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Ordered subalgebras of ordered BCI-algebras based on the MBJ-neutrosophic structure

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Abstract: The neutrosophic set consists of three fuzzy sets called true membership function, false membership function and indeterminate membership function. MBJ-neutrosophic structure is a structure constructed using interval-valued fuzzy set instead of indeterminate membership function in the neutrosophic set. In general, the indeterminate part appears in a wide range. So instead of treating the indeterminate part as a single value, it is treated as an interval value, allowing a much more comprehensive processing. In an attempt to apply the MBJ-neutrosophic structure to ordered BCI-algebras, the notion of MBJ-neutrosophic (ordered) subalgebras is introduced and their properties are studied. The relationship between MBJ-neutrosophic subalgebra and MBJ-neutrosophic ordered subalgebra is established, and MBJ-neutrosophic ordered subalgebra is formed using (intuitionistic) fuzzy ordered subalgebra. Given an MBJ-neutrosophic set, its $(q, \tilde{\mathfrak{c}}, p)$-translative MBJ-neutrosophic set is introduced and its characterization is considered. An MBJ-neutrosophic ordered subalgebra is created using $(q, \tilde{\mathfrak{c}}, p)$-translative MBJ-neutrosophic set.

Keywords: Ordered BCI-algebra, ordered subalgebra, MBJ-neutrosophic ordered subalgebra, MBJ-ordered subalgebras, $(q, \tilde{\mathfrak{c}}, p)$-translative MBJ-neutrosophic set.

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1 Introduction

The classical set theory contains only two components: true and false, which means that an element can either belong to the set (true) or not belong to the set (false). However, in many cases, there is an indeterminate state which means it is not clear that elements are in or out of a set. In other words, there is a lot of incomplete or uncertain information, which is an indeterminate state that cannot be expressed as true or false. Neutrosophic logic is an extension of classical and fuzzy logic, which is a good tool for processing uncertain and indeterminate information in a more versatile way. The neutrosophic set is a mathematical concept introduced by Florentin Smarandash in the late 1990s, which is particularly useful for dealing with the indeterminate states that classical set theory cannot address. It is applied to various fields such as artificial intelligence, decision-making, expert systems, and information management that require coping with ambiguity and inaccuracy. In addition, neutrosophic set theory is applied to various algebraic structures including logical algebras (see [1], [3], [4], [5], [6], [7],[9], [13], [16], [17]). The neutrosophic set gives three values, i.e., membership degree (T), non-membership degree (F), and indeterminacy degree (I), for each element. In [11], Mohseni Takallo et al. extended the indeterminacy degree (F) to the interval value to introduce the MBJ-n-set, and it is applied to several logical algebras (see [2], [8], [10], [12], [15]). The “MBJ” stands for the initials of the three researchers, R. A. Borzooei, M. Mohseni Takallo and Y. B. Jun.

This paper applies the MBJ-neutrosophic structure to OBCI-algebras. We first introduce the notion of MBJ-neutrosophic (ordered) subalgebras in OBCI-algebras and then investigate their properties. We look at the relations of the MBJ-neutrosophic subalgebra to the MBJ-neutrosophic ordered subalgebra (O-subalgebra for simplicity). Using (intuitionistic) fuzzy ordered subalgebras, we establish an MBJ-neutrosophic ordered subalgebra. We discuss the characterization of MBJ-neutrosophic O-subalgebras. Given an MBJ-n-set, we introduce its $(q, \tilde{a_{13}}, p)$-translative MBJ-n-set and consider its characterization. We use $(q, \tilde{a_{13}}, p)$-translative MBJ-n-set to generate an MBJ-neutrosophic O-subalgebra.

2 Preliminaries

Definition 2.1 ([18]). An *OBCI-algebra* is defined to be a set W together with a binary relation “ $\leq_W$ ”, a constant “$\check{e}$” and a binary operation “ $\rightsquigarrow$ ” that satisfies:

$$\check{e} \leq_W (a_{11} \rightsquigarrow a_{12}) \rightsquigarrow ((a_{12} \rightsquigarrow a_{13}) \rightsquigarrow (a_{11} \rightsquigarrow a_{13})), \tag{2.1}$$

$$\check{e} \leq_W a_{11} \rightsquigarrow ((a_{11} \rightsquigarrow a_{12}) \rightsquigarrow a_{12}), \tag{2.2}$$

$$\check{e} \leq_W a_{11} \rightsquigarrow a_{11}, \tag{2.3}$$

$$\check{e} \leq_W a_{11} \rightsquigarrow a_{12},\ \check{e} \leq_W a_{12} \rightsquigarrow a_{11} \ \Rightarrow \ a_{11} = a_{12}, \tag{2.4}$$

$$a_{11} \leq_W a_{12} \ \Leftrightarrow \ \check{e} \leq_W a_{11} \rightsquigarrow a_{12}, \tag{2.5}$$

$$\check{e} \leq_W a_{11},\ a_{11} \leq_W a_{12} \ \Rightarrow \ \check{e} \leq_W a_{12} \tag{2.6}$$

for all $a_{11}, a_{12}, a_{13} \in W$.

Obviously $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ with $W = \{\check{e}\}$ is an OBCI-algebra, which is said to be the *trivial OBCI-algebra*.

Proposition 2.2 ([18]). *If $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ is an OBCI-algebra, it satisfies:*

$$\check{e} \rightsquigarrow a_{11} = a_{11}, \tag{2.7}$$
$$a_{13} \rightsquigarrow (a_{12} \rightsquigarrow a_{11}) = a_{12} \rightsquigarrow (a_{13} \rightsquigarrow a_{11}), \tag{2.8}$$
$$\check{e} \leq_W a_{11} \rightsquigarrow a_{12} \;\Rightarrow\; \check{e} \leq_W (a_{12} \rightsquigarrow a_{13}) \rightsquigarrow (a_{11} \rightsquigarrow a_{13}), \tag{2.9}$$
$$\check{e} \leq_W a_{11} \rightsquigarrow a_{12},\, \check{e} \leq_W a_{12} \rightsquigarrow a_{13} \;\Rightarrow\; \check{e} \leq_W a_{11} \rightsquigarrow a_{13}, \tag{2.10}$$
$$\check{e} \leq_W (a_{13} \rightsquigarrow (a_{12} \rightsquigarrow a_{11})) \rightsquigarrow (a_{12} \rightsquigarrow (a_{13} \rightsquigarrow a_{11})), \tag{2.11}$$
$$\check{e} \leq_W a_{13} \rightsquigarrow (a_{12} \rightsquigarrow a_{11}) \;\Rightarrow\; \check{e} \leq_W a_{12} \rightsquigarrow (a_{13} \rightsquigarrow a_{11}), \tag{2.12}$$
$$((a_{11} \rightsquigarrow a_{12}) \rightsquigarrow a_{12}) \rightsquigarrow a_{12} = a_{11} \rightsquigarrow a_{12}, \tag{2.13}$$
$$(a_{11} \rightsquigarrow a_{11}) \rightsquigarrow a_{11} = a_{11}, \tag{2.14}$$
$$\check{e} \leq_W (a_{12} \rightsquigarrow a_{13}) \rightsquigarrow ((a_{11} \rightsquigarrow a_{12}) \rightsquigarrow (a_{11} \rightsquigarrow a_{13})), \tag{2.15}$$
$$\check{e} \leq_W a_{11} \rightsquigarrow a_{12} \;\Rightarrow\; \check{e} \leq_W (a_{13} \rightsquigarrow a_{11}) \rightsquigarrow (a_{13} \rightsquigarrow a_{12}) \tag{2.16}$$

for all $a_{11}, a_{12}, a_{13} \in W$.

