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The Computing of Pythagoras Triples in Symbolic 2-Plithogenic Rings

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Abstract:

This paper is dedicated to finding a general algorithm for generating different solutions for Pythagoras' non-linear Diophantine equation in four variables $x^2 + y^2 = z^2$ in symbolic 2-plithogenic rings, which are known as Pythagoras triples.

Also, we present some examples of those triples in some finite symbolic 2-plithogenic rings.

Keywords: symbolic 2-plithogenic ring, Pythagoras triples, Diophantine equations

Introduction and Preliminaries

Symbolic n-plithogenic algebraic structures are a new generalization of classical algebraic structures, as they have serious algebraic properties to study.

In the previous literature, we can clearly note several algebraic studies that were interested in discovering the properties of these algebraic structures, for example, we can find some applications of plithogenic structures in probability, ring theory, linear spaces, matrices, and equations [1-10].

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Researchers have studied Pythagorean triples in the ring of ordinary algebraic numbers [11-14]. Several efficient algorithms for calculating these quadruples have been presented, as solutions to the corresponding Diophantine equation.

This has motivated us to study Pythagoras triples in the symbolic 2-plithogenic commutative case, where we find a general algorithm for generating different solutions for Pythagoras non-linear Diophantine equation in four variables $\boldsymbol{x^2} + \boldsymbol{y^2} = \boldsymbol{z^2}$ in symbolic 2-plithogenic rings.

Definition.

The symbolic 2-plithogenic ring of real numbers is defined as follows:

$$2 - SP_R = \left\{t_0 + t_1P_1 + t_2P_2; t_i \in R, P_1 \times P_2 = P_2 \times P_1 = P_2, {P_1}^2 = {P_2}^2 = P_2\right\}$$

The addition operation on $2 - SP_R$ is defined as follows:

$$(t_0 + t_1P_1 + t_2P_2) + (t_0' + t_1'P_1 + t_2'P_2) = (t_0 + t_0') + (t_1 + t_1')P_1 + (t_2 + t_2')P_2$$

The multiplication on $2 - SP_R$ is defined as follows:

$$(t_0 + t_1P_1 + t_2P_2)(t_0' + t_1'P_1 + t_2'P_2)$$
$$= t_0t_0' + (t_0t_1' + t_1t_0' + t_1t_1')P_1 + (t_0t_2' + t_1t_2' + t_2t_2' + t_2t_0' + t_2t_1')P_2$$

Main Discussion

Definition.

Let R be a ring, then (t, s, k) is called a Pythagoras triple if and only if $t^2 + s^2 = k^2; t, s, k \in R$..

Theorem.

Let $T = t_0 + t_1P_1 + t_2P_2, S = s_0 + s_1P_1 + s_2P_2, K = k_0 + k_1P_1 + k_2P_2$ are three arbitrary symbolic 3-plithogenic elements $T, S, K \in 2 - SP_R$, then (T, S, K) are Pythagoras triple in $2 - SP_R$ if and only if:

$$\begin{cases} (t_0, s_0, k_0), (t_0 + t_1, s_0 + s_1, k_0 + k_1)\ are\ pythagoras\ triples\ in\ R \\ (t_0 + t_1 + t_2, s_0 + s_1 + s_2, k_0 + k_1 + k_2)\ is\ pythagoras\ triple\ in\ R \end{cases}$$

Proof.

According to [ ], we have:

$$T^2 = {t_0}^2 + [(t_0 + t_1)^2 - {t_0}^2]P_1 + [(t_0 + t_1 + t_2)^2 - (t_0 + t_1)^2]P_2$$

$$S^2 = {s_0}^2 + [(s_0 + s_1)^2 - {s_0}^2]P_1 + [(s_0 + s_1 + s_2)^2 - (s_0 + s_1)^2]P_2$$

$T^2 = {k_0}^2 + \left[(k_0 + k_1)^2 - {k_0}^2\right]P_1 + [(k_0 + k_1 + k_2)^2 - (k_0 + k_1)^2]P_2$

The equation $T^2 + S^2 = K^2$ is equivalent to:

${t_0}^2 + {s_0}^2 = {k_0}^2$ $(equation\ 1)$,

$(t_0 + t_1)^2 + (s_0 + s_1)^2 = (k_0 + k_1)^2$ $(equation\ 2)$,

$(t_0 + t_1 + t_2)^2 + (s_0 + s_1 + s_2)^2 = (k_0 + k_1 + k_2)^2$ $(equation\ 3)$,

Equation (1) implies that (t_0, s_0, k_0) is a Pythagoras triple in R.

