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Embedding Norms into Neutrosophic Multi Fuzzy Subrings

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Abstract. We have embedded the concept norm with the proposed notion Neutrosophic multifuzzy Subrings. This conception was manipulated with Neutrosophic multi fuzzy ideals and level sets. Furthermore, some propositions and theorems related to them were explored. Eventually, direct product and homomorphic properties of Neutrosophic multifuzzy Subrings were derived.

Keywords: Neutrosophic fuzzy set (NFS); Neutrosophic multisets (NMS); Neutrosophic multi fuzzy set (NMFS); T norms (T_n) and T conorms (T_c) ; Neutrosophic multifuzzy subring (NMFSR); Neutrosophic multifuzzy left(right) ideals (NMFL(R)I).

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1. **Introduction**

There is a lack of certainty that couldnt be manipulated by classical set. To overcome the complication, fuzzy set was enlightened by L.A.Zadeh [4]. Smarandache [5] initiated Neutrosophic set to build upon the thought of Atanassovs [11] intuitionistic fuzzy sets very convenient and effectively which is the part of philosophy. In Neutrosophic logic every hypothesis having degree of validity, neutral and non-validity is represented independently. The notion norm is a sort of dual operation tracking down numerous applications in fuzzy set, probability and statistics and other areas. A t-norm interprets intersection of fuzzy sets and conjunction in logics. There were some essential properties like Archimedean, strict and nilpotent t-norm that exist.

The Application of group theory to fuzzy set was originated by Rosenfield [10]. In view of the fuzzy set hypothesis, Multifuzzy set was initiated by Sebastian and Ramakrishnan [8]. The unified notions of Multifuzzy set and Group called as multifuzzy group was examined by

Muthuraj [1]. Also, he has discussed its Level Subgroups. The combined concepts Intuitionistic Fuzzy sets and Fuzzy Multisets together were developed as Intuitionistic Fuzzy multisets by Shinoj [9].

The thought of Intuitionistic fuzzy groups along with homomorphism and direct product had been explored by Sharma [15, 16]. Rasul Rasuli [2, 7, 18, 19] investigated his thought on Intuitionistic fuzzy subgroups and subrings regarding norms and reached out into fuzzy Multi-groups. Abu Osman [12] explored products of fuzzy subgroups. Intuitionistic fuzzy multiset was initiated by Shinoj and John [9]. Then, Wang [14] gave the comparative activities and outcomes of single esteemed neutrosophic set hypothesis. To elaborate the neutrosophic set theory, the conception neutrosophic multiset was originated by Deli [13] and Ye [21, 22] for modelling vagueness and uncertainty. VakkasUlucay [3] proposed the notion of Neutrosophic Multi Groups. Hemabala [6] gave the thought of gamma near ring applied into Anti Neutrosophic Multi fuzzy set. The extension principle was defined by Sahin[20] using neutrosophic multi-sets.

The scope of this work is predicated upon the notion of Neutrosophic set and multifuzzy set together with rings .We have characterized here a thought of Neutrosophic multifuzzy subrings along with triangular norms and made sense of certain outcomes connected with them.

2. Preliminaries

This part consists of, fundamental definitions are referred to that are essential.

Definition 2.1. [5] A NFS$\mathcal{A}$ on the space of points X is characterized by a truth membership $\mu_{\mathcal{A}}(x)$, an indeterminancy $\mathcal{N}_{\mathcal{A}}(x)$, and falsity membership $\digamma_{\mathcal{A}}(x)$ is defined as

$\mathcal{A} = \langle x, \mu_{\mathcal{A}}(x), \mathcal{N}_{\mathcal{A}}(x), \digamma_{\mathcal{A}}(x) : x \in X\rangle$ where $\mu_{\mathcal{A}}, \mathcal{N}_{\mathcal{A}}, \digamma_{\mathcal{A}} : X \to [0,1]$ and

$$0 \leq \mu_{\mathcal{A}}(x) + \mathcal{N}_{\mathcal{A}}(x) + \digamma_{\mathcal{A}}(x) \leq 3$$

Definition 2.2. [13] A NMS $\mathcal{A}$ on X be defined as follows:
$\mathcal{A} = \{< x, (\mu^1_{\mathcal{A}}(x),\ \mu^2_{\mathcal{A}}(x), \ldots, \mu^n_{\mathcal{A}}(x)),\ (\mathcal{N}^1_{\mathcal{A}}(x),\ \mathcal{N}^2_{\mathcal{A}}(x), \ldots, \mathcal{N}^n_{\mathcal{A}}(x),\ (\digamma^1_{\mathcal{A}}(x),\ \digamma^2_{\mathcal{A}}(x), \ldots, \digamma^n_{\mathcal{A}}(x)) >: x \in X\}$,
where, $\mu^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(x),\ \digamma^i_{\mathcal{A}}(x) : X \to [0,1]$, $0 \leq \sup \mu^i_{\mathcal{A}}(x) + \sup \mathcal{N}^i_{\mathcal{A}}(x) + \sup \digamma^i_{\mathcal{A}}(x) \leq 3$ $(i = 1, 2, \ldots, n)$ and for any x, truth membership $\mu^1_{\mathcal{A}}(x) \geq \mu^2_{\mathcal{A}}(x) \geq \cdots \geq \mu^n_{\mathcal{A}}(x)$ as decreasing order but no restrictions for indeterminacy and falsity membership. Futher more, n is called the dimension of $\mathcal{A}$, denoted $d(\mathcal{A})$.

Definition 2.3. [12] A function T_n **:** [0,1] $\times[0,1] \to [0,1]$ is a t-norm possess the following axioms.

1.$T_n\ (x, 1) = x$

2.$T_n\ (x, y) = T_n\ (x, z)$ if $y \leq z$

3.$T_n\ (x, y) = T_n\ (y, x)$

4.$T_n\ (x, T_n\ (y, z)) = T_n\ (T_n\ ((x, y)\ , z))\ \forall x, y, z \in [0, 1]$

Definition 2.4. [17] A function T_c **:** [0,1] $\times[0,1] \rightarrow [0,1]$ is a t-conorm possess the following axioms

1.$T_c\ (x, 0) = x$

2.$T_c\ (x, y) = T_c\ (x,$ z) if $y \leq z$

3.$T_c\ (x, y) = T_c\ (y, x)$

4.$T_c\ (x, T_c\ (y, z)) = T_c\ (T_c\ ((x, y)\ , z))\ \forall x, y, z \in [0, 1]$

Recollect if T_n is idempotent function $T_n\ (x, x) = x$. Similarly, if T_c is idempotent function $T_c\ (x, x) = x, \forall x \in [0, 1]$.

3. Neutrosophic Multifuzzy Subring with respect to T_n and T_c

Definition 3.1. A NMFS $\mathcal{A} = \{< (x, \mu^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}, \digamma^i_{\mathcal{A}}(x)) >, x \in R,\ i = 1, 2, \ldots, n\ \}$ of a ring R is said to be NMFSR with respect to T_n and T_c of R if

(i) $\mu^i_{\mathcal{A}}(x - y) \geq T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right)$; $\mathcal{N}^i_{\mathcal{A}}(x - y) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right)$;
$\digamma^i_{\mathcal{A}}(x - y) \leq T_c\left(\digamma^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{A}}(y)\right)$

(ii) $\mu^i_{\mathcal{A}}(xy) \geq T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right)$; $\mathcal{N}^i_{\mathcal{A}}(xy) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right)$;
$\digamma^i_{\mathcal{A}}(xy) \leq T_c\left(\digamma^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{A}}(y)\right)$

$\forall x, y \in R,\ i = 1, 2 \ldots, n.$

Example 3.2. Let ($Z_3, +, \cdot$) be a ring. For all $x \in Z_3$, we define a NMFS $\mathcal{A}$ over T_n and T_c of Z_3 as

$\mathcal{A}$= $(< 0(0.9, 0.7, 0.5), (0.2.0.4, 0.8), (0.3, 0.4, 0.6) >,$
$< 1(0.9, 0.5, 0.4), (0.2, 0.5, 0.7),(0.3, 0.5, 0.7) >,< 2(0.8, 0.5, 0.4), (0.2, 0.5, 0.7), (0.4, 0.5, 0.7) >).$

Let $T_n(x, y) = xy$ and $T_c(x, y) = x + y - xy, \quad \forall x, y \in Z_3$ then $\mathcal{A}$ is a NMFSR of Z_3 over T_n and T_c

Proposition 3.3. *If $\mathcal{A}$ is a NMFSR of R with T_n and T_c, where T_n , T_c are idempotent then* $\forall \mathbf{x} \in R\ \& i = 1, 2, \ldots, n$

(i) $\mu^i_{\mathcal{A}}(0) \geq \mu^i_{\mathcal{A}}(x)$; $\mathcal{N}^i_{\mathcal{A}}(0) \leq \mathcal{N}^i_{\mathcal{A}}(x)$ *;* $\digamma^i_{\mathcal{A}}(0) \leq \digamma^i_{\mathcal{A}}(x)$

(ii) $\mu^i_{\mathcal{A}}(-x) = \mu^i_{\mathcal{A}}(x)$ *;* $\mathcal{N}^i_{\mathcal{A}}(-x) = \mathcal{N}^i_{\mathcal{A}}(x)$ *;* $\digamma^i_{\mathcal{A}}(-x) = \digamma^i_{\mathcal{A}}(x)$

Proof. If $x \in$ R.

(i) $\mu^i_{\mathcal{A}}(0) = \mu^i_{\mathcal{A}}(x - x) \geq T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(x)\right) = \mu^i_{\mathcal{A}}(x)$

$\mathcal{N}^i_{\mathcal{A}}(0) = \mathcal{N}^i_{\mathcal{A}}(x - x) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(x)\right) = \mathcal{N}^i_{\mathcal{A}}(x)$

Similarly, $\digamma^i_{\mathcal{A}}(0) \leq \digamma^i_{\mathcal{A}}(x)$

$(ii) \qquad \mu^i_{\mathcal{A}}(-x) = \mu^i_{\mathcal{A}}(0{-}x)$

$$\geq T_n\left(\mu^i_{\mathcal{A}}(0), \mu^i_{\mathcal{A}}(x)\right)$$
$$\geq T_n\left(\mu^i_{A}(x), \mu^i_{A}(x)\right)$$
$$= \mu^i_{A}(x) = \mu^i_{A}(0-(-\text{x}))$$
$$\geq T_n(\mu^i_{A}(0), \mu^i_{A}(-x)$$
$$\geq T_n\left(\mu^i_{A}(-x), \mu^i_{A}(-x)\right)$$
$$\geq T^i_{A}(-x))$$

So that, $\mu^i_{\mathcal{A}}(x) = \mu^i_{\mathcal{A}}(-x)$

$$\mathcal{N}^i_{\mathcal{A}}(-x) = \mathcal{N}^i_{\mathcal{A}}(0-x)$$
$$\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(0), \mathcal{N}^i_{\mathcal{A}}(x)\right)$$
$$\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(x)\right)$$
$$= \mathcal{N}^i_{\mathcal{A}}(x) = \mathcal{N}^i_{\mathcal{A}}(0-(-x))$$
$$\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(0), \mathcal{N}^i_{\mathcal{A}}(-x)\right)$$
$$\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(-x), \mathcal{N}^i_{\mathcal{A}}(-x)\right)$$
$$\leq \mathcal{N}^i_{\mathcal{A}}(-x)$$

So that, $\mathcal{N}^i_{\mathcal{A}}(x) = \mathcal{N}^i_{\mathcal{A}}(-x)$.

