetial estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{Pe}_{ft_{(N\Phi)}}$ are given in Table [12].

(3) When $d = 3$ in the values of A, B and C in equation (36) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{DR}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1_{(N\Phi)}} - f\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}} - \bar{x}_{(N\Phi)}}\right] exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}} - \bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}} + \bar{x}_{(N\Phi)}) + 2b}\right\}$$

which is generalised dual to ratio type exponetial estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{DR}_{ft_{(N\Phi)}}$ are given in Table [13].

(4) When $d = 4$ in the values of A, B and C in equation (36) estimator $\Im^{yp}_{ft_{(N\Phi)}}$ takes the form as

$$\Im^{exp}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} exp\left\{\frac{a(\bar{x}_{1_{(N\Phi)}} - \bar{x}_{(N\Phi)})}{a(\bar{x}_{1_{(N\Phi)}} + \bar{x}_{(N\Phi)}) + 2b}\right\}$$

which is generalised ratio type exponetial ratio estimators in two phase sampling. For suitable values of (a≠0, b), members of proposed neutrosophic estimators $\Im^{exp}_{ft_{(N\Phi)}}$ are given in Table [14].

4. Bias and MSE

To derive the expressions of Bias and MSE we have following two cases for the proposed class of estimators.

Case I: A large sample, designated as S', is drawn over the population employing SRSWOR, having a size of $n'_{(N\Phi)}$, ($n'_{(N\Phi)}$ being less than $N'_{(N\Phi)}$). This sample is used to collect observations that are entirely connected to the auxiliary variable $x_{(N\Phi)}$, with the goal of estimating the population mean $\bar{X}_{(N\Phi)}$ associated with this auxiliary variable.

Case II: A sample with the symbol S and a size of $n_{(N\Phi)}$ is chosen, where ($n_{(N\Phi)} < N_{(N\Phi)}$). This sample is taken directly from the population, which has the size $N_{(N\Phi)}$, or from the set of S' characters. This sample's goal is to collect data on both the primary neutrosophic study variable and the secondary neutrosophic auxiliary variable.

4.1. *Case I*

To derive the expression for Bias and MSE for Case I, consider the following transformations as follows

$\bar{y}_{(N\Phi)} = \bar{Y}_{(N\Phi)}(1 + e_{0_{(N\Phi)}})$, $\bar{x}_{(N\Phi)} = \bar{X}_{(N\Phi)}(1 + e_{1_{(N\Phi)}})$, and $\bar{x}_{1_{(N\Phi)}} = \bar{X}_{(N\Phi)}(1 + e_{2_{(N\Phi)}})$
such that $E(e_{0_{(N\Phi)}}) = E(e_{1_{(N\Phi)}}) = E(e_{2_{(N\Phi)}}) = 0$ and $E(e^2_{0_{(N\Phi)}}) = \gamma_{(N\Phi)} C^2_{y_{(N\Phi)}}$,
$E(e^2_{1_{(N\Phi)}}) = \gamma_{(N\Phi)} C^2_{x_{(N\Phi)}}$, $E(e^2_{2_{(N\Phi)}}) = \gamma^*_{(N\Phi)} C^2_{x_{(N\Phi)}}$, $E(e_{0_{(N\Phi)}} e_{1_{(N\Phi)}}) = \gamma_{(N\Phi)} C_{(N\Phi)} C^2_{x_{(N\Phi)}}$,
$E(e_{0_{(N\Phi)}} e_{2_{(N\Phi)}}) = \gamma^*_{(N\Phi)} C_{(N\Phi)} C^2_{x_{(N\Phi)}}$, $E(e_{1_{(N\Phi)}} e_{2_{(N\Phi)}}) = \gamma^*_{(N\Phi)} C^2_{x_{(N\Phi)}}$,

$\gamma_{(N\Phi)} = \left(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\right)$, $\gamma^{*}_{(N\Phi)} = \left(\frac{1}{n1_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\right)$, $\gamma^{**}_{(N\Phi)} = \gamma_{(N\Phi)} - \gamma^{*}_{(N\Phi)} = \left(\frac{1}{n_{(N\Phi)}} - \frac{1}{n1_{(N\Phi)}}\right)$, $C_{(N\Phi)} = \rho_{y_{(N\Phi)}x_{(N\Phi)}} \frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}$ and $g_{(N\Phi)} = \frac{n_{(N\Phi)}}{n1_{(N\Phi)} - n_{(N\Phi)}}$
under the above transformations and from equation (34) expressing estimators in terms of e's, we get

$$\Im^{yp}_{ft_{(N\Phi)}} = \bar{Y}_{(N\Phi)}(1+e_{0_{(N\Phi)}}) \left[\frac{(A+c)\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}}) + f_{(N\Phi)}B\bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}})}{(A+f_{(N\Phi)}B)\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}}) + C\bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}})}\right]^{\alpha} exp\left\{\frac{a(\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}}) - \bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}}))}{a(\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}}) + \bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}})) + 2b}\right\} \tag{37}$$

on simplyfying we get

$$\begin{aligned}\Im^{yp}_{ft_{(N\Phi)}} - \bar{Y}_{(N\Phi)} = & \bar{Y}_{(N\Phi)}\Big[e_{0_{(N\Phi)}} + e_{1_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big) - e_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big) \\ & + e^2_{1_{(N\Phi)}}\Big(- \alpha\xi_{(N\Phi)}\phi_{2_{(N\Phi)}} + \frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)} - \frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) \\ & + e^2_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} + \frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)} - \frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big) \\ & + e_{0_{(N\Phi)}}e_{1_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big) - e_{0_{(N\Phi)}}e_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big) \\ & + e_{1_{(N\Phi)}}e_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} - \alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} - \alpha(\alpha-1)\xi^2_{(N\Phi)} \\ & + \alpha\xi_{(N\Phi)}k_{(N\Phi)} - \frac{k^2_{(N\Phi)}}{4}\Big)\Big]\end{aligned} \tag{38}$$

where $\phi_{1_{(N\Phi)}} = \frac{f_{(N\Phi)}B}{A+f_{(N\Phi)}B+c}$, $\phi_{2_{(N\Phi)}} = \frac{C}{A+f_{(N\Phi)}B+c}$, $\phi_{3_{(N\Phi)}} = \frac{A+C}{A+f_{(N\Phi)}B+c}$, $\phi_{4_{(N\Phi)}} = \frac{A+f_{(N\Phi)}B}{A+f_{(N\Phi)}B+c}$, $\xi_{(N\Phi)} = \phi_{1_{(N\Phi)}} - \phi_{2_{(N\Phi)}} = \phi_{4_{(N\Phi)}} - \phi_{3_{(N\Phi)}}$ and $k_{(N\Phi)} = \left(\frac{a\bar{X}_{(N\Phi)}}{a\bar{X}_{(N\Phi)}+b}\right)$