Equation (2) implies that $(t_0 + t_1, s_0 + s_1, k_0 + k_1)$ is a Pythagoras triple in R.

Equation (3) implies that $(t_0 + t_1 + t_2, s_0 + s_1 + s_2, k_0 + k_1 + k_2)$ is a Pythagoras triple in R.

Thus the proof is complete.

Theorem.

Let $(t_0, s_0, k_0), (t_1, s_1, k_1), (t_2, s_2, k_2)$be three Pythagoras triples in the ring R, then (T, S, K) Pythagoras triple in $3 - SP_R$, where:

$T = t_0 + [t_1 - t_0]P_1 + [t_2 - t_1]P_2$.

$S = s_0 + [s_1 - s_0]P_1 + [s_2 - s_1]P_2$.

$K = k_0 + [k_1 - k_0]P_1 + [k_2 - k_1]P_2$.

Proof.

We have: $t_0 + (t_1 - t_0) = t_1, t_0 + (t_1 - t_0) + (t_2 - t_1) = t_2$.

$s_0 + (s_1 - s_0) = s_1, s_0 + (s_1 - s_0) + (s_2 - s_1) = s_2$,

$k_0 + (k_1 - k_0) = k_1, k_0 + (k_1 - k_0) + (k_2 - k_1) = k_2$.

This implies that (T, S, K) Pythagoras triple in $2 - SP_R$ according to the theorem.

Examples.

We have:

$$\begin{cases} (t_0, s_0, k_0) = (3,4,5) \\ (t_1, s_1, k_1) = (6,8,10) \\ (t_2, s_2, k_2) = (4,3,5) \end{cases}$$

Are three Pythagoras triples in Z.

The corresponding symbolic 2-plithogenic Pythagoras triple is (T, S, K), where:

$T = 3 + [6 - 3]P_1 + [4 - 6]P_2 = 3 + 3P_1 - 2P_2$

$S = 4 + [8 - 4]P_1 + [3 - 8]P_2 = 4 + 4P_1 - 5P_2$

$K = 5 + [10 - 5]P_1 + [5 - 10]P_2 = 5 + 5P_1 - 5P_2$

Example.

Find all Pythagoras triples in $2 - SP_{Z_2}$, where Z_2 I the ring of integers module 2.

First, we find all Pythagoras triples in Z.

$L_1 = (0,0,0), L_2 = (1,0,1), L_3 = (0,1,1), L_4 = (1,1,0)$

Remark that for every permutation of the set $\{L_1, L_2, L_3, L_4\}$, we get a different symbolic 2-plithogenic Pythagoras triple.

We discuss all possible cases:

Permutation (1).

$$\begin{cases} Y_1 = P_1 - P_2 = P_1 + P_2 \\ \acute{Y}_1 = P_2 \\ Y_1'' = P_1 = P_1 \end{cases}$$

Permutation (2).

$$\begin{cases} Y_2 = P_2 \\ \acute{Y}_2 = P_1 - P_2 = P_1 + P_2 \\ Y_2'' = P_1 = P_1 \end{cases}$$

Permutation (3).

$$\begin{cases} Y_3 = P_1 \\ \acute{Y}_3 = P_1 + P_2 \\ Y_3'' = P_2 \end{cases}$$

Permutation (4).

$$\begin{cases} Y_4 = P_1 + P_2 \\ \acute{Y}_4 = P_1 \\ Y_4'' = P_2 \end{cases}$$

Permutation (5).

$$\begin{cases} Y_5 = P_1 \\ \acute{Y}_5 = P_2 \\ Y_5'' = P_1 + P_2 \end{cases}$$

Permutation (6).

$$\begin{cases} Y_6 = P_2 \\ \acute{Y}_6 = P_1 \\ Y_6'' = P_1 + P_2 \end{cases}$$

Permutation (7).

$$\begin{cases} Y_7 = 1 + P_2 \\ Y_7' = P_1 \\ Y_7'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (8).

$$\begin{cases} Y_8 = P_2 \\ Y_8' = 1 + P_1 + P_2 \\ Y_8'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (9).

$$\begin{cases} Y_9 = 1 + P_1 \\ Y_9' = P_1 + P_2 \\ Y_9'' = 1 + P_2 \end{cases}$$

