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# Neutrosophic B-spline Surface Approximation Model for 3-Dimensional Data Collection

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**Abstract:** Since there are three membership functions: truth, false, and indeterminacy, geometrical modeling for B-spline surface approximation including neutrosophic data is particularly difficult to construct. Using neutrosophic set theory, this study introduces a neutrosophic B-spline for 3-dimensional data collecting. The neutrosophic control net was first introduced using the neutrosophic set notion. The control net is then merged with the B-spline basis function, and the approximation approach is used to display the B-spline surface. Following this work, there is a numerical demonstration of how to create the surface. As a result, the primary goal of this study is to offer the mathematical formulation and visualization of the neutrosophic B-spline surface approximation model for 3D data collecting.

**Keywords:** Neutrosophic B-spline Surface; Neutrosophic Control Net; Neutrosophic Set Theory, Neutrosophic B-spline Surface Approximation

## 1. Introduction

In modeling and addressing real-world situations, several mathematical tools have been established. Several researchers have been drawn to the concept of fuzzy set developed by Lotfi Zadeh [1] for problems involving imprecision, ambiguity, and uncertainty because of its potential for the recreation of human thinking as well as perception using linguistic information. Numerous hypotheses were created afterward to address the issue of impreciseness but in various structural forms. When fuzzy sets and fuzzy logic proposed by Zadeh cannot express false membership data, the neutrosophic theory proposed by Florentin Smarandache [2] is newly offered as an improved alternative; meanwhile, intuitionistic fuzzy sets and intuitionistic fuzzy logic proposed by Krasimir Atanassov [3] cannot handle data indeterminacy or imperfect information [17]. In 2014, Smarandache extended his neutrosophic logic study to n-valued refined neutrosophic logic for use in physics [32]. A neutrosophic multiset is an n-valued refined neutrosophic set. The neutrosophic multiset is expanded by Chatterjee to a single-valued neutrosophic multiset [33]. Following that, a combination of neutrosophic multisets and other uncertainty methods, such as rough multisets [34] and soft multisets [35], is introduced. This is due to the information's ambiguity and impreciseness, which always combines opposing and neutral knowledge. As a result, some academics have covered a few applications in their work that use fuzzy set, intuitionistic fuzzy set, and neutrosophic set theory [26-31].

The randomness of data collection has an impact on curve and surface design. This data is used as a control point for approximate and interpolate approaches in geometric modeling [4]. The data set is required for the creation of curves and surfaces, as well as the procedure itself. Uncertainty data affecting the curve and surface is frequently disregarded or discarded. Thus, for any problem to be addressed, data sets with some variability must be filtered before being utilized to generate surfaces and curves. In geometric modeling, there are three models: Bézier, B-spline, and non-uniform rational B-splines (NURBS). However, this study focuses on the B-spline surface model. Bernstein basis, or Bézier basis, is a specific case of B-spline basis (from Basis Spline). This foundation is not worldwide [22]. B-spline surfaces are non-global because each vertex has a basis function. The B-spline basis permits changing the basis function order and surface degree without changing the control polygon vertices. Piegl and Tiller introduced the mathematical representation for the B-spline surface approximation model [22].

Atanassov enhanced the fuzzy set theory with truth, falsehood, and uncertainty degrees in 1986 [3]. As fuzzy sets only accept full membership data, they can be employed when there is inadequate data for categorization and processing [5]. To cope with uncertain data, several academics employ geometric modeling with the fuzzy set and intuitionistic fuzzy set approach [6-14]. Meanwhile, Tas and Topal [15-16] have employed a study for neutrosophic geometric modeling but only focused on the Bézier curve and surface using the approximation method generally. Rosli and Zulkifly introduced the neutrosophic B-spline curve by using the interpolation method [23], neutrosophic bicubic surface interpolation [24], and the 3-dimensional neutrosophic quartic Bézier curve approximation method [25]. However, the papers motivate the authors to produce and focus on the B-spline surface approximation method to visualize the 3-dimensional data collection. As a result, the novelty of this study is the mathematical representations of the neutrosophic B-spline surface approximation method and its visualization for truth, indeterminacy, and falsity memberships.