Definition 2.3 ([18]). A subset A of W is said to be

- a *subalgebra* of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if it satisfies:

$$(\forall a_{11}, a_{12} \in W)(a_{11}, a_{12} \in A \;\Rightarrow\; a_{11} \rightsquigarrow a_{12} \in A). \tag{2.17}$$

- an *ordered subalgebra* (briefly, O-subalgebra) of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if it satisfies:

$$(\forall a_{11}, a_{12} \in W)(a_{11}, a_{12} \in A,\, \check{e} \leq_W a_{11},\, \check{e} \leq_W a_{12} \;\Rightarrow\; a_{11} \rightsquigarrow a_{12} \in A). \tag{2.18}$$

A function $\mu : W \to [0, 1]$ is said to be a *fuzzy set* (f-set for brevity) in a set W.

Definition 2.4 ([19]). An f-set μ in W is said to be a *fuzzy ordered subalgebra* (briefly, FO-subalgebra) of an OBCI-algebra $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if it satisfies:

$$(\forall x_{11}, x_{12} \in W)(\tilde{e} \leq_W x_{11},\, \tilde{e} \leq_W x_{12} \;\Rightarrow\; \mu(x_{11} \rightsquigarrow x_{12}) \geq \min\{\mu(x_{11}), \mu(x_{12})\}). \tag{2.19}$$

Definition 2.5 ([14]). An intuitionistic f-set $\mathcal{I} := \{\langle x_{11}, \mu_I, \nu_I\rangle \mid x_{11} \in W\}$ is said to be an *intuitionistic fuzzy ordered subalgebra* (briefly, IFO-subalgebra) of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if it satisfies:

$$(\forall x_{11}, x_{12} \in W)\left(\begin{array}{l} \tilde{e} \leq_W x_{11},\; \tilde{e} \leq_W x_{12} \\ \quad \Rightarrow \left\{\begin{array}{l} \mu_I(x_{11} \rightsquigarrow x_{12}) \geq \min\{\mu_I(x_{11}), \mu_I(x_{12})\} \\ \nu_I(x_{11} \rightsquigarrow x_{12}) \leq \max\{\nu_I(x_{11}), \nu_I(x_{12})\} \end{array}\right. \end{array}\right). \tag{2.20}$$

The neutrosophic set (n-set for brevity) is an extension of traditional set theory and has the advantage of handling uncertain, indeterminate and inconsistent information in a more flexible way than classical sets.

Given a non-empty set W, an *n-set* in W is a structure of the form:

$$\mathcal{M}_{\circledcirc} := \{\langle x_{11}; \mathcal{M}_T^{\circledcirc}(x_{11}), \mathcal{M}_F^{\circledcirc}(x_{11}), \mathcal{M}_I^{\circledcirc}(x_{11})\rangle \mid x_{11} \in W\}$$

where $\mathcal{M}_T^{\circledcirc} : W \to [0,1]$ is a true membership function, $\mathcal{M}_F^{\circledcirc} : W \to [0,1]$ is a false membership function, and $\mathcal{M}_I^{\circledcirc} : W \to [0,1]$ is an indeterminate membership function. For brevity, we use the symbol $\mathcal{M}_{\circledcirc} := (\mathcal{M}_T^{\circledcirc}, \mathcal{M}_I^{\circledcirc}, \mathcal{M}_F^{\circledcirc})$ for the n-set

$$\mathcal{M}_{\circledcirc} := \{\langle x_{11}; \mathcal{M}_T^{\circledcirc}(x_{11}), \mathcal{M}_I^{\circledcirc}(x_{11}), \mathcal{M}_F^{\circledcirc}(x_{11})\rangle \mid x_{11} \in W\}.$$

Given an n-set $\mathcal{M}_{\circledcirc} := (\mathcal{M}_T^{\circledcirc}, \mathcal{M}_I^{\circledcirc}, \mathcal{M}_F^{\circledcirc})$ in W, we consider the following sets.

$$\begin{aligned}
\mathcal{W}(\mathcal{M}_T^{\circledcirc}; \alpha) &:= \{x_{11} \in W \mid \mathcal{M}_T^{\circledcirc}(x_{11}) \geq \alpha\}, \\
\mathcal{W}(\mathcal{M}_I^{\circledcirc}; \beta) &:= \{x_{11} \in W \mid \mathcal{M}_I^{\circledcirc}(x_{11}) \geq \beta\}, \\
\mathcal{W}(\mathcal{M}_F^{\circledcirc}; \gamma) &:= \{x_{11} \in W \mid \mathcal{M}_F^{\circledcirc}(x_{11}) \leq \gamma\},
\end{aligned}$$

which are said to be *neutrosophic level subsets* of W where $\alpha, \beta, \gamma \in [0,1]$.

The interval-valued f-set is an extension of f-set theory and is a mathematical tool that serves to more subtly address uncertainty.

By an *interval number*, a closed subinterval $\tilde{c} = [c^-, c^+]$ of I, where $0 \leq c^- \leq c^+ \leq 1$, is meant and by $[I]$, the set of all interval numbers is denoted. We define what is known as a *refined minimum* (briefly, rmin) and a *refined maximum* (briefly, rmax) of two elements in $[I]$, and the symbols "$\succeq$", "$\preceq$", "$=$" in case of two elements in $[I]$. Take two interval numbers $\tilde{c}_1 := \left[c_1^-, c_1^+\right]$ and $\tilde{c}_2 := \left[c_2^-, c_2^+\right]$. Then

$$\begin{aligned}
&\operatorname{rmin}\{\tilde{c}_1, \tilde{c}_2\} = \left[\min\left\{c_1^-, c_2^-\right\}, \min\left\{c_1^+, c_2^+\right\}\right], \\
&\operatorname{rmax}\{\tilde{c}_1, \tilde{c}_2\} = \left[\max\left\{c_1^-, c_2^-\right\}, \max\left\{c_1^+, c_2^+\right\}\right], \\
&\tilde{c}_1 \succeq \tilde{c}_2 \;\Leftrightarrow\; c_1^- \geq c_2^-,\; c_1^+ \geq c_2^+.
\end{aligned}$$

Analogously one has $\tilde{c}_1 \preceq \tilde{c}_2$ and $\tilde{c}_1 = \tilde{c}_2$. By $\tilde{c}_1 \succ \tilde{c}_2$ (resp. $\tilde{c}_1 \prec \tilde{c}_2$), $\tilde{c}_1 \succeq \tilde{c}_2$ and $\tilde{c}_1 \neq \tilde{c}_2$ (resp. $\tilde{c}_1 \preceq \tilde{c}_2$ and $\tilde{c}_1 \neq \tilde{c}_2$) are meant. For $\tilde{c}_i \in [I]$, where $i \in \Lambda$, define:

$$\operatorname*{rinf}_{i\in\Lambda} \tilde{c}_i = \left[\inf_{i\in\Lambda} c_i^-, \inf_{i\in\Lambda} c_i^+\right] \quad \text{and} \quad \operatorname*{rsup}_{i\in\Lambda} \tilde{c}_i = \left[\sup_{i\in\Lambda} c_i^-, \sup_{i\in\Lambda} c_i^+\right].$$

Given a nonempty set X, a mapping $A : X \to [I]$ is said to be an *interval-valued f-set* (briefly, an *IVF set*) in X. By $[I]^X$, we denote the set of all IVF sets in X. For all $A \in [I]^X$ and $a_{12} \in X$, $A(a_{12}) = [A^-(a_{12}), A^+(a_{12})]$ is said to be the *degree* of membership of an element x to A, where $A^- : X \to I$ and $A^+ : X \to I$ are f-sets in X which are said to be a *lower f-set* and an *upper f-et* in X, respectively. For brevity, we denote $A(a_{12}) = [A^-(a_{12}), A^+(a_{12})]$ by $A = [A^-, A^+]$.

