



Ranking of Neutrosophic number based on values and ambiguities and its application to linear programming problem

Manas Karak¹, Pramodh Bharati², Animesh Mahata^{3,*}, Subrata paul⁴, Santosh Biswas⁵, Supriya Mukherjee⁶, Said Broumi⁷, Mahendra Rong⁸ and Banamali Roy⁸

¹Department of Mathematics, Umeschandra College, 13 Surya Sen Street, Kolkata - 700012, West Bengal, India; karakmanas659@gmail.com

²Department of Mathematics, Swami Vivekananda University Of Telinipara, Barasat-Barrackpore Rd, Bara-Kanthalia 700121, West Bengal, India; pramodbharatinilu@gmail.com

³Department of Mathematics, Sri Ramkrishna Sarada Vidya Mahapitha, Kamarpukur - 712612, West Bengal, India; animeshmahata8@gmail.com

⁴Department of Mathematics, Arambagh Govt. Polytechnic, Arambagh - 712602, West Bengal, India; paulsubrata564@gmail.com

⁵Department of Mathematics, Jadavpur University, 188 Raja S.C. Mallik Road Kolkata - 700032, West Bengal, India; sbiswas.math@jadavpuruniversity.in

⁶Department of Mathematics, Gurudas College, 1/1 Suren Sarkar Road, Kolkata - 700054, West Bengal, India; supriyaskbu2013@gmail.com

⁷Labratory of Information Processing, Faculty of Science Ben M'Sik, University of HassanII, Casablanca, Morocco; broumisaid78@gmail.com

⁸Department of Mathematics, Bangabasi Evening College, Kolkata - 700009, West Bengal, India; mahendrarong@gmail.com, banamaliroy@yahoo.com

*Correspondence: animeshmahata8@gmail.com

Abstract. The goal of this article is to establish a methodology for ordering of single-valued neutrosophic numbers (SVN-numbers) on the basis of values and ambiguities. First of all, the idea of neutrosophic numbers is discussed, and (α, β, γ) -cut and arithmetic operations defined over SVN-numbers are examined. Thereafter, corresponding to each component, the values and ambiguities are defined and using these definitions, the ratio ranking function is constructed. Then, for the stability of the ratio ranking function, some examples are provided for comparing this method with other approaches. Applying this ratio ranking function, neutrosophic linear programming problem (Neu-LPP) converts to the crisp linear programming problems (CLP-Problems) and solved it by computational lingo method. At last, Neu-LPP is illustrated by two numerical real-life examples.

Keywords: Neutrosophic number, Value and ambiguity, Ranking function, Neu-LPP, C-LPP, Computational Lingo method.

1. Introduction

In operation research, LP is one of the most significant and valuable optimizations methods. LP-models expand in a variety of decision problems that happen in economics, engineering, industry, and government. The practical decision problems are described not only by these models but also find applications in science. Some variation in this data must impact on optimal solution, and hence the opinion of decision maker's, that we need to investigate for a new scientific algorithm that gives us optimal solutions useful for all conditions and accepts all variations that may happen. In the work environment, we search for applications of the idea of neutrosophic science that take into consideration variations that can happen in the work environment through the indeterminacy of neutrosophic values. Hence, applying the idea of neutrosophic science, we define many practical problems.

In 1965, First of all, Zadeh [1] presented a fuzzy set (FS), which was classified through only the membership component, and then in 1986, K. Atanassov [2] presented an intuitionistic fuzzy set (IFS), which was classified by two components: membership and non-membership simultaneously. Regularly, to manage uncertainty, FS and IFS perform a vital role. In 1998, Smarandache [3] presented neutrosophic set (NS) to manage some incomplete and inconsistent information in philosophical sense. The components truth, indeterminacy, and falsity independently classified on NS. Sometimes a few suitable decisions are impossible to take by IFS, and hence the indeterminacy of NS plays a vital role. Because some real-world problems such as politics, law, medicine, industry, psychology, and economics, are completely indeterminate. The ordering of SVN-number has vital role in the application of sequential problems, linear and non linear programming problems and multi-attribute selection making problems, etc. Lately, some writers [4–6, 8, 9, 14–16, 19] researched IFS models for applications and some writers [10–13, 17, 20, 22–29, 36] have researched NS models for applications. For the importance

of the LP-method, we introduce the neutrosophic linear model [30]. We presented the neutrosophic linear programming method and applied it in the field of education [31]. We applied the neutrosophic linear programming method to determine optimal agricultural land use [32]. Chakraborty et al. [33] use the removal area method and apply it to time cost optimization. Jdid and Smarandache [34] used the neutrosophic method and applied it to management and corporate work. Karak et al. [21] established a ranking technique between SVN-numbers using the newly developed sign distance method and applied it to the transportation problem.

The structure of the paper is given step by step. Firstly, in section 2, some essential definitions, such as NS, single valued trapezoidal and triangular neutrosophic number (SVTN-numbers, SVTrN-numbers), and arithmetic operation are given. In section 3, the value and ambiguity indexes of SVN-numbers were designed, and we presented a new ratio ranking function primarily based on expanding values and ambiguities. In this subsection, for the validity and feasibility of the ratio ranking function, we satisfied some reasonable properties. In section 4, a set of six examples is given, using these examples, the ranking results of proposed method are compared with other approaches [4, 8, 10, 12, 13, 19, 20]. In section 5, based on the ranking algorithm, Neu-LPP with neutrosophic constraints transferred to C-LPP with real constraints and solved by computational lingo method. In section 6, the concept of Neu-LPP is illustrated by two suitable real-life numerical examples. In the last section, the conclusion is stated briefly.

2. Preliminaries

Let's remind ourselves of a few fundamental definitions that are essential to reaching the main idea of this paper.

Definition 2.1. [3] Let us take ξ as an arbitrary element of X , the universe of discourse. Then \tilde{N} is called NS over X if it is classified through three independent components, namely $T_{\tilde{N}}$, $I_{\tilde{N}}$, and $F_{\tilde{N}}$, which were said to be truth, indeterminacy and falsity neutrosophic components, respectively. These components are maps from X to $]^{-0, 1^+}$ i.e., $T_{\tilde{N}}(\xi), I_{\tilde{N}}(\xi), F_{\tilde{N}}(\xi) \in]^{-0, 1^+}$ where $]^{-0, 1^+}$ is called non-standard unit interval. Thus, \tilde{N} is described by $\tilde{N} = \{ \langle \xi; T_{\tilde{N}}(\xi), I_{\tilde{N}}(\xi), F_{\tilde{N}}(\xi) \rangle : \xi \in X \}$, with $^{-0} \leq \sup T_{\tilde{N}}(\xi) + \sup I_{\tilde{N}}(\xi) + \sup F_{\tilde{N}}(\xi) \leq 3^+$.