This project focuses on the construction of a geometric model capable of dealing with data collection; specifically, the model's primary focus will be the neutrosophic B-spline surface approximation (NB-sSA) model. The neutrosophic control point must be defined before building the NB-sSA, utilizing neutrosophic set theories and the features they provide. These control points, along with the B-spline basis function, are used to construct NB-sSA models, which are then visualized using an approximation method. This paper is organized as follows: The first section of this chapter provides background information on the topic. In the second section, the neutrosophic point relation (NPR) and the neutrosophic control net relation (NCNR) are introduced. The third section discusses the method used for the NB-sSA using NCNR. The fourth section includes a numerical example as well as a graphical representation of NB-sSA. The investigation will be completed with the fifth section as the conclusion of this study.

## 2. Preliminaries

In fuzzy systems, the intuitionistic set can tolerate imperfect information but not indeterminate or inconsistent information [17]. There are three membership functions in a neutrosophic set. With the addition of the parameter "indeterminacy" to the neutrosophic set (NS) specification, there are three types of membership functions: a membership function, denoted by the letter *T*; an indeterminacy membership function, denoted by the letter *I*; and a non-membership function, denoted by the letter *F*.

**Definition 1** [18]

Let $Y$ be the collection of universal space, with the element $y \in Y$. The neutrosophic set is an object in the form.

$$\hat{B} = \{\left\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \right\rangle | \; y \in Y\} \tag{1}$$

where, the functions $T, I, F : Y \to ]^{-}0,1^{+}[$ define, respectively, the degree of truth membership, the degree of indeterminacy, and the degree of false membership of the element $y \in Y$ to the set $\hat{B}$ with the condition;

$$0^{-} \leq T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \leq 3^{+} \tag{2}$$

There is no limit to the amount of $T_{\hat{B}}(y), I_{\hat{B}}(y)$ and $F_{\hat{B}}(y)$.

A value is chosen by NS from one of the real standard subsets or one of the non-standard subsets of $]^{-}0,1^{+}[$. The actual value of the interval $[0,1]$, on the other hand, $]^{-}0,1^{+}[$ will be utilized in technical applications since its utilization in real data, such as the resolution of scientific challenges, will be physically impossible. As a direct consequence of this, membership value utilization is increased.

$$\hat{B} = \{\left\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \right\rangle \mid y \in Y\} \text{ and } T_{\hat{B}}(y), I_{\hat{B}}(y), F_{\hat{B}}(y) \in [0,1] \tag{3}$$

There is no restriction on the sum of $T_{\hat{B}}(y), I_{\hat{B}}(y), F_{\hat{B}}(y)$ .Therefore,

$$0 \leq T_{\hat{B}}(y) + I_{\hat{B}}(y) + F_{\hat{B}}(y) \leq 3 \tag{4}$$

**Definition 2** [15-16]
Let $\hat{B} = \{\left\langle y : T_{\hat{B}(y)}, I_{\hat{B}(y)}, F_{\hat{B}(y)} \right\rangle \mid y \in Y\}$ and $\hat{C} = \{\left\langle z : T_{\hat{C}(z)}, I_{\hat{C}(z)}, F_{\hat{C}(z)} \right\rangle \mid z \in Z\}$ be neutrosophic elements. Thus, $NR = \{\left\langle (y,z) : T_{(y,z)}, I_{(y,z)}, F_{(y,z)} \right\rangle \mid y \in \hat{B}, z \in \hat{C}\}$ is a neutrosophic relation between $\hat{B}$ and $\hat{C}$.