Definition 2.6 ([11])**.** For a non-empty set W, an *MBJ-neutrosophic set* (briefly, MBJn-set) in W is defined to be a structure of the form:

$$\mathcal{G} := \{\langle a_{11}; M_G(a_{11}), \tilde{B}_G(a_{11}), J_G(a_{11})\rangle \mid a_{11} \in W\}$$

where M_G and J_G are f-sets in W, which are said to be a true membership function and a false membership function, respectively, and $\tilde{B}_G$ is an IVF set in W which is said to be an indeterminate interval-valued membership function.

For brevity, we use the symbol $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ for the MBJn-set

$$\mathcal{G} := \{\langle a_{11}; M_G(a_{11}), \tilde{B}_G(a_{11}), J_G(a_{11})\rangle \mid a_{11} \in W\}.$$

In an MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ in W, if we take

$$\tilde{B}_G : W \to [I],\ a_{12} \mapsto [B_G^-(a_{12}), B_G^+(a_{12})]$$

with $B_G^-(a_{12}) = B_G^+(a_{12})$, then $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an n-set in W.

Given an MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ in a set W, we consider the following sets.

$$\begin{aligned} &U(M_G; s) := \{x_{11} \in W \mid M_G(x_{11}) \geq s\},\\ &U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2]) := \{x_{11} \in W \mid \tilde{B}_G(x_{11}) \succeq [\varepsilon_1, \varepsilon_2]\},\\ &L(J_G; t) := \{x_{11} \in W \mid J_G(x_{11}) \leq t\} \end{aligned}$$

where $s, t \in [0, 1]$ and $[\varepsilon_1, \varepsilon_2] \in [I]$.

3 MBJ-neutrosophic O-subalgebras

Unless specified otherwise, in what follows, $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ denotes an OBCI-algebra.

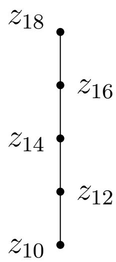
Definition 3.1. An MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ is said to be an *MBJ-neutrosophic subalgebra* (briefly, MBJn-subalgeba) of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if it satisfies:

$$(\forall x_{11}, x_{12} \in W) \left(\begin{array}{l} M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\}, \\ \tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}, \\ J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), J_G(x_{12})\}. \end{array} \right) \tag{3.1}$$

Definition 3.2. An MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ is said to be an *MBJ-neutrosophic O-subalgebra* (briefly, MBJnO-subalgebra) of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if it satisfies:

$$(\forall x_{11}, x_{12} \in W) \left(\tilde{e} \leq_W x_{11},\ \tilde{e} \leq_W x_{12}\ \Rightarrow \left\{ \begin{array}{l} M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\}, \\ \tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}, \\ J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), J_G(x_{12})\}. \end{array} \right. \right). \tag{3.2}$$

Example 3.3. Let $W = \{z_{10}, z_{12}, z_{14}, z_{16}, z_{18}\}$ be a set with the Hasse diagram and Table as follows:



Hassee diagram of $(W, \leq_W)$

$\rightsquigarrow$	z_{10}	z_{12}	z_{14}	z_{16}	z_{18}
z_{10}	z_{18}	z_{18}	z_{18}	z_{18}	z_{18}
z_{12}	z_{10}	z_{16}	z_{16}	z_{16}	z_{18}
z_{14}	z_{10}	z_{12}	z_{14}	z_{16}	z_{18}
z_{16}	z_{10}	z_{12}	z_{14}	z_{16}	z_{18}
z_{18}	z_{10}	z_{10}	z_{10}	z_{10}	z_{18}

Table for " $\rightsquigarrow$ "

Then $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, where $\tilde{e} = z_{12}$, is an OBCI-algebra (see [18]).

(i) Let $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ be an MBJn-set in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ provided by Table 1.

Table 1: Table for $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$

W	$M_G(w)$	$\tilde{B}_G(w)$	$J_G(w)$
z_{18}	0.93	$[0.56, 0.89]$	0.25
z_{16}	0.67	$[0.47, 0.56]$	0.42
z_{14}	0.54	$[0.38, 0.47]$	0.58
z_{12}	0.44	$[0.29, 0.42]$	0.69
z_{10}	0.87	$[0.51, 0.82]$	0.36

It can be eaily verified that $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJn-subalgeba of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$.

(ii) Let $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ be an MBJn-set in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ given by Table 2.

Table 2: Table for $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$

W	$M_G(w)$	$\tilde{B}_G(w)$	$J_G(w)$
z_{18}	0.37	$[0.53, 0.85]$	0.81
z_{16}	0.94	$[0.65, 0.94]$	0.28
z_{14}	0.82	$[0.49, 0.76]$	0.46
z_{12}	0.66	$[0.37, 0.67]$	0.53
z_{10}	0.54	$[0.53, 0.85]$	0.62

It can be eaily verified that $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$.

It is certain that every MBJn-subalgeba is an MBJnO-subalgebra, but the converse may not be true as shown in the following example. In light of this view, it can be said that the MBJnO-subalgebra is a generalization of the MBJ-neutrosophic subalgebra.

Example 3.4. (i) The MBJnO-subalgebra $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ in Example 3.3(ii) is not an MBJn-subalgeba of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ since

$$M_G(z_{10} \rightsquigarrow z_{10}) = M_G(z_{11}) = 0.37 \ngeq 0.54 = \min\{M_G(z_{10}), M_G(z_{10})\}$$

and/or $J_G(z_{10} \rightsquigarrow z_{10}) = J_G(z_{11}) = 0.81 \nleq 0.62 = \max\{J_G(z_{10}), J_G(z_{10})\}$.

(ii) Consider the OBCI-algebra $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ in Example 3.3, and let $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ be an MBJn-set in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ given by Table 3.