Definition 2.2. [7] Performing non-standard analysis of neutrosophic components in real ground is too tough. So for real application, only their standard subset is taken. When three neutrosophic components take the values on $[0, 1]$, NS is said to be SVN-Set. Thus an SVN-Set \tilde{N} is designed as : $\tilde{N} = \{ \langle \xi, T_{\tilde{N}}(\xi), I_{\tilde{N}}(\xi), F_{\tilde{N}}(\xi) \rangle : \xi \in X; T_{\tilde{N}}(\xi), I_{\tilde{N}}(\xi), F_{\tilde{N}}(\xi) \in [0, 1] \text{ and } 0 \leq \sup T_{\tilde{N}}(\xi) + \sup I_{\tilde{N}}(\xi) + \sup F_{\tilde{N}}(\xi) \leq 3 \}$.

Definition 2.3. [18] Let \tilde{N} be defined as NS over \mathbb{R} , which is called a neutrosophic number if it fulfils three characteristics given below:

1. $T_{\tilde{N}}(\xi_0) = 1$ and $I_{\tilde{N}}(\xi_0) = F_{\tilde{N}}(\xi_0) = 0$ for some $\xi_0 \in \mathbb{R}$ i.e., \tilde{N} is normal.
2. $T_{\tilde{N}}(\nu\xi_1 + (1 - \nu)\xi_2) \geq \min(T_{\tilde{N}}(\xi_1), T_{\tilde{N}}(\xi_2)), \forall \xi_1, \xi_2 \in \mathbb{R}$, and $\nu \in [0, 1]$ i.e., \tilde{N} is convex for $T_{\tilde{N}}(\xi)$.
3. $I_{\tilde{N}}(\nu\xi_1 + (1 - \nu)\xi_2) \geq \max(I_{\tilde{N}}(\xi_1), I_{\tilde{N}}(\xi_2))$, and $F_{\tilde{N}}(\nu\xi_1 + (1 - \nu)\xi_2) \geq \max(F_{\tilde{N}}(\xi_1), F_{\tilde{N}}(\xi_2)), \forall \xi_1, \xi_2 \in \mathbb{R}$, and $\nu \in [0, 1]$ i.e., \tilde{N} is concave for $I_{\tilde{N}}(\xi)$ and $F_{\tilde{N}}(\xi)$.

Definition 2.4. [12] A NS $\tilde{m} = \langle ([l, m, n]; t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}}) \rangle$ defined on \mathbb{R} , where $t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \in [0, 1]$ and $l, m, n \in \mathbb{R}$ satisfy the condition $l \leq m \leq n$ is called SVTrN-number whose truth, indeterminacy, and falsity component are denoted by $T_{\tilde{m}} : \mathbb{R} \mapsto [0, t_{\tilde{m}}]$, $I_{\tilde{m}} : \mathbb{R} \mapsto [i_{\tilde{m}}, 1]$, and $F_{\tilde{m}} : \mathbb{R} \mapsto [f_{\tilde{m}}, 1]$ as described below:

$$T_{\tilde{m}}(\xi) = \begin{cases} \frac{(\xi-l)t_{\tilde{m}}}{(m-l)}, & l \leq \xi \leq m, \\ \frac{(n-\xi)t_{\tilde{m}}}{(n-m)}, & m \leq \xi \leq n, \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\tilde{m}}(\xi) = \begin{cases} \frac{(m-\xi)+i_{\tilde{m}}(\xi-l)}{(m-l)}, & l \leq \xi \leq m, \\ \frac{(\xi-m)+i_{\tilde{m}}(n-\xi)}{(n-m)}, & m \leq \xi \leq n, \\ 0, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{m}}(\xi) = \begin{cases} \frac{(m-\xi)+f_{\tilde{m}}(\xi-l)}{(m-l)}, & l \leq \xi \leq m, \\ \frac{(\xi-m)+f_{\tilde{m}}(n-\xi)}{(n-m)}, & m \leq \xi \leq n, \\ 0, & \text{otherwise.} \end{cases}$$

respectively.

For example, let us take SVTrN-number $\tilde{A}_1 = \langle [1, 4, 8]; 0.9, 0.3, 0.5 \rangle$. Then the graphical representation of \tilde{A}_1 is given below:

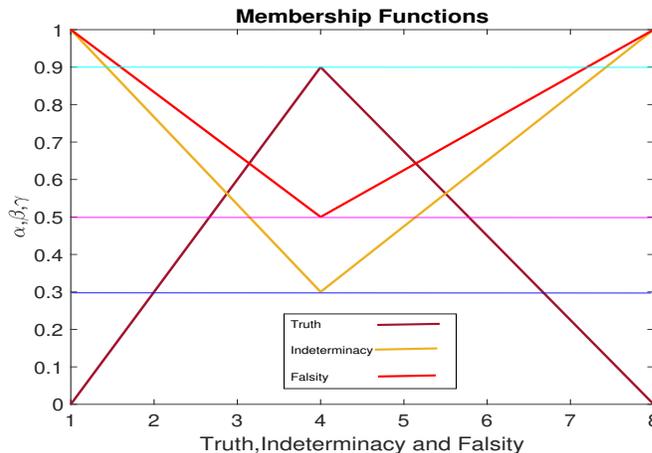


FIGURE 1. Graphical representation of single valued triangular neutrosophic number(SVTN) \tilde{A}_1 .

Definition 2.5. [12] Let $\tilde{m} = \langle ([l, m, n, p]; t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}}) \rangle$ be NS on \mathbb{R} where $l, m, n, p \in \mathbb{R}$, and $t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \in [0, 1]$ having condition $l \leq m \leq n \leq p$ is called SVTN-numbers whose truth, indetereminacy, and falsity component are denoted by $T_{\tilde{m}} : \mathbb{R} \mapsto [0, t_{\tilde{m}}]$, $I_{\tilde{m}} : \mathbb{R} \mapsto [i_{\tilde{m}}, 1]$, and $F_{\tilde{m}} : \mathbb{R} \mapsto [f_{\tilde{m}}, 1]$ as described below.

$$T_{\tilde{m}}(\xi) = \begin{cases} \frac{(\xi-l)t_{\tilde{m}}}{(m-l)}, & l \leq \xi < m, \\ t_{\tilde{m}}, & m \leq \xi \leq n, \\ \frac{(p-\xi)t_{\tilde{m}}}{(p-n)}, & n < \xi \leq p, \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\tilde{m}}(\xi) = \begin{cases} \frac{(m-\xi)+i_{\tilde{m}}(\xi-l)}{(m-l)}, & l \leq \xi < m, \\ i_{\tilde{m}}, & m \leq \xi \leq n, \\ \frac{(\xi-n)+i_{\tilde{m}}(p-\xi)}{(p-n)}, & n < \xi \leq p, \\ 0, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{m}}(\xi) = \begin{cases} \frac{(m-\xi)+f_{\tilde{m}}(\xi-l)}{(m-l)}, & l \leq \xi < m, \\ f_{\tilde{m}}, & m \leq \xi \leq n, \\ \frac{(\xi-n)+f_{\tilde{m}}(p-\xi)}{(p-n)}, & n < \xi \leq p, \\ 0, & \text{otherwise.} \end{cases}$$

respectively.

