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The symbolic plithogenic differentials calculus

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Abstract: In this paper, we were keen to present the concept of the symbolic plithogenic differentials calculus, Where the symbolic plithogenic differentiable is defined. In addition, properties of the symbolic plithogenic differentiation are introduced. Also, we prove the derivation rules of the symbolic plithogenic functions.

Keywords: symbolic plithogenic differentials; symbolic plithogenic functions; derivative symbolic plithogenic functions.

1. Introduction and Preliminaries

To The genesis, origination, formation, development, and evolution of new entities through dynamics of contradictory and/or neutral and/or noncontradictory multiple old entities is known as plithogenic. Plithogeny advocates for the integration of theories from several fields. We use numerous "knowledges" from domains like soft sciences, hard sciences, arts and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on plithogeny, plithogenic set, logic, probability, and statistics [2], in addition to presenting introduction to the symbolic plithogenic algebraic structures (revisited), through which he discussed several ideas, including mathematical operations on plithogenic numbers [1]. Also, an overview of plithogenic set and symbolic plithogenic algebraic structures was discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-5-6-7].

Alhasan also presented several papers on calculus, in which he discussed neutrosophic definite and indefinite integrals. He also presented the most important applications of definite integrals in neutrosophic logic [8-9].

Integration is important in human life, and one of its most important applications is the calculation of area, size and arc length. In our reality we find things that cannot be precisely defined, and that contain an indeterminacy part. This is the reason for studying neutrosophic integration and methods of its integration in this paper.

Smarandache presented the division operation in the plithogenic field as follows [1]:

Division of Symbolic Plithogenic Components

$$\frac{P_i}{P_j} = \begin{cases} x_0 + x_1P_1 + x_2P_2 + \cdots + x_jP_j + P_i & x_0 + x_1 + x_2 + \cdots + x_j = 0 \quad i > j \\ x_0 + x_1P_1 + x_2P_2 + \cdots + x_iP_i & x_0 + x_1 + x_2 + \cdots + x_i = 1 \quad i = j \\ \emptyset & i < j \end{cases}$$

where all coefficients $x_0, x_1, x_2, \dots, x_i, \dots \in$ SPS.

Division of Symbolic Plithogenic Numbers

Let consider two symbolic plithogenic numbers as below:

$$PN_r = a_0 + a_1P_1 + a_2P_2 + \cdots + a_rP_r$$

$$PN_s = b_0 + b_1P_1 + b_2P_2 + \cdots + b_sP_s$$

$$\frac{PN_r}{PN_s} = \begin{cases} none, one \;\; many & r \geq s \\ \emptyset & r < s \end{cases}$$

This study covered a number of subjects; in the first, which included an introduction and information of plithogenic filed. We presented the symbolic plithogenic differentials calculusin the main discussion section. The paper's conclusion is provided in the final.

Main Discussion

The symbolic plithogenic differentials

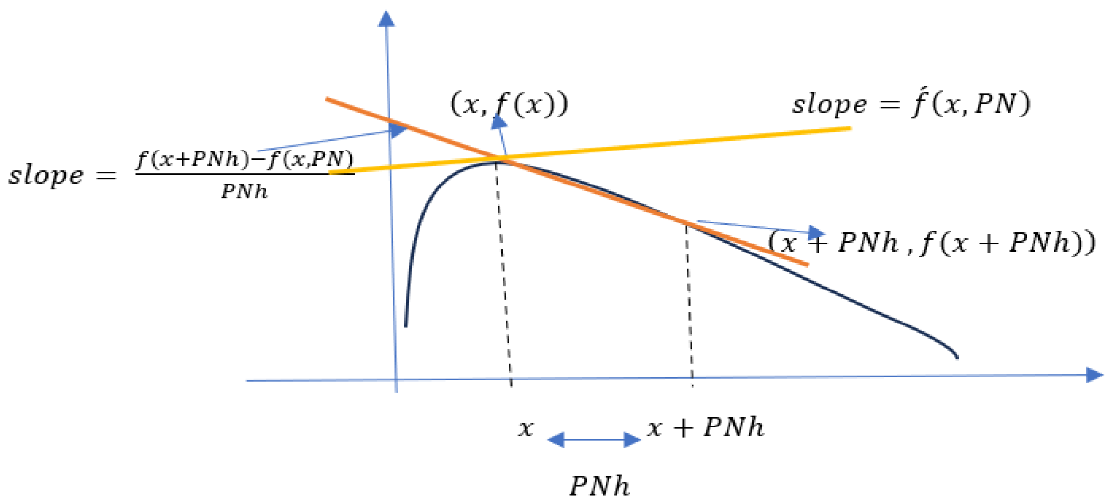
Definition 1

Let $f: SPS \to SPS$, if:

$$\lim_{PNh \to 0} \frac{f(x + PNh) - f(x, PN)}{PNh}$$

exist, then we say that the function $f(x, PN)$ is differentiable with respect to x and it is given by the formula:

$$\acute{f}(x, PN) = \lim_{PNh \to 0} \frac{f(x + PNh) - f(x, PN)}{PNh}$$



where $PNh = h_0 + h_1P_1 + h_2P_2 + \cdots + h_nP_n \in SPS$ is amount of small change in x.

then, $PNh \to 0$ is equivalent to: $h_0 \to 0$, $h_1 \to 0$, $h_2 \to 0$, …. , and $h_n \to 0$

Notes:

1) The tangent slop to $f(x, PN)$ at $x_0 \in SPS$, where $x_0 = a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$, is:

$$m_{PN} = \acute{f}(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n).$$

2) The equation of the tangent to $f(x, PN)$ at $x_0 = a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$ is:

$$y - f(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n) = \acute{f}(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)(x - a_0 - a_1P_1 - a_2P_2 - \cdots - a_nP_n)$$

Example 1

Differentiate $f(x, PN) = (3P_2 + 2)x^2$ with respect to x using definition, and find an equation of the tangent line to the curve at $x_0 = P_1 + 1$

solution:

$$\acute{f}(x, PN) = \lim_{PNh \to 0} \frac{f(x + PNh) - f(x, PN)}{PNh}$$

$$\acute{f}(x, PN) = \lim_{PNh \to 0} \frac{(3P_2 + 2)(x + PNh)^2 - (3P_2 + 2)x^2}{PNh}$$

$$\acute{f}(x, PN) = \lim_{PNh \to 0} \frac{(3P_2 + 2)(x^2 + 2(PNh)x + (PNh)^2) - (3P_2 + 2)x^2}{PNh}$$

$$= \lim_{PNh \to 0} \frac{(3P_2 + 2)x^2 + (3P_2 + 2)(2(PNh)x + (PNh)^2) - (3P_2 + 2)x^2}{PNh}$$

$$= \lim_{PNh \to 0} \frac{PNh(3P_2 + 2)(2x + PNh)}{PNh}$$

$$= \lim_{PNh \to 0} (3P_2 + 2)(2x + PNh)$$

$$= (3P_2 + 2)(2x + 0)$$

$$\Rightarrow \qquad \acute{f}(x, PN) = (6P_2 + 4)x$$

Let's find the tangent equation:

$$m_{PN} = \acute{f}(P_1 + 1) = (6P_2 + 4)(P_1 + 1) = 12P_2 + 4P_1 + 4$$

$$f(P_1 + 1) = (P_1 + 1)^2 = 3P_1 + 1$$

then:

$$y - f(P_1 + 1) = \acute{f}(P_1 + 1)(x - P_1 - 1)$$

$$y - 3P_1 - 1 = (12P_2 + 4P_1 + 4)(x - P_1 - 1)$$

$$y - 3P_1 - 1 = (12P_2 + 4P_1 + 4)x + (12P_2 + 4P_1 + 4)(-P_1 - 1)$$

$$y-3P_1-1=(12P_2+4P_1+4)x-12P_2-4P_1-4P_1-12P_2-4P_1-4$$

$$y-3P_1-1=(12P_2+4P_1+4)x-24P_2-12P_1-4$$

$$y=(12P_2+4P_1+4)x-24P_2-9P_1-4$$

Example 2
Differentiate $f(x,PN)=sin\big((P_4-5P_1+7)x+4P_1\big)$ with respect to x using definition.
solution:

$$\acute{f}(x,PN)=\lim_{PNh\to 0}\frac{f(x+PNh)-f(x,PN)}{PNh}$$

$$\acute{f}(x,I)=\lim_{PNh\to 0}\frac{sin\big((P_4-5P_1+7)(x+PNh)+4P_1\big)\ -sin\big((P_4-5P_1+7)x+4P_1\big)}{PNh}$$

$$=\lim_{PNh\to 0}\frac{sin\big((P_4-5P_1+7)x+4P_1+(P_4-5P_1+7)(x+PNh)\big)\ -sin\big((P_4-5P_1+7)x+4P_1\big)}{PNh}$$

$$=\lim_{PNh\to 0}\frac{cos\left((P_4-5P_1+7)x+4P_1+\frac{(P_4-5P_1+7)}{2}PNh\right)sin\left(\frac{(P_4-5P_1+7)}{2}PNh\right)}{\frac{PNh}{2}}$$

$$=\lim_{PNh\to 0}cos\left((P_4-5P_1+7)x+4P_1+\frac{(P_4-5P_1+7)}{2}PNh\right)\lim_{PNh\to 0}\frac{sin\left(\frac{(P_4-5P_1+7)}{2}PNh\right)}{\frac{PNh}{2}}$$

$$=\lim_{PNh\to 0}cos\left((P_4-5P_1+7)x+4P_1+\frac{(P_4-5P_1+7)}{2}PNh\right)\lim_{PNh\to 0}\frac{(P_4-5P_1+7)sin\left(\frac{(P_4-5P_1+7)}{2}PNh\right)}{\frac{(P_4-5P_1+7)PNh}{2}}$$

$$=cos\big((P_4-5P_1+7)x+4P_1\big)(P_4-5P_1+7)\quad (1)$$

$$\Rightarrow\qquad \acute{f}(x,PN)=(P_4-5P_1+7)\,cos\big((P_4-5P_1+7)x+4P_1\big)$$

Example 3
Differentiate $f(x,I)=\sqrt{7P_5x-3P_1+1}$ with respect to x using definition.
solution:

$$\acute{f}(x,PN)=\lim_{PNh\to 0}\frac{f(x+PNh)-f(x,PN)}{PNh}$$

$$\acute{f}(x,PN)=\lim_{PNh\to 0}\frac{\sqrt{7P_5(x+PNh)-3P_1+1}-\sqrt{7P_5x-3P_1+1}}{PNh}$$

$$= \lim_{PNh\to 0} \frac{\sqrt{7P_5(x+PNh)-3P_1+1}-\sqrt{7P_5x-3P_1+1}}{PNh} \frac{\sqrt{7P_5(x+PNh)-3P_1+1}+\sqrt{7P_5x-3P_1+1}}{\sqrt{7P_5(x+PNh)-3P_1+1}+\sqrt{7P_5x-3P_1+1}}$$

$$= \lim_{PNh\to 0} \frac{7P_5(x+PNh)-3P_1+1-7P_5x+3P_1-1}{PNh\left(\sqrt{7P_5(x+PNh)-3P_1+1}+\sqrt{7P_5x-3P_1+1}\right)}$$

$$= \lim_{PNh\to 0} \frac{7P_5x+(7P_5)PNh-3P_1+1-7P_5x+3P_1-1}{PNh\left(\sqrt{7P_5(x+PNh)-3P_1+1}+\sqrt{7P_5x-3P_1+1}\right)}$$

$$= \lim_{PNh\to 0} \frac{(7P_5)PNh}{PNh\left(\sqrt{7P_5(x+PNh)-3P_1+1}+\sqrt{7P_5x-3P_1+1}\right)}$$

$$= \lim_{PNh\to 0} \frac{7P_5}{\sqrt{7P_5(x+PNh)-3P_1+1}+\sqrt{7P_5x-3P_1+1}}$$

$$\Rightarrow \qquad \acute{f}(x,PN) = \frac{7P_5}{2\sqrt{7P_5x-3P_1+1}}$$

The rules of the symbolic plithogenic derivatives

Let $PN_s = a_0 + a_1P_1 + a_2P_2 + \cdots + a_rP_r$, $PN_r = b_0 + b_1P_1 + b_2P_2 + \cdots + b_sP_s \in SPS$, then we can prove each of the following, using the definition 1:

1) $\frac{d}{dx}[PC] = 0$; where $PC = c_0 + c_1P_1 + c_2P_2 + \cdots + c_rP_r$ is symbolic plithogenic constant.