Table 3: Table for $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$

W	$M_G(w)$	$\tilde{B}_G(w)$	$J_G(w)$
z_{10}	0.67	$[0.57, 0.88]$	0.28
z_{18}	0.35	$[0.36, 0.77]$	0.61
z_{16}	0.67	$[0.57, 0.88]$	0.28
z_{14}	0.35	$[0.36, 0.77]$	0.61
z_{12}	0.35	$[0.36, 0.77]$	0.61

It can be easily checked that $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJn-subalgeba of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. But it is not an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ since

$$\tilde{B}_G(z_{10} \rightsquigarrow z_{12}) = \tilde{B}_G(z_{11}) = [0.36, 0.77] \nsucceq [0.57, 0.88] = \text{rmin}\{\tilde{B}_G(z_{10}), \tilde{B}_G(z_{12})\}$$

and/or $J_G(z_{10} \rightsquigarrow z_{12}) = J_G(z_{11}) = 0.61 \nleq 0.28 = \max\{J_G(z_{10}), J_G(z_{12})\}$.

Using (intuitionistic) fuzzy O-subalgebras, we establish an MBJnO-subalgebra.

Theorem 3.5. *Given an MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, if (M_G, J_G) is an IFO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, and $\tilde{B}_G^-$ and $\tilde{B}_G^+$ are FO-subalgebras of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, then $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$.*

Proof. Let $x_{11}, x_{12} \in W$ be such that $\breve{e} \leq_W x_{11}$ and $\breve{e} \leq_W x_{12}$. Then

$$\begin{aligned}\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) &= [B_G^-(x_{11} \rightsquigarrow x_{12}), B_G^+(x_{11} \rightsquigarrow x_{12})]\\ &\succeq [\min\{B_G^-(x_{11}), B_G^-(x_{12})\}, \min\{B_G^+(x_{11}), B_G^+(x_{12})\}]\\ &= \text{rmin}\{[B_G^-(x_{11}), B_G^+(x_{11})], [B_G^-(x_{12}), B_G^+(x_{12})]\\ &= \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}.\end{aligned}$$

Since (M_G, J_G) is an IFO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, it is clear that

$$M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\} \text{ and } J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), M_G(x_{12})\}.$$

Therefore $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJn-subalgeba of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. □

If $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$, then

$$\begin{aligned}[B_G^-(x_{11} \rightsquigarrow x_{12}), B_G^+(x_{11} \rightsquigarrow x_{12})] &= \tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\} \\ &= \text{rmin}\{[B_G^-(x_{11}), B_G^+(x_{11}), [B_G^-(x_{12}), B_G^+(x_{12})]\} \\ &= [\min\{B_G^-(x_{11}), B_G^-(x_{12})\}, \min\{B_G^+(x_{11}), B_G^+(x_{12})\}]\end{aligned}$$

for all $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. It follows that

$$B_G^-(x_{11} \rightsquigarrow x_{12}) \geq \min\{B_G^-(x_{11}), B_G^-(x_{12})\} \text{ and } B_G^+(x_{11} \rightsquigarrow x_{12}) \geq \min\{B_G^+(x_{11}), B_G^+(x_{12})\}.$$

Thus B_G^- and B_G^+ are FO-subalgebras of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$. But (M_G, J_G) is not an IFO-subalgebra of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$ as one can see in Example 3.3(ii). It verifies that the converse of Theorem 3.5 is not true.

We can observe that if $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$ that satisfies $M_G(x_{11}) + J_G(x_{11}) \leq 1$ for all $x_{11} \in W$, then (M_G, J_G) is an IFO-subalgebra of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$.

Theorem 3.6. *An MBJn-set $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ in $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$ is an MBJnO-subalgebra of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$ if and only if the non-empty sets $U(M_G; s)$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L(J_G; t)$ are O-subalgebras of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$ for all $s, t \in [0, 1]$ and $[\varepsilon_1, \varepsilon_2] \in [I]$.*

The O-subalgebras $U(M_G; s)$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L(J_G; t)$ are said to be *MBJ-ordered subalgebras* of $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$.

Proof. Suppose that $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ is an MBJnO-subalgebra of W. Let $s, t \in [0, 1]$ and $[\varepsilon_1, \varepsilon_2] \in [I]$ be such that $U(M_G; s)$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L(J_G; t)$ are non-empty. Let $x_{11}, x_{12} \in W$ be such that $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. If $x_{11}, x_{12} \in U(M_G; s) \cap U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2]) \cap L(J_G; t)$, then

$$\begin{aligned}&M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\} \geq \min\{s, s\} = s, \\ &\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\} \succeq \text{rmin}\{[\varepsilon_1, \varepsilon_2], [\varepsilon_1, \varepsilon_2]\} = [\varepsilon_1, \varepsilon_2], \\ &J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), J_G(x_{12})\} \leq \min\{t, t\} = t,\end{aligned}$$

and so $x_{11} \rightsquigarrow x_{12} \in U(M_G; s) \cap U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2]) \cap L(J_G; t)$. Therefore $U(M_G; s)$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L(J_G; t)$ are O-subalgebras of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$.

Conversely, suppose that the non-empty sets $U(M_G; s)$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L(J_G; t)$ are O-subalgebras of $\mathbf{W} := (W,\, \rightsquigarrow,\, \tilde{e},\, \leq_W)$ for all $s, t \in [0, 1]$ and $[\varepsilon_1, \varepsilon_2] \in [I]$. If $M_G(a_{11} \rightsquigarrow a_{12}) < \min\{M_G(a_{11}), M_G(a_{12})\}$ for some $a_{11}, a_{12} \in W$ with $\check{e} \leq_W a_{11}$ and $\check{e} \leq_W a_{12}$, then $a_{11}, a_{12} \in U(M_G; s_0)$ but $a_{11} \rightsquigarrow a_{12} \notin U(M_G; s_0)$ for $s_0 := \min\{M_G(a_{11}), M_G(a_{12})\}$. This is a contradiction, and thus $M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\}$ for all $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. Similarly, we can show that $J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), J_G(x_{12})\}$ for all $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. Suppose that $\tilde{B}_G(a_{11} \rightsquigarrow a_{12}) \prec \text{rmin}\{\tilde{B}_G(a_{11}), \tilde{B}_G(a_{12})\}$ for some $a_{11}, a_{12} \in W$ with $\check{e} \leq_W a_{11}$ and $\check{e} \leq_W a_{12}$. Let $\tilde{B}_G(a_{11}) = [\varrho_1, \varrho_2]$, $\tilde{B}_G(a_{12}) = [\varrho_3, \varrho_4]$ and $\tilde{B}_G(a_{11} \rightsquigarrow a_{12}) = [\varepsilon_1, \varepsilon_2]$. Then

$$[\varepsilon_1, \varepsilon_2] \prec \text{rmin}\{[\varrho_1, \varrho_2], [\varrho_3, \varrho_4]\} = [\min\{\varrho_1, \varrho_3\}, \min\{\varrho_2, \varrho_4\}],$$

and so $\varepsilon_1 < \min\{\varrho_1, \varrho_3\}$ and $\varepsilon_2 < \min\{\varrho_2, \varrho_4\}$. Taking

$$[\lambda_1, \lambda_2] := \tfrac{1}{2}\left(\tilde{B}_G(a_{11} \rightsquigarrow a_{12}) + \mathrm{rmin}\{\tilde{B}_G(a_{11}), \tilde{B}_G(a_{12})\}\right)$$

implies that

$$\begin{aligned}[\lambda_1, \lambda_2] &= \tfrac{1}{2}\left([\varepsilon_1, \varepsilon_2] + [\min\{\varrho_1, \varrho_3\}, \min\{\varrho_2, \varrho_4\}]\right) \\ &= \left[\tfrac{1}{2}(\varepsilon_1 + \min\{\varrho_1, \varrho_3\}, \tfrac{1}{2}(\varepsilon_2 + \min\{\varrho_2, \varrho_4\}\right].\end{aligned}$$