For example, let us take SVTN-number $\tilde{A}_2 = \langle [1, 3, 6, 9]; 0.7, 0.5, 0.6 \rangle$. Then the graphical representation of \tilde{A}_2 is given below:

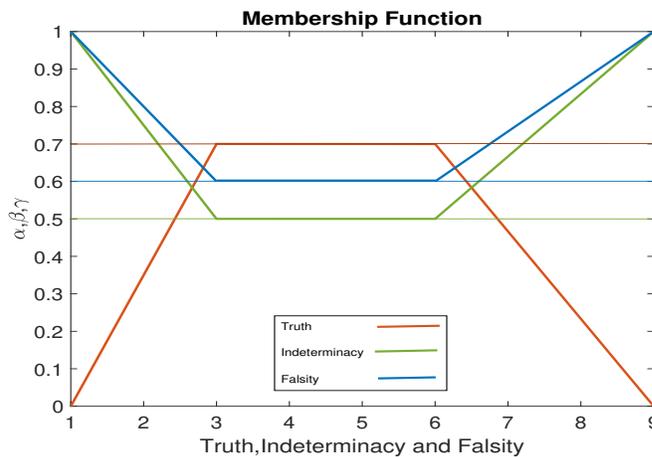


FIGURE 2. Graphical representation of single valued trapezoidal neutrosophic number(SVTN) \tilde{A}_2 .

Definition 2.6. [12] For \tilde{N} defined in 2.3, (α, β, γ) -cut is designed as : $\tilde{N}_{(\alpha, \beta, \gamma)} = \{\xi \in X : T_{\tilde{N}}(\xi) \geq \alpha, I_{\tilde{N}}(\xi) \leq \beta, F_{\tilde{N}}(\xi) \leq \gamma\}$ where $0 \leq \alpha, \beta, \gamma \leq 1$.

Then for SVTrN-number \tilde{m} defined in 2.4, the (α, β, γ) cuts are respectively

$$\begin{aligned} \tilde{m}_\alpha &= [L_{\tilde{m}}(\alpha), R_{\tilde{m}}(\alpha)] = \left[\frac{(t_{\tilde{m}} - \alpha)l + \alpha m}{t_{\tilde{m}}}, \frac{(t_{\tilde{m}} - \alpha)n + \alpha m}{t_{\tilde{m}}} \right], \\ \tilde{m}_\beta &= [L'_{\tilde{m}}(\beta), R'_{\tilde{m}}(\beta)] = \left[\frac{(1 - \beta)m + (\beta - i_{\tilde{m}})l}{1 - i_{\tilde{m}}}, \frac{(1 - \beta)m + (\beta - i_{\tilde{m}})n}{1 - i_{\tilde{m}}} \right], \\ \text{and } \tilde{m}_\gamma &= [L''_{\tilde{m}}(\gamma), R''_{\tilde{m}}(\gamma)] = \left[\frac{(1 - \gamma)m + (\gamma - f_{\tilde{m}})l}{1 - f_{\tilde{m}}}, \frac{(1 - \gamma)m + (\gamma - f_{\tilde{m}})n}{1 - f_{\tilde{m}}} \right]. \end{aligned}$$

Here $L_{\tilde{m}}, R'_{\tilde{m}}$, and $R''_{\tilde{m}}$ are non-decreasing and continuous functions, and $R_{\tilde{m}}, L'_{\tilde{m}}$, and $L''_{\tilde{m}}$ are non-increasing continuous functions in their respectively intervals.

Similarly (α, β, γ) cut of SVTN-number \tilde{m} defined in 2.5, are respectively

$$\begin{aligned} \tilde{m}_\alpha &= [L_{\tilde{m}}(\alpha), R_{\tilde{m}}(\alpha)] = \left[\frac{(t_{\tilde{m}} - \alpha)l + \alpha m}{t_{\tilde{m}}}, \frac{(t_{\tilde{m}} - \alpha)p + \alpha n}{t_{\tilde{m}}} \right], \\ \tilde{m}_\beta &= [L'_{\tilde{m}}(\beta), R'_{\tilde{m}}(\beta)] = \left[\frac{(1 - \beta)m + (\beta - i_{\tilde{m}})l}{1 - i_{\tilde{m}}}, \frac{(1 - \beta)n + (\beta - i_{\tilde{m}})p}{1 - i_{\tilde{m}}} \right], \\ \tilde{m}_\gamma &= [L''_{\tilde{m}}(\gamma), R''_{\tilde{m}}(\gamma)] = \left[\frac{(1 - \gamma)m + (\gamma - f_{\tilde{m}})l}{1 - f_{\tilde{m}}}, \frac{(1 - \gamma)n + (\gamma - f_{\tilde{m}})p}{1 - f_{\tilde{m}}} \right]. \end{aligned}$$

Definition 2.7. [11] Let us take two SVTN-numbers $\tilde{m} = \langle \langle [l, m, n, p]; t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle \rangle$ and $\tilde{n} = \langle \langle [u, v, w, x]; t_{\tilde{n}}, i_{\tilde{n}}, f_{\tilde{n}} \rangle \rangle$, and $\delta (\neq 0) \in \mathbb{R}$. Then

- (i) $\tilde{m} \oplus \tilde{n} = \langle \langle (+u, m + v, n + w, p + x); t_{\tilde{m}} \wedge t_{\tilde{n}}, i_{\tilde{m}} \vee i_{\tilde{n}}, f_{\tilde{m}} \vee f_{\tilde{n}} \rangle \rangle$.
- (ii) $\delta \tilde{m} = \begin{cases} \langle \langle (\delta l, \delta m, \delta n, \delta p); t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle \rangle & (\delta > 0). \\ \langle \langle (\delta p, \delta n, \delta m, \delta l); t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle \rangle & (\delta < 0). \end{cases}$
- (iii) $\tilde{m} \ominus \tilde{n} = \langle \langle (l - x, m - w, n - v, p - u); t_{\tilde{m}} \wedge t_{\tilde{n}}, i_{\tilde{m}} \vee i_{\tilde{n}}, f_{\tilde{m}} \vee f_{\tilde{n}} \rangle \rangle$.

Definition 2.8. [11] Let us take two SVTrN-numbers $\tilde{m} = \langle \langle [l, m, n]; t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle \rangle$ and $\tilde{n} = \langle \langle [u, v, w]; t_{\tilde{n}}, i_{\tilde{n}}, f_{\tilde{n}} \rangle \rangle$, and $\delta (\neq 0) \in \mathbb{R}$. Then

- (i) $\tilde{m} \oplus \tilde{n} = \langle \langle (l + u, m + v, n + w); t_{\tilde{m}} \wedge t_{\tilde{n}}, i_{\tilde{m}} \vee i_{\tilde{n}}, f_{\tilde{m}} \vee f_{\tilde{n}} \rangle \rangle$.
- (ii) $\delta \tilde{m} = \begin{cases} \langle \langle (\delta l, \delta m, \delta n); t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle \rangle & (\delta > 0). \\ \langle \langle (\delta n, \delta m, \delta l); t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle \rangle & (\delta < 0). \end{cases}$
- (iii) $\tilde{m} \ominus \tilde{n} = \langle \langle (l - w, m - v, n - u); t_{\tilde{m}} \wedge t_{\tilde{n}}, i_{\tilde{m}} \vee i_{\tilde{n}}, f_{\tilde{m}} \vee f_{\tilde{n}} \rangle \rangle$.