2) $\frac{d}{dx}[PN_sx + PN_r] = PN_s$

3) $\frac{d}{dx}[PN_sx^n] = nPN_sx^{n-1}; n \ \ is$ real number.

4) $\frac{d}{dx}[e^{PN_sx+PN_r}] = PN_se^{PN_sx+PN_r}$

5) $\frac{d}{dx}(PN_r)^x = (PN_r)^x \ \ ln(PN_r)$; where $PN_r > 0$

6) $\frac{d}{dx}\left[PN_r \, log_{PN_s} x\right] = \frac{PN_r}{x \ ln(PN_s)}$; where $PN_s > 0$, and $\frac{PN_r}{ln(PN_s)}$ is divisible.

7) $\frac{d}{dx}[ln(PN_sx + PN_r)] = \frac{PN_s}{PN_sx + PN_r}$

8) $\frac{d}{dx}\left[\sqrt{PN_sx + PC}\right] = \frac{PN_s}{2\sqrt{PN_sx + PC}}$

9) $\frac{d}{dx}[sin(PN_sx + PN_r)] = PN_scos(PN_sx + PN_r)$

10) $\frac{d}{dx}[cos(PN_sx + PN_r)] = -PN_ssin(PN_sx + PN_r)$

11) $\frac{d}{dx}[tan(PN_s x + PN_r)] = PN_s sec^2(PN_s x + PN_r)$

12) $\frac{d}{dx}[cot(PN_s x + PN_r)] = -PN_s csc^2(PN_s x + PN_r)$

13) $\frac{d}{dx}[sec(PN_s x + PN_r)] = PN_s sec(PN_s x + PN_r)tan(PN_s x + PN_r)$

14) $\frac{d}{dx}[csc(PN_s x + PN_r)] = -PN_s csc(PN_s x + PN_r)cot(PN_s x + PN_r)$

Proof (3)

$$\frac{d}{dx}[PN_s x^n] = \lim_{PNh \to 0} \frac{f(x + PNh) - f(x, PN)}{PNh}$$

$$= \lim_{PNh \to 0} \frac{PN_s(x + PNh)^n - PN_s x^n}{PNh}$$

$$= \lim_{PNh \to 0} \frac{\left[PN_s x^n + nPN_s x^{n-1}(PNh) + \frac{n(n-1)}{2!} PN_s x^{n-2}(PNh)^2 + \cdots + nPN_s x(PNh)^{n-1} + (PNh)^n\right] - PN_s x^n}{PNh}$$

$$= \lim_{PNh \to 0} \left[\frac{nPN_s x^{n-1}(PNh) + \frac{n(n-1)}{2!} PN_s x^{n-2}(PNh)^2 + \cdots + nPN_s x(PNh)^{n-1} + (PNh)^n}{PNh}\right]$$

$$= \lim_{PNh \to 0} \left[nPN_s x^{n-1} + \frac{n(n-1)}{2!} PN_s x^{n-2}(PNh) + \cdots + nPN_s x(PNh)^{n-2} + (PNh)^{n-1}\right]$$

$$= nPN_s x^{n-1} + 0 + \cdots\cdots + 0 + 0$$

$$\Rightarrow \quad \frac{d}{dx}[PN_s x^n] = nPN_s x^{n-1}$$

Example 4

1) $\frac{d}{dx}(P_7 - 8P_4 + 1) = 0$

2) $\frac{d}{dx}[(-4P_3 - 3P_1)x - 7P_5 - 3P_1 + 5] = -4P_3 - 3P_1$

3) $\frac{d}{dx}[(3P_1 + 5)x^5] = (15P_1 + 25)x^4$

4) $\frac{d}{dx}\left[e^{(P_6+53P_3)x+73P_2+4}\right] = (P_6 + 53P_3)e^{(P_6+53P_3)x+73P_2+4}$