To obtain Bias of the estimators we will take expectation of equation (38) and then by substituting the value of the considered transformations, we get

$$Bias[\Im^{yp}_{ft_{(N\Phi)}}] =\bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\alpha\xi_{(N\Phi)}\phi_{2_{(N\Phi)}}-\frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)} -\frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2}+\frac{3}{8}k^2_{(N\Phi)}\Big)+\Big(\frac{1}{n_{1_{(N\Phi)}}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} +\frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)}-\frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2}-\frac{k^2_{(N\Phi)}}{8}\Big) +\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{N_{(N\Phi)}}\Big)\rho C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}-\frac{k_{(N\Phi)}}{2}\Big) -\Big(\frac{1}{n_{1_{(N\Phi)}}}-\frac{1}{N_{(N\Phi)}}\Big)\rho C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}-\frac{k_{(N\Phi)}}{2}\Big) +\Big(\frac{1}{n_{1_{(N\Phi)}}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\alpha(\alpha-1)\xi^2_{(N\Phi)}+\alpha\xi_{(N\Phi)}k_{(N\Phi)} -\frac{k^2_{(N\Phi)}}{4}\Big)\Big] \tag{39}$$

Squaring both sides of the equation (38) and then taking expectation on both sides, the MSE will take the structure as

$$MSE[\Im^{yp}_{ft_{(N\Phi)}}] =\bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}}+\Big(\alpha\xi_{(N\Phi)}-\frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} -\frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}}+2\big(\alpha\xi_{(N\Phi)}-\frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} -\frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \tag{40}$$

Now , we can obtain the optimal value of α by differentiating equation (40) with respect to α and equating its to zero we will get

$$\alpha=\frac{1}{\xi_{(N\Phi)}}\left\{\frac{k_{(N\Phi)}}{2}-\rho_{(N\Phi)}\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}\right\} \tag{41}$$

we can get the minimum MSE of $\Im^{yp}_{ft_{(N\Phi)}}$ by substituting the value of α in equation (40)

$$MSE[\Im^{yp}_{ft}]_{min_{(N\Phi)}}=\bar{Y}^2_{(N\Phi)}\Big[C^2_{y_{(N\Phi)}}(\gamma_{(N\Phi)}-\gamma^{**}_{(N\Phi)}\rho^2_{(N\Phi)})\Big] \tag{42}$$

4.1.1. *Properties of the particular cases of the proposed estimators*

(i) When $d = 1$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{Re}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{Re}_{ft_{(N\Phi)}}$:

$$\begin{aligned} Bias[\Im^{Re}_{ft_{(N\Phi)}}] =& \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\alpha - \frac{\alpha(\alpha-1)}{2} + \frac{\alpha k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\frac{\alpha(\alpha-1)}{2} + \frac{\alpha k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big) \\ &+ \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-\alpha - \frac{k_{(N\Phi)}}{2}\Big) \\ &- \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-\alpha - \frac{k_{(N\Phi)}}{2}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\alpha(\alpha-1) - \alpha k_{(N\Phi)} - \frac{k^2_{(N\Phi)}}{4}\Big)\Big] \end{aligned} \quad (43)$$

MSE of the estimator $\Im^{Re}_{ft_{(N\Phi)}}$:

$$\begin{aligned} MSE[\Im^{Re}_{ft_{(N\Phi)}}] =& \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(-\alpha - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} \\ &+ 2\Big(-\alpha - \frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \end{aligned} \quad (44)$$

Optimal values of α

$$\alpha = -\left\{\frac{k_{(N\Phi)}}{2} - \rho_{(N\Phi)}\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}\right\} \quad (45)$$

Minimum MSE of the estimator $\Im^{Re}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{Re}_{ft}]_{min_{(N\Phi)}} = \bar{Y}^2_{(N\Phi)}\Big[C^2_{y_{(N\Phi)}}(\gamma_{(N\Phi)} - \gamma^{**}_{(N\Phi)}\rho^2_{(N\Phi)})\Big] \quad (46)$$

(ii) When $d = 2$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{Pe}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{Pe}_{ft_{(N\Phi)}}$:

$$\begin{aligned} Bias[\Im^{Pe}_{ft_{(N\Phi)}}] =& \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{\alpha(\alpha-1)}{2} - \frac{\alpha k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\alpha + \frac{\alpha(\alpha-1)}{2} - \frac{\alpha k}{2} - \frac{k^2_{(N\Phi)}}{8}\Big) \\ &+ \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\alpha - \frac{k_{(N\Phi)}}{2}\Big) \\ &- \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\alpha - \frac{k_{(N\Phi)}}{2}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\alpha(\alpha-1) + \alpha k_{(N\Phi)} - \frac{k^2_{(N\Phi)}}{4}\Big)\Big] \end{aligned} \quad (47)$$

MSE of the estimator $\Im^{Pe}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{Pe}_{ft_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(\alpha - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\big(\alpha - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (48)$$

Optimal values of α

$$\alpha = \left\{\frac{k_{(N\Phi)}}{2} - \rho_{(N\Phi)}\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}\right\} \quad (49)$$

Minimum MSE of the estimator $\Im^{Pe}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{Pe}_{ft}]_{min_{(N\Phi)}} = \bar{Y}^2_{(N\Phi)}\Big[C^2_{y_{(N\Phi)}}(\gamma_{(N\Phi)} - \gamma^{**}_{(N\Phi)}\rho^2_{(N\Phi)})\Big] \quad (50)$$

(iii) When $d = 3$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{DR}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{DR}_{ft_{(N\Phi)}}$:

$$Bias[\Im^{DR}_{ft_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(- \frac{\alpha(\alpha-1)}{2}\aleph^2 - \frac{\alpha\aleph k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\alpha\aleph + \frac{\alpha(\alpha-1)}{2}\aleph^2 - \frac{\alpha\aleph k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big) + \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\alpha\aleph - \frac{k_{(N\Phi)}}{2}\Big) - \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\alpha\aleph - \frac{k_{(N\Phi)}}{2}\Big) + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(- \alpha(\alpha-1)\aleph^2 + \alpha\aleph k - \frac{k^2_{(N\Phi)}}{4}\Big)\Big] \quad (51)$$

Where $\aleph = \frac{-n_{(N\Phi)}}{n_{1_{(N\Phi)}} - n_{(N\Phi)}}$.