3. Neutrosophic numbers and their ordering method

In this part, we presented an ordering method for SVN-numbers depending on values and ambiguities in a new direction.

Definition 3.1. If \tilde{m} is any arbitrary SVN-number, then

1. the value and ambiguity of \tilde{m} for truth component, are symbolised by $V_T(\tilde{m})$ and $A_T(\tilde{m})$ and described as follows:

$$(i) V_T(\tilde{m}) = \int_0^{t_{\tilde{m}}} \{L_{\tilde{m}}(\alpha) + R_{\tilde{m}}(\alpha)\}f(\alpha)d\alpha.$$

$$(ii) A_T(\tilde{m}) = \int_0^{t_{\tilde{m}}} \{R_{\tilde{m}}(\alpha) - L_{\tilde{m}}(\alpha)\}f(\alpha)d\alpha.$$

Where $f(\alpha) \in [0, 1]$ ($\alpha \in [0, t_{\tilde{m}}]$), $f(0) = 0$, and $f(\alpha)$ is non decreasing monotonic continuous function of α .

2. the value and ambiguity of \tilde{m} for indeterminacy component, are symbolised by $V_I(\tilde{m})$ and $A_I(\tilde{m})$ and described as follows:

$$(i) V_I(\tilde{m}) = \int_{i_{\tilde{m}}}^1 \{L'_{\tilde{m}}(\beta) + R'_{\tilde{m}}(\beta)\}g(\beta)d\beta.$$

$$(ii) A_I(\tilde{m}) = \int_{i_{\tilde{m}}}^1 \{R'_{\tilde{m}}(\beta) - L'_{\tilde{m}}(\beta)\}g(\beta)d\beta.$$

Where $g(\beta) \in [0, 1]$ ($\beta \in [i_{\tilde{m}}, 1]$), $g(1)=0$, and $g(\beta)$ is non increasing monotonic continuous function of β .

3. the value and ambiguity of \tilde{m} for falsity component, are symbolised by $V_F(\tilde{m})$ and $A_F(\tilde{m})$ and described as follows:

$$(i) V_F(\tilde{m}) = \int_{f_{\tilde{m}}}^1 \{L''_{\tilde{m}}(\gamma) + R''_{\tilde{m}}(\gamma)\}h(\gamma)d\gamma.$$

$$(ii) A_F(\tilde{m}) = \int_{f_{\tilde{m}}}^1 \{R''_{\tilde{m}}(\gamma) - L''_{\tilde{m}}(\gamma)\}h(\gamma)d\gamma.$$

Where $h(\gamma) \in [0, 1]$ ($\gamma \in [f_{\tilde{m}}, 1]$), $h(1)=0$, and $h(\gamma)$ is non increasing monotonic continuous function of γ .

Definition 3.2. For an arbitrary SVN-number \tilde{m} , the value and ambiguity of \tilde{m} are symbolised as $V(\tilde{m})$ and $A(\tilde{m})$ and expressed as follows:

$$(i) V(\tilde{m}) = \frac{1}{3} [V_T + V_I + V_F], \text{ and}$$

$$(ii) A(\tilde{m}) = \frac{1}{3} [A_T + A_I + A_F].$$

From now on we take $f(\alpha) = \frac{\alpha}{t_{\tilde{m}}}$, $\alpha \in [0, t_{\tilde{m}}]$ ($t_{\tilde{m}} \in (0, 1]$), $g(\beta) = \frac{1-\beta}{1-i_{\tilde{m}}}$, $\beta \in [i_{\tilde{m}}, 1]$ ($i_{\tilde{m}} \in [0, 1)$), $h(\gamma) = \frac{1-\gamma}{1-f_{\tilde{m}}}$, $\gamma \in [f_{\tilde{m}}, 1]$ ($f_{\tilde{m}} \in [0, 1)$) for the SVN-number \tilde{m} , and similarly for other SVN-numbers throughout the paper.

Remark 1. It is easily derived that the value function $V(\tilde{m})$ should be maximized, whereas the ambiguity function should be minimised.

Corollary 3.1. For arbitrary SVTrN-number $\tilde{m} = \langle [l, m, n]; t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle$, the value and ambiguity are given by

$$(i) V(\tilde{m}) = \frac{1}{18} [(l + 4m + n) \times (2 + t_{\tilde{m}} - i_{\tilde{m}} - f_{\tilde{m}})], \text{ and}$$

$$(ii) A(\tilde{m}) = \frac{1}{18} [(n - l) \times (2 + t_{\tilde{m}} - i_{\tilde{m}} - f_{\tilde{m}})].$$

Corollary 3.2. for arbitrary SVTrN-number $\tilde{m} = \langle [l, m, n, p]; t_{\tilde{m}}, i_{\tilde{m}}, f_{\tilde{m}} \rangle$, the value and ambiguity are given by

- (i) $V(\tilde{m}) = \frac{1}{18} [(l + 2m + 2n + p) \times (2 + t_{\tilde{m}} - i_{\tilde{m}} - f_{\tilde{m}})]$, and
- (ii) $A(\tilde{m}) = \frac{1}{18} [\{p - l - 2(m - n)\} \times (2 + t_{\tilde{m}} - i_{\tilde{m}} - f_{\tilde{m}})]$.

Property 1. For any SVN-number \tilde{m} and $\delta (\neq 0) \in \mathbb{R}$,

- (i) $V(\delta\tilde{m}) = \delta V(\tilde{m})$.
- (ii) $A(\delta\tilde{m}) = \delta A(\tilde{m})$.

Proof: (i),(ii) are obvious (see definitions 2.7,2.8, and 3.1).

Theorem 3.1. For two SNTTrN-numbers \tilde{m} and \tilde{n} with $t_{\tilde{n}} = t_{\tilde{m}}, i_{\tilde{n}} = i_{\tilde{m}}, f_{\tilde{n}} = f_{\tilde{m}}$,

- (i) $V(\tilde{m} \oplus \tilde{n}) = V(\tilde{m}) + V(\tilde{n})$.
- (ii) $A(\tilde{m} \oplus \tilde{n}) = A(\tilde{m}) + A(\tilde{n})$.

Proof:

- (i) By the definition 2.8 and given condition, we get

$$\begin{aligned} V(\tilde{m} \oplus \tilde{n}) &= \frac{1}{18} [\{(l + u) + 2(m + v) + 2(n + w) + (p + x)\} \times (2 + t_{\tilde{n}} - i_{\tilde{n}} - f_{\tilde{n}})] \\ &= V(\tilde{m}) + V(\tilde{n}). \end{aligned}$$

Hence, the proof.

- (ii) Similarly, it can be proved.

NOTE: The theorem is also true for SNTTrN-numbers.