Permutation (10).

$$: \begin{cases} Y_{10} = P_1 + P_2 \\ Y_{10}' = 1 + P_1 \\ Y_{10}'' = 1 + P_2 \end{cases}$$

Permutation (11).

$$\begin{cases} Y_{11} = 1 + P_2 \\ Y_{11}' = 1 + P_1 \\ Y_{11}'' = P_1 + P_2 \end{cases}$$

Permutation (12).

$$\begin{cases} Y_{12} = 1 + P_1 + P_2 \\ Y_{12}' = 1 + P_1 \\ Y_{12}'' = P_2 \end{cases}$$

Permutation (13).

$$\begin{cases} Y_{13} = 1 + P_1 + P_2 \\ Y_{13}' = 1 + P_2 \\ Y_{13}'' = P_1 \end{cases}$$

Permutation (14).

$$\begin{cases} Y_{14} = 1 + P_1 \\ Y_{14}' = 1 + P_2 \\ Y_{14}'' = P_1 + P_2 \end{cases}$$

Permutation (15).

$$\begin{cases} Y_{15} = 1 + P_1 \\ Y_{15}' = 1 + P_1 + P_2 \\ Y_{15}'' = P_2 \end{cases}$$

Permutation (16).

$$\begin{cases} Y_{16} = 1 + P_2 \\ \acute{Y_{16}} = 1 + P_1 + P_2 \\ Y_{16}'' = P_1 \end{cases}$$

Permutation (17).

$$\begin{cases} Y_{17} = 1 + P_2 \\ \acute{Y_{17}} = P_1 + P_2 \\ Y_{17}'' = 1 + P_1 \end{cases}$$

Permutation (18).

$$\begin{cases} Y_{18} = 1 + P_2 \\ \acute{Y_{18}} = P_1 \\ Y_{18}'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (19).

$$: \begin{cases} Y_{19} = P_1 \\ \acute{Y_{19}} = 1 + P_1 + P_2 \\ Y_{19}'' = 1 + P_2 \end{cases}$$

Permutation (20).

$$\begin{cases} Y_{20} = P_1 \\ \acute{Y_{20}} = 1 + P_1 + P_2 \\ Y_{20}'' = 1 + P_2 \end{cases}$$

Permutation (21).

$$: \begin{cases} Y_{21} = P_1 + P_2 \\ \acute{Y_{21}} = 1 + P_2 \\ Y_{21}'' = 1 + P_1 \end{cases}$$

Permutation (22).

$$\begin{cases} Y_{22} = 1 + P_1 \\ \acute{Y_{22}} = P_1 + P_2 \\ Y_{22}'' = 1 + P_2 \end{cases}$$

Permutation (23).

$$\begin{cases} Y_{23} = P_1 \\ \acute{Y_{23}} = 1 + P_2 \\ Y_{23}'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (24).

$$\begin{cases} Y_{24} = P_1 + P_2 \\ \acute{Y_{24}} = 1 + P_1 \\ Y_{24}'' = 1 + P_2 \end{cases}$$

Also, other quadraples $(L_i, L_j, L_k, L_s); 1 \leq i, j, k, s \leq 4$ give Pythagoras triples with i, j, k, s are not distinct at all.

We continuo our discussions.

Permutation (25).

$$\begin{cases} Y_{25} = (0,0,0) \\ Y_{25}' = (0,0,0) \\ Y_{25}'' = (0,0,0) \end{cases}$$

Permutation (26).

$$\begin{cases} Y_{26} = P_1 + P_2 \\ Y_{26}' = 0 \\ Y_{26}'' = P_1 + P_2 \end{cases}$$

Permutation (27).

$$\begin{cases} Y_{27} = 0 \\ Y_{27}' = P_1 + P_2 \\ Y_{27}'' = P_1 + P_2 \end{cases}$$

Permutation (28).

$$\begin{cases} Y_{28} = P_1 + P_2 \\ Y_{28}' = P_1 + P_2 \\ Y_{28}'' = 0 \end{cases}$$

Permutation (29).

$$\begin{cases} Y_{29} = P_2 \\ Y_{29}' = 0 \\ Y_{29}'' = P_2 \end{cases}$$

Permutation (30).

$$\begin{cases} Y_{30} = 0 \\ Y_{30}' = P_2 \\ Y_{30}'' = P_2 \end{cases}$$

Permutation (31).