MSE of the estimator $\Im^{DR}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{DR}_{ft_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(\alpha\aleph - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\big(\alpha\aleph - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (52)$$

Optimal values of α

$$\alpha = \frac{1}{\aleph}\left\{\frac{k_{(N\Phi)}}{2} - \rho_{(N\Phi)}\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}\right\} \quad (53)$$

Minimum MSE of the estimator $\Im^{DR}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{DR}_{ft}]_{min_{(N\Phi)}} = \bar{Y}^2_{(N\Phi)}\Big[C^2_{y_{(N\Phi)}}(\gamma_{(N\Phi)} - \gamma^{**}_{(N\Phi)}\rho^2_{(N\Phi)})\Big] \quad (54)$$

(iv) When $d = 4$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{exp}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{exp}_{ft_{(N\Phi)}}$:

$$Bias[\Im^{exp}_{ft_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\frac{3}{8}k^2_{(N\Phi)}\Big) + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{k^2_{(N\Phi)}}{8}\Big) + \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-\frac{k_{(N\Phi)}}{2}\Big) - \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-\frac{k_{(N\Phi)}}{2}\Big) + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{k^2_{(N\Phi)}}{4}\Big)\Big] \quad (55)$$

MSE of the estimator $\Im^{exp}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{yp}_{ft_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(-\frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\Big(-\frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (56)$$

Remarks: Similarly, For the proposed Estimator when $\alpha = 1$

(i) When $d = 1$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{Re}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{Re}_{vk_{(N\Phi)}}$:

$$Bias[\Im^{Re}_{vk_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(1 + \frac{k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\frac{k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big) + \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-1 - \frac{k_{(N\Phi)}}{2}\Big) - \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-1 - \frac{k_{(N\Phi)}}{2}\Big) + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-k_{(N\Phi)} - \frac{k^2_{(N\Phi)}}{4}\Big)\Big] \quad (57)$$

MSE of the estimator $\Im^{Re}_{vk_{(N\Phi)}}$:

$$MSE[\Im^{Re}_{vk_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(-1 - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\Big(-1 - \frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (58)$$

(ii) When $d = 2$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{Pe}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{Pe}_{vk_{(N\Phi)}}$:

$$\begin{aligned} Bias[\Im^{Pe}_{vk_{(N\Phi)}}] =& \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(1 - \frac{k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big) \\ &+ \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(1 - \frac{k_{(N\Phi)}}{2}\Big) \\ &- \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(1 - \frac{k_{(N\Phi)}}{2}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(k_{(N\Phi)} - \frac{k^2_{(N\Phi)}}{4}\Big)\Big] \end{aligned} \tag{59}$$

MSE of the estimator $\Im^{Pe}_{vk_{(N\Phi)}}$:

$$\begin{aligned} MSE[\Im^{Pe}_{vk_{(N\Phi)}}] =& \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(1 - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} \\ &+ 2\big(1 - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \end{aligned} \tag{60}$$

(iii) When $d = 3$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{DR}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{DR}_{vk_{(N\Phi)}}$:

$$\begin{aligned} Bias[\Im^{DR}_{vk_{(N\Phi)}}] =& \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{\aleph k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\aleph - \frac{\aleph k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big) \\ &+ \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\aleph - \frac{k_{(N\Phi)}}{2}\Big) \\ &- \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(\aleph - \frac{k_{(N\Phi)}}{2}\Big) \\ &+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\aleph k_{(N\Phi)} - \frac{k^2_{(N\Phi)}}{4}\Big)\Big] \end{aligned} \tag{61}$$

Where $\aleph = \frac{-n_{(N\Phi)}}{n_{1_{(N\Phi)}} - n_{(N\Phi)}}$.

MSE of the estimator $\Im^{DR}_{vk_{(N\Phi)}}$:

$$\begin{aligned} MSE[\Im^{DR}_{vk_{(N\Phi)}}] =& \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(\aleph - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} \\ &+ 2\big(\aleph - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \end{aligned} \tag{62}$$

(iv) When $d=4$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{exp}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{exp}_{vk_{(N\Phi)}}$:

$$Bias[\Im^{exp}_{vk_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\frac{3}{8}k^2_{(N\Phi)}\Big)+\Big(\frac{1}{n_{1_{(N\Phi)}}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{k^2_{(N\Phi)}}{8}\Big)+\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-\frac{k_{(N\Phi)}}{2}\Big)$$
$$-\Big(\frac{1}{n_{1_{(N\Phi)}}}-\frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big(-\frac{k_{(N\Phi)}}{2}\Big)+\Big(\frac{1}{n_{1_{(N\Phi)}}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{k^2_{(N\Phi)}}{4}\Big)\Big] \quad (63)$$

MSE of the estimator $\Im^{exp}_{vk_{(N\Phi)}}$:

$$MSE[\Im^{yp}_{vk_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}}+\Big(-\frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}}$$
$$+2\Big(-\frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}}-\frac{1}{n_{1_{(N\Phi)}}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (64)$$

4.2. *Case II*

To derive the expression for Bias and MSE for Case I, consider the following transformations as follows

$\bar{y}_{(N\Phi)}=\bar{Y}_{(N\Phi)}(1+e_{0_{(N\Phi)}})$, $\bar{x}_{(N\Phi)}=\bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}})$, and $\bar{x}_{1_{(N\Phi)}}=\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}})$
such that $E(e_{0_{(N\Phi)}})=E(e_{1_{(N\Phi)}})=E(e_{2_{(N\Phi)}})=0$ and $E(e^2_{0_{(N\Phi)}})=\gamma_{(N\Phi)}C^2_{y_{(N\Phi)}}$,
$E(e^2_{1_{(N\Phi)}})=\gamma_{(N\Phi)}C^2_{x_{(N\Phi)}}$, $E(e^2_{2_{(N\Phi)}})=\gamma^*_{(N\Phi)}C^2_{x_{(N\Phi)}}$,
$E(e_{0_{(N\Phi)}}e_{1_{(N\Phi)}})=\gamma_{(N\Phi)}C_{(N\Phi)}C^2_{x_{(N\Phi)}}$, $E(e_{0_{(N\Phi)}}e_{2_{(N\Phi)}})=0$,
$E(e_{1_{(N\Phi)}}e_{2_{(N\Phi)}})=0$, $\gamma_{(N\Phi)}=\left(\frac{1}{n_{(N\Phi)}}-\frac{1}{N_{(N\Phi)}}\right)$, $\gamma^*_{(N\Phi)}=\left(\frac{1}{n_{1_{(N\Phi)}}}-\frac{1}{N_{(N\Phi)}}\right)$,
$\gamma^{**}_{(N\Phi)}=\gamma_{(N\Phi)}-\gamma^*_{(N\Phi)}=\left(\frac{1}{n_{(N\Phi)}}-\frac{1}{n_{1_{(N\Phi)}}}\right)$,
$\gamma^{***}_{(N\Phi)}=\gamma_{(N\Phi)}+\gamma^*_{(N\Phi)}$, $C_{(N\Phi)}=\rho_{y_{(N\Phi)}x_{(N\Phi)}}\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}$ and $g_{(N\Phi)}=\frac{n_{(N\Phi)}}{n_{1_{(N\Phi)}}-n_{(N\Phi)}}$
under the above transformations and from equation (34) expressing estimators in terms of e's, we get

$$\Im^{yp}_{ft_{(N\Phi)}} = \bar{Y}_{(N\Phi)}(1+e_{0_{(N\Phi)}})\left[\frac{(A+c)\bar{X}(1+e_{2_{(N\Phi)}})+f_{(N\Phi)}B\bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}})}{(A+f_{(N\Phi)}B)\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}})+C\bar{X}_{(N\Phi)}(1+e_1)}\right]^{\alpha} \quad (65)$$
$$exp\left\{\frac{a(\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}})-\bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}}))}{a(\bar{X}_{(N\Phi)}(1+e_{2_{(N\Phi)}})+\bar{X}_{(N\Phi)}(1+e_{1_{(N\Phi)}}))+2b}\right\}$$

