Neutrosophic Spherical Cubic Sets

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Abstract: In this paper, a new concept of Neutrosophic Spherical Cubic Set (NSCS) is introduced as an amalgamation of sets such as Neutrosophic, Interval valued, cubic and spherical sets. We studied the concepts of internal and external neutrosophic spherical cubic sets and discussed their basic properties. Further P-order, P-union, P-intersection as well as R-order, R-union, R-intersection are discussed for NSCSs.

Keywords: Neutrosophic set(NS); Neutrosophic spherical set (NSS),Neutrosophic cubic set(NCS);Neutrosophic spherical cubic sets(NSCSs)internalneutrosophic spherical cubic set (IntNSCS) and externalneutrosophic spherical cubic set(ExtNSCS). Truth Internal/External- $\mathcal{R}-$ Int/Ext , Inderterminacy Internal/External- $\mathcal{J}$ Int/Ext ,Falsity Internal/External $-\mathcal{S}$ Int/Ext

1. Introduction

Zadeh [12] established the fuzzy set notion in 1965 to cope with probabilistic uncertainty associated with inaccuracy of events, observations and desires.By the idea of fuzziness, the value of 1 is allocated to an object that is fully within the set and value of 0 is allocated to an object that is totally outside the set, then

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any item partially inside the set will have a value ranging between 0 and 1, Fuzzy set along with it generalizations has many real life applications [9,10,11].

Jun et al. [2] proposed cubic set which is a hybrid of fuzzy sets and interval valued fuzzy sets. They also examined internal (external) cubic sets. By adding the falsehood (f), the degree of non-membership, and various properties.

In 1995,Smarandache [7,8] presented the concept of neutrosophicsets and neutrosophic logic. Neutrosophy lays the groundwork for plenty of new mathematical theories that encompass classical and fuzzy analogues. There are three defining functions in neutrosophic set they are truth T, indeterminate I and false membership function F all of which are defined on a universe of discourse X.These three functions are totally self-contained. The formation, nature, and extent of neutralities are all investigated in the Neutrosophicset. The idea ofneutrosophic set is a generalization of idea of a classical fuzzy set and so on.

KutluGundogu, Fatmaa, Kahraman, Cengiz [5,6] developed spherical fuzzy sets and spherical fuzzy TOPSIS method. They introduced generalized three dimensional spherical fuzzy sets (SFS) including some essential differences from the other fuzzy sets.

The spherical fuzzy set is a more dominant structures for coping with these situations. The idea behind spherical fuzzy set is to let decision makers to generalize other extensions of fuzzy sets by defining a membership function on a spherical surfaces and independently assign the parameters of that function with a larger domain.

The motive of the paper is to introduce a new concept called NSCSs and to study theINSCSs and ENSCSs that is truth, indeterminacy,falsity internaland truth, indeterminacy, falsity external respectively. Also, we have investigated their properties. We showed that P-union and the P-intersection of INSCSs are also theINSCSs. Examples are given to show that P-union and the P-intersection of ENSCSs may not be ENSCSs. R-union and the R- intersection of INSCSsmaynotbe INSCS.Also, we have given theconditionsfortheR-unionoftwoT-INSCSs(resp. I-INSCSsandF-INSCSs)tobeaT-INSCSs(resp.I-INSCSs and F-INSCSs) NSCS.Some of the fundamental properties of NSCSs were also investigated.

2. PRELIMINARIES

FUZZY SET [12]

A fuzzy set in a set S is defined to be a function $\mu : S \to [0,1]$. Define a relation $\leq$ on $[0,1]^S$ as follows $\left(\forall \mu, \lambda \in [0,1]^S\right), \left(\mu \leq \lambda \Leftrightarrow (\forall s \in S)(\mu(s) \leq \lambda(s)\right)$.

NEUTROSOPHIC SET [7]

Let *S*be a non-empty set. A neutrosophic set (NS)isastructureoftheform:

$$\Omega = \left\{ \langle s, \gamma_T(s), \gamma_I(s), \gamma_F(s) \rangle / s \in S \right\}$$

where $\gamma_T : S \to [0,1]$ is a truth membership function, $\underset{\to}{\gamma_I} : S \to [0,1]$ is an indeterminate membership function, and $\gamma_F : S \to [0,1]$is a false membership function.

CUBIC SETS [2]

Let Sbe a non-empty set. A cubic set in Y is a structure of the form

$$\mathscr{C} = \left\{ y, C(y), \lambda(y) / y \in S \right\}$$

where A is an interval valued fuzzy set in S and λ is a fuzzy set in S.

NEUTROSOPHIC CUBIC SETS [4]

Let S be a non-empty set. A neutrosophic cubic set (NCS) in S is a pair $\mathscr{A} = (C, \lambda)$ where $C = \left\{ \langle s, C_T(s), C_I(s), C_F(s) \rangle / s \in S \right\}$ is an intervalneutrosophic set in S and $\lambda = \left\{ \langle s; \lambda_T(s), \lambda_I(s), \lambda_F(s) \rangle / s \in X \right\}$ is a neutrosophic set inS.

SPHERICAL FUZZY SETS [5]

A Spherical Fuzzy Set $\mathscr{A}_s^{\sim}$ of the universe U is given by

$$\mathscr{A}_s^{\sim} = \left\{ \left\langle u, \mu_{A_s^{\sim}}(u), \gamma_{A_s^{\sim}}(u), \pi_{A_s^{\sim}}(u) \right\rangle / u \in U \right\} \text{where } \mu_{A_s^{\sim}}, \gamma_{A_s^{\sim}}, \pi_{A_s^{\sim}}, U \to [0,1]$$

and $0 \leq \mu^2_{A_s^{\sim}}(u) + \gamma^2_{A_s^{\sim}}(u) + \pi^2_{A_s^{\sim}} \leq 1 \ \forall u \in U$.

3. NEUTROSOPHIC SPHERICAL SETS

DEFINITION 3.1

Let S be a non-empty set. A Neutrosophic Spherical set in NS is of the form

$$A_s = \{\langle s: T_{A_s}(s), I_{A_S}(s), F_{A_s}(s)\rangle / s \in S\}$$

where T_{A_s} is truth degree membership

I_{A_s} is indeterminate degree membership

F_{A_s} is false degree membership.

where

$$T_{A_S}(s), I_{A_S}(s), F_{A_S}(s) / s \in S \to [0,1]$$

$$0 \leq [T_{A_S}(s)]^2 + [I_{A_S}(s)]^2 + [F_{A_S}(s)]^2 \leq \sqrt{3}$$

INTERVAL VALUED NEUTROSOPHIC SPHERICAL SETS

DEFINITION 3.2

Let S be a non-empty set. An interval-valued Neutrosophic Spherical set is of the form

$$\mathscr{A}_s = \left\{ s : \left[T^-_{A_s(s)}, T^+_{A_s(s)} \right] \left[I^-_{A_s(s)}, I^+_{A_s(s)} \right] \left[F^-_{A_s(s)}, F^+_{A_s(s)} \right] / s \in S \right\}$$

Where $T^-_{A_s}(s), I^-_{A_s}(s), F^-_{A_s}(s) / s \in S \to [0,1]$

$$0 \leq [T^-_{A_s}(s)]^2 + [I^-_{A_s}(s)]^2 + [F^-_{A_s}(s)]^2 \leq \sqrt{3}$$

and$T^+_{A_s}(s), I^+_{A_s}(s), F^+_{A_s}(s) / s \in S \to [0,1]$

$$0 \leq [T^+_{A_s}(s)]^2 + [I^+_{A_s}(s)]^2 + [F^+_{A_s}(s)]^2 \leq \sqrt{3}$$

NEUTROSOPHIC SPHERICAL CUBIC SETS

DEFINITION 3.3

A non-empty set $\mathcal{V}_{NSC}$ of NSCS is defined by

$$\mathcal{C}_{NSC} = \{\langle v, A_s(v), \lambda_s(v)\rangle / v \in \mathcal{V}_{NSC}\}$$

where $A_s(v)$ is an IVNSS in $\mathcal{V}_{NSC}$ and $\lambda_s(v)$ is a NSS in $\mathcal{V}_{NSC}$.

EXAMPLE 3.1

For $\mathcal{V}_{NSC} = \{v_1, v_2, v_3\}$, the pair $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ with the tabular representation in Table 0.2 is an NSCS in $\mathcal{V}_{NSC}$.

