MBJ-neutrosophic structures and its applications in BCK/BCI-algebras

M. Mohseni Takallo[1], R.A. Borzooei[1], Young Bae Jun[1,2]

[1]Department of Mathematics, Shahid Beheshti University, Tehran, Iran

E-mail: mohammad.mohseni1122@gmail.com (M. Mohseni Takallo), borzooei@sbu.ac.ir (R.A. Borzooei)

[2]Department of Mathematics Education, Gyeongsang National University, Jinju 52828, Korea.

E-mail: skywine@gmail.com

*Correspondence: Young Bae Jun (skywine@gmail.com)

Abstract: Smarandache (F. Smarandache. Neutrosophy, neutrosophic probability, set, and logic, ProQuest Information & Learning, Ann Arbor,Michigan, USA, 105 p., 1998) initiated neutrosophic sets which can be used as a mathematical tool for dealing with indeterminates and inconsistent information. As a generalization of a neutrosophic set, the notion of MBJ-neutrosophic sets is introduced, and it is applied to BCK/BCI-algebras. The concept of MBJ-neutrosophic subalgebras in BCK/BCI-algebras is introduced, and related properties are investigated. A characterization of MBJ-neutrosophic subalgebra is provided. Using an MBJ-neutrosophic subalgebra of a BCI-algebra, a new MBJ-neutrosophic subalgebra is established. Homomorphic inverse image of MBJ-neutrosophic subalgebra is considered. Translation of MBJ-neutrosophic subalgebra is discussed. Conditions for an MBJ-neutrosophic set to be an MBJ-neutrosophic subalgebra are provided.

Keywords: MBJ-neutrosophic set; MBJ-neutrosophic subalgebra; MBJ-neutrosophic S-extension.

1 Introduction

In many practical situations and in many complex systems like biologial, behavioral and chemical etc., different types of uncertainties are encountered. The fuzzy set was introduced by L.A. Zadeh [19] in 1965 to handle uncertainties in many real applications, and the intuitionistic fuzzy set on a universe X was introduced by K. Atanassov in 1983 as a generalization of fuzzy set. The notion of neutrosophic set is developed by Smarandache ([14], [15] and [16]), and is a more general platform that extends the notions of classic set, (intuitionistic) fuzzy set and interval valued (intuitionistic) fuzzy set. Neutrosophic set theory is applied to various part which is refered to the site http://fs.gallup.unm.edu/neutrosophy.htm. Neutrosophic algebraic structures in BCK/BCI-algebras are discussed in the papers [1], [2], [6], [7], [8], [9], [10], [12], [13], [17] and [18]. We know that there are many generalizations of Smarandache's neutrosophic sets. The aim of this article is to consider another generalization of a neutrosophic set. In the neutrosophic set, the truth, false and indeterminate membership functions are fuzzy sets. In considering a generalization of neutrosophic set, we use the interval valued fuzzy set as the indeterminate membership function because interval valued fuzzy set is a generalization of a fuzzy set. We introduce the notion of MBJ-neutrosophic sets, and we apply it to BCK/BCI-algebras. We introduce the concept of MBJ-neutrosophic subalgebras in BCK/BCI-algebras, and investigate related properties. We provide a characterization of MBJ-neutrosophic subalgebra, and establish a new MBJ-neutrosophic subalgebra by using an MBJ-neutrosophic subalgebra of a BCI-algebra. We consider the homomorphic inverse image of MBJ-neutrosophic subalgebra, and discuss translation of MBJ-neutrosophic subalgebra. We provide conditions for an MBJ-neutrosophic set to be an MBJ-neutrosophic subalgebra.

2 Preliminaries

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Is´eki (see [4] and [5]) and was extensively investigated by several researchers.

By a *BCI-algebra*, we mean a set X with a special element 0 and a binary operation $*$ that satisfies the following conditions:

(I) $(\forall x, y, z \in X)\ (((x * y) * (x * z)) * (z * y) = 0)$,

(II) $(\forall x, y \in X)\ ((x * (x * y)) * y = 0)$,

(III) $(\forall x \in X)\ (x * x = 0)$,

(IV) $(\forall x, y \in X)\ (x * y = 0,\ y * x = 0 \ \Rightarrow\ x = y)$.

If a BCI-algebra X satisfies the following identity:

(V) $(\forall x \in X)\ (0 * x = 0)$,

then X is called a *BCK-algebra.* Any BCK/BCI-algebra X satisfies the following conditions:

$$(\forall x \in X)\,(x * 0 = x)\,, \tag{2.1}$$
$$(\forall x, y, z \in X)\,(x \le y \ \Rightarrow\ x * z \le y * z,\ z * y \le z * x)\,, \tag{2.2}$$
$$(\forall x, y, z \in X)\,((x * y) * z = (x * z) * y)\,, \tag{2.3}$$
$$(\forall x, y, z \in X)\,((x * z) * (y * z) \le x * y) \tag{2.4}$$

where $x \le y$ if and only if $x * y = 0$. Any BCI-algebra X satisfies the following conditions (see [3]):

$$(\forall x, y \in X)(x * (x * (x * y)) = x * y), \tag{2.5}$$
$$(\forall x, y \in X)(0 * (x * y) = (0 * x) * (0 * y)). \tag{2.6}$$

A nonempty subset S of a BCK/BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$.