**Definition 3** [15-16]
The neutrosophic set of $\hat{B}$ in space $Y$ is neutrosophic point (NP) and $\hat{B} = \{\hat{B}_i\}$ where $i = 0,...,n$ is a set of NPs where there exists $T_{\hat{B}} : Y \to [0,1]$ as truth membership, $I_{\hat{B}} : Y \to [0,1]$ as indeterminacy membership, and $F_{\hat{B}} : \hat{Y} \to [0,1]$ as false membership with,

$$T_{\hat{B}}(\hat{B}) = \begin{cases} 0 & if\ \hat{B}_i \notin \hat{B} \\ a \in (0,1) & if\ \hat{B}_i \mathrel{\hat{\in}} \hat{B} \\ 1 & if\ \hat{B}_i \in \hat{B} \end{cases}$$

$$I_{\hat{B}}(\hat{B}) = \begin{cases} 0 & if\ \hat{B}_i \notin \hat{B} \\ b \in (0,1) & if\ \hat{B}_i \mathrel{\hat{\in}} \hat{B} \\ 1 & if\ \hat{B}_i \in \hat{B} \end{cases} \tag{5}$$

$$F_{\hat{B}}(\hat{B}) = \begin{cases} 0 & if\ \hat{B}_i \notin \hat{B} \\ c \in (0,1) & if\ \hat{B}_i \mathrel{\hat{\in}} \hat{B} \\ 1 & if\ \hat{B}_i \in \hat{B} \end{cases}$$

### 2.1 *Neutrosophic Point Relation*

Neutrosophic point relation (NPR) is based on the concept of a neutrosophic set, which was discussed in the previous section. If $P,Q$ is a collection of Euclidean universal space points and $P,Q \in \mathbf{R}^2$, then NPR is defined as follows:

**Definition 4** [23]
Let $X,Y$ be a collection of universal space points with a non-empty set and $P,Q,I \in \mathbf{R}\times\mathbf{R}\times\mathbf{R}$, then NPR is defined as:

$$\hat{R} = \left\{ \left\langle \left(p_i, q_j\right), T_R(p_i,q_j), I_R(p_i,q_j), F_R(p_i,q_j) \right\rangle \mid T_R(p_i,q_j), I_R(p_i,q_j), F_R(p_i,q_j) \in I \right\} \tag{6}$$

where $\left(p_i, q_j\right)$ is an ordered pair of coordinates and $\left(p_i, q_j\right) \in P \times Q$ while $T_R(p_i, q_j), I_R(p_i, q_j), F_R(p_i, q_j)$ are the truth membership, indeterminacy membership, and false membership that follow the condition of the neutrosophic set which is $0 \le T_{\hat{B}}(\hat{y}) + I_{\hat{B}}(\hat{y}) + F_{\hat{B}}(\hat{y}) \le 3$.

*2.2 Neutrosophic Control Net Relation*

The geometry of a spline surface can only be described by all the points required to build the surface. The control net plays an important role in the development, control, and manufacture of smooth surfaces. The neutrosophic control point relation (NCPR) is first defined in this section by using the notion of control point from the research published in [19-21] in the following way:

**Definition 5** [23]
Let $\hat{R}$ be an NPR, then NCPR is defined as a set of points $n+1$ that indicates the positions and coordinates of a location and is used to describe the curve and is denoted by:

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_0^T, \hat{p}_1^T, \ldots, \hat{p}_n^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_0^I, \hat{p}_1^I, \ldots, \hat{p}_n^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_0^F, \hat{p}_1^F, \ldots, \hat{p}_n^F \right\} \end{aligned} \tag{7}$$

where $\hat{P}_i^T$, $\hat{P}_i^I$, and $\hat{P}_i^F$ are neutrosophic control points for truth membership, indeterminacy membership, and falsity membership, and $i$ is one less than $n$. Thus, the NCNR can be defined as follows.