Table 1: $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\lambda_s(v)$
v_1	([0.3,0.4], [0.4,1.0], [0.3,0.5])	(0.2,0.4,0.4)
v_2	([0.4,0.7], [0.2,1.0], [0.2,0.4])	(0.5,0.2,0.3)
v_3	([0.7,0.6], [0.0,1.0], [0.3,0.8])	(0.4,0.1,0.5)

DEFINITION 3.4

A non-empty set of $\mathcal{V}_{NSC}$ of NSCS, $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ in $\mathcal{V}_{NSC}$ is said to

- Truth Int (briefly $\mathcal{R} -$ Int) is defined by

$$(\forall \quad v \in \mathcal{V}_{NSC})(\mathcal{R}^-_{A_s}(v) \leq \mathcal{R}_{\lambda_s}(v) \leq \mathcal{R}^+_{A_s}(v)) \tag{1}$$

- Indeterminacy-Int (briefly $\mathcal{J} -$ Int) is defined by

$$(\forall \quad v \in \mathcal{V}_{NSC})(\mathcal{J}^-_{A_s}(v) \leq \mathcal{J}_{\lambda_s}(v) \leq \mathcal{J}^+_{A_s}(v)) \tag{2}$$

- Falsity-int(briefly $\mathcal{S}$ Int) is defined by

$$(\forall \quad v \in \mathcal{V}_{NSC})(\mathcal{S}^-_{A_s}(v) \leq \mathcal{S}_{\lambda_s}(v) \leq \mathcal{S}^+_{A_s}(v)) \tag{3}$$

If a NSCS, $\mathcal{C}_{NSC}$ in $\mathcal{V}_{NSC}$ satisfies above inequalities then $\mathcal{C}_{NSC}$ is an Int NSCS in $\mathcal{V}_{NSC}$.

EXAMPLE 3.2

For $\mathcal{V}_{NSC} = \{v_1, v_2, v_3\}$, the pair $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ with the tabular representation in Table 0.4 is an Int NSCS in $\mathcal{V}_{NSC}$.

Table 2: $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\lambda_s(v)$
v_1	([0.3,0.4], [0.2,1.0], [0.5,0.6])	(0.35,0.2,0.55)
v_2	([0.5,0.6], [0.1,1.0], [0.4,0.6])	(0.5,0.1,0.4)

v_3	$([0.6,0.7],[0.1,1.0],[0.2,0.4])$	$(0.65,0.1,0.25)$

DEFINITION 3.5

A non-empty set $\mathcal{V}_{NSC}$ of NSCS, $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ is said to

- Truth Ext (briefly $\mathcal{R}$ Ext) is defined by

$$(\forall \ v \in \mathcal{V}_{NSC})(\mathcal{R}_{\lambda_s}(v) \not\subseteq (\mathcal{R}^-_{A_s}(v), \mathcal{R}^+_{A_s}(v)) \quad (4)$$

- Indeterminacy-Ext (briefly $\mathcal{J}$ Ext) is defined by

$$(\forall \ v \in \mathcal{V}_{NSC})(\mathcal{J}_{\lambda_s}(v) \not\subseteq (\mathcal{J}^-_{A_s}(v), \mathcal{J}^+_{A_s}(v)) \quad (5)$$

- Falsity-Ext (briefly $\mathcal{S}$ Ext) is defined by

$$(\forall \ v \in \mathcal{V}_{NSC})(\mathcal{S}_{\lambda_s}(v) \not\subseteq (\mathcal{S}^-_{A_s}(v), \mathcal{S}^+_{A_s}(v)) \quad (6)$$

If a NSCS, $\mathcal{C}_{NSC}$ in $\mathcal{V}_{NSC}$ satisfies above inequalities then $\mathcal{C}_{NSC}$ is an Ext NSCS in $\mathcal{V}_{NSC}$.

EXAMPLE 3.3

For $\mathcal{V}_{NSC} = \{v_1, v_2, v_3\}$, the pair $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ with the tabular representation in Table 0.6 is an Ext NSCS in $\mathcal{V}_{NSC}$.

Table 3: $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\lambda_s(v)$
v_1	$([0.3,0.4],[0.2,1.0],[0.4,0.5])$	$(0.45,0.1,0.65)$
v_2	$([0.5,0.6],[0.1,1.0],[0.4,0.6])$	$(0.4,0.0,0.7)$
v_3	$([0.6,0.7],[0.1,1.0],[0.2,0.4])$	$(0.5,0.0,0.45)$

Theorem 3.4 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ be a NSCS in $\mathcal{V}_{NSC}$ is not Ext then there exists $v \in \mathcal{V}_{NSC}$ such that $\mathcal{R}_{\lambda_s}(v) \in (\mathcal{R}^-_{A_s}(v), \mathcal{R}^+_{A_s}(v))$, $\mathcal{J}_{\lambda_s}(v) \in (\mathcal{J}^-_{A_s}(v), \mathcal{J}^+_{A_s}(v))$ or $\mathcal{S}_{\lambda_s}(v) \in (\mathcal{S}^-_{A_s}(v), \mathcal{S}^+_{A_s}(v))$.

Proof. From the definition of an Ext NSCS ,

$$\mathcal{R}_{\lambda_s}(v) \notin [\mathcal{R}^-_{A_s}(v), \mathcal{R}^+_{A_s}(v)],$$
$$\mathcal{J}_{\lambda_s}(v) \notin [\mathcal{J}^-_{A_s}(v), \mathcal{J}^+_{A_s}(v)],$$
$$\mathcal{S}_{\lambda_s}(v) \notin [\mathcal{S}^-_{A_s}(v), \mathcal{S}^+_{A_s}(v)]$$

for $v \in \mathcal{V}_{NSC}$. But given that $\mathcal{C}_{NSC}$ is not Ext NSCS, such that

$$\mathcal{R}^-_{A_s}(v) \leq \mathcal{R}_{\lambda_s}(v) \leq \mathcal{R}^+_{A_s}(v)$$
$$\mathcal{J}^-_{A_s}(v) \leq \mathcal{J}_{\lambda_s}(v) \leq \mathcal{J}^+_{A_s}(v)$$
$$\mathcal{S}^-_{A_s}(v) \leq \mathcal{S}_{\lambda_s}(v) \leq \mathcal{S}^+_{A_s}(v)$$

Hence the result.

Theorem 3.5 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s)(v)$ be a NSCS in $\mathcal{V}_{NSC}$, if $\mathcal{C}_{NSC}$ is both $\mathcal{R}$ Int and $\mathcal{R}$ Ext then

$$(\forall \;\; v \in \mathcal{V}_{NSC})(\mathcal{R}_{\lambda_s}(v) \in \{\mathcal{R}^-_{A_s}(v)/v \in \mathcal{V}_{NSC}\} \cup \{\mathcal{R}^+_{A_s}(v)/v \in \mathcal{V}_{NSC}\}).$$

Proof. Two conditions (1) and (4) which implies that

$$\mathcal{R}^-_{A_s}(v) \leq \mathcal{R}_{\lambda_s}(v) \leq \mathcal{R}^+_{A_s}(v) \qquad and$$

$$\mathcal{R}_{\lambda_s}(v) \not\subseteq (\mathcal{R}^-_{A_s}(v), \mathcal{R}^+_{A_s}(v)) \forall \;\; v \in \mathcal{V}_{NSC}.$$

Then $\mathcal{R}_{\lambda_s}(x) = (\mathcal{R}^-_{A_s}(v)$ or $\mathcal{R}_{\lambda_s}(v) = (\mathcal{R}^+_{A_s}(v)$ so that

$$\mathcal{R}_{\lambda_s}(v) \in \{\mathcal{R}^-_{A_s}(v)/v \in \mathcal{V}_{NSC}\} \cup \{\mathcal{R}^+_{A_s}(v)/v \in \mathcal{V}_{NSC}\}.$$

Hence proved.

Theorem 3.6 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ be a NSCS in $\mathcal{V}_{NSC}$, if $\mathcal{C}_{NSC}$ is both $\mathcal{J}$ Int and $\mathcal{J}$ Ext then

$$(\forall \;\; v \in \mathcal{V}_{NSC})(\mathcal{J}_{\lambda_s}(v) \in \{\mathcal{J}^-_{A_s}(v)/v \in \mathcal{V}_{NSC}\} \cup \{\mathcal{J}^+_{A_s}(v)/v \in \mathcal{V}_{NSC}\}).$$

Proof. Two conditions (2) and (5) which implies that

$$\mathcal{J}^-_{A_s}(v) \leq \mathcal{J}_{\lambda_s}(v) \leq \mathcal{J}^+_{A_s}(v) \qquad and$$

$$\mathcal{J}_{\lambda_s}(v) \not\subseteq (\mathcal{J}^-_{A_s}(v), \mathcal{J}^+_{A_s}(v)) \forall \;\; v \in \mathcal{V}_{NSC}.$$

Then $\mathcal{J}_{\lambda_s}(x) = (\mathcal{J}^-_{A_s}(v)$ or $\mathcal{J}_{\lambda_s}(v) = (\mathcal{J}^+_{A_s}(v)$ so that

$$\mathcal{J}_{\lambda_s}(v) \in \{\mathcal{J}^-_{A_s}(v)/v \in \mathcal{V}_{NSC}\} \cup \{\mathcal{J}^+_{A_s}(v)/v \in \mathcal{V}_{NSC}\}.$$

Hence proved.