5) $\frac{d}{dx}(1 + P_1 + 2P_2 + P_3)^x = (1 + 2P_2 + P_1)^x \;\; ln(2P_2 + P_1)$

$$= (1 + 2P_2 + P_1 + P_3)^x \ \ [ln1 + (ln3 - ln1)P_1 + (ln4 - ln3)P_2 + (ln5 - ln4)P_3]$$

$$= (1 + 2P_2 + P_1 + P_3)^x \ \ \left[(ln3)P_1 + \left(ln\frac{4}{3}\right)P_2 + \left(ln\frac{5}{4}\right)P_2\right]$$

6) $$\frac{d}{dx}\left[P_4 \, log_{(1+2P_1)} \, x\right] = \frac{P_4}{x \, ln(1 + 2P_1)} = \left(\frac{P_4}{ln(1 + 2P_1)}\right)\frac{1}{x}$$

$$= \left(\frac{P_4}{(ln3)P_1}\right)\frac{1}{x} = \frac{1}{(ln3)}(x_0 + x_1P_1 + P_4)\frac{1}{x}$$

$$= \left(\frac{x_0 + x_1P_1 + P_4}{ln3}\right)\frac{1}{x}$$

where:

$$\frac{P_4}{P_1} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 \qquad \Rightarrow \qquad P_4 = x_0P_1 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4$$

$$\Rightarrow \qquad P_2 = (x_0 + x_1)P_1 + x_2P_2 + x_3P_3 + x_4P_4$$, then:

$$x_0 + x_1 = 0 \ , \ x_2 = 0 \ , x_3 = 0 \text{ and } x_4 = 1$$

hence: $\frac{P_4}{P_1} = x_0 + x_1P_1 + P_4$, where: $x_0 + x_1 = 0$

7) $$\frac{d}{dx}\left[ln\big((7 + 5P_2 + P_3)x + 6 + 7P_1 + P_4\big)\right] = \frac{7 + 5P_2 + P_3}{(7 + 5P_2 + P_3)x + 6 + 7P_1 + P_4}$$

8) $$\frac{d}{dx}\left[\sqrt{(4 + 8P_7 + P_4)x + 2 + P_3}\right] = \frac{4 + 8P_7 + P_4}{2\sqrt{(4 + 8P_7 + P_4)x + 2 + P_3}}$$

9) $$\frac{d}{dx}\left[sin\big((9 - P_7)x + P_4\big)\right] = (9 - P_7)cos\big((9 - P_7)x + P_4\big)$$

10) $$\frac{d}{dx}\left[cos\big((5P_2 + P_1 - 4)x + P_8 + 2\big)\right] = (-5P_2 - P_1 + 4)sin\big((5P_2 + P_1 - 4)x + P_8 + 2\big)$$

11) $$\frac{d}{dx}\left[tan\big((P_7 + 4P_5 + 6)x + 6\big)\right] = (P_7 + 4P_5 + 6)sec^2\big((P_7 + 4P_5 + 6)x + 6\big)$$

12) $$\frac{d}{dx}\left[csc\big((8P_6 + 6)x + 7P_5 + 3\big)\right]$$
$$= (-8P_6 - 6)csc\big((8P_6 + 6)x + 7P_5 + 3\big)cot\big((8P_6 + 6)x + 7P_5 + 3\big)$$

Properties of the symbolic plithogenic differentiation:

I. Derivative of sum or difference of the symbolic plithogenic functions.

Suppose that $f(x, PN)$ and $g(x, PN)$ are any two differentiable symbolic plithogenic functions, then:

$$\frac{d}{dx}[f(x, PN) \ \pm g(x, PN)] = \frac{d}{dx}[f(x, PN) \ ] \pm \frac{d}{dx}[g(x, PN)]$$

Proof:

$$\frac{d}{dx}[f(x,PN)+g(x,PN)] =$$

$$= \lim_{PNh\to 0}\frac{f(x+PNh)\pm g(x+PNh)-[f(x,PN)+g(x,PN)]}{PNh}$$

$$= \lim_{PNh\to 0}\frac{[f(x+PNh)-f(x,PN)]\pm[g(x+PNh)-g(x,PN)]}{PNh}$$

$$= \lim_{PNh\to 0}\left[\frac{f(x+PNh)-f(x,PN)}{PNh}\pm\frac{f(x+PNh)-f(x,PN)}{PNh}\right]$$

$$= \lim_{PNh\to 0}\frac{f(x+PNh)-f(x,PN)}{PNh}\pm\lim_{PNh\to 0}\frac{f(x+PNh)-f(x,PN)}{PNh}$$

$$= \frac{d}{dx}[f(x,PN)\ \ ]\pm\frac{d}{dx}[g(x,PN)]$$

Example 5

1) $\frac{d}{dx}\left[5P_2x^4+cot\left((7+P_2+P_3)x\right)\right]=20P_2x^3-(7+P_2+P_3)csc^2\left((7+P_2+P_3)x\right)$