**Definition 6** [24]
Let $\hat{P}$ be an NCPR, and then define an NCNR as points $n+1$ and $m+1$ for $\hat{P}$ in their direction, and it can be denoted by $\hat{P}_{i,j}$ that represents the locations of points used to describe the surface and may be written as:

$$\hat{P}_{i,j} = \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,j} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,j} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{i,0} & \hat{P}_{i,1} & \cdots & \hat{P}_{i,j} \end{bmatrix} \tag{8}$$

where $\hat{P}_{i,j}$ are also the points that make up a polygon's control net.

## 3. Neutrosophic B-spline Surface Approximation

Surface is a two-parameter vector value function that governs how the plane is projected into the Euclidean three-dimensional frame [22]. The NCNR and **Definition 1** are used to construct the neutrosophic B-spline surface approximation (NB-sSA), which is then utilized to embed the B-spline blending function in a geometric model. The model, which stands for approximation approach, is mathematically represented as follows:

**Definition 7**
Let $\hat{\hat{P}}_{i,j}^{T,I,F} = \left\{ \hat{\hat{P}}_{i,j}^{T,I,F} \right\}_{i=0,j=0}^{n,m}$ where $i = 0,1,\ldots,n$ and $j = 0,1,\ldots,m$ is NCNR for truth, indeterminacy, and falsity memberships. The neutrosophic B-spline surface approximation (NB-sSA) is denoted as $BsS(u,w)$ and represented as follows:

$$BsS(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}\hat{P}_{i,j}N_i^k(u)M_j^l(w) \tag{9}$$

where $N_i^k(u)$ and $M_j^l(w)$ are the Bernstein function in the $u$ and $w$ parametric directions.

$$\begin{aligned} N_i^1(u)&=\begin{cases}1 & if \quad \mathrm{u}_i \le u < u_{i+1}\\ 0 & otherwise\end{cases}\\ N_i^1(u)&=\frac{(u-u_i)}{u_{i+k-1}-u_i}N_i^{k-1}(u)+(7)\frac{(u_{i+k}-u)}{u_{i+k}-u_{i+1}}N_{i+1}^{k-1}(u)\end{aligned} \tag{10}$$

$$\begin{aligned} M_j^1(w)&=\begin{cases}1 & if \quad \mathrm{w}_j \le w < w_{j+1}\\ 0 & otherwise\end{cases}\\ M_j^1(w)&=\frac{(w-w_j)}{w_{j+l-1}-u_i}M_i^{l-1}(w)+(8)\frac{(w_{j+l}-w)}{w_{j+l}-w_{j+1}}M_{j+1}^{l-1}(w)\end{aligned} \tag{11}$$

The parametric function NB-sSA in **Equation (9)** is defined as follows and is made up of three surfaces: a membership surface, a non-membership surface, and an indeterminacy surface.

$$BsS^T(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}\hat{P}_{i,j}{}^T N_i^k(u)M_j^l(w) \tag{12}$$

$$BsS^F(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}\hat{P}_{i,j}{}^F N_i^k(u)M_j^l(w) \tag{13}$$

$$BsS^I(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}\hat{P}_{i,j}{}^I N_i^k(u)M_j^l(w) \tag{14}$$

Each $BsS(u,w)$ can be expressed as a matrix product in the following way [24]:

$$BsS(u_i,w_j)=\begin{bmatrix}N_0^k(u_i) & N_1^k(u_i) & \cdots & N_i^k(u_i)\end{bmatrix}\times\begin{bmatrix}\hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,j}\\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,j}\\ \vdots & \vdots & \ddots & \vdots\\ \hat{P}_{i,0} & \hat{P}_{i,1} & \cdots & \hat{P}_{i,j}\end{bmatrix}\times\begin{bmatrix}M_0^l(w_j)\\ M_1^l(w_j)\\ \vdots\\ M_i^l(w_j)\end{bmatrix} \tag{15}$$

All the independent equations can be combined to form a single matrix equation:

$$BsS = N^T\hat{P}M \tag{16}$$

*3.1. Properties of Neutrosophic B-Spline Surface Approximation (NB-sSA)*

By using a B-spline basis to define a B-spline surface, many characteristics beyond those already mentioned become obviously clear:

- In each parametric, the degree of NB-sSA is one less than the number of NCNR vertices in that direction.
- The NCNR shape is generally followed by the NB-sSA.
- The NCNR corner point and the resulting NB-sSA coincide.
- The NCNR's shape is generally followed by the NB-sSA.
- The NB-sSA is contained within NCNR's convex hull.
- The NB-sSA has a continuity in each parametric direction that is two less than the number of NCNR vertices in that direction.
- An affine transformation does not change the NB-sSA.