By an *interval number* we mean a closed subinterval $\tilde{a} = [a^-, a^+]$ of I, where $0 \le a^- \le a^+ \le 1$. Denote by $[I]$ the set of all interval numbers. Let us define what is known as *refined minimum* (briefly, rmin) and *refined maximum* (briefly, rmax) of two elements in $[I]$. We also define the symbols "$\succeq$", "$\preceq$", "$=$" in case of two elements in $[I]$. Consider two interval numbers $\tilde{a}_1 := \left[a_1^-, a_1^+\right]$ and $\tilde{a}_2 := \left[a_2^-, a_2^+\right]$. Then

$$\begin{aligned}
&\operatorname{rmin}\{\tilde{a}_1, \tilde{a}_2\} = \left[\min\left\{a_1^-, a_2^-\right\}, \min\left\{a_1^+, a_2^+\right\}\right], \\
&\operatorname{rmax}\{\tilde{a}_1, \tilde{a}_2\} = \left[\max\left\{a_1^-, a_2^-\right\}, \max\left\{a_1^+, a_2^+\right\}\right], \\
&\tilde{a}_1 \succeq \tilde{a}_2 \ \Leftrightarrow\ a_1^- \ge a_2^-,\ a_1^+ \ge a_2^+,
\end{aligned}$$

and similarly we may have $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. To say $\tilde{a}_1 \succ \tilde{a}_2$ (resp. $\tilde{a}_1 \prec \tilde{a}_2$) we mean $\tilde{a}_1 \succeq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$ (resp. $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$). Let $\tilde{a}_i \in [I]$ where $i \in \Lambda$. We define

$$\operatorname{rinf}_{i\in\Lambda} \tilde{a}_i = \left[\inf_{i\in\Lambda} a_i^-, \inf_{i\in\Lambda} a_i^+\right] \quad \text{and} \quad \operatorname{rsup}_{i\in\Lambda} \tilde{a}_i = \left[\sup_{i\in\Lambda} a_i^-, \sup_{i\in\Lambda} a_i^+\right].$$

Let X be a nonempty set. A function $A : X \to [I]$ is called an *interval-valued fuzzy set* (briefly, an *IVF set*) in X. Let $[I]^X$ stand for the set of all IVF sets in X. For every $A \in [I]^X$ and $x \in X$, $A(x) = [A^-(x), A^+(x)]$ is called the *degree* of membership of an element x to A, where $A^- : X \to I$ and $A^+ : X \to I$ are fuzzy sets in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Let X be a non-empty set. A *neutrosophic set* (NS) in X (see [15]) is a structure of the form:

$$A := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}$$

where $A_T : X \to [0, 1]$ is a truth membership function, $A_I : X \to [0, 1]$ is an indeterminate membership function, and $A_F : X \to [0, 1]$ is a false membership function. For the sake of simplicity, we shall use the symbol $A = (A_T, A_I, A_F)$ for the neutrosophic set

$$A := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}.$$

We refer the reader to the books [3, 11] for further information regarding BCK/BCI-algebras, and to the site "http://fs.gallup.unm.edu/neutrosophy.htm" for further information regarding neutrosophic set theory.

3 MBJ-neutrosophic structures with applications in BCK/BCI-algebras

Definition 3.1. Let X be a non-empty set. By an *MBJ-neutrosophic set* in X, we mean a structure of the form:

$$\mathcal{A} := \{\langle x; M_A(x), \tilde{B}_A(x), J_A(x)\rangle \mid x \in X\}$$

where M_A and J_A are fuzzy sets in X, which are called a truth membership function and a false membership function, respectively, and $\tilde{B}_A$ is an IVF set in X which is called an indeterminate interval-valued membership function.

For the sake of simplicity, we shall use the symbol $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ for the MBJ-neutrosophic set

$$\mathcal{A} := \{\langle x; M_A(x), \tilde{B}_A(x), J_A(x)\rangle \mid x \in X\}.$$

In an MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X, if we take

$$\tilde{B}_A : X \to [I],\ x \mapsto [B_A^-(x), B_A^+(x)]$$

with $B_A^-(x) = B_A^+(x)$, then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is a neutrosophic set in X.

Definition 3.2. Let X be a BCK/BCI-algebra. An MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X is called an *MBJ-neutrosophic subalgebra* of X if it satisfies:

$$(\forall x, y \in X) \begin{pmatrix} M_A(x * y) \geq \min\{M_A(x), M_A(y)\}, \\ \tilde{B}_A(x * y) \succeq \mathrm{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}, \\ J_A(x * y) \leq \max\{J_A(x), J_A(y)\}. \end{pmatrix} \tag{3.1}$$

Example 3.3. Consider a set $X = \{0, a, b, c\}$ with the binary operation $*$ which is given in Table 1. Then

Table 1: Cayley table for the binary operation "$*$"

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

$(X; *, 0)$ is a BCK-algebra (see [11]). Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X defined by Table 2. It is routine to verify that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X.

Table 2: MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$

X	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$
0	0.7	$[0.3, 0.8]$	0.2
a	0.3	$[0.1, 0.5]$	0.6
b	0.1	$[0.3, 0.8]$	0.4
c	0.5	$[0.1, 0.5]$	0.7

Example 3.4. Consider a BCI-algebra $(\mathbb{Z}, -, 0)$ and let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in $\mathbb{Z}$ defined by

$$M_A : \mathbb{Z} \to [0, 1],\ x \mapsto \begin{cases} 0.6 & \text{if } x \in 4\mathbb{Z}, \\ 0.4 & \text{if } x \in 2\mathbb{Z} \setminus 4\mathbb{Z}, \\ 0.3 & \text{otherwise}, \end{cases}$$

$$\tilde{B}_A : \mathbb{Z} \to [I],\ x \mapsto \begin{cases} [0.6, 0.8] & \text{if } x \in 6\mathbb{Z}, \\ [0.4, 0.5] & \text{if } x \in 3\mathbb{Z} \setminus 6\mathbb{Z}, \\ [0.2, 0.3] & \text{otherwise}, \end{cases}$$

$$J_A : \mathbb{Z} \to [0,1],\ x \mapsto \begin{cases} 0.2 & \text{if } x \in 8\mathbb{Z}, \\ 0.4 & \text{if } x \in 4\mathbb{Z} \setminus 8\mathbb{Z}, \\ 0.5 & \text{otherwise.} \end{cases}$$

It is routine to verify that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of $(\mathbb{Z}, -, 0)$.