Theorem 3.7 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ be a NSCS $\mathcal{V}_{NSC}$, if $\mathcal{C}_{NSC}$ is both $\mathcal{S}$ Int and $\mathcal{S}$ Ext then

$$(\forall \;\; v \in \mathcal{V}_{NSC})(\mathcal{S}_{\lambda_s}(v) \in \{\mathcal{S}^-_{A_s}(v)/v \in \mathcal{V}_{NSC}\} \cup \{\mathcal{S}^+_{A_s}(v)/v \in \mathcal{V}_{NSC}\}).$$

Proof. Two conditions (3) and (6) which implies that

$$\mathcal{S}^-_{A_s}(v) \leq \mathcal{S}_{\lambda_s}(v) \leq \mathcal{S}^+_{A_s}(v) \qquad and$$

$$\mathcal{S}_{\lambda_s}(v) \not\in (\mathcal{S}^-_{A_s}(v), \mathcal{S}^+_{A_s}(v)) \forall \ \ v \in \mathcal{V}_{NSC}.$$

Then $\mathcal{S}_{\lambda_s}(x) = (\mathcal{S}^-_{A_s}(v)$ or $\mathcal{S}_{\lambda_s}(v) = (\mathcal{S}^+_{A_s}(v)$ so that

$$\mathcal{S}_{\lambda_s}(v) \in \{\mathcal{S}^-_{A_s}(v)/v \in \mathcal{V}_{NSC}\} \cup \{\mathcal{S}^+_{A_s}(v)/v \in \mathcal{V}_{NSC}\}.$$

Hence proved.

Definition 3.8 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be a NSCS in $\mathcal{V}_{NSC}$ where

$$\mathcal{C}_{NSC} = \{\mathcal{V}_{NSC}: [\mathcal{R}^-_{A_s(v)}, \mathcal{R}^+_{A_s(v)}][\mathcal{J}^-_{A_s(v)}, \mathcal{J}^+_{A_s(v)}][\mathcal{S}^-_{A_s(v)}, \mathcal{S}^+_{A_s(v)}]/v \in \mathcal{V}_{NSC}\}$$

$$\lambda_s: \{(\mathcal{V}_{NSC}, \mathcal{R}_{\lambda_s}(v), \mathcal{J}_{\lambda_s}(v), \mathcal{S}_{\lambda_s}(v))/v \in \mathcal{V}_{NSC}\}$$

$$\mathcal{B}_{NSC} = \{\mathcal{V}_{NSC}: [\mathcal{R}^-_{B_s(v)}, \mathcal{R}^+_{B_s(v)}][\mathcal{J}^-_{B_s(v)}, \mathcal{J}^+_{B_s(v)}][\mathcal{S}^-_{B_s(v)}, \mathcal{S}^+_{B_s(v)}]/v \in \mathcal{V}_{NSC}\}$$

$$\psi_s: \{(\mathcal{V}_{NSC}, \mathcal{R}_{\psi_s}(v), \mathcal{J}_{\psi_s}(v), \mathcal{S}_{\psi_s}(v))/v \in \mathcal{V}_{NSC}\}.$$

Then

$\mathcal{C}_{NSC} = \mathcal{B}_{NSC}$ iff

$A_s(v) = B_s(v)$ and $\lambda_s(v) = \psi_s(v)$ for all $v \in \mathcal{V}_{NSC}$

If $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be any two NSCSs, then P −order is defined by

$\mathcal{C}_{NSC} \subseteq_P \mathcal{B}_{NSC}$ iff $A_s(v) \subseteq B_s(v)$ and $\lambda_s(v) \leq \psi_s(v)$ for all $v \in \mathcal{V}_{NSC}$

If $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be any two NSCSs, then R −order is defined by

$\mathcal{C}_{NSC} \subseteq_R \mathcal{B}_{NSC}$ iff $A_s(v) \subseteq B_s(v)$ and $\lambda_s(v) \geq \psi_s(v)$ for all $v \in \mathcal{V}_{NSC}$.

Definition3.9 Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be two NSCS in $\mathcal{V}_{NSC}$, then P-union is defined by

$$\mathcal{C}_{NSC} \cup_P \mathcal{B}_{NSC} = \{\langle v, max(A_s(v), B_s(v)), (\lambda_s(v) \vee_P \psi_s(v))\rangle: v \in \mathcal{V}_{NSC}\}$$

where $A_s(v), B_s(v)$ represent IVNSSs and $\lambda_s(v), \psi_s(v)$ represent NSSs. Hence

$$\mathcal{R}_{\mathcal{C}_{NSC}} \vee_P \mathcal{R}_{\mathcal{B}_{NSC}} = \{\langle v, max(\mathcal{R}_{A_s(v)}, B_s(v)), (\mathcal{R}_{\lambda_s}(v) \vee_P \mathcal{R}_{\psi_s}(v))\rangle: v \in \mathcal{V}_{NSC}\}$$

$$\mathcal{J}_{\mathcal{C}_{NSC}} \vee_P \mathcal{J}_{\mathcal{B}_{NSC}} = \{\langle v, max(\mathcal{J}_{A_s(v)}, B_s(v)), (\mathcal{J}_{\lambda_s}(v) \vee_P \mathcal{J}_{\psi_s}(v))\rangle: v \in \mathcal{V}_{NSC}\}$$

$$\mathcal{S}_{\mathcal{C}_{NSC}} \vee_P \mathcal{S}_{\mathcal{B}_{NSC}} = \{\langle v, max(\mathcal{S}_{A_s(v)}, B_s(v)), (\mathcal{S}_{\lambda_s}(v) \vee_P \mathcal{S}_{\psi_s}(v))\rangle: v \in \mathcal{V}_{NSC}\}$$

Definition 3.10 Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be two NSCS in $\mathcal{V}_{NSC}$, then P-intersection is defined by

$$\mathcal{C}_{NSC} \cap_P \mathcal{B}_{NSC} = \{\langle v, min(A_s(v), B_s(v)), (\lambda_s(v) \wedge_P \psi_s(v))\rangle: v \in \mathcal{V}_{NSC}\}$$

where $A_s(v), B_s(v)$ represent IVNSSs and $\lambda_s(v), \psi_s(v)$ represent NSSs. Hence

$$\mathcal{R}_{\mathcal{C}_{NSC}} \wedge_P \mathcal{R}_{\mathcal{B}_{NSC}} = \{\langle v, min(\mathcal{R}_{A_s(v)}, B_s(v)), (\mathcal{R}_{\lambda_s}(v) \wedge_P \mathcal{R}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$
$$\mathcal{J}_{\mathcal{C}_{NSC}} \wedge_P \mathcal{J}_{\mathcal{B}_{NSC}} = \{\langle v, min(\mathcal{J}_{A_s(v)}, B_s(v)), (\mathcal{J}_{\lambda_s}(v) \wedge_P \mathcal{J}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$
$$\mathcal{S}_{\mathcal{C}_{NSC}} \wedge_P \mathcal{S}_{\mathcal{B}_{NSC}} = \{\langle v, min(\mathcal{S}_{A_s(v)}, B_s(v)), (\mathcal{S}_{\lambda_s}(v) \wedge_P \mathcal{S}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$

Example 3.11 For $\mathcal{V}_{NSC} = \{v_1, v_2, v_3\}$, let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be two NSCSs over $\mathcal{V}_{NSC}$ is defined by

$\mathcal{C}_{NSC} = \{\langle v, A_s(v), \lambda_s(v)\rangle v \in \mathcal{V}_{NSC}\}$ is

Table 4: $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\lambda_s(v)$
v_1	([0.3,0.4], [0.5,1.0], [0.2,0.4])	(0.35,0.55,0.25)
v_2	([0.2,0.3], [0.4,1.0], [0.4,0.6])	(0.35,0.45,0.55)
v_3	([0.4,0.5], [0.2,1.0], [0.4,0.6])	(0.55,0.35,0.45)