- The NB-sSA lacks the variation-diminishing property. For bivariant NB-sSA, the variation-diminishing property is both undefined and unknown.

## 4. Numerical Example with Its Visualizations

To demonstrate a 3-dimensional neutrosophic B-spline surface using the approximation approach, suppose a four-by-four NCNR with the following degrees of membership, non-membership, and indeterminacy:

$$\begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \hat{P}_{0,2} & \hat{P}_{0,3} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \hat{P}_{1,2} & \hat{P}_{1,3} \\ \hat{P}_{2,0} & \hat{P}_{2,1} & \hat{P}_{2,2} & \hat{P}_{2,3} \\ \hat{P}_{3,0} & \hat{P}_{3,1} & \hat{P}_{3,2} & \hat{P}_{3,3} \end{bmatrix}$$

The NB-sSA is fourth order in the $u$ direction $(k=4)$ and third order in the $w$ direction $(l=3)$ based on the **Equation (10)** and **Equation (11).** Therefore, by using **Equation (16)**, the NB-sSA can be derived as follows:

$$\begin{aligned} BsS &= \sum_{i=1}^{4}\sum_{j=1}^{4}\hat{P}_{i,j}N_{i,4}(u)M_{j,3}(w) \\ &= N_{1,4}(u)\left(\hat{P}_{1,1}M_{1,3}(w)+\hat{P}_{1,2}M_{2,3}(w)+\hat{P}_{1,3}M_{3,3}(w)+\hat{P}_{1,4}M_{4,3}(w)\right) \\ &+ N_{2,4}(u)\left(\hat{P}_{2,1}M_{1,3}(w)+\hat{P}_{2,2}M_{2,3}(w)+\hat{P}_{2,3}M_{3,3}(w)+\hat{P}_{2,4}M_{4,3}(w)\right) \\ &+ N_{3,4}(u)\left(\hat{P}_{3,1}M_{1,3}(w)+\hat{P}_{3,2}M_{2,3}(w)+\hat{P}_{3,3}M_{3,3}(w)+\hat{P}_{3,4}M_{4,3}(w)\right) \\ &+ N_{4,4}(u)\left(\hat{P}_{4,1}M_{1,3}(w)+\hat{P}_{4,2}M_{2,3}(w)+\hat{P}_{4,3}M_{3,3}(w)+\hat{P}_{4,4}M_{4,3}(w)\right) \end{aligned}$$

Every column is labeled $\langle T, F, I\rangle$ with its respective value and degree. Based on example below for $\hat{P}_{0,0}$ for $i=0$ and $j=0$, the value of truth membership denoted as *T* is 0.4, the value of falsity membership denoted as *F* is 0.7, and the value of indeterminacy membership denoted as *I* is 0.2.

$$\begin{bmatrix} \hat{P}_{0,0} \\ \hat{P}_{1,0} \\ \hat{P}_{2,0} \\ \hat{P}_{3,0} \end{bmatrix} = \begin{bmatrix} \langle(-16,16);0.4,0.7,0.2\rangle \\ \langle(-6,16);0.6,0.4,0.3\rangle \\ \langle(6,16);0.6,0.2,0.5\rangle \\ \langle(16,16);0.7,0.3,0.3\rangle \end{bmatrix}$$