Similarly, $F^i_{\mathcal{A}}(x) = F^i_{\mathcal{A}}(-x)$. $\forall x \in R$ and $i = 1, 2 \ldots n$ Hence the result. □

Proposition 3.4. *Let $\mathcal{A}$ be a NMFSR of R over T_n and T_c , $x \in R$ $\forall i = 1, 2 \ldots n$ then*

$$\mu^i_{\mathcal{A}}(x-y) = 1 \Rightarrow \mu^i_{\mathcal{A}}(x) \geq \mu^i_{\mathcal{A}}(y); \mathcal{N}^i_{\mathcal{A}}(x-y) = 0 \Rightarrow \mathcal{N}^i_{\mathcal{A}}(\mathbf{x}) \leq \mathcal{N}^i_{\mathcal{A}}(y)$$

$$F^i_{\mathcal{A}}(x-y) = 0 \Rightarrow F^i_{\mathcal{A}}(x) \leq F^i_{\mathcal{A}}(y)$$

Proof. Let $x, y \in R$ and $i = 1, 2 \ldots n$. Then

(i) $\mu^i_{\mathcal{A}}(x) = \mu^i_{\mathcal{A}}(x-y+y) \geq T_n(\mu^i_{\mathcal{A}}(x-y), \mu^i_{\mathcal{A}}(y)) = T_n(1, \mu^i_{\mathcal{A}}(y)) = \mu^i_{\mathcal{A}}(y)$

(ii) $\mathcal{N}^i_{\mathcal{A}}(x) = \mathcal{N}^i_{\mathcal{A}}(x-y+y) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x-y), \mathcal{N}^i_{\mathcal{A}}(y)\right) = T_c\left(0, \mathcal{N}^i_{\mathcal{A}}(y)\right) = \mathcal{N}^i_{\mathcal{A}}(x)$

Similarly, $F^i_{\mathcal{A}}(x) \leq F^i_{\mathcal{A}}(y)$.

Hence the result. □

Proposition 3.5. *Let $\mathcal{A}$ be a NMFSR of R with respect to T_n and T_c where T_n , T_c are idempotent. Then $\mathcal{A}(x-y) = \mathcal{A}(y)$ iff $\mathcal{A}(x) = \mathcal{A}(0)$, $\forall x, y \in R$ and $i = 1, 2, 3 \ldots n$.*

Proof. Let $\mathcal{A}(x-y) = \mathcal{A}(y)$. If $y = 0, \Rightarrow \mathcal{A}(x) = \mathcal{A}(0)$

Conversely, if $\mathcal{A}(x) = \mathcal{A}(0)$, Then,

(i).$\mu^i_{\mathcal{A}}(x) = \mu^i_{\mathcal{A}}(0) \geq \mu^i_{\mathcal{A}}(x-y)$

$\mu^i_{\mathcal{A}}(x) = \mu^i_{\mathcal{A}}(0) \geq \mu^i_{\mathcal{A}}(y)$ (by proposition 3.3)

$$\begin{aligned}
\text{Now,} \qquad \mu^i_{\mathcal{A}}(x-y) &\geq T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right) \\
&\geq T_n\left(\mu^i_{\mathcal{A}}(y), \mu^i_{\mathcal{A}}(y)\right) \\
&= \mu^i_{\mathcal{A}}(y) \\
&= \mu^i_{\mathcal{A}}(-y) \\
&= \mu^i_{\mathcal{A}}(x-y-x) \\
&\geq T_n\left(\mu^i_{\mathcal{A}}(x-y), \mu^i_{\mathcal{A}}(x)\right) \\
&\geq T_n\left(\mu^i_{\mathcal{A}}(x-y), \mu^i_{\mathcal{A}}(x-y)\right) \\
&= \mu^i_{\mathcal{A}}(x-y)
\end{aligned}$$

So, we get $\mu^i_{\mathcal{A}}(x-y) = \mu^i_{\mathcal{A}}(y)$

(ii). $\mathcal{N}^i_{\mathcal{A}}(x) = \mathcal{N}^i_{\mathcal{A}}(0) \leq \mathcal{N}^i_{\mathcal{A}}(x-y)$

$\mathcal{N}^i_{\mathcal{A}}(x) = \mathcal{N}^i_{\mathcal{A}}(0) \leq \mathcal{N}^i_{\mathcal{A}}(y)$

Now,

$$\begin{aligned}
\mathcal{N}^i_{\mathcal{A}}(x-y) &\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right) \\
&\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(y), \mathcal{N}^i_{\mathcal{A}}(y)\right) \\
&= \mathcal{N}^i_{\mathcal{A}}(y) \\
&= \mathcal{N}^i_{\mathcal{A}}(-y) \ \text{ (by theorem 3.3)} \\
&= \mathcal{N}^i_{\mathcal{A}}(x-y-x) \\
&\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x-y), \mathcal{N}^i_{\mathcal{A}}(x)\right) \\
&\leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x-y), \mathcal{N}^i_{\mathcal{A}}(x-y)\right) \\
&= \mathcal{N}^i_{\mathcal{A}}(x-y) \\
\therefore \mathcal{N}^i_{\mathcal{A}}(x-y) &= \mathcal{N}^i_{\mathcal{A}}(y)
\end{aligned}$$

Similarly, $\digamma^i_{\mathcal{A}}(x-y) = \digamma^i_{\mathcal{A}}(y)$

$\therefore \mathcal{A}(x-y) = \mathcal{A}(y)$ if $\mathcal{A}(x) = \mathcal{A}(0) \forall \mathbf{x}, \mathbf{y} \in R$ and $i = 1, 2 \ldots, n$. □

4. Neutrosophic Multifuzzy ideal and level set

Definition 4.1. Let $\mathcal{A}$ and $\mathcal{B}$ be two NMS of R. Define

$\mathcal{A} \cap \mathcal{B} = \left(\mu^i_{\mathcal{A}\cap\mathcal{B}}, \mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}, \digamma^i_{\mathcal{A}\cap\mathcal{B}}\right)$ as $\mu^i_{\mathcal{A}\cap\mathcal{B}}(x) = T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{B}}(x))$

$\mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(x) = T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{B}}(x))$; $\digamma^i_{\mathcal{A}\cap\mathcal{B}}(x) = T_c(\digamma^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{B}}(x))$;

$\mathcal{A} \cup \mathcal{B} = (\mu^i_{\mathcal{A}\cup\mathcal{B}}, \mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}, \digamma^i_{\mathcal{A}\cup\mathcal{B}})$ as $\mu^i_{\mathcal{A}\cup\mathcal{B}}(x) = T_c\ (\ \mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{B}}(x)\ )$

$\mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}(x) = T_n\ (\ \mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{B}}(x)\ )$; $\digamma^i_{\mathcal{A}\cup\mathcal{B}}(x\ ) = T_n\ (\ \digamma^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{B}}(x)\ )$, $\forall\ x \in R$.

Example 4.2. Consider the ring $(Z_2, +, \cdot)$. For all $\mathbf{x} \in Z_2$, we define NMFS $\mathcal{A}$ and $\mathcal{B}$ of Z_2 as $\mathcal{A} = (< 0(0.9,0.7), (0.1.0.3), (0.4,0.6) >; < 1(0.8,0.6), (0.1,0.4), (0.4,0.7) >$
$\mathcal{B} = (< 0(0.9,0.6), (0.2.0.1), (0.5,0.4) >; < 1(0.7,0.4), (0.3,0.4), (0.6,0.7) >$
Let $T_n(x,y) = xy$ and $T_c(x,y) = x + y - xy, \quad \forall x, y \in Z_2$.Then
$\mathcal{A} \cup \mathcal{B} = \{< 0, (0.98,0.88),(0.02,0.03), (0.20,0.24) > < 1(0.94,0.76), (0.03,0.16), (0.24,0.0.49) >\}$
$\mathcal{A} \cap \mathcal{B} = (<0(0.72,0.43), (0.28.0.37), (0.7,0.76)>; <1(0.56,0.24), (0.37,0.64), (0.76,0.91) >)$.