In what follows, let X be a BCK/BCI-algebra unless otherwise specified.

Proposition 3.5. *If $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X, then $M_A(0) \geq M_A(x)$, $\tilde{B}_A(0) \succeq \tilde{B}_A(x)$ and $J_A(0) \leq J_A(x)$ for all $x \in X$.*

Proof. For any $x \in X$, we have

$$M_A(0) = M_A(x * x) \geq \min\{M_A(x), M_A(x)\} = M_A(x),$$

$$\begin{aligned} \tilde{B}_A(0) &= \tilde{B}_A(x * x) \succeq \mathrm{rmin}\{\tilde{B}_A(x), \tilde{B}_A(x)\} \\ &= \mathrm{rmin}\{[B_A^-(x), B_A^+(x)], [B_A^-(x), B_A^+(x)]\} \\ &= [B_A^-(x), B_A^+(x)] = \tilde{B}_A(x), \end{aligned}$$

and

$$J_A(0) = J_A(x * x) \leq \max\{J_A(x), J_A(x)\} = J_A(x).$$

This completes the proof. □

Proposition 3.6. *Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic subalgebra of X. If there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} M_A(x_n) = 1,\ \lim_{n\to\infty} \tilde{B}_A(x_n) = [1,1] \text{ and } \lim_{n\to\infty} J_A(x_n) = 0, \tag{3.2}$$

then $M_A(0) = 1$, $\tilde{B}_A(0) = [1,1]$ and $J_A(0) = 0$.

Proof. Using Proposition 3.5, we know that $M_A(0) \geq M_A(x_n)$, $\tilde{B}_A(0) \succeq \tilde{B}_A(x_n)$ and $J_A(0) \leq J_A(x_n)$ for every positive integer n. Note that

$$\begin{aligned} &1 \geq M_A(0) \geq \lim_{n\to\infty} M_A(x_n) = 1, \\ &[1,1] \succeq \tilde{B}_A(0) \succeq \lim_{n\to\infty} \tilde{B}_A(x_n) = [1,1], \\ &0 \leq J_A(0) \leq \lim_{n\to\infty} J_A(x_n) = 0. \end{aligned}$$

Therefore $M_A(0) = 1$, $\tilde{B}_A(0) = [1,1]$ and $J_A(0) = 0$. □

Theorem 3.7. *Given an MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X, if (M_A, J_A) is an intuitionistic fuzzy subalgebra of X, and B_A^- and B_A^+ are fuzzy subalgebras of X, then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X.*

Proof. It is sufficient to show that $\tilde{B}_A$ satisfies the condition

$$(\forall x, y \in X)(\tilde{B}_A(x * y) \succeq \mathrm{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}). \tag{3.3}$$

For any $x, y \in X$, we get

$$\begin{aligned}\tilde{B}_A(x * y) &= [B_A^-(x * y), B_A^+(x * y)] \\ &\succeq [\min\{B_A^-(x), B_A^-(y)\}, \min\{B_A^+(x), B_A^+(y)\}] \\ &= \mathrm{rmin}\{[B_A^-(x), B_A^+(x)], [B_A^-(y), B_A^+(y)]\} \\ &= \mathrm{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}.\end{aligned}$$

Therefore $\tilde{B}_A$ satisfies the condition (3.3), and so $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X. □

If $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X, then

$$\begin{aligned}[B_A^-(x * y), B_A^+(x * y)] &= \tilde{B}_A(x * y) \succeq \mathrm{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\} \\ &= \mathrm{rmin}\{[B_A^-(x), B_A^+(x), [B_A^-(y), B_A^+(y)]\} \\ &= [\min\{B_A^-(x), B_A^-(y)\}, \min\{B_A^+(x), B_A^+(y)\}]\end{aligned}$$

for all $x, y \in X$. It follows that $B_A^-(x * y) \geq \min\{B_A^-(x), B_A^-(y)\}$ and $B_A^+(x * y) \geq \min\{B_A^+(x), B_A^+(y)\}$. Thus B_A^- and B_A^+ are fuzzy subalgebras of X. But (M_A, J_A) is not an intuitionistic fuzzy subalgebra of X as seen in Example 3.3. This shows that the converse of Theorem 3.7 is not true.

Given an MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X, we consider the following sets.

$$\begin{aligned}&U(M_A; t) := \{x \in X \mid M_A(x) \geq t\}, \\ &U(\tilde{B}_A; [\delta_1, \delta_2]) := \{x \in X \mid \tilde{B}_A(x) \succeq [\delta_1, \delta_2]\}, \\ &L(J_A; s) := \{x \in X \mid J_A(x) \leq s\}\end{aligned}$$

where $t, s \in [0, 1]$ and $[\delta_1, \delta_2] \in [I]$.