$$\begin{bmatrix} \hat{P}_{0,1} \\ \hat{P}_{1,1} \\ \hat{P}_{2,1} \\ \hat{P}_{3,1} \end{bmatrix} = \begin{bmatrix} \langle(-16,6);0.9,0.3,0.1\rangle \\ \langle(-6,6);0.8,0.2,0.3\rangle \\ \langle(6,6);0.8,0.4,0.1\rangle \\ \langle(16,6);0.4,0.6,0.3\rangle \end{bmatrix}$$

$$\begin{bmatrix} \hat{P}_{0,2} \\ \hat{P}_{1,2} \\ \hat{P}_{2,2} \\ \hat{P}_{3,2} \end{bmatrix} = \begin{bmatrix} \langle(-16,-16);0.6,0.5,0.2\rangle \\ \langle(-6,-16);0.7,0.4,0.2\rangle \\ \langle(6,-16);0.5,0.3,0.5\rangle \\ \langle(16,-16);0.7,0.4,0.2\rangle \end{bmatrix}$$

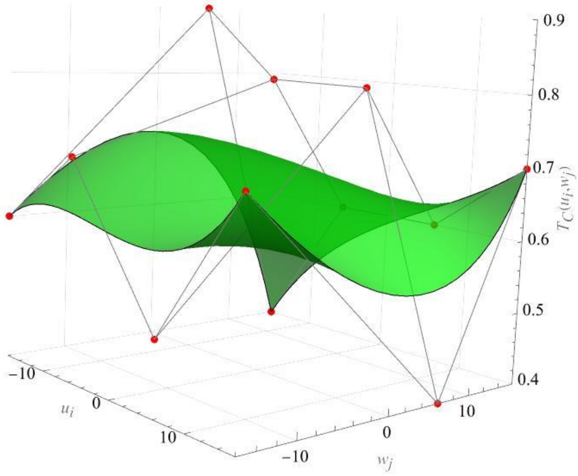


**Figure 1.** NB-sSA for truth membership.

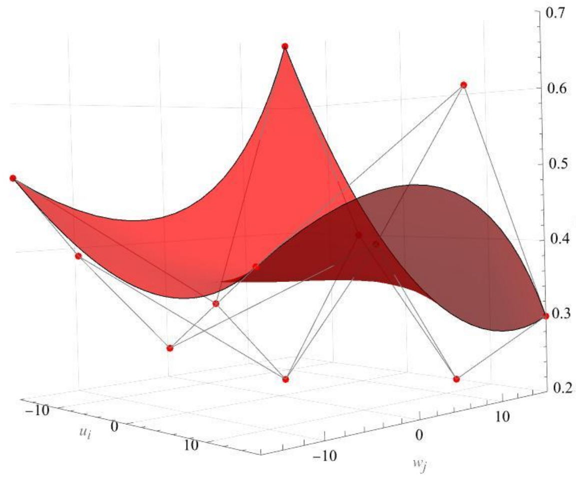


**Figure 2.** NB-sSA for false membership.

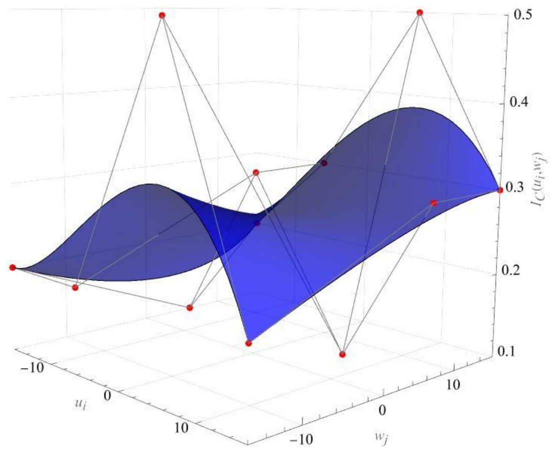


**Figure 3.** NB-sSA for indeterminacy membership.

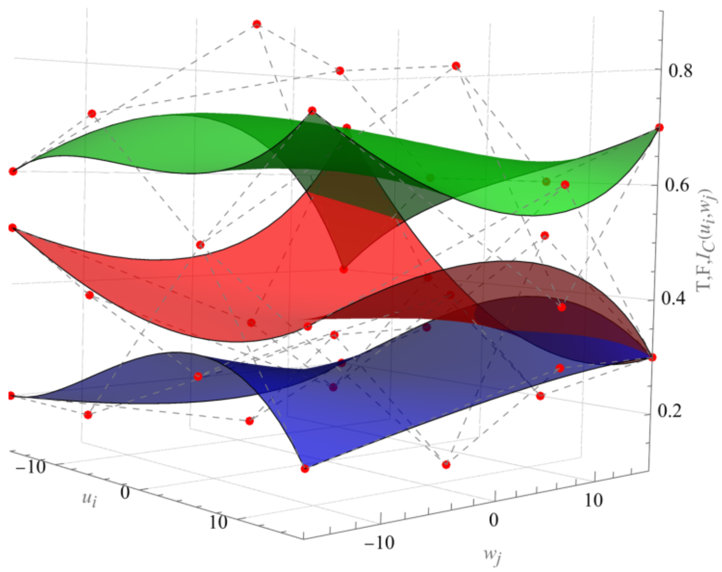


**Figure 4.** NB-sSA for truth, false, and indeterminacy membership.

This study employs the original formula of the B-spline basis function and then blends it with the NS theory, which uses the NCNR to approximate the surface. It differs from the interpolation approach, which requires determining the inverse of the formula B-spline basis function for the control net to find the interpolated data as introduced and visualized in Rosli and Zulkifly [23]'s study. Therefore, this study uses the B-spline surface approximation model that was introduced by Piegl and Tiller [22] and blends it with NCNR. However, one of the difficulties when constructing this model is ensuring that the random data collection adheres to the criterion of neutrosophic set theory, which is $0 \le T_{\hat{B}}(\hat{y}) + I_{\hat{B}}(\hat{y}) + F_{\hat{B}}(\hat{y}) \le 3$. The 3-dimensional neutrosophic B-spline surface approximation model is depicted in **Figures 1** to **4**. **Figure 1** shows a 3D surface for truth membership, **Figure 2** shows a 3D surface for false membership, **Figure 3** shows an indeterminacy surface, and **Figure 