Theorem 4.3. *If $\mathcal{A}$ and $\mathcal{B}$ are NMFSR of ring R, then $\mathcal{A} \cap \mathcal{B}$ also a NMFSR of R with respect to T_n and T_c, where T_n and T_c are idempotent.*

Proof. Let $x, y \in R$ and $i = 1, 2, 3, \ldots, n$

$$\begin{aligned}
(i)\ \mu^i_{\mathcal{A}\cap\mathcal{B}}(x-y) &= T_n(\mu^i_{\mathcal{A}}(x-y), \mu^i_{\mathcal{B}}(x-y)) \\
&\geq T_n\left\{T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)), T_n(\mu^i_{\mathcal{B}}(x), \mu^i_{\mathcal{B}}(\mathbf{y}))\right\} \\
&= T_n\left\{T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{B}}(x)), T_n(\mu^i_{A}(y), \mu^i_{\mathcal{B}}(y))\right\} \\
&= T_n(\mu^i_{\mathcal{A}\cap\mathcal{B}}(x), \mu^i_{\mathcal{A}\cap\mathcal{B}}(y)) \\
\mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(x-y) &= T_c(\mathcal{N}^i_{\mathcal{A}}(x-y), \mathcal{N}^i_{\mathcal{B}}(x-y)) \\
&\leq T_c\left\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)), T_c(\mathcal{N}^i_{\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{B}}(y))\right\} \\
&= T_c\left\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{B}}(x)), T_c(\mathcal{N}^i_{\mathcal{A}}(y), \mathcal{N}^i_{\mathcal{B}}(y))\right\} \\
&= T_c(\mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(y))
\end{aligned}$$

Similarly, $\digamma^i_{\mathcal{A}\cap\mathcal{B}}(x-y) \leq T_c(\digamma^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{B}}(y))$

$$\begin{aligned}
(ii)\ \mu^i_{\mathcal{A}\cap\mathcal{B}}(xy) &= T_n(\mu^i_{\mathcal{A}}(xy), \mu^i_{\mathcal{B}}(xy)) \\
&\geq T_n\left\{T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)), T_n(\mu^i_{\mathcal{B}}(x), \mu^i_{\mathcal{B}}(y))\right\} \\
&= T_n\left\{T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{B}}(x)), T_n(\mu^i_{\mathcal{A}}(y), \mu^i_{\mathcal{B}}(y))\right\} \\
&= T_n(\mu^i_{\mathcal{A}\cap\mathcal{B}}(x), \mu^i_{\mathcal{A}\cap\mathcal{B}}(y)) \\
\mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(xy) &= T_c(\mathcal{N}^i_{\mathcal{A}}(xy), \mathcal{N}^i_{\mathcal{B}}(xy)) \\
&\leq T_c\left\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{A}(y)), T_c(\mathcal{N}^i_{\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{B}}(y))\right\} \\
&= T_c\left\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{B}}(x)), T_c(\mathcal{N}^i_{\mathcal{A}}(y), \mathcal{N}^i_{\mathcal{B}}(y))\right\} \\
&= T_c(\mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(y))
\end{aligned}$$

Similarly, $\digamma^i_{\mathcal{A}\cap\mathcal{B}}(xy) \leq T_c(\digamma^i_{A}(x), \digamma^i_{\mathcal{B}}(y))$
Hence $\mathcal{A} \cap \mathcal{B}$ is a NMFSR of R w.r.t T_n and T_c $\forall x, y \in R$ and $i = 1, 2, \ldots, n$. □

Example 4.4. Consider the ring $(Z_2, +, \cdot)$. For all $\mathbf{x} \in Z_2$, we define NMFSR $\mathcal{A}$ and $\mathcal{B}$ of Z_2 as $\mathcal{A} = (<0(0.9,0.7), (0.1.0.3), (0.4,0.6)>; <1(0.8,0.6), (0.1,0.4), (0.4,0.7)>)$
$\mathcal{B} = (<0(0.8,0.6), (0.2.0.1), (0.5,0.4)>; <1(0.7,0.4), (0.3,0.4), (0.6,0.7)>)$

$\mathcal{A} \cap \mathcal{B}$=(<0(0.7,0.3), (0.3.0.4), (0.9,1)>; <1(0.5,0), (0.4,0.3), (1,1) >). Let $T_n(\mathbf{x}, \mathbf{y}) = \max(\mathbf{x} + \mathbf{y} - 1, 0)$ and $T_c(x, y) = \min(1, x + y) \forall x, y \in Z_2$ then $\mathcal{A} \cap \mathcal{B}$ is NMFSR of Z_2 over T_n & T_c.

Remark 4.5. In general, if $\mathcal{A}, \mathcal{B}$ are NMFSR of R with respect to T_n and T_c, then $\mathcal{A} \cup \mathcal{B}$ will always not be a NMFSR of R with respect to T_n and T_c. The accompanying example will show our case.

Example 4.6. Let $(Z_4, +, \cdot)$ be a ring of integers.
Let us define $\mathcal{A}$ = {(<0(0.9,0.6,0.4) (0.2,0.4,0.4) (0.3,0.5,0.6)>, <1(0.7,0.5,0.4) (0.2,0.5,0.6) (0.3,0.6,0.7)>, <2(0.6,0.5,0.4) (0.3,0.6,0.7) (0.3,0.6,0.7)>, <3(0.9,0.5,0.3) (0.2,0.5,0.7) (0.3,0.6,0.7)>}
$\mathcal{B}$ = {<(0 (0.9,0.8,0.7), (0.1,0.2,0.3), (0.2,0.4,0.6), <1 (0.8,0.4,0.3), (0.2,0.3,0.3), (0.3,0.5,0.6)>, <2 (0.9,0.5,0.4), (0.3,0.4,0.5), (0.4,0.5,0.6)> , <3 (0.5,0.2,0.1), (0.3,0.4,0.5), (0.4,0.5,0.6)> }
be two NMFSR of Z_4 under T_n and T_c.
Let us consider $T_n(x, y) = \min(x, y); T_c(x, y) = \max(x, y)$ then $\mathcal{A}$, $\mathcal{B}$ are NMFSR of Z_4.
$\mathcal{A} \cup \mathcal{B}$ = { < 0, (0.9,0.8,0.7), (0.1,0.2,0.3), (0.2,0.4,0.6) > < 1(0.8,0.5,0.4), (0.2,0.3,0.3), (0.3,0.5,0.7) >, < 2(0.9,0.5,0.4), (0.3,0.4,0.5), (0.3,0.5,0.6) >, 3(0.9,0.5,0.3), (0.2,0.4,0.5), (0.3,0.5,0.6) > }
Then for $x = 3; y = 2$. $\mu^i_{\mathcal{A}\cup\mathcal{B}}(3-2) = (0.8, 0.5, 0.4)$
Again, if $\mathcal{A}$ is a NMFSR with respect to T_n and T_c of R then $\forall\ x, y \in Z_4$;
$\mu^i_{\mathcal{A}\cup\mathcal{B}}(x-y) \geq T_n(\mu^i_{\mathcal{A}\cup\mathcal{B};}(x), \mu^i_{\mathcal{A}\cup\mathcal{B}}(y))$
But for $x = 3; y = 2$

$$T_n\left\{\mu^i_{\mathcal{A}\cup\mathcal{B}}(x), \mu^i_{\mathcal{A}\cup\mathcal{B}}(y)\right\} = T_n(\mu^i_{\mathcal{A}\cup\mathcal{B}}(3), \mu^i_{\mathcal{A}\cup\mathcal{B}}(2)) = T_n\{(0.9, 0.5, 0.3), (0.9, 0.5, 0.4)\} = (0.9, 0.5, 0.3)$$

$\therefore \mu^i_{\mathcal{A}\cup\mathcal{B}}(3-2) = (0.8, 0.5, 0.4); T_n\{\mu^i_{A}(3), \mu^i_{A}(2)\} = (0.9, 0.5, 0.3)$
$\mu^i_{\mathcal{A}\cup\mathcal{B}}(3-2) \ngeq T_n\{\mu^i_{\mathcal{A}\cup\mathcal{B}}(2), \mu^i_{\mathcal{A}\cup\mathcal{B}}(3)\}$
Hence $\mathcal{A} \cup \mathcal{B}$ is not NMFSR of Z_4 over T_n and T_c.

Corollary 4.7. *If $\mathcal{A}, \mathcal{B}$ are NMFSR of R then $\mathcal{A} \cup \mathcal{B}$ is a NMFSR of R if one is contained in other.*

Proof. Let $\mathbf{x}, \mathbf{y} \in R$ and $i = 1, 2, 3 \ldots, n$

$$\begin{aligned}(i)\ \mu^i_{\mathcal{A}\cup\mathcal{B}}(x-y) &= T_c(\mu^i_{\mathcal{A}}(x-y), \mu^i_{\mathcal{B}}(x-y)) \\ &\geq T_c\{T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)), T_n(\mu^i_{\mathcal{B}}(x), \mu^i_{\mathcal{B}}(y))\}\end{aligned}$$

$$= T_n\{T_c(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{B}}(x)), T_c(\mu^i_{\mathcal{A}}(y), \mu^i_{\mathcal{B}}(y))\}$$
$$= T_n(\mu^i_{\mathcal{A}\cup\mathcal{B}}(x), \mu^i_{\mathcal{A}\cup\mathcal{B}}(x))$$
$$\mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}(x-y) = T_c(\mathcal{N}^i_{\mathcal{A}}(x-y), \mathcal{N}^i_{\mathcal{B}}(x-y))$$
$$\leq T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)), T_c(\mathcal{N}^i_{\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{B}}(y))\}$$
$$= T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{B}}(x)), T_c(\mathcal{N}^i_{\mathcal{A}}(y), \mathcal{N}^i_{\mathcal{B}}(y))\}$$
$$= T_c(\mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}(y))$$

Similarly, $\digamma^i_{\mathcal{A}\cup\mathcal{B}}(x-y) \leq T_c(\digamma^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{B}}(y))$

$$(ii)\ \mu^i_{\mathcal{A}\cup\mathcal{B}}(xy) = T_c(\mu^i_{\mathcal{A}}(xy), \mu^i_{\mathcal{B}}(xy))$$
$$\geq T_c\{T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)), T_n(\mu^i_{\mathcal{B}}(\mathbf{x}), \mu^i_{\mathcal{B}}(\mathbf{y}))\}$$
$$= T_n\{T_c(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{B}}(x)), T_c(\mu^i_{\mathcal{A}}(y), \mu^i_{\mathcal{B}}(y))\}$$
$$= T_n(\mu^i_{\mathcal{A}\cup\mathcal{B}}(x), \mu^i_{\mathcal{A}\cup\mathcal{B}}(y))$$
$$\mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}(xy) = T_c(\mathcal{N}^i_{\mathcal{A}}(xy), \mathcal{N}^i_{\mathcal{B}}(xy))$$
$$\leq T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)), T_c(\mathcal{N}^i_{\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{B}}(y))\}$$
$$= T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{B}}(x)), T_c(\mathcal{N}^i_{\mathcal{A}}(y), \mathcal{N}^i_{\mathcal{B}}(y))\}$$
$$= T_c(\mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}(x), \mathcal{N}^i_{\mathcal{A}\cup\mathcal{B}}(y))$$

Similarly, $\digamma^i_{\mathcal{A}\cup\mathcal{B}}(xy) \leq T_c(\digamma^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{B}}(y))$
Hence $\mathcal{A}\cup\mathcal{B}$ is a NMFSR of R w.r.t T_n and T_c $\forall x, y \in R$ and $i = 1, 2 \ldots, n$ □