Table 5: $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\psi_s(v)$
v_1	([0.1,0.2], [0.5,1.0], [0.4,0.6])	(0.45,0.55,0.45)
v_2	([0.2,0.4], [0.3,1.0], [0.5,0.7])	(0.45,0.65,0.65)
v_3	([0.3,0.5], [0.4,1.0], [0.3,0.5])	(0.65,0.55,0.75)

Table 6: $\mathcal{C}_{NSC} \cup_P \mathcal{B}_{NSC} = (A_s(v) \cup_P B_s(v), \lambda_s(v) \vee_P \psi_s(v))$

$\mathcal{V}_{NSC}$	$(A_s(v) \cup_P B_s(v)$	$(\lambda_s(v) \vee_P \psi_s(v)$
v_1	([0.3,0.4], [0.5,1.0], [0.4,0.6])	(0.45,0.55,0.45)
v_2	([0.2,0.4], [0.4,1.0], [0.5,0.7])	(0.45,0.65,0.65)
v_3	([0.4,0.5], [0.4,1.0], [0.4,0.6])	(0.65,0.55,0.75)

Table 7: $\mathcal{C}_{NSC} \cap_P \mathcal{B}_{NSC} = (A_s(v) \cap_P B_s(v), \lambda_s(v) \wedge_P \psi_s(v))$

$\mathcal{V}_{NSC}$	$(A_s(v) \cap_P B_s(v)$	$(\lambda_s(v) \wedge_P \psi_s(v)$
v_1	$([0.1,0.2],[0.5,1.0],[0.2,0.4])$	$(0.35,0.55,0.15)$
v_2	$([0.2,0.3],[0.3,1.0],[0.4,0.6])$	$(0.35,0.45,0.55)$
v_3	$([0.3,0.5],[0.2,1.0],[0.3,0.5])$	$(0.55,0.35,0.5)$

Definition 3.12 Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be two NSCS in $\mathcal{V}_{NSC}$, then R-union is defined by

$$\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = \{\langle v, max(A_s(v), B_s(v)), (\lambda_s(v) \vee_R \psi_s(v))\rangle : v \in \mathcal{V}_{NSC}\}$$

where $A_s(v), B_s(v)$ represent IVNSSs and $\lambda_s(v), \psi_s(v)$ represent NSSs. Hence

$$\mathcal{R}_{\mathcal{C}_{NSC}} \vee_R \mathcal{R}_{\mathcal{B}_{NSC}} = \{\langle v, max(\mathcal{R}_{A_s(v)}, B_s(v)), (\mathcal{R}_{\lambda_s}(v) \vee_R \mathcal{R}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$
$$\mathcal{J}_{\mathcal{C}_{NSC}} \vee_R \mathcal{J}_{\mathcal{B}_{NSC}} = \{\langle v, max(\mathcal{J}_{A_s(v)}, B_s(v)), (\mathcal{J}_{\lambda_s}(v) \vee_R \mathcal{J}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$
$$\mathcal{S}_{\mathcal{C}_{NSC}} \vee_R \mathcal{S}_{\mathcal{B}_{NSC}} = \{\langle v, max(\mathcal{S}_{A_s(v)}, B_s(v)), (\mathcal{S}_{\lambda_s}(v) \vee_R \mathcal{S}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$

Definition 3.13 Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be two NSCS in $\mathcal{V}_{NSC}$, then R-intersection is defined by

$$\mathcal{C}_{NSC} \cap_R \mathcal{B}_{NSC} = \{\langle v, min(A_s(v), B_s(v)), (\lambda_s(v) \wedge_R \psi_s(v))\rangle : v \in \mathcal{V}_{NSC}\}$$

where $A_s(v), B_s(v)$ represent IVNSSs and $\lambda_s(v), \psi_s(v)$ represent NSSs. Hence

$$\mathcal{R}_{\mathcal{C}_{NSC}} \wedge_R \mathcal{R}_{\mathcal{B}_{NSC}} = \{\langle v, min(\mathcal{R}_{A_s(v)}, B_s(v)), (\mathcal{R}_{\lambda_s}(v) \wedge_R \mathcal{R}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$
$$\mathcal{J}_{\mathcal{C}_{NSC}} \wedge_R \mathcal{J}_{\mathcal{B}_{NSC}} = \{\langle v, min(\mathcal{J}_{A_s(v)}, B_s(v)), (\mathcal{J}_{\lambda_s}(v) \wedge_R \mathcal{J}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$
$$\mathcal{S}_{\mathcal{C}_{NSC}} \wedge_R \mathcal{S}_{\mathcal{B}_{NSC}} = \{\langle v, min(\mathcal{S}_{A_s(v)}, B_s(v)), (\mathcal{S}_{\lambda_s}(v) \wedge_R \mathcal{S}_{\psi_s}(v))\rangle : v \in \mathcal{V}_{NSC}\}$$

Example 3.14 For $\mathcal{V}_{NSC} = \{v_1, v_2, v_3\}$. Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be two NSCSs over $\mathcal{V}_{NSC}$ is defined by

$\mathcal{C}_{NSC} = \{\langle v, A_s(v), \lambda_s(v)\rangle v \in \mathcal{V}_{NSC}\}$ is

Table 8: $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\lambda_s(v)$
v_1	$([0.2,0.3],[0.4,1.0],[0.4,0.6])$	$(0.55,0.65,0.75)$
v_2	$([0.1,0.5],[0.6,1.0],[0.3,0.6])$	$(0.65,0.55,0.85)$
v_3	$([0.2,0.5],[0.4,1.0],[0.4,0.6])$	$(0.75,0.85,0.65)$

Table 9: $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\psi_s(v)$
v_1	([0.2,0.3], [0.4,1.0], [0.5,0.6])	(0.35,0.65,0.75)
v_2	([0.1,0.5], [0.6,1.0], [0.3,0.6])	(0.65,0.55,0.85)
v_3	([0.2,0.5], [0.4,1.0], [0.4,0.6])	(0.75,0.85,0.65)

Table 10: $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s(v) \cup_R B_s, \lambda_s(v) \vee_R \psi_s(v))$

$\mathcal{V}_{NSC}$	$(A_s(v) \cup_R B_s(v)$	$(\lambda_s(v) \vee_R \psi_s(v)$
v_1	([0.4,0.5], [0.4,1.0], [0.4,0.6])	(0.35,0.45,0.55)
v_2	([0.2,0.4], [0.6,1.0], [0.3,0.6])	(0.25,0.45,0.65)
v_3	([0.4,0.7], [0.4,1.0], [0.4,0.6])	(0.45,0.55,0.45)

Table 11: $\mathcal{C}_{NSC} \cap_R \mathcal{B}_{NSC} = (A_s(v) \cap_R B_s(v), \lambda_s(v) \wedge_R \psi_s(v))$

$\mathcal{V}_{NSC}$	$(A_s(v) \cap_R B_s(v)$	$(\lambda_s(v) \wedge_R \psi_s(v)$
v_1	([0.2,0.3], [0.3,1.0], [0.3,0.6])	(0.55,0.65,0.75)
v_2	([0.1,0.5], [0.5,1.0], [0.3,0.6])	(0.65,0.55,0.85)
v_3	([0.2,0.5], [0.4,1.0], [0.4,0.6])	(0.75,0.85,0.65)

Theorem 3.15 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ be a NSCS over $\mathcal{V}_{NSC}$. [i)] If $\mathcal{C}_{NSC}$ is an Int NSCS, then the complement $\mathcal{C}_{NSC}^c$ is also an Int NSCS. [ii)]If $\mathcal{C}_{NSC}$ is an Ext NSCS, then the complement $\mathcal{C}_{NSC}^c$ is also an Ext NSCS.

Proof. [i)] Given $\mathcal{C}_{NSC} = \{\langle v, A_s(v), \lambda_s(v)\rangle : v \in \mathcal{V}_{NSC}\}$ is an Int NSCS this implies
$\mathcal{R}_{A_s}^-(v) \leq \mathcal{R}_{\lambda_s}(v) \leq \mathcal{R}_{A_s}^+(v)$,
$\mathcal{J}_{A_s}^-(v) \leq \mathcal{J}_{\lambda_s}(v) \leq \mathcal{J}_{A_s}^+(v)$,
$\mathcal{S}_{A_s}^-(v) \leq \mathcal{S}_{\lambda_s}(v) \leq \mathcal{S}_{A_s}^+(v)$ for all $v \in \mathcal{V}_{NSC}$
This implies $1 - \mathcal{R}_{A_s}^-(v) \leq 1 - \mathcal{R}_{\lambda_s}(v) \leq 1 - \mathcal{R}_{A_s}^+(v)$,
$1 - \mathcal{J}_{A_s}^-(v) \leq 1 - \mathcal{J}_{\lambda_s}(v) \leq 1 - \mathcal{J}_{A_s}^+(v)$,
$1 - \mathcal{S}_{A_s}^-(v) \leq 1 - \mathcal{S}_{\lambda_s}(v) \leq 1 - \mathcal{S}_{A_s}^+(v)$ for all $v \in \mathcal{V}_{NSC}$.