Theorem 3.8. *An MBJ-neutrosophic set $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ in X is an MBJ-neutrosophic subalgebra of X if and only if the non-empty sets $U(M_A; t)$, $U(\tilde{B}_A; [\delta_1, \delta_2])$ and $L(J_A; s)$ are subalgebras of X for all $t, s \in [0, 1]$ and $[\delta_1, \delta_2] \in [I]$.*

Proof. Suppose that $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X. Let $t, s \in [0, 1]$ and $[\delta_1, \delta_2] \in [I]$ be such that $U(M_A; t)$, $U(\tilde{B}_A; [\delta_1, \delta_2])$ and $L(J_A; s)$ are non-empty. For any $x, y, a, b, u, v \in X$, if $x, y \in U(M_A; t)$, $a, b \in U(\tilde{B}_A; [\delta_1, \delta_2])$ and $u, v \in L(J_A; s)$, then

$$\begin{aligned}&M_A(x * y) \geq \min\{M_A(x), M_A(y)\} \geq \min\{t, t\} = t, \\ &\tilde{B}_A(a * b) \succeq \mathrm{rmin}\{\tilde{B}_A(a), \tilde{B}_A(b)\} \succeq \mathrm{rmin}\{[\delta_1, \delta_2], [\delta_1, \delta_2]\} = [\delta_1, \delta_2], \\ &J_A(u * v) \leq \max\{J_A(u), J_A(v)\} \leq \min\{s, s\} = s,\end{aligned}$$

and so $x * y \in U(M_A; t)$, $a * b \in U(\tilde{B}_A; [\delta_1, \delta_2])$ and $u * v \in L(J_A; s)$. Therefore $U(M_A; t)$, $U(\tilde{B}_A; [\delta_1, \delta_2])$ and $L(J_A; s)$ are subalgebras of X.

Conversely, assume that the non-empty sets $U(M_A;t)$, $U(\tilde{B}_A;[\delta_1,\delta_2])$ and $L(J_A;s)$ are subalgebras of X for all $t,s \in [0,1]$ and $[\delta_1,\delta_2] \in [I]$. If $M_A(a_0 * b_0) < \min\{M_A(a_0), M_A(b_0)\}$ for some $a_0, b_0 \in X$, then $a_0, b_0 \in U(M_A;t_0)$ but $a_0 * b_0 \notin U(M_A;t_0)$ for $t_0 := \min\{M_A(a_0), M_A(b_0)\}$. This is a contradiction, and thus $M_A(a*b) \geq \min\{M_A(a), M_A(b)\}$ for all $a,b \in X$. Similarly, we can show that $J_A(a*b) \leq \max\{J_A(a), J_A(b)\}$ for all $a,b \in X$. Suppose that $\tilde{B}_A(a_0 * b_0) \prec \operatorname{rmin}\{\tilde{B}_A(a_0), \tilde{B}_A(b_0)\}$ for some $a_0, b_0 \in X$. Let $\tilde{B}_A(a_0) = [\lambda_1, \lambda_2]$, $\tilde{B}_A(b_0) = [\lambda_3, \lambda_4]$ and $\tilde{B}_A(a_0 * b_0) = [\delta_1, \delta_2]$. Then

$$[\delta_1,\delta_2] \prec \operatorname{rmin}\{[\lambda_1,\lambda_2],[\lambda_3,\lambda_4]\} = [\min\{\lambda_1,\lambda_3\}, \min\{\lambda_2,\lambda_4\}],$$

and so $\delta_1 < \min\{\lambda_1,\lambda_3\}$ and $\delta_2 < \min\{\lambda_2,\lambda_4\}$. Taking

$$[\gamma_1,\gamma_2] := \tfrac{1}{2}\left(\tilde{B}_A(a_0 * b_0) + \operatorname{rmin}\{\tilde{B}_A(a_0), \tilde{B}_A(b_0)\}\right)$$

implies that

$$\begin{aligned}[\gamma_1,\gamma_2] &= \tfrac{1}{2}\left([\delta_1,\delta_2] + [\min\{\lambda_1,\lambda_3\}, \min\{\lambda_2,\lambda_4\}]\right)\\ &= \left[\tfrac{1}{2}(\delta_1 + \min\{\lambda_1,\lambda_3\}, \tfrac{1}{2}(\delta_2 + \min\{\lambda_2,\lambda_4\}\right].\end{aligned}$$

It follows that

$$\min\{\lambda_1,\lambda_3\} > \gamma_1 = \tfrac{1}{2}(\delta_1 + \min\{\lambda_1,\lambda_3\} > \delta_1$$

and

$$\min\{\lambda_2,\lambda_4\} > \gamma_2 = \tfrac{1}{2}(\delta_2 + \min\{\lambda_2,\lambda_4\} > \delta_2.$$

Hence $[\min\{\lambda_1,\lambda_3\}, \min\{\lambda_2,\lambda_4\}] \succ [\gamma_1,\gamma_2] \succ [\delta_1,\delta_2] = \tilde{B}_A(a_0 * b_0)$, and therefore $a_0 * b_0 \notin U(\tilde{B}_A;[\gamma_1,\gamma_2])$. On the other hand,

$$\tilde{B}_A(a_0) = [\lambda_1,\lambda_2] \succeq [\min\{\lambda_1,\lambda_3\}, \min\{\lambda_2,\lambda_4\}] \succ [\gamma_1,\gamma_2]$$

and

$$\tilde{B}_A(b_0) = [\lambda_3,\lambda_4] \succeq [\min\{\lambda_1,\lambda_3\}, \min\{\lambda_2,\lambda_4\}] \succ [\gamma_1,\gamma_2],$$

that is, $a_0, b_0 \in U(\tilde{B}_A;[\gamma_1,\gamma_2])$. This is a contradiction, and therefore $\tilde{B}_A(x*y) \succeq \operatorname{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}$ for all $x,y \in X$. Consequently $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X. □

Using Proposition 3.5 and Theorem 3.8, we have the following corollary.