on simplyfying we get

$$\begin{aligned}\Im^{yp}_{ft_{(N\Phi)}} - \bar{Y}_{(N\Phi)} =&\bar{Y}_{(N\Phi)}\Big[e_{0_{(N\Phi)}} + e_{1_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big) - e_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big)\\&+ e^2_{1_{(N\Phi)}}\Big(-\alpha\xi_{(N\Phi)}\phi_{2_{(N\Phi)}} + \frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)} - \frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big)\\&+ e^2_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} + \frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)} - \frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big)\\&+ e_{0_{(N\Phi)}}e_{1_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big) - e_{0_{(N\Phi)}}e_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big)\\&+ e_{1_{(N\Phi)}}e_{2_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} - \alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} - \alpha(\alpha-1)\xi^2_{(N\Phi)}\\&+ \alpha\xi_{(N\Phi)}k_{(N\Phi)} - \frac{k^2_{(N\Phi)}}{4}\Big)\Big] \qquad (66)\end{aligned}$$

To obtain Bias of the estimators we will take expectation of equation (66) and then by substituting the value of the considered transformations, we get

$$\begin{aligned}Bias[\Im^{yp}_{ft_{(N\Phi)}}] =&\bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C_{x_{(N\Phi)}}\Big\{C_{x_{(N\Phi)}}\Big(-\alpha\xi_{(N\Phi)}\phi_{2_{(N\Phi)}} + \frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)}\\&- \frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)}C_{y_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big)\Big\}\\&+ \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\alpha\xi_{(N\Phi)}\phi_{4_{(N\Phi)}} + \frac{\alpha(\alpha-1)}{2}\xi^2_{(N\Phi)} - \frac{\alpha\xi_{(N\Phi)}k_{(N\Phi)}}{2}\\&- \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \qquad (67)\end{aligned}$$

Squaring both sides of the equation (66) and then taking expectation on both sides, the MSE will take the structure as

$$\begin{aligned}MSE[\Im^{yp}_{ft_{(N\Phi)}}] =&\bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}}\\&- \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\big(\alpha\xi_{(N\Phi)} - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}}\\&- \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \qquad (68)\end{aligned}$$

Now , we can obtain the optimal value of α by differentiating equation (68) with respect to α and equating its to zero we will get

$$\alpha = \frac{1}{\xi_{(N\Phi)}}\left\{\frac{k_{(N\Phi)}}{2} - \rho_{(N\Phi)}\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}\right\} \qquad (69)$$

we can get the minimum MSE of $\Im^{yp}_{ft_{(N\Phi)}}$ by substituting the value of α in equation (68)

$$MSE[\Im^{yp}_{ft_{(N\Phi)}}]_{min} = \bar{Y}^2_{(N\Phi)}C^2_{y_{(N\Phi)}}\gamma_{(N\Phi)}\Big[1 - \frac{\gamma_{(N\Phi)}\rho^2}{\gamma^{**}_{(N\Phi)}}\Big] \qquad (70)$$

4.2.1. *Properties of the particular cases of the proposed estimators*

(i) When $d = 1$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{Re}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{Re}_{ft_{(N\Phi)}}$:

$$Bias[\Im^{yp}_{ft_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C_{x_{(N\Phi)}}\Big\{C_{x_{(N\Phi)}}\Big(\alpha + \frac{\alpha(\alpha-1)}{2} + \frac{\alpha k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)}C_{y_{(N\Phi)}}\Big(-\alpha - \frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(+ \frac{\alpha(\alpha-1)}{2} + \frac{\alpha k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \tag{71}$$

MSE of the estimator $\Im^{Re}_{ft}$:

$$MSE[\Im^{Re}_{ft_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(-\alpha - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\big(-\alpha - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \tag{72}$$

Optimal values of α

$$\alpha = -\left\{\frac{k_{(N\Phi)}}{2} - \rho_{(N\Phi)}\frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}}\right\} \tag{73}$$

Minimum MSE of the estimator $\Im^{Re}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{Re}_{ft}]_{min_{(N\Phi)}} = \bar{Y}^2_{(N\Phi)}C^2_{y_{(N\Phi)}}\gamma_{(N\Phi)}\Big[1 - \frac{\gamma_{(N\Phi)}\rho^2}{\gamma^{**}_{(N\Phi)}}\Big] \tag{74}$$

(ii) When $d = 2$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{Pe}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{Pe}_{ft_{(N\Phi)}}$:

$$Bias[\Im^{Pe}_{ft_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C_{x_{(N\Phi)}}\Big\{C_{x_{(N\Phi)}}\Big(\frac{\alpha(\alpha-1)}{2} - \frac{\alpha k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)}C_{y_{(N\Phi)}}\Big(\alpha - \frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\alpha + \frac{\alpha(\alpha-1)}{2} - \frac{\alpha k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \tag{75}$$

MSE of the estimator $\Im^{Pe}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{Pe}_{ft_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(\alpha - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\big(\alpha - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \tag{76}$$

Optimal values of α

$$\alpha = \left\{ \frac{k_{(N\Phi)}}{2} - \rho_{(N\Phi)} \frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}} \right\} \tag{77}$$

Minimum MSE of the estimator $\Im^{Pe}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{Pe}_{ft}]_{min_{(N\Phi)}} = \bar{Y}^2_{(N\Phi)} C^2_{y_{(N\Phi)}} \gamma_{(N\Phi)} \Big[1 - \frac{\gamma_{(N\Phi)}\rho^2}{\gamma^{**}_{(N\Phi)}}\Big] \tag{78}$$

(iii) When $d = 3$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{DR}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{DR}_{ft_{(N\Phi)}}$:

$$\begin{aligned} Bias[\Im^{DR}_{ft_{(N\Phi)}}] =& \bar{Y}_{(N\Phi)} \Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big) C_{x_{(N\Phi)}} \Big\{ C_{x_{(N\Phi)}} \Big(\frac{\alpha(\alpha-1)}{2}\aleph^2 - \frac{\alpha\aleph k_{(N\Phi)}}{2} \\ &+ \frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)} C_{y_{(N\Phi)}} \Big(\alpha\aleph - \frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} \\ &- \frac{1}{N_{(N\Phi)}}\Big) C^2_{x_{(N\Phi)}} \Big(\alpha\aleph + \frac{\alpha(\alpha-1)}{2}\aleph^2 - \frac{\alpha\aleph k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \end{aligned} \tag{79}$$

Where $\aleph = \frac{-n_{(N\Phi)}}{n_{1_{(N\Phi)}} - n_{(N\Phi)}}$.