Definition 4.8. Let $\mathcal{A}$=$\{< (x, \mu^i_{\mathcal{A}}(y), \mathcal{N}^i_{\mathcal{A}}(x), \digamma^i_{\mathcal{A}}(x) >; x \in R$ and $i = 1, 2 \ldots, n\}$ be a NMFSR of R. Let $\alpha_i, \beta_i, \gamma_i \in [0, 1]$. With $0 \leq \alpha_i + \beta_i + \gamma_i \leq 3$. Then the set $\mathcal{A}_{\alpha,\beta,\gamma}$ is called a level set of $\mathcal{A}$, where for any $x \in \mathcal{A}_{\alpha,\beta,\gamma}$ the following inequalities hold $\mu^i_{\mathcal{A}}(x) \geq \alpha_i$; $\mathcal{N}^i_{\mathcal{A}}(x) \leq \beta_i$; $\digamma^i_{\mathcal{A}}(x) \leq \gamma_i$;

Theorem 4.9. *If $\mathcal{A}$ is said to be a NMFSR of R with respect to T_n and T_c iff $\mathcal{A}_{\alpha,\beta,\gamma}$ is a subring of R with respect to T_n and T_c for all $\alpha_i, \beta_i, \gamma_i \in [0, 1]$ with $\mu_{\mathcal{A}}(x) \geq \alpha_i$; $\mathcal{N}_{\mathcal{A}}(x) \leq \beta_i$; $\digamma_{\mathcal{A}}(x) \leq \gamma_i$; $i = 1, 2, \ldots, n$ and assume that T_n and T_C are idempotent.*

Proof. Since $\mu_{\mathcal{A}}(x) \geq \alpha$; $\mathcal{N}_{\mathcal{A}}(x) \leq \beta$; $\digamma_{\mathcal{A}}(x) \leq \gamma$; $\forall x \in \mathcal{A}_{\alpha,\beta,\gamma}$.
(ie) $A_{\alpha,\beta,\gamma}$ is non-empty.
Then for all i, $\mu^i_{A}(x) \geq \alpha_i$; $\mathcal{N}^i_{A}(x) \leq \beta_i$; $\digamma^i_{A}(x) \leq \gamma_i$;
Now, let $\mathcal{A}$ be NMFSR of R with respect to T_n and T_c and $x, y \in \mathcal{A}_{\alpha,\beta,\gamma}$
To show that, $x - y, xy \in \mathcal{A}_{\alpha,\beta,\gamma}$.
(i) $\mu^i_{\mathcal{A}}(x-y) \geq T\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right) \geq T_n(\alpha_i, \alpha_i) = \alpha_i$
Again, $\mu^i_{\mathcal{A}}(xy) \geq T\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right) \geq T_n(\alpha_i, \alpha_i) = \alpha_i$

(ii) $\mathcal{N}^i_{\mathcal{A}}(x-y) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right) \leq T_c(\ \beta_i, \beta_i\ ) = \beta_i$

Again, $\mathcal{N}^i_{\mathcal{A}}(x) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right) \leq T_c(\ \beta_i, \beta_i\ ) = \beta_i$

Similarly, $F^i_{\mathcal{A}}(x-y) \leq \gamma_i$; $F^i_{\mathcal{A}}(xy) \leq \gamma_i$

$\therefore \mu_{\mathcal{A}}(x) \geq \alpha$; $\mathcal{N}_{\mathcal{A}}(x) \leq \beta$; $F_{\mathcal{A}}(x) \leq \gamma$;

Thus $x-y, xy \in \mathcal{A}_{\alpha,\beta,\gamma}$ is a subring of R.

Conversely, let $\mathcal{A}_{\alpha,\beta,\gamma}$ be a subring of R.

To show that, $\mathcal{A}$ is a NMFSR of R with respect to T_n and T_c .

Let $x, y \in R$ then there exist $\alpha_i \in [0,1]$ such that $T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right) = \alpha_i$

So, $\mu^i_{\mathcal{A}}(x) \geq \alpha_i$; $\mu^i_{\mathcal{A}}(y) \geq \alpha_i$

Also, let there exist $\beta_i, \gamma_i \in$ [0,1] such that $T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right) = \beta_i$; $T_c\left(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y)\right) = \gamma_i$.

Then $x, y \in \mathcal{A}_{\alpha,\beta,\gamma}$.

Again as $\mathcal{A}_{\alpha,\beta,\gamma}$ is a subring of R. $x-y, xy \in \mathcal{A}_{\alpha,\beta,\gamma}$

Hence,

$$\mu^i_{\mathcal{A}}(x-y) \geq \alpha_i = T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right)$$
$$\mu^i_{\mathcal{A}}(xy) \geq \alpha_i = T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right)$$

Similarly,

$$\mathcal{N}^i_{\mathcal{A}}(x-y) \leq \beta_i = T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right); \mathcal{N}^i_{\mathcal{A}}(xy) \leq \beta_i = T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right)$$
$$F^i_{\mathcal{A}}(x-y) \leq \gamma_i = T_c\left(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y)\right); F^i_{\mathcal{A}}(xy) \leq \gamma_i = T_c\left(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y)\right)$$

$\therefore \mathcal{A}$ is a NMFSR of R with respect to T_n and T_c . □

Proposition 4.10. *Let $\mathcal{A}$ be a NMFSR of R w.r. t. T_n and T_c where T_n, T_c are idempotent then S= $\{\ x \in R/\mu^i_A(x) = 1, \mathcal{N}^i_A(x) = 0, F^i_A(x) = 0;\ i = 1, 2 \ldots, n\}$ is a subring of R.*

Proof. Let $x, y \in$ S.Then,

(i) $\mu^i_{\mathcal{A}}(x-y) \geq T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right) = T(1,1) = 1$

$\mathcal{N}^i_{\mathcal{A}}(x-y) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right) = T_c(0,0) = 0$

Similarly, $F^i_{\mathcal{A}}(x-y) \leq 0$. hence $x-y \in S$.

Also,

(ii) $\mu^i_{\mathcal{A}}(xy) \geq T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right) = T(1,1) = 1$

$\mathcal{N}^i_{\mathcal{A}}(xy) \leq T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right) = T_c(0,0) = 0$

Similarly, $F^i_{\mathcal{A}}(xy) \leq 0$. Hence $xy \in S$.

Thus $S = \{x \in R/\mu^i_{\mathcal{A}}(\mathbf{x}) = 1, \mathcal{N}^i_{\mathcal{A}}(x) = 0, F^i_{\mathcal{A}}(x) = 0\}$ is a subring of R w. r. t T_n and T_c . □

Definition 4.11. Let $\mathcal{A}$ be a NMFS of R. Then $\mathcal{A}$ is Said to be NMFLI of R w.r.t, T_n and T_c if

(i) $\mu^i_{\mathcal{A}}(x-y) \geq T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y))$; $\mathcal{N}^i_{\mathcal{A}}(x-y) \leq T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y))$;
$F^i_{\mathcal{A}}(x-y) \leq T_c(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y))$
(ii) $\mu^i_{\mathcal{A}}(xy) \geq \mu^i_{\mathcal{A}}(y)$; $\mathcal{N}^i_{\mathcal{A}}(xy) \leq \mathcal{N}^i_{\mathcal{A}}(y)$; $F^i_{\mathcal{A}}(xy) \leq F^i_{\mathcal{A}}(y) \, \forall x, y \in R, \quad i = 1, 2 \ldots, n$

Example 4.12. Let $(Z_2, +, \cdot)$ be a ring. Define
$\mathcal{A} = \{(<$ (0,(0.9,0.7),(0.1,0.5),(0.2,0.3)), (1,(0.8,0.6),(0.2,0.5),(0.3,0.6)) $> \}$
Let us consider $T_n(x, y) = xy$; $T_c(x, y) = x + y - xy$. Then $\mathcal{A}$ is NMFLI of Z_2 with T_n and T_c

Definition 4.13. Let $\mathcal{A}$ be a NMFS of R w. r. t T_n and T_c. Then $\mathcal{A}$ is NMFRI of R w. r. t T_n and T_c if
(i) $\mu^i_{\mathcal{A}}(x-y) \geq T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right)$; $\mathcal{N}^i_{\mathcal{A}}(x-y) \leq T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y))$;
$F^i_{\mathcal{A}}(x-y) \leq T_c(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y))$
(ii) $\mu^i_{\mathcal{A}}(xy) \geq \mu^i_{\mathcal{A}}(x)$; $\mathcal{N}^i_{\mathcal{A}}(xy) \leq \mathcal{N}^i_{\mathcal{A}}(\mathbf{x})$; $F^i_{\mathcal{A}}(xy) \leq F^i_{\mathcal{A}}(x)$, $\forall x, y \in R$, $i = 1, 2 \ldots, n$.

Example 4.14. Consider the ring $(Z_3, +, \cdot)$. For all $x \in Z_3$, we define NMFS $\mathcal{A}$ of Z_3 as $\mathcal{A} =$ ($<$ 0(0.9,0.7), (0.1.0.3), (0.4,0.6) $>$; $<$ 1(0.8,0.6), (0.1,0.4), (0.4,0.7) $>$;$<$ 2(0.7,0.4), (0.1.0.4), (0.4,0.6) $>$)
Let us consider $T_n(x, y) = \min(x, y)$; $T_c(x, y) = \max(x, y)$ then $\mathcal{A}$ is NMFRI of Z_3 over T_n & T_c.

Definition 4.15. Let $\mathcal{A}$ be a NMFS of R with respect to T_n and T_c. Then $\mathcal{A}$ is Said to be NMFI with respect to T_n and T_c of R if
(i) $\mu^i_{\mathcal{A}}(x-y) \geq T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y))$; $\mathcal{N}^i_{\mathcal{A}}(x-y) \leq T_c(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y))$
$F^i_{\mathcal{A}}(x-y) \leq T_c(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y))$
(ii) $\mu^i_{\mathcal{A}}(xy) \geq T_c(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y))$; $\mathcal{N}^i_{\mathcal{A}}(xy) \leq T_n(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y))$
$F^i_{\mathcal{A}}(xy) \leq T_n(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y))$, $\forall x, y \in R$.

Example 4.16. Consider the ring $(Z_2, +, \cdot)$. For all $x \in Z_2$, we define NMFS $\mathcal{A}$ of Z_2 as $\mathcal{A} =$ ($<$ 0(0.8,0.7), (0.2.0.3), (0.1,0.4) $>$; $<$ 1(0.7,0.6), (0.2,0.3), (0.2,0.5) $>$
Let us consider $T_n(x, y) = \min(x, y)$; $T_c(x, y) = \max(x, y)$ then $\mathcal{A}$ is NMFI of Z_2 over T_n & T_c.