Hence $\mathcal{C}_{NSC}^c$ is an INSCS. [ii)] Given $\mathcal{C}_{NSC} = \{\langle v, A_s(v), \lambda_s(v)\rangle : v \in \mathcal{V}_{NSC}\}$ is an Ext NSCS. This implies

$\mathcal{R}_{\lambda_s}(v) \notin (\mathcal{R}_{A_s}^-(v), \mathcal{R}_{A_s}^+(v)), \mathcal{J}_{\lambda_s}(v) \notin (\mathcal{J}_{A_s}^-(v), \mathcal{J}_{A_s}^+(v)), \mathcal{S}_{\lambda_s}(v) \notin (\mathcal{S}_{A_s}^-(v), \mathcal{S}_{A_s}^+(v))$ for all $v \in \mathcal{V}_{NSC}$

Since,

$$\mathcal{R}_{\lambda_s}(v) \notin (\mathcal{R}_{A_s}^-(v), \mathcal{R}_{A_s}^+(v)),$$

and $0 \leq \mathcal{R}_{A_s}^-(v) \leq \mathcal{R}_{A_s}^+(v) \leq 1$

$$\mathcal{J}_{\lambda_s}(v) \notin (\mathcal{J}_{A_s}^-(v), \mathcal{J}_{A_s}^+(v)),$$

and $0 \leq \mathcal{J}_{A_s}^-(v) \leq \mathcal{J}_{A_s}^+(v) \leq 1$

$$\mathcal{S}_{\lambda_s}(v) \notin (\mathcal{S}_{A_s}^-(v), \mathcal{S}_{A_s}^+(v))$$

and $0 \leq \mathcal{S}_{A_s}^-(v) \leq \mathcal{S}_{A_s}^+(v) \leq 1$

So we have
$\mathcal{R}_{\lambda_s}(v) \leq \mathcal{R}_{A_s}^-(v)$ or $\mathcal{R}_{A_s}^+(v) \leq \mathcal{R}_{\lambda_s}(v)$
$\mathcal{J}_{\lambda_s}(v) \leq \mathcal{J}_{A_s}^-(v)$ or $\mathcal{J}_{A_s}^+(v) \leq \mathcal{J}_{\lambda_s}(v)$
$\mathcal{S}_{\lambda_s}(v) \leq \mathcal{S}_{A_s}^-(v)$ or $\mathcal{S}_{A_s}^+(v) \leq \mathcal{S}_{\lambda_s}(v)$
This implies
$1 - \mathcal{R}_{\lambda_s}(v) \geq 1 - \mathcal{R}_{A_s}^-(v)$ or $1 - \mathcal{R}_{A_s}^+(v) \geq 1 - \mathcal{R}_{\lambda_s}(v)$
$1 - \mathcal{J}_{\lambda_s}(v) \geq 1 - \mathcal{J}_{A_s}^-(v)$ or $1 - \mathcal{J}_{A_s}^+(v) \geq 1 - \mathcal{J}_{\lambda_s}(v)$
$1 - \mathcal{S}_{\lambda_s}(v) \leq 1 - \mathcal{S}_{A_s}^-(v)$ or $1 - \mathcal{S}_{A_s}^+(v) \leq 1 - \mathcal{S}_{\lambda_s}(v)$ for all $v \in \mathcal{V}_{NSC}$.

Thus $1 - \mathcal{R}_{\lambda_s}(v) \notin (1 - \mathcal{R}_{A_s}^-(v), 1 - \mathcal{R}_{A_s}^+(v)), 1 - \mathcal{J}_{\lambda_s}(v) \notin (1 - \mathcal{J}_{A_s}^-(v), 1 - \mathcal{J}_{A_s}^+(v)), 1 - \mathcal{S}_{\lambda_s}(v) \notin (1 - \mathcal{S}_{A_s}^-(v), 1 - \mathcal{S}_{A_s}^+(v))$ for all $v \in \mathcal{V}_{NSC}$

Hence $\mathcal{C}_{NSC}^c = (A_s(v), \lambda_s(v))$ is an Ext NSCS.

Remark 3.16 The below example shows that $P-$ union and $P-$ intersection of $\mathcal{R}$ Ext (resp. $\mathcal{J}$ Ext and $\mathcal{S}$ Ext) NSCSs may not be $\mathcal{R}$ Ext (resp. $\mathcal{J}$ Ext and $\mathcal{S}$ Ext) NSCSs.

Example 3.17 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be NSCSs in $\mathcal{V}_{NSC}$ where

$$A_s(v) = \{(v, (0.3,0.5), (0.5,0.7), (0.3,0.5))/v \in \mathcal{V}_{NSC}\}$$

$\lambda_s(v) = \{(v, 0.4,0.4,0.8)/v \in \mathcal{V}_{NSC}\}$

$B_s(v) = \{(v, (0.7,0.9), (0.6,0.7), (0.7,0.9))/v \in \mathcal{V}_{NSC}\}$

$\psi_s(v) = \{(v, 0.8,0.3,0.8)/v \in \mathcal{V}_{NSC}\}$

Then $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ are $\mathcal{S}$ Ext NSCSs in $\mathcal{V}_{NSC}$.

$\mathcal{C}_{NSC} \cup_P \mathcal{B}_{NSC} = (A_s \cup_P B_s, \lambda_s \vee_P \psi_s)$ of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is given as follows

$A_s(v) \cup_P B_s(v) = \{(v, (0.7,0.9), (0.6,0.7), (0.7,0.9))/v \in \mathcal{V}_{NSC}\}$

$\lambda_s(v) \vee_P \psi_s(v) = \{(v, 0.8,0.4,0.8)/v \in \mathcal{V}_{NSC}\}$

is not an $\mathcal{S}$ Ext NSCSs in $\mathcal{V}_{NSC}$.

Since

$(\mathcal{S}_{\lambda_s} \vee_P \mathcal{S}_{\psi_s})(v) = 0.8 \in (0.7,0.9)$

$= (\mathcal{S}_{A_s} \cup_P \mathcal{S}_{B_s})^-(v), (\mathcal{S}_{A_s} \cup_P \mathcal{S}_{B_s})^+(v)$

also $A_s \cap_P B_s = (A_s \cap B_s, \lambda_s \wedge_P \psi_s)$ with

$A_s \cap B_s = \{(v, (0.3,0.5), (0.4,0.7), (0.3,0.5))/v \in \mathcal{V}_{NSC}\}$

$\lambda_s \wedge \psi_s = \{\langle v, 0.4,0.3,0.8\rangle/v \in \mathcal{V}_{NSC}\}$

is not an $\mathcal{S}$ Ext NSCS in $\mathcal{V}_{NSC}$ since

$(\mathcal{S}_{\lambda_s} \wedge_P \mathcal{S}_{\psi_s})(v) = 0.4 \in (0.4,0.7)$

$= (\mathcal{S}_{A_s} \cap_P \mathcal{S}_{B_s})^-(v), (\mathcal{S}_{A_s} \cap_P \mathcal{S}_{B_s})^+(v)$

Example 3.18 For $\mathcal{V}_{NSC} = \{v_1, v_2, v_3\}$, let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be NSCSs in $\mathcal{V}_{NSC}$ with the Table 0.21 and 0.21, respectively.

Table 12: $\mathcal{C}_{NSC} = (A_s(v), \lambda_s)(v)$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\lambda_s(v)$
v_1	$([0.3,0.4], [0.5,1.0], [0.2,0.3])$	$(0.35,0.55,0.25)$
v_2	$([0.2,0.3], [0.4,1.0], [0.4,0.6])$	$(0.25,0.45,0.45)$
v_3	$([0.6,0.7], [0.1,1.0], [0.3,0.4])$	$(0.65,0.15,0.35)$

Table 13: $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$

$\mathcal{V}_{NSC}$	$A_s(v)$	$\lambda_s(v)$
v_1	$([0.2,0.4], [0.7,1.0], [0.1,0.2])$	$(0.20,0.75,0.15)$
v_2	$([0.5,0.6], [0.2,1.0], [0.3,0.4])$	$(0.55,0.25,0.25)$
v_3	$([0.4,0.6], [0.4,1.0], [0.2,0.4])$	$(0.55,0.45,0.25)$

Then $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ are both $\mathcal{R}$ Ext and $\mathcal{J}$ Ext NSCSs in $\mathcal{V}_{NSC}$. $\mathcal{C}_{NSC} \cup_P \mathcal{B}_{NSC} = (A_s \cup_P B_s, \lambda_s \vee_P \psi_s)$ and $\mathcal{C}_{NSC} \cap_P \mathcal{B}_{NSC} = (A_s \cap_P B_s, \lambda_s \wedge_P \psi_s)$ are given below Tables 0.21 and 0.21 .