$$\begin{cases} Y_{31} = P_2 \\ Y_{31}' = P_2 \\ Y_{31}'' = 0 \end{cases}$$

Permutation (32).

$$\begin{cases} Y_{32} = 1 \\ Y_{32}' = 0 \\ Y_{32}'' = 1 \end{cases}$$

Permutation (33).

$$: \begin{cases} Y_{33} = 1 + P_2 \\ Y_{33}' = 0 \\ Y_{33}'' = 1 + P_2 \end{cases}$$

Permutation (34).

$$\begin{cases} Y_{34} = 1 + P_2 \\ Y_{34}' = P_2 \\ Y_{34}'' = 1 \end{cases}$$

Permutation (35).

$$\begin{cases} Y_{35} = 1 \\ Y_{35}' = P_2 \\ Y_{35}'' = 1 + P_2 \end{cases}$$

Permutation (36).

$$\begin{cases} Y_{36} = 1 + P_1 \\ Y_{36}' = 0 \\ Y_{36}'' = 1 + P_1 \end{cases}$$

Permutation (37).

$$\begin{cases} Y_{37} = 1 \\ Y_{37}' = P_1 + P_2 \\ Y_{37}'' = 1 + P_2 + P_1 \end{cases}$$

Permutation (38).

$$\begin{cases} Y_{38} = 1 + P_2 \\ Y_{38}' = P_1 + P_2 \\ Y_{38}'' = 1 + P_1 \end{cases}$$

Permutation (39).

$$\begin{cases} Y_{39} = 0 \\ Y_{39}' = 1 \\ Y_{39}'' = 1 \end{cases}$$

Permutation (40).

$$\begin{cases} Y_{40} = P_2 \\ Y_{40}' = 1 + P_2 \\ Y_{40}'' = 1 \end{cases}$$

Permutation (41).

$$\begin{cases} Y_{41} = P_2 \\ Y_{41}' = 1 \\ Y_{41}'' = 1 + P_2 \end{cases}$$

Permutation (42).

$$\begin{cases} Y_{42} = 0 \\ \acute{Y_{42}} = 1 + P_1 + P_2 \\ Y_{42}'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (43).

$$\begin{cases} Y_{43} = P_1 + P_2 \\ \acute{Y_{43}} = 1 + P_1 + P_2 \\ Y_{43}'' = 1 \end{cases}$$

Permutation (44).

$$\begin{cases} Y_{44} = P_1 + P_2 \\ \acute{Y_{44}} = 1 \\ Y_{44}'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (45).

$$\begin{cases} Y_{45} = 1 \\ \acute{Y_{45}} = 1 \\ Y_{45}'' = 0 \end{cases}$$

Permutation (46).

$$\begin{cases} Y_{46} = 1 + P_2 \\ \acute{Y_{46}} = 1 + P_2 \\ Y_{46}'' = 0 \end{cases}$$

Permutation (47).

$$\begin{cases} Y_{47} = 1 \\ \acute{Y_{47}} = 1 + P_2 \\ Y_{47}'' = P_2 \end{cases}$$

Permutation (48).

$$\begin{cases} Y_{48} = 1 + P_2 \\ \acute{Y_{48}} = 1 \\ Y_{48}'' = P_2 \end{cases}$$

Permutation (49).

$$\begin{cases} Y_{49} = 1 + P_1 + P_2 \\ \acute{Y_{49}} = 1 + P_1 + P_2 \\ Y_{49}'' = 0 \end{cases}$$

Permutation (50).

$$\begin{cases} Y_{50} = 1 \\ \acute{Y_{50}} = 1 + P_1 + P_2 \\ Y_{50}'' = P_1 + P_2 \end{cases}$$

Permutation (51).

$$\begin{cases} Y_{51} = 1 + P_1 + P_2 \\ Y_{51}' = 1 \\ Y_{51}'' = P_1 + P_2 \end{cases}$$

By continuing this argument, we can get all Pythagoras triples in $2 - SP_{Z_2}$

Conclusion.

In this paper, we have studied Pythagoras triples in symbolic 2-plithogenic commutative rings, where necessary and sufficient conditions for a symbolic 2-plithogenic triple (x, y, z) to be a Pythagoras triple.

Also, we have presented some related examples that explain how to find 2-plithogenic triples from classical triples.

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