Theorem 4.17. *Let $\mathcal{A}$ be a NMFS of R with respect to T_n and T_c where, T_n, T_c are idempotent. Then $\mathcal{A}$ is said to be NMFLI(NMFRI) of R with T_n and T_c iff $\mathcal{A}_{\alpha,\beta,\gamma}$ is a LI(RI) of R, $\forall$ $\alpha_i, \beta_i, \gamma_i \in$ [0,1]. with $\mu^i_{\mathcal{A}}(\mathbf{x}) \geq \alpha_i$; $\mathcal{N}^i_{\mathcal{A}}(x) \leq \beta_i$; $F^i_{\mathcal{A}}(x) \leq \gamma_i$ and $\alpha_i + \beta_i + \gamma_i \leq 3$, where $\mu^i_{\mathcal{A}}(0) \geq \alpha_i$; $\mathcal{N}^i_{\mathcal{A}}(0) \leq \beta_i$; $F^i_{\mathcal{A}}(0) \leq \gamma_i$, $i = 1, 2 \ldots, n$.*

Proof. Let $\mathcal{A}$ be a NMFLI of R with respect to T_n and T_c .
If $x, y \in \mathcal{A}_{\alpha,\beta,\gamma}$, $i = 1, 2, \ldots, n$
Then by $\mu^i_{\mathcal{A}}(x-y) \geq T_n(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)) \geq T_n(\alpha_i, \alpha_i) = \alpha_i$
$\mathcal{N}^i_{\mathcal{A}}(x-y) \leq T_C(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(x)) \leq T_C(\beta_i, \beta_i) = \beta_i$

Similarly, $F^i_{\mathcal{A}}(x-y) \leq \gamma_i$. $\therefore \mu_{\mathcal{A}}(x-y) \geq \alpha$; $\mathcal{N}_{\mathcal{A}}(x-y) \leq \beta; F_{\mathcal{A}}(x-y) \leq \gamma$;
We obtain that $x-y \in \mathcal{A}_{\alpha,\beta,\gamma}$
Now let $x \in \mathcal{A}_{\alpha,\beta,\gamma}$ and r $\in$ R. Then from $\mu^i_{\mathcal{A}}(rx) \geq \mu^i_{\mathcal{A}}(x) \geq \alpha_i$

$$\mathcal{N}^i_{\mathcal{A}}(rx) \leq \mathcal{N}^i_{\mathcal{A}}(x) \leq \beta_i$$

$$F^i_{\mathcal{A}}(rx) \leq F^i_{\mathcal{A}}(x) \leq \gamma_i$$

Therefore $rx \in \mathcal{A}_{\alpha,\beta,\gamma}$. Hence $\mathcal{A}_{\alpha,\beta,\gamma}$ is a LI of R.
Similarly, we can prove it for right ideal (ie) $xr \in \mathcal{A}_{\alpha,\beta,\gamma}$.
Conversely, let $\mathcal{A}_{\alpha,\beta,\gamma}$ be a LI of R and x, $y \in \mathcal{A}_{\alpha,\beta,\gamma}$ such that
$\mu^i_{\mathcal{A}}(x) = \mu^i_{\mathcal{A}}(y) = \alpha_i; \mathcal{N}^i_{\mathcal{A}}(x) = \mathcal{N}^i_{\mathcal{A}}(y) = \beta_i; F^i_{\mathcal{A}}(x) = F^i_{\mathcal{A}}(y) = \gamma_i$
$\because x-y \in \mathcal{A}_{\alpha,\beta,\gamma}$ so

$$\mu^i_{\mathcal{A}}(x-y) \geq \alpha_i = T(\alpha_i,\alpha_i) = T_n\left(\mu^i_{\mathcal{A}}(x), \mu^i_{\mathcal{A}}(y)\right)$$

$$\mathcal{N}^i_{\mathcal{A}}(x-y) \leq \beta_i = T_c(\beta_i,\beta_i) = T_c\left(\mathcal{N}^i_{\mathcal{A}}(x), \mathcal{N}^i_{\mathcal{A}}(y)\right)$$

Similarly, we get $F^i_{\mathcal{A}}(x-y) \leq T_c\left(F^i_{\mathcal{A}}(x), F^i_{\mathcal{A}}(y)\right)$. Also $\because xy \in \mathcal{A}_{\alpha,\beta,\gamma}$ then

$$\mu^i_{\mathcal{A}}(xy) \geq \alpha_i = \mu^i_{\mathcal{A}}(y)$$

$$\mathcal{N}^i_{\mathcal{A}}(xy) \leq \beta_i = \mathcal{N}^i_{\mathcal{A}}(y)$$

$$F^i_{\mathcal{A}}(xy) \leq \gamma_i = F^i_{\mathcal{A}}(y), x, y \in \mathcal{A}_{\alpha,\beta,\gamma}.$$

$\therefore \mathcal{A}$ is a NMFLR of R with T_n and T_c are idempotent. Similarly, we can prove it for RI. □

Theorem 4.18. *Let $\mathcal{A}$ be a NMFS of R with respect to T_n and T_c where T_n, T_c be idempotent. Then $\mathcal{A}$ is said to be NMFI of R with T_n and T_c iff $\mathcal{A}_{\alpha,\beta,\gamma}$ is an ideal of R $\forall$ $\alpha_i, \beta_i, \gamma_i \in$ [0,1] with $\mu^i_{\mathcal{A}}(x) \geq \alpha_i$; $\mathcal{N}^i_{\mathcal{A}}(x) \leq \beta_i$; $F^i_{\mathcal{A}}(x) \leq \gamma_i$ and $0 \leq \alpha_i + \beta_i + \gamma_i \leq 3$, where $\mu^i_{\mathcal{A}}(0) \geq \alpha_i$; $\mathcal{N}^i_{\mathcal{A}}(0) \leq \beta_i$; $F^i_{\mathcal{A}}(0) \leq \gamma_i$,i=1, 2. . . , n.*

Proof. Follows from the above theorem. □

Theorem 4.19. *If $\mathcal{A}$ and $\mathcal{B}$ are NMFLI(NMFRI) of R with respect to T_n and T_c then $\mathcal{A} \cap \mathcal{B}$ also a NMFLI(NMFRI) of R with respect to T_n and T_c where, T_n and T_c are idempotent.*

Proof. Let $x, y \in R.\mathcal{A} \cap \mathcal{B}$ is NMFSR with respect to T_n and T_c . (By theorem 4.2).
It is enough to show,

$$
\begin{aligned}
\text{(i)}\mu^i_{\mathcal{A}\cap\mathcal{B}}(xy) &= T_n(\mu^i_{\mathcal{A}}(xy), \mu^i_{\mathcal{B}}(xy)) \\
&\geq T_n(\mu^i_{\mathcal{A}}(y)), \mu^i_{\mathcal{B}}(y)) \\
&= T_n(\mu^i_{\mathcal{A}\cap\mathcal{B}}(y)) \\
\mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(xy) &= T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{xy}), \mathcal{N}^i_{\mathcal{B}}(\mathbf{xy})) \\
&\leq T_c(\mathcal{N}^i_{\mathcal{A}}(y), \mathcal{N}^i_{\mathcal{B}}(y)) \\
&= \mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(\mathbf{y})
\end{aligned}
$$

Similarly, $F^i_{\mathcal{A}\cap\mathcal{B}}((\mathbf{xy}) \leq F^i_{\mathcal{A}\cap\mathcal{B}}(\mathbf{y})$. Therefore $\mathcal{A}\cap\mathcal{B}$ is a NMFLI with respect to T_n and T_c In the similar way we can easily prove for NMFRI. □

Remark 4.20. In general, if $\mathcal{A}, \mathcal{B}$ are NMFLI(NMFRI) of R with respect to T_n and T_c, then $\mathcal{A}\cup\mathcal{B}$ will always not be a NMFLI(NMFRI) of R with respect to T_n and T_c. The accompanying example will show our case.

Example 4.21. Let $(Z_4, +, \cdot)$ be a ring of integers.
Let us define $\mathcal{A}$ = {(<0(0.9,0.6) (0.2,0.4) (0.3,0.5)>, <1(0.8,0.5) (0.3,0.6) (0.3,0.6)>, <2(0.8,0.5) (0.3,0.6) (0.3,0.6)>, <3(0.9,0.5) (0.2,0.5) (0.3,0.6)>}
$\mathcal{B}$ = {<(0 (0.9,0.8), (0.1,0.2), (0.2,0.4), <1 (0.8,0.4), (0.3,0.4), (0.4,0.5)>, <2 (0.9,0.5), (0.3,0.4), (0.4,0.5)> , <3 (0.8,0.4), (0.3,0.4), (0.4,0.5)> } be two NMFS of Z_4 under T_n and T_c.
Let us consider $T_n(\mathbf{x}, \mathbf{y}) = \min(\mathbf{x}, \mathbf{y}); T_c(\mathbf{x}, \mathbf{y}) = \max(\mathbf{x}, \mathbf{y})$ then $\mathcal{A}$,and $\mathcal{B}$ be NMFSR of Z_4.
$\mathcal{A}\cup\mathcal{B}$ = { < 0, (0.9,0.8), (0.1,0.2), (0.2,0.4) > < 1(0.8,0.5), (0.2,0.3), (0.3,0.5) >, < 2(0.9,0.5), (0.3,0.4), (0.3,0.5) >, 3(0.9,0.5), (0.2,0.4), (0.3,0.5) > }
Then for $\mathbf{x} = 3; \mathbf{y} = 2$. $\mu^i_{\mathcal{A}\cup\mathcal{B}}(3-2) = (0.8, 0.5)$
Again, if $\mathcal{A}$ is a NMFLI with respect to T_n and T_c of R then $\forall\ \mathbf{x}, \mathbf{y} \in Z_4$
$\mu^i_{\mathcal{A}\cup\mathcal{B}}(\mathbf{x}-\mathbf{y}) \geq T_n(\mu^i_{\mathcal{A}\cup\mathcal{B}}(\mathbf{x}), \mu^i_{\mathcal{A}\cup\mathcal{B}}(y))\ \mu^i_{\mathcal{A}}(\mathbf{xy}) \geq \mu^i_{\mathcal{A}}(\mathbf{y});\ \ \mathcal{N}^i_{\mathcal{A}}(\mathbf{x}y) \leq \mathcal{N}^i_{\mathcal{A}}(\mathbf{y});\ \ F^i_{\mathcal{A}}(\mathbf{x}y) \leq F^i_{\mathcal{A}}(\mathbf{y})\, \forall \mathbf{x}, y \in R,\ \ i = 1, 2 \ldots, n$
But for $\mathbf{x} = 3; \mathbf{y} = 2$

$$T_n\left\{\mu^i_{\mathcal{A}\cup\mathcal{B}}(\mathbf{x}), \mu^i_{\mathcal{A}\cup\mathcal{B}}(y)\right\} = T_n(\mu^i_{\mathcal{A}\cup\mathcal{B}}(3), \mu^i_{\mathcal{A}\cup\mathcal{B}}(2)) = T_n\{(0.9, 0.5), (0.9, 0.5)\} = (0.9, 0.5)$$

$\therefore \mu^i_{\mathcal{A}\cup\mathcal{B}}(3-2) = (0.8, 0.5); T_n\{\mu^i_{\mathcal{A}}(3), \mu^i_{\mathcal{A}}(2)\} = (0.9, 0.5)$
$\mu^i_{\mathcal{A}\cup\mathcal{B}}(3-2) \ngeq T_n\{\mu^i_{\mathcal{A}\cup\mathcal{B}}(3), \mu^i_{\mathcal{A}\cup\mathcal{B}}(2)\}$
Hence $\mathcal{A}\cup\mathcal{B}$ is not NMFLI of Z_4 over T_n and T_c.