Definition 3.3. Let us consider a ratio ranking function ϕ that maps from $N(\mathbb{R})$ to \mathbb{R} and is described by $\phi(\tilde{m}) = \frac{V(\tilde{m})}{1+A(\tilde{m})} \forall \tilde{m} \in N(\mathbb{R})$, where $N(\mathbb{R})$ indicates set of all SVN-numbers on \mathbb{R} whose truth component $\in (0, 1]$, indeterminacy component $\in [0, 1)$, and falsity component $\in [0, 1)$.

For any $\tilde{m}, \tilde{n} \in N(\mathbb{R})$, we define ordering of \tilde{m}, \tilde{n} by

- (1) $\tilde{m} \prec_{\phi} \tilde{n}$ iff $\phi(\tilde{m}) < \phi(\tilde{n})$.
- (2) $\tilde{m} \succ_{\phi} \tilde{n}$ iff $\phi(\tilde{m}) > \phi(\tilde{n})$.
- (3) $\tilde{m} \approx_{\phi} \tilde{n}$ iff $\phi(\tilde{m}) = \phi(\tilde{n})$.

Then the order \preceq_{ϕ} is formulated as $\tilde{m} \preceq_{\phi} \tilde{n}$ iff $\tilde{m} \approx_{\phi} \tilde{n}$ or $\tilde{m} \prec_{\phi} \tilde{n}$.

Corollary 3.3. Let $\tilde{m} \in N(\mathbb{R})$ be SVTrN-number defined in definition 2.4. Then the ranking functional value of SVTrN-number \tilde{m} is described by $\phi(\tilde{m}) = \frac{(l+4m+n) \times (2+t_{\tilde{m}}-i_{\tilde{m}}-f_{\tilde{m}})}{18+(n-l) \times (2+t_{\tilde{m}}-i_{\tilde{m}}-f_{\tilde{m}})}$

Corollary 3.4. Let $\tilde{m} \in N(\mathbb{R})$ be SVTN-number defined in definition 2.5. Then the ranking functional value of SVTN-number \tilde{m} is described by $\phi(\tilde{m}) = \frac{(l+2m+2n+n) \times (2+t_{\tilde{m}}-i_{\tilde{m}}-f_{\tilde{m}})}{18+(p-l-2m+2n) \times (2+t_{\tilde{m}}-i_{\tilde{m}}-f_{\tilde{m}})}$.

Remark 2. It is easily seen that $\phi(\tilde{m})$ is not linear function of a SVN-number \tilde{m} although $V(\tilde{m})$ and $A(\tilde{m})$ are linear on \tilde{m} . In other words, $\phi(\tilde{m} \oplus \tilde{n}) \neq \phi(\tilde{m}) + \phi(\tilde{n})$

Example 1. Let $\tilde{m} = \langle [1, 4, 7]; 0.6, 0.1, 0.4 \rangle, \tilde{n} = \langle [3, 5, 6]; 0.7, 0.1, 0.2 \rangle \in N(\mathbb{R})$.

Then, by definition 3.5, $\phi(\tilde{m}) = 1.1667$, and $\phi(\tilde{n}) = 2.19005$.

So, $\phi(\tilde{m}) < \phi(\tilde{n})$ and hence the ranking of SVTrN-numbers \tilde{m} and \tilde{n} is $\tilde{m} \prec_{\phi} \tilde{n}$.

Example 2. Let $\tilde{m} = \langle [1, 2, 4, 7]; 0.7, 0.1, 0.3 \rangle$, $\tilde{n} = \langle [1, 3, 5, 6]; 0.6, 0.2, 0.4 \rangle \in N(\mathbb{R})$

Then, by definition 3.5, $\phi(\tilde{m}) = 1.1219$, and $\phi(\tilde{n}) = 1.2778$.

So, $\phi(\tilde{m}) < \phi(\tilde{n})$ and hence the ranking of SVTN-numbers \tilde{m} and \tilde{n} is $\tilde{m} \prec_{\phi} \tilde{n}$.

Property 2. The relations \preceq_{ϕ} is total ordering on $N(\mathbb{R})$.

Proof: If the relation \preceq_{ϕ} is total ordering on $N(\mathbb{R})$, then we need to prove the following:

(a) \preceq is a partial order i.e., \preceq_{ϕ} is reflexive, anti symmetric, and transitive.

(b) any two element in $N(\mathbb{R})$ are comparable.

We now prove the condition (a) and (b).

(a) By definition 3.6 , it is clear that the relation \preceq_{ϕ} is reflexive i.e., $\tilde{m} \preceq_{\phi} \tilde{m}$, $\forall \tilde{m} \in N(\mathbb{R})$

let $\tilde{m}, \tilde{n} \in N(\mathbb{R})$ with $\tilde{m} \preceq_{\phi} \tilde{n}$ and $\tilde{n} \preceq_{\phi} \tilde{m}$

Then by definition 3.6, $\phi(\tilde{m}) - \phi(\tilde{n}) \leq 0$ and $\phi(\tilde{n}) - \phi(\tilde{m}) \geq 0$, and hence $\phi(\tilde{m}) - \phi(\tilde{n}) = 0$.

Therefore, $\tilde{m} \approx_{\phi} \tilde{n}$ i.e., the relation \preceq_{ϕ} is anti symmetric.

let $\tilde{m}, \tilde{n}, \tilde{p} \in N(\mathbb{R})$ with $\tilde{m} \preceq_{\phi} \tilde{n}$ and $\tilde{n} \preceq_{\phi} \tilde{p}$.

Then by definition 3.6 , $\phi(\tilde{m}) - \phi(\tilde{n}) \leq 0$ and $\phi(\tilde{n}) - \phi(\tilde{p}) \leq 0$, and hence $\phi(\tilde{m}) - \phi(\tilde{p}) \leq 0$.

Therefore, $\tilde{m} \preceq_{\phi} \tilde{p}$ i.e., the relation \preceq_{ϕ} is transitive.

Therefore, the relation \preceq_{ϕ} satisfy all the condition of partial ordering on $N(\mathbb{R})$.

(b) By the definition 3.6, we can say that any two element in $N(\mathbb{R})$ are comparable.

Therefore, the relation \preceq_{ϕ} is total ordering.

3.1. Rationality of validation of the ratio ranking algorithm

Seven axioms $A_1 - A_7$ proposed by Wang and Kerre [35] have reasonable properties for the validation of ratio ranking algorithm for ordering fuzzy numbers. In this article, the introduced ratio ranking method fulfils the the properties A_1, A_2, A_3 , and A_5 easily. However, the properties A_4, A_6 , and A_7 are not satisfied by the ratio ranking method because this method is not linear according to Remark 2. By the Remark 1, the value index $V(\tilde{m})$ should be maximized, whereas the ambiguity index $A(\tilde{m})$ should be minimised, i.e., $V(\tilde{m})$ and $A(\tilde{m})$ are in conflict. Hence, the ranking algorithm should be established dependant on the above two functions and applied it to solve Neu-LPP. Even, in general, Neu-LPP are not easily solved. Hence, the ratio ranking algorithm is used to aggregate $V(\tilde{m})$ and $A(\tilde{m})$. As a consequence, the ordering of SVN-numbers is dependant on the ratio of $V(\tilde{m})$ and $1 + A(\tilde{m})$ rather than either $V(\tilde{m})$ and $A(\tilde{m})$.