MSE of the estimator $\Im^{DR}_{ft_{(N\Phi)}}$:

$$\begin{aligned} MSE[\Im^{DR}_{ft_{(N\Phi)}}] =& \bar{Y}^2_{(N\Phi)} \Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big) C^2_{y_{(N\Phi)}} + \Big(\alpha\aleph - \frac{k_{(N\Phi)}}{2}\Big)^2 \Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big) C^2_{x_{(N\Phi)}} \\ &+ 2\Big(\alpha\aleph - \frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big) \rho_{(N\Phi)} C_{y_{(N\Phi)}} C_{x_{(N\Phi)}}\Big] \end{aligned} \tag{80}$$

Optimal values of α

$$\alpha = \frac{1}{\aleph}\left\{ \frac{k_{(N\Phi)}}{2} - \rho_{(N\Phi)} \frac{C_{y_{(N\Phi)}}}{C_{x_{(N\Phi)}}} \right\} \tag{81}$$

Minimum MSE of the estimator $\Im^{DR}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{yp}_{ft}]_{min_{(N\Phi)}} = \bar{Y}^2_{(N\Phi)} C^2_{y_{(N\Phi)}} \gamma_{(N\Phi)} \Big[1 - \frac{\gamma_{(N\Phi)}\rho^2}{\gamma^{**}_{(N\Phi)}}\Big] \tag{82}$$

(iv) When $d = 4$ in the values of A, B and C, $\Im^{yp}_{ft_{(N\Phi)}}$ becomes $\Im^{exp}_{ft_{(N\Phi)}}$

Bias of the estimator $\Im^{exp}_{ft_{(N\Phi)}}$:

$$\begin{aligned} Bias[\Im^{exp}_{ft_{(N\Phi)}}] =& \bar{Y}_{(N\Phi)} \Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big) C_{x_{(N\Phi)}} \Big\{ C_{x_{(N\Phi)}} \Big(\frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)} C_{y_{(N\Phi)}} \Big(\\ &- \frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big) C^2_{x_{(N\Phi)}} \Big(- \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \end{aligned} \tag{83}$$

MSE of the estimator $\Im^{exp}_{ft_{(N\Phi)}}$:

$$MSE[\Im^{exp}_{ft_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(-\frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\Big(-\frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (84)$$

Remarks: Similarly, For the proposed Estimator when $\alpha = 1$

(i) When $d = 1$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{Re}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{Re}_{vk_{(N\Phi)}}$:

$$Bias[\Im^{Re}_{vk_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C_{x_{(N\Phi)}}\Big\{C_{x_{(N\Phi)}}\Big(1 + \frac{k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)}C_{y_{(N\Phi)}}\Big(-1 - \frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\frac{k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \quad (85)$$

MSE of the estimator $\Im^{Re}_{vk_{(N\Phi)}}$:

$$MSE[\Im^{Re}_{vk_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(-1 - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\Big(-1 - \frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (86)$$

(ii) When $d = 2$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{Pe}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{Pe}_{vk_{(N\Phi)}}$:

$$Bias[\Im^{Pe}_{vk_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C_{x_{(N\Phi)}}\Big\{C_{x_{(N\Phi)}}\Big(-\frac{k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)}C_{y_{(N\Phi)}}\Big(1 - \frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(1 + \frac{k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \quad (87)$$

MSE of the estimator $\Im^{Pe}_{vk_{(N\Phi)}}$:

$$MSE[\Im^{Pe}_{vk_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(1 - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\Big(1 - \frac{k_{(N\Phi)}}{2}\Big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \quad (88)$$

(iii) When $d = 3$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{DR}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{DR}_{vk_{(N\Phi)}}$:

$$Bias[\Im^{DR}_{vk_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C_{x_{(N\Phi)}}\Big\{C_{x_{(N\Phi)}}\Big(-\frac{\aleph k_{(N\Phi)}}{2} + \frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)}C_{y_{(N\Phi)}}\Big(\aleph - \frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(\aleph + \frac{\aleph k_{(N\Phi)}}{2} - \frac{k^2_{(N\Phi)}}{8}\Big)\Big] \tag{89}$$

Where $\aleph = \frac{-n_{(N\Phi)}}{n_{1_{(N\Phi)}} - n_{(N\Phi)}}$.

MSE of the estimator $\Im^{DR}_{vk_{(N\Phi)}}$:

$$MSE[\Im^{DR}_{vk_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(\aleph - \frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\big(\aleph - \frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \tag{90}$$

(iv) When $d = 4$ in the values of A, B and C, $\Im^{yp}_{vk_{(N\Phi)}}$ becomes $\Im^{exp}_{vk_{(N\Phi)}}$

Bias of the estimator $\Im^{exp}_{vk_{(N\Phi)}}$:

$$Bias[\Im^{exp}_{vk_{(N\Phi)}}] = \bar{Y}_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C_{x_{(N\Phi)}}\Big\{C_{x_{(N\Phi)}}\Big(\frac{3}{8}k^2_{(N\Phi)}\Big) + \rho_{(N\Phi)}C_{y_{(N\Phi)}}\Big(-\frac{k_{(N\Phi)}}{2}\Big)\Big\} + \Big(\frac{1}{n_{1_{(N\Phi)}}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{x_{(N\Phi)}}\Big(-\frac{k^2_{(N\Phi)}}{8}\Big)\Big] \tag{91}$$

MSE of the estimator $\Im^{exp}_{vk_{(N\Phi)}}$:

$$MSE[\Im^{exp}_{vk_{(N\Phi)}}] = \bar{Y}^2_{(N\Phi)}\Big[\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)C^2_{y_{(N\Phi)}} + \Big(-\frac{k_{(N\Phi)}}{2}\Big)^2\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{n_{1_{(N\Phi)}}}\Big)C^2_{x_{(N\Phi)}} + 2\big(-\frac{k_{(N\Phi)}}{2}\big)\Big(\frac{1}{n_{(N\Phi)}} - \frac{1}{N_{(N\Phi)}}\Big)\rho_{(N\Phi)}C_{y_{(N\Phi)}}C_{x_{(N\Phi)}}\Big] \tag{92}$$

5. Numerical Study

The study goal of this in-depth investigation study is to investigate and compare several Neutrosophic estimators for estimating the population mean within the context of Neutrosophic two-phase sampling. To accomplish this goal, we chose real-world data from the open public website "https://data.gov.in/resource/seasonal-and-annual-minimum-maximum-temperature-series-1901-2019." The dataset includes complete data on All India Seasonal and Annual Temperature the series of circuits including minimum, maximum, and mean temperatures over a long period of time.

The information was made available under the National Data Sharing and Accessibility Policy (NDSAP) by prestigious organisations such as the Ministry of Earth Sciences and the

India Meteorological Department (IMD), Pune. The diversity of temperature data enables us to obtain significant insights into temperature variations and patterns throughout seasons and years.

Our major goal is to compare the performance of the proposed Neutrosophic estimators to those existing estimators in the unique situation of two-phase sampling. We aim to test the efficiency, accuracy, and robustness of these estimators in calculating population means by doing this research using real data from India.