Theorem 4.22. *If $\mathcal{A}$ and $\mathcal{B}$ are NMFI of ring R with respect to T_n and T_c then $\mathcal{A}\cap\mathcal{B}$ also a NMFI of R w. r. t T_n and T_c where T_n and T_c are idempotent.*

Proof. Follows from above theorem. □

Corollary 4.23. *Let* $\{\mathcal{A}_i, i = 1, 2 \ldots, n\}$ *be a NMFSR of* R *with respect to* T_n *and* T_c. *Then* $\cap\mathcal{A}_i$ *is also NMFSR of* R .

Definition 4.24. Let $\mathcal{A}$ and $\mathcal{B}$ be the two NMFS in R. Then $\mathcal{A} \circ \mathcal{B}$ is defined as , $\forall \mathbf{x}, \mathbf{y} \in$ R,

$$(\mathcal{A} \circ \mathcal{B})(\mathbf{x}) = \begin{cases} \underbrace{sup}_{\mathbf{x}=\mathbf{y}z} & T_n\left(\mu^i_{\mathcal{A}}(\mathbf{y}), \mu^i_{\mathcal{B}}(z)\right) \\ \underbrace{inf}_{\mathbf{x}=\mathbf{y}z} & T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}), \mathcal{N}^i_{\mathcal{B}}(z)) \quad \text{if } \mathbf{x} = \mathbf{yz} \\ \underbrace{inf}_{\mathbf{x}=\mathbf{y}z} & T_c(\digamma^i_{\mathcal{A}}(\mathbf{y}), \digamma^i_{\mathcal{B}}(z)) \\ (0,1,1) & \qquad\qquad\qquad \text{if } \mathbf{x} \neq \mathbf{y}z \end{cases}$$

Theorem 4.25. *Let* $\mathcal{A}$, $\mathcal{B}$ *be the two NMS in R. If* $\mathcal{A}$ *and* $\mathcal{B}$ *are NMFI of R with respect to* T_n *and* T_c *then* $\mathcal{A} \circ\mathcal{B} \subset \mathcal{A} \cap \mathcal{B}$.

Proof. Let $\mathbf{x} \in R$. Suppose $\mathcal{A} \circ \mathcal{B} = (0, 1, 1)$ then there is nothing to prove.
Suppose $\mathcal{A} \circ \mathcal{B} \neq (0, 1, 1)$
Then,

$$(\mathcal{A} \circ \mathcal{B})(\mathbf{x}) = \begin{cases} \underbrace{sup}_{x=yz} & T_n\left(\mu^i_{\mathcal{A}}(\mathbf{y}), \mu^i_{\mathcal{B}}(z)\right) \\ \underbrace{inf}_{x=yz} & T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}), \mathcal{N}^i_{\mathcal{B}}(z) \quad \text{if } \mathbf{x} = \mathbf{yz} \\ \underbrace{inf}_{x=yz} & T_c(\digamma^i_{\mathcal{A}}(\mathbf{y}), \digamma^i_{\mathcal{B}}(z) \end{cases}$$

Since $\mathcal{A}$, $\mathcal{B}$ are NMFI of R with T_n and T_c .
(i) $\mu^i_{\mathcal{A}}(\mathbf{y}) \leq \mu^i_{\mathcal{A}}(\mathbf{y}z) = \mu^i_{\mathcal{A}}(\mathbf{x})$; $\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}) \geq \mathcal{N}^i_{\mathcal{A}}(\mathbf{y}z) = \mathcal{N}_{\mathcal{A}}(\mathbf{x})$; $\digamma^i_{\mathcal{A}}(\mathbf{y}) \geq \digamma^i_{\mathcal{A}}(\mathbf{y}z) = \digamma^i_{\mathcal{A}}(\mathbf{x})$
(ii) $\mu^i_{\mathcal{B}}(z) \leq \mu^i_{\mathcal{B}}(\mathbf{y}z) = \mu^i_{\mathcal{B}}(\mathbf{x})$; $\mathcal{N}^i_{\mathcal{B}}(z) \geq \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}z) = \mathcal{N}^i_{\mathcal{B}}(\mathbf{x})$; $\digamma^i_{\mathcal{B}}(z) \geq \digamma^i_{\mathcal{B}}(\mathbf{y}z) = \digamma^i_{\mathcal{B}}(\mathbf{x})$
Thus,

$$\begin{aligned} \mu^i_{\mathcal{A}\circ\mathcal{B}}(\mathbf{x}) &= \underbrace{sup}_{x=\mathbf{y}z}\{T_n(\mu^i_{\mathcal{A}}(\mathbf{y}), \mu^i_{\mathcal{B}}(z)\} \\ &\leq T_n(\mu^i_{\mathcal{A}}(\mathbf{x}), \mu^i_{\mathcal{B}}(\mathbf{x})) \\ &= \mu^i_{\mathcal{A}\cap\mathcal{B}}(\mathbf{x}) \\ \mathcal{N}^i_{\mathcal{A}\circ\mathcal{B}}(\mathbf{x}) &= \underbrace{inf}_{x=yz}\{T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}), \mathcal{N}^i_{\mathcal{B}}(z)\} \\ &\geq T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}), \mathcal{N}^i_{\mathcal{B}}(z)) \\ &= \mathcal{N}^i_{\mathcal{A}\cap\mathcal{B}}(\mathbf{x}) \end{aligned}$$

Similarly, $\digamma^i_{\mathcal{A}\circ\mathcal{B}}(\mathbf{x}) \geq \digamma^i_{\mathcal{A}\cap\mathcal{B}}(\mathbf{x})$. Hence $\mathcal{A} \circ \mathcal{B} \subset \mathcal{A} \cap \mathcal{B}$. □

5. Direct product and Homomorphism on Neutrosophic Multifuzzy subrings over norms

Definition 5.1. Let R_1 and R_2 be the two rings. Let $\mathcal{A}$ and $\mathcal{B}$ be the two NMFS of R_1 and R_2 respectively with T_n and T_c. Then $\mathcal{A}\times\mathcal{B}$=$\{< ((\mathbf{x},\mathbf{y}), \mu^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x},\mathbf{y}), \mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x},\mathbf{y}), F^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x},\mathbf{y}) > / \mathbf{x}\in R_1, \mathbf{y}\in R_2, i = 1, 2, \ldots, n\}$
Where $\mu^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x},\mathbf{y}) = T_n(\mu^i_{\mathcal{A}}(\mathbf{x}), \mu^i_B(\mathbf{y}))$
$\mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x},\mathbf{y}) = T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{x}), \mathcal{N}^i_B(\mathbf{y}))$, $F^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x},\mathbf{y}) = T_c(F^i_{\mathcal{A}}(\mathbf{x}), F^i_{\mathcal{B}}(\mathbf{y}))$

Theorem 5.2. *Let R_1 and R_2 be the two rings with $\mathcal{A}$ and B are respectively NMFSR of R_1 and R_2 over T_n and T_c .Then $\mathcal{A}\times\mathcal{B}$ is also a NMFSR of $R_1\times R_2$ With respect to T_n and T_c .*

Proof. Let $\mathcal{A}$ and B are respectively NMFSR of R_1 and R_2 respectively over T_n and T_c .
Let $(\mathbf{x}_1,\mathbf{y}_1)(\mathbf{x}_2,\mathbf{y}_2)\in\mathcal{A}\times B$.

$$
\begin{aligned}
\text{Then, } \mu^i_{\mathcal{A}\times\mathcal{B}}[(\mathbf{x}_1,\mathbf{y}_1)-(\mathbf{x}_2,\mathbf{y}_2)] &= \mu^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1-\mathbf{x}_2),(\mathbf{y}_1-\mathbf{y}_2))\\
&= T_n\left\{\mu^i_{\mathcal{A}}((\mathbf{x}_1-\mathbf{x}_2)), \mu^i_{\mathcal{B}}(\mathbf{y}_1-\mathbf{y}_2))\right\}\\
&\geq T_n\{T_n(\mu^i_{A}(\mathbf{x}_1), \mu^i_{A}(\mathbf{x}_2)), T_n(\mu^i_{\mathcal{B}}(\mathbf{y}_1), \mu^i_{\mathcal{B}}(\mathbf{y}_2))\}\\
&\geq T_n\{T_n(\mu^i_{A}(\mathbf{x}_1), \mu^i_{\mathcal{B}}(\mathbf{y}_1)), T_n(\mu^i_{A}(\mathbf{x}_2), \mu^i_{\mathcal{B}}(\mathbf{y}_2))\}\\
&\geq T_n\left\{\mu^i_{\mathcal{A}\times\mathcal{B}}((x_1,\mathbf{y}_1)), \mu^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x},\mathbf{y}_2))\right\}.\\
\mu^i_{\mathcal{A}\times\mathcal{B}}[(\mathbf{x}_1,\mathbf{y}_1)\cdot(\mathbf{x}_2,\mathbf{y}_2)] &= \mu^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1.\mathbf{x}_2),(\mathbf{y}_1.\mathbf{y}_2))\\
&= T_n\left\{\mu^i_{\mathcal{A}}((\mathbf{x}_1.\mathbf{x}_2)), \mu^i_{\mathcal{B}}(\mathbf{y}_1.\mathbf{y}_2))\right\}\\
&\geq T_n\{T_n(\mu^i_{A}(\mathbf{x}), \mu^i_{\mathcal{A}}(\mathbf{x}_2)), T_n(\mu^i_{\mathcal{B}}(\mathbf{y}_1), \mu^i_{\mathcal{B}}(\mathbf{y}_2))\}\\
&\geq T_n\{T_n(\mu^i_{A}(\mathbf{x}_1), \mu^i_{\mathcal{B}}(\mathbf{y}_1)), T_n(\mu^i_{\mathcal{A}}(\mathbf{x}_2), \mu^i_{\mathcal{B}}(\mathbf{y}_2))\}\\
&\geq T_n\left\{\mu^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1,\mathbf{y}_1)), \mu^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))\right\}.
\end{aligned}
$$