Corollary 3.9. *If $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X, then the sets $X_{M_A} := \{x \in X \mid M_A(x) = M_A(0)\}$, $X_{\tilde{B}_A} := \{x \in X \mid \tilde{B}_A(x) = \tilde{B}_A(0)\}$, and $X_{J_A} := \{x \in X \mid J_A(x) = J_A(0)\}$ are subalgebras of X.*

We say that the subalgebras $U(M_A;t)$, $U(\tilde{B}_A;[\delta_1,\delta_2])$ and $L(J_A;s)$ are *MBJ-subalgebras* of $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$.

Theorem 3.10. *Every subalgebra of X can be realized as MBJ-subalgebras of an MBJ-neutrosophic subalgebra of X.*

Proof. Let K be a subalgebra of X and let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X defined by

$$M_A(x) = \begin{cases} t & \text{if } x \in K, \\ 0 & \text{otherwise,} \end{cases} \quad \tilde{B}_A(x) = \begin{cases} [\gamma_1, \gamma_2] & \text{if } x \in K, \\ [0,0] & \text{otherwise,} \end{cases} \quad J_A(x) = \begin{cases} s & \text{if } x \in K, \\ 1 & \text{otherwise,} \end{cases} \tag{3.4}$$

where $t \in (0, 1]$, $s \in [0, 1)$ and $\gamma_1, \gamma_2 \in (0, 1]$ with $\gamma_1 < \gamma_2$. It is clear that $U(M_A; t) = K$, $U(\tilde{B}_A; [\gamma_1, \gamma_2]) = K$ and $L(J_A; s) = K$. Let $x, y \in X$. If $x, y \in K$, then $x * y \in K$ and so

$$\begin{aligned} &M_A(x * y) = t = \min\{M_A(x), M_A(y)\} \\ &\tilde{B}_A(x * y) = [\gamma_1, \gamma_2] = \text{rmin}\{[\gamma_1, \gamma_2], [\gamma_1, \gamma_2]\} = \text{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}, \\ &J_A(x * y) = s = \max\{J_A(x), J_A(y)\}. \end{aligned}$$

If any one of x and y is contained in K, say $x \in K$, then $M_A(x) = t$, $\tilde{B}_A(x) = [\gamma_1, \gamma_2]$, $J_A(x) = s$, $M_A(y) = 0$, $\tilde{B}_A(y) = [0, 0]$ and $J_A(y) = 1$. Hence

$$\begin{aligned} &M_A(x * y) \geq 0 = \min\{t, 0\} = \min\{M_A(x), M_A(y)\} \\ &\tilde{B}_A(x * y) \succeq [0, 0] = \text{rmin}\{[\gamma_1, \gamma_2], [0, 0]\} = \text{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}, \\ &J_A(x * y) \leq 1 = \max\{s, 1\} = \max\{J_A(x), J_A(y)\}. \end{aligned}$$

If $x, y \notin K$, then $M_A(x) = 0 = M_A(y)$, $\tilde{B}_A(x) = [0, 0] = \tilde{B}_A(y)$ and $J_A(x) = 1 = J_A(y)$. It follows that

$$\begin{aligned} &M_A(x * y) \geq 0 = \min\{0, 0\} = \min\{M_A(x), M_A(y)\} \\ &\tilde{B}_A(x * y) \succeq [0, 0] = \text{rmin}\{[0, 0], [0, 0]\} = \text{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}, \\ &J_A(x * y) \leq 1 = \max\{1, 1\} = \max\{J_A(x), J_A(y)\}. \end{aligned}$$

Therefore $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X. □

Theorem 3.11. *For any non-empty subset K of X, let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X which is given in (3.4). If $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X, then K is a subalgebra of X.*

Proof. Let $x, y \in K$. Then $M_A(x) = t = M_A(y)$, $\tilde{B}_A(x) = [\gamma_1, \gamma_2] = \tilde{B}_A(y)$ and $J_A(x) = s = J_A(y)$. Thus

$$\begin{aligned} &M_A(x * y) \geq \min\{M_A(x), M_A(y)\} = t, \\ &\tilde{B}_A(x * y) \succeq \text{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\} = [\gamma_1, \gamma_2], \\ &J_A(x * y) \leq \max\{J_A(x), J_A(y)\} = s, \end{aligned}$$

and therefore $x * y \in K$. Hence K is a subalgebra of X. □

Using an MBJ-neutrosophic subalgebra of a BCI-algera, we establish a new MBJ-neutrosophic subalgebra.

Theorem 3.12. *Given an MBJ-neutrosophic subalgebra $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ of a BCI-algebra X, let $\mathcal{A}^* = (M_A^*, \tilde{B}_A^*, J_A^*)$ be an MBJ-neutrosophic set in X defined by $M_A^*(x) = M_A(0 * x)$, $\tilde{B}_A^*(x) = \tilde{B}_A(0 * x)$ and $J_A^*(x) = J_A(0 * x)$ for all $x \in X$. Then $\mathcal{A}^* = (M_A^*, \tilde{B}_A^*, J_A^*)$ is an MBJ-neutrosophic subalgebra of X.*

Proof. Note that $0 * (x * y) = (0 * x) * (0 * y)$ for all $x, y \in X$. We have

$$\begin{aligned} M_A^*(x * y) &= M_A(0 * (x * y)) = M_A((0 * x) * (0 * y)) \\ &\geq \min\{M_A(0 * x), M_A(0 * y)\} \\ &= \min\{M_A^*(x), M_A^*(y)\}, \end{aligned}$$

$$\begin{aligned} \tilde{B}_A^*(x * y) &= \tilde{B}_A(0 * (x * y)) = \tilde{B}_A((0 * x) * (0 * y)) \\ &\succeq \operatorname{rmin}\{\tilde{B}_A(0 * x), \tilde{B}_A(0 * y)\} \\ &= \operatorname{rmin}\{\tilde{B}_A^*(x), \tilde{B}_A^*(y)\} \end{aligned}$$

and

$$\begin{aligned} J_A^*(x * y) &= J_A(0 * (x * y)) = J_A((0 * x) * (0 * y)) \\ &\leq \max\{J_A(0 * x), J_A(0 * y)\} \\ &= \max\{J_A^*(x), J_A^*(y)\} \end{aligned}$$

for all $x, y \in X$. Therefore $\mathcal{A}^* = (M_A^*, \tilde{B}_A^*, J_A^*)$ is an MBJ-neutrosophic subalgebra of X. □