It follows that $\min\{\varrho_1, \varrho_3\} > \lambda_1 = \frac{1}{2}(\varepsilon_1 + \min\{\varrho_1, \varrho_3\} > \varepsilon_1$ and

$$\min\{\varrho_2, \varrho_4\} > \lambda_2 = \tfrac{1}{2}(\varepsilon_2 + \min\{\varrho_2, \varrho_4\} > \varepsilon_2.$$

Hence $[\min\{\varrho_1, \varrho_3\}, \min\{\varrho_2, \varrho_4\}] \succ [\lambda_1, \lambda_2] \succ [\varepsilon_1, \varepsilon_2] = \tilde{B}_G(a_{11} \rightsquigarrow a_{12})$, and therefore $a_{11} \rightsquigarrow a_{12} \notin U(\tilde{B}_G; [\lambda_1, \lambda_2])$. On the other hand, $\tilde{B}_G(a_{11}) = [\varrho_1, \varrho_2] \succeq [\min\{\varrho_1, \varrho_3\}, \min\{\varrho_2, \varrho_4\}] \succ [\lambda_1, \lambda_2]$ and

$$\tilde{B}_G(a_{12}) = [\varrho_3, \varrho_4] \succeq [\min\{\varrho_1, \varrho_3\}, \min\{\varrho_2, \varrho_4\}] \succ [\lambda_1, \lambda_2],$$

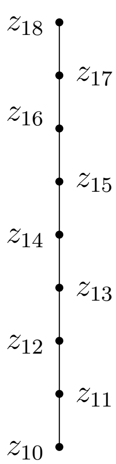
that is, $a_{11}, a_{12} \in U(\tilde{B}_G; [\lambda_1, \lambda_2])$, a contradiction. Therefore

$$\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \mathrm{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}$$

for all $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. Consequently $\mathcal{G} = (M_G,\ \tilde{B}_G,\ J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. □

The example below illustrates Theorem 3.6.

Example 3.7. Let $W = \{z_{10}, z_{11}, z_{12}, z_{13}, z_{14}, z_{15}, z_{16}, z_{17}, z_{18}\}$ be a set with the Hasse diagram and Table as follows:



Hassee diagram of $(W, \leq_W)$

$\rightsquigarrow$	z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}	z_{16}	z_{17}	z_{18}
z_{10}	z_{10}	z_{18}	z_{18}	z_{18}	z_{18}	z_{18}	z_{18}	z_{18}	z_{18}
z_{11}	z_{10}	z_{17}	z_{17}	z_{17}	z_{17}	z_{17}	z_{17}	z_{17}	z_{18}
z_{12}	z_{10}	z_{11}	z_{16}	z_{16}	z_{16}	z_{16}	z_{16}	z_{17}	z_{18}
z_{13}	z_{10}	z_{11}	z_{12}	z_{15}	z_{15}	z_{15}	z_{16}	z_{17}	z_{18}
z_{14}	z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}	z_{16}	z_{17}	z_{18}
z_{15}	z_{10}	z_{11}	z_{12}	z_{13}	z_{13}	z_{15}	z_{16}	z_{17}	z_{18}
z_{16}	z_{10}	z_{11}	z_{12}	z_{12}	z_{12}	z_{12}	z_{16}	z_{17}	z_{18}
z_{17}	z_{10}	z_{11}	z_{11}	z_{11}	z_{11}	z_{11}	z_{11}	z_{17}	z_{18}
z_{18}	z_{10}	z_{10}	z_{10}	z_{10}	z_{10}	z_{10}	z_{10}	z_{10}	z_{18}

Table for " $\rightsquigarrow$ "

Then $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, where $\tilde{e} = z_{14}$, is an OBCI-algebra. Let $\mathcal{G} = (M_G,\ \tilde{B}_G,\ J_G)$ be an MBJn-set in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ given by Table 4.

Table 4: Table for $\mathcal{G} = (M_G,\ \tilde{B}_G,\ J_G)$

W	$M_G(w)$	$\tilde{B}_G(w)$	$J_G(w)$
z_{10}	0.36	$[0.58, 0.79]$	0.61
z_{11}	0.49	$[0.52, 0.73]$	0.67
z_{12}	0.53	$[0.47, 0.66]$	0.46
z_{13}	0.75	$[0.39, 0.58]$	0.27
z_{14}	0.87	$[0.63, 0.85]$	0.27
z_{15}	0.75	$[0.39, 0.58]$	0.27
z_{16}	0.53	$[0.47, 0.66]$	0.46
z_{17}	0.49	$[0.52, 0.73]$	0.67
z_{18}	0.36	$[0.58, 0.79]$	0.61

Then $\mathcal{G} = (M_G,\ \tilde{B}_G,\ J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W,\ \rightsquigarrow,\ \tilde{e},\ \leq_W)$. The sets $U(M_G; s)$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L(J_G; t)$ are given as follows:

$$U(M_G; s) = \begin{cases} \emptyset & \text{if } \ 0.87 < s \leq 1, \\ \{z_{14}\} & \text{if } \ 0.75 < s \leq 0.87, \\ \{z_{14}, z_{15}, z_{13}\} & \text{if } \ 0.53 < s \leq 0.75, \\ \{z_{14}, z_{15}, z_{13}, z_{16}, z_{12}\} & \text{if } \ 0.49 < s \leq 0.53, \\ \{z_{14}, z_{15}, z_{13}, z_{16}, z_{12}, z_{17}, z_{11}\} & \text{if } \ 0.36 < s \leq 0.49, \\ W & \text{if } \ 0.00 \leq s \leq 0.36, \end{cases}$$

$$U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2]) = \begin{cases} \emptyset & \text{if } \ [0.63, 0.85] \prec [\varepsilon_1, \varepsilon_2] \preceq [1, 1], \\ \{z_{14}\} & \text{if } \ [0.58, 0.79] \prec [\varepsilon_1, \varepsilon_2] \preceq [0.63, 0.85], \\ \{z_{14}, z_{18}, z_{10}\} & \text{if } \ [0.52, 0.73] \prec [\varepsilon_1, \varepsilon_2] \preceq [0.58, 0.79], \\ \{z_{14}, z_{18}, z_{10}, z_{17}, z_{11}\} & \text{if } \ [0.47, 0.66] \prec [\varepsilon_1, \varepsilon_2] \preceq [0.52, 0.73], \\ \{z_{14}, z_{18}, z_{10}, z_{17}, z_{11}, z_{16}, z_{12}\} & \text{if } \ [0.39, 0.58] \prec [\varepsilon_1, \varepsilon_2] \preceq [0.47, 0.66], \\ W & \text{if } \ [0, 0] \prec [\varepsilon_1, \varepsilon_2] \preceq [0.39, 0.58], \end{cases}$$

and

$$L(J_G; t) = \begin{cases} W & \text{if } \ 0.67 \leq t \leq 1, \\ \{z_{14}, z_{15}, z_{13}, z_{16}, z_{12}, z_{18}, z_{10}\} & \text{if } \ 0.61 \leq t < 0.67, \\ \{z_{14}, z_{15}, z_{13}, z_{16}, z_{12}\} & \text{if } \ 0.46 \leq t < 0.61, \\ \{z_{14}, z_{15}, z_{13}\} & \text{if } \ 0.27 \leq t < 0.46, \\ \emptyset & \text{if } \ 0.00 \leq t < 0.27. \end{cases}$$

It is routine to verify that $U(M_G; s)$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L(J_G; t)$ are O-subalgebras of $\mathbf{W} := (W,\ \rightsquigarrow,\ \tilde{e},$

$\leq_W$) for all $s, t \in [0,1]$ and $[\varepsilon_1, \varepsilon_2] \in [I]$ whenever they are nonempty.