$$
\begin{aligned}
\text{Again, } \mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}[(\mathbf{x},\mathbf{y}_1)-(\mathbf{x}_2,\mathbf{y}_2)] &= \mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1-\mathbf{x}_2),(\mathbf{y}_1-\mathbf{y}_2))\\
&= T_c\left\{\mathcal{N}^i_{\mathcal{A}}((\mathbf{x}_1-\mathbf{x}_2)), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_1-\mathbf{y}_2))\right\}\\
&\leq T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{x}), \mathcal{N}^i_{\mathcal{A}}(\mathbf{x}_2)), T_c(\mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_1), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_2))\}\\
&\leq T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{x}_1), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_1)), T_c(\mathcal{N}^i_{A}(\mathbf{x}_2), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_2))\}\\
&\leq T_c\left\{\mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x}_1,\mathbf{y}_1), \mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}(\mathbf{x}_2,\mathbf{y}_2)\right\}.\\
\mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}[(\mathbf{x}_1,\mathbf{y}_1).(\mathbf{x}_2,\mathbf{y}_2)] & \mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1.\mathbf{x}_2),(\mathbf{y}_1.\mathbf{y}_2))\\
&= T_c\left\{\mathcal{N}^i_{\mathcal{A}}((\mathbf{x}_1.\mathbf{x}_2)), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_1.\mathbf{y}_2))\right\}\\
&\leq T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{x}_1), \mathcal{N}^i_{\mathcal{A}}(\mathbf{x}_2)), T_c(\mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_1), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_2))\}\\
&\leq T_c\{T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{x}_1), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_1)), T_c(\mathcal{N}^i_{\mathcal{A}}(\mathbf{x}_2), \mathcal{N}^i_{\mathcal{B}}(\mathbf{y}_2))\}
\end{aligned}
$$

$$\leq T_c \left\{\mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1,\mathbf{y}_1)), \mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))\right\}.$$

Similarly, we get,

$$F^i_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1)-(\mathbf{x}_2,\mathbf{y}_2)\right] \leq T_c \left\{F^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1,\mathbf{y}_1)), F^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))\right\}$$

$$F^i_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1).(\mathbf{x}_2,\mathbf{y}_2)\right] \leq T_c \left\{F^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_1,\mathbf{y}_1)), F^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))\right\}$$

Hence $\mathcal{A}\times\mathcal{B}$ is also a NMFSR of $R_1 \times R_2$ over T and T_c. □

Remark 5.3. However, $\mathcal{A}\times\mathcal{B}$ is a NMFSR of $R_1 \times R_2$ over T_n and T_c. Then both $\mathcal{A}$ and $\mathcal{B}$ are need not be NMFSR of R_1 and R_2 respectively over T_n and T_c which is obvious from the accompanying case.

Example 5.4. Let $(Z_4,+,\cdot)$ and $(Z_2,+,\cdot)$ be a ring. Let $T_n\,(\mathbf{x},\mathbf{y}) = \min(\mathbf{x},\mathbf{y})$ and $T_c(\mathbf{x},\mathbf{y}) = \max(\mathbf{x},\mathbf{y})$. We define a NMFS $\mathcal{A}$ and $\mathcal{B}$ of Z_4 and Z_2 as

$\mathcal{A}$ = ($<$ 0(0.9,0.8), (0.1.0.2), (0.5,0.6) $>$; $<$ 1(0.9,0.7), (0.1,0.2), (0.5,0.6) $>$ $<$ 2(0.8,0.6), (0.2,0.3), (0.6,0.7) $>$, $<$ 3(0.7,0.5), (0.3,0.2), (0.7,0.6)

$\mathcal{B}$ = ($<$ 0(0.8,0.7), (0.2.0.3), (0.6,0.7) $>$; $<$ 1(0.7,0.7), (1,0) (0.3,0.4), (0.7,0.8) $>$).

$\mathcal{A}\times\mathcal{B}$ = { $<$ (0,0) (0.8,0.7) $>$, $<$ (0,1) (0.7,0.7) $>$, $<$ (1,0) (0.8,0.7) $>$, $<$ (1,1) (0.7,0.7) $>$, $<$ (2,0) (0.8,0.6) $>$, $<$ (2,1) (0.7,0.6) $>$, $<$ (3,0) (0.7,0.5) $>$, $<$ (3,1) (0.7,0.5) $>$)

It is clear that $\mathcal{A}\times\mathcal{B}$ a NMFSR of $Z_4\times Z_2$. But $\mathcal{A}$ is not a NMFSR of Z_2 as $\mathcal{N}^i_{\mathcal{A}}\,(1\cdot 0) = (0.1, 0.3)$; $T_c\{\mathcal{N}^i_{\mathcal{A}}\,(1)\,, \mathcal{N}^i_{\mathcal{A}}\,(0)\} = (0.1, 0.2)$ $\mathcal{N}^i_{\mathcal{A}}\;(1\cdot 0) \nleq T_c\{\mathcal{N}^i_{\mathcal{A}}\,(1)\,, \mathcal{N}^i_{\mathcal{A}}\,(0)\ \}$

Corollary 5.5. *Let, for all $i \in \{1,\ 2.\ldots\ n\}$,$(R_i,+,\cdot\ )$ are rings and $\mathcal{A}_i$ is a NMFSR of R_i over T_n and T_c .Then $\mathcal{A}_1\times\mathcal{A}_2\times\ldots.\mathcal{A}_n$ is a NMFSR of $R_1\times R_2\ldots.\times R_n$ over T_n and T_c, where $n\in N$*

Theorem 5.6. *If $\mathcal{A}$ and $\mathcal{B}$ are NMFLI(NMFRI) of R_1 and R_2 over T_n and T_c. Then $\mathcal{A}\times\mathcal{B}$ is also a NMFLI(NMFRI) of $R_1\times R_2$ With respect to T_n and T_c.*

Proof. Let $(x_1,y_1)\,(x_2,y_2) \in \mathcal{A}\times\mathcal{B}$. Assume $\mathcal{A}_1$ and $\mathcal{A}_2$ are NMFLI of R_1 and R_2 respectively over T_n and T_c.

We have to show that $\mathcal{A}\times\mathcal{B}$ is also a NMFLI of $R_1\times R_2$ over T_n and T_c

By theorem 5.2,

$\mathcal{A}\times\mathcal{B}$ is also a NMFSR of $R_1\times R_2$ over T_n and T_c .

It is enough to show

$$\mu^i_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1)\,(\mathbf{x}_2,\mathbf{y}_2)\right] \geq \mu^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))$$

$$\mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1)\,(\mathbf{x}_2,\mathbf{y}_2)\right] \leq \mathcal{N}^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))$$

$$F^i_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1)\,(\mathbf{x}_2,\mathbf{y}_2)\right] \leq F^i_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))$$

$$\begin{aligned}(ie)\mu^{i}_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1)(\mathbf{x}_2,\mathbf{y}_2)\right] &= \mu^{i}_{\mathcal{A}\times\mathcal{B}}\left[((\mathbf{x}_1\mathbf{x}_2,\mathbf{y}_1\mathbf{y}_2))\right]\\ &= T_n(\mu^{i}_{\mathcal{A}}(\mathbf{x}_1\mathbf{x}_2),\mu^{i}_{\mathcal{B}}(\mathbf{y}_1\mathbf{y}_2))\\ &\geq T_n(\mu^{i}_{\mathcal{A}}(\mathbf{x}_2),\mu^{i}_{\mathcal{B}}(\mathbf{y}_2))\\ &= \mu^{i}_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))\\ \mathcal{N}^{i}_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1)(\mathbf{x}_2,\mathbf{y}_2)\right] &= \mathcal{N}^{i}_{\mathcal{A}\times\mathcal{B}}\left[((\mathbf{x}_1\mathbf{x}_2,\mathbf{y}_1\mathbf{y}_2))\right]\\ &= T_c(\mathcal{N}^{i}_{\mathcal{A}}(\mathbf{x}_1\mathbf{x}_2),\mathcal{N}^{i}_{\mathcal{B}}(\mathbf{y}_1\mathbf{y}_2))\\ &\leq T_c(\mathcal{N}^{i}_{\mathcal{A}}(\mathbf{x}_2),\mathcal{N}^{i}_{\mathcal{B}}(\mathbf{y}_2))\\ &= \mathcal{N}^{i}_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))\end{aligned}$$

Similarly, $F^{i}_{\mathcal{A}\times\mathcal{B}}\left[(\mathbf{x}_1,\mathbf{y}_1)(\mathbf{x}_2,\mathbf{y}_2)\right] \leq F^{i}_{\mathcal{A}\times\mathcal{B}}((\mathbf{x}_2,\mathbf{y}_2))$

Hence $\mathcal{A}\times\mathcal{B}$ is also a NMFLI of $R_1\times R_2$ over T_n and T_c

Similarly, we can show it for NMFRI. □

Theorem 5.7. *If $\mathcal{A}$ and $\mathcal{B}$ are NMFI of R_1 and R_2 over T_n and T_c .Then $\mathcal{A}\times\mathcal{B}$ is also a NMFI of $R_1\times R_2$ with respect to T_n and T_c.*

Proof. Follows from above theorem. □

Example 5.8. Let $(Z_2,+,\cdot)$ be a ring. Define

$\mathcal{A} = \{(<$ (0,(0.9,0.7),(0.1,0.5),(0.2,0.3)), (1,(0.8,0.6),(0.2,0.5),(0.3,0.6)) $>\}$

$\mathcal{B}$ =$\{<$ (0,(0.8,0.7),(0.2,0.3),(0.1,0.4)), (1,(0.7,0.6),(0.2,0.3),(0.2,0.5)) $>\}$ be two NMFS of Z_2 under T_n and T_c. Let us consider $T_n(\mathbf{x},\mathbf{y}) = \mathbf{xy}$; $T_c(\mathbf{x},\mathbf{y}) = \mathbf{x}+\mathbf{y}-\mathbf{xy}$. Then $\mathcal{A}\times\mathcal{B}$ is NMFI with T_n and T_c of $Z_2\times Z_2$.