The consequences of our research go beyond the specific dataset, since the findings may have broader applicability in a variety of domains that use two-phase sampling and Neutrosophic statistics. We think that by adding to the knowledge base in this field, we will improve understanding and practical use of Neutrosophic estimators in real-world settings, allowing for better informed decision-making and developing statistical approaches.

Discriptions of Datasets are given in Table [1]. The auxiliary variable $X_{(N\Phi)}$ for Population A reflects the minimum and highest temperatures reported in January and February. The study variable $Y_{(N\Phi)}$, on the other hand, reflects the lowest and highest temperatures reported from March to May.

The auxiliary variable $X_{(N\Phi)}$ represents the minimum and maximum temperatures measured from June to September for Population B, whereas the study variable $Y_{(N\Phi)}$ reflects the minimum and maximum temperatures reported from October to December.

6. **Simulation Study**

The fundamental goal of this advanced scientific research was to validate and assess the effectiveness of proposed Neutrosophic esitimators and up against adapted Neutrosophic estimators for the study variable $Y_{(N\Phi)}$. To achieve this purpose, the researchers employed Neutrosophic data with known auxiliary parameters and the Neutrosophic normal distribution in a rigorous simulation workouts. Neutrosophic normal distributions were used to create the study Neutrosophic variable, designated as $Y_{(N\Phi)}$ as neutrosophic study variable, and the auxiliary variable, labelled as $X_{(N\Phi)}$.

The parameters for $Y_{(N\Phi)} \sim \text{NN}(\mu_{y_{(N\Phi)}}, \sigma^2_{y_{(N\Phi)}})$, where $\mu_{y_{(N\Phi)}}$ stood for the mean, and $\sigma_{y_{(N\Phi)}}$ for the standard deviation. Similar to this, the parameters for $X_{(N\Phi)}\sim \text{NN}(\mu_{x_{(N\Phi)}},\ \sigma^2_{x_{(N\Phi)}})$, where $\mu_{x_{(N\Phi)}}$ stood for mean, and $\sigma_{x_{(N\Phi)}}$ for standard deviation. Specific parameter values for $Y_{(N\Phi)}$ and $X_{(N\Phi)}$ were chosen in order to aid numerical demonstration, creating a simulated dataset with 100 normally distributed observations for each variable. The parameters were set for $Y_{(N\Phi)} \sim \text{NN}([76.0, 54.9], [(12.9)^2, (17.2)^2])$, and for $X_{(N\Phi)}\sim \text{NN}([17.2, 18.4], [(5.8)^2, (6.7)^2])$.

For the simulated Neutrosophic data, the researchers produced descriptive statistics, giving a thorough breakdown of the dataset's features. The researchers intended to carefully assess the efficiency of the suggested Neutrosophic estimators and compare them to other estimators, so they carried out this complex simulation study. The paper offers important new understandings on the use of Neutrosophic statistics to handle uncertainties and indeterminacies in challenging real-world data analysis. In scenarios involving neutrosophic data and two-phase sampling, the results of this study have the potential to expand statistical approaches and improve knowledge of population parameter estimate.

7. Conclusions

We introduced a novel family of neutrosophic factor-type exponential estimators in two phase sampling for estimating neutrosophic population mean ($\bar{Y}_{(N\Phi)}$) using known neutrosophic auxiliary parameters in this thorough study. We thoroughly explored the neutrosophic sampling characteristics, specifically the bias and mean squared error (MSE), with an emphasis on degree one approximation.

We determined the neutrosophic lowest MSE by doing thorough investigations to determine the characterising scalars' neutrosophic optimal values for the proposed estimator. Several adapted neutrosophic competing estimators, such as $t_{0_{(N\Phi)}}$ $t^{d}_{R_{(N\Phi)}}$, $t^{d}_{P_{(N\Phi)}}$, $t^{d}_{Re_{(N\Phi)}}$, $t^{d}_{Pe_{(N\Phi)}}$, $t^{*d}_{R_{(N\Phi)}}$, $t^{*d}_{P_{(N\Phi)}}$, $t^{*d}_{Re_{(N\Phi)}}$, $t^{*d}_{Pe_{(N\Phi)}}$, $\jmath^{*d}_{Re_{(N\Phi)}}$, $\jmath^{*d}_{Pe_{(N\Phi)}}$ estimators were used to compare the performance of our proposed estimators, $\Im^{yp}_{ft_{(N\Phi)}}$.

Our investigations indisputably showed that our suggested estimators $\Im^{yp}_{ft_{(N\Phi)}}$, as shown by their reduced bias and MSE values, exhibited superior efficiency than the current estimators. Tables [5, 6, 7, 8, 9, 10, 11, 12, 13, and 14] provide specific examples of our suggested estimators for various values of (a, b). Further evidence that our proposed estimator performed better than any other neutrosophic competing estimators for Neutosophic proposed estimators can be seen in Tables [2, 3, 4].

We strongly advise their use for the estimate of neutrosophic population mean $\bar{Y}_{(N\Phi)}$ in numerous domains of application based on the excellent performance and efficiency proven by our introduced class of estimators. It is significant to highlight that neutrosophic estimators offer better population mean estimate in situations when study variable data are nondeterministic. It is recognised that neutrosophic estimators may still be better to classical estimators in circumstances when study variable data are indeterministic.

TABLE 1. Descriptive statistics of the Population - 1, Population - 2 and Simulated Data

Parameter	Population-1 Neutrosophic Value	Population-2 Neutrosophic Value	Simulated Data Neutrosophic Value
$N_{(N\Phi)}$	[119, 119]	[119, 119]	[100, 100]
$n_{(N\Phi)}$	[24, 24]	[24, 24]	[24, 24]
$n_{1_{(N\Phi)}}$	[44, 44]	[44, 44]	[44, 44]
$\bar{Y}_{(N\Phi)}$	[20.6937, 3055933]	[16.58437, 27.23613]	[20.6937, 31.55933]
$\bar{X}_{(N\Phi)}$	[13.90807, 24.6737]	[23.30966, 31.21807]	[13.90807, 24.6737]
$C_{y_{(N\Phi)}}$	[0.02657039, 0.02539073]	[0.03440212, 0.02580739]	[0.02657039, 0.02539073]
$C_{x_{(N\Phi)}}$	[0.04133301, 0.03918572]	[0.01435458, 0.01423702]	[0.04133301, 0.03918572]
$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	[0.6273783, 0.7201808]	[0.6737446, 0.7504361]	[0.01838219, -0.05842247]

TABLE 2. Mean Square Error of the Neutrosophic Estimators based on Population - 1.