4. Comparison Analysis

Here the ranking of neutrosophic numbers is compared with other approaches with the proposed method by six set of examples given in the following:

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Set-1: $\tilde{m} = \langle ([1, 5, 7, 8]; 0.9, 0.3, 0.4) \rangle$, $\tilde{n} = \langle ([2, 4, 6, 7]; 0.8, 0.4, 0.5) \rangle$

Then, by definition 3.5, $\phi(\tilde{m}) = 0.8601896$, $\phi(\tilde{n}) = 0.7849003$

So, $\phi(\tilde{n}) < \phi(\tilde{m})$, and hence $\tilde{n} \prec_{\phi} \tilde{m}$.

Set-2: $\tilde{m} = \langle ([2, 4, 7, 9]; 0.4, 0.1, 0.3) \rangle$, $\tilde{n} = \langle ([1, 4, 5, 9]; 0.8, 0.2, 0.5) \rangle$

Then, by definition 3.5, $\phi(\tilde{m}) = 0.75$, $\phi(\tilde{n}) = 0.7538462$

So, $\phi(\tilde{m}) < \phi(\tilde{n})$, and hence $\tilde{m} \prec_{\phi} \tilde{n}$.

Set-3: $\tilde{m} = \langle ([1, 3, 6, 8]; 0.7, 0.2, 0.5) \rangle$, $\tilde{n} = \langle ([3, 6, 8, 9]; 0.9, 0.1, 0.3) \rangle$

Then, by definition 3.5, $\phi(\tilde{m}) = 0.6136364$, $\phi(\tilde{n}) = 1.162791$

So, $\phi(\tilde{m}) < \phi(\tilde{n})$, and hence $\tilde{m} \prec_{\phi} \tilde{n}$.

Set-4: $\tilde{m} = \langle ([1, 2, 3, 4]; 0.5, 0.1, 0.2) \rangle$, $\tilde{n} = \langle ([2, 4, 5, 6]; 0.6, 0.2, 0.3) \rangle$, $\tilde{p} = \langle ([3, 4, 6, 7]; 0.7, 0.2, 0.4) \rangle$

Then, by definition 3.5, $\phi(\tilde{m}) = 0.5689655$, $\phi(\tilde{n}) = 0.8921569$, $\phi(\tilde{p}) = 0.9051724$

So, $\phi(\tilde{m}) < \phi(\tilde{n}) < \phi(\tilde{p})$, and hence $\tilde{m} \prec_{\phi} \tilde{n} \prec_{\phi} \tilde{p}$.

Set-5: $\tilde{m} = \langle ([2, 5, 8, 9]; 0.7, 0.1, 0.2) \rangle$, $\tilde{n} = \langle ([1, 3, 6, 8]; 0.6, 0.2, 0.3) \rangle$, $\tilde{p} = \langle ([3, 4, 5, 7]; 0.5, 0.1, 0.3) \rangle$

Then, by definition 3.5, $\phi(\tilde{m}) = 0.9024390$, $\phi(\tilde{n}) = 0.6258278$, $\phi(\tilde{p}) = 0.9607843$

So, $\phi(\tilde{n}) < \phi(\tilde{m}) < \phi(\tilde{p})$, and hence $\tilde{n} \prec_{\phi} \tilde{m} \prec_{\phi} \tilde{p}$.

Set-6: $\tilde{m} = \langle ([4, 5, 6, 7]; 0.5, 0.1, 0.4) \rangle$, $\tilde{n} = \langle ([2, 4, 6, 8]; 0.6, 0.2, 0.3) \rangle$, $\tilde{p} = \langle ([3, 5, 7, 9]; 0.7, 0.2, 0.5) \rangle$

Then, by definition 3.5, $\phi(\tilde{m}) = 1.178571$, $\phi(\tilde{n}) = 0.8076923$, $\phi(\tilde{p}) = 0.9473684$

So, $\phi(\tilde{n}) < \phi(\tilde{p}) < \phi(\tilde{m})$, and hence $\tilde{n} \prec_{\phi} \tilde{p} \prec_{\phi} \tilde{m}$.

We now compare the ranking results of the above six set of examples with other approaches. In the articles [10,12,13,20] on NS, for ranking of these examples, we directly apply the respective approaches. But in the articles [4,8,19] on IFS, for the ranking of these examples, we must reject the hesitancy part and then apply the respective methods.

Table-1 : A Comparison of ordering for several approaches

Source	Set - 1	Set - 2	Set - 3	Set - 4	Set - 5	Set - 6
Deli et al. [12]	$\tilde{n} < \tilde{m}$	$\tilde{n} < \tilde{m}$	$\tilde{m} < \tilde{n}$	$\tilde{m} < \tilde{n} < \tilde{p}$	$\tilde{n} < \tilde{p} < \tilde{m}$	$\tilde{n} < \tilde{m} < \tilde{p}$
Peng et al. [13]	$\tilde{n} \prec \tilde{m}$	$\tilde{m} \prec \tilde{n}$	$\tilde{m} \prec \tilde{n}$	$\tilde{n} \prec \tilde{p} \prec \tilde{m}$	$\tilde{p} \prec \tilde{n} \prec \tilde{m}$	$\tilde{m} \approx \tilde{p} \prec \tilde{n}$
Ye. [10]	$\tilde{n} \prec \tilde{m}$	$\tilde{m} \prec \tilde{n}$	$\tilde{m} \prec \tilde{n}$	$\tilde{p} \prec \tilde{n} \prec \tilde{m}$	$\tilde{p} \prec \tilde{n} \prec \tilde{p}$	$\tilde{n} \prec \tilde{m} \prec \tilde{p}$
Fahad A.Alzahrani et al. [20]	$\tilde{n} < \tilde{m}$	$\tilde{n} < \tilde{m}$	$\tilde{m} < \tilde{n}$	$\tilde{m} < \tilde{n} < \tilde{p}$	$\tilde{n} < \tilde{p} < \tilde{m}$	$\tilde{n} < \tilde{m} < \tilde{p}$
Qiang and Zhong [4]	$\tilde{n} < \tilde{m}$	$\tilde{m} < \tilde{n}$	$\tilde{m} < \tilde{n}$	$\tilde{m} < \tilde{n} < \tilde{p}$	$\tilde{p} < \tilde{n} < \tilde{m}$	$\tilde{m} < \tilde{p} < \tilde{n}$
De and Das [8]	$\tilde{n} < \tilde{m}$	$\tilde{n} < \tilde{m}$	$\tilde{m} < \tilde{n}$	$\tilde{m} < \tilde{n} < \tilde{p}$	$\tilde{p} < \tilde{n} < \tilde{m}$	$\tilde{m} < \tilde{n} < \tilde{p}$
Suresh Mohan et al. [19]	$\tilde{n} < \tilde{m}$	$\tilde{n} < \tilde{m}$	$\tilde{m} < \tilde{n}$	$\tilde{m} < \tilde{n} < \tilde{p}$	$\tilde{n} < \tilde{m} < \tilde{m}$	$\tilde{n} < \tilde{m} < \tilde{p}$
proposed method	$\tilde{n} \prec \tilde{m}$	$\tilde{m} \prec \tilde{n}$	$\tilde{m} \prec \tilde{n}$	$\tilde{m} \prec \tilde{n} \prec \tilde{p}$	$\tilde{n} \prec \tilde{m} \prec \tilde{p}$	$\tilde{n} \prec \tilde{p} \prec \tilde{m}$