Given a non-empty subset A of W, consider an MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ defined by

$$M_G(x_{11}) = \begin{cases} s & \text{if } x_{11} \in A, \\ 0 & \text{otherwise,} \end{cases} \quad \tilde{B}_G(x_{11}) = \begin{cases} [\varepsilon_1, \varepsilon_2] & \text{if } x_{11} \in A, \\ [0,0] & \text{otherwise,} \end{cases} \quad J_G(x_{11}) = \begin{cases} t & \text{if } x_{11} \in A, \\ 1 & \text{otherwise,} \end{cases} \tag{3.3}$$

where $(s,t) \in (0,1] \times [0,1)$ and $\varepsilon_1, \varepsilon_2 \in (0,1]$ with $\varepsilon_1 < \varepsilon_2$.

Theorem 3.8. *Given a non-empty subset A of W, the MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ given in (3.3) is an MBJnO-subalgebra of $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if and only if A is an O-subalgebra of $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. Moreover $U(M_G; s) = A$, $U(\tilde{B}_G; [\varepsilon_1, \varepsilon_2]) = A$ and $L(J_G; t) = A$.*

Proof. Assume that $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. Let $x_{11}, x_{12} \in W$ be such that $\breve{e} \leq_W x_{11}$, $\breve{e} \leq_W x_{12}$ and $x_{11}, x_{12} \in A$. Then $M_G(x_{11}) = s = M_G(x_{12})$, $\tilde{B}_G(x_{11}) = [\varepsilon_1, \varepsilon_2] = \tilde{B}_G(x_{12})$, and $J_G(x_{11}) = t = J_G(x_{12})$. Hence $M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\} = s$, $\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\} = [\varepsilon_1, \varepsilon_2]$, $J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), J_G(x_{12})\} = t$, which imply that $M_G(x_{11} \rightsquigarrow x_{12}) = s$, $\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) = [\varepsilon_1, \varepsilon_2]$ and $J_G(x_{11} \rightsquigarrow x_{12}) = t$. Hence $x_{11} \rightsquigarrow x_{12} \in A$, and therefore A is an O-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$.

Suppose conversely that A is an O-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. Let $x_{11}, x_{12} \in W$ be such that $\breve{e} \leq_W x_{11}$ and $\breve{e} \leq_W x_{12}$. If $x_{11}, x_{12} \in A$, then $x_{11} \rightsquigarrow x_{12} \in A$ and thus

$$\begin{aligned} &M_G(x_{11} \rightsquigarrow x_{12}) = s = \min\{M_G(x_{11}), M_G(x_{12})\}, \\ &\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) = [\varepsilon_1, \varepsilon_2] = \text{rmin}\{[\varepsilon_1, \varepsilon_2], [\varepsilon_1, \varepsilon_2]\} = \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}, \\ &J_G(x_{11} \rightsquigarrow x_{12}) = t = \max\{J_G(x_{11}), J_G(x_{12})\}. \end{aligned}$$

If $x_{11}, x_{12} \notin A$, then $M_G(x_{11}) = 0 = M_G(x_{12})$, $\tilde{B}_G(x_{11}) = [0,0] = \tilde{B}_G(x_{12})$ and $J_G(x_{11}) = 1 = J_G(x_{12})$. Hence

$$\begin{aligned} &M_G(x_{11} \rightsquigarrow x_{12}) \geq 0 = \min\{0,0\} = \min\{M_G(x_{11}), M_G(x_{12})\}, \\ &\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq [0,0] = \text{rmin}\{[0,0],[0,0]\} = \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}, \\ &J_G(x_{11} \rightsquigarrow x_{12}) \leq 1 = \max\{1,1\} = \max\{J_G(x_{11}), J_G(x_{12})\}. \end{aligned}$$

If $x_{11} \in A$ and $x_{12} \notin A$, then $M_G(x_{11}) = s$, $\tilde{B}_G(x_{11}) = [\varepsilon_1, \varepsilon_2]$, $J_G(x_{11}) = t$, $M_G(x_{12}) = 0$, $\tilde{B}_G(x_{12}) = [0,0]$, and $J_G(x_{12}) = 1$. Thus

$$\begin{aligned} &M_G(x_{11} \rightsquigarrow x_{12}) \geq 0 = \min\{s, 0\} = \min\{M_G(x_{11}), M_G(x_{12})\}, \\ &\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq [0,0] = \text{rmin}\{[\varepsilon_1, \varepsilon_2], [0,0]\} = \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}, \\ &J_G(x_{11} \rightsquigarrow x_{12}) \leq 1 = \max\{t, 1\} = \max\{J_G(x_{11}), J_G(x_{12})\}. \end{aligned}$$

Similarly, if $x_{11} \notin A$ and $x_{12} \in A$, then $M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\}$, $\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}$, and $J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), J_G(x_{12})\}$. Therefore $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. □

Let $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ be an MBJn-set in $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. We denote

$$\begin{aligned}
&\bot := \inf\{J_G(a_{11}) \mid a_{11} \in W\},\\
&\top := 1 - \sup\{M_G(a_{11}) \mid a_{11} \in W\},\\
&\Omega := [1,1] - \text{rsup}\,\{\tilde{B}_G(a_{11}) \mid a_{11} \in W\}.
\end{aligned}$$

For any $q \in [0,\top]$, $\tilde{a_{13}} \in [[0,0],\Omega]$ and $p \in [0,\bot]$, we define $\mathcal{G}^T = (M_G^q,\, \tilde{B}_G^{\tilde{a_{13}}},\, J_G^p)$ by $M_G^q(a_{11}) = M_G(a_{11}) + q$, $\tilde{B}_G^{\tilde{a_{13}}}(a_{11}) = \tilde{B}_G(a_{11}) + \tilde{a_{13}}$ and $J_G^p(a_{11}) = J_G(a_{11}) - p$. Then $\mathcal{G}^T = (M_G^q,\, \tilde{B}_G^{\tilde{a_{13}}},\, J_G^p)$ is an MBJn-set in W, and it is said to be a $(q,\tilde{a_{13}},p)$-*translative MBJn-set* of $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$.