Table 14: $\mathcal{C}_{NSC} \cup_P \mathcal{B}_{NSC} = (A_s \cup_P B_s, \lambda_s \vee_P \psi_s)$

$\mathcal{V}_{NSC}$	$(A_s \cup B_s(v)$	$(\lambda_s \vee \psi_s(v)$
v_1	([0.3,0.4], [0.7,1.0], [0.2,0.3])	(0.35,0.75,0.25)
v_2	([0.5,0.6], [0.4,1.0], [0.4,0.6])	(0.55,0.45,0.45)
v_3	([0.6,0.7], [0.4,1.0], [0.4,0.5])	(0.65,0.45,0.35)

Table 15: $\mathcal{C}_{NSC} \cap_P \mathcal{B}_{NSC} = (A_s \cap_P B_s, \lambda_s \wedge_P \psi_s)$

$\mathcal{V}_{NSC}$	$(A_s \cap_P B_s(v)$	$(\lambda_s \wedge_P \psi_s(v)$
v_1	([0.2,0.4], [0.5,1.0], [0.1,0.2])	(0.30,0.55,0.15)
v_2	([0.2,0.3], [0.2,1.0], [0.3,0.4])	(0.25,0.25,0.35)
v_3	([0.4,0.6], [0.1,1.0], [0.2,0.4])	(0.55,0.15,0.35)

Then $\mathcal{C}_{NSC} \cup_P \mathcal{B}_{NSC}$ is neither an $\mathcal{J}$ Ext NSCS nor a $\mathcal{R}$ Ext NSCS in $\mathcal{V}_{NSC}$ since

$$(\mathcal{J}_{\lambda_s} \vee_P \mathcal{J}_{\psi_s})(c) = 1.0 \in (0.2,1.0) = ((\mathcal{J}_{A_s} \cup_P \mathcal{J}_{B_s})^-(c), (\mathcal{J}_{A_s} \cup_P \mathcal{J}_{B_s})^+(c))$$

and

$$(\mathcal{R}_{\lambda_s} \vee_P \mathcal{R}_{\psi_s})(a) = 0.35 \in (0.3,0.4) = ((\mathcal{R}_{A_s} \cup_P \mathcal{R}_{B_s})^-(a), (\mathcal{R}_{A_s} \cup_P \mathcal{R}_{B_s})^+(a)).$$

Remark 3.19 $R-$ union and $R-$ intersection of $\mathcal{R}$ Int (resp. $\mathcal{J}$ Int and $\mathcal{S}$ Int) NSCSs may not be $\mathcal{A}$ Int (resp. $\mathcal{J}$ Int and $\mathcal{S}$ Int) NSCSs in the below examples.

Example 3.20 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be NSCSs in $\mathcal{V}_{NSC}$ where

$$A_s(v) = \{(v, (0.3,0.5), (0.4,1.0), (0.3,0.4))/v \in \mathcal{V}_{NSC}\}$$
$$\lambda_s(v) = \{(v, 0.4,0.2,0.4)/v \in \mathcal{V}_{NSC}\}$$
$$B_s(v) = \{(v, (0.5,0.6), (0.2,1.0), (0.3,0.2))/v \in \mathcal{V}_{NSC}\}$$
$$\psi_s(v) = \{(v, 0.5,0.3,0.2)/v \in \mathcal{V}_{NSC}\}$$

Then $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ are $\mathcal{R}$ Int NSCSs in $\mathcal{V}_{NSC}$ and $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup_R B_s, \lambda_s \wedge_R \psi_s)$ with

$$A_s \cup_R B_s = \{\langle v, [0.5,0.6], [0.4,1.0], [0.3,0.4]\rangle/v \in \mathcal{V}_{NSC}\},$$
$$\lambda_s \wedge_R \psi_s = \{\langle v, 0.4,0.2,0.2\rangle/v \in \mathcal{V}_{NSC}\}.$$

Note that $(\mathcal{R}_{\lambda_s} \wedge_R \mathcal{R}_{\psi_s})(v) = 0.4 < 0.5 = (\mathcal{R}_{A_s} \cup_R \mathcal{R}_{B_s})^-(v)$ and $(\mathcal{R}_{\lambda_s} \wedge_R \mathcal{J}_{\psi_s})(v) = 0.2 < 0.3 = (\mathcal{J}_{A_s} \cup_R \mathcal{J}_{B_s})^-(v)$. Hence $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup_R B_s, \lambda_s \wedge_R \psi_s)$ is neither a $\mathcal{R}$ Int NSCS nor a $\mathcal{J}$ Int NSCS in $\mathcal{V}_{NSC}$. But we know that $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup B_s, \lambda_s \wedge \psi_s)$ is an $\mathcal{S}$ Int NSCS in $\mathcal{V}_{NSC}$.

The R −intersection $\mathcal{C}_{NSC} \cap_R \mathcal{B}_{NSC} = (A_s \cap B_s, \lambda_s \vee \psi_s)$ with

$$A_s \cap_R B_s = \{\langle v, [0.3,0.5], [0.2,1.0], [0.3,0.2]\rangle/v \in \mathcal{V}_{NSC}\},$$
$$\lambda_s \vee_R \psi_s = \{\langle v, 0.5,0.3,0.4\rangle/v \in \mathcal{V}_{NSC}\}.$$

Since $(\mathcal{J}_{A_s} \cap_R \mathcal{J}_{B_s})^-(v) \leq (\mathcal{J}_{\lambda_s} \vee_R \mathcal{J}_{\psi_s})(v) \leq (\mathcal{J}_{A_s} \cap_R \mathcal{J}_{B_s})^+(v)$ for all $v \in \mathcal{V}_{NSC}$. $\mathcal{C}_{NSC} \cap_R \mathcal{B}_{NSC} = (A_s \cap B_s, \lambda_s \vee_R \psi_s)$ is an $\mathcal{J}$ Int NSCS in $\mathcal{V}_{NSC}$.

But neither a $\mathcal{R}$ Int NSCS $\mathcal{S}$ Int NSCSs in $\mathcal{V}_{NSC}$.

Example 3.21 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be NSCSs in $\mathcal{V}_{NSC}$ where

$$A_s = \{(v, (0.3,0.5), (0.5,1.0), (0.2,0.4))/v \in \mathcal{V}_{NSC}\}$$
$$\lambda_s = \{(v, 0.4,0.2,0.4)/v \in \mathcal{V}_{NSC}\}$$
$$B_s = \{(v, (0.1,0.5), (0.6,1.0), (0.3,0.5))/v \in \mathcal{V}_{NSC}\}$$
$$\psi_s = \{(v, 0.3,0.2,0.5)/v \in \mathcal{V}_{NSC}\}$$

Then $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ are $\mathcal{J}$ Int NSCSs in $\mathcal{V}_{NSC}$ and $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup_R B_s, \lambda_s \wedge_R \psi_s)$ with

$$A_s \cup_R B_s = \{\langle v, [0.3,0.5], [0.6,1.0], [0.3,0.5]\rangle/v \in \mathcal{V}_{NSC}\},$$
$$\lambda_s \wedge_R \psi_s = \{\langle v, 0.3,0.2,0.4\rangle/v \in \mathcal{V}_{NSC}\}.$$

Since $(\mathcal{J}_{\lambda_s} \wedge_R \mathcal{J}_{\psi_s})(v) = 0.2 < 0.6 = (\mathcal{J}_A \cup_R \mathcal{J}_B)^-(v)$ we know that and $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC}$ is not an $\mathcal{J}$ Int NSCS in $\mathcal{V}_{NSC}$.

Also, the R −intersection $\mathcal{C}_{NSC} \cap_R \mathcal{B}_{NSC} = (A_s \cap B_s, \lambda_s \vee_R \psi_s)$ with

$$A_s \cap_R B_s = \{\langle v, [0.1,0.5], [0.5,1.0], [0.2,0.4]\rangle/v \in \mathcal{V}_{NSC}\},$$

$\lambda_s \vee_R \psi_s = \{\langle v, 0.4,0.2,0.5\rangle / v \in \mathcal{V}_{NSC}\}$

and it is not an $\mathcal{J}$ Int NSCS in $\mathcal{V}_{NSC}$.