Theorem 3.13. *Let $f : X \to Y$ be a homomorphism of BCK/BCI-algebras. If $\mathcal{B} = (M_B, \tilde{B}_B, J_B)$ is an MBJ-neutrosophic subalgebra of Y, then $f^{-1}(\mathcal{B}) = (f^{-1}(M_B), f^{-1}(\tilde{B}_B), f^{-1}(J_B))$ is an MBJ-neutrosophic subalgebra of X, where $f^{-1}(M_B)(x) = M_B(f(x))$, $f^{-1}(\tilde{B}_B)(x) = \tilde{B}_B(f(x))$ and $f^{-1}(J_B)(x) = J_B(f(x))$ for all $x \in X$.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned} f^{-1}(M_B)(x * y) &= M_B(f(x * y)) = M_B(f(x) * f(y)) \\ &\geq \min\{M_B(f(x)), M_B(f(y))\} \\ &= \min\{f^{-1}(M_B)(x), f^{-1}(M_B)(y)\}, \end{aligned}$$

$$\begin{aligned} f^{-1}(\tilde{B}_B)(x * y) &= \tilde{B}_B(f(x * y)) = \tilde{B}_B(f(x) * f(y)) \\ &\succeq \operatorname{rmin}\{\tilde{B}_B(f(x)), \tilde{B}_B(f(y))\} \\ &= \operatorname{rmin}\{f^{-1}(\tilde{B}_B)(x), f^{-1}(\tilde{B}_B)(y)\}, \end{aligned}$$

and

$$\begin{aligned} f^{-1}(J_B)(x * y) &= J_B(f(x * y)) = J_B(f(x) * f(y)) \\ &\leq \max\{J_B(f(x)), J_B(f(y))\} \\ &= \max\{f^{-1}(J_B)(x), f^{-1}(J_B)(y)\}. \end{aligned}$$

Hence $f^{-1}(\mathcal{B}) = (f^{-1}(M_B), f^{-1}(\tilde{B}_B), f^{-1}(J_B))$ is an MBJ-neutrosophic subalgebra of X. □

Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in a set X. We denote

$$\begin{aligned}
&\top := 1 - \sup\{M_A(x) \mid x \in X\},\\
&\Pi := [1,1] - \operatorname{rsup}\{\tilde{B}_A(x) \mid x \in X\},\\
&\bot := \inf\{J_A(x) \mid x \in X\}.
\end{aligned}$$

For any $p \in [0, \top]$, $\tilde{a} \in [[0,0], \Pi]$ and $q \in [0, \bot]$, we define $\mathcal{A}^T = (M_A^p, \tilde{B}_A^{\tilde{a}}, J_A^q)$ by $M_A^p(x) = M_A(x) + p$, $\tilde{B}_A^{\tilde{a}}(x) = \tilde{B}_A(x) + \tilde{a}$ and $J_A^q(x) = J_A(x) - q$. Then $\mathcal{A}^T = (M_A^p, \tilde{B}_A^{\tilde{a}}, J_A^q)$ is an MBJ-neutrosophic set in X, which is called a $(p, \tilde{a}, q)$*-translative MBJ-neutrosophic set* of $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$.

Theorem 3.14. *If $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X, then the $(p, \tilde{a}, q)$-translative MBJ-neutrosophic set of $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is also an MBJ-neutrosophic subalgebra of X.*

Proof. For any $x, y \in X$, we get

$$\begin{aligned}
M_A^p(x * y) &= M_A(x * y) + p \geq \min\{M_A(x), M_A(y)\} + p\\
&= \min\{M_A(x) + p, M_A(y) + p\} = \min\{M_A^p(x), M_A^p(y)\},
\end{aligned}$$

$$\begin{aligned}
\tilde{B}_A^{\tilde{a}}(x * y) &= \tilde{B}_A(x * y) + \tilde{a} \succeq \operatorname{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\} + \tilde{a}\\
&= \operatorname{rmin}\{\tilde{B}_A(x) + \tilde{a}, \tilde{B}_A(y) + \tilde{a}\} = \operatorname{rmin}\{\tilde{B}_A^{\tilde{a}}(x), \tilde{B}_A^{\tilde{a}}(y)\},
\end{aligned}$$

and

$$\begin{aligned}
J_A^q(x * y) &= J_A(x * y) - q \leq \max\{J_A(x), J_A(y)\} - q\\
&= \max\{J_A(x) - q, J_A(y) - q\} = \max\{J_A^q(x), J_A^q(y)\}.
\end{aligned}$$

Therefore $\mathcal{A}^T = (M_A^p, \tilde{B}_A^{\tilde{a}}, J_A^q)$ is an MBJ-neutrosophic subalgebra of X. □

We consider the converse of Theorem 3.14.