Estimators	Case-I MSE	Case-II MSE
$t_{0_{(N\Phi)}}$	[0.01005628, 0.02135852]	[0.01005628, 0.02135852]
$t^{d}_{R_{(N\Phi)}}$	[0.01273596, 0.02677414]	[0.02524190, 0.05277647]
$t^{d}_{P_{(N\Phi)}}$	[0.03508849, 0.07387339]	[0.0644997, 0.1354970]
$t^{d}_{Re_{(N\Phi)}}$	[0.007932134, 0.016825020]	[0.008945461, 0.018872943]
$t^{d}_{Pe_{(N\Phi)}}$	[0.01910840, 0.04037465]	[0.02857436, 0.06023321]
$t^{*d}_{R_{(N\Phi)}}$	[0.01659733, 0.03480892]	[0.03663452, 0.07652684]
$t^{*d}_{P_{(N\Phi)}}$	[0.04342036, 0.09132803]	[0.08374387, 0.17579148]
$t^{*d}_{Re_{(N\Phi)}}$	[0.008338662, 0.017656234]	[0.01081217, 0.02274252]
$t^{*d}_{Pe_{(N\Phi)}}$	[0.02175018, 0.04591579]	[0.03436685, 0.07237484]
$\jmath^{*d}_{Re_{(N\Phi)}}$	[0.007802574, 0.016571881]	[0.007289519, 0.015482204]
$\jmath^{*d}_{Pe_{(N\Phi)}}$	[0.007802574, 0.016571881]	[0.007289519, 0.015482204]
$\Im^{yp}_{ft_{(N\Phi)}}$	**[0.007802574, 0.015051057]**	**[0.003104505, 0.001902523]**

TABLE 3. Mean Square Error of the Neutrosophic Estimators based on Population - 2.

	Case-I	**Case-II**
Estimators	**MSE**	**MSE**
$t_{0_{(N\Phi)}}$	[0.01082764, 0.01643405]	[0.01082764, 0.01643405]
$t^{d}_{R_{(N\Phi)}}$	[0.008434704, 0.012325983]	[0.007436709, 0.011372767]
$t^{d}_{P_{(N\Phi)}}$	[0.01536729, 0.02623753]	[0.01961242, 0.03580566]
$t^{d}_{Re_{(N\Phi)}}$	[0.009362831, 0.013668091]	[0.00845794, 0.01211462]
$t^{d}_{Pe_{(N\Phi)}}$	[0.01282912, 0.02062387]	[0.01454580, 0.02433106]
$t^{*d}_{R_{(N\Phi)}}$	[0.008213724, 0.012187818]	[0.007405786, 0.012077748]
$t^{*d}_{P_{(N\Phi)}}$	[0.01653282, 0.02888168]	[0.02201664, 0.04139722]
$t^{*d}_{Re_{(N\Phi)}}$	[0.009134271, 0.013285761]	[0.008145817, 0.011680043]
$t^{*d}_{Pe_{(N\Phi)}}$	[0.01329382, 0.02163269]	[0.01545124, 0.02633978]
$\jmath^{*d}_{Re_{(N\Phi)}}$	[0.008029139, 0.012186527]	[0.007392062, 0.011219580]
$\jmath^{*d}_{Pe_{(N\Phi)}}$	[0.008029139, 0.012186527]	[0.007392062, 0.011219580]
$\Im^{yp}_{ft_{(N\Phi)}}$	**[0.008029139, 0.011164511]**	**[0.0021953943, 0.0001796326]**

TABLE 4. Mean Square Error of the Neutrosophic Estimators based on Simulated Data.

	Case-I	**Case-II**
Estimators	**MSE**	**MSE**
$t_{0_{(N\Phi)}}$	[5.636972, 9.212937]	[5.636972, 9.212937]
$t^{d}_{R_{(N\Phi)}}$	[14.86116, 16.35115]	[27.39777, 26.10238]
$t^{d}_{P_{(N\Phi)}}$	[15.27578, 16.81980]	[28.09102, 26.88596]
$t^{d}_{Re_{(N\Phi)}}$	[7.89119, 10.93891]	[10.99052, 13.33735]
$t^{d}_{Pe_{(N\Phi)}}$	[8.098501, 11.173233]	[11.33714, 13.72914]
$t^{*d}_{R_{(N\Phi)}}$	[18.96955, 19.54820]	[37.05571, 33.62776]
$t^{*d}_{P_{(N\Phi)}}$	[19.46710, 20.11058]	[37.88761, 34.56805]
$t^{*d}_{Re_{(N\Phi)}}$	[8.907923, 11.726456]	[13.38767, 15.19911]
$t^{*d}_{Pe_{(N\Phi)}}$	[9.156696, 12.007645]	[5.635613, 9.210716]
$\jmath^{*d}_{Re_{(N\Phi)}}$	[5.635832, 9.211075]	[5.635613, 9.210716]
$\jmath^{*d}_{Pe_{(N\Phi)}}$	[5.635832, 9.211075]	[5.635613, 9.210716]
$\Im^{yp}_{ft_{(N\Phi)}}$	**[5.635832, 9.194130]**	**[5.633787, 9.160360]**

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

TABLE 5. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{ft_{(N\Phi)}}$.

S.No.	a	b	Estimator
1.	1	0	$\Im^{yp(1)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{yp(2)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{yp(3)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{yp(4)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{yp(5)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(6)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{yp(7)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{yp(8)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{yp(9)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(10)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{yp(11)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{yp(12)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{yp(13)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(14)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{yp(15)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{yp(16)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{yp(17)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(18)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{yp(19)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{yp(20)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{yp(21)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{yp(22)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{(A+c)\bar{x}_{1_{(N\Phi)}}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}_{1_{(N\Phi)}}+C\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 6. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{ft_{(N\Phi)}}$ for $d = 1$.

S.No.	a	b	Estimator
1.	1	0	$\Im^{Re(1)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{Re(2)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{Re(3)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(4)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{Re(5)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(6)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{Re(7)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(8)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x)_{(N\Phi)}}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{Re(9)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(10)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{Re(11)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{Re(12)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{Re(13)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(14)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{Re(15)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{Re(16)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(17)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(18)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{Re(19)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{Re(20)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(21)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	C_x	$\Im^{Re(22)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}}{\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 7. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{ft_{(N\Phi)}}$ for $d = 2$.

S.No.	a	b	Estimator
1.	1	0	$\Im^{Pe(1)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{Pe(2)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{Pe(3)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(4)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{Pe(5)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(6)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{Pe(7)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(8)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{Pe(9)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(10)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{Pe(11)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{Pe(12)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{Pe(13)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(14)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{Pe(15)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{Pe(16)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(17)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(18)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{Pe(19)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{Pe(20)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(21)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{Pe(22)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}_{1_{(N\Phi)}}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 8. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{ft_{(N\Phi)}}$ for $d = 3$.

S.No.	a	b	Estimator
1.	1	0	$\Im^{DR(1)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{DR(2)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{DR(3)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(4)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{DR(5)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(6)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{DR(7)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(8)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{DR(9)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(10)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{DR(11)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{DR(12)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{DR(13)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(14)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{DR(15)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{DR(16)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(17)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(18)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{DR(19)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{DR(20)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(21)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{DR(22)}_{ft_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}1_{(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)}}\right]^{\alpha} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 9. Members of the Neutrosophic Proposed Class of Estimators $\Im_{ft_{(N\Phi)}}^{yp}$ for $d = 4$.