Corollary 5.9. *Let, for all $i\in\{1,2\ldots,n\}$, $(R_i,+,\cdot)$ are rings and A_i is a NMFI of R_i. Then $\mathcal{A}_1\times\mathcal{A}_2\times\ldots\times\mathcal{A}_n$ is a NMFI of $R_1\times R_2\ldots\times R_n$ where $n\in N$.*

Definition 5.10. If $\mathcal{A} = (\mu^{i}_{\mathcal{A}},\mathcal{N}^{i}_{\mathcal{A}},F^{i}_{\mathcal{A}})$ is a NMFS in R, then $\mathcal{F}(\mathcal{A}) = \mathcal{B}$, is the NMFS defined by

$$\mathcal{F}\left(T^{i}_{\mathcal{A}}\right)(\mathbf{y}) = \begin{cases}\sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})}\left(\mu^{i}_{\mathcal{A}}\right)(\mathbf{x}), & \text{if } \mathcal{F}^{-1}(\mathbf{y})\neq 0\\ 0, & \text{otherwise}\end{cases}$$

$$\mathcal{F}\left(\mathcal{N}^{i}_{\mathcal{A}}\right)(\mathbf{y}) = \begin{cases}\sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})}\left(\mathcal{N}^{i}_{\mathcal{A}}\right)(\mathbf{x}), & \text{if } \mathcal{F}^{-1}(\mathbf{y})\neq 0\\ 1, & \text{otherwise}\end{cases}$$

$$\mathcal{F}\left(F^{i}_{\mathcal{A}}\right)(\mathbf{y}) = \begin{cases}\inf_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})}\left(F^{i}_{\mathcal{A}}\right)(\mathbf{x}), & \text{if} \mathcal{F}^{-1}(\mathbf{y})\neq 0\\ 1, & \text{otherwise}\end{cases}$$

Where $\mathcal{F}$ is ring homomorphism of R onto R_1. Also $\mathcal{F}^{-1}(\mathcal{B}) = \left\{\left\langle\mathbf{x},\mathcal{F}^{-1}\left(\mu^{i}_{\mathcal{B}}\right)(\mathbf{x}),\right.\right.$ $\left.\left.\mathcal{F}^{-1}\left(\mathcal{N}^{i}_{\mathcal{B}}\right)(\mathbf{x}),\mathcal{F}^{-1}\left(F^{i}_{\mathcal{B}}\right)(\mathbf{x})\right\rangle : \mathbf{x}\in\mathcal{A}\right\}$ where $\mathcal{F}^{-1}(\mathcal{B})(\mathbf{x}) = (\mathcal{B})(\mathcal{F}(\mathbf{x}))$.

Theorem 5.11. *Let R and R_1 be any two rings and $\mathcal{F}$ be a homomorphism from R onto R_1. If $\mathcal{A} \in$ NMFSR of R under T_n and T_c then $\mathcal{F}(\mathcal{A}) \in$ NMFSR of R_1 over T_n and T_c.*

Proof. Let $\mathbf{x}_1, \mathbf{x}_2 \in R$ and $\mathbf{y}_1, \mathbf{y}_2 \in R_1$. If $\mathcal{A} \in$ NMFSR of R . Then

$$\begin{aligned}
(i)\ \mathcal{F}\left(\left(\mu^i_{\mathcal{A}}\right)(\mathbf{y}_1 - \mathbf{y}_2)\right) &= \sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} \mu^i_{\mathcal{A}}(\mathbf{x}_1 - \mathbf{x}_2) \\
&\geq \sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} \left(T_n(\mu^i_{\mathcal{A}}(\mathbf{x}_1), \mu^i_{\mathcal{A}}(\mathbf{x}_2)\right) \\
&= T_n(\sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} (\mu^i_{\mathcal{A}}(\mathbf{x}_1), \sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} \mu^i_{\mathcal{A}}(\mathbf{x}_2)) \\
&= T_n\left(\mathcal{F}\left(\mu^i_{\mathcal{A}}(\mathbf{y}_1)\right), \mathcal{F}\left(\mu^i_{\mathcal{A}}(\mathbf{y}_2)\right)\right) \\
\text{Similarly, } \mathcal{F}\left(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}_1 - \mathbf{y}_2)\right) &\leq T_c\left(\mathcal{F}\left(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}_1)\right), \mathcal{F}\left(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}_2)\right)\right) \\
\mathcal{F}\left(F^i_{\mathcal{A}}(\mathbf{y}_1 - \mathbf{y}_2)\right) &\leq T_c\left(\mathcal{F}\left(F^i_{\mathcal{A}}(\mathbf{y}_1)\right), \mathcal{F}\left(F^i_{\mathcal{A}}(\mathbf{y}_2)\right)\right).
\end{aligned}$$

$$\begin{aligned}
(ii)\ \mathcal{F}\left(\left(\mu^i_{\mathcal{A}}\right)(\mathbf{y}_1\mathbf{y}_2)\right) &= \sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} \mu^i_{\mathcal{A}}(\mathbf{x}_1\mathbf{x}_2) \\
&\geq \sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} T_n\left(\mu^i_{\mathcal{A}}(\mathbf{x}_1), \mu^i_{\mathcal{A}}(\mathbf{x}_2)\right) \\
&= T_n(\sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} \mu^i_{A}(\mathbf{x}_1), \sup_{\mathbf{x}\in\mathcal{F}^{-1}(\mathbf{y})} \mu^i_{A}(\mathbf{x}_2)) \\
&= T_n\left(\mathcal{F}\left(\mu^i_{\mathcal{A}}(\mathbf{y}_1)\right), \mathcal{F}\left(\mu^i_{\mathcal{A}}(\mathbf{y}_2)\right)\right) \\
\textit{Similarly, } \mathcal{F}\left(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}_1\mathbf{y}_2)\right) &\leq T_c\left(\mathcal{F}\left(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}_1)\right), \mathcal{F}\left(\mathcal{N}^i_{\mathcal{A}}(\mathbf{y}_2)\right)\right) \\
\mathcal{F}\left(F^i_{\mathcal{A}}(\mathbf{y}_1\mathbf{y}_2)\right) &\leq T_c\left(\mathcal{F}\left(F^i_{\mathcal{A}}(\mathbf{y}_1)\right), \mathcal{F}\left(F^i_{\mathcal{A}}(\mathbf{y}_2)\right)\right)
\end{aligned}$$

Hence then $\mathcal{F}(\mathcal{A}) \in$ NMFSR of R_1 over T_n and T_c. □

Theorem 5.12. *Let R and R_1 be any two rings and $\mathcal{F}$ be a homomorphism from R onto R_1. If $\mathcal{B} \in$ NMFSR of R_1 under T_n and T_c then $\mathcal{F}^{-1}(\mathcal{B})$ NMFSR of R under T and T_c*

Proof. Let $\mathbf{x}, \mathbf{y} \in R$. Let $B \in$ NMFSR of R_1. Then

$$\begin{aligned}
(i)\ \ \mathcal{F}^{-1}\left(\left(\mu^i_{\mathcal{B}}\right)(\mathbf{x} - \mathbf{y})\right) &= \mu^i_{\mathcal{B}}(\mathcal{F}(\mathbf{x} - \mathbf{y})) \\
&= T^i_{\mathcal{B}}(\mathcal{F}(\mathbf{x}) - \mathcal{F}(\mathbf{y})) \\
&\geq T_n(\mu^i_{\mathcal{B}}(\mathcal{F}(\mathbf{x})), \mu^i_{\mathcal{B}}(\mathcal{F}(\mathbf{y}))) \\
&= T_n(\mathcal{F}^{-1}\left(\mu^i_{\mathcal{B}}\right)(\mathbf{x}), \mathcal{F}^{-1}\left(\mu^i_{\mathcal{B}}\right)(\mathbf{y})).
\end{aligned}$$

Similarly, $\mathcal{F}^{-1}\left(\mathcal{N}^i_{\mathcal{B}}\right)(\mathbf{x} - \mathbf{y}) \leq T_c\left(\mathcal{F}^{-1}\left(\mathcal{N}^i_{\mathcal{B}}\right)(\mathbf{x}), \mathcal{F}^{-1}\left(\mathcal{N}^i_{\mathcal{B}}\right)(\mathbf{y})\right)$

$$\mathcal{F}^{-1}\left(F^i_{\mathcal{B}}\right)(\mathbf{x} - \mathbf{y}) \leq T_c\left(\mathcal{F}^{-1}\left(F^i_{\mathcal{B}}\right)(\mathbf{x}), \mathcal{F}^{-1}\left(F^i_{\mathcal{B}}\right)(\mathbf{y})\right)$$

$$(ii)\quad \mathcal{F}^{-1}\left(\left(\mu^{i}_{\mathcal{B}}\right)(\mathbf{xy})\right) = \mu^{i}_{\mathcal{B}}\left(\mathcal{F}\left(\mathbf{xy}\right)\right)$$
$$= \mu^{i}_{\mathcal{B}}\left(\mathcal{F}\left(\mathbf{x}\right)\mathcal{F}(\mathbf{y})\right)$$
$$\geq T_n\left(\mu^{i}_{\mathcal{B}}\left(\mathcal{F}\left(\mathbf{x}\right)\right), \mu^{i}_{\mathcal{B}}(\mathcal{F}\left(\mathbf{y}\right),)\right)$$
$$= T_n\left(\mathcal{F}^{-1}\left(\mu^{i}_{\mathcal{B}}\right)(\mathbf{x}), \mathcal{F}^{-1}(\mu^{i}_{\mathcal{B}})(\mathbf{y})\right)$$

Similarly, $\mathcal{F}^{-1}\left(\mathcal{N}^{i}_{\mathcal{B}}\right)(\mathbf{xy}) \leq T_c\left(\mathcal{F}^{-1}\left(\mathcal{N}^{i}_{\mathcal{B}}\right)(\mathbf{x}), \mathcal{F}^{-1}\left(\mathcal{N}^{i}_{\mathcal{B}}\right)(\mathbf{y})\right)$

$$\mathcal{F}^{-1}\left(F^{i}_{\mathcal{B}}\right)(\mathbf{xy}) \leq T_c\left(\mathcal{F}^{-1}\left(F^{i}_{\mathcal{B}}\right)(\mathbf{x}), \mathcal{F}^{-1}\left(F^{i}_{\mathcal{B}}\right)(\mathbf{y})\right)$$

Hence $\mathcal{F}^{-1}(\mathcal{B})$ is a NMFSR of R under T_n and T_c. □

6. Conclusion

We deliberated neutrosophic multifuzzy subrings and ideals along with triangular norm and made use of the concepts of direct product, image and inverse image of homomorphism. We have established some theorems and results. This study will give base for our upcoming work.

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