Theorem 3.15. *Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X such that its $(p, \tilde{a}, q)$-translative MBJ-neutrosophic set is an MBJ-neutrosophic subalgebra of X for $p \in [0, \top]$, $\tilde{a} \in [[0,0], \Pi]$ and $q \in [0, \bot]$. Then $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X.*

Proof. Assume that $\mathcal{A}^T = (M_A^p, \tilde{B}_A^{\tilde{a}}, J_A^q)$ is an MBJ-neutrosophic subalgebra of X for $p \in [0, \top]$, $\tilde{a} \in [[0,0], \Pi]$ and $q \in [0, \bot]$. Let $x, y \in X$. Then

$$\begin{aligned}
M_A(x * y) + p = M_A^p(x * y) &\geq \min\{M_A^p(x), M_A^p(y)\}\\
&= \min\{M_A(x) + p, M_A(y) + p\}\\
&= \min\{M_A(x), M_A(y)\} + p,
\end{aligned}$$

$$\begin{aligned}
\tilde{B}_A(x * y) + \tilde{a} = \tilde{B}_A^{\tilde{a}}(x * y) &\succeq \operatorname{rmin}\{\tilde{B}_A^{\tilde{a}}(x), \tilde{B}_A^{\tilde{a}}(y)\}\\
&= \operatorname{rmin}\{\tilde{B}_A(x) + \tilde{a}, \tilde{B}_A(y) + \tilde{a}\}\\
&= \operatorname{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\} + \tilde{a},
\end{aligned}$$

and

$$\begin{aligned} J_A(x*y) - q = J_A^q(x*y) &\leq \max\{J_A^q(x), J_A^q(y)\} \\ &= \max\{J_A(x) - q, J_A(y) - q\} \\ &= \max\{J_A(x), J_A(y)\} - q. \end{aligned}$$

It follows that $M_A(x*y) \geq \min\{M_A(x), M_A(y)\}$, $\tilde{B}_A(x*y) \succeq \mathrm{rmin}\{\tilde{B}_A(x), \tilde{B}_A(y)\}$ and $J_A(x*y) \leq \max\{J_A(x), J_A(y)\}$ for all $x, y \in X$. Hence $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ is an MBJ-neutrosophic subalgebra of X. □

Definition 3.16. Let $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ and $\mathcal{B} = (M_B,\, \tilde{B}_B,\, J_B)$ be MBJ-neutrosophic sets in X. Then $\mathcal{B} = (M_B,\, \tilde{B}_B,\, J_B)$ is called an *MBJ-neutrosophic S-extension* of $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ if the following assertions are valid.

(1) $M_B(x) \geq M_A(x)$, $\tilde{B}_B(x) \succeq \tilde{B}_A(x)$ and $J_B(x) \leq J_A(x)$ for all $x \in X$,

(2) If $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ is an MBJ-neutrosophic subalgebra of X, then $\mathcal{B} = (M_B,\, \tilde{B}_B,\, J_B)$ is an MBJ-neutrosophic subalgebra of X.

Theorem 3.17. *Given $p \in [0, \top]$, $\tilde{a} \in [[0,0], \Pi]$ and $q \in [0, \perp]$, the $(p, \tilde{a}, q)$-translative MBJ-neutrosophic set $\mathcal{A}^T = (M_A^p,\, \tilde{B}_A^{\tilde{a}},\, J_A^q)$ of an MBJ-neutrosophic subalgebra $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ is an MBJ-neutrosophic S-extension of $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$.*

Proof. Straightforward. □

Given an MBJ-neutrosophic set $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ in X, we consider the following sets.

$$\begin{aligned} &U_p(M_A; t) := \{x \in X \mid M_A(x) \geq t - p\}, \\ &U_{\tilde{a}}(\tilde{B}_A; [\delta_1, \delta_2]) := \{x \in X \mid \tilde{B}_A(x) \succeq [\delta_1, \delta_2] - \tilde{a}\}, \\ &L_q(J_A; s) := \{x \in X \mid J_A(x) \leq s + q\} \end{aligned}$$

where $t, s \in [0,1]$, $[\delta_1, \delta_2] \in [I]$, $p \in [0, \top]$, $\tilde{a} \in [[0,0], \Pi]$ and $q \in [0, \perp]$ such that $t \geq p$, $[\delta_1, \delta_2] \succeq \tilde{a}$ and $s \leq q$.

Theorem 3.18. *Let $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ be an MBJ-neutrosophic set in X. Given $p \in [0, \top]$, $\tilde{a} \in [[0,0], \Pi]$ and $q \in [0, \perp]$, the $(p, \tilde{a}, q)$-translative MBJ-neutrosophic set of $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ is an MBJ-neutrosophic subalgebra of X if and only if $U_p(M_A; t)$, $U_{\tilde{a}}(\tilde{B}_A; [\delta_1, \delta_2])$ and $L_q(J_A; s)$ are subalgebras of X for all $t \in Im(M_A)$, $[\delta_1, \delta_2] \in Im(\tilde{B}_A)$ and $s \in Im(J_A)$ with $t \geq p$, $[\delta_1, \delta_2] \succeq \tilde{a}$ and $s \leq q$.*