2) $\frac{d}{dx}[(7+P_5)x+ln(P_2x)]=7+P_5+\frac{P_2}{P_2x}=7+P_5+(x_0+x_1P_1+x_2P_2)\frac{1}{x}$

where:

$$\frac{P_2}{P_2}=x_0+x_1P_1+x_2P_2 \quad\Longrightarrow\quad P_2=x_0P_2+x_1P_2+x_2P_2$$

$$\Longrightarrow\quad P_2=(x_0+x_1+x_2)P_2 \text{ , then:}$$

$$x_0+x_1+x_2=1$$

hence: $\frac{P_2}{P_2}=x_0+x_1P_1+x_2P_2$, where: $x_0+x_1+x_2=1$

II. Derivative of product of a symbolic plithogenic constant and the symbolic plithogenic function

$$\frac{d}{dx}[PC.f(x,PN)]=PC.\frac{d}{dx}\ \ [f(x,PN)]$$

Proof:

$$\frac{d}{dx}[PC.f(x,PN)]=\lim_{PNh\to 0}\frac{PC.f(x+PNh)-PC.f(x,PN)}{PNh}$$

$$= \lim_{PNh\to 0}PC\left[\frac{f(x+PNh)-f(x,PN)}{PNh}\right]$$

$$= PC\lim_{PNh\to 0}\left[\frac{f(x+PNh)-f(x,PN)}{PNh}\right]$$

$$= PC\frac{d}{dx}\ \ [f(x,PN)]$$

III. Derivative of product of two the symbolic plithogenic functions

$$\frac{d}{dx}[f(x,PN).\,g(x,PN)] = f(x,PN)\frac{d}{dx}[g(x,PN)] + g(x,PN)\frac{d}{dx}[f(x,PN)]$$

Proof:

$$\frac{d}{dx}[f(x,PN).\,g(x,PN)] =$$

$$= \lim_{PNh\to 0}\frac{f(x+PNh).\,g(x+PNh) - f(x,PN).\,g(x,PN)}{PNh}$$

$$= \lim_{PNh\to 0}\frac{f(x+PNh).\,g(x+PNh) - f(x+PNh)g(x,PN) + f(x+PNh)g(x,PN) - f(x,PN).\,g(x,PN)}{PNh}$$

$$= \lim_{PNh\to 0}\left[f(x+PNh)\frac{g(x+PNh) - g(x,PN)}{PNh} + g(x,PN)\frac{f(x+PNh) - f(x,PN)}{PNh}\right]$$

$$= \lim_{PNh\to 0}f(x+PNh)\lim_{PNh\to 0}\frac{g(x+PNh) - g(x,PN)}{PNh} + \lim_{PNh\to 0}g(x,PN)\lim_{PNh\to 0}\frac{f(x+PNh) - f(x,PN)}{PNh}$$

$$= f(x,PN)\frac{d}{dx}[g(x,PN)] + g(x,PN)\frac{d}{dx}[f(x,PN)]$$

Example 6

1) $\frac{d}{dx}\left[4P_4x^2sin\big((P_2+P_3)x\big)\ \ \right] = 8x.\,sin\big((P_2+P_3)x\big) + (4P_2+4P_3)cos\big((P_2+P_3)x\big)$

2) $\frac{d}{dx}\left[x\sqrt{(P_7-3)x+P_5}\right] = \sqrt{(P_7-3)x+P_5} + \frac{P_7-3}{2\sqrt{(P_7-3)x+P_5}}$

V. Derivative of quotient of two the symbolic plithogenic functions

$$\frac{d}{dx}\left[\frac{f(x,PN)}{g(x,PN)}\right] = \frac{g(x,PN)\frac{d}{dx}[f(x,PN)] - f(x,PN)\frac{d}{dx}[g(x,PN)]}{\big(g(x,PN)\big)^2}$$