4** shows a 3D neutrosophic B-spline surface approximation for all membership in one axis, with green representing 3D truth membership, red representing 3D false membership, and blue representing 3D indeterminacy membership. In **Figures 1** to **4**, the red dot represents their individual NCNR, and the gray line indicates their respective control polygons that hold the NCNR. As this study uses an approximation strategy and adheres to the criterion of the neutrosophic set, each NCNR approximates its surfaces, and the memberships are not dependent on the others. An algorithm for constructing the NB-sSA will be discussed as follows:

**Step 1:** Introduce the NCNR by using **Definition 6.**
**Step 2:** Blend the NCPR with B-spline Basis function as in **Definition 7.**
**Step 3:** Collect the coefficients of $N_i^k(u)$ and $M_j^l(w)$. The coefficients of the parameter terms are collected and rewritten in matrix form as in the given example.
**Step 4:** Repeat step 1 to 3 for indeterminacy and falsity memberships cases.

## 5. Conclusions

By introducing NCNR, this paper introduced the NB-sSA model. This study can be expanded to produce better findings by incorporating non-uniform rational B-splines (NURBS) functions for surfaces and curves. The suggested 3-dimensional model can handle surface data visualization challenges such as modelling geographical regions with unclear borders in geoinformation systems (GIS), remote sensing, object reconstruction from an aerial laser scanner, bathymetric data visualization, and many more. Implementing this strategy has the impact or benefit of ensuring that no data is wasted throughout the data collection process in any application. The NB-sSA model can be used to address and solve difficulties characterized by uncertainty. The NCNR and NB-sSA models can provide a comprehensive analysis and description of a modelling issue in which each surface is modelled separately.

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