Here, Deli and Subas [12] applies score and accuracy to determine the ranking of SVN-numbers, and the ranking results of this method are very close to the ranking results of the introduced method. Peng et al. [13] and Ye. [10] designed score and accuracy to determine the ordering

of neutrosophic numbers, and applying the score function, the ordering of six set of examples is given in Table-1, which is almost unequal to the ordering results of the introduced method. Alzahrani et al. [20] use de-neutrosophication method to determine the ordering of SVN-numbers, and the ranking results of this method are almost equal to the ranking results of the introduced method. De and Das [8] define a ranking function using value and ambiguity in IFS, and usins this ranking function, the ordering of given SVTN numbers is given in Table-1, which is very close to the ranking results of proposed method and has few difference because it has no hesitancy part. Qiang and Zhong [4] described accuracy and score functions and using this ordering of SVTN-numbers are given in Table-1, which has the same reason as De and Das for the comparison of ranking results with the proposed method. Suresh Mohan et al. [19] define magnitude to define the ordering of neutrosophic numbers and using this magnitude, the ordering of above set of examples are given in Table-1 which is almost equal to the ranking results of proposed method and has few difference because it has no hesitancy part.

5. Neutrosophic linear programming problem and its solution

In this section, we propose the idea of Neu-LPP in a new direction using the ranking function. First, we recall the concept of linear programming problems with crisp data, i.e., C-LPP. Usually, C-LPP is expressed as:

$$\text{Maximize } Z = C\xi$$

$$\text{subject to } A\xi \leq B, \xi \geq 0$$

$$\text{Where } C \in \mathbb{R}^s, B^t \in \mathbb{R}^r, \xi \in \mathbb{R}^s \text{ and } A = (a_{ij})_{r \times s}$$

Here, the constraints of C-LPP are crisp numbers. Next, we designed Neu-LPP.

Definition 5.1. The Neu-LPP with constraints in terms of SVN-numbers is defined in the following below:

$$\text{Maximize } \tilde{Z} \approx_{\phi} \tilde{C}\xi$$

$$\text{subject to } \tilde{A}\xi \preceq_{\phi} \tilde{B}, \xi \geq 0$$

$$\text{Where } \tilde{A} = (\tilde{a}_{ij})_{r \times s} \in (N(\mathbb{R}))^s, \tilde{B} \in (N(\mathbb{R}))^r, \tilde{C}^t \in (N(\mathbb{R}))^s, \xi \in \mathbb{R}^s.$$

METHODOLOGY

There are four steps to reaching the optimal solution, and the steps are given below.

Step-1: First of all, the given Neu-LPP with SVN-numbers can be wriiten in the form of a mathematically formulation.

Step-2. Using ranking function $\phi(\tilde{m}) = \frac{V(\tilde{m})}{1+A(\tilde{m})}$ convert the mentioned SVN-numbers to crisp numbers.

Step-3. Formulate the C-LPP.

Step-4. Solve the C-LPP by Computational Lingo method.

6. Numerical Example

In this section, we give two examples of Neu-LPP with constraints SVN-numbers. In the first examples, we take Neu-LPP with constraints in terms of SVTN-numbers, and in second example, we take Neu-LPP with constraints in terms of SVTrN-numbers. **Example 3.** A firm produces three products I, II, and III. The per unit profits are Rs. \tilde{c}_1 and Rs. \tilde{c}_2 and Rs. \tilde{c}_3 respectively, they are uncertain in nature, assuming as SVTN-numbers. The firm has two machines and each product is processed on two machines X and Y. The processing time required in hours in terms of SNTN-numbers on each product is given below the table.

Machines	Product – I	Product – II	Product – III
X	\tilde{a}_1	\tilde{a}_2	\tilde{a}_3
Y	\tilde{a}'_1	\tilde{a}'_2	\tilde{a}'_3

The machines X and Y have \tilde{b}_1 and \tilde{b}_2 machine hours in terms of SVTN-numbers, respectively. We have to maximize the profit of the company.

Where,

$$\begin{aligned} \tilde{c}_1 &= \langle\langle [6, 8, 11, 14]; 0.7, 0.2, 0.5 \rangle\rangle, & \tilde{c}_2 &= \langle\langle [5, 8, 9, 10]; 0.6, 0.1, 0.2 \rangle\rangle, & \tilde{c}_3 &= \langle\langle [7, 10, 14, 17]; 0.8, 0.3, 0.4 \rangle\rangle \\ \tilde{a}_1 &= \langle\langle [3, 7, 9, 15]; 0.6, 0.1, 0.3 \rangle\rangle, & \tilde{a}_2 &= \langle\langle [7, 9, 12, 16]; 0.6, 0.2, 0.5 \rangle\rangle, & \tilde{a}_3 &= \langle\langle [3, 8, 12, 14]; 0.5, 0.3, 0.4 \rangle\rangle, \\ \tilde{a}'_1 &= \langle\langle [4, 7, 10, 13]; 0.4, 0.1, 0.2 \rangle\rangle, & \tilde{a}'_2 &= \langle\langle [4, 7, 10, 13]; 0.4, 0.1, 0.2 \rangle\rangle, & \tilde{a}'_3 &= \langle\langle [5, 9, 12, 15]; 0.5, 0.4, 0.1 \rangle\rangle, \\ \tilde{a}'_3 &= \langle\langle [5, 10, 13, 15]; 0.7, 0.3, 0.5 \rangle\rangle, & \tilde{b}_1 &= \langle\langle [35, 38, 47, 58]; 0.9, 0.1, 0.3 \rangle\rangle, & \tilde{b}_2 &= \langle\langle [35, 50, 56, 63]; 0.8, 0.2, 0.4 \rangle\rangle. \end{aligned}$$

Solution:

Step-1: Let the company produce the quantity ξ_1, ξ_2, ξ_3 of the products A, B, and C respectively. Then the mathematical form of the above Neu-LPP is

$$\begin{aligned} & \text{Maximize } \tilde{Z} \approx_{\phi} \tilde{c}_1 \xi_1 \oplus \tilde{c}_2 \xi_2 \oplus \tilde{c}_3 \xi_3 \\ & \text{subject to, } \tilde{a}_1 \xi_1 \oplus \tilde{a}_2 \xi_2 \oplus \tilde{a}_3 \xi_3 \preceq_{\phi} \tilde{b}_1 \\ & \tilde{a}'_1 \xi_1 \oplus \tilde{a}'_2 \xi_2 \oplus \tilde{a}'_3 \xi_3 \preceq_{\phi} \tilde{b}_2 \\ & \text{and } \xi_i \geq 0, i = 1, 2, 3. \end{aligned}$$