Example 3.22 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be NSCSs in $\mathcal{V}_{NSC}$ where

$A_s = \{(v, (0.2,0.3), (0.4,1.0), (0.4,0.5))/v \in \mathcal{V}_{NSC}\}$

$\lambda_s = \{(v, 0.4,0.2,0.2)/v \in \mathcal{V}_{NSC}\}$

$B_s = \{(v, (0.4,0.6), (0.3,1.0), (0.3,0.4))/v \in \mathcal{V}_{NSC}\}$

$\psi_s = \{(v, 0.3,0.2,0.1)/v \in \mathcal{V}_{NSC}\}$

Then $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ are $\mathcal{S}$ Int NSCSs in $\mathcal{V}_{NSC}$ and $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup_R B_s, \lambda_s \wedge_R \psi_s)$ with

$A_s \cup_R B_s = \{\langle v, [0.4,0.6], [0.4,1.0], [0.4,0.5]\rangle / v \in \mathcal{V}_{NSC}\}$,

$\lambda_s \wedge_R \psi_s = \{\langle v, 0.3,0.2,0.1\rangle / v \in \mathcal{V}_{NSC}\}$.

which is not an $\mathcal{S}$ Int NSCS in $\mathcal{V}_{NSC}$. If $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be NSCSs in $\mathcal{R}$ where

$A_s(v) = \{(v, (0.2,0.3), (0.6,1.0), (0.2,0.4))/v \in \mathcal{V}_{NSC}\}$

$\lambda_s(v) = \{(v, 0.5,0.4,0.1)/v \in \mathcal{R}\}$

$B_s(v) = \{(v, (0.1,0.2), (0.5,1.0), (0.4,0.5))/v \in \mathcal{V}_{NSC}\}$

$\psi_s(v) = \{(v, 0.6,0.2,0.2)/v \in \mathcal{V}_{NSC}\}$

then $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ are $\mathcal{S}$ Int NSCSs in $\mathcal{V}_{NSC}$ and the $R-$ intersection $\mathcal{C}_{NSC} \cap_R \mathcal{B}_{NSC} = (A_s \cap_R B_s, \lambda_s \vee_R \psi_s)$ of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ which is given as follows:

$A_s \cap_R B_s = \{\langle x, [0.1,0.2], [0.5,1.0], [0.2,0.4]\rangle / v \in \mathcal{V}_{NSC}\}$,

$\lambda_s \vee_R \psi_s = \{\langle v, 0.6,0.4,0.2\rangle / v \in \mathcal{V}_{NSC}\}$

and it is not an $\mathcal{S}$ Int NSCS in $\mathcal{V}_{NSC}$.

Theorem 3.23 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be $\mathcal{R}$ Int NSCSs in $\mathcal{V}_{NSC}$ such that

$$(\forall \ v \in \mathcal{V}_{NSC})(\max\{\mathcal{R}_A^-(v), \mathcal{R}_B^-(v)\} \leq (\mathcal{R}_{\lambda_s} \wedge_R \mathcal{R}_{\psi_s})(v)). \tag{7}$$

Then the $R-$ union of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is a $\mathcal{R}$ Int NSCSs in $\mathcal{V}_{NSC}$.

Proof. Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be $\mathcal{R}$ Int NSCSs in $\mathcal{V}_{NSC}$ it satisfy the condition (7). Then $\mathcal{R}_{A_s}^-(v) \leq \mathcal{R}_{\lambda_s}(v) \leq \mathcal{R}_{A_s}^+(v)$ and $\mathcal{R}_{B_s}^-(v) \leq \mathcal{R}_{\psi_s}(v) \leq \mathcal{R}_{B_s}^+(v)$.

And so, $(\mathcal{R}_{\lambda_s} \wedge_R \mathcal{R}_{\psi_s})(v) \leq (\mathcal{R}_{A_s} \cup_R \mathcal{R}_{B_s})^+(v)$.

It follows from (7) that,

$(\mathcal{R}_{A_s} \cup_R \mathcal{R}_{B_s})^-(v) = \max\{\mathcal{R}_{A_s})^-(v), \mathcal{R}_{B_s})^-(v)\}$

$\leq (\mathcal{R}_{\lambda_s} \wedge_R \mathcal{R}_{\psi_s})(v) \leq (\mathcal{R}_{A_s} \cup_R \mathcal{R}_{B_s})^+(v)$.

Hence $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup_R B_s, \lambda_s \wedge_R \psi_s)$ is a $\mathcal{R}$ Int NSCS in $\mathcal{V}_{NSC}$.

Theorem 3.24 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be $\mathcal{J}$ Int NSCSs in $\mathcal{V}_{NSC}$ such that

$$(\forall \;\; v \in \mathcal{V}_{NSC})(\max\{\mathcal{J}_A^-(v), \mathcal{J}_B^-(v)\} \leq (\mathcal{J}_{\lambda_s} \wedge_R \mathcal{J}_{\psi_s})(v)). \quad (8)$$

Then the $R-$ union of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is a $\mathcal{J}$ Int NSCSs in a non-empty set $\mathcal{V}_{NSC}$.

Proof. Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be $\mathcal{J}$ Int NSCSs in $\mathcal{V}_{NSC}$ it satisfy the condition (8). Then $\mathcal{J}_{A_s}^-(v) \leq \mathcal{J}_{\lambda_s}(v) \leq \mathcal{J}_{A_s}^+(v)$ and $\mathcal{J}_{B_s}^-(v) \leq \mathcal{J}_{\psi_s}(v) \leq \mathcal{J}_{B_s}^+(v)$.

And so, $(\mathcal{J}_{\lambda_s} \wedge_R \mathcal{J}_{\psi_s})(v) \leq (\mathcal{J}_{A_s} \cup_R \mathcal{J}_{B_s})^+(v)$.

It follows from (8) that,

$$(\mathcal{J}_{A_s} \cup_R \mathcal{J}_{B_s})^-(v) = \max\{\mathcal{J}_{A_s})^-(v), \mathcal{J}_{B_s})^-(v)\}$$
$$\leq (\mathcal{J}_{\lambda_s} \wedge_R \mathcal{J}_{\psi_s})(v) \leq (\mathcal{J}_{A_s} \cup_R \mathcal{J}_{B_s})^+(v).$$

Hence $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup_R B_s, \lambda_s \wedge_R \psi_s)$ is a $\mathcal{J}$ Int NSCS in $\mathcal{V}_{NSC}$.

Theorem 3.25 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be $\mathcal{S}$ Int NSCSs in $\mathcal{V}_{NSC}$ such that

$$(\forall \;\; v \in \mathcal{V}_{NSC})(\max\{\mathcal{S}_A^-(v), \mathcal{S}_B^-(v)\} \leq (\mathcal{S}_{\lambda_s} \wedge_R \mathcal{S}_{\psi_s})(v)). \quad (9)$$

Then the $R-$ union of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is a $\mathcal{S}$ Int NSCSs in $\mathcal{V}_{NSC}$.

Proof. Let $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ be $\mathcal{S}$ Int NSCSs in $\mathcal{V}_{NSC}$ it satisfy the condition (9). Then $\mathcal{S}_{A_s}^-(v) \leq \mathcal{S}_{\lambda_s}(v) \leq \mathcal{S}_{A_s}^+(v)$ and $\mathcal{S}_{B_s}^-(v) \leq \mathcal{A}_{\psi_s}(v) \leq \mathcal{S}_{B_s}^+(v)$.

And so, $(\mathcal{S}_{\lambda_s} \wedge_R \mathcal{S}_{\psi_s})(v) \leq (\mathcal{S}_{A_s} \cup_R \mathcal{S}_{B_s})^+(v)$.

It follows from (9) that,

$$(\mathcal{S}_{A_s} \cup_R \mathcal{S}_{B_s})^-(v) = \max\{\mathcal{S}_{A_s})^-(v), \mathcal{S}_{B_s})^-(v)\}$$
$$\leq (\mathcal{S}_{\lambda_s} \wedge_R \mathcal{S}_{\psi_s})(v) \leq (\mathcal{S}_{A_s} \cup_R \mathcal{S}_{B_s})^+(v).$$

Hence $\mathcal{C}_{NSC} \cup_R \mathcal{B}_{NSC} = (A_s \cup_R B_s, \lambda_s \wedge_R \psi_s)$ is a $\mathcal{S}$ Int NSCS in $\mathcal{V}_{NSC}$.