Proof:

$$\frac{d}{dx}\left[\frac{f(x,PN)}{g(x,PN)}\right] = \lim_{PNh\to 0}\frac{\frac{f(x+PNh)}{g(x+PNh)} - \frac{f(x,PN)}{g(x,PN)}}{PNh}$$

$$= \lim_{PNh\to 0}\frac{f(x+PNh).\,g(x,PN) - f(x,PN).\,g(x,PN) - f(x,PN).\,g(x+PNh) + f(x,PN).\,(x,PN)}{PNh.\,g(x,PN).\,g(x+PNh)}$$

$$= \lim_{PNh\to 0}\left[\frac{g(x,PN)\frac{f(x+PNh) - f(x,PN)}{PNh} - f(x,PN)\frac{g(x+PNh) - g(x,PN)}{PNh}}{g(x,PN).\,g(x+PNh)}\right]$$

$$= \frac{\lim\limits_{PNh \to 0}\left[g(x,PN)\frac{f(x+PNh)-f(x,PN)}{PNh}\right] - \lim\limits_{PNh \to 0}\left[f(x,PN)\frac{g(x+PNh)-g(x,PN)}{PNh}\right]}{\lim\limits_{PNh \to 0}[g(x,PN).\,g(x+PNh)]}$$

$$= \frac{\lim\limits_{PNh \to 0} g(x,PN) \lim\limits_{PNh \to 0}\frac{f(x+PNh)-f(x,PN)}{PNh} - \lim\limits_{PNh \to 0} f(x,PN) \lim\limits_{PNh \to 0}\frac{g(x+PNh)-g(x,I)}{PNh}}{\lim\limits_{PNh \to 0} g(x,PN).\, \lim\limits_{PNh \to 0} g(x+PNh)}$$

$$= \frac{g(x,PN)\frac{d}{dx}[f(x,PN)] - f(x,PN)\frac{d}{dx}[g(x,PN)]}{g(x,PN).\,g(x,PN)}$$

$$= \frac{d}{dx}\left[\frac{f(x,PN)}{g(x,PN)}\right] = \frac{g(x,PN)\frac{d}{dx}[f(x,PN)] - f(x,PN)\frac{d}{dx}[g(x,PN)]}{\left(g(x,PN)\right)^2}$$

Example 7

1) $\frac{d}{dx}\left[\frac{e^{P_3x+2P_7-3}}{P_2x}\right] = \frac{P_3xe^{P_3x+2P_7-3} - P_2e^{P_3x+2P_7-3}}{P_2x^2}$

$$= \frac{(3+I)xe^{(3+I)x+5I} - e^{(3+I)x+5I}}{(3+4I)x^2}$$

$$= (\frac{1}{3} - \frac{4}{21}I)\left[\frac{(3+I)xe^{(3+I)x+5I} - e^{(3+I)x+5I}}{x^2}\right]$$

2) $\frac{d}{dx}\left[\frac{P_3}{P_2x}\right] = \frac{-P_3P_2}{P_2x^2} = \frac{-P_3}{P_2x^2} = (x_0 + x_1P_1 + x_2P_2 - P_3)\frac{1}{x^2}$

where:

$$\frac{-P_3}{P_2} = x_0 + x_1P_1 + x_2P_2 + x_3P_3 \quad \Longrightarrow \quad -P_3 = x_0P_2 + x_1P_2 + x_2P_2 + x_3P_3$$

$$\Longrightarrow \quad -P_3 = (x_0 + x_1 + x_2)P_2 + x_3P_3 \text{ , then:}$$

$$x_0 + x_1 + x_2 = 0 \ , \ x_3 = -1$$

hence: $\frac{-P_3}{P_2} = x_0 + x_1P_1 + x_2P_2 - P_3$, where: $x_0 + x_1 = 0$

5. Conclusions

In our daily lives, derivatives are crucial for tasks like figuring out how to calculate acceleration, displacement, and velocity as a function of time in rectilinear motion, other. In this article, we discussed the concept of The symbolic plithogenic differentials calculus, where we presented the rules of The symbolic plithogenic differentials, taking into account the mathematical operations on them.

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