Theorem 3.9. *An MBJn-set $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ in $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ is an MBJnO-subalgebra of $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if and only if its $(q,\tilde{a_{13}},p)$-translative MBJn-set $\mathcal{G}^T = (M_G^q,\, \tilde{B}_G^{\tilde{a_{13}}},\, J_G^p)$ is an MBJnO-subalgebra of $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ for $q \in [0,\top]$, $\tilde{a_{13}} \in [[0,0],\Omega]$ and $p \in [0,\bot]$.*

Proof. Assume that $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, and let $q \in [0,\top]$, $\tilde{a_{13}} \in [[0,0],\Omega]$ and $p \in [0,\bot]$. For every $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$, we obtain

$$\begin{aligned}
M_G^q(x_{11} \rightsquigarrow x_{12}) &= M_G(x_{11} \rightsquigarrow x_{12}) + q \geq \min\{M_G(x_{11}), M_G(x_{12})\} + q\\
&= \min\{M_G(x_{11}) + q, M_G(x_{12}) + q\} = \min\{M_G^q(x_{11}), M_G^q(x_{12})\},
\end{aligned}$$

$$\begin{aligned}
\tilde{B}_G^{\tilde{a_{13}}}(x_{11} \rightsquigarrow x_{12}) &= \tilde{B}_G(x_{11} \rightsquigarrow x_{12}) + \tilde{a_{13}} \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\} + \tilde{a_{13}}\\
&= \text{rmin}\{\tilde{B}_G(x_{11}) + \tilde{a_{13}}, \tilde{B}_G(x_{12}) + \tilde{a_{13}}\} = \text{rmin}\{\tilde{B}_G^{\tilde{a_{13}}}(x_{11}), \tilde{B}_G^{\tilde{a_{13}}}(x_{12})\},
\end{aligned}$$

and

$$\begin{aligned}
J_G^p(x_{11} \rightsquigarrow x_{12}) &= J_G(x_{11} \rightsquigarrow x_{12}) - p \leq \max\{J_G(x_{11}), J_G(x_{12})\} - p\\
&= \max\{J_G(x_{11}) - p, J_G(x_{12}) - p\} = \max\{J_G^p(x_{11}), J_G^p(x_{12})\}.
\end{aligned}$$

Hence $\mathcal{G}^T = (M_G^q,\, \tilde{B}_G^{\tilde{a_{13}}},\, J_G^p)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$.

Suppose conversely that the $(q,\tilde{a_{13}},p)$-translative MBJn-set $\mathcal{G}^T = (M_G^q,\, \tilde{B}_G^{\tilde{a_{13}}},\, J_G^p)$ of $\mathcal{G} = (M_G,\, \tilde{B}_G,\, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ for all $q \in [0,\top]$, $\tilde{a_{13}} \in [[0,0],\Omega]$ and $p \in [0,\bot]$. Let $x_{11}, x_{12} \in W$ be such that $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. Then

$$\begin{aligned}
M_G(x_{11} \rightsquigarrow x_{12}) + q = M_G^q(x_{11} \rightsquigarrow x_{12}) &\geq \min\{M_G^q(x_{11}), M_G^q(x_{12})\}\\
&= \min\{M_G(x_{11}) + q, M_G(x_{12}) + q\}\\
&= \min\{M_G(x_{11}), M_G(x_{12})\} + q,
\end{aligned}$$

$$\begin{aligned}
\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) + \tilde{a_{13}} = \tilde{B}_G^{\tilde{a_{13}}}(x_{11} \rightsquigarrow x_{12}) &\succeq \text{rmin}\{\tilde{B}_G^{\tilde{a_{13}}}(x_{11}), \tilde{B}_G^{\tilde{a_{13}}}(x_{12})\}\\
&= \text{rmin}\{\tilde{B}_G(x_{11}) + \tilde{a_{13}}, \tilde{B}_G(x_{12}) + \tilde{a_{13}}\}\\
&= \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\} + \tilde{a_{13}},
\end{aligned}$$

and

$$\begin{aligned} J_G(x_{11} \rightsquigarrow x_{12}) - p &= J_G^p(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G^p(x_{11}), J_G^p(x_{12})\} \\ &= \max\{J_G(x_{11}) - p, J_G(x_{12}) - p\} \\ &= \max\{J_G(x_{11}), J_G(x_{12})\} - p. \end{aligned}$$

It follows that $M_G(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G(x_{11}), M_G(x_{12})\}$, $\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G(x_{11}), \tilde{B}_G(x_{12})\}$ and $J_G(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G(x_{11}), J_G(x_{12})\}$. Therefore $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. □

Theorem 3.10. *Given an MBJn-set $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ in $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$, consider the following sets:*

$$\begin{aligned} &U_q(M_G; s) := \{x_{11} \in W \mid M_G(x_{11}) \geq s - q\}, \\ &U_{\tilde{a_{13}}}(\tilde{B}_G; [\varepsilon_1, \varepsilon_2]) := \{x_{11} \in W \mid \tilde{B}_G(x_{11}) \succeq [\varepsilon_1, \varepsilon_2] - \tilde{a_{13}}\}, \\ &L_p(J_G; t) := \{x_{11} \in W \mid J_G(x_{11}) \leq t + p\} \end{aligned}$$

where $s, t \in [0, 1]$, $[\varepsilon_1, \varepsilon_2] \in [I]$, $q \in [0, \top]$, $\tilde{a_{13}} \in [[0, 0], \Omega]$ and $p \in [0, \bot]$ such that $p \leq t$, $[\varepsilon_1, \varepsilon_2] \succeq \tilde{a_{13}}$ and $q \geq s$. Then the $(q, \tilde{a_{13}}, p)$-translative MBJn-set of $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ is an MBJnO-subalgebra of $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ if and only if $U_q(M_G; s)$, $U_{\tilde{a_{13}}}(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L_p(J_G; t)$ are O-subalgebras of $\boldsymbol{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ for all $s \in Im(M_G)$, $[\varepsilon_1, \varepsilon_2] \in Im(\tilde{B}_G)$ and $t \in Im(J_G)$ satisfying $s \geq q$, $[\varepsilon_1, \varepsilon_2] \succeq \tilde{a_{13}}$ and $t \leq p$.

Proof. Let the $(q, \tilde{a_{13}}, p)$-translative MBJn-set of $\mathcal{G} = (M_G, \tilde{B}_G, J_G)$ be an MBJ-neutro-sophic O-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. Let's have $s \in Im(M_G)$, $[\varepsilon_1, \varepsilon_2] \in Im(\tilde{B}_G)$ and $t \in Im(J_G)$ that satisfy $s \geq q$, $[\varepsilon_1, \varepsilon_2] \succeq \tilde{a_{13}}$ and $t \leq p$, respectively. Let $x_{11}, x_{12} \in W$ be such that $\breve{e} \leq_W x_{11}$ and $\breve{e} \leq_W x_{12}$. If $x_{11}, x_{12} \in U_q(M_G; s)$, then $M_G(x_{11}) \geq s - q$ and $M_G(x_{12}) \geq s - q$, which imply that $M_G^q(x_{11}) \geq s$ and $M_G^q(x_{12}) \geq s$. It follows that

$$M_G^q(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G^q(x_{11}), M_G^q(x_{12})\} \geq s.$$

Hence $M_G(x_{11} \rightsquigarrow x_{12}) \geq s - q$, and so $x_{11} \rightsquigarrow x_{12} \in U_q(M_G; s)$. If $x_{11}, x_{12} \in U_{\tilde{a_{13}}}(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$, then $\tilde{B}_G(x_{11}) \succeq [\varepsilon_1, \varepsilon_2] - \tilde{a_{13}}$ and $\tilde{B}_G(x_{12}) \succeq [\varepsilon_1, \varepsilon_2] - \tilde{a_{13}}$. Hence

$$\tilde{B}_G^{\tilde{a_{13}}}(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G^{\tilde{a_{13}}}(x_{11}), \tilde{B}_G^{\tilde{a_{13}}}(x_{12})\} \succeq [\varepsilon_1, \varepsilon_2],$$

and so $\tilde{B}_G(x_{11} \rightsquigarrow x_{12}) \succeq [\varepsilon_1, \varepsilon_2] - \tilde{a_{13}}$. Thus $x_{11} \rightsquigarrow x_{12} \in U_{\tilde{a_{13}}}(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$. If $x_{11}, x_{12} \in L_p(J_G; t)$, then $J_G(x_{11}) \leq t + p$ and $J_G(x_{12}) \leq t + p$. It follows that

$$J_G^p(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G^p(x_{11}), J_G^p(x_{12})\} \leq t,$$

that is, $J_G(x_{11} \rightsquigarrow x_{12}) \leq t + p$. Thus $x_{11} \rightsquigarrow x_{12} \in L_p(J_G; t)$. Therefore $U_q(M_G; s)$, $U_{\tilde{a_{13}}}(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L_p(J_G; t)$ are O-subalgebras of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$.

Conversely, suppose that $U_q(M_G; s)$, $U_{\tilde{a_{13}}}(\tilde{B}_G; [\varepsilon_1, \varepsilon_2])$ and $L_p(J_G; t)$ are ordered subalgebras of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$ for all $s \in Im(M_G)$, $[\varepsilon_1, \varepsilon_2] \in Im(\tilde{B}_G)$ and $t \in Im(J_G)$ with $s \geq q$, $[\varepsilon_1, \varepsilon_2] \succeq \tilde{a_{13}}$ and $t \leq p$. Assume that $M_G^q(a_{11} \rightsquigarrow a_{12}) < \min\{M_G^q(a_{11}), M_G^q(a_{12})\}$ for some $a_{11}, a_{12} \in W$ with $\breve{e} \leq_W a_{11}$ and

$\check{e} \leq_W a_{12}$. Then $a_{11}, a_{12} \in U_q(M_G; s_0)$ and $a_{11} \rightsquigarrow a_{12} \notin U_q(M_G; s_0)$ for $s_0 = \min\{M_G^q(a_{11}), M_G^q(a_{12})\}$. This is a contradiction, and so $M_G^q(x_{11} \rightsquigarrow x_{12}) \geq \min\{M_G^q(x_{11}), M_G^q(x_{12})\}$ for all $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. If $\tilde{B}_G^{\tilde{a_{13}}}(a_{11} \rightsquigarrow a_{12}) \prec \text{rmin}\{\tilde{B}_G^{\tilde{a_{13}}}(a_{11}), M_G^{\tilde{a_{13}}}(a_{12})\}$ for some $a_{11}, a_{12} \in W$ with $\check{e} \leq_W a_{11}$ and $\check{e} \leq_W a_{12}$, then there exists $\tilde{a_{12}} \in [I]$ such that $\tilde{B}_G^{\tilde{a_{13}}}(a_{11} \rightsquigarrow a_{12}) \prec \tilde{a_{12}} \preceq \text{rmin}\{\tilde{B}_G^{\tilde{a_{13}}}(a_{11}), M_G^{\tilde{a_{13}}}(a_{12})\}$. Hence $a_{11}, a_{12} \in U_{\tilde{a_{13}}}(\tilde{B}_G; \tilde{a_{12}})$ but $a_{11} \rightsquigarrow a_{12} \notin U_{\tilde{a_{13}}}(\tilde{B}_G; \tilde{a_{12}})$, which is a contradiction. Thus $\tilde{B}_G^{\tilde{a_{13}}}(x_{11} \rightsquigarrow x_{12}) \succeq \text{rmin}\{\tilde{B}_G^{\tilde{a_{13}}}(x_{11}), M_G^{\tilde{a_{13}}}(x_{12})\}$ for all $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. Suppose that $J_G^p(a_{11} \rightsquigarrow a_{12}) > \max\{J_G^p(a_{11}), J_G^p(a_{12})\}$ for some $a_{11}, a_{12} \in W$ with $\check{e} \leq_W a_{11}$ and $\check{e} \leq_W a_{12}$. Taking $t_0 := \max\{J_G^p(a_{11}), J_G^p(a_{12})\}$ implies that $J_G(a_{11}) \leq t_0 + p$ and $J_G(a_{12}) \leq t_0 + p$ but $J_G(a_{11} \rightsquigarrow a_{12}) > t_0 + p$. This shows that $a_{11}, a_{12} \in L_p(J_G; t_0)$ and $a_{11} \rightsquigarrow a_{12} \notin L_p(J_G; t_0)$. This is a contradiction, and therefore $J_G^p(x_{11} \rightsquigarrow x_{12}) \leq \max\{J_G^p(x_{11}), J_G^p(x_{12})\}$ for all $x_{11}, x_{12} \in W$ with $\check{e} \leq_W x_{11}$ and $\check{e} \leq_W x_{12}$. Consequently, the $(q, \tilde{a_{13}}, p)$-translative MBJn-set $\mathcal{G}^T = (M_G^q,\ \tilde{B}_G^{\tilde{a_{13}}},\ J_G^p)$ of $\mathcal{G} = (M_G,\ \tilde{B}_G,\ J_G)$ is an MBJnO-subalgebra of $\mathbf{W} := (W, \rightsquigarrow, \tilde{e}, \leq_W)$. □

Before we conclude this paper, we raise the following question.

Question. *Is the inverse of Theorem 3.5 true or false?*

4 Conclusion

In a classical set, an element can either belong to the set (true) or not belong to the set (false). Neutrosophy is a branch of philosophy that deals with the study of indeterminacy and includes three components: true, false, and indeterminate. Neutrosophy introduces the idea that an element can have an indeterminate state, which means it is not clear that the element is in or out of a set. The neutrosophic set consists of three f-sets called true membership function, false membership function and indeterminate membership function. MBJ-neutrosophic structure is a structure constructed using interval-valued f-set instead of indeterminate membership function in the n-set. In general, the indeterminate part appears in a wide range. So instead of treating the indeterminate part as a single value, it is treated as an interval value, allowing a much more comprehensive processing.

The aim of this study is to apply such MBJ-neutrosophic structure to logical algebra, especially OBCI-algebra. We first introduced the notion of MBJ-neutrosophic (ordered) subalgebras in OBCI-algebras and then addressed severl elated properties. We looked at the relationship between the MBJn-subalgeba and the MBJ-neutrosophic ordered subalgebra. We established an MBJ-neutrosophic ordered subalgebra by using (intuitionistic) fuzzy ordered subalgebras, and discussed the characterization of MBJ-neutrosophic ordered subalgebras. We introduced the $(q, \tilde{a_{13}}, p)$-translative MBJn-set based on a MBJn-set, and considered its characterization. We generated an MBJ-neutrosophic ordered subalgebra by using $(q, \tilde{a_{13}}, p)$-translative MBJn-set. The ideas and results covered in this paper will be applied to logical algebras in the future and contribute to producing various results.

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