Proof. Assume that the $(p, \tilde{a}, q)$-translative MBJ-neutrosophic set of $\mathcal{A} = (M_A,\, \tilde{B}_A,\, J_A)$ is an MBJ-neutrosophic subalgebra of X. Let $x, y \in U_p(M_A; t)$. Then $M_A(x) \geq t - p$ and $M_A(y) \geq t - p$, which imply that $M_A^p(x) \geq t$ and $M_A^p(y) \geq t$. It follows that

$$M_A^p(x*y) \geq \min\{M_A^p(x), M_A^p(y)\} \geq t$$

and so that $M_A(x*y) \geq t - p$. Hence $x*y \in U_p(M_A; t)$. If $x, y \in U_{\tilde{a}}(\tilde{B}_A; [\delta_1, \delta_2])$, then $\tilde{B}_A(x) \succeq [\delta_1, \delta_2] - \tilde{a}$ and $\tilde{B}_A(y) \succeq [\delta_1, \delta_2] - \tilde{a}$. Hence

$$\tilde{B}_A^{\tilde{a}}(x*y) \succeq \mathrm{rmin}\{\tilde{B}_A^{\tilde{a}}(x), \tilde{B}_A^{\tilde{a}}(y)\} \succeq [\delta_1, \delta_2],$$

and so $\tilde{B}_A(x*y) \succeq [\delta_1, \delta_2] - \tilde{a}$. Thus $x*y \in U_{\tilde{a}}(\tilde{B}_A; [\delta_1, \delta_2])$. Let $x, y \in L_q(J_A; s)$. Then $J_A(x) \leq s + q$ and $J_A(y) \leq s + q$. It follows that

$$J_A^q(x*y) \leq \max\{J_A^q(x), J_A^q(y)\} \leq s,$$

that is, $J_A(x*y) \leq s + q$. Thus $x*y \in L_q(J_A; s)$. Therefore $U_p(M_A; t)$, $U_{\tilde{a}}(\tilde{B}_A; [\delta_1, \delta_2])$ and $L_q(J_A; s)$ are subalgebras of X.

Conversely, suppose that $U_p(M_A; t)$, $U_{\tilde{a}}(\tilde{B}_A; [\delta_1, \delta_2])$ and $L_q(J_A; s)$ are subalgebras of X for all $t \in Im(M_A)$, $[\delta_1, \delta_2] \in Im(\tilde{B}_A)$ and $s \in Im(J_A)$ with $t \geq p$, $[\delta_1, \delta_2] \succeq \tilde{a}$ and $s \leq q$. Assume that $M_A^p(a*b) < \min\{M_A^p(a), M_A^p(b)\}$ for some $a, b \in X$. Then $a, b \in U_p(M_A; t_0)$ and $a*b \notin U_p(M_A; t_0)$ for $t_0 = \min\{M_A^p(a), M_A^p(b)\}$. This is a contradiction, and so $M_A^p(x*y) \geq \min\{M_A^p(x), M_A^p(y)\}$ for all $x, y \in X$. If $\tilde{B}_A^{\tilde{a}}(x_0*y_0) \prec \text{rmin}\{\tilde{B}_A^{\tilde{a}}(x_0), M_A^{\tilde{a}}(y_0)\}$ for some $x_0, y_0 \in X$, then there exists $\tilde{b} \in [I]$ such that $\tilde{B}_A^{\tilde{a}}(x_0*y_0) \prec \tilde{b} \preceq \text{rmin}\{\tilde{B}_A^{\tilde{a}}(x_0), M_A^{\tilde{a}}(y_0)\}$. Hence $x_0, y_0 \in U_{\tilde{a}}(\tilde{B}_A; \tilde{b})$ but $x_0 * y_0 \notin U_{\tilde{a}}(\tilde{B}_A; \tilde{b})$, which is a contradiction. Thus $\tilde{B}_A^{\tilde{a}}(x*y) \succeq \text{rmin}\{\tilde{B}_A^{\tilde{a}}(x), M_A^{\tilde{a}}(y)\}$ for all $x, y \in X$. Suppose that $J_A^q(a*b) > \max\{J_A^q(a), J_A^q(b)\}$ for some $a, b \in X$. Taking $s_0 := \max\{J_A^q(a), J_A^q(b)\}$ implies that $J_A(a) \leq s_0 + q$ and $J_A(b) \leq s_0 + q$ but $J_A(a*b) > s_0 + q$. This shows that $a, b \in L_q(J_A; s_0)$ and $a*b \in L_q(J_A; s_0)$. This is a contradiction, and therefore $J_A^q(x*y) \leq \max\{J_A^q(x), J_A^q(y)\}$ for all $x, y \in X$. Consequently, the $(p, \tilde{a}, q)$-translative MBJ-neutrosophic set $\mathcal{A}^T = (M_A^p, \tilde{B}_A^{\tilde{a}}, J_A^q)$ of $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ is an MBJ-neutrosophic subalgebra of X. □

4 Conclusion

This paper is written during the third author visit Shahid Beheshti University. In the study of Smarandache's neutrosophic sets, the authors of this article had a periodical research meeting three times per week, and tried to get a generalization of Smarandache's neutrosophic sets. In the neutrosophic set, the truth, false and indeterminate membership functions are fuzzy sets. In considering a generalization of neutrosophic set, we used the interval valued fuzzy set as the indeterminate membership function because interval valued fuzzy set is a generalization of a fuzzy set, and we called it MBJ-neutrosophic set where "MBJ" is the initial of authors's surname, that is, Mohseni, Borzooei and Jun, respectively. We also use M_A, $\tilde{B}_A$ and J_A as the truth membership function, the indeterminate membership function and the false membership function, respectively. We know that there are many generalizations of Smarandache's neutrosophic sets. In this article, we have made up a generalization of neutrosophic set, called an MBJ-neutrosophic set, and have applied it to BCK/BCI-algebras. We have introduced the concept of MBJ-neutrosophic subalgebras in BCK/BCI-algebras, and investigated related properties. We have provided a characterization of MBJ-neutrosophic subalgebra, and established a new MBJ-neutrosophic subalgebra by using an MBJ-neutrosophic subalgebra of a BCI-algebra. We have considered the homomorphic inverse image of MBJ-neutrosophic subalgebra, and discussed translation of MBJ-neutrosophic subalgebra. We also have found conditions for an MBJ-neutrosophic set to be an MBJ-neutrosophic subalgebra.

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