S.No.	a	b	Estimator
1.	1	0	$\Im_{ft_{(N\Phi)}}^{exp(1)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im_{ft_{(N\Phi)}}^{exp(2)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(3)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im_{ft_{(N\Phi)}}^{exp(4)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(5)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(6)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im_{ft_{(N\Phi)}}^{exp(7)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im_{ft_{(N\Phi)}}^{exp(8)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(9)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(10)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im_{ft_{(N\Phi)}}^{exp(11)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(12)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x)_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(13)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(14)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im_{ft_{(N\Phi)}}^{exp(15)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(16)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im_{ft_{(N\Phi)}}^{exp(17)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(18)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im_{ft_{(N\Phi)}}^{exp(19)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(20)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{yx_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im_{ft_{(N\Phi)}}^{exp(21)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im_{ft_{(N\Phi)}}^{exp(22)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 10. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{vk_{(N\Phi)}}$ for $\alpha = 1$.

S.No.	a	b	Estimator
1.	1	0	$\Im^{yp(1)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{yp(2)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{yp(3)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{yp(4)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{yp(5)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(6)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{yp(7)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{yp(8)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{yp(9)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(10)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{yp(11)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{yp(12)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{yp(13)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(14)}_{vk} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{yp(15)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{yp(16)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{yp(17)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{yp(18)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{yp(19)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{yp(20)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x)}$	$\Im^{yp(21)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{yp(22)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{(A+c)\bar{x}1_{(N\Phi)}+f_{(N\Phi)}B\bar{x}_{(N\Phi)}}{(A+f_{(N\Phi)}B)\bar{x}1_{(N\Phi)}+C\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 11. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{vk(N\Phi)}$ for $\alpha = 1$ and $d = 1$.

S.No.	a	b	Estimator
1.	1	0	$\Im^{Re(1)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{Re(2)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{Re(3)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(4)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{Re(5)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(6)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{Re(7)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(8)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{Re(9)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(10)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{Re(11)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{Re(12)}_{vk} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{Re(13)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(14)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{Re(15)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{Re(16)}_{vk} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(17)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Re(18)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{Re(19)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{Re(20)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Re(21)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{Re(22)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{1(N\Phi)}}{\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 12. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{vk_{(N\Phi)}}$ for $\alpha = 1$ and $d = 2$.

S.No.	a	b	**Estimator**
1.	1	0	$\Im^{Pe(1)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{Pe(2)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{Pe(3)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(4)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{Pe(5)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(6)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{Pe(7)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(8)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{Pe(9)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(10)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{Pe(11)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{Pe(12)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{Pe(13)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(14)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{Pe(15)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{Pe(16)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(17)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{Pe(18)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{Pe(19)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	S_x	$\Im^{Pe(20)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{Pe(21)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{Pe(22)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)}\left[\frac{\bar{x}_{(N\Phi)}}{\bar{x}1_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}1_{(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 13. Members of the Neutrosophic Proposed Class of Estimators $\Im^{yp}_{vk_{(N\Phi)}}$ for $\alpha = 1$ and $d = 3$.

S.No.	a	b	Estimator
1.	1	0	$\Im^{DR(1)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im^{DR(2)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im^{DR(3)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(4)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im^{DR(5)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(6)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im^{DR(7)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(8)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_1-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{DR(9)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(10)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im^{DR(11)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im^{DR(12)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x_{(N\Phi)})}$	$C_{x_{(N\Phi)}}$	$\Im^{DR(13)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(14)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im^{DR(15)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{DR(16)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(17)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im^{DR(18)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+x_{(\bar{N}\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im^{DR(19)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im^{DR(20)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im^{DR(21)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im^{DR(22)}_{vk_{(N\Phi)}} = \bar{y}_{(N\Phi)} \left[\frac{\bar{x}_{1(N\Phi)}-f_{(N\Phi)}\bar{x}_{(N\Phi)}}{\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)}}\right] exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1(N\Phi)}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

TABLE 14. Members of the Neutrosophic Proposed Class of Estimators $\Im_{vk_{(N\Phi)}}^{yp}$ for $\alpha = 1$ and $d = 4$.

S.No.	a	b	Estimator
1.	1	0	$\Im_{vk_{(N\Phi)}}^{exp(1)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})}\right\}$
2.	1	1	$\Im_{vk_{(N\Phi)}}^{exp(2)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
3.	1	$S_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(3)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
4.	1	$\beta_{2(x_{(N\Phi)})}$	$\Im_{vk_{(N\Phi)}}^{exp(4)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
5.	1	$C_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(5)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
6.	1	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(6)} = \bar{y}_{(N\Phi)} exp\left\{\frac{(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
7.	$S_{x_{(N\Phi)}}$	1	$\Im_{vk_{(N\Phi)}}^{exp(7)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
8.	$S_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im_{vk_{(N\Phi)}}^{exp(8)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
9.	$S_{x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(9)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x})}{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
10.	$S_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(10)} = \bar{y}_{(N\Phi)} exp\left\{\frac{S_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{S_{x_{(N\Phi)}}(\bar{x}_1+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
11.	$\beta_{2(x_{(N\Phi)})}$	1	$\Im_{vk_{(N\Phi)}}^{exp(11)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
12.	$\beta_{2(x_{(N\Phi)})}$	$S_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(12)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
13.	$\beta_{2(x)_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(13)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$
14.	$\beta_{2(x_{(N\Phi)})}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(14)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\beta_{2(x_{(N\Phi)})}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
15.	$C_{x_{(N\Phi)}}$	1	$\Im_{vk_{(N\Phi)}}^{exp(15)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
16.	$C_{x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(16)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
17.	$C_{x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im_{vk_{(N\Phi)}}^{exp(17)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
18.	$C_{x_{(N\Phi)}}$	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(18)} = \bar{y}_{(N\Phi)} exp\left\{\frac{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{C_{x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\rho_{y_{(N\Phi)}x_{(N\Phi)}}}\right\}$
19.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	1	$\Im_{vk_{(N\Phi)}}^{exp(19)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2}\right\}$
20.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$S_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(20)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{yx_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2S_{x_{(N\Phi)}}}\right\}$
21.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$\beta_{2(x_{(N\Phi)})}$	$\Im_{vk_{(N\Phi)}}^{exp(21)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2\beta_{2(x_{(N\Phi)})}}\right\}$
22.	$\rho_{y_{(N\Phi)}x_{(N\Phi)}}$	$C_{x_{(N\Phi)}}$	$\Im_{vk_{(N\Phi)}}^{exp(22)} = \bar{y}_{(N\Phi)} exp\left\{\frac{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}-\bar{x}_{(N\Phi)})}{\rho_{y_{(N\Phi)}x_{(N\Phi)}}(\bar{x}_{1_{(N\Phi)}}+\bar{x}_{(N\Phi)})+2C_{x_{(N\Phi)}}}\right\}$

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