Step-2: In this step, we will apply ranking function to convert SVTN-numbers to real numbers. $\phi(\tilde{c}_1) = 2.521739, \phi(\tilde{c}_2) = 3.304985, \phi(\tilde{c}_3) = 2.709677, \phi(\tilde{a}_1) = 2.067669, \phi(\tilde{a}_2) = 2.655914, \phi(\tilde{a}_3) = 1.965517, \phi(\tilde{a}'_1) = 2.163636, \phi(\tilde{a}'_2) = 2.480000, \phi(\tilde{a}'_3) = 2.590909, \phi(\tilde{b}_1) = 5.456432, \phi(\tilde{b}_2) = 6.433962.$

Step-3: Therefore, the C-LPP with constraints in terms of crisp number is

Maximize $Z = 2.521739\xi_1 + 3.304985\xi_2 + 2.709677\xi_3$

subject to

$$2.067669\xi_1 + 2.655914\xi_2 + 1.965517\xi_3 \leq 5.456432$$

$$2.163636\xi_1 + 2.480000\xi_2 + 2.590909\xi_3 \leq 6.433962$$

Step-4: By Lingo method, the optimal feasible solition is $\xi_1 = 0$, $\xi_2 = 0.7430211$, $\xi_3 = 1.772069$ and $Z_{max} = 7.257408$.

Example 4. At a cattle breeding firm it is prescribed that the food ration for one animal must contain at least \tilde{b}_1 , \tilde{b}_2 and \tilde{b}_3 respectively, they are uncertain in nature, assuming as SVTrN-numbers. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of the three nutrients in terms of SVTrN-numbers:

	Fodder – 1	Fodder – 2
Nutrient-A	\tilde{a}_1	\tilde{a}_2
Nutrient-B	\tilde{a}'_1	\tilde{a}'_2
Nutrient-C	\tilde{a}''_1	\tilde{a}''_2

It is given that the costs of unit quantity of Fodder-1 and Fodder-2 are \tilde{c}_1 and \tilde{c}_2 monetary units respectively. Pose a linear programming problem in terms of minimizing the cost of purchasing the fodders for the above cattle breeding firm.

Where,

$$\tilde{c}_1 = \langle\langle [4, 7, 8]; 0.9, 0.2, 0.5 \rangle\rangle, \tilde{c}_2 = \langle\langle [2, 3, 5]; 0.5, 0.3, 0.4 \rangle\rangle, \tilde{a}_1 = \langle\langle [1, 6, 7]; 0.6, 0.2, 0.5 \rangle\rangle,$$

$$\tilde{a}_2 = \langle\langle [4, 8, 9]; 0.5, 0.1, 0.4 \rangle\rangle, \tilde{a}'_1 = \langle\langle [1, 2, 4]; 0.6, 0.2, 0.3 \rangle\rangle, \tilde{a}'_2 = \langle\langle [2, 3, 6]; 0.4, 0.3, 0.2 \rangle\rangle$$

$$\tilde{a}''_1 = \langle\langle [3, 4, 7]; 0.5, 0.4, 0.2 \rangle\rangle, \tilde{a}''_2 = \langle\langle [4, 5, 6]; 0.6, 0.3, 0.4 \rangle\rangle, \tilde{b}_1 = \langle\langle [1, 3, 5]; 0.5, 0.3, 0.1 \rangle\rangle,$$

$$\tilde{b}_2 = \langle\langle [1, 2, 3]; 0.5, 0.3, 0.5 \rangle\rangle, \tilde{b}_3 = \langle\langle [2, 4, 6]; 0.6, 0.3, 0.4 \rangle\rangle.$$

Solution:

Step-1: Let ξ_1 unit of Fodder-1 and ξ_2 unit of Fodder-2 are to be purchased to fulfil the requirement and minimizing the cost of purchasing.

Therefore, the mathematical formulation of the abpve Neu-LPP is

$$\begin{aligned} \text{Minimize } \tilde{Z} \approx_{\phi} \quad & \tilde{c}_1\xi_1 \oplus \tilde{c}_2\xi_2 \\ \text{subject to,} \quad & \tilde{a}_1\xi_1 \oplus \tilde{a}_2\xi_2 \succeq_{\phi} \tilde{b}_1 \\ & \tilde{a}'_1\xi_1 \oplus \tilde{a}'_2\xi_2 \succeq_{\phi} \tilde{b}_2 \\ & \tilde{a}''_1\xi_1 \oplus \tilde{a}''_2\xi_2 \succeq_{\phi} \tilde{b}_3 \\ \text{and} \quad & \xi_i \geq 0, i = 1, 2. \end{aligned}$$

Step-2: In this step, we will apply ranking function to convert SVTrN-numbers to real numbers. $\phi(\tilde{c}_1) = 3.283582$, $\phi(\tilde{c}_2) = 1.461538$, $\phi(\tilde{a}_1) = 2.068027$, $\phi(\tilde{a}_2) = 3.214286$,

$\phi(\tilde{a}'_1) = 1.123457, \phi(\tilde{a}'_2) = 1.484375, \phi(\tilde{a}''_1) = 1.929688, \phi(\tilde{a}''_2) = 2.614679, \phi(\tilde{b}_1) = 1.431818,$
 $\phi(\tilde{b}_2) = 0.9532710, \phi(\tilde{b}_3) = 1.781250.$

Step-3: Therefore, the C-LPP with constraints in terms of crisp number is

Minimize $Z = 3.283582\xi_1 + 1.461538\xi_2$

subject to

$2.068027\xi_1 + 3.214286\xi_2 \geq 1.431818,$

$1.123457\xi_1 + 1.484375\xi_2 \geq 0.9532710,$

$1.929688\xi_1 + 2.614679\xi_2 \geq 1.781250.$

Step-4: By Computational Lingo method, the optimal feasible solution is $\xi_1 = 0, \xi_2 = 0.6812500$ and $Z_{min} = 0.9956727.$

7. Conclusions

In this article, we describe the ranking system of neutrosophic numbers in a new direction based on value and ambiguity. We also developed some properties and theorems about value and ambiguity. Here, we generalised C-LPP by considering the constraints in terms of SVN-numbers, and the generalised C-LPP is called Neu-LPP. Then, to solve such Neu-LPP, we proposed a simplex algorithm, and finally, this newly developed algorithm is used in real-life problems. The proposed ranking method is applied to convert the Neu-LPP with constraints in terms of SVTN-numbers to the C-LPP with constraints in terms of real numbers and solves it by the computational Lingo method. The idea has been explained by two numerical examples using both SVTN-numbers and SVTrN-numbers. For the stability and feasibility of this methodology, we also compared different existing methodologies with the proposed method. In the future, the idea of Neu-LPP may be more generalised way.

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