Corollary 3.26 If two Int NSCSs $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ satisfy (7) ,(8)and (9) then the $R-$ union of $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ is an Int NSCSs.

Theorem 3.27 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be $\mathcal{J}$ Int NSCSs in $\mathcal{V}_{NSC}$ such that

$$(\forall \;\; v \in \mathcal{V}_{NSC})((\mathcal{J}_{\lambda_s} \vee_R \mathcal{J}_{\psi_s})(v) \leq \min\{\mathcal{J}_A^+(v), \mathcal{J}_B^+(v)\}). \quad (10)$$

Then the $R-$ intersection of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is a $\mathcal{J}$ Int NSCS in $\mathcal{V}_{NSC}$.

Proof. If 10 is valid. Then $\mathcal{J}_{A_s}^-(v) \leq \mathcal{J}_{\lambda_s}(v) \leq \mathcal{J}_{A_s}^+(v)$ and $\mathcal{J}_{B_s}^-(v) \leq \mathcal{J}_{\psi_s}(v) \leq \mathcal{J}_{B_s}^+(v)$ for all $\mathcal{V}_{NSC}$. It follows from 10 that

$(\mathcal{J}_{A_s} \cap_R \mathcal{J}_{B_s})^{-}(v) \leq (\mathcal{J}_{\lambda_s} \cap_R \mathcal{J}_{\psi_s})^{-}(v) \leq \min\{\mathcal{J}^{+}_{A_s}(v), \mathcal{J}^{+}_{B_s}(v)\} \leq (\mathcal{J}_{A_s} \cap_R \mathcal{J}_{B_s})^{+}(v)$
for all $v \in \mathcal{V}_{NSC}$.

$\therefore$ $A_s \cap_R B_s = (A_s \cap_R B_s, \lambda_s \vee_R \psi_s)$ is an $\mathcal{J}$ Int NSCS in $\mathcal{V}_{NSC}$.

Theorem 3.28 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be $\mathcal{R}$ Int NSCSs in $\mathcal{V}_{NSC}$ such that

$$(\forall\ v \in \mathcal{V}_{NSC})((\mathcal{R}_{\lambda_s} \vee_R \mathcal{R}_{\psi_s})(v) \leq \min\{\mathcal{R}^{+}_{A}(v), \mathcal{R}^{+}_{B}(v)\}). \quad (11)$$

Then the $R-$ intersection of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is a $\mathcal{R}$ Int NSCS in $\mathcal{V}_{NSC}$.

Proof. If 11 is valid. Then $\mathcal{R}^{-}_{A_s}(v) \leq \mathcal{R}_{\lambda_s}(v) \leq \mathcal{R}^{+}_{A_s}(v)$ and $\mathcal{R}^{-}_{B_s}(v) \leq \mathcal{R}_{\psi_s}(v) \leq \mathcal{R}^{+}_{B_s}(v)$ for all $\mathcal{V}_{NSC}$. It follows from 11 that
$(\mathcal{R}_{A_s} \cap \mathcal{R}_{B_s})^{-}(v) \leq (\mathcal{R}_{\lambda_s} \cap_R \mathcal{R}_{\psi_s})^{-}(v) \leq \min\{\mathcal{R}^{+}_{A_s}(v), \mathcal{R}^{+}_{B_s}(v)\} \leq (\mathcal{R}_{A_s} \cap_R \mathcal{R}_{B_s})^{+}(v)$
for all $v \in \mathcal{V}_{NSC}$.

$\therefore$ $A_s \cap_R B_s = (A_s \cap_R B_s, \lambda_s \vee_R \psi_s)$ is an $\mathcal{R}$Int NSCS in $\mathcal{V}_{NSC}$.

Theorem 3.29 Let $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ be $\mathcal{S}$ Int NSCSs in $\mathcal{V}_{NSC}$ such that

$$(\forall\ v \in \mathcal{V}_{NSC})((\mathcal{S}_{\lambda_s} \vee_R \mathcal{S}_{\psi_s})(v) \leq \min\{\mathcal{S}^{+}_{A}(v), \mathcal{S}^{+}_{B}(v)\}). \quad (12)$$

Then the $R-$ intersection of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is a $\mathcal{S}$ Int NSCS in $\mathcal{V}_{NSC}$.

Proof. If 12 is valid. Then $\mathcal{S}^{-}_{A_s}(v) \leq \mathcal{S}_{\lambda_s}(v) \leq \mathcal{S}^{+}_{A_s}(v)$ and $\mathcal{S}^{-}_{B_s}(v) \leq \mathcal{S}_{\psi_s}(v) \leq \mathcal{S}^{+}_{B_s}(v)$ for all $\mathcal{V}_{NSC}$. It follows from 12 that
$(\mathcal{S}_{A_s} \cap_R \mathcal{S}_{B_s})^{-}(v) \leq (\mathcal{S}_{\lambda_s} \cap \mathcal{S}_{\psi_s})^{-}(v) \leq \min\{\mathcal{S}^{+}_{A_s}(v), \mathcal{S}^{+}_{B_s}(v)\} \leq (\mathcal{S}_{A_s} \cap_R \mathcal{S}_{B_s})^{+}(v)$
for all $v \in \mathcal{V}_{NSC}$.

$\therefore$ $A_s \cap_R B_s = (A_s \cap_R B_s, \lambda_s \vee_R \psi_s)$ is an $\mathcal{S}$ Int NSCS in $\mathcal{V}_{NSC}$.

Corollary 3.30 If two Int NSCSs $\mathcal{C}_{NSC} = (A_s(v), \lambda_s(v))$ and $\mathcal{B}_{NSC} = (B_s(v), \psi_s(v))$ satisfy conditions (10), (11), (12) then the $R-$intersection of $\mathcal{C}_{NSC}$ and $\mathcal{B}_{NSC}$ is an Int NSCS in $\mathcal{V}_{NSC}$.

Conclusions

In this paper we have introduced the notion of Neutrosophic spherical cubic sets .We have discussed properties of Neutrosophic spherical cubic sets. For the future prospects, we will extend this work by using topological structures and commit to exploring the real life applications.

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REFERENCES

1. Broumi, S. ;Smarandache, F. Correlation coefficient of interval neutrosophic sets, *AppliedMechanics and Materials*, *436***(2013)** 511-517.
2. Jun, Y.B. ; Kim, C.S. ; and Yang, K.O. Cubic sets, *Annals of Fuzzy Mathematics and Informatics*, 4(3) **(2012)** 83-98.
3. Jun, Y.B. ; Kim, C.S. ; and Kang, M.S. Cubic subalgebra and ideals of BCK/BCI-algebras, *Far East.J.Math. Sci.(FJMS),* 44 **(2010)**239-250.
4. Jun, Y.B. ; Kim, C.S. ; and Yang, K.O. Smarandache, F. Neutrosophic Cubic sets, *NewMathematics and Natural Computation,* **(2015)** .
5. KutluGündoğdu, Fatmaa ; * | Kahraman, Cengiz Spherical fuzzy sets and spherical fuzzy TOPSIS method *Journal of Intelligent & Fuzzy Systems*, **(2019)** vol. 36, no. 1, pp. 337-352**.**
6. Gündogdu FK, Kahraman C A novel VIKOR method using spherical fuzzy sets and its application to warehouse site selection. *J Intell Fuzzy Syst,***(2019)** 37(1):1197–1211.
7. Smarandache, F. A Unifying Field in Logics. Neutrosophy ,Neutrosophic Probability, Set and Logic, Rehoboth: *American Research Press*,(**1999).**
8. Smarandache, F. Neutrosophic set-a generalization of the intuitionistic fuzzy set, *Journal of Defense Resource Management*, 1(1)**(2010)** .
9. TanmoyMahapatra, ;Sankarsahoo, ; Ganesh Ghorai, ; Madhumangal Pal Interval valued m-polar fuzzy planar graph and its applications, *Artificial Intelligence Review,***(2021)** 54:1649–1675.
10. TanmoyMahapatra, ; Ganesh Ghorai, ; Madhumangal Pal Fuzzy fractional coloring of fuzzy graphs with its applications , *Journal of Ambient Intelligence and Humanized Computing,* **(2020)** 11:5771–5784 .
11. TanmoyMahapatra, ; Ganesh Ghorai, ; Madhumangal Pal Fuzzy coloring of fuzzy m-polar fuzzy planar graph and its applications, *Journal of Intelligent & fuzzy systems,***(2018)** vol 35, no 6, pp. 6379-6391.
12. Zadeh, L. A. Fuzzy sets, *Information and Control 8***(1965